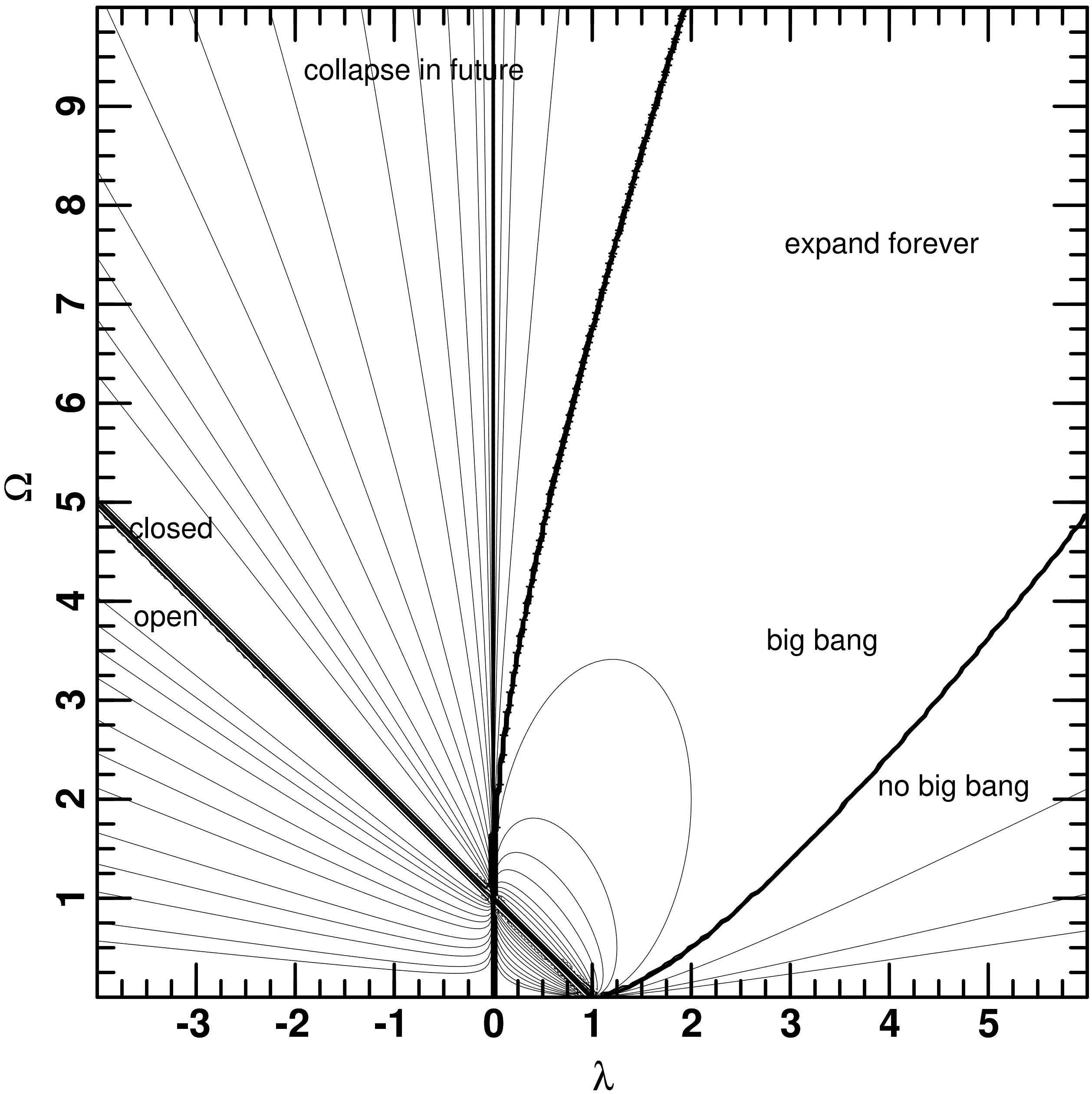
Conserved quantities in cosmology

Phillip Helbig

Parameters of Friedmann–Robertson–Walker models

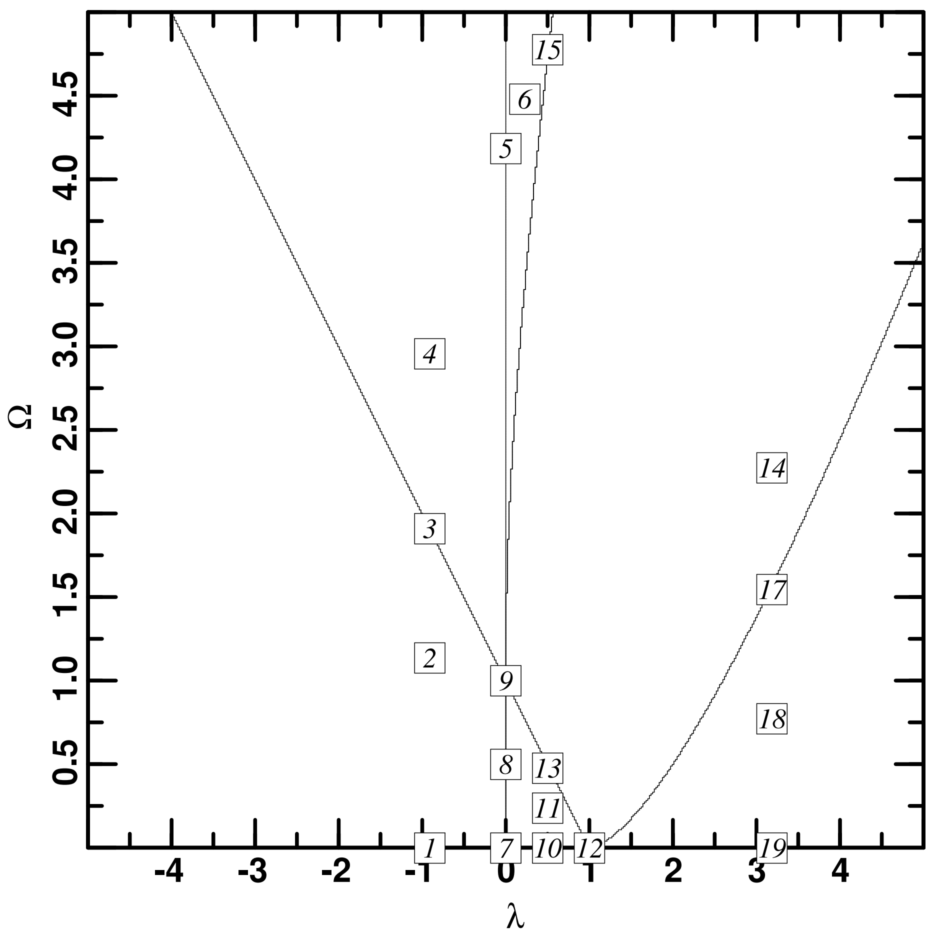
- Homogeneous and isotropic models based on GR
- Only dust, cosmological constant, both, or neither
- scale factor R
- $H := \frac{\dot{R}}{R}$
- $\Omega := \frac{8\pi G\rho}{3H^2}$
- $\lambda := \frac{\Lambda}{3H^2}$
- $K := \Omega + \lambda - 1$; $k :=$ sign(K)
- $R = R(t)$, $H = H(t)$, $\Omega = \Omega(t)$, $\lambda = \lambda(t)$, $K = K(t)$
- Values today: R_0, H_0, Ω_0, λ_0, K_0

Cosmic evolution



Evolutionary trajectories in the λ–Ω plane

Types of world models



The 19 types of FRW models with (at most) dust and a cosmological constant

Motivation

- Lake (*PRL*, **94**, 201102, 2005)
- Use constant parameters to assess likelihood of universes
- Type 14: $k = +1$, $\lambda > 0$, expand forever (our Universe?)
- $\alpha := \frac{27\Omega^2\lambda}{4|K|^3}$ is constant of motion
- α must be fine-tuned to *avoid* $K \approx 0$
- Argument against one aspect of flatness problem

What is α?

- $\alpha \sim \Lambda M^2$, where M mass of universe
- Obviously constant in time
- Lake: for type 14
- For all types with $k = +1$ (hence finite mass of dust) (4–6, 14–19)

Other models?

