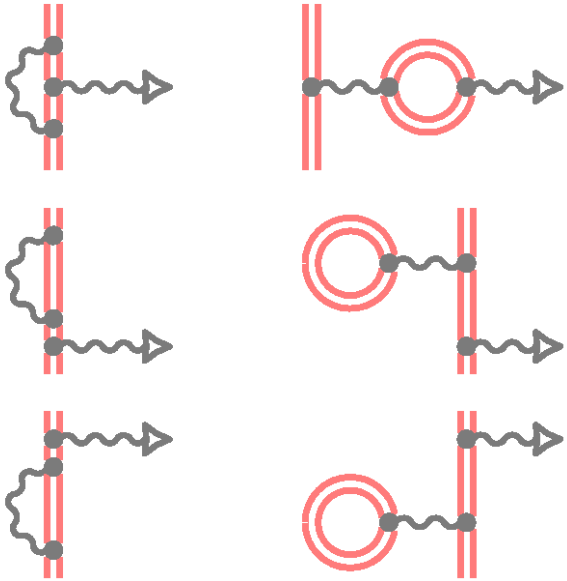
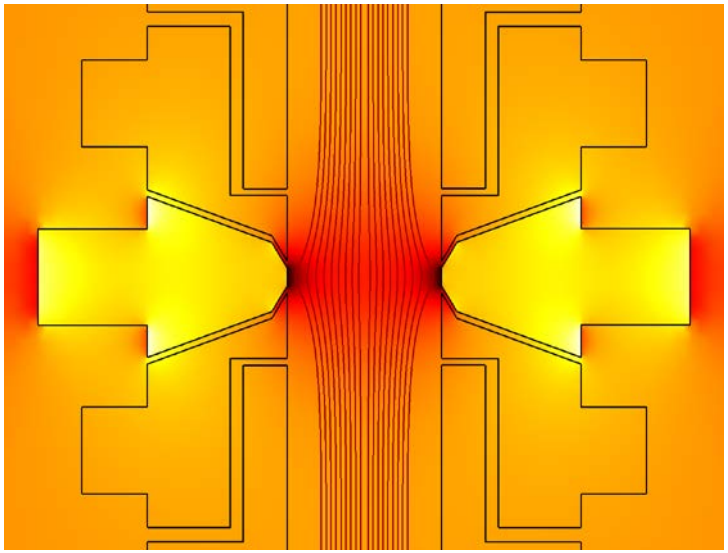
Electron Mass and Proton Magnetic Moment

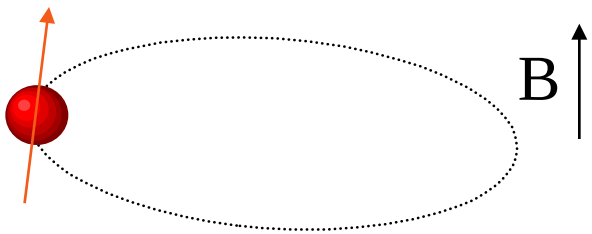
g-Factor Measurements and Fundamental Interactions





Wolfgang Quint
GSI Darmstadt and Univ. Heidelberg

g-Factor of the electron



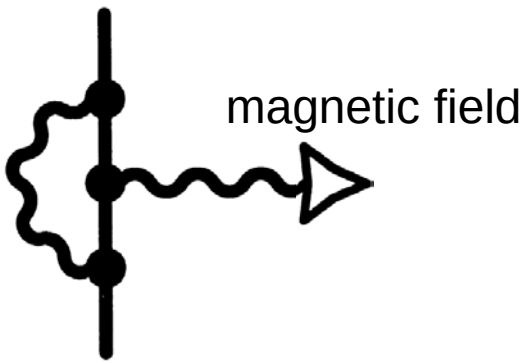
$$\frac{|\vec{\mu}|}{\mu_B} = g \cdot \frac{|\vec{s}|}{\hbar}$$

μ: magnetic moment
g: g-factor
s: spin
μ_B: Bohr magneton

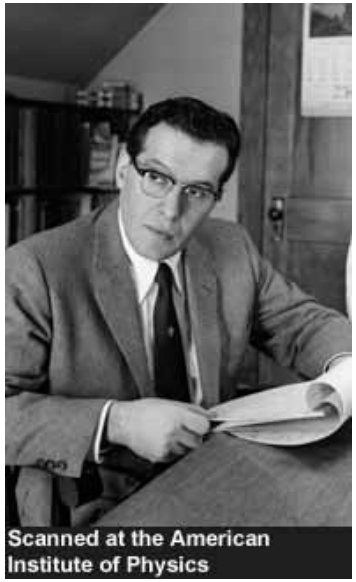
$g = 2 + \alpha / \pi$

QED contributions to the g-factor of the free electron

$$g_{free} = 2\,(1 + C_1\alpha/\pi + C_2(\alpha/\pi)^2 + C_3(\alpha/\pi)^3 + C_4(\alpha/\pi)^4 + C_5(\alpha/\pi)^5 + \ldots$$



1st order in α:
Schwinger term
$C_1 = ½$



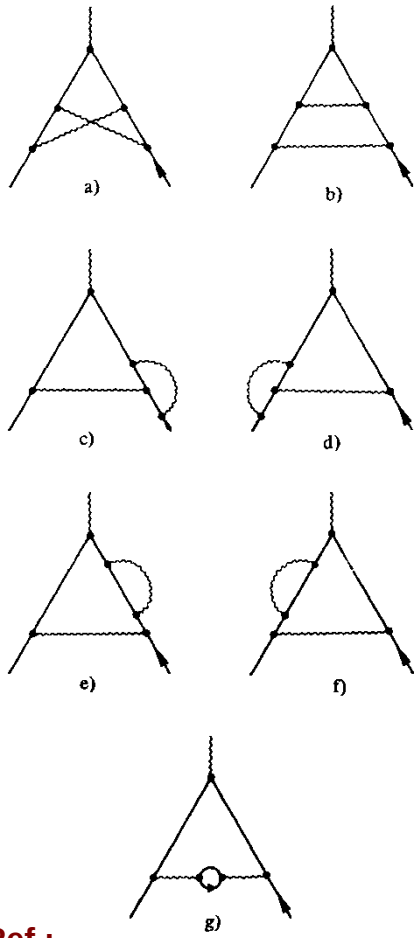
The theory of quantum electrodynamics is, I would say, the jewel of physics - our proudest possession.

R. Feynman

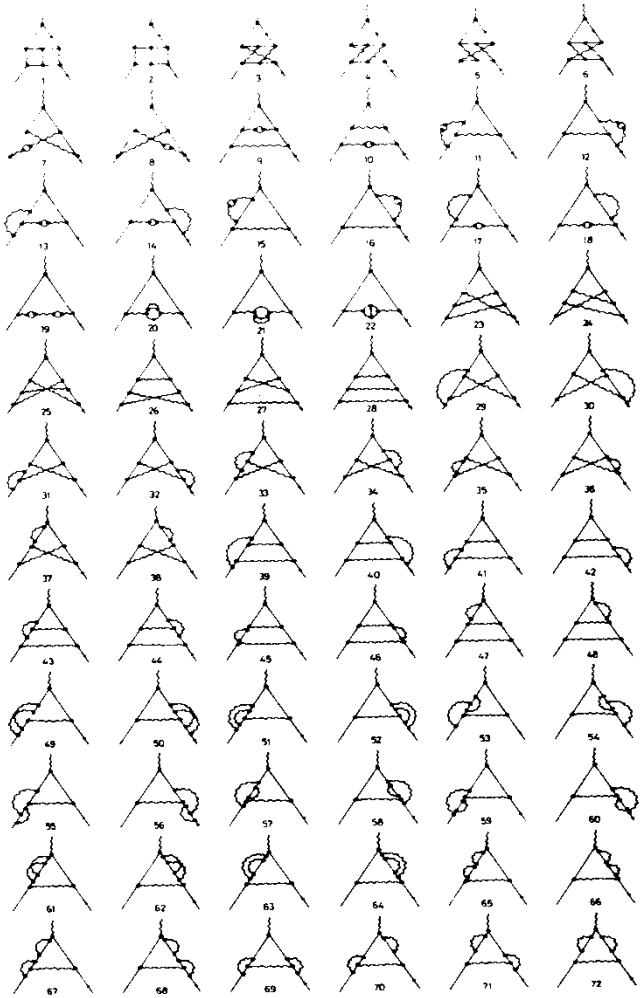
Ref.:
J. Schwinger, Phys. Rev. 73, 416 (1948); Hanneke et al., PRL 100, 120801 (2008)

Free electron: QED contributions of 2nd and 3rd order

$$g_{free} = 2\,(1 + C_1\alpha/\pi + C_2(\alpha/\pi)^2 + C_3(\alpha/\pi)^3 + C_4(\alpha/\pi)^4 + C_5(\alpha/\pi)^5 + \ldots$$



2nd order in α:
C_2 = - 0.328 478 966
7 graphs



3rd order in α:
C_3 = 1.1765
72 graphs

not shown:
4th order in α:
C_4 = -1.9108
891 graphs

Ref.:
B. Lautrup et al., Phys. Rep. 3, 193 (1972)

Free electron: QED contributions of 5th order

$$g_{free} = 2\,(1 + C_1\alpha/\pi + C_2(\alpha/\pi)^2 + C_3(\alpha/\pi)^3 + C_4(\alpha/\pi)^4 + C_5(\alpha/\pi)^5 + \ldots$$

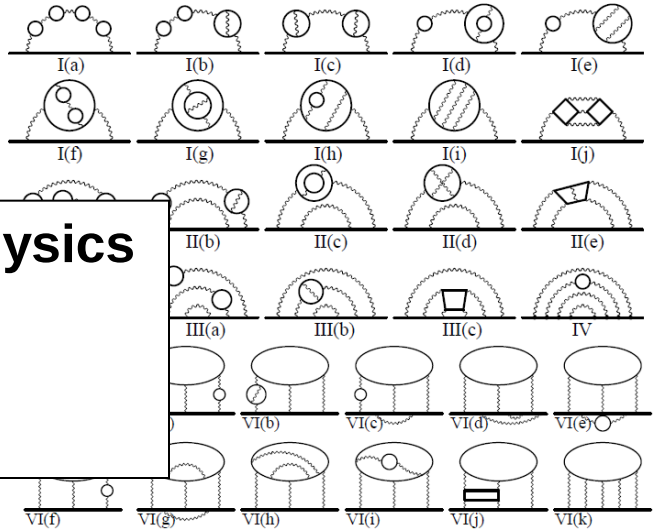
Harvard g-2 measurement 2008:

g_{free} **= 2 (1.001 159 652 180 73 (28)) → determination of α**

5th order in α:

C_5 = 9.16

12672 graphs



„I am digging at the roots of physics to see whether there is some treasure there."
Toichiro Kinoshita

Ref.:
Kinoshita et al., arXiv:1205.5368v1 [hep-ph] 24 May 2012

g-Factor of the electron
Harvard 2008

PRL **100,** 120801 (2008) PHYSICAL REVIEW LETTERS week ending 28 MARCH 2008

New Measurement of the Electron Magnetic Moment and the Fine Structure Constant

D. Hanneke, S. Fogwell, and G. Gabrielse*

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 4 January 2008; published 26 March 2008)

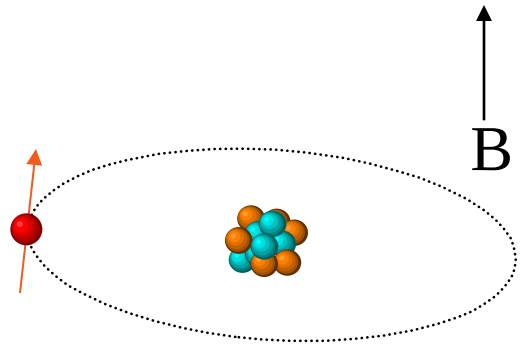
A measurement using a one-electron quantum cyclotron gives the electron magnetic moment in Bohr magnetons, $g/2 = 1.001\,159\,652\,180\,73\,(28)$ [0.28 ppt], with an uncertainty 2.7 and 15 times smaller than for previous measurements in 2006 and 1987. The electron is used as a magnetometer to allow line shape statistics to accumulate, and its spontaneous emission rate determines the correction for its interaction with a cylindrical trap cavity. The new measurement and QED theory determine the fine structure constant, with $\alpha^{-1} = 137.035\,999\,084\,(51)$ [0.37 ppb], and an uncertainty 20 times smaller than for any independent determination of α.

g-Factor of the bound electron in a hydrogen-like ion

(nucleus has no spin, e.g. $^{12}C^{5+}$, $^{16}O^{7+}$, $^{28}Si^{13+}$, $^{40}Ca^{19+}$)

Larmor precession frequency of the bound electron:

$$\omega_L^{\,e} = \frac{g_J}{2}\frac{e}{m_e}B$$



Ion cyclotron frequency:

$$\omega_c^{\,ion} = \frac{Q}{M_{ion}}B$$

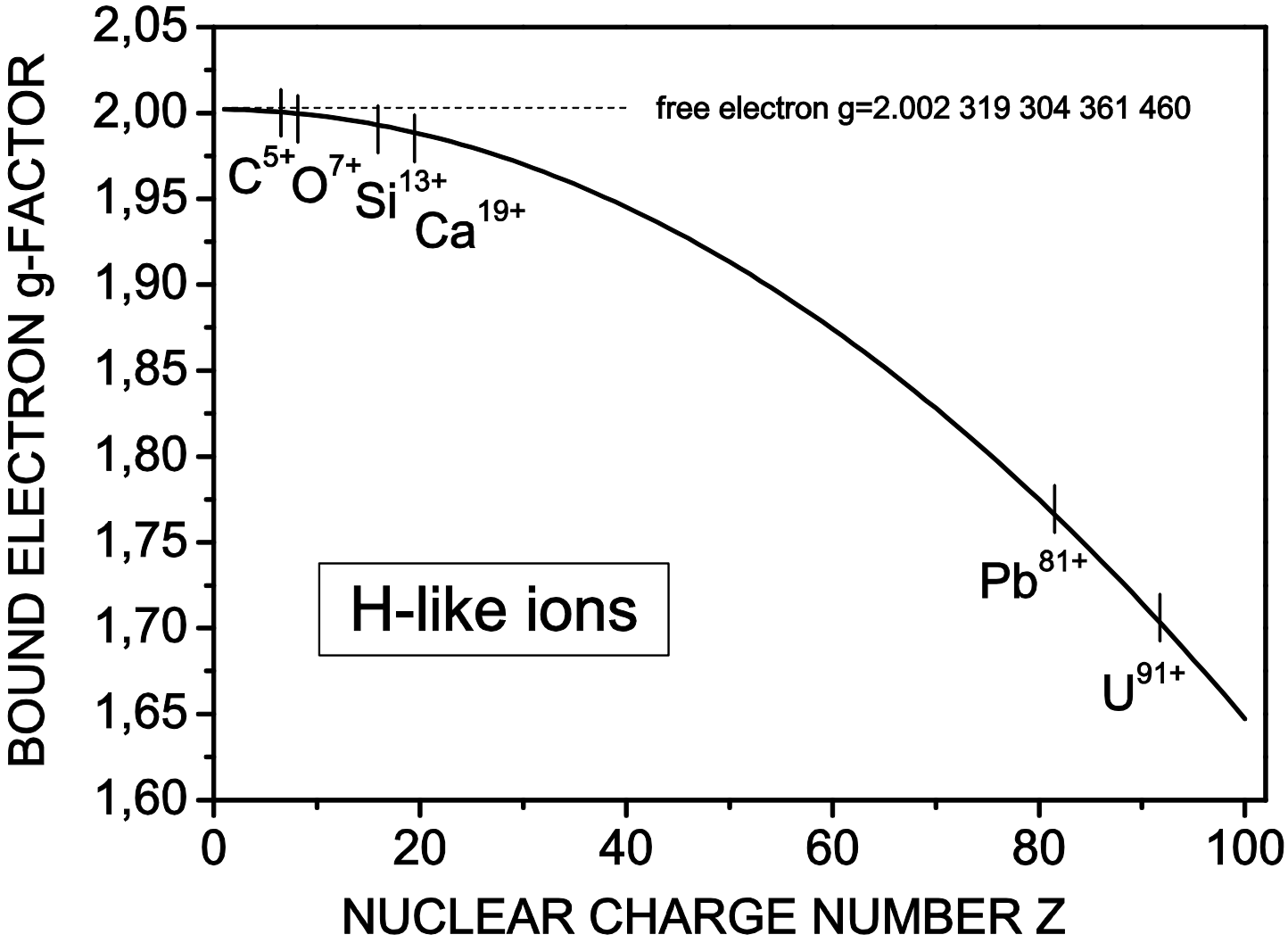
$$g_J = 2\cdot\frac{\omega_L^{\,e}}{\omega_c^{\,ion}}\cdot\frac{m_e}{M_{ion}}\cdot\frac{Q^{ion}}{e}$$

