The pion transition form factor

Why it is interesting and how to calculate it

Stefan Leupold

Uppsala, Nov. 2014

Collaborators

- Martin Hoferichter (Darmstadt)
- Bastian Kubis (Bonn)
- Franz Niecknig (Bonn)
- Sebastian Schneider (Bonn, now industry)

related previous work of the Bonn group:

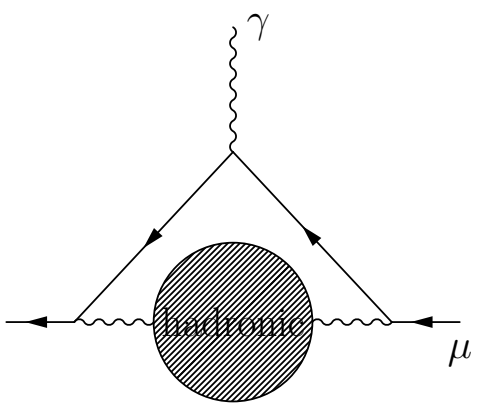
F. Niecknig, B. Kubis and S. P. Schneider, Eur. Phys. J. C 72, 2014 (2012)
S. P. Schneider, B. Kubis and F. Niecknig, Phys. Rev. D 86, 054013 (2012)
M. Hoferichter, B. Kubis and D. Sakkas, Phys. Rev. D 86, 116009 (2012)

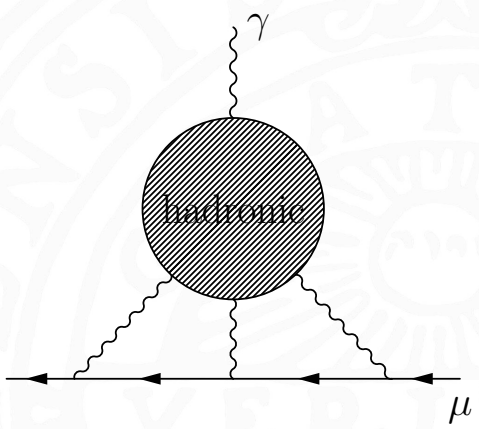
Pion transition form factor

Why is it interesting?

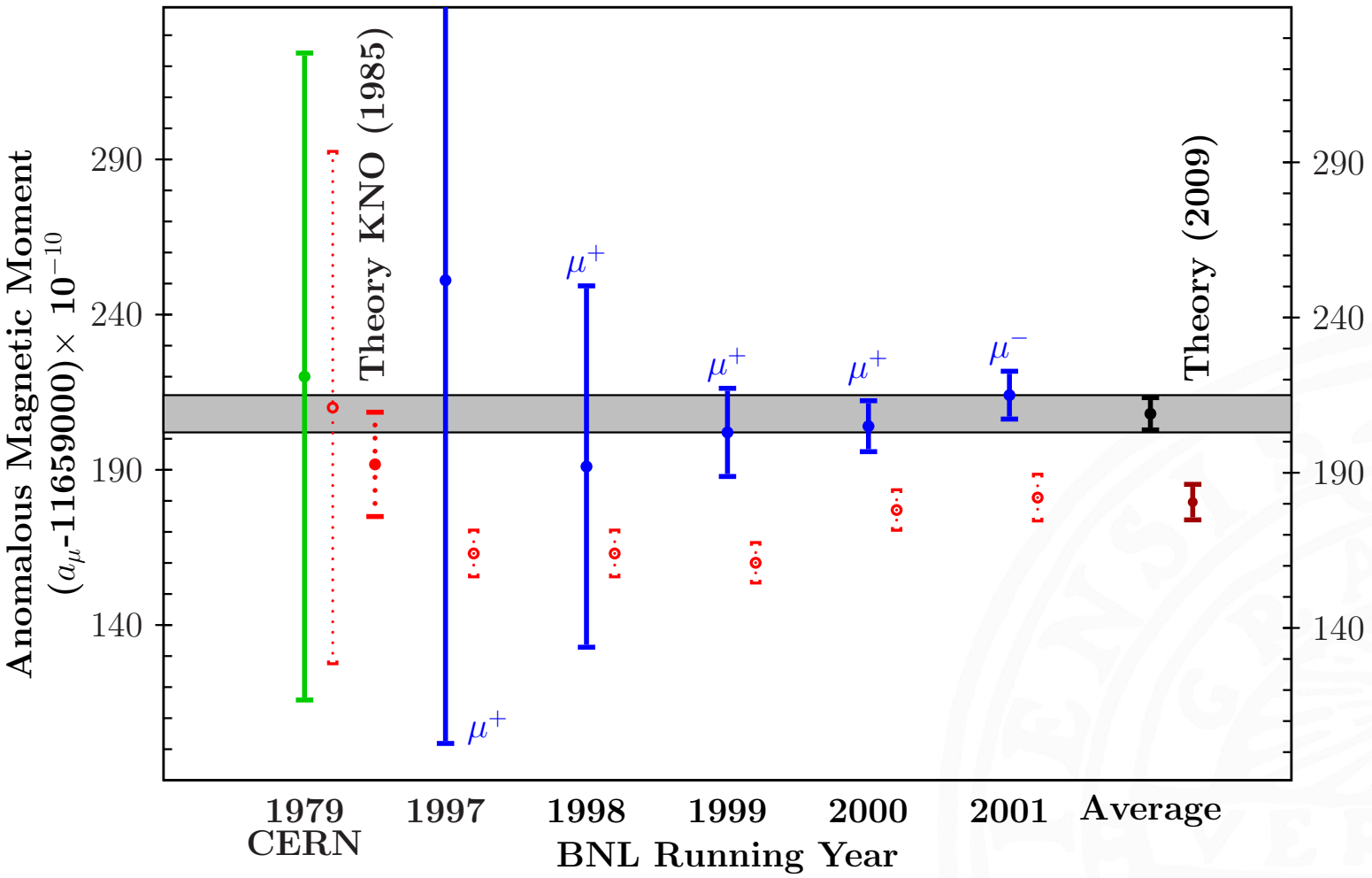
- explore intrinsic structure of lightest hadron = pion
  - $\rightsquigarrow$ form factors
  - $\rightsquigarrow$ for instance $\gamma^* \to \pi^0 \gamma^{(*)}$
- background for physics beyond standard model