- α not useful if $\lambda = 0$ and/or $\Omega = 0$
- Other constants of motion
- More generally, characteristic properties of universes
- Relation to α (or, if α not useful, to other parameters)
- No time for derivation, but just simple algebra

Type 5

- $\lambda = 0$, $\Omega > 1$
- preferred by Einstein after he gave up Λ since spatially finite
- $R_{\mathrm{max}} = \frac{4GM}{3\pi c^2}$
- No surprise; mass is conserved

Type 1

- $\lambda < 0$, $\Omega = 0$
- $R^2_{\mathrm{max}} = \frac{3c^2}{|\Lambda|}$
- No surprise that related to Λ

Type 19

- $\lambda > 1$, $\Omega = 0$; Lanczos model
- $R^2_{\mathrm{min}} = \frac{3c^2}{\Lambda}$
- No surprise that related to Λ
- $z = \frac{R_0}{R} - 1 \to z_{\mathrm{max}} = \sqrt{\frac{\lambda}{\lambda - 1}} - 1$

Types 11, 13, and 14

- Big-bang models with dust and Λ with $R \to \infty$
- $\rho \sim R^{-3}$, $\Lambda \sim R^0$
- First deceleration, then acceleration
- $R(t)$ has point of inflection
- $R^2_{\mathrm{infl}} = \frac{c^2 \sqrt[3]{\alpha}}{\Lambda}$
- Similar for R_{max} in 15 and R_{min} in 17

Type 12

- $\lambda = 1$, $\Omega = 0$
- de Sitter model
- $H^2 = \frac{\Lambda}{3}$

Types 15–17

- 15 maximum scale factor R, 17 minimum
- 15 $\rightarrow$ 16 (static Einstein model)
- 16 (static Einstein model) $\rightarrow$ 17
- $\frac{R^2_{\{\text{max}|\text{min}|\text{Einstein}\}}\Lambda}{c^2} = 1$
- $\alpha = \sqrt[3]{\alpha}$ for $\alpha = 1$, hence R same as above for inflection point
- $\Lambda M^2 = \left(\frac{\pi^2 c^6}{4G^2}\right)\alpha$
- $\Lambda M^2 = \left(\frac{\pi^2 c^6}{4G^2}\right)$
- $\Lambda = 4\pi G\rho = \frac{c^2}{R^2}$
- $R_{\{\text{max}|\text{min}|\text{Einstein}\}} = \frac{GM}{2\pi c^2}$

Types 1-6

- Such models have a maximum scale factor R
- 1–6 collapse (15 $\rightarrow$ static Einstein model)
- R_{max} is characteristic quantity
- R_{max} constant along trajectory since just the same model over time
- α constant along trajectory
- Is there a simple relation between R_{max} and α?

A practical use
for redshift drift

Phillip Helbig

FRW cosmology, notation

- Homogeneous and isotropic models based on GR
- Only dust, cosmological constant, both, or neither
- $\dot{R}^2 = \frac{8\pi G\rho R^2}{3} + \frac{\Lambda R^2}{3} - kc^2$
- Scale factor R
- $H := \frac{\dot{R}}{R}$
- $\Omega := \frac{8\pi G\rho}{3H^2}$
- $\lambda := \frac{\Lambda}{3H^2}$
- $K := \Omega + \lambda - 1$; $k :=$ sign(K)
- $R = R(t)$, $H = H(t)$, $\Omega = \Omega(t)$, $\lambda = \lambda(t)$, $K = K(t)$
- Values today: R_0, H_0, Ω_0, λ_0, K_0

What is redshift drift?

- Usually: suffix 0 refers to time of observation, assumed constant
- Change in redshift with time: time of *observation*
- Neither lookback time from now ($t = t_0$) nor cosmic time from big bang
- $z := \frac{R_0}{R} - 1$
- $z_A := \frac{R_A}{R_1} - 1$ emitted at t_1, received at t_A
- $z_B := \frac{R_B}{R_2} - 1$ emitted at t_2, received at t_B
- $t_1 < t_2 \ll t_A < t_B$
- $\dot{z} = H_0 \left(1 + z - \frac{H(z)}{H_0}\right) = H_0 Z(z)$
- $\dot{z} = H_0(1 + z) - H(z)$
- $H(z) = H_0 \sqrt{\Omega_0(1 + z)^3 - K_0(1 + z)^2 + \lambda_0}$