→ 'experimental g-factor'
→ comparison with theory

our measurement

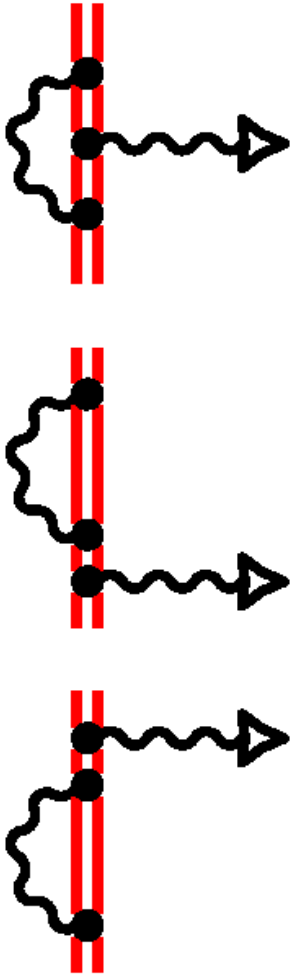
external input parameter

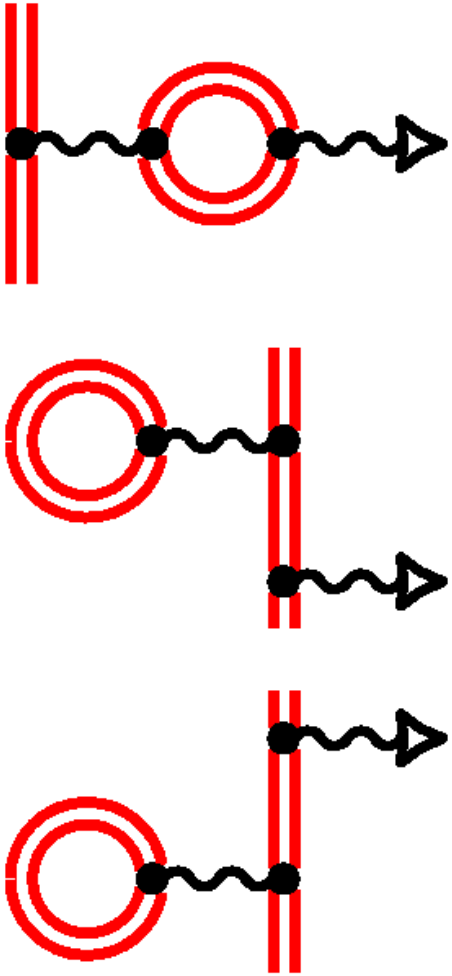
Bound-electron g-factor



Theory: Feynman graphs 1st order in α

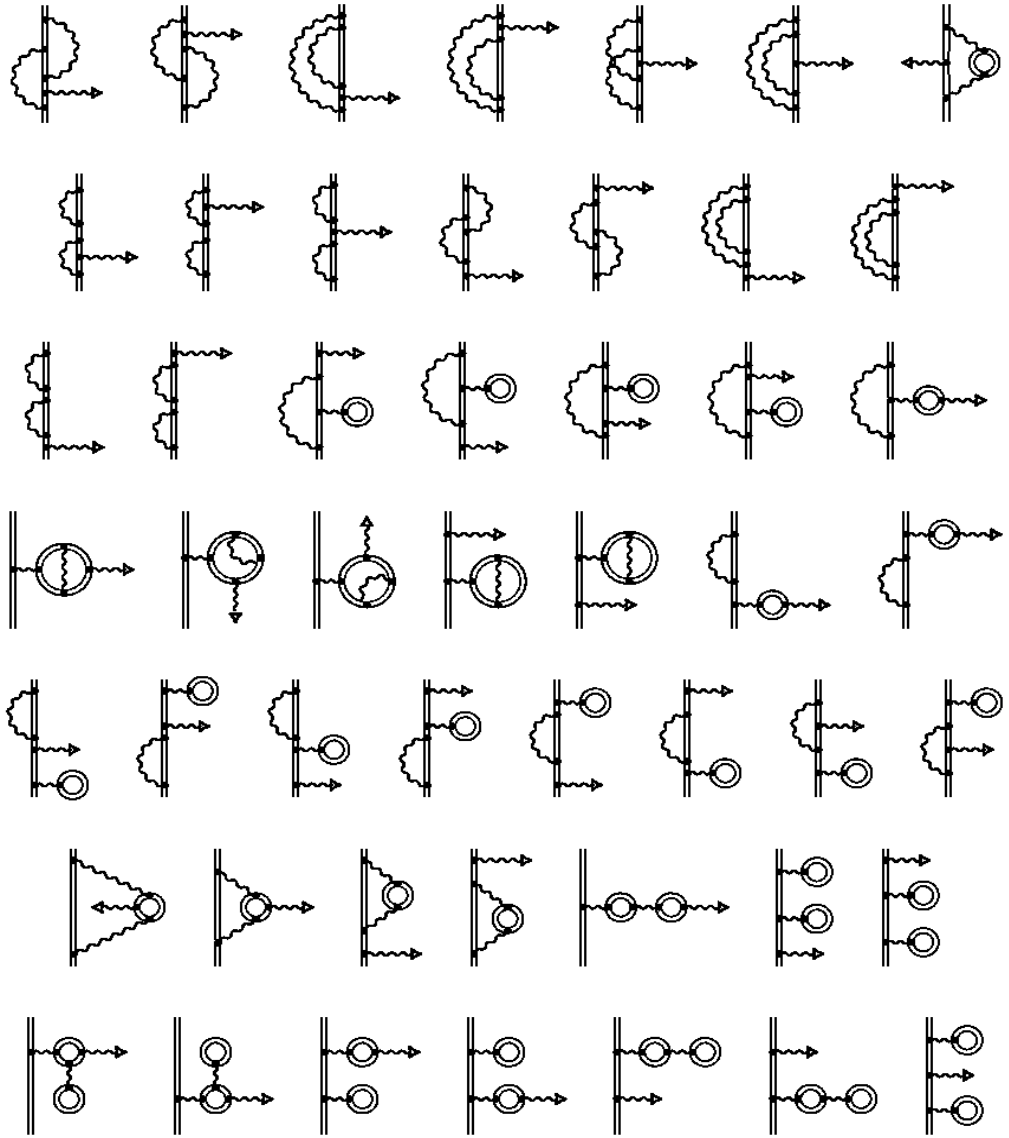
SELF ENERGY





VACUUM POLARIZATION

Theory: Feynman graphs 2nd order in α

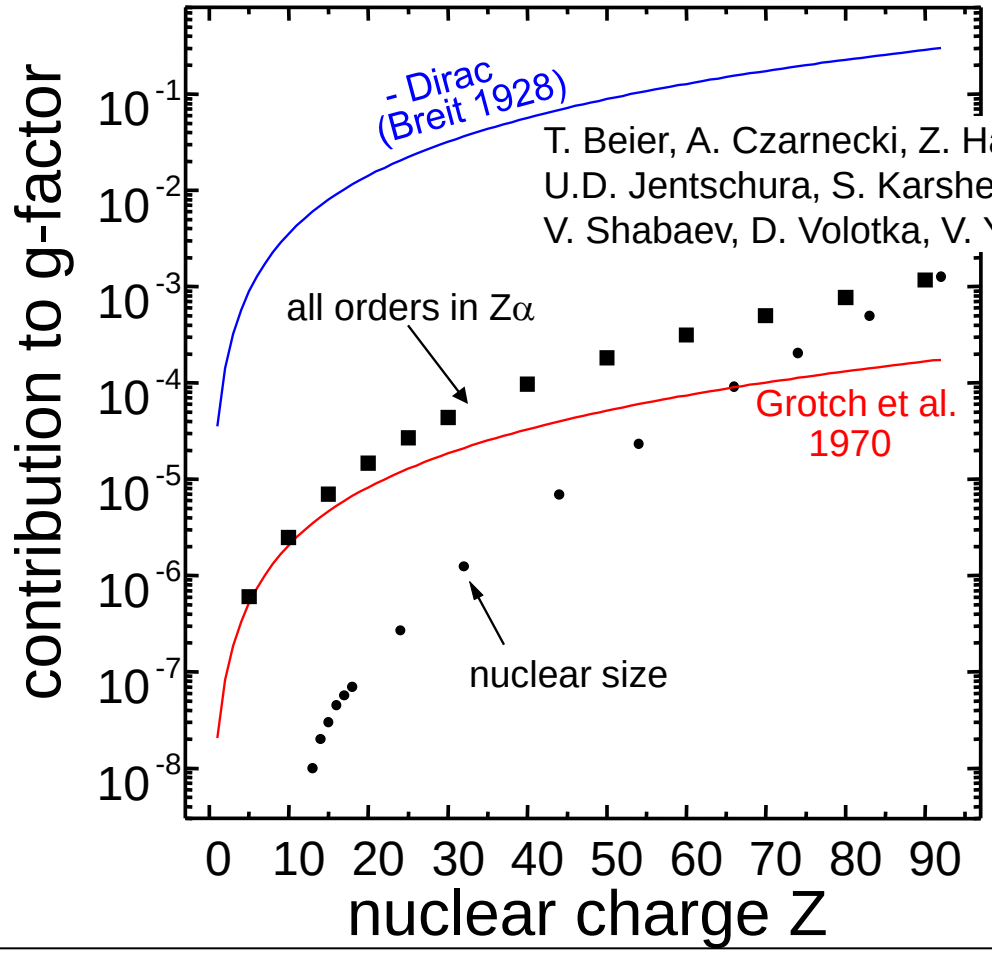


Bound-state QED of electron g-factor

$$g_{bound}/g_{free} \approx 1 - (Z\alpha)^2/3 + \alpha(Z\alpha)^2/4\pi + \ldots$$