$\rightsquigarrow$ $g-2$ of muon





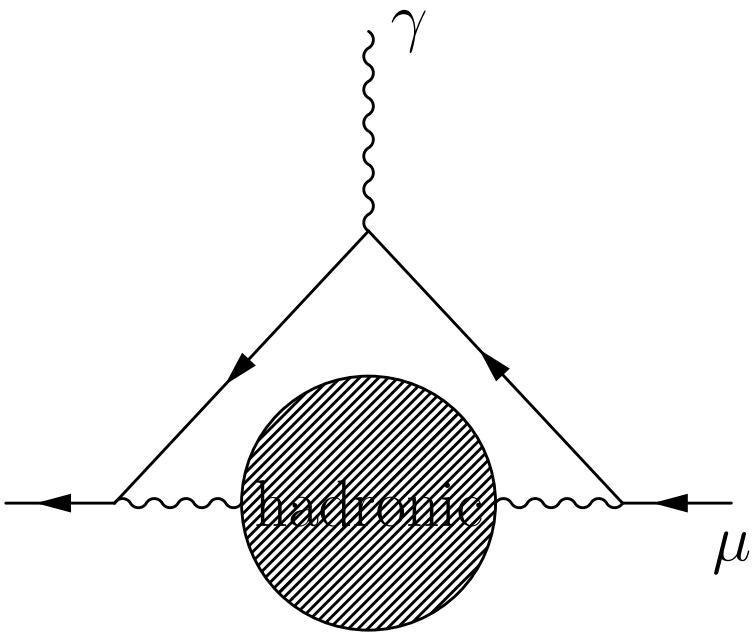
$g-2$ of the muon — status



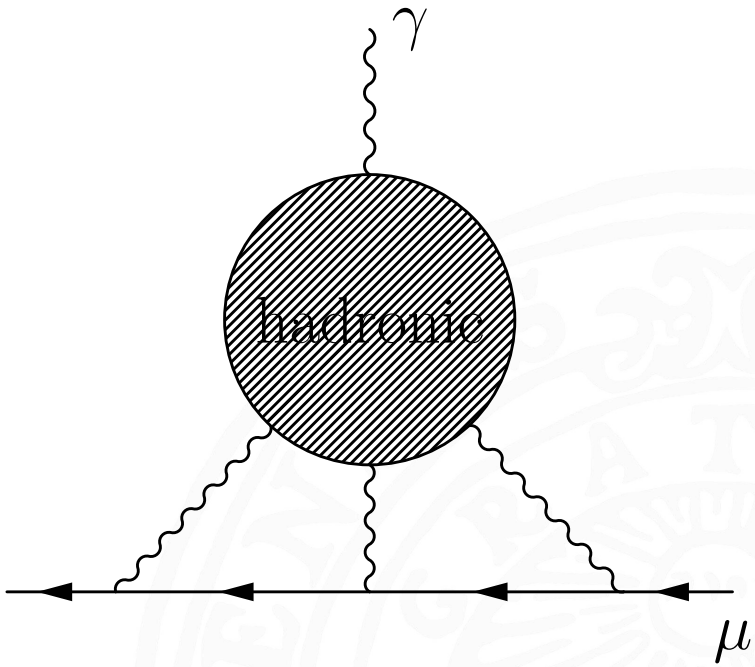
Jegerlehner/Nyffeler, Phys. Rept. 477, 1 (2009)

$g-2$ of the muon — theory

Largest uncertainty of standard model: hadronic contributions



vacuum polarization
$\sim \alpha^2$



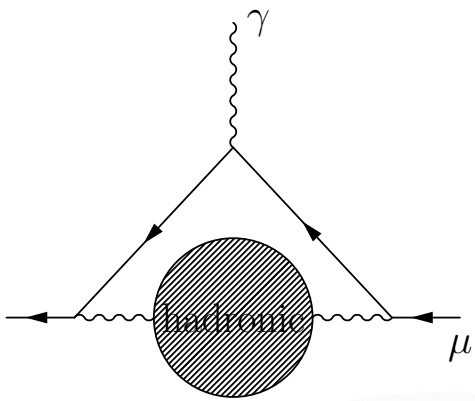
light-by-light scattering
$\sim \alpha^3$