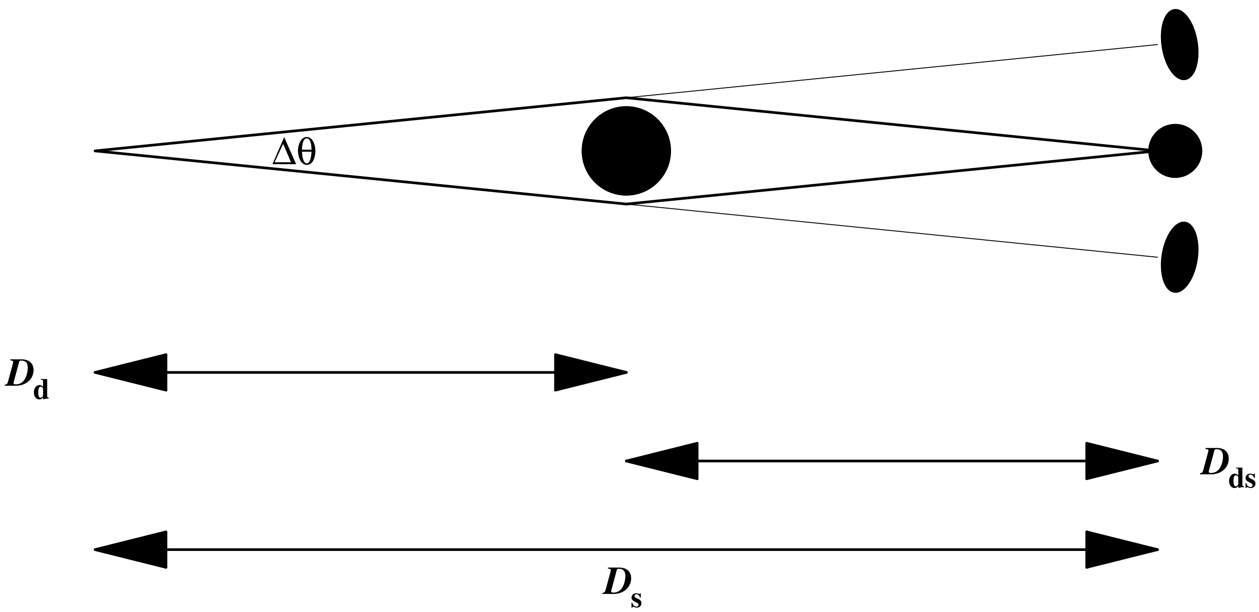
History

- Sandage (1962): $\lambda = 0$, all values of k, steady-state model
- 263 citations: 19 before 2006 (almost all from very well known cosmologists) and 244 since then
- Appendix by McVittie: $\lambda \neq 0$, general expression
- Lake (1981): adds radiation (but found to be unimportant); measure with QSO absorption lines in not-too-distant future
- Lake (2007): explores equation for $\dot{z}$ in more detail
- Loeb (1998): revives interest, 'Sandage–Loeb effect'
- After 1998: mostly applications, little new theory
- Liske *et al.* (2007): spectroscopy for extremely large telescopes
- Li *et al.* (2008): spectroscopic precision $v/c \approx 1$ cm/s feasible

Practical use?

- Sandage: '. . . a precision redshift catalogue must be stored away for the order of 10^7 years . . . .'
- Same paper, different topic: apparent galaxy luminosities will decrease with time due to *cosmological* effects
- '. . . galaxies will recede beyond the limit of easy observation . . . '
- '. . . data for extragalactic astronomy must be collected from ancient literature.'
- Very optimistic about humanity, pessimistic about observations
- Even if measurable, is it useful?
- Can determine cosmological parameters without worrying about source evolution
- Consistency check
- Rule out some non-standard models