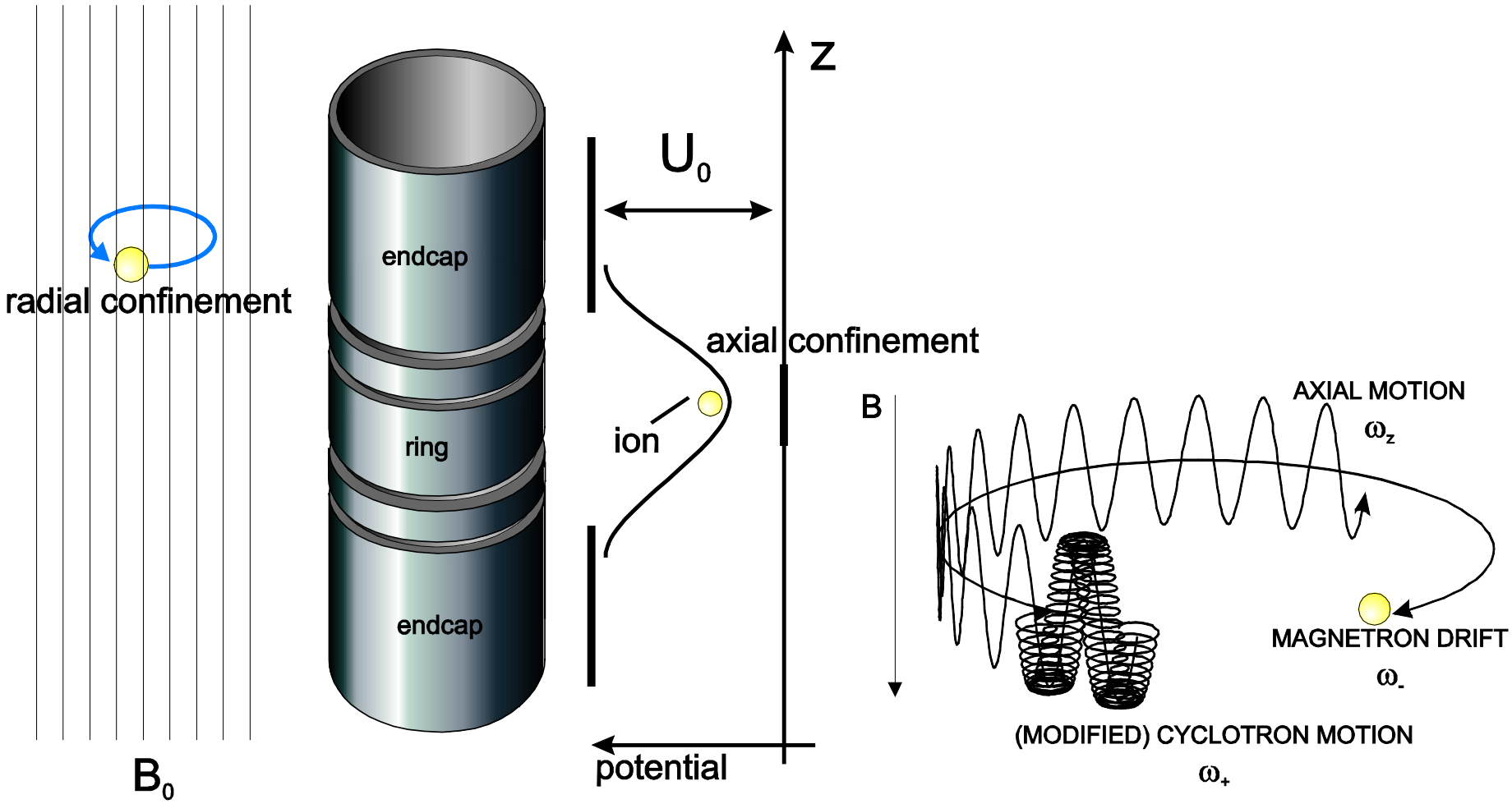
Dirac theory

bound-state QED



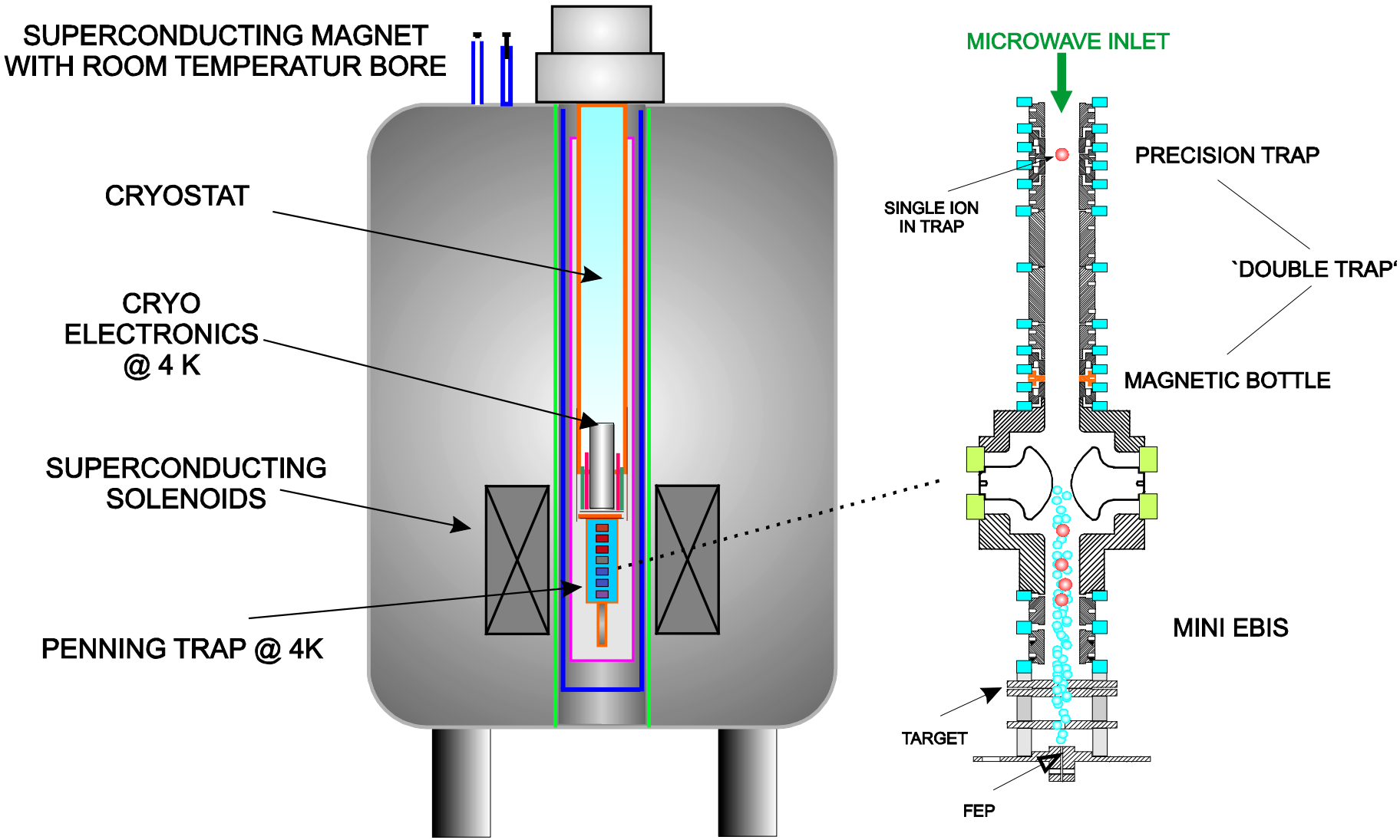
T. Beier, A. Czarnecki, Z. Harman, D. Glazov,
U.D. Jentschura, S. Karshenboim, H. Persson,
V. Shabaev, D. Volotka, V. Yerokhin, J. Zatorski, et al.

A single highly charged ion stored in a Penning trap



magnetic | physical | electric | combined ion motion

HCI g-factor apparatus



Penning Trap System

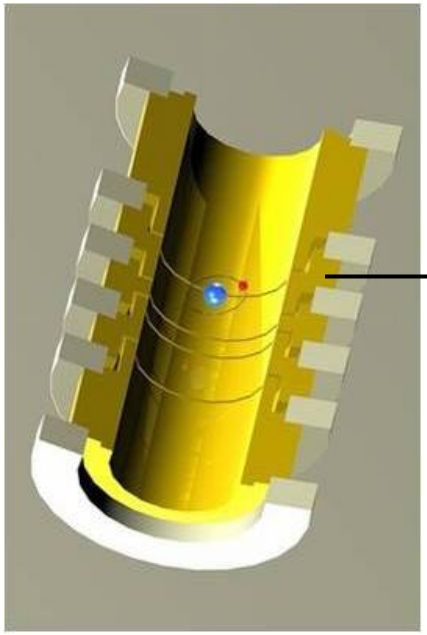
Precision trap

- Very homogeneous magnetic field

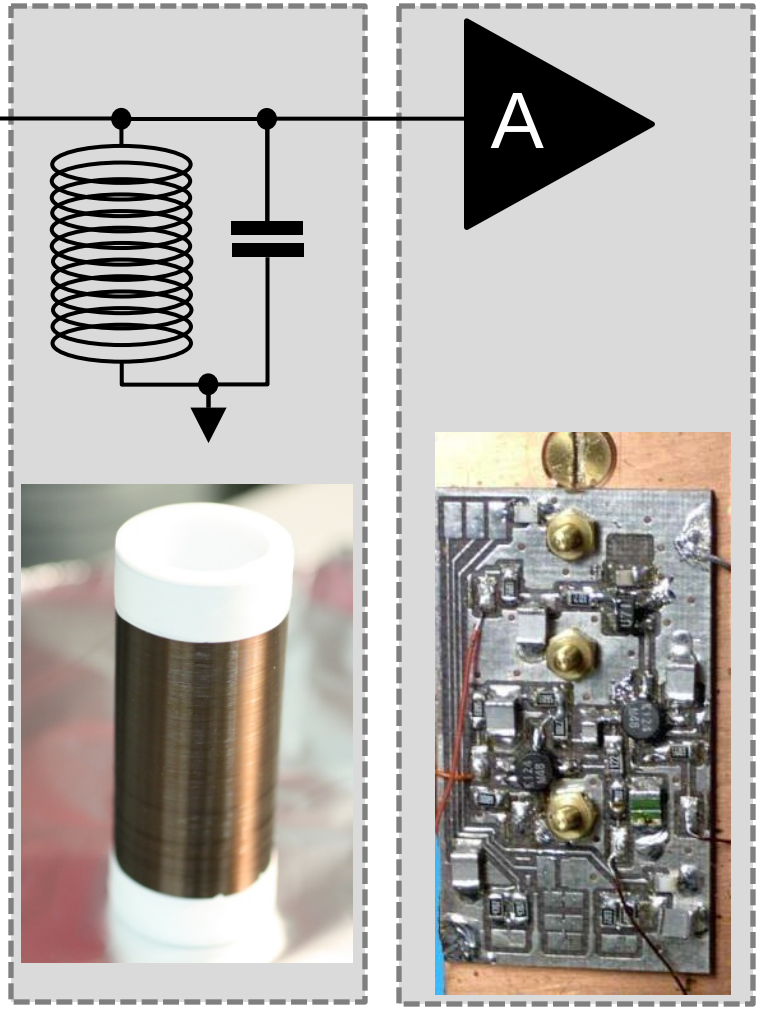
Analysis trap

- Magnetic bottle for spin detection

Ion oscillation frequency measurement



- Measurement of the tiny image currents (~fA) in the trap electrodes requires:

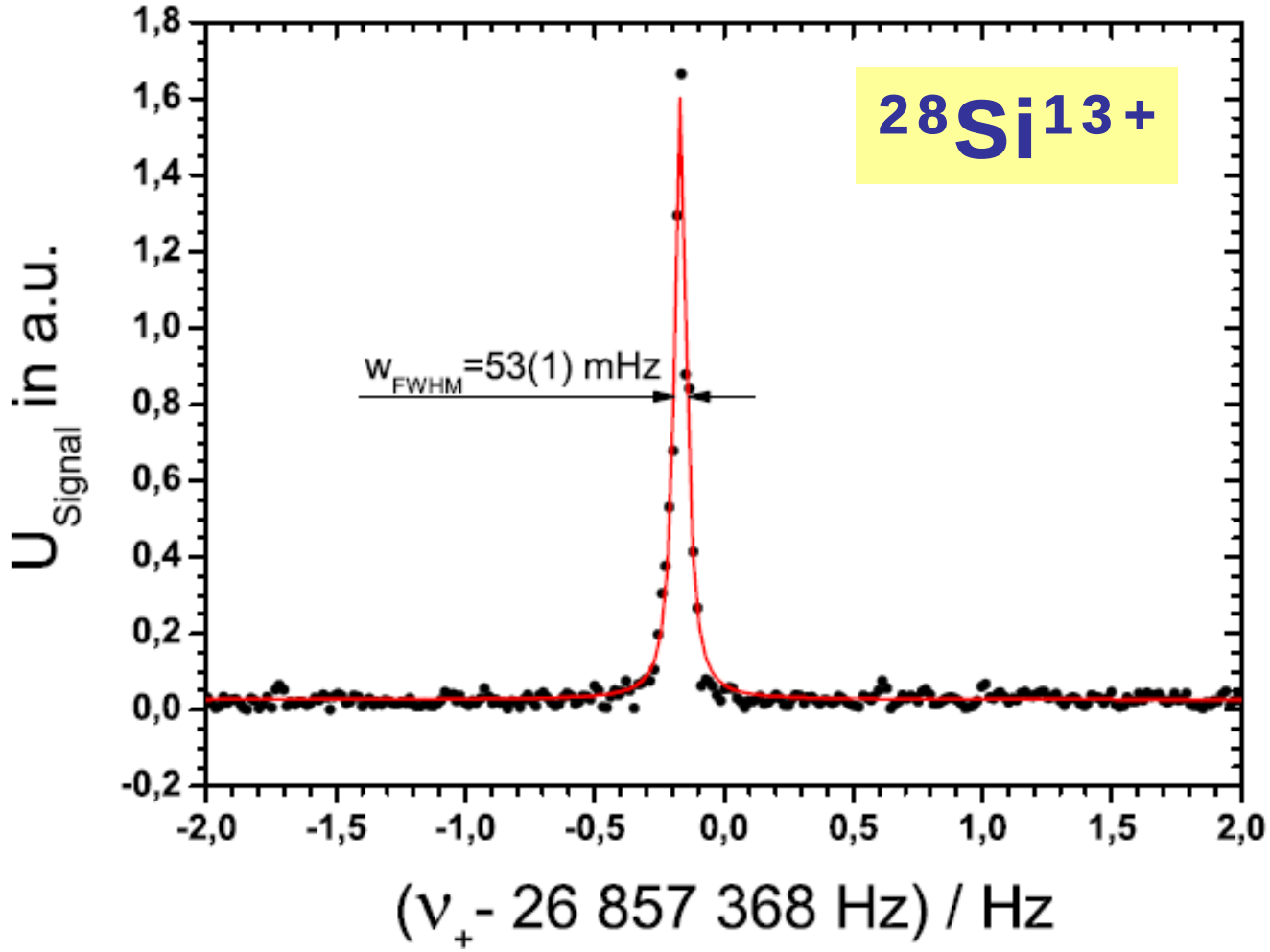


- ✓ Superconducting resonance circuit
- ✓ Ultra-low noise cryogenic amplifiers

Superconducting helical resonator
R_p = 50 MΩ
Q = 3200

e_n = 400 pV/√Hz
i_n ≤ 10 fA/ √Hz

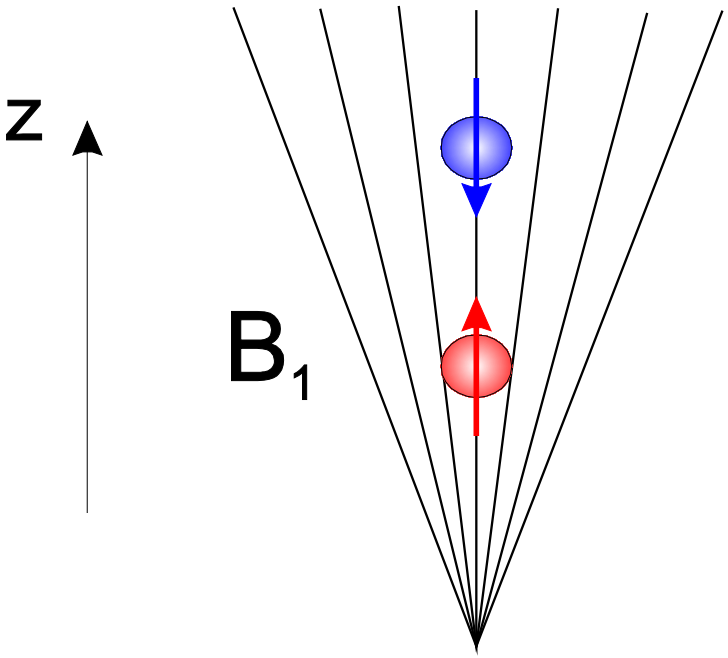
High-resolution cyclotron frequency measurement of a single highly charged silicon ion