Hadronic contribution to $g-2$ of the muon

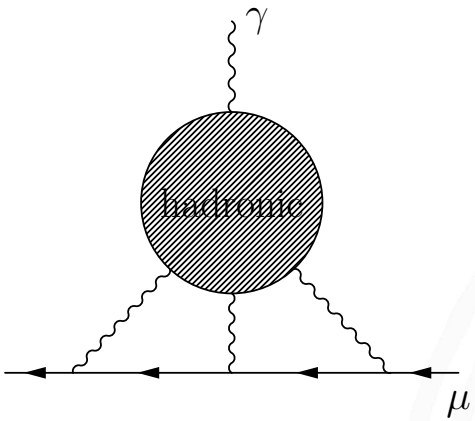
vacuum polarization

- related to cross section $e^+e^- \to$ hadrons

$\rightsquigarrow$ measurable



light-by-light scattering



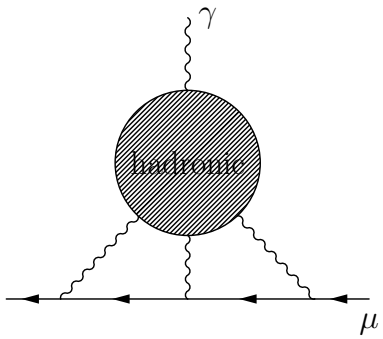
- $\gamma^*\gamma^* \leftrightarrow$ hadron(s) is not easily accessible by experiment

$\hookrightarrow$ need good theory with reasonable estimate of uncertainty (ideally an effective field theory)

$\hookrightarrow$ need experiments to constrain such hadronic theories

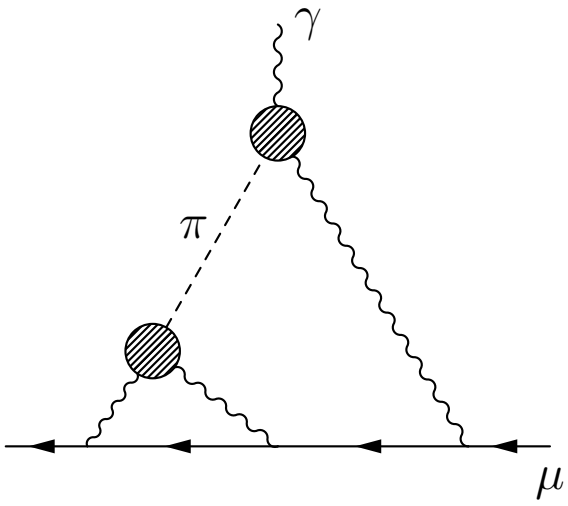
Hadronic light-by-light contribution

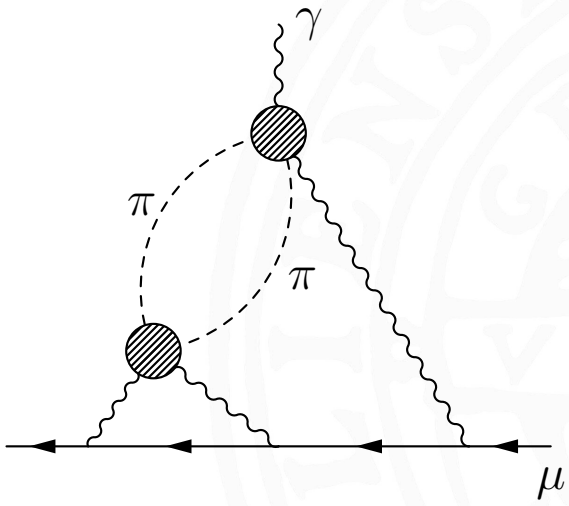
true for all hadronic contributions:



- the lighter the hadronic system, the more important
  (though high-energy contributions not unimportant for light-by-light)

$\hookrightarrow$ $\gamma^{(*)}\gamma^{(*)} \leftrightarrow \pi^0$ $\qquad$ $\gamma^{(*)}\gamma^{(*)} \leftrightarrow 2\pi, \ldots$





Unitarity and analyticity

- constraints from quantum field theory:
  partial-wave amplitudes for reactions/decays must be
  - unitary:

$$S\,S^\dagger = 1\,, \quad S = 1 + iT \quad \Rightarrow \quad 2\,\mathrm{Im}\,T = T\,T^\dagger$$

$\hookrightarrow$ note that this is a matrix equation:
$\mathrm{Im}\,T_{A\to B} = \sum_X T_{A\to X}\,T^\dagger_{X\to B}$