Strong gravitational lensing

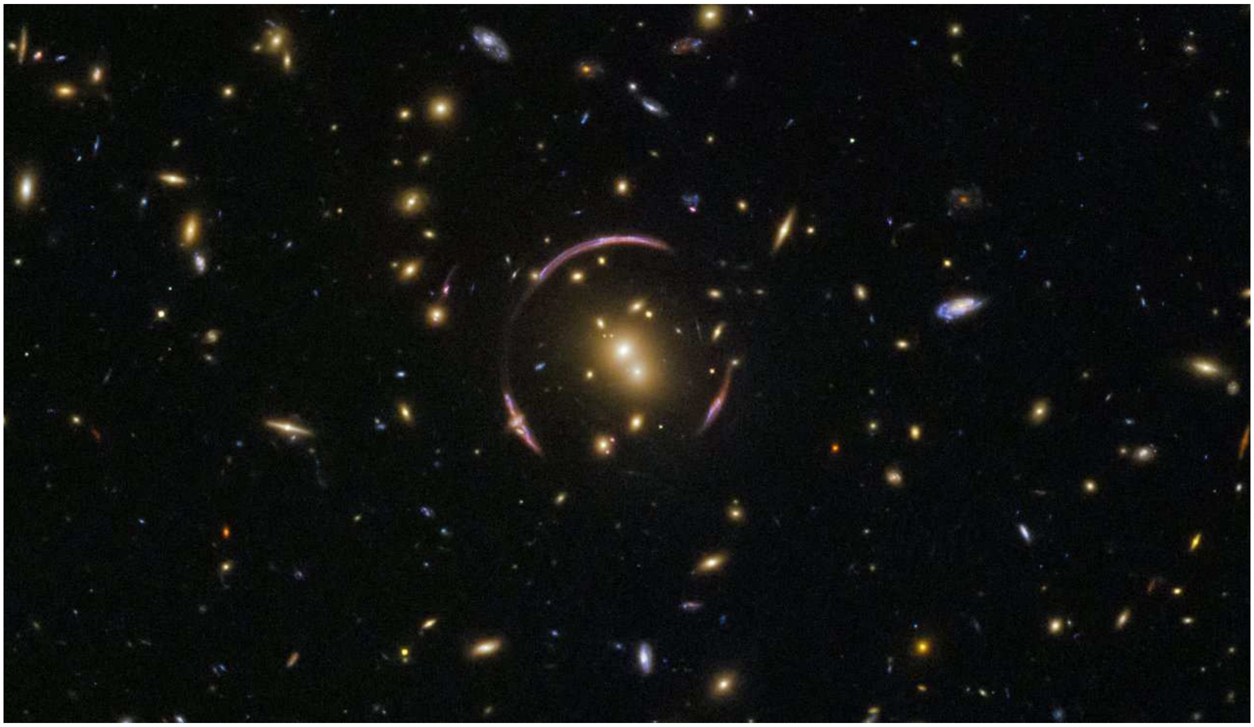


Strong gravitational lensing: redshift drift

- Definition: more than one image of each source
- Of course, normal redshift drift observable in each image
- Different scenarios
- Supernova Refsdal: one time of emission, different times of observation
- Time delay known
- Still mainly consistency check
- $\Delta t = \left(4\pi\frac{\sigma^2}{c^2}\right)^2 \frac{D_\mathrm{d}D_\mathrm{ds}}{cD_\mathrm{s}}\,(1+z_\mathrm{d})\,2y$
- Cluster lens: velocity dispersion $\sigma \approx 1000$–$1500\,\mathrm{km/s}$
- $\Delta t \approx 100$–1000 years

Strong gravitational lensing:

cluster lensing



SDSSJ0146−0929

Credit: ESA/Hubble & NASA

Acknowledgement: Judy Schmidt

Strong gravitational lensing:

Conventional scenario

- Observe several images at once
- Same time of observation
- Different times of emission due to time delay
- Difference in redshift between images due to redshift drift
- Measuring difference equivalent to measuring time delay
- Can measure time delay *even if source is not variable*
- Can measure time delay *even if hundreds or thousands of years*
- The longer it is, the easier it is to measure

Strong gravitational lensing: Details

- Thought of this while on holiday in August
- Of course someone else must have thought of it, but who?
- Loeb (1998) (I remembered his paper but not that part)
- But doesn't work because transverse velocity will also cause a redshift difference, possibly larger
- Einstein ring: due to symmetry, light-travel time from all parts must be the same
- Any difference thus not due to redshift drift
- Use measurements of Einstein ring to determine transverse velocity
- Correct for this effect for other redshifts
- Additional constraints on lens model
- Use precisely measured redshifts to match images

Conclusions

- Now feasible to measure redshift drift on time scale of several years
- Measure it at only one epoch *via* measuring gravitational-lens time delays
- Works for non-variable sources
- Works for time delays of hundreds or thousands of years
- The longer the time delay, the easier it is to measure
- Additional constraints for mass models of clusters
- Transverse velocity needed for correction interesting in itself

Would normally stop here

- Sent abstract to Anish on 4 October
- 13 October: automatic email from Google Scholar Alerts
- Few papers and citations so only once every week or two
- Biggs *et al.* (1999): 'Time delay for the gravitational lens system B0218+357'
- Often cited as example of time-delay measurement
- Recognized two of the authors, so looked more closely
- 'Well, boys, we've been scooped', said Bob Dicke upon learning that Penzias and Wilson had discovered the CMB
- My future work should be more obscure