Continuous Stern-Gerlach effect: Determination of spin direction

CLASSICAL STERN-GERLACH

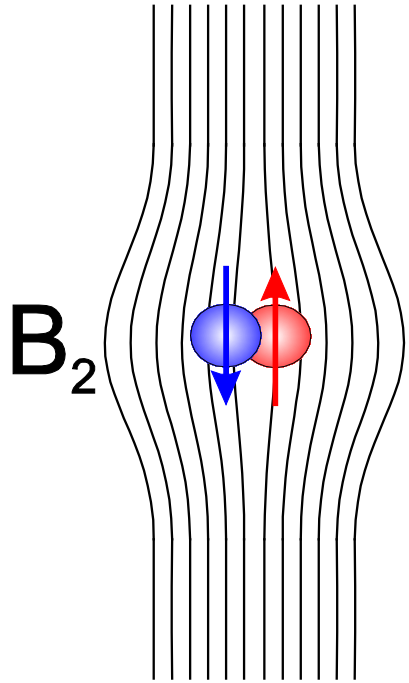
SEPARATION IN POSITION SPACE



$$\Delta z = \frac{\mu L^2}{2KE} B_1$$

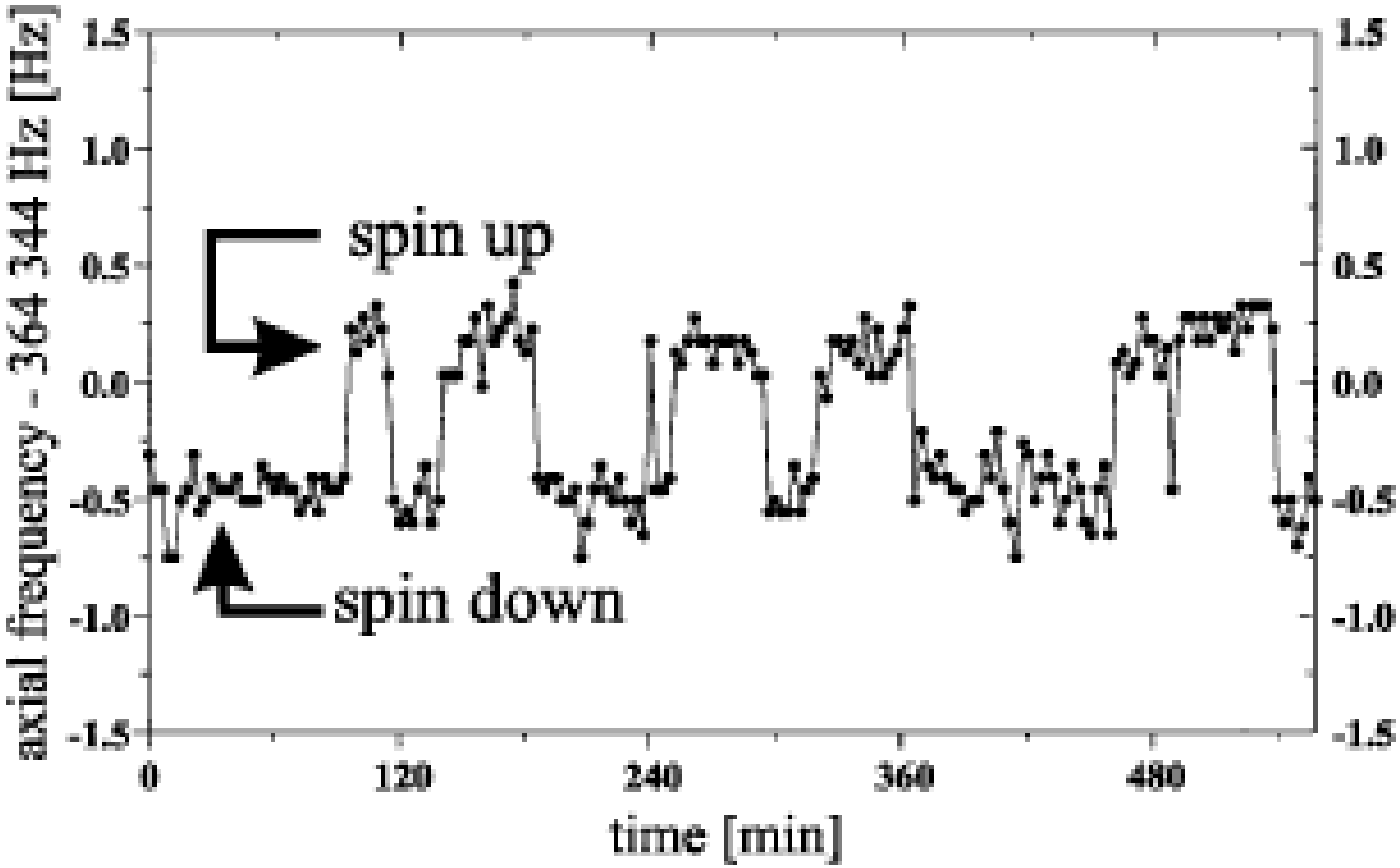
CONTINUOUS STERN-GERLACH

SEPARATION IN FREQUENCY SPACE

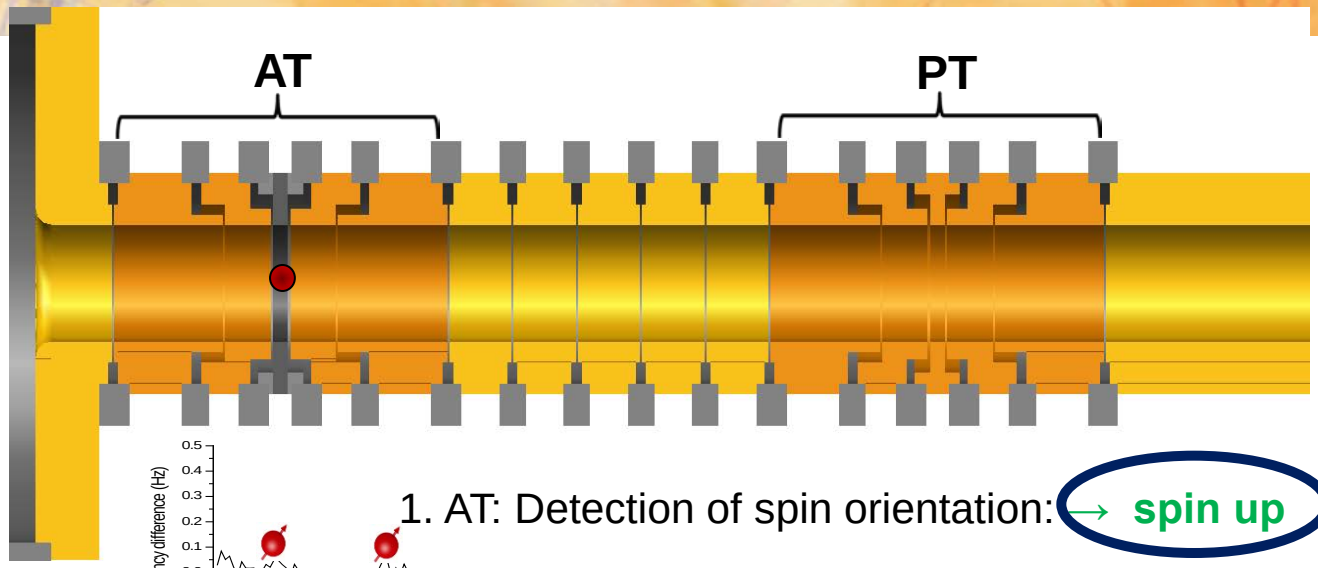


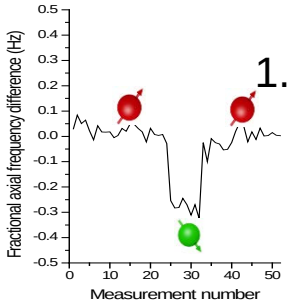
$$\Delta\omega_z = \frac{\mu}{m\omega_z} B_2$$

Quantum jump spectroscopy: Spin-flip transitions in the analysis trap

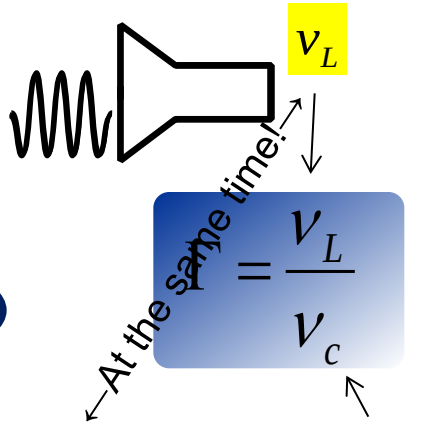


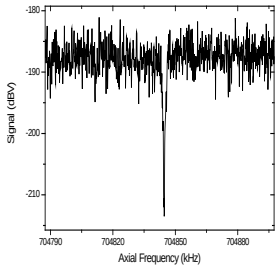
Measurement Cycle

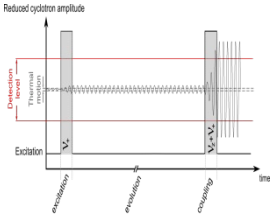




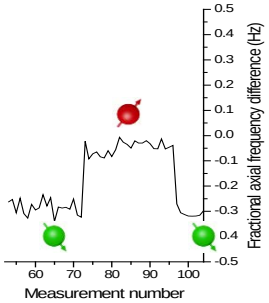
1. AT: Detection of spin orientation: → **spin up**







2. PT: Measurement of eigenfrequencies and simultaneous irradiation with microwaves



3. AT: Detection of spin orientation → **spin down**

4. Spin flip in PT? → **spin has flipped!**

Bound electron magnetic moment measurement on hydrogen-like silicon $^{28}Si^{13+}$

PRL 107, 023002 (2011) PHYSICAL REVIEW LETTERS week ending 8 JULY 2011

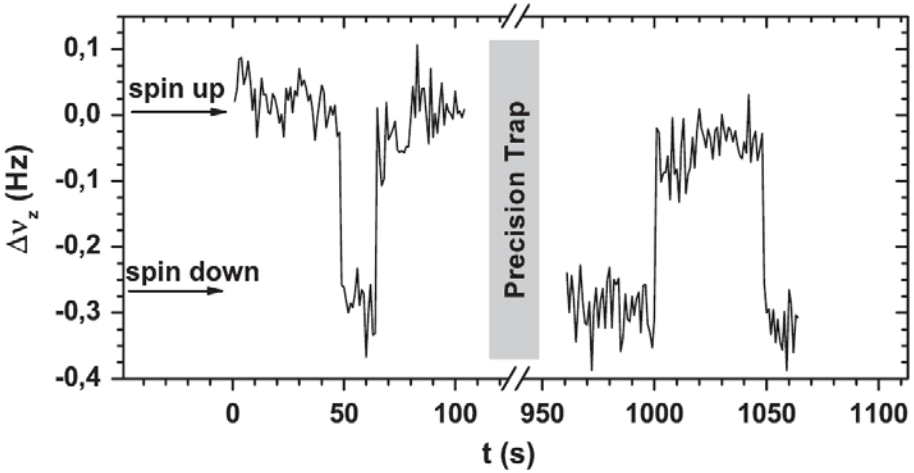
g Factor of Hydrogenlike $^{28}Si^{13+}$

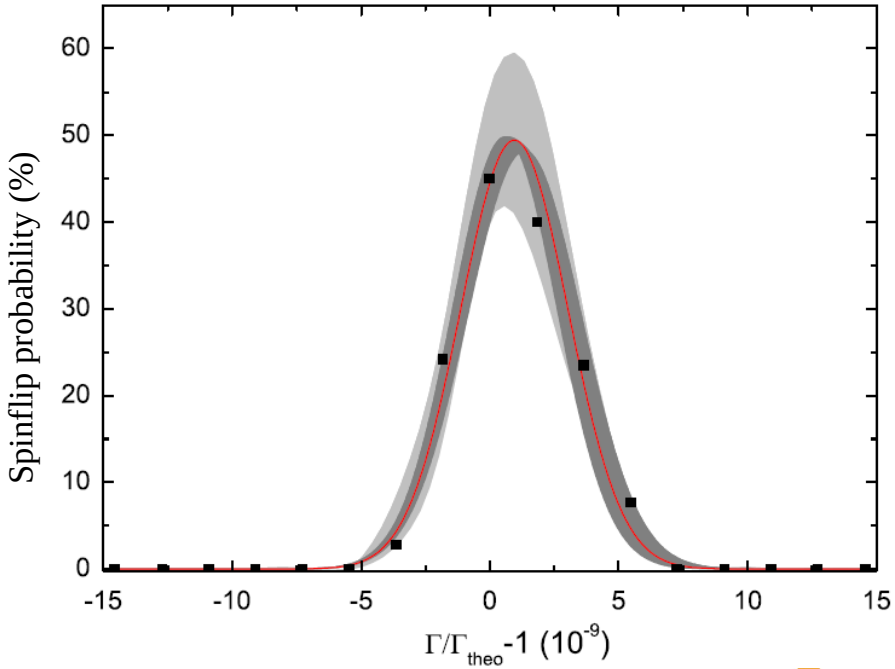
S. Sturm,[1,2] A. Wagner,[1] B. Schabinger,[1,2] J. Zatorski,[1] Z. Harman,[1,3] W. Quint,[4] G. Werth,[2] C. H. Keitel,[1] and K. Blaum[1]

[1]*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*
[2]*Institut für Physik, Johannes Gutenberg-Universität, 55099 Mainz, Germany*
[3]*ExtreMe Matter Institute EMMI, Planckstraße 1, 64291 Darmstadt, Germany*
[4]*GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany*

(Received 6 May 2011; published 7 July 2011)

We determined the experimental value of the g factor of the electron bound in hydrogenlike $^{28}Si^{13+}$ by using a single ion confined in a cylindrical Penning trap. From the ratio of the ion's cyclotron frequency and the induced spin flip frequency, we obtain $g = 1.995\,348\,958\,7(5)(3)(8)$. It is in excellent agreement with the state-of-the-art theoretical value of $1.995\,348\,958\,0(17)$, which includes QED contributions up to the two-loop level of the order of $(Z\alpha)^2$ and $(Z\alpha)^4$ and represents a stringent test of bound-state quantum electrodynamics calculations.



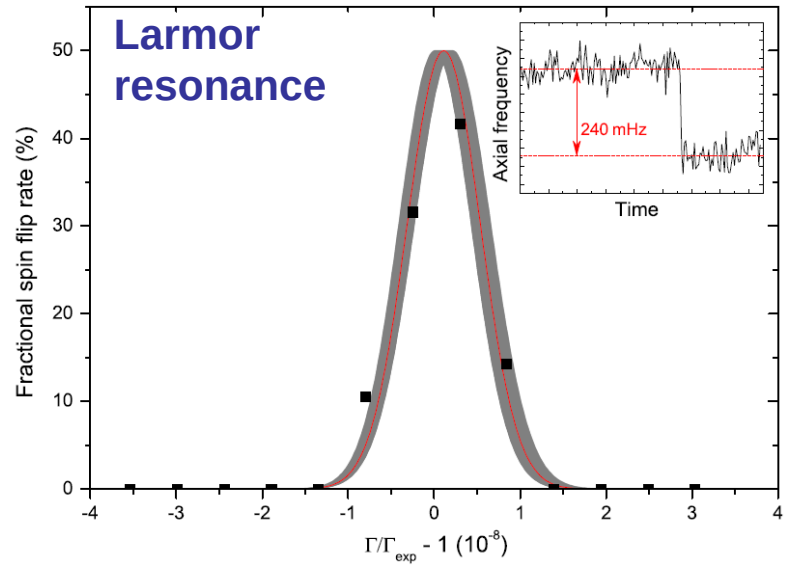


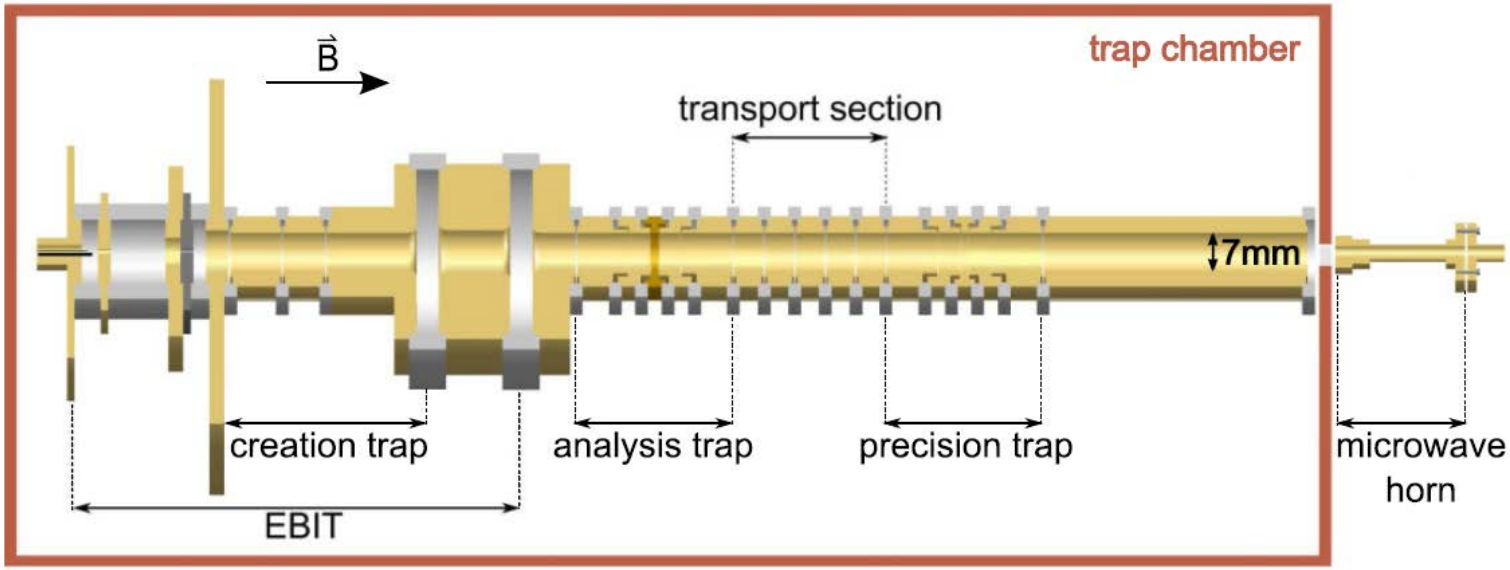
Bound electron magnetic moment measurement on lithium-like silicon $^{28}Si^{11+}$

$g_{exp}(^{28}Si^{11+})$ = 2.000 889 889 9(21)
$g_{theo}(^{28}Si^{11+})$ = 2.000 889 909 (51)

theoretical calculations by D.A. Glazov,
A.V. Volotka, V.M. Shabaev

Precision test of
- electron-electron interaction
- screened QED contributions



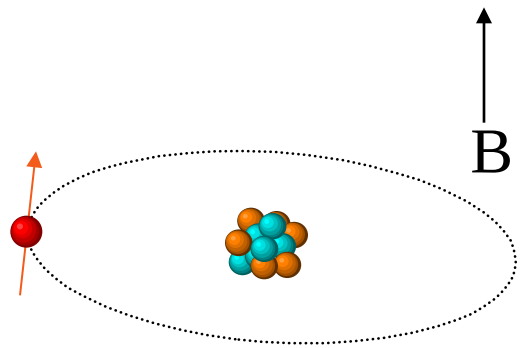


Ref.:
A. Wagner et al.
PRL 110, 033003 (2013)

