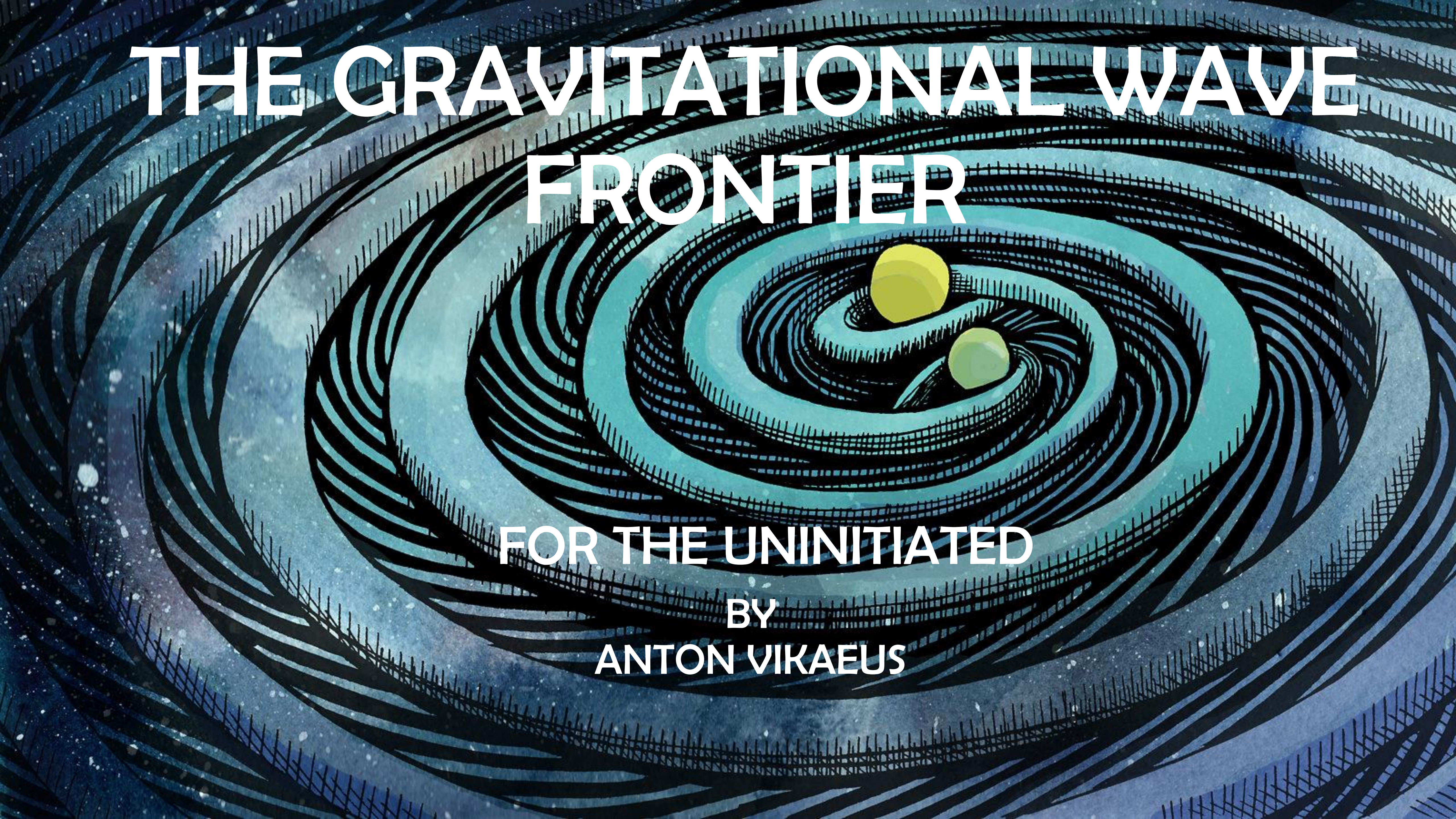

THE GRAVITATIONAL WAVE FRONTIER

FOR THE UNINITIATED

BY
ANTON VIKAEUS

Todays topic: Gravitational Waves (GWs)

This work is an incomplete overview of the state of gravitational wave physics and observations – and how it relates to astronomy and astrophysics

The project involved a visit to the Kavli institute for theoretical physics (KITP) in Santa Barbara, USA

The visit was funded by a Liljewalchs travel stipend

So, what are gravitational waves?

Solutions to linearized gravity:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

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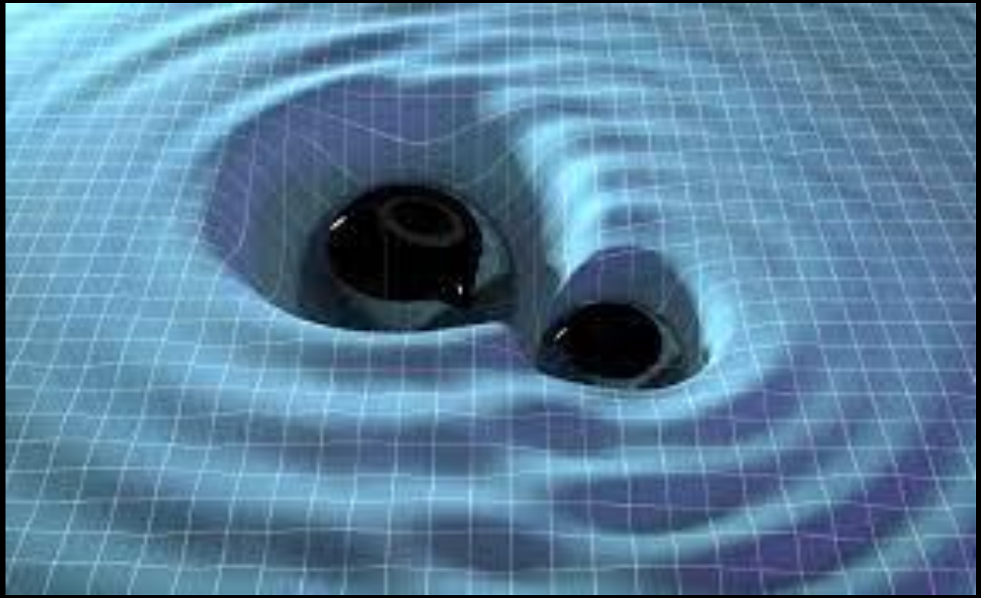
First order wavelike solutions appear with a time varying quadrupolar moment

Strain amplitude:

$$h_{jk} = \frac{2G}{c^4}\frac{1}{D}\frac{\mathrm{d}^2 I_{jk}}{\mathrm{d}t^2}$$

Typical source: Merging binary black holes



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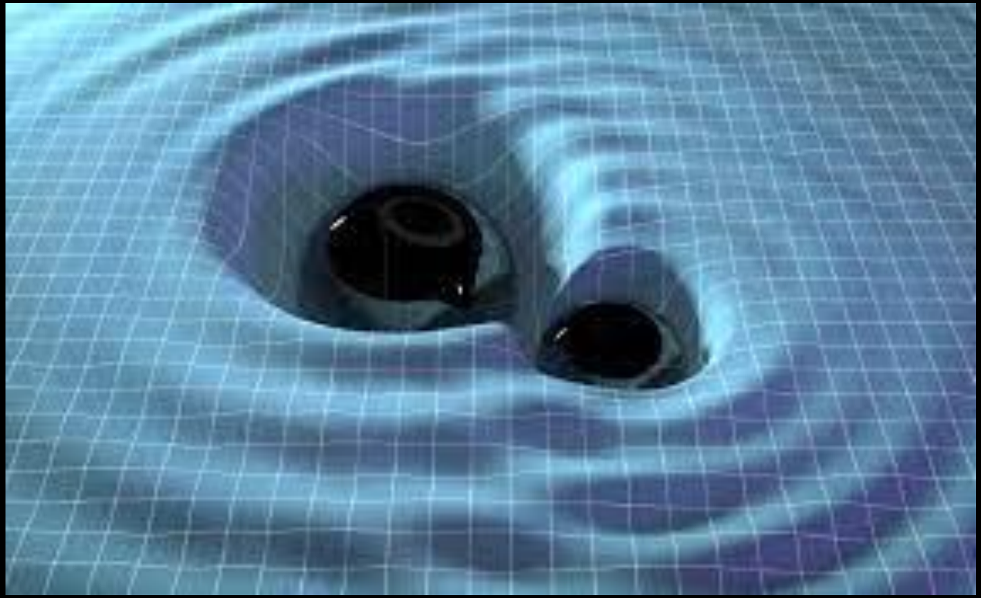


mass monopole = total mass (conserved with time)

mass dipole = center of mass (r)
(1:st derivative relates to momentum; $p = m\dot{r}$)
(2:nd derivative conserved for isolated system)

STRAIN AMPLITUDE

The strain amplitude for a given polarization depends on a series of variables

$$h_+ = \frac{2c}{D}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}\Omega^{2/3}\left(1+\cos^2 i\right)\cos 2\Phi(t) \qquad (2)$$

Chirp mass:

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

(redshifted)

Luminosity distance: D
Calculated quantity

Binary orbital frequency: Ω

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}\Omega^{11/3} \qquad \text{(to leading order)}$$