$\rightsquigarrow$ in practice: use most relevant intermediate states X

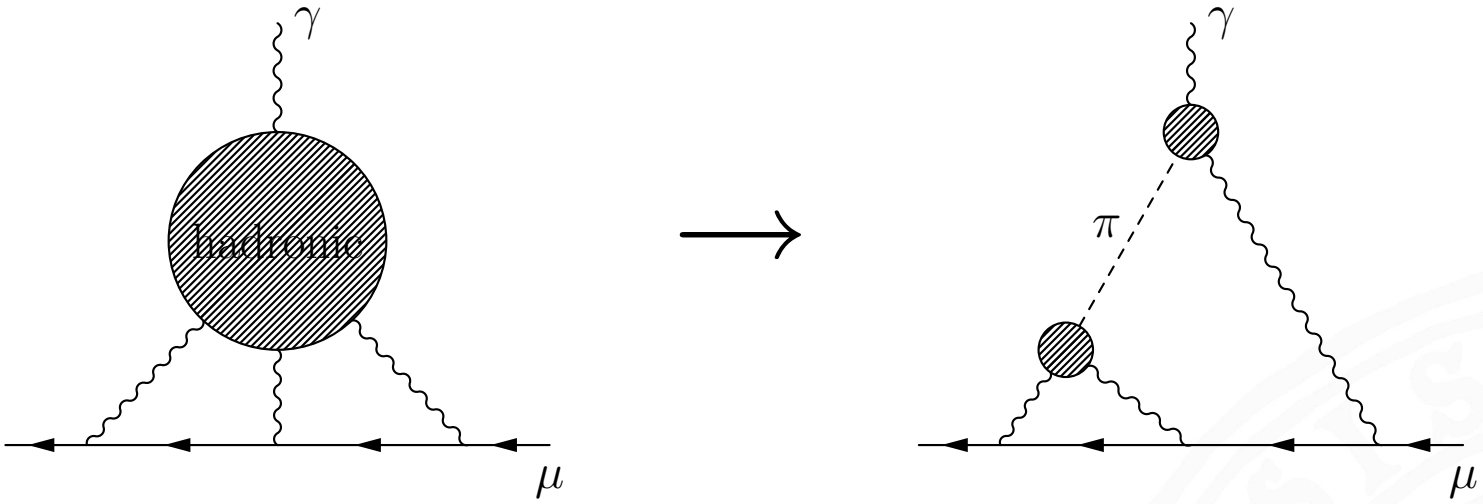
  - analytical (dispersion relations):

$$T(s) = T(0) + \frac{s}{\pi}\int\limits_{-\infty}^{\infty} ds'\,\frac{\mathrm{Im}\,T(s')}{s'\,(s'-s-i\epsilon)}\,,$$

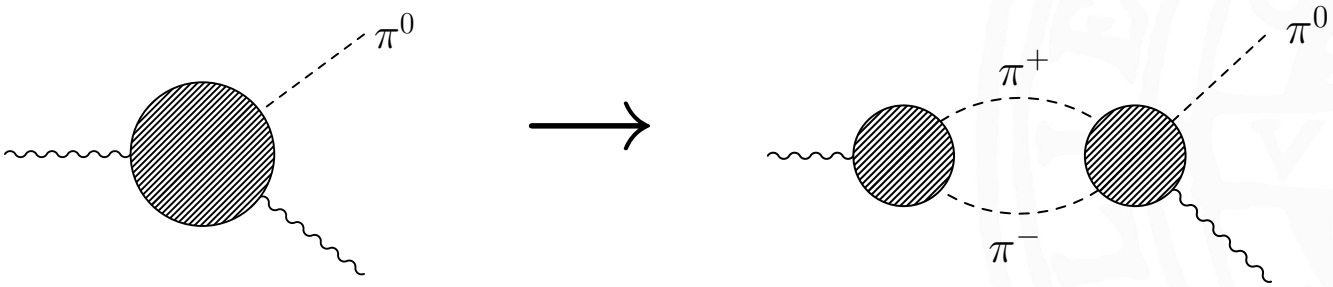
$\rightsquigarrow$ can be used to calculate whole amplitude from imaginary part