Electron mass

Larmor precession frequency of the bound electron:

$$\omega_L^{\,e} = \frac{g_J}{2}\frac{e}{m_e}B$$



Ion cyclotron frequency:

$$\omega_c^{\,ion} = \frac{Q}{M_{ion}}B$$

$$\frac{m_e}{M_{ion}} = \frac{g_J}{2}\cdot\frac{\omega_c^{\,ion}}{\omega_L^{\,e}}\cdot\frac{e}{Q}$$

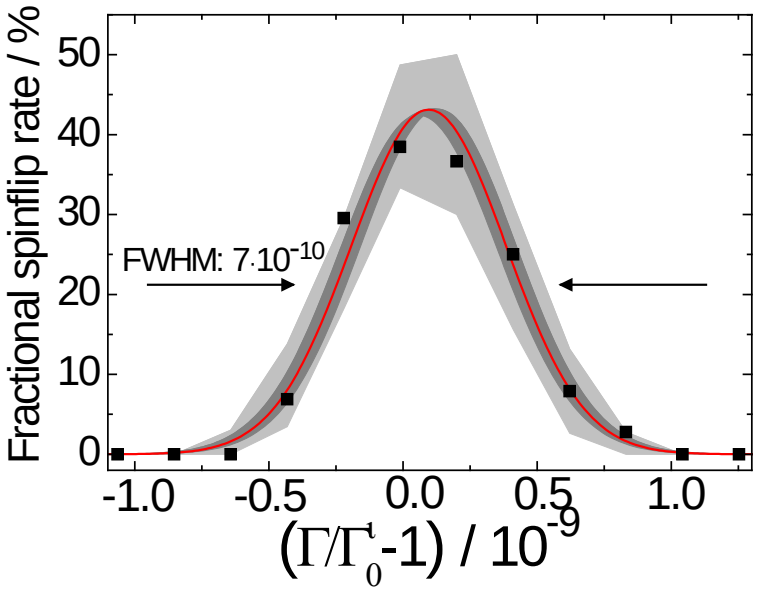
→ determination of electron mass

theory as input parameter

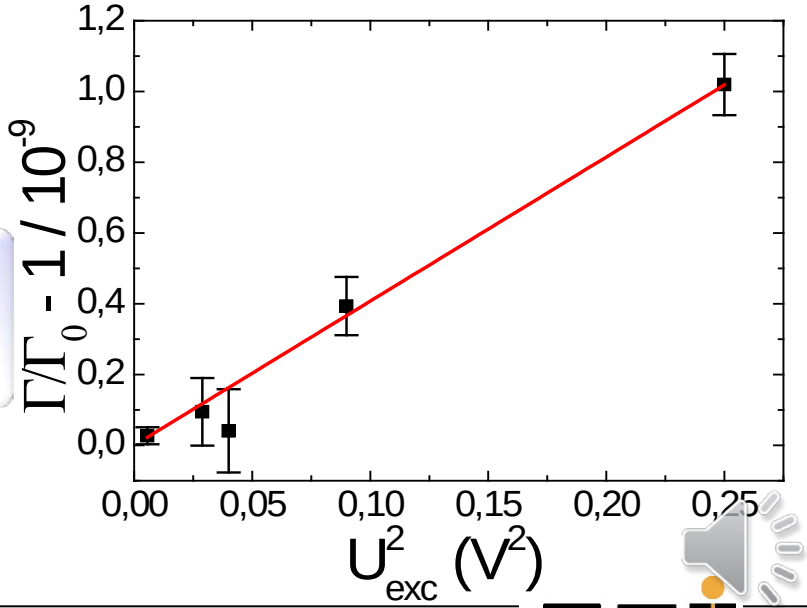
our measure-ment

Experimental Result

- Resonance width and thus statistical error limited by magnetic field fluctuations



- Several Γ-resonances at different cyclotron energies to check systematics → extrapolation to zero energy



(stat.) (syst.)

$$\Gamma_0 = 4\,736.\,210\,500\,89\,(11)(7)$$

- Dominant systematics:
  - image charge shift: -282 (14) ppt

New electron mass

Experiment

Γ_0 = 4 736. 210 500 89 (11)(7)
(stat.) (syst.)

Theory

g_{theo} = 2. 001 041 590 176 (6)

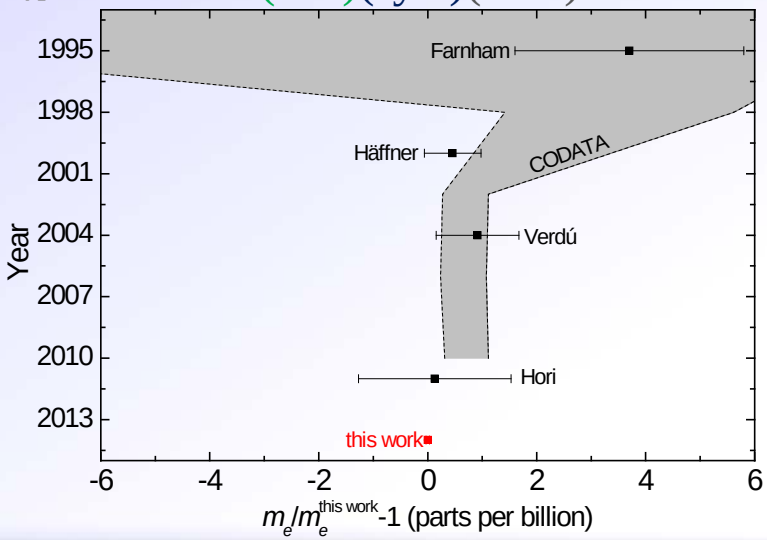
$$m_e = m_{ion} \frac{g_{theo}}{2} \frac{e}{q_{ion}} \frac{1}{\Gamma_0}$$

m_e=0,000 548 579 909 067 (14)(9)(2) u

[S. Sturm et al., Nature 506, 467-470 (2014)]

(stat)(syst)(theo)

$$\delta m_e/m_e = 3 \cdot 10^{-11}$$



Result

Experiment | Theory

Γ_0 = 4 736 ... 176 (6)

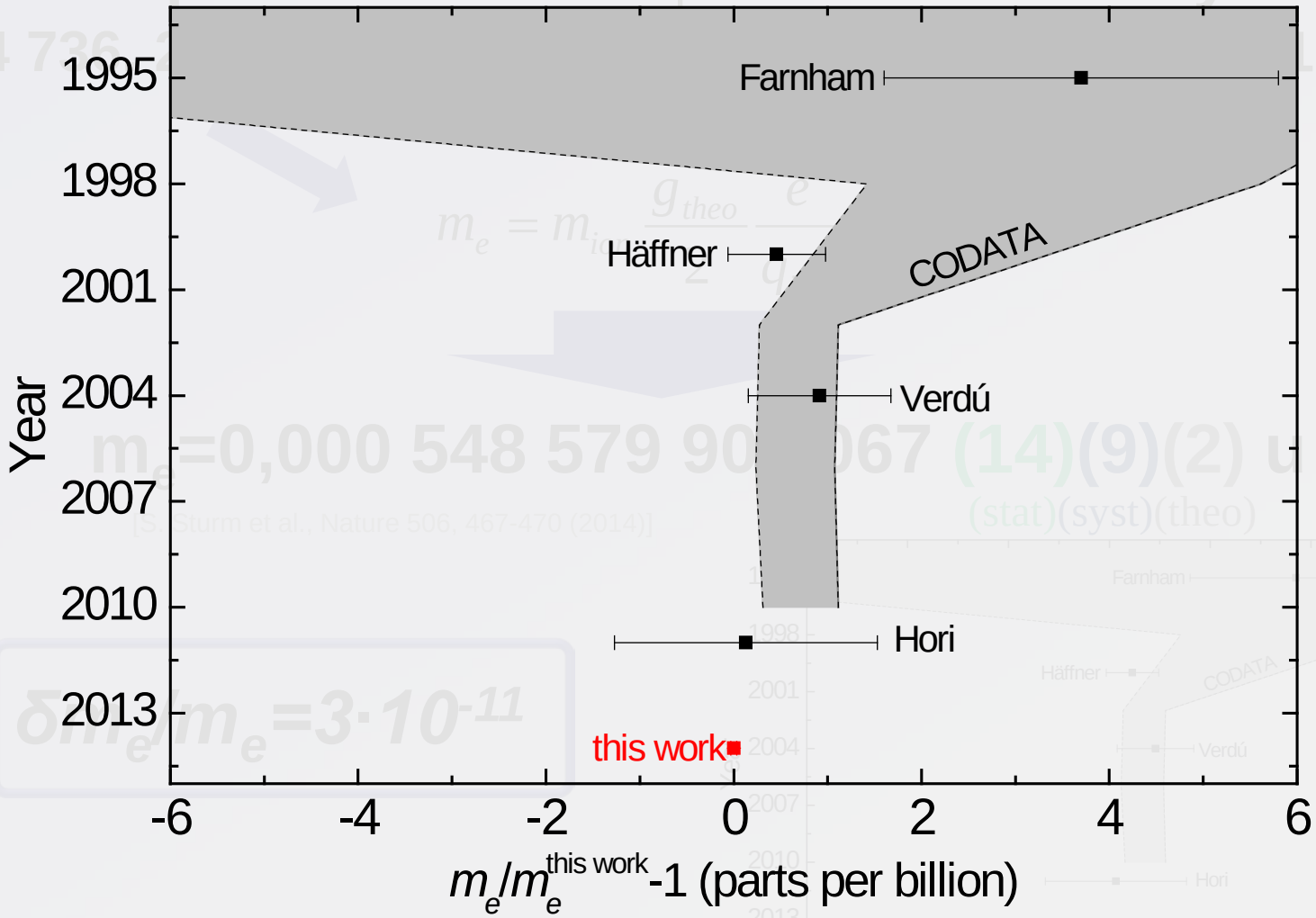
$m_e = m_{ion} \frac{g_{theo}}{2} \frac{e}{q}$

m_e=0,000 548 579 90... 067 (14)(9)(2) u

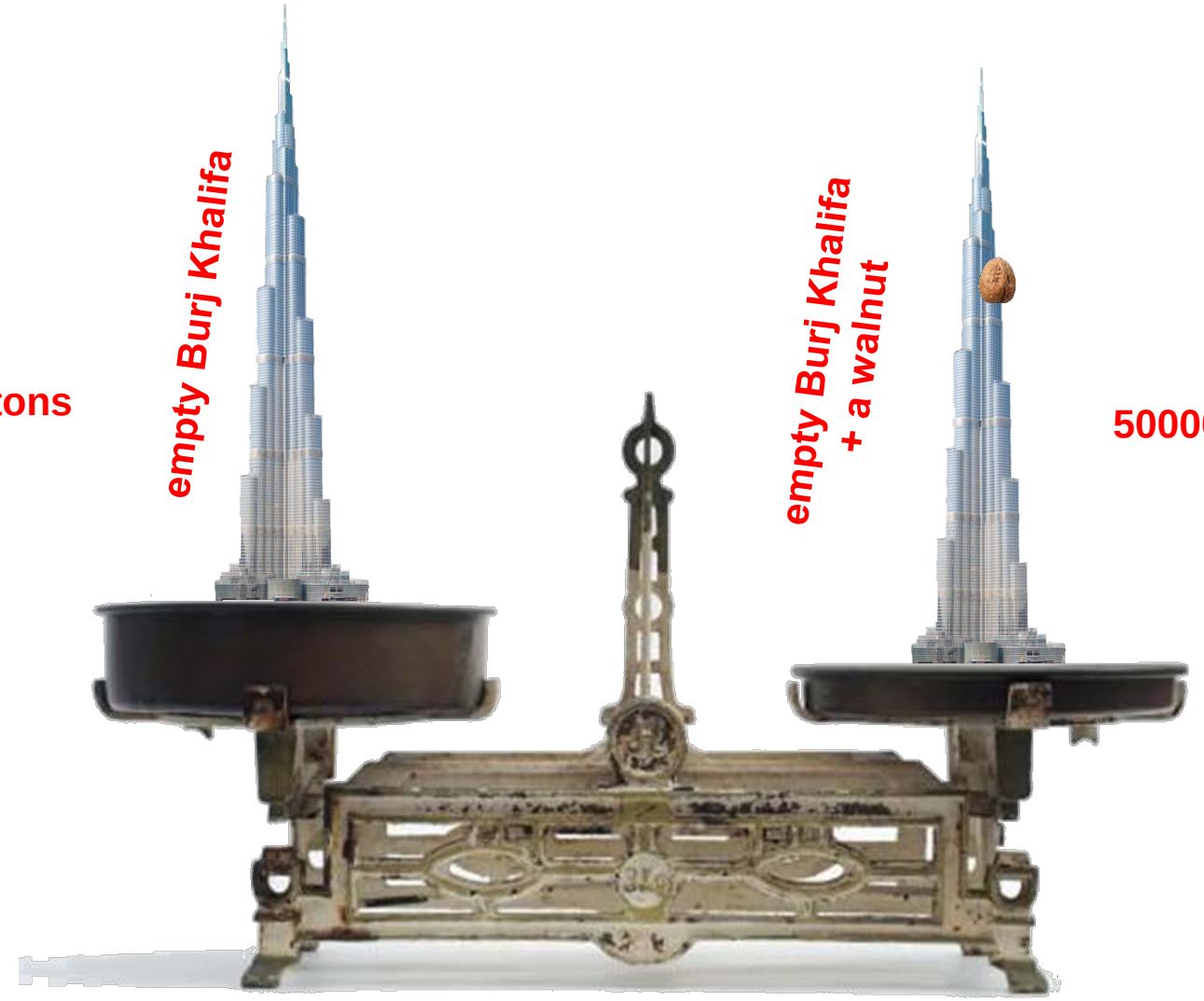
(stat)(syst)(theo)

[S. Sturm et al., Nature 506, 467-470 (2014)]

$\delta m_e/m_e = 3 \cdot 10^{-11}$



Who has forgotten the walnut?