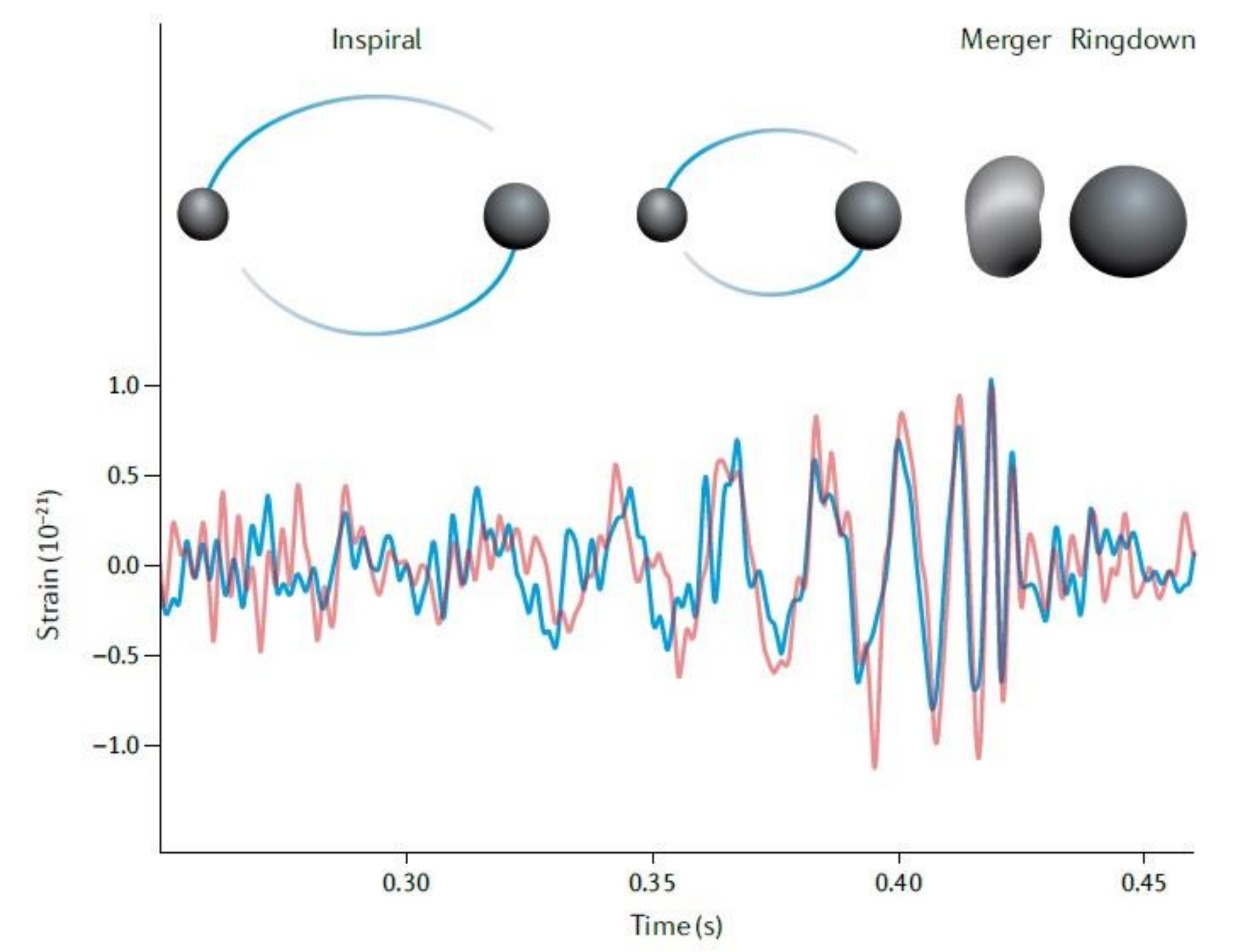
Inclination: i
Calculated quantity

STRAIN AMPLITUDE

The previous quantities describes the inspiral phase

The frequency, its chirp and the phase evolution can be measured directly from the signal

Chirp mass from template fitting



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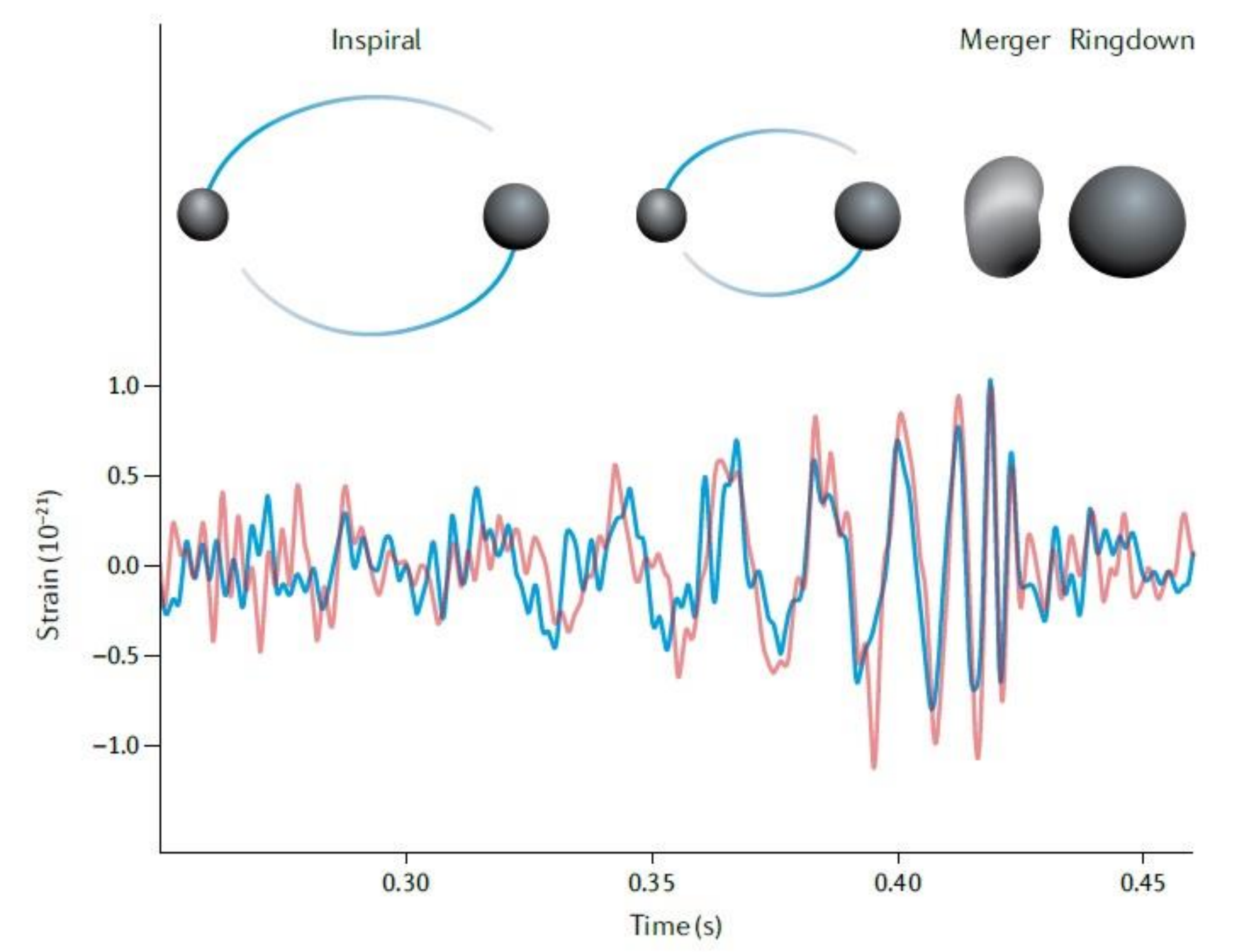
Chirp mass from template fitting

In order to estimate the distance we must constrain the inclination – notice that a single passing GW signal is sufficient to estimate D

Any of the two variables could be responsible for an up/down scaling of the amplitude

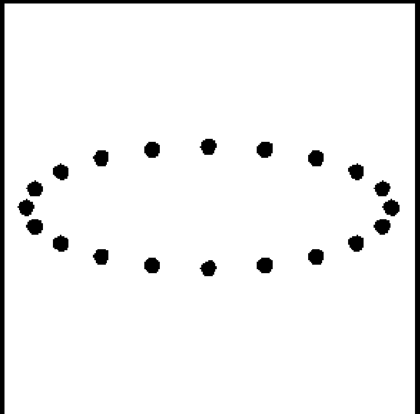


POLARIZATION

Similarly to electromagnetic waves, GWs comes with two polarizations

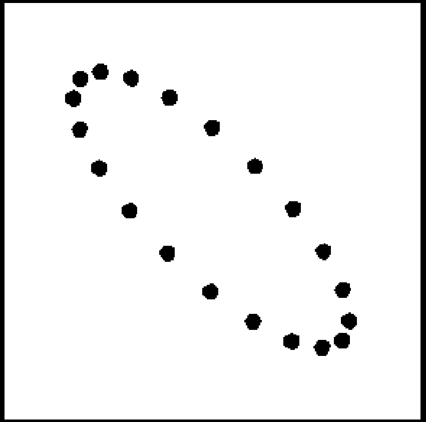
Plus-polarized:

$$h_+ = \frac{2c}{D}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}\Omega^{2/3}\left(1+\cos^2 i\right)\cos 2\Phi(t) \tag{2}$$



Cross-polarized:

$$h_\times = \frac{4c}{D}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}\Omega^{2/3}\cos i\sin 2\Phi(t) \tag{3}$$