Using lowest-mass states

hadronic light-by-light contribution

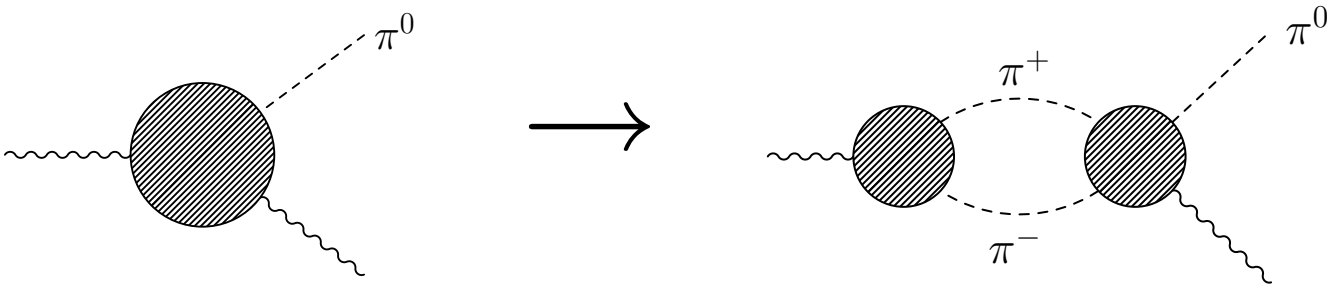


⇝ need pion transition form factor

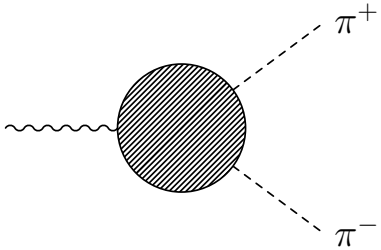


Dispersive reconstruction I

pion transition form factor

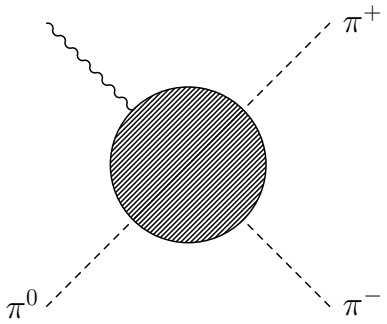


⇝ need pion vector form factor



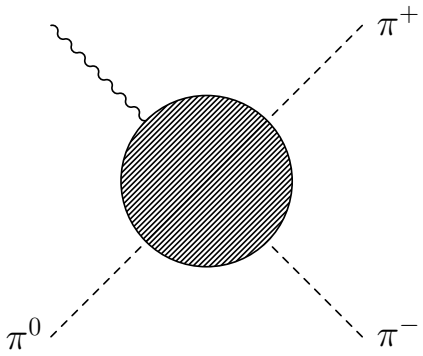
very well measured

and amplitude γ^*–3-pion

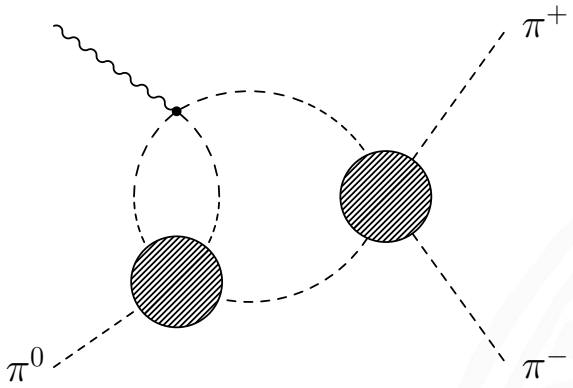


Dispersive reconstruction II

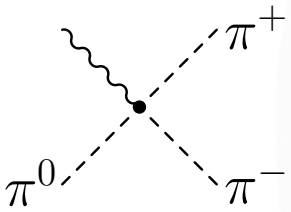
amplitude γ^*–3-pion



contains two-body correlations (depend on s, t, u), e.g. $\rightsquigarrow$

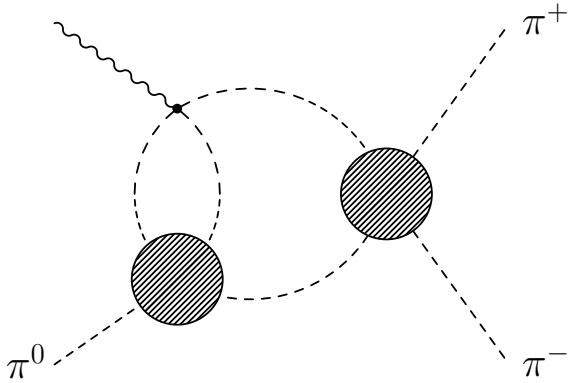


and genuine three-body correlations (depend on $m_{3\pi}^2 = m_{\gamma^*}^2$)

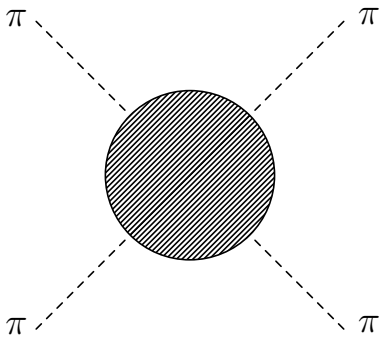


Required input

for

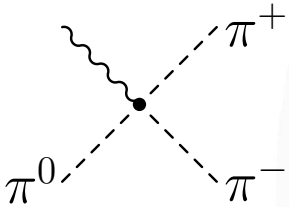


need pion phase shift



$\rightsquigarrow$ very well measured

and genuine three-body correlations
(one-parameter function!)



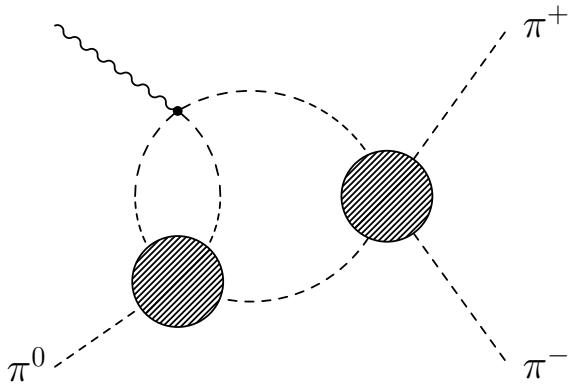
$\rightsquigarrow$ fit to cross section of $e^+e^- \longrightarrow \pi^+\pi^-\pi^0$