Relative Precision: $3 \cdot 10^{-11}$

Profit of an improved electron mass m_e

- Important ingredient in determination of fine-structure constant α:

$$\alpha \equiv \frac{e^2}{2\varepsilon_0 hc}$$

5 ppt, CODATA (T. Hänsch) 2010

115 ppt, E. Myers 2010

$$\alpha_{recoil}^2 = \frac{2R_\infty h}{cm_e} = \frac{2R_\infty}{c}\frac{M_{Rb}}{m_e}\frac{h}{M_{Rb}}$$

1241 ppt, F. Biraben 2011

$$v_{recoil} = \frac{\hbar k}{M_{Rb}}$$

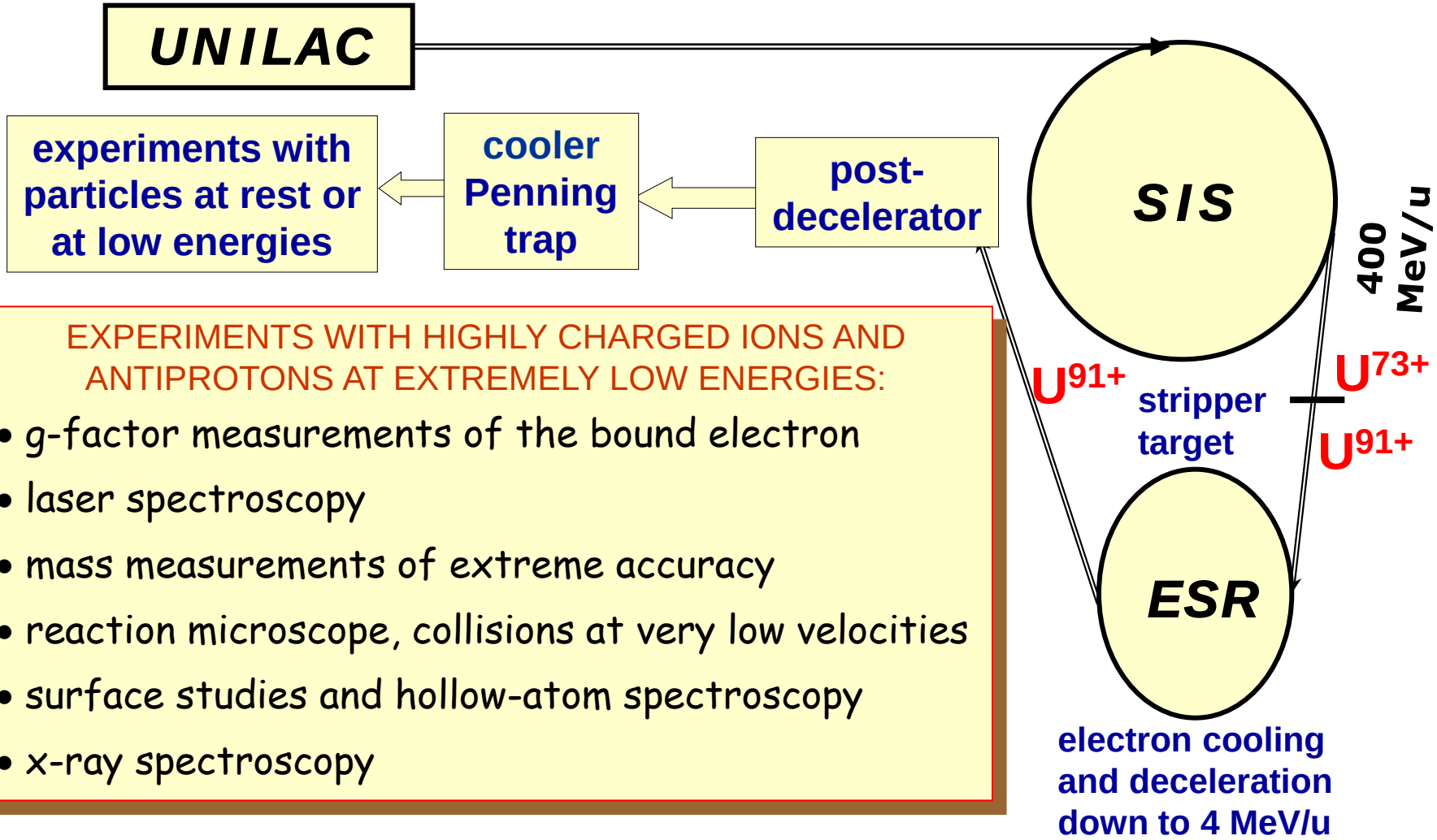
30 ppt, our value

Hint for physics beyond SM: **2.5 σ discrepancy at muon *g*-2 (0.54 ppm)**

- enhanced sensitivity to „new physics“ due to masses: $(m_\mu/m_e)^2=40000$;
- with a precision of 37 ppt for α you could check this effect with the electron:
  - α from the free electron *g*-factor and theory has to improve by a factor of 8
  - α_{recoil} has to improve by a factor of 20

→ precision of m_e (30ppt) now sufficient

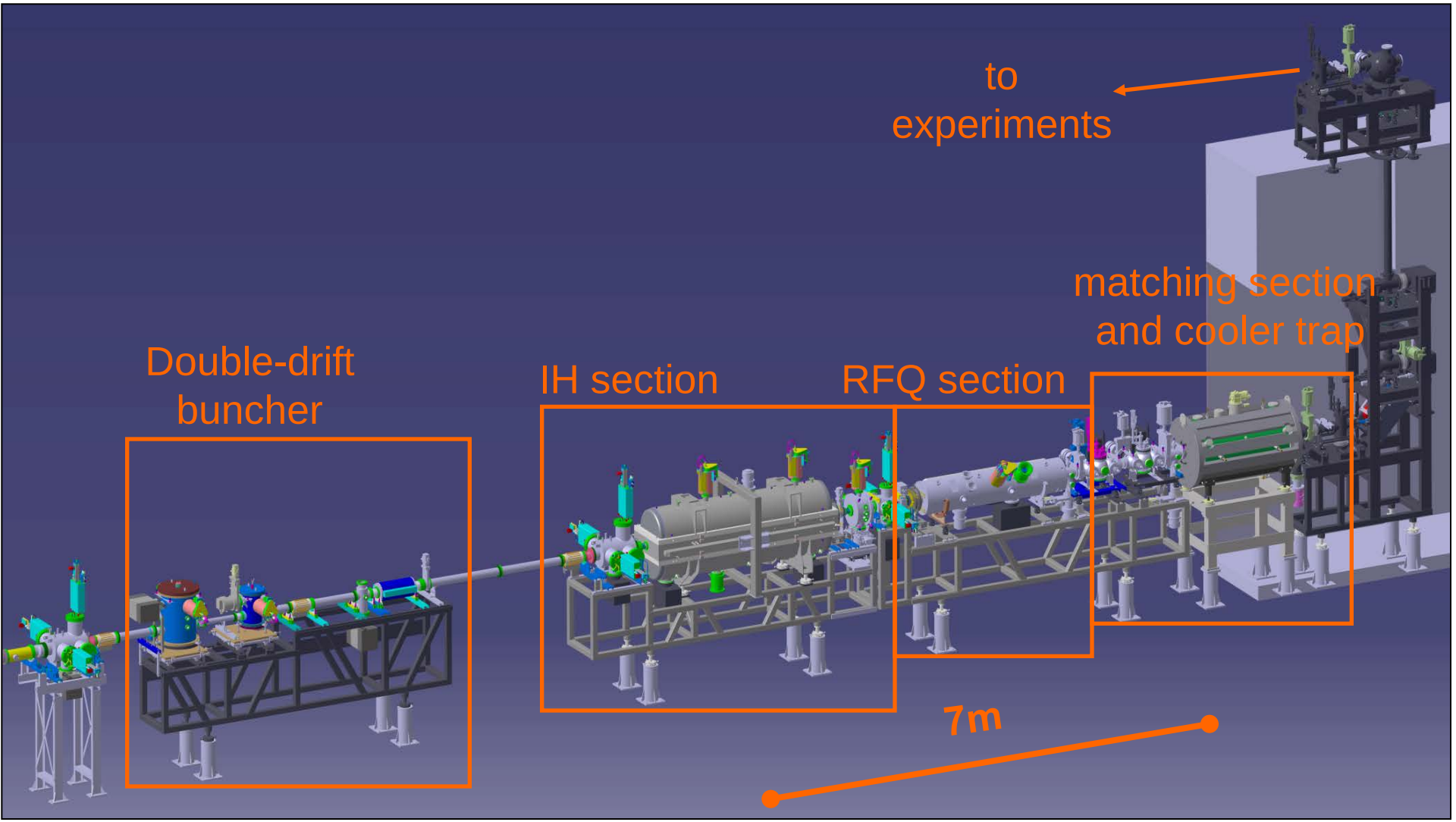
HITRAP at the Experimental Storage Ring ESR



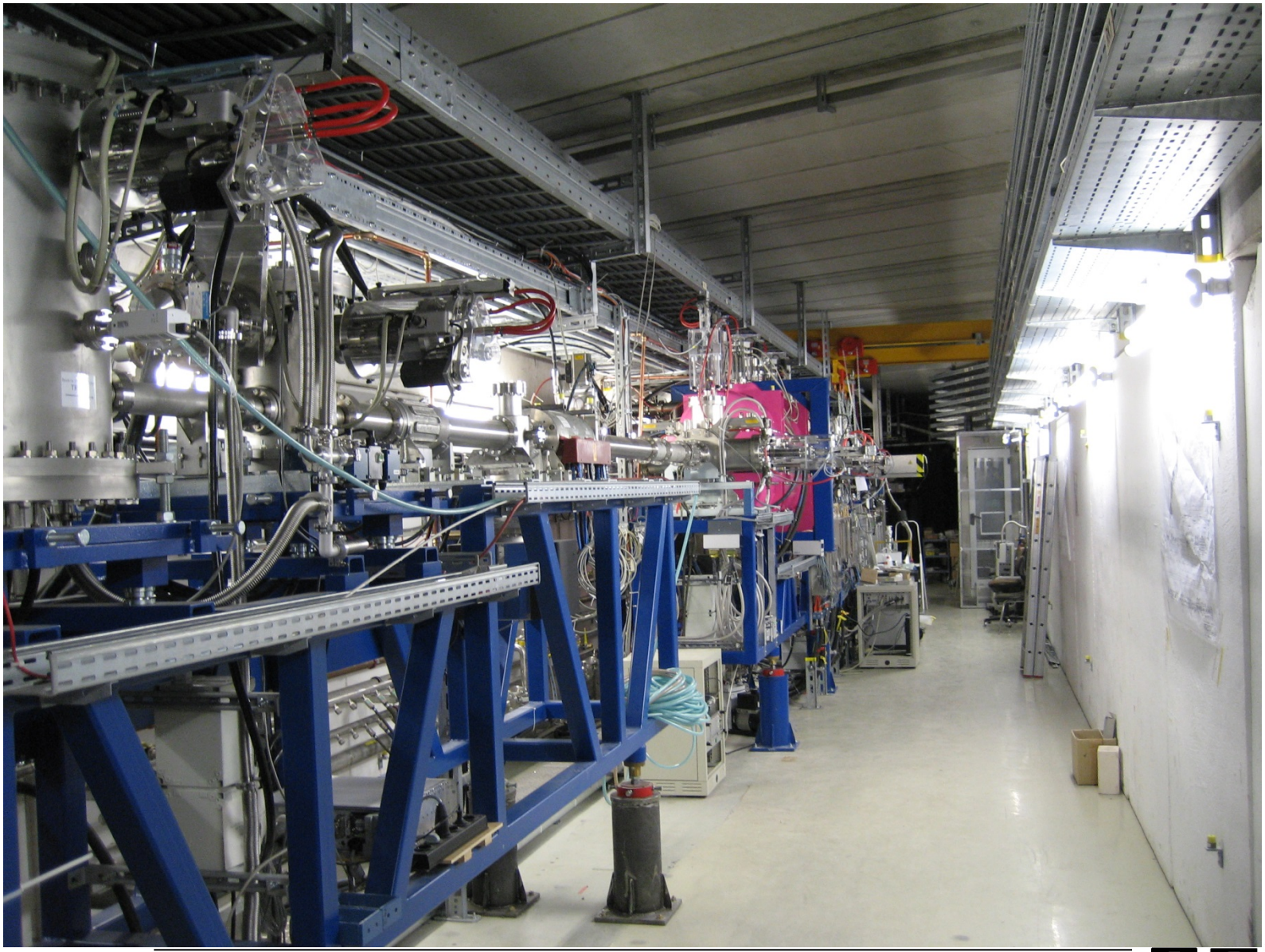
EXPERIMENTS WITH HIGHLY CHARGED IONS AND ANTIPROTONS AT EXTREMELY LOW ENERGIES:

- g-factor measurements of the bound electron
- laser spectroscopy
- mass measurements of extreme accuracy
- reaction microscope, collisions at very low velocities
- surface studies and hollow-atom spectroscopy
- x-ray spectroscopy