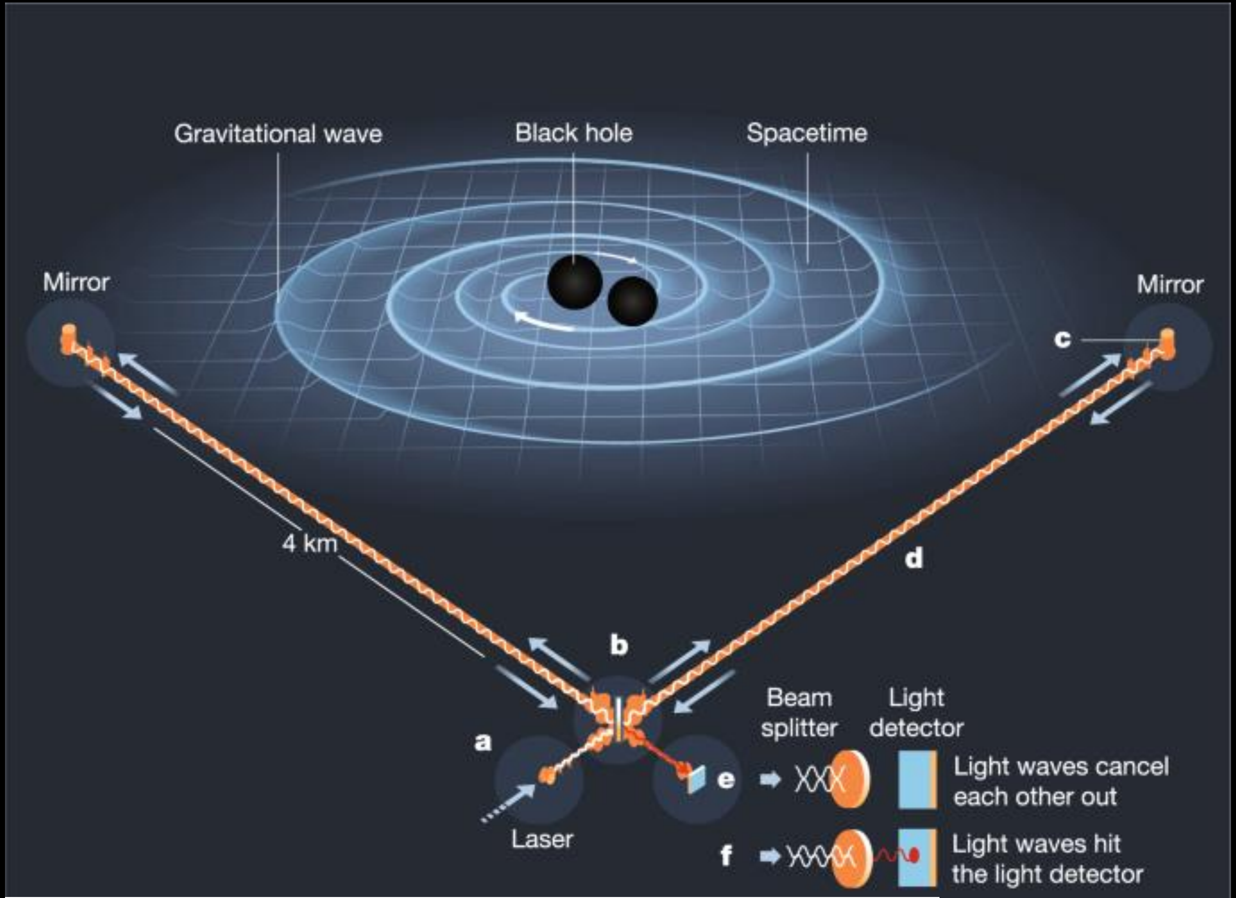
POLARIZATION

These two polarization states make up the combined signal impinging on the gravitational wave detector

The orientation of the detector affects the strength of the signal

A system of multiple detectors improves the areal coverage and sensitivity

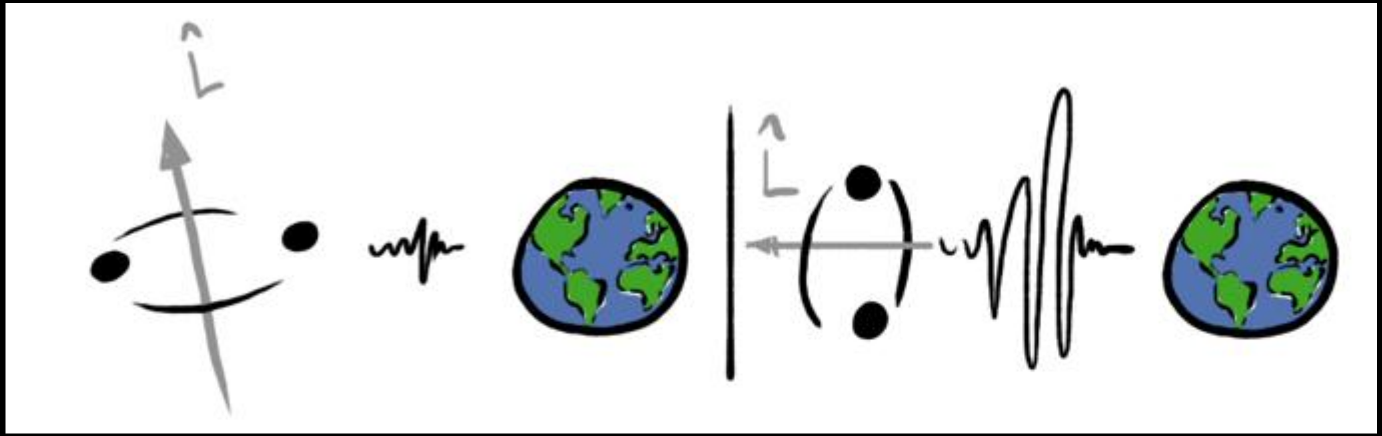
Three or more allows for triangulation of the source location



$$\frac{\delta L(t)}{L} = F_{+}(\theta,\, \phi,\, \psi)h_{+}(t) + F_{\times}(\theta,\, \phi,\, \psi)h_{\times}(t),$$

INCLINATION

One of the variables determining the signal strength is the inclination (i)



Without exact constraints on the inclination, distance estimates suffer!

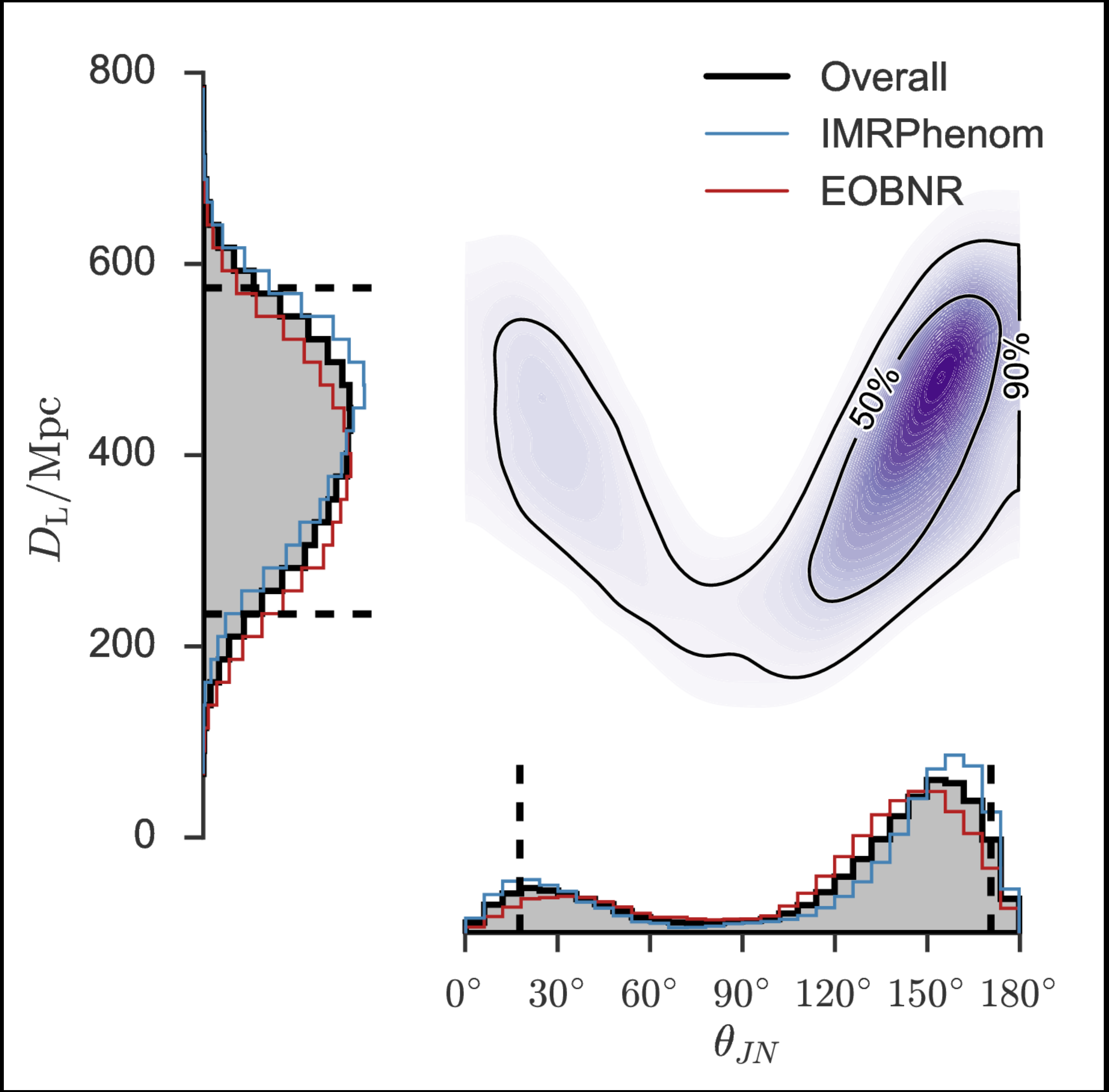
INCLINATION

Right-hand figure display the degeneracy with D and i

$$h_+ = \frac{2c}{D}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}\Omega^{2/3}\left(1+\cos^2 i\right)\cos 2\Phi(t)$$

If polarizations can be separated:

$$\frac{h_+}{h_\times} = \frac{1+\cos^2 i}{2\cos i},$$



DISTANCES

Unlike electromagnetic waves, GWs are not generated by 1:st or 2:nd order multipole moments in its basic charge (for GWs, the mass charge)

This results in the interesting phenomena:

$$h \propto \frac{1}{D}$$

I.e., luminosity distance is in principle measurable directly from the waveform, but GW luminosity is not required!