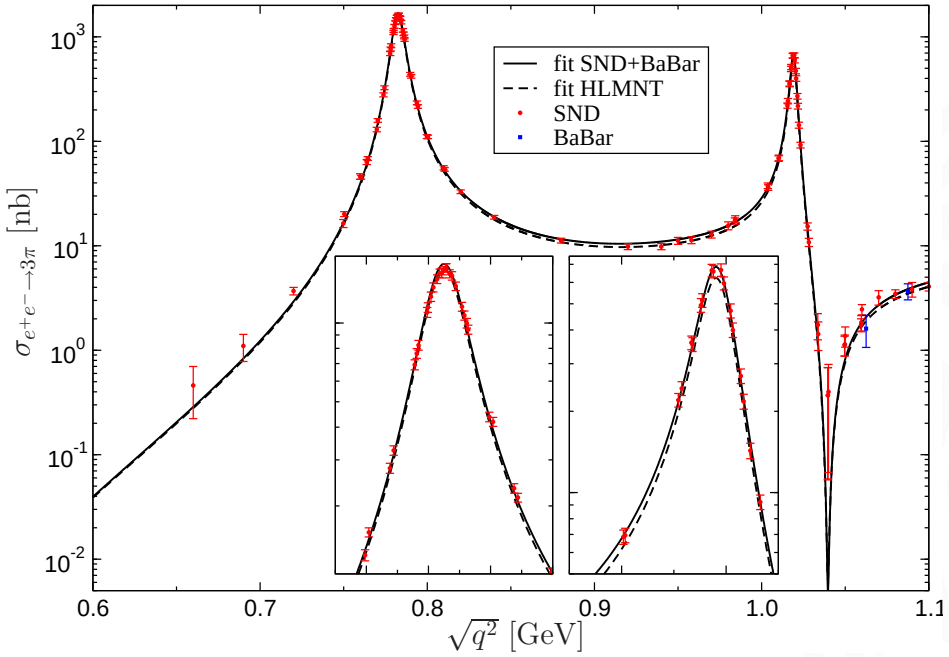
Fit to $e^+e^- \to \pi^+\pi^-\pi^0$

- dominated by narrow resonances ω, ϕ

$\hookrightarrow$ use Breit-Wigners plus background for genuine three-body correlations

$\hookrightarrow$ fully include cross-channel rescattering of pion pairs (two-body correlations)



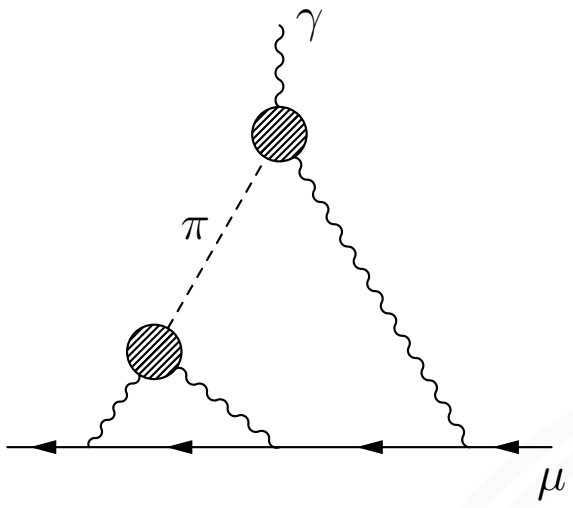


M. Hoferichter, B. Kubis, S.L., F. Niecknig and S. P. Schneider, arXiv:1410.4691 [hep-ph]

Results

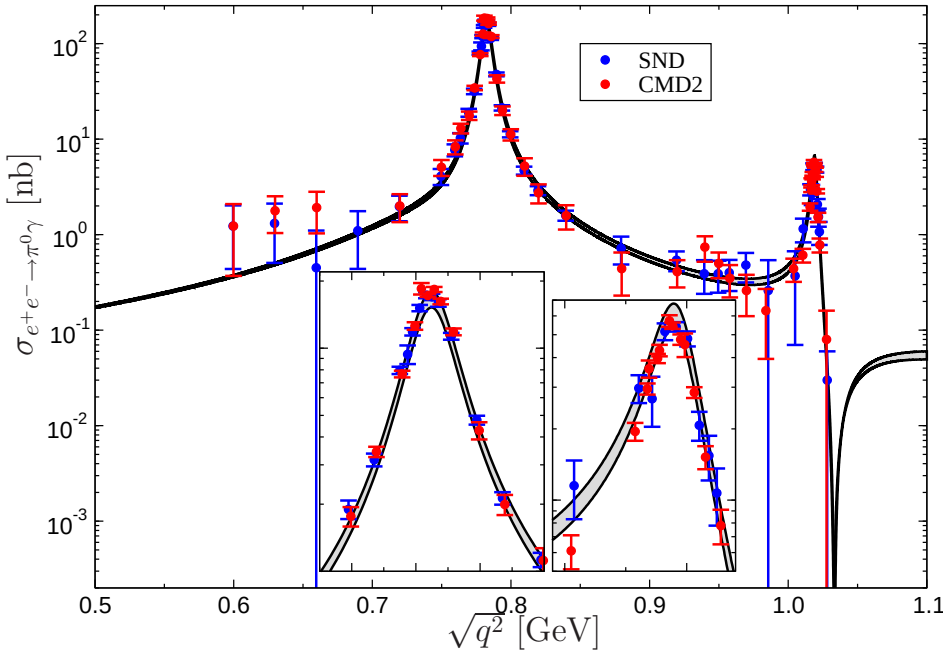
- final aim: double-virtual pion transition form factor

  $\hookrightarrow$ relevant for $g-2$



- so far: single-virtual pion transition form factor
  - time-like: cross section $e^+e^- \to \pi^0\gamma$
    $\hookrightarrow$ compare to experimental data
  - space-like: reaction $\gamma^*\gamma \to \pi^0$
    $\hookrightarrow$ prediction for low energies