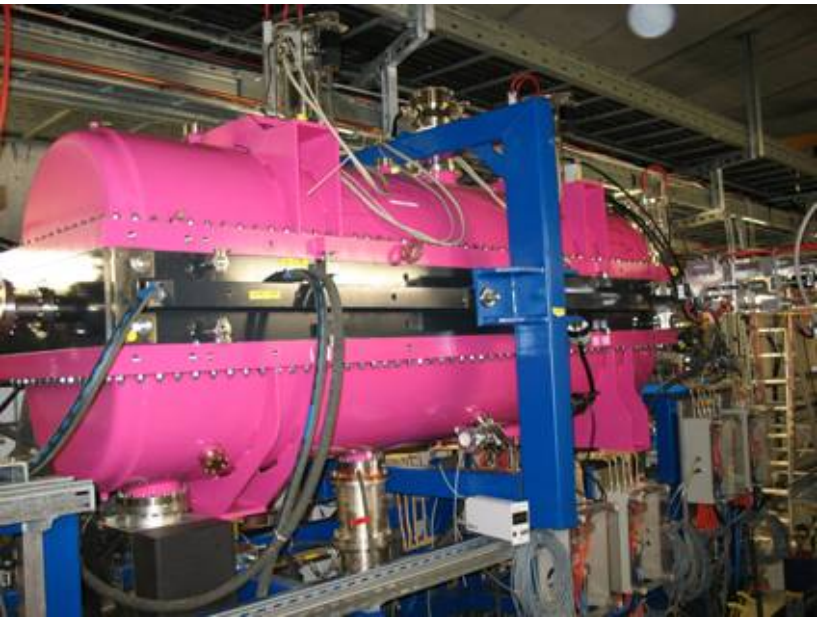
HITRAP: Technical design



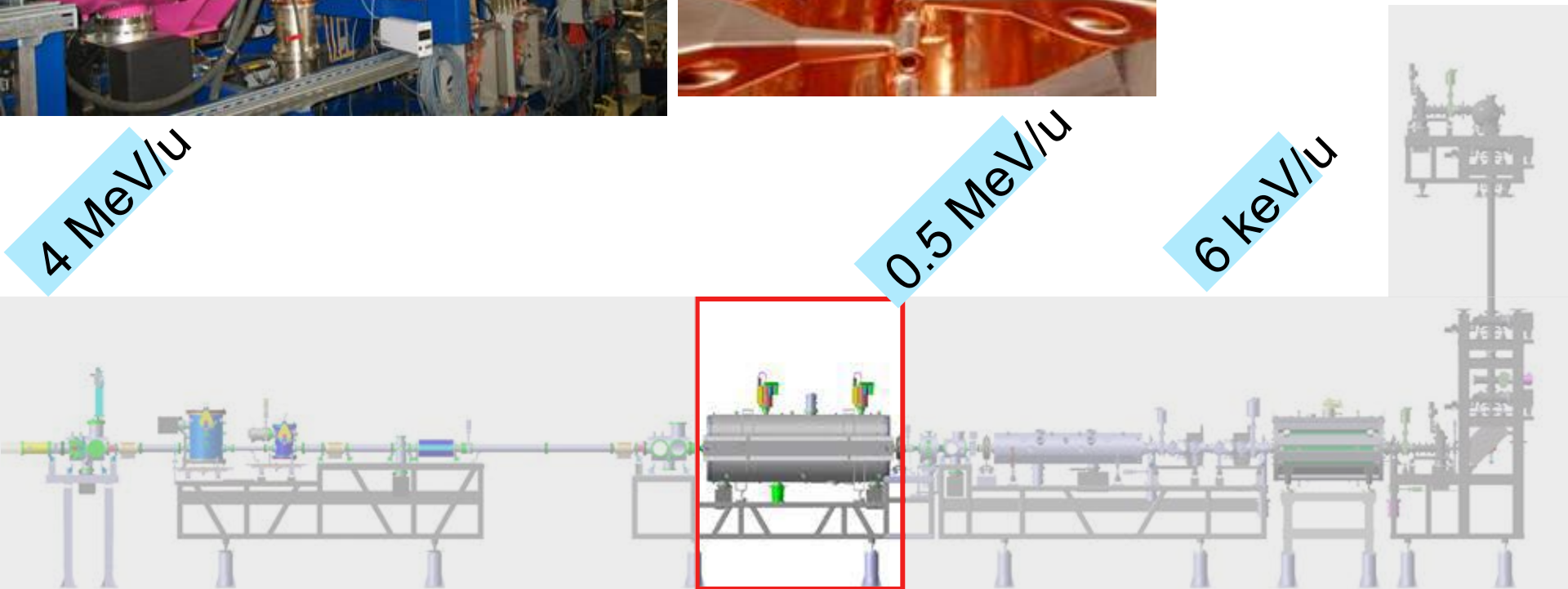
HITRAP facility at ESR



HITRAP: IH structure

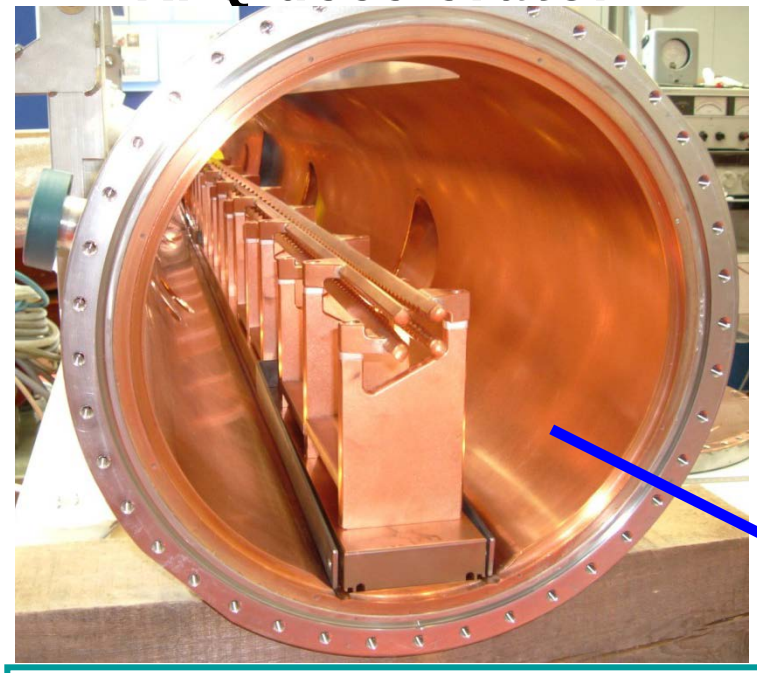


deceleration from
4 MeV/u to
0.5 MeV/u

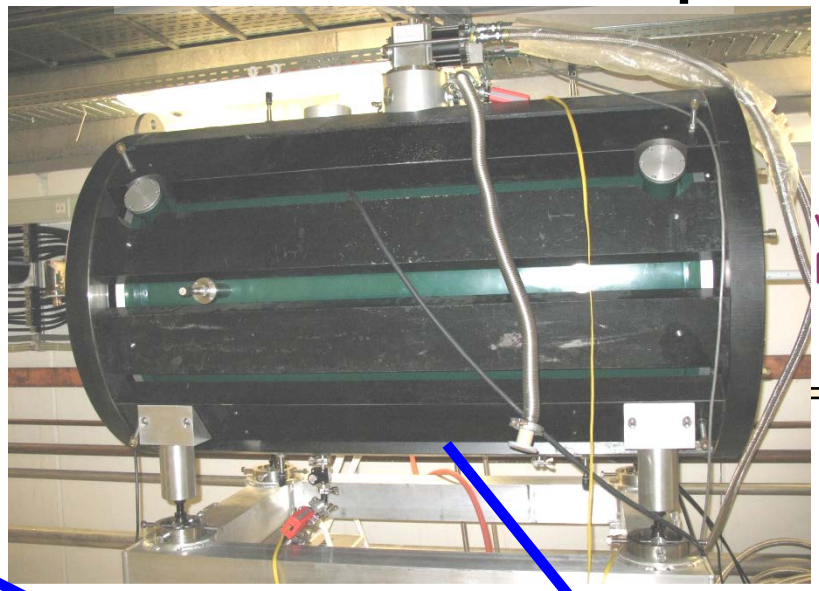


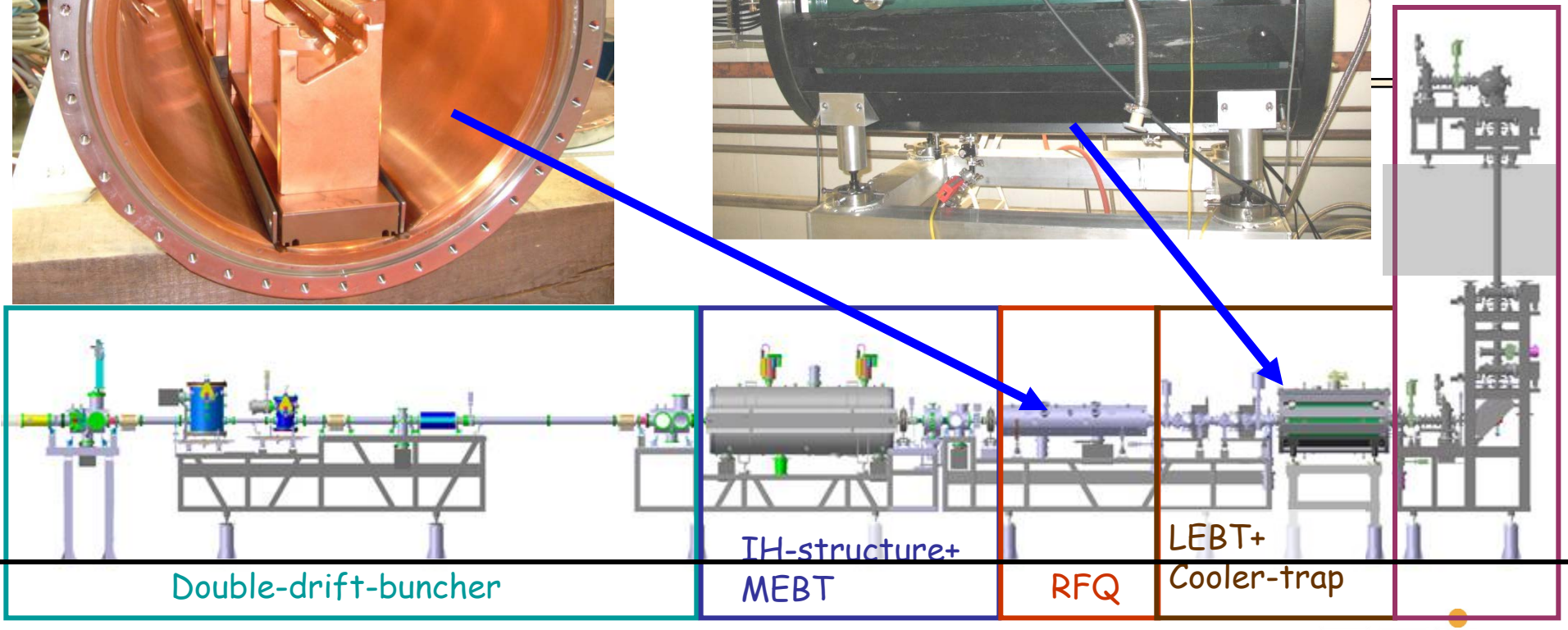
Commissioning of RFQ decelerator and of cooler trap in progress

RFQ decelerator



HITRAP cooler trap

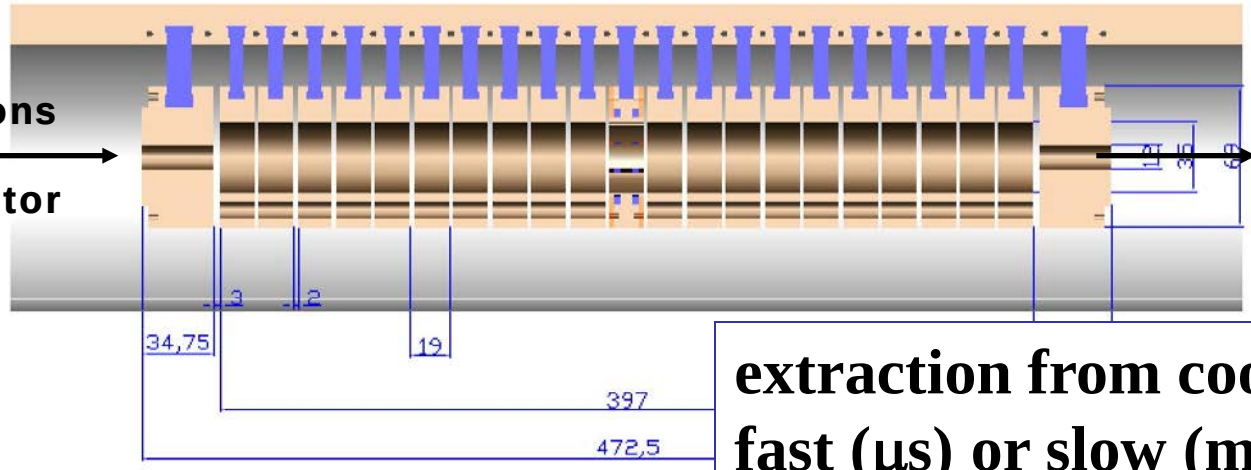




HITRAP Cooler Penning trap

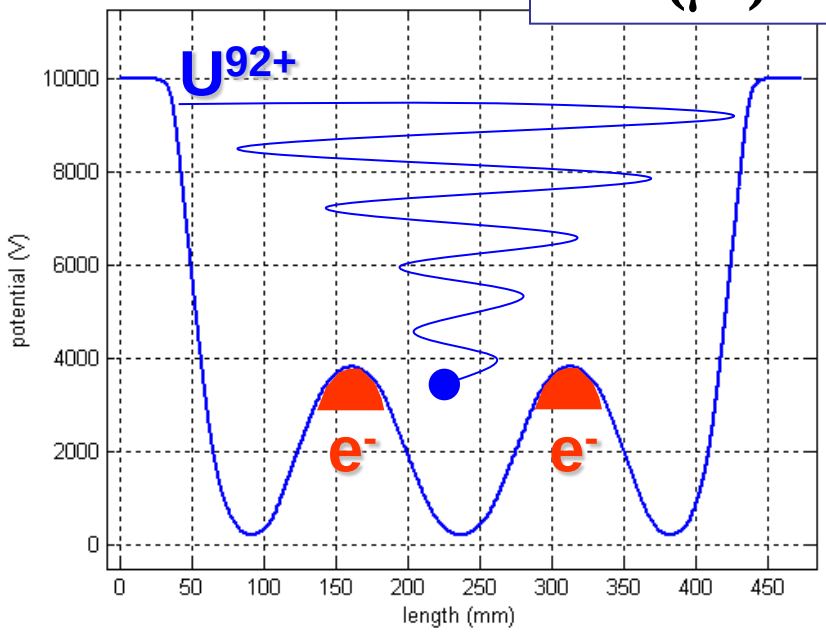
- electron cooling
- resistive cooling to T = 4 K

highly charged ions from HITRAP decelerator



cooled HCI to HITRAP experiments

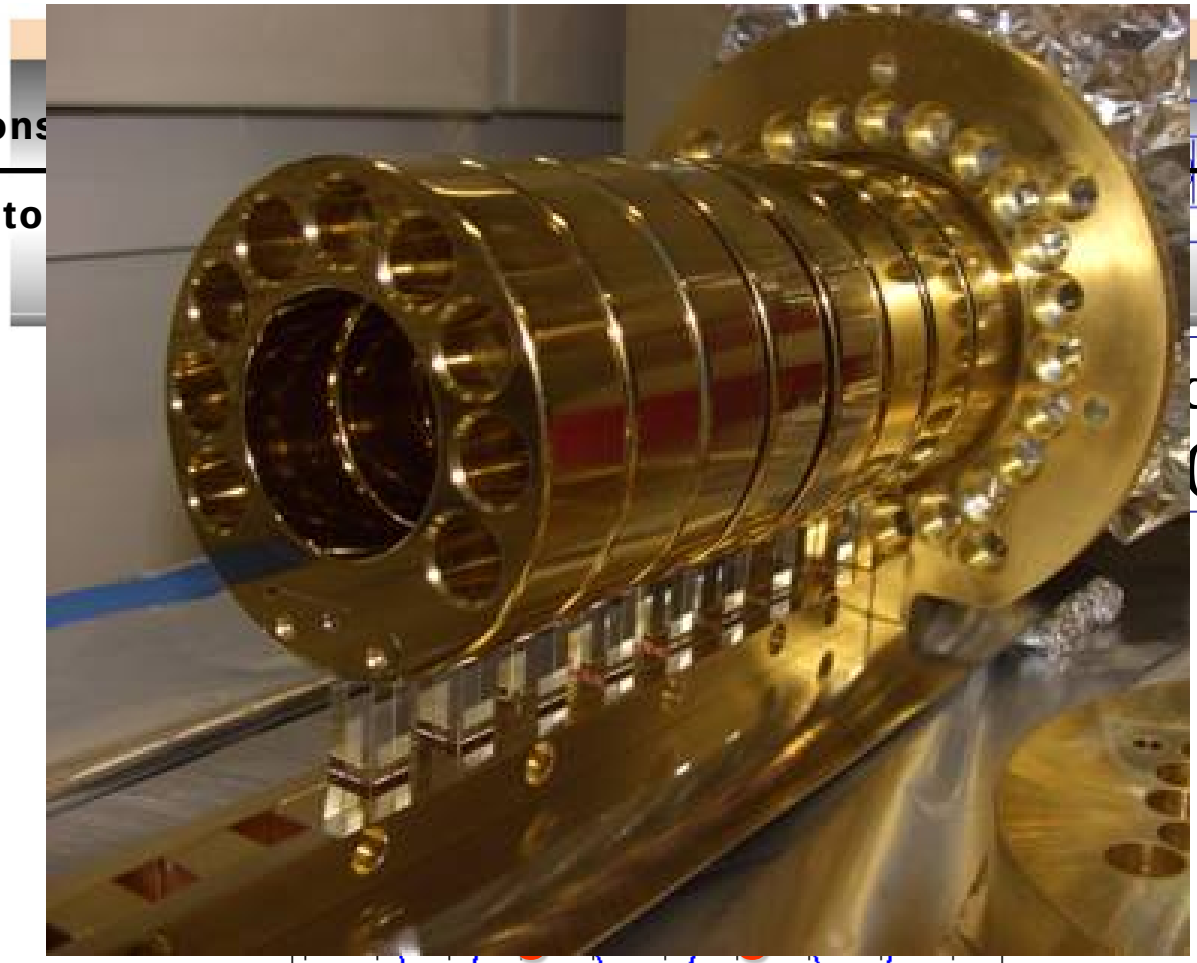
extraction from cooler trap: fast (µs) or slow (ms...sec)



HITRAP Cooler Penning trap

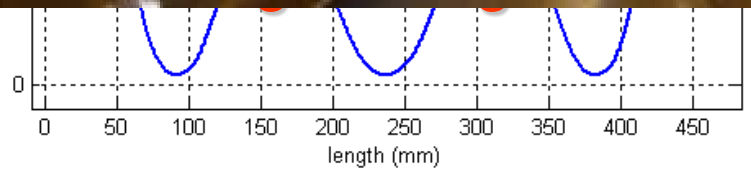
- electron cooling
- resistive cooling to T = 4 K

highly charged ions from HITRAP decelerato



cooled HCI to HITRAP experiments

cooler trap: (ms...sec)



LETTER

doi:10.1038/nature13388

Direct high-precision measurement of the magnetic moment of the proton

A. Mooser[1,2]†, S. Ulmer[3], K. Blaum[4], K. Franke[3,4], H. Kracke[1,2], C. Leiteritz[1], W. Quint[5,6], C. C. Rodegheri[1,4], C. Smorra[3] & J. Walz[1,2]