This is truly helpful when trying to reach more distant sources

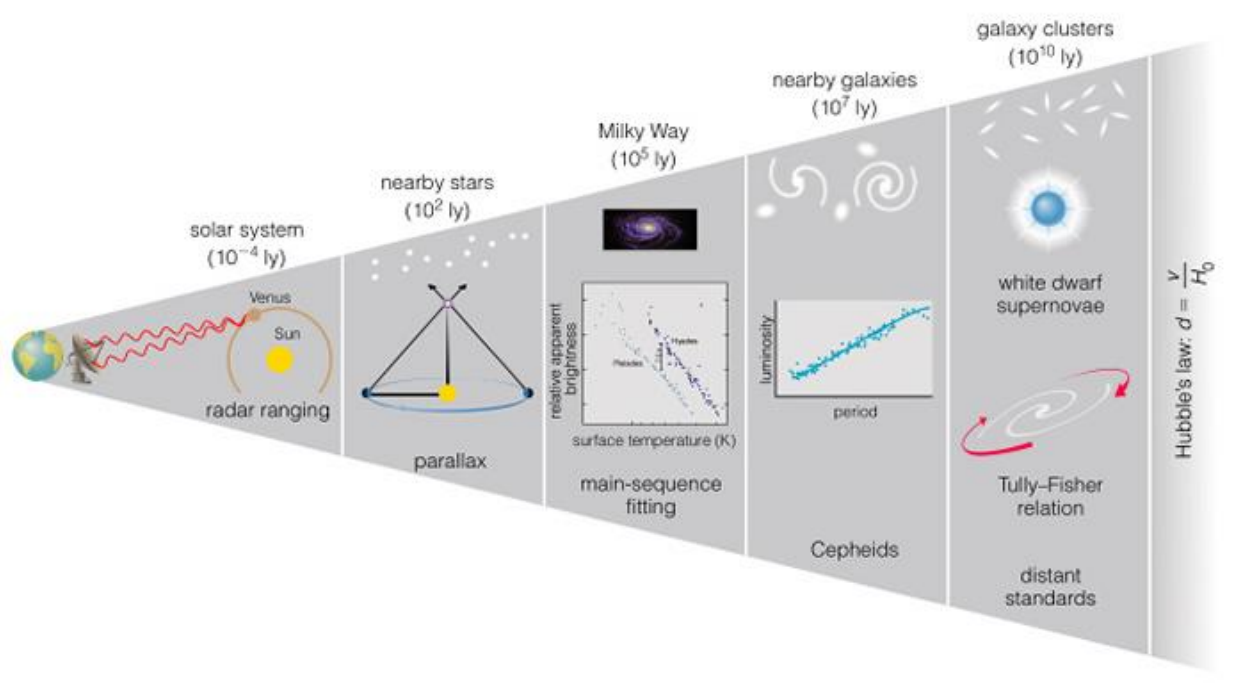
DISTANCES

GWs offer a unique means to measure distance without reference to the distance ladder

A lot like redshift being the ultimate measurable quantity for cosmological distance estimates with electromagnetic waves:

$$cz = H_0 D_C$$

(requires a distance ladder)



DISTANCES

GWs on the other hand, directly measures luminosity distance from the waveform!

It does not, by any means, measure redshift!

A problem: All measured entities are subject to redshift:

$$\mathcal{M}_o = (1+z)\mathcal{M}$$

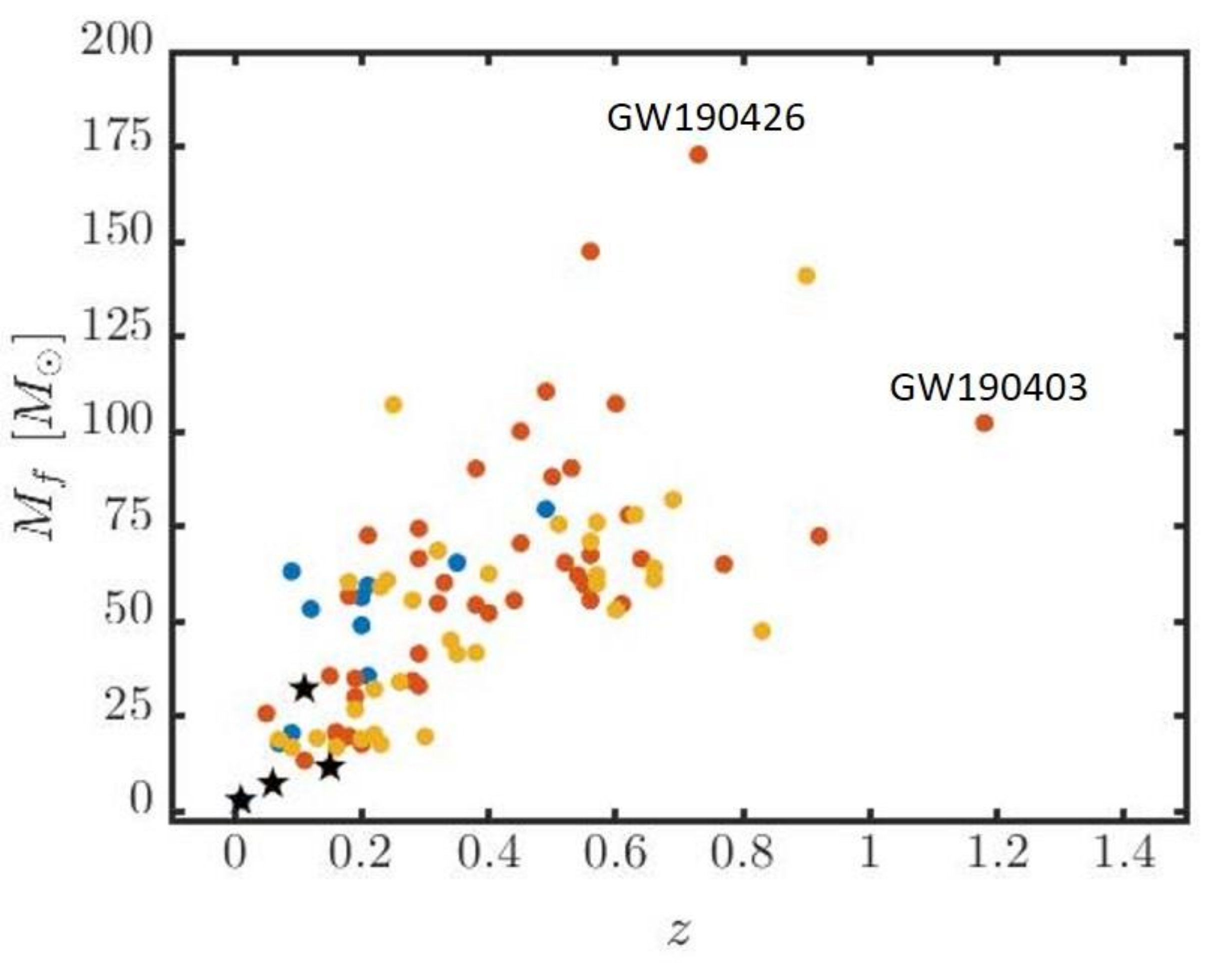
You have to correct for the redshift in order to not overestimate your distances

DISTANCES

Final remnant mass VS redshift for current available datasets

Includes highly massive and distant remnants!

NS-NS mergers as well as NS-BH merger



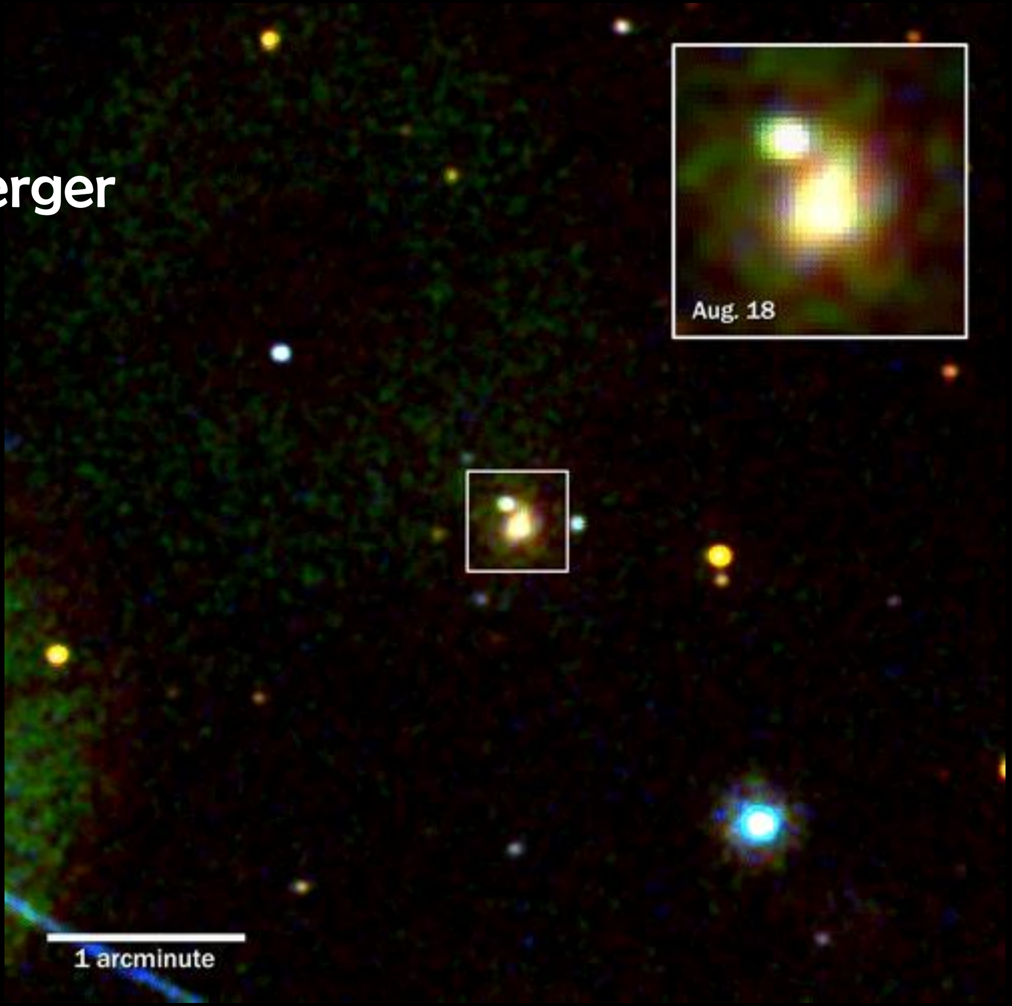
STANDARD SIRENS

For an astronomer, a means of directly measuring distances and current expansion rate H_0, is a (should be!) dream worth dying for!