Time-like pion transition form factor



theory uncertainties from

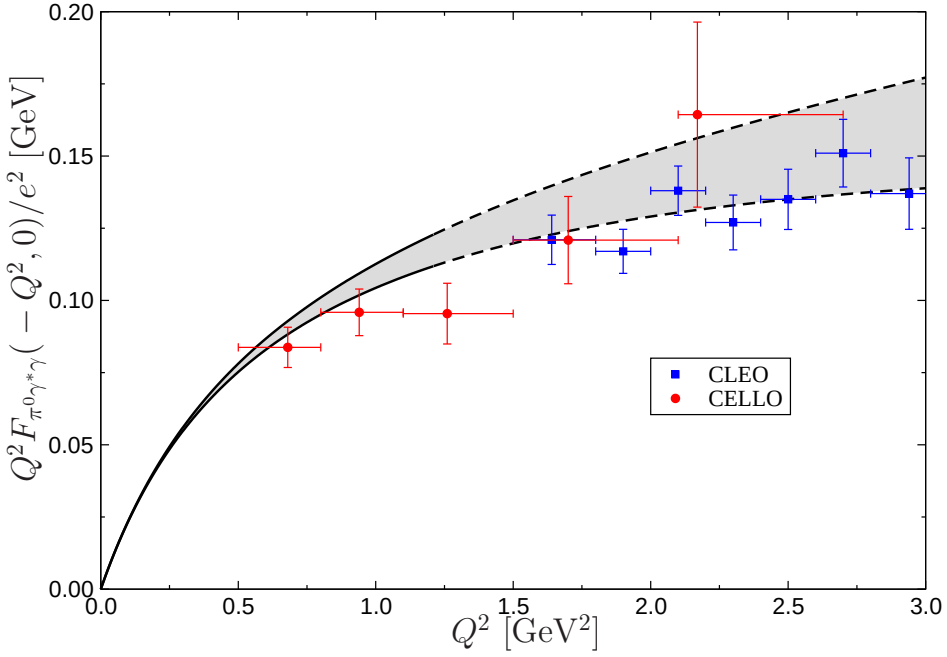
- different data sets for $e^+e^- \to 3\pi$
- different pion phase shifts
- other intermediate states than 2π neglected

  $\hookrightarrow$ explored by different cutoff for range where 2π dominates

$\rightsquigarrow$ excellent agreement

M. Hoferichter, B. Kubis, S.L., F. Niecknig and S. P. Schneider, arXiv:1410.4691 [hep-ph]

Space-like pion transition form factor



- this is a prediction, no data yet at low energies
- expect new measurements from BESIII
- final aim: double virtual transition form factor

$\hookrightarrow$ relevant for $g-2$

M. Hoferichter, B. Kubis, S.L., F. Niecknig and S. P. Schneider, arXiv:1410.4691 [hep-ph]

Summary and Outlook

- form factors allow access to intrinsic structure of hadrons
- in addition input for standard-model baseline calculations for high-precision determinations (muon's $g-2$)

$\hookrightarrow$ requires model-independent hadronic calculations with reliable uncertainty estimates

$\hookrightarrow$ we are sharpening our theory tools to improve the accuracy of predictions

---

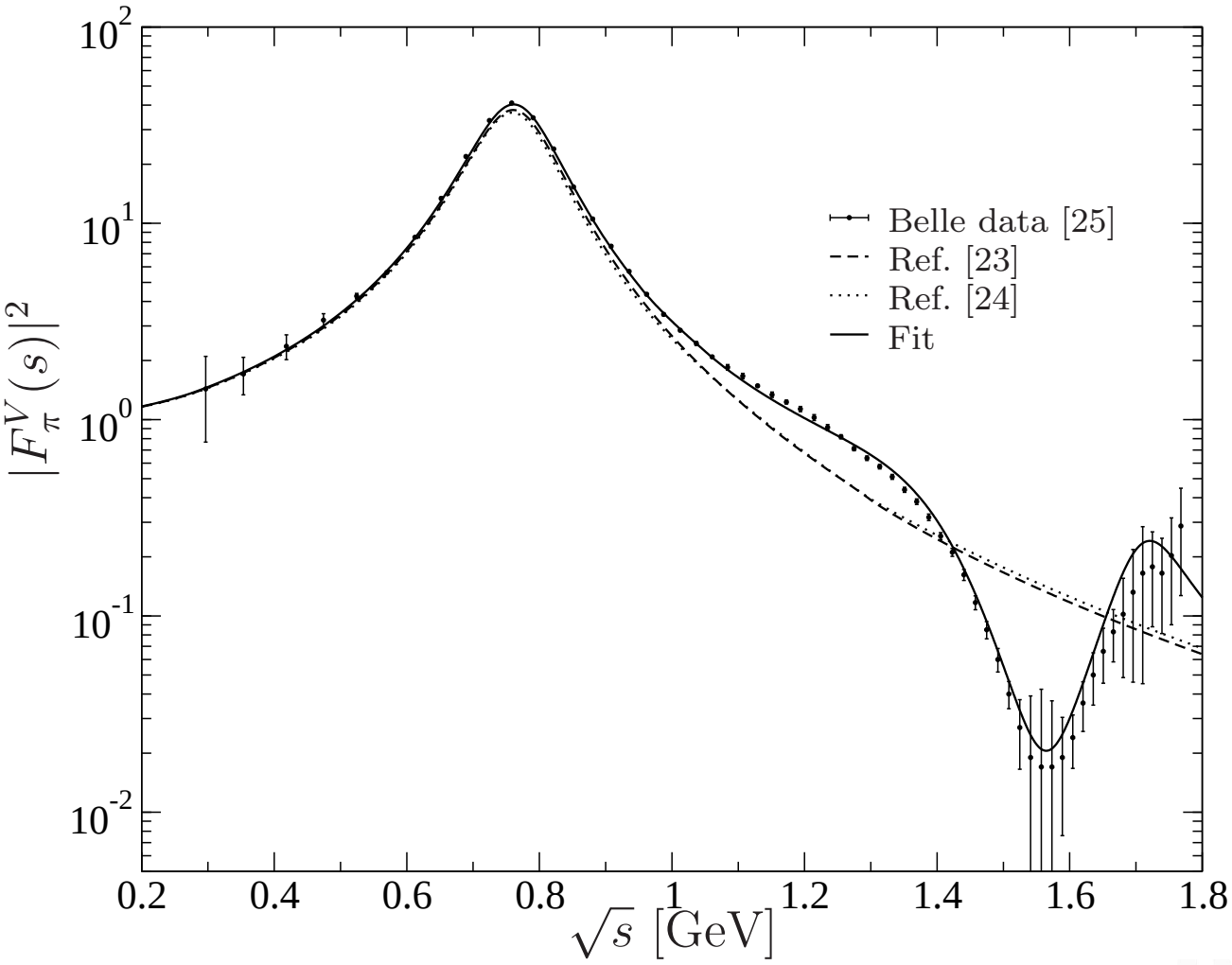
- same dispersive framework will be applied to strange-baryon transition form factors (Carlos Granados and SL, work in progress)
  - $\Sigma \to \Lambda\, e^+ e^-$
  - $\Sigma^* \to \Lambda\, e^+ e^-$ (decuplet-to-octet transition)