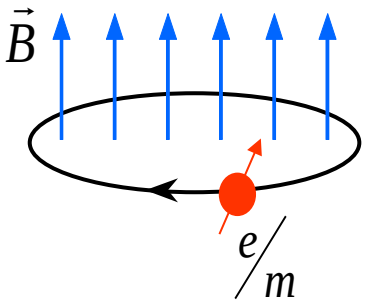
$$\omega_c = \frac{e}{m_p} B$$

Cyclotron frequency

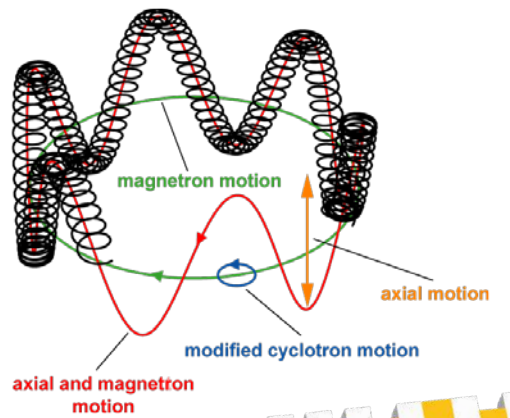
$$g = 2\frac{\omega_L}{\omega_c}$$

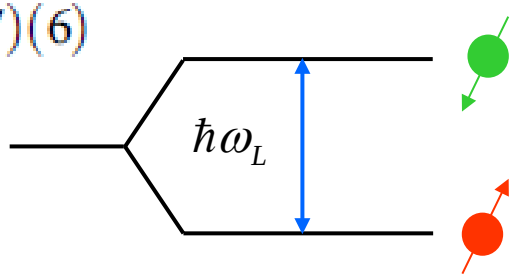
$$\omega_L = g\frac{e}{2m_p} B$$

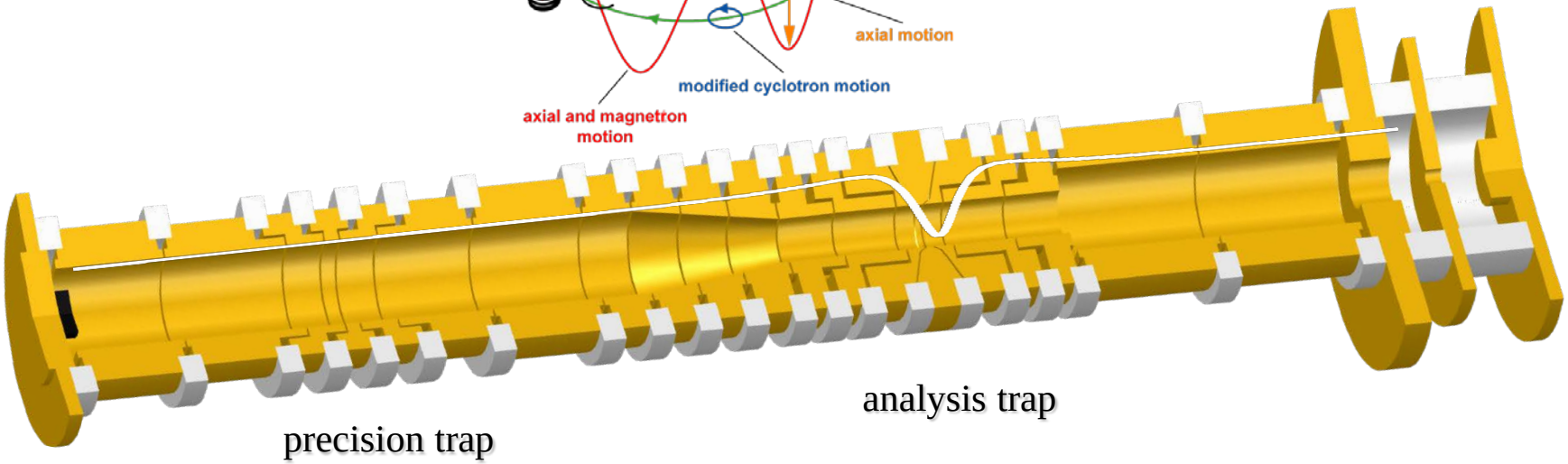
Larmor frequency



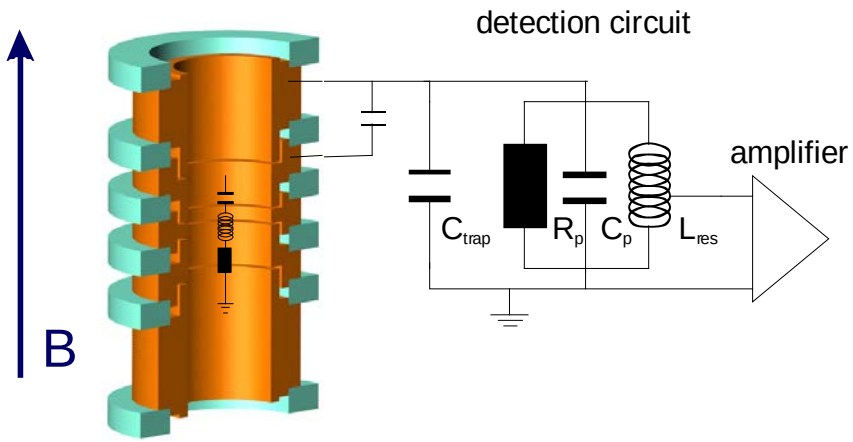
$$\frac{\mu_p}{\mu_N} = \frac{g_p}{2} = 2.792\,847\,350(7)(6)$$



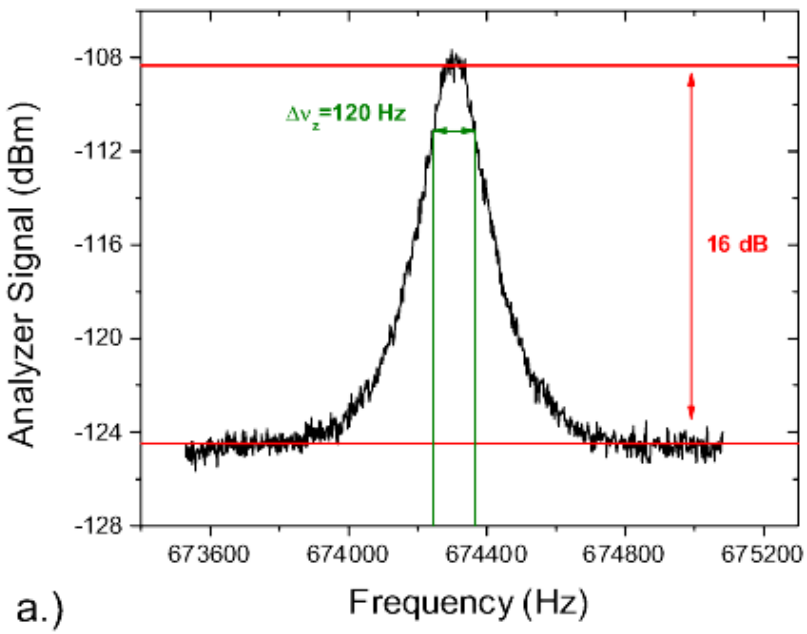


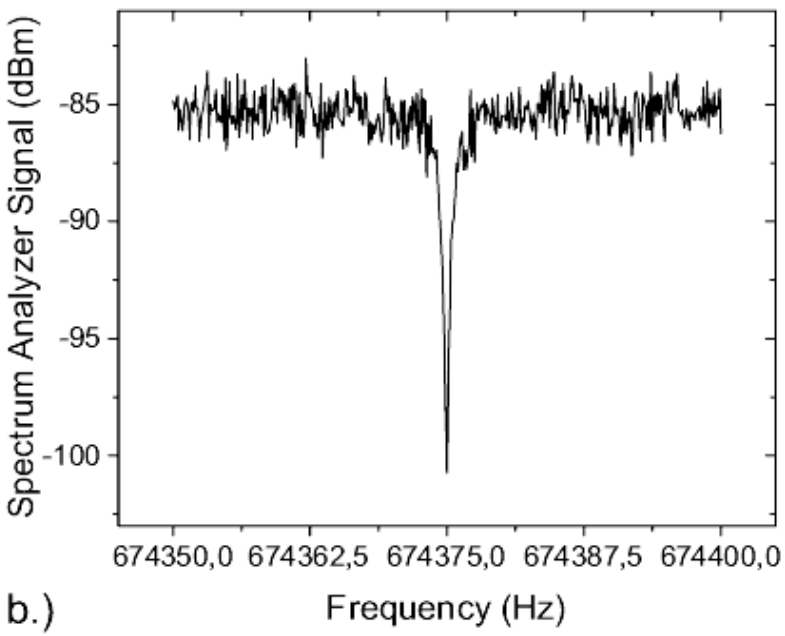


Measurement of the Axial Frequency of a Single Trapped Proton



- proton in thermal equilibrium at cryogenic temperature
- proton = series LC-circuit
- proton shorts detector noise at ν_z
- minimum in FFT spectrum



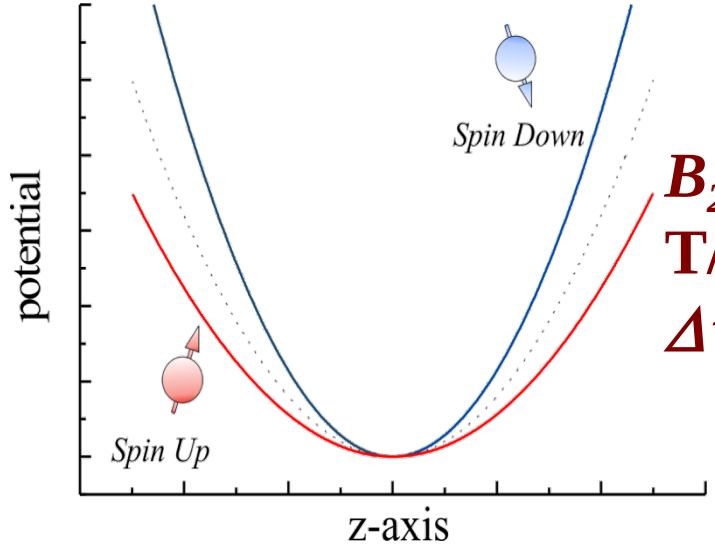


A single trapped proton and the continuous Stern-Gerlach effect

axial frequency shift due to spinflip:

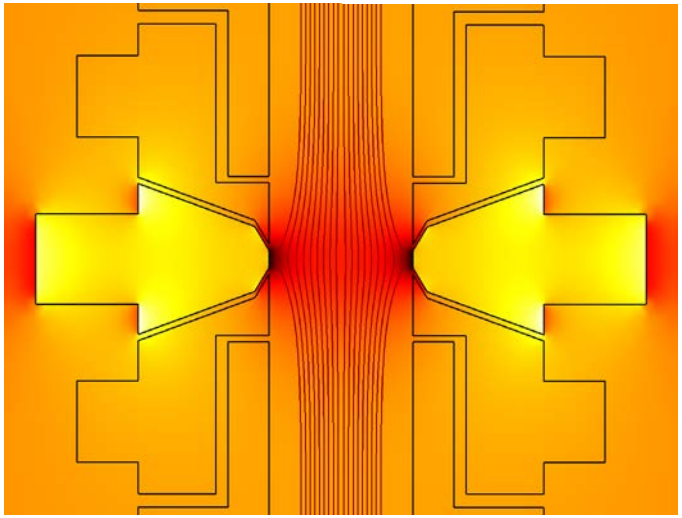
$$\Delta\nu_z \approx \frac{1}{2\pi^2}\frac{\mu_z B_2}{m\nu_z}$$

Proton measurement is 10 000 times harder compared to electron g-2 measurement.

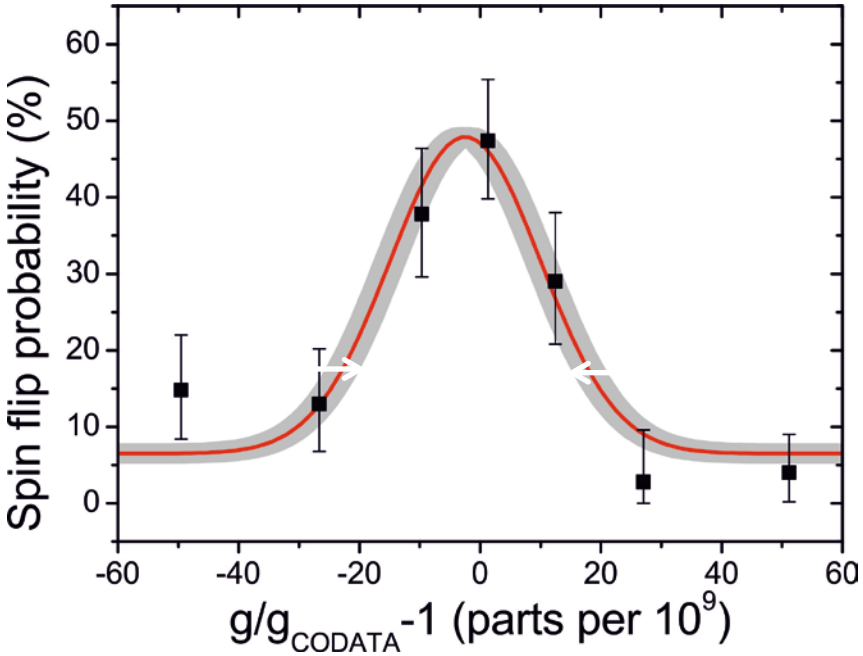


B_2 = 0.3 T/mm^2

$\Delta\nu_z$= 190 mHz



The g -factor of the proton



g/2 = 2.792 847 350 (7) (6)

- **First direct high-precision measurement of the proton magnetic moment.**
- **Improves 42 year old Maser measurement by factor of 2.5 (D. Kleppner, MIT)**
- **Value in agreement with accepted CODATA value, but 2.5 times more precise**

The *g*-factor of the proton

Systematic errors

Parameter	Relative Shift of $g_p/2$	Uncertainty
Trapping Potential	0	0.2 ppb
Relativistic Shift	0.030 ppb	<0.003 ppb
Image-Charge Shift	-0.088 ppb	<0.010 ppb
Cyclotron Cooling	-0.51 ppb	0.08 ppb
Nonlinear Magnetic Field Drift	0	2 ppb
Voltage Stability	-0.07 ppb	0.35 ppb
Total Systematic Shift	**-0.64 ppb**	**2 ppb**

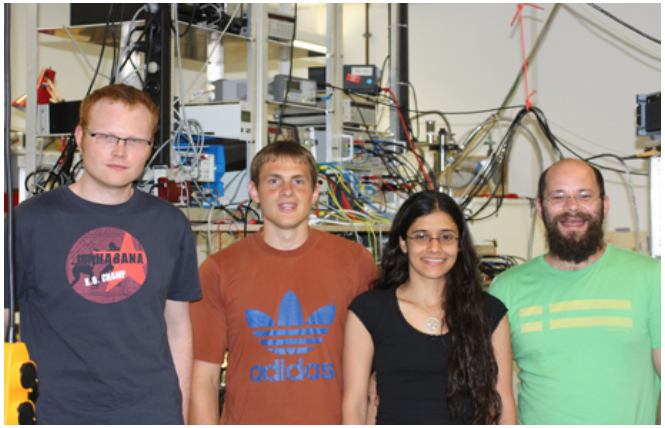
Highly charged ions: special thanks to

Experiment: Jiamin Hou, Florian Köhler, Sven Sturm,
Anke Wagner, Günter Werth, Klaus Blaum
Theory: Jacek Zatorski, Zoltán Harman, Christoph Keitel

- ***Atomic Physics Division at GSI Helmholtzzentrum, Darmstadt***
- ***MATS group within QUANTUM at the Institut für Physik, Mainz***
- ***MPI-K Heidelberg***
- ***International Max Planck Research School – Quantum Dynamics***

Helmholtz Alliance (HA216)

Proton/Antiproton: special thanks to



BASE Collaboration: Stefan Ulmer, Christian Smorra, Takashi Higuchi, Andreas Mooser, Kurt Franke, Peter Koss, Nathan Leefer, Clemens Leiteritz, Hiroki Nagahama, Georg Schneider, Simon Van Gorp, Klaus Blaum, Yasuyuki Matsuda, Christian Ospelkaus, Wolfgang Quint, Jochen Walz, Yasunori Yamazaki

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