Theoretical upper bounds on detector's response to DM-e^- interactions in direct searches for sub-GeV dark matter

Michał Iglicki

in collaboration with Riccardo Catena

arXiv:2410.xxxxx

CHALMERS

UNIVERSITY OF TECHNOLOGY

Partikeldagarna 2024

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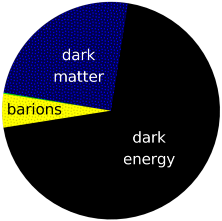
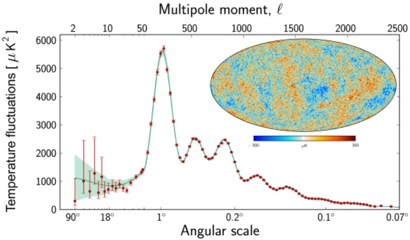
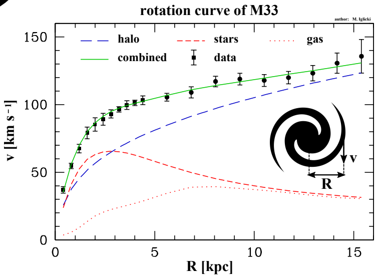
Dark matter

https://apod.nasa.gov/apod

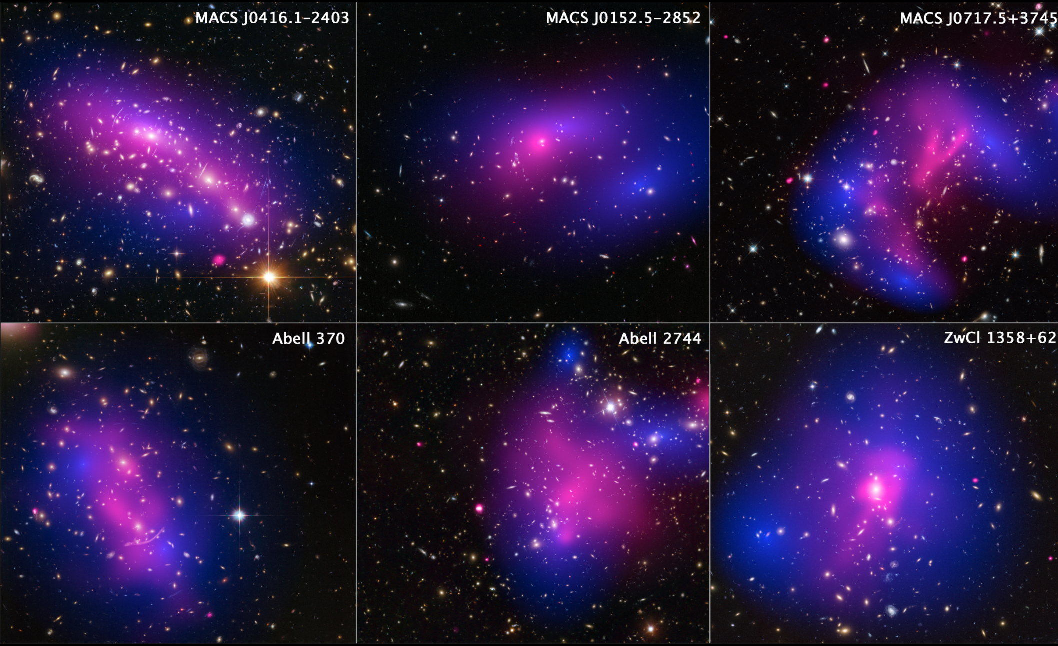




http://sci.esa.int/planck ,
https://wiki.cosmos.esa.int/planck-legacy-archive



data from: arXiv:astro-ph/9909252



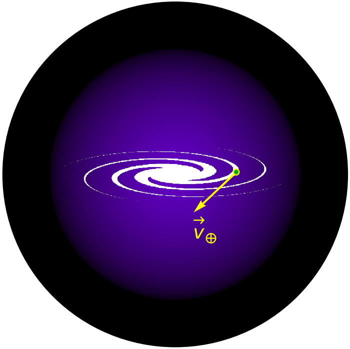
https://hubblesite.org

Local distribution of DM

Baxter et al., 2105.00599

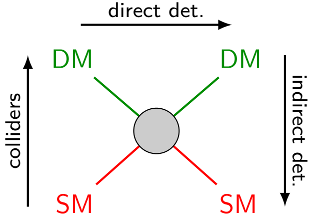
$$n_{\mathsf{DM}} \simeq \frac{1\ \mathsf{GeV}}{m_{\mathsf{DM}}} \times 0.3\ \ \mathsf{cm}^{-3}$$