$\hookrightarrow$ access to spin properties from decay pattern of strange baryons

$\hookrightarrow$ predictions for PANDA

backup slides

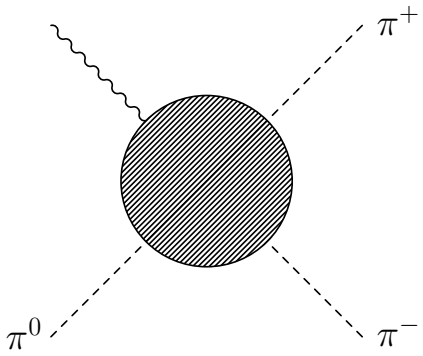
Pion vector form factor



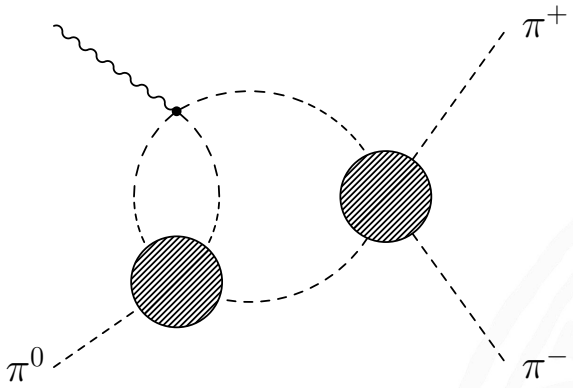
Sebastian P. Schneider, Bastian Kubis, Franz Niecknig, Phys.Rev.D86:054013,2012

Dispersive reconstruction II

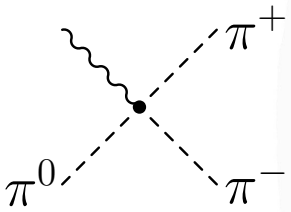
amplitude γ^*–3-pion



contains two-body correlations (depend on s, t, u), e.g. $\rightsquigarrow$

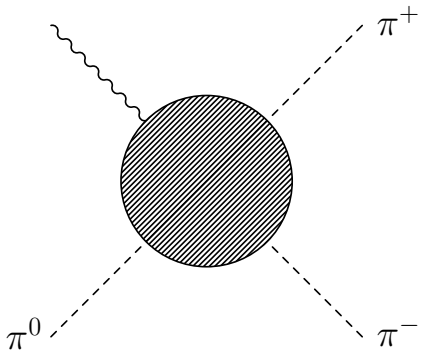


and genuine three-body correlations (depend on $m_{3\pi}^2 = m_{\gamma^*}^2$)

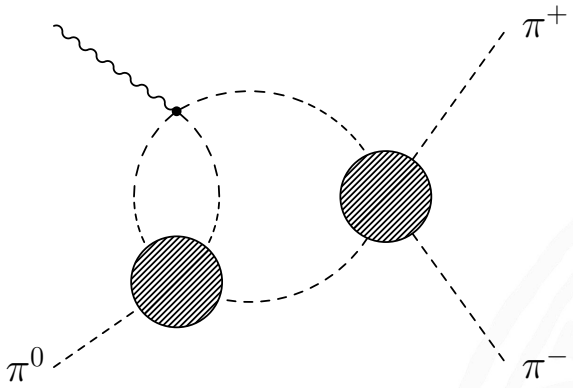


Dispersive reconstruction II

amplitude γ^*–3-pion



contains two-body correlations (depend on s, t, u), e.g. $\rightsquigarrow$



and genuine three-body correlations (depend on $m_{3\pi}^2 = m_{\gamma^*}^2$)