Detection of GW170817 – a binary neutron star merger

Simultaneous measurement of D and z

One single event provides a value for H_0



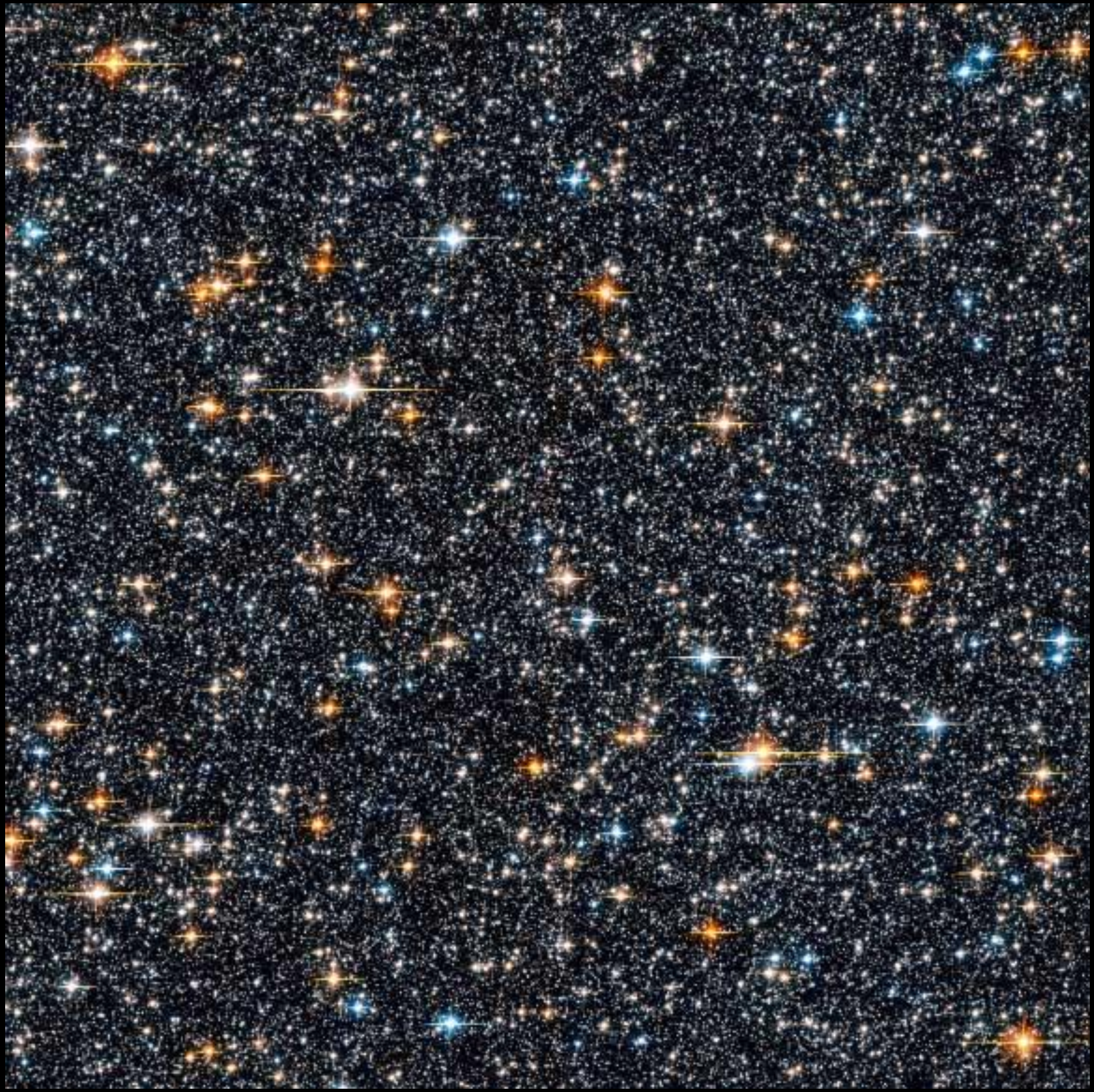
DARK SIRENS

BH-BH merger can also provide standard sirens

Improved localization with multiple fascilities could pinpoint host galaxy with high precision

LISA will itself be able to improve localisation significantly (for some sources)

Statistical inference on the location of GW source combined with <u>complete</u> galaxy catalogues



ASTROPHYSICS

Gravitational waves inform us of several astrophyscial phenomena

- Black hole mergers
- Merger rates and mass distribution of BH
- Formation mechanisms for BHs

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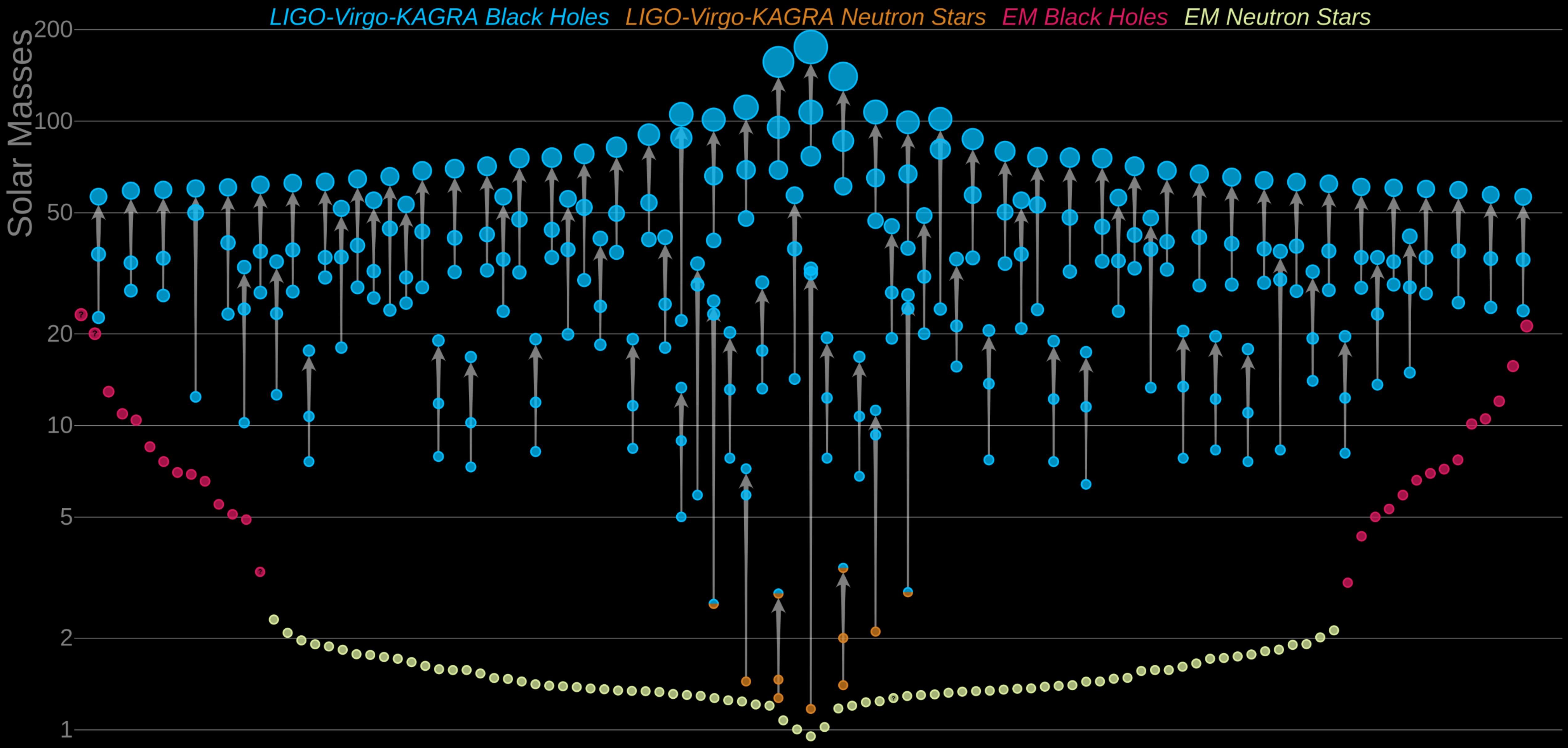
- White dwarf mergers
- Type 1A supernovae?

Masses in the Stellar Graveyard



LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

ASTROPHYSICS

Recent detections of massive remnants GW190521 – first directly observed IMBH

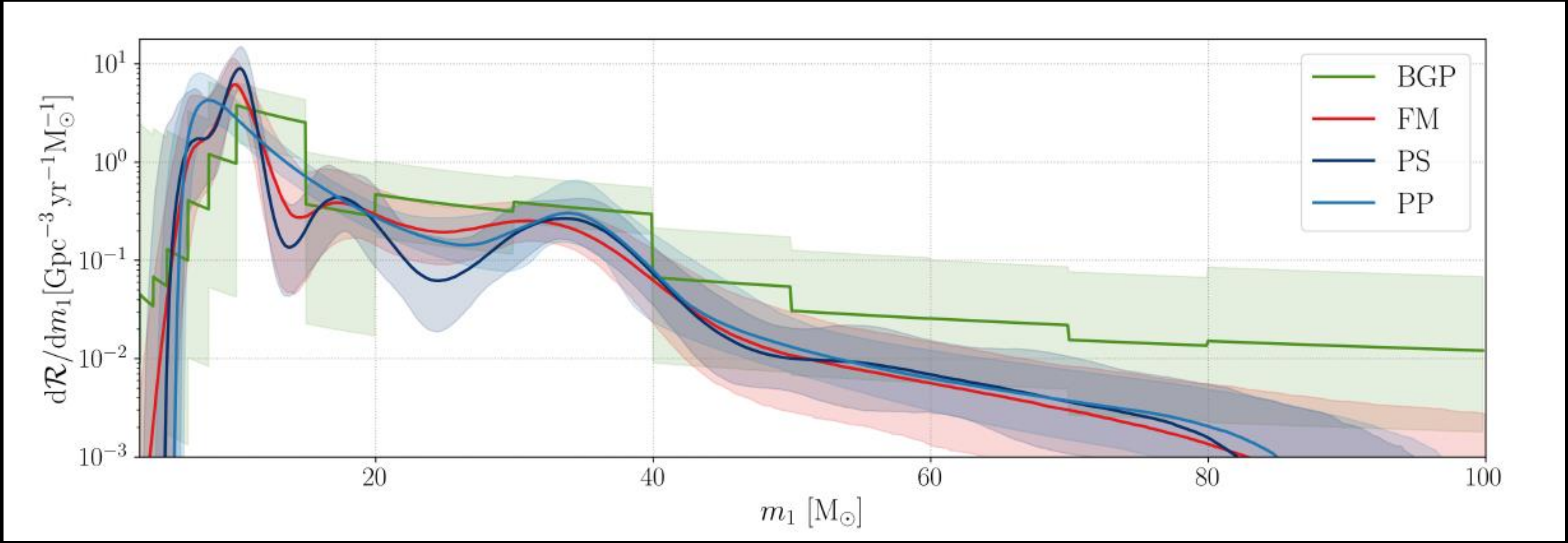
$$m_1 = 85\ M_\odot, \qquad m_2 = 66\ M_\odot$$
$$M_f = 142\ M_\odot$$

Likely not formed directly from stars

The pair-Instability supernovae mechanism suggests a mass limit for stellar BH:
$m < 150\ M_\odot$ - more massive stars simply leaves no remnant

Pulsational pair-instability suggests stars form black holes of at most $\sim 50\ M_\odot$

→ The black hole mass gap: $\sim 50 - 150\ M_\odot$



Substructures in the black hole mass distribution

A large catalogue of black hole mergers allows us to constrain BH formation mechanisms, merger rates etc.

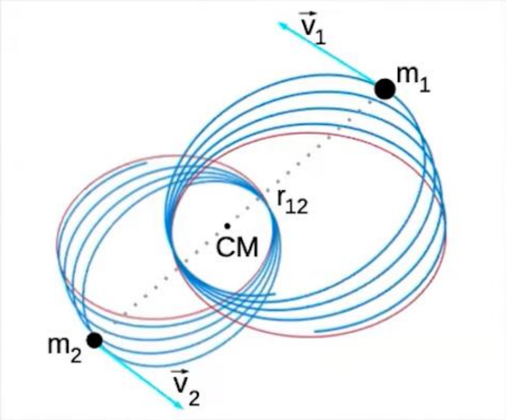
APPROXIMATIVE METHODS AND NUMERICAL RELATIVITY

Post-Newtonian approximation – fundamental framework allowing solutions to GR

Describes the inspiral where velocities are small and gravitational field is ”weak”

expansion in orders of $\sim (v/c)^n$

Famous example:
Precession of Mercury



$$\frac{\mathrm{d}\boldsymbol{v}_1}{\mathrm{d}t} = -\frac{Gm_2}{r_{12}^2}\boldsymbol{n}_{12} + \overbrace{\frac{1}{c^2}\left\{\left[\frac{5G^2m_1m_2}{r_{12}^3}+\frac{4G^2m_2^2}{r_{12}^3}+\cdots\right]\boldsymbol{n}_{12}+\cdots\right\}}^{\text{1PN}}$$

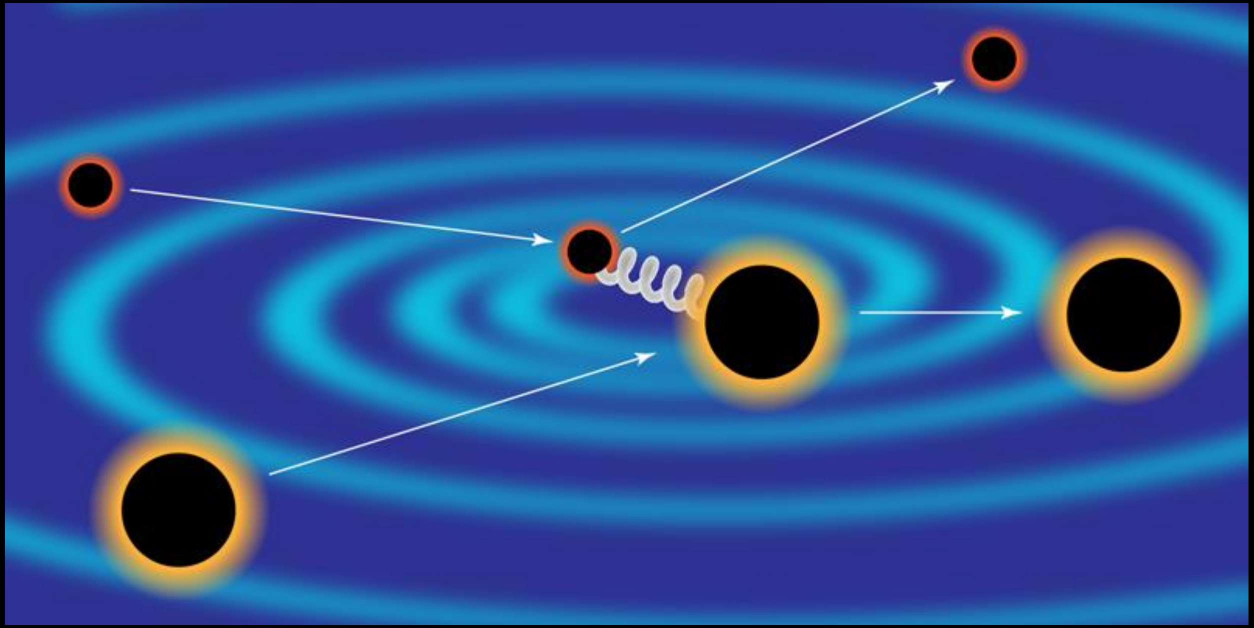
$$+\underbrace{\frac{1}{c^4}[\cdots]}_{\text{2PN}}+\underbrace{\frac{1}{c^5}[\cdots]}_{\substack{\text{2.5PN}\\ \text{radiation reaction}}}+\underbrace{\frac{1}{c^6}[\cdots]}_{\text{3PN}}+\underbrace{\frac{1}{c^7}[\cdots]}_{\substack{\text{3.5PN}\\ \text{radiation reaction}}}+\underbrace{\frac{1}{c^8}[\cdots]}_{\substack{\text{4PN}\\ \text{conservative \& dissipative (tail)}}}+\mathcal{O}\left[\left(\frac{v}{c}\right)^9\right]$$

APPROXIMATIVE METHODS AND NUMERICAL RELATIVITY

Post-Minkowskian approximation – more generic strong field limits with large velocities

Higher order deviations from the linearized metric – expansion in orders of G^n

Allows solutions to be tracked closer to the merger event and is frequently used to describe black hole scattering



Famous example: Effective one-body with reduced mass μ moving in combined potential

APPROXIMATIVE METHODS AND NUMERICAL RELATIVITY

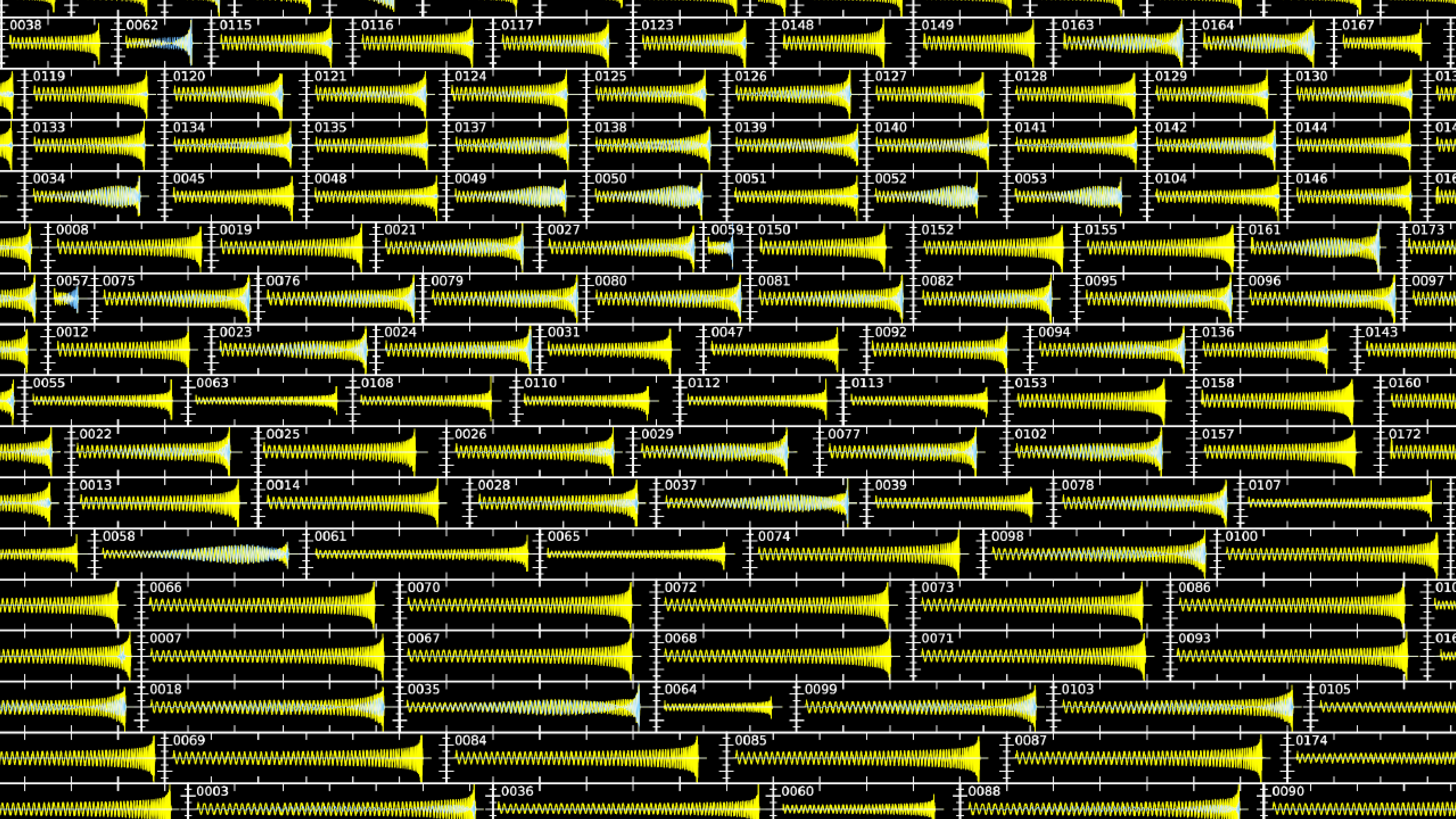
Numerical relativity is computationally expansive but solves the Einstein fields equations ”exactly”

Used to model the merger and final plunge

Used to calibrate the PN and PM approximations – fixing coefficients and testing

Behind much of the recent success due to numerical breakthrough of Pretorius 2005

More and more sophisticated numerical models required to keep up with nascent detector fascilites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Credit: Teresita Ramirez / Geoffrey Lovelace / SXS Collaboration / LIGO Virgo Collaboration.

TIDAL DEFORMATION

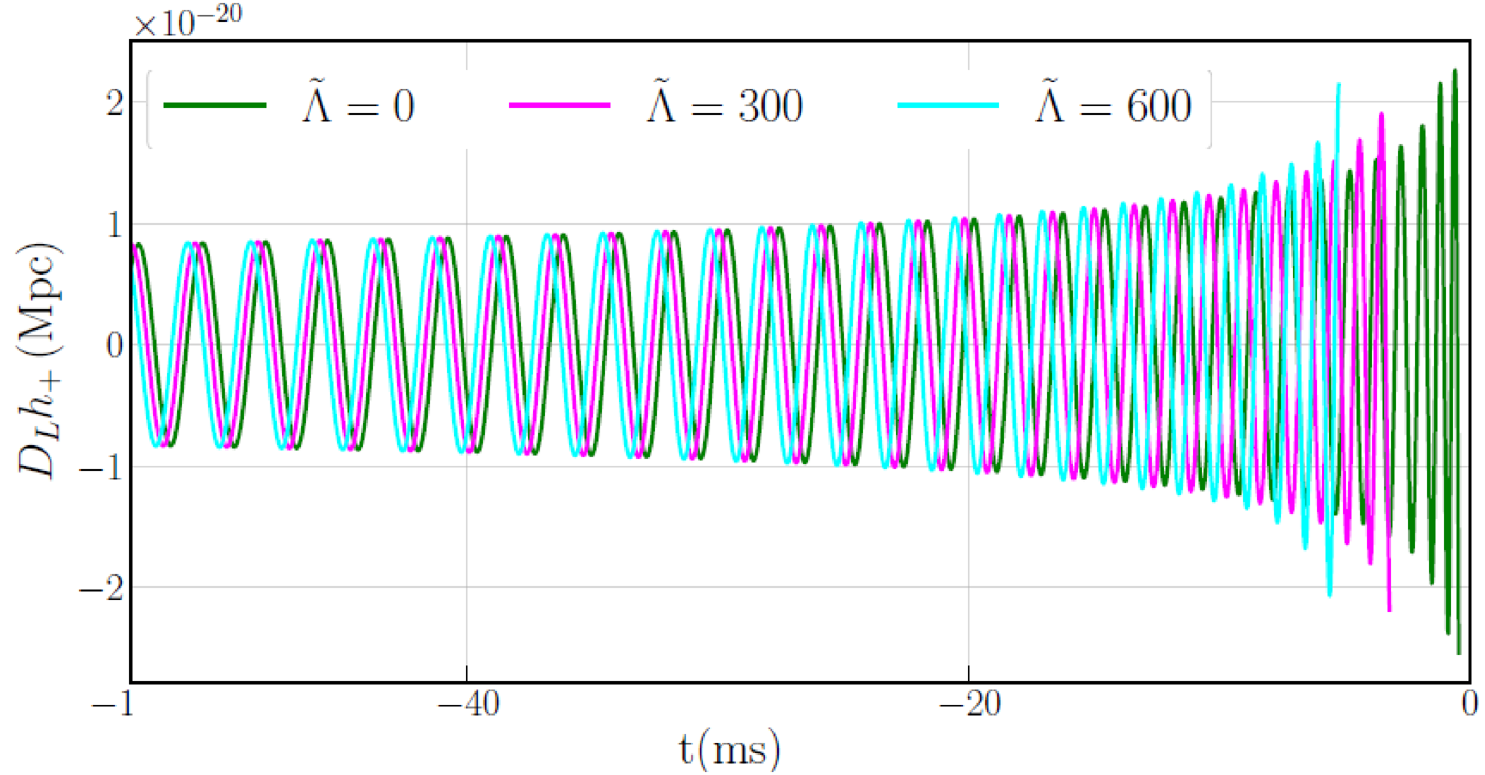
Neutron stars – tidal deformation

Induced quadrupolar moment:

$$Q_{ij} = -\Lambda m^5 \varepsilon_{ij}$$

$$\Lambda = \lambda / m^5 = \frac{2}{3} k_2 R^5$$

k_2 = Love number



Increased quadruplar moment due to tidal deformation

FUTURE PROSPECTS

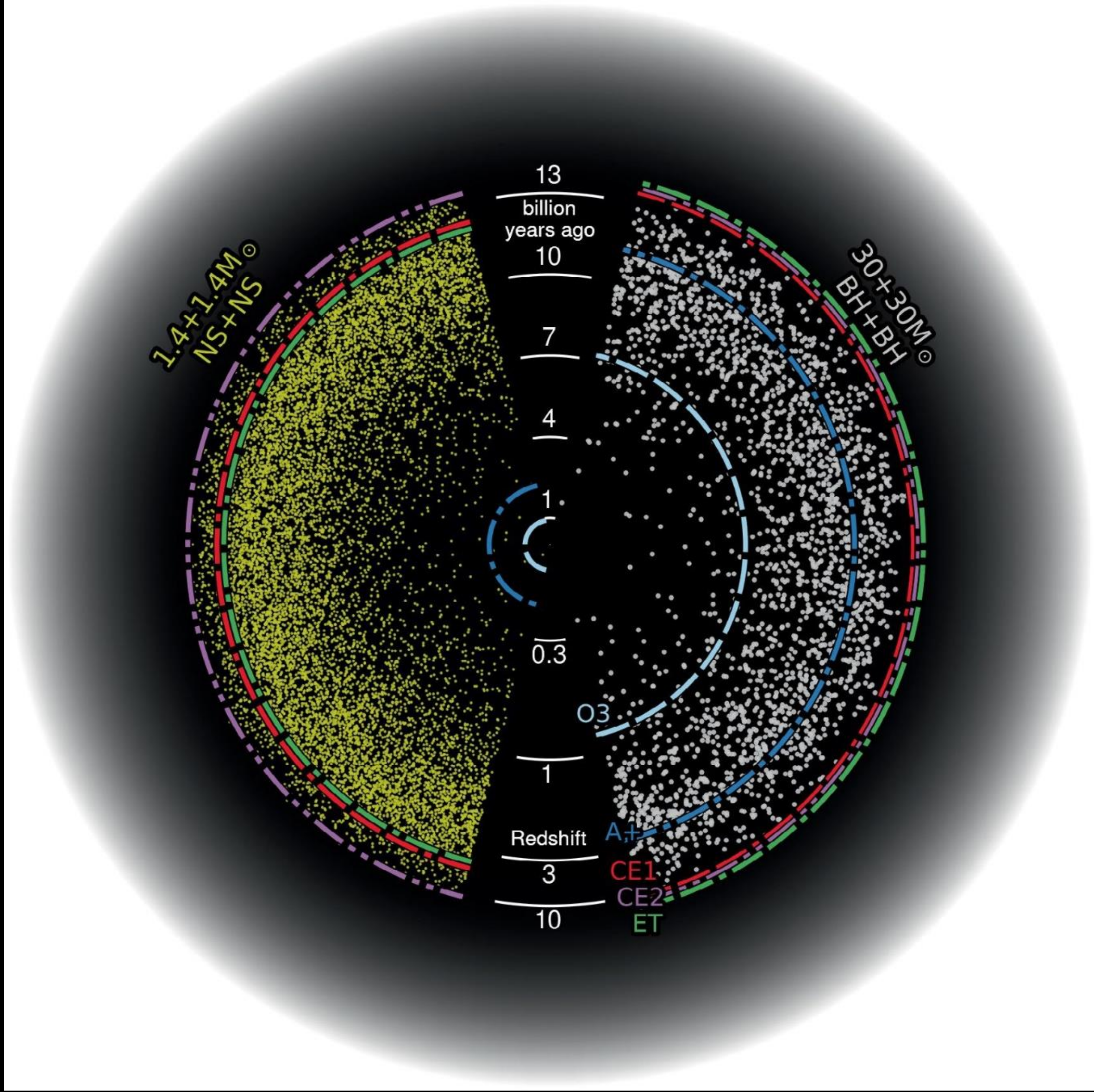
New facilities like:

- Cosmic explorer
- Einstein telescope
- LISA

The horizon for detection pushes to high redshift

High detection rates, even for NS-NS mergers

BH-BH mergers at even smaller masses is soon within reach



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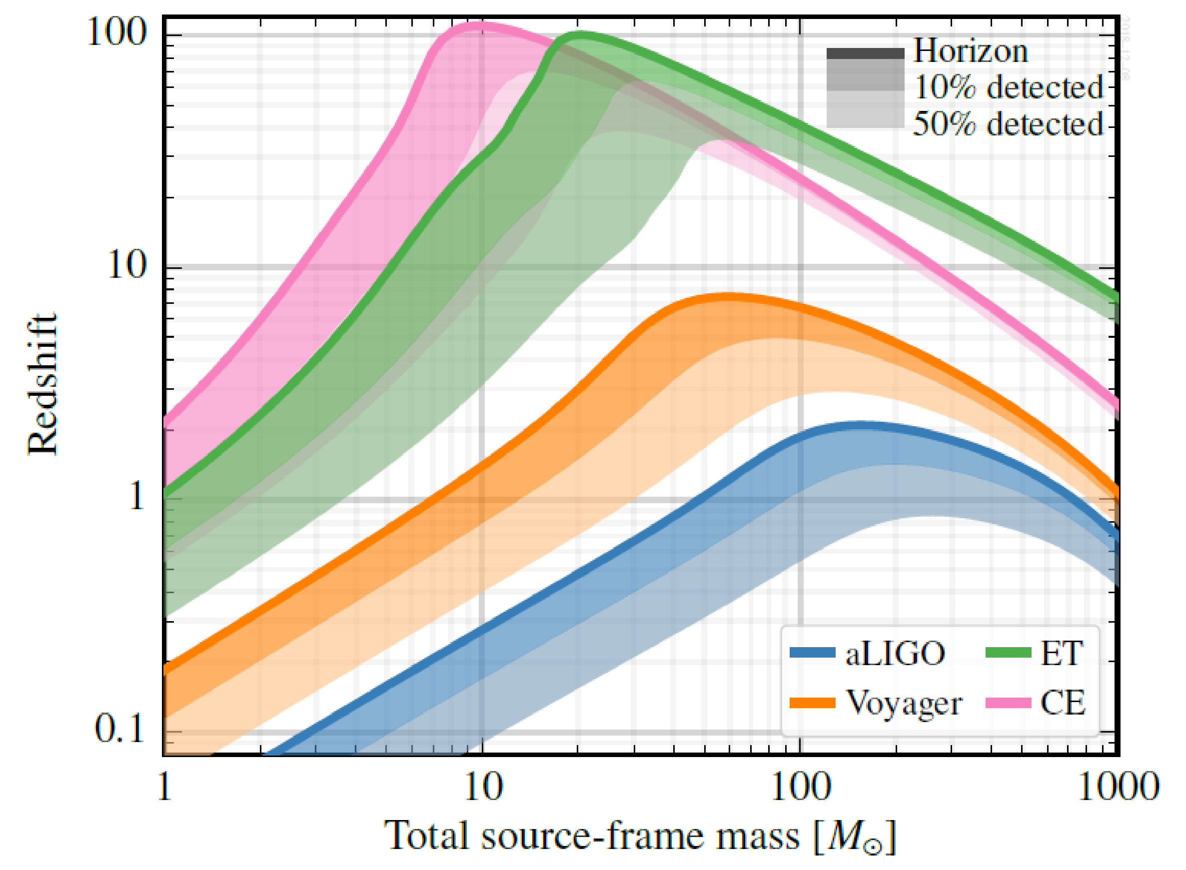
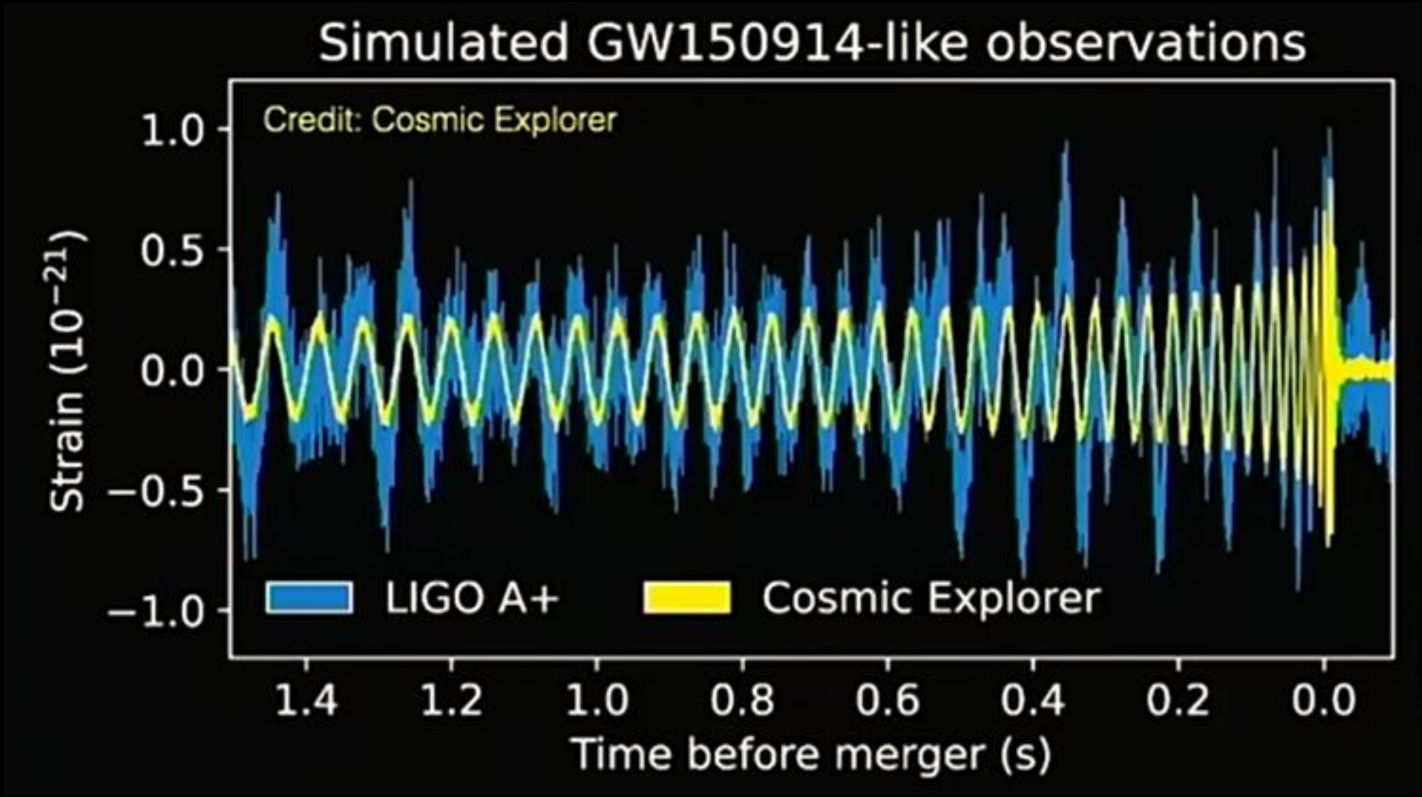


FUTURE PROSPECTS

The increased sensitivity of detectors challenges the numerical models



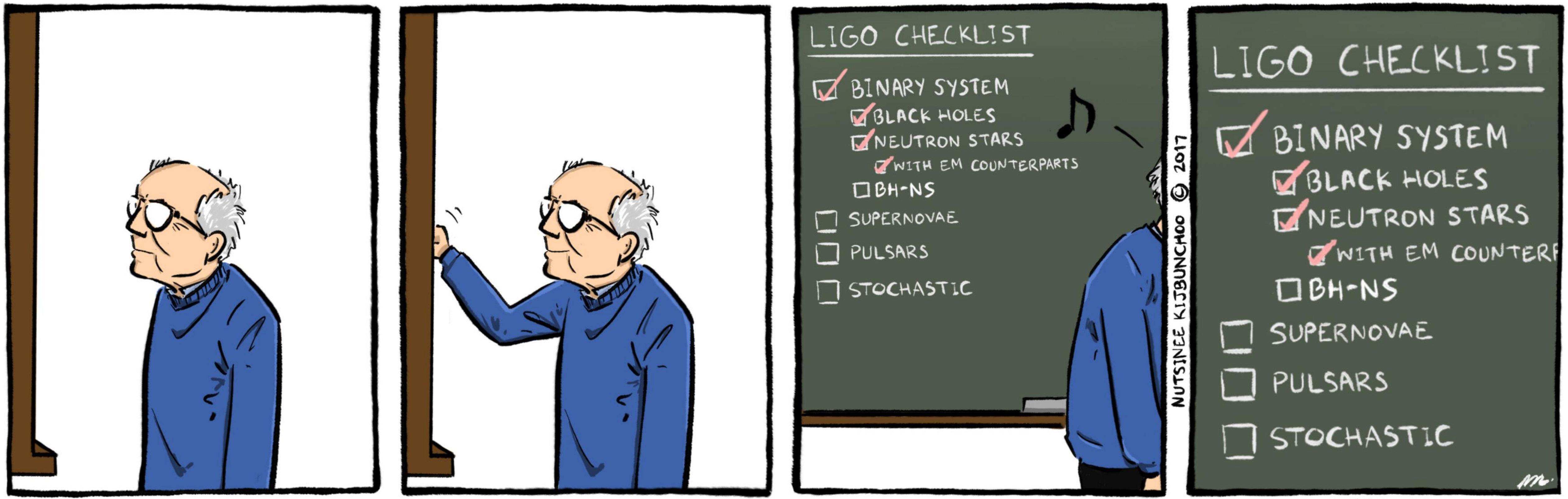
PN & PM approximations can push to higher orders

Detailed physics like:

- Mass ratios
- Tidal deformation
- Ellipticity
- Spin

Can be studied in detail with high SNR

FUTURE PROSPECTS



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