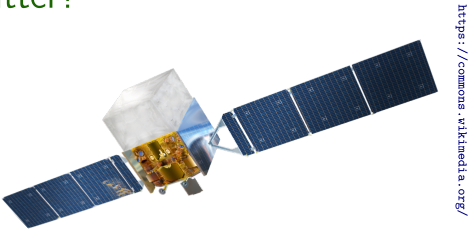
$$\rho(\boldsymbol{v}) = \mathcal{N}\ \exp\left[-\frac{(\boldsymbol{v}+\boldsymbol{v}_\oplus)^2}{v_0^2}\right]\theta(v_{\mathsf{esc}} - |\boldsymbol{v}+\boldsymbol{v}_\oplus|)$$

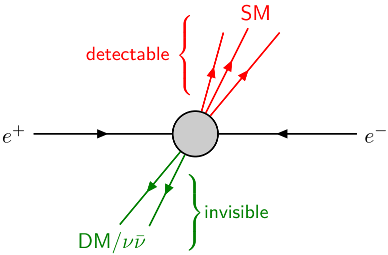


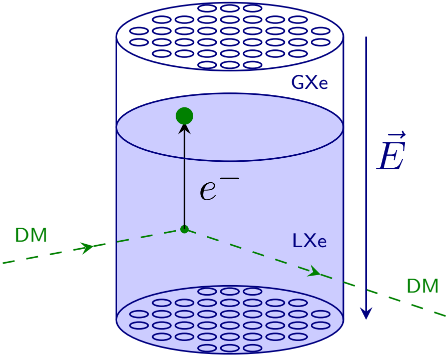
$$v_{\mathsf{esc}} = 544\,\mathsf{km/s}$$
$$v_\oplus = 250.5\,\mathsf{km/s}$$
$$v_0 = 238\,\mathsf{km/s}$$

How to detect particle dark matter?



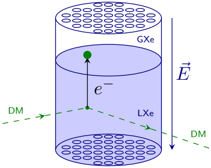






Direct detection experiments

- LUX-ZEPLIN, PandaX-4T, XENONnT, SuperCDMS, ...



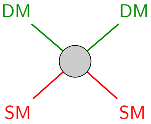
- GeV+ range of masses (WIMPs):
  no success so far
  $\Rightarrow$ sub-GeV DM?

- nuclear vs. electronic recoil of non-relativistic DM

$$\Delta E_{\text{SM}} \leq \frac{4\,\mu}{(1+\mu)^2}\, E_{\text{DM}}^{\text{in}} \qquad \leftarrow \text{ maximized for } \mu \equiv m_{\text{SM}}/m_{\text{DM}} = 1$$

$\Rightarrow$ m_{SM} should be as close to m_{DM} as possible!

$\Rightarrow$ **electrons** preferable for light DM



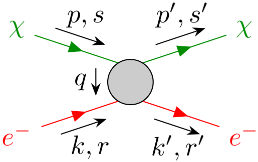
- what material to use?

Effective non-relativistic theory for spin-$^1\!/_2$ DM

Catena et al., 2105.02233

$$\boldsymbol{v}^{\perp} \equiv \frac{\boldsymbol{p}+\boldsymbol{p}'}{2\,m_\chi} - \frac{\boldsymbol{k}+\boldsymbol{k}'}{2\,m_e}$$

$$\boldsymbol{q}\cdot\boldsymbol{v}^{\perp} \xrightarrow{\text{en. cons.}} 0$$



$\left.\begin{array}{l}\text{non-relativistic limit}\\ \text{Lorentz (Galilean) invariance}\end{array}\right\} \Rightarrow$ $\mathcal{M} = \sum_i c_i\,\mathcal{O}_i$

14 simple operators
in the leading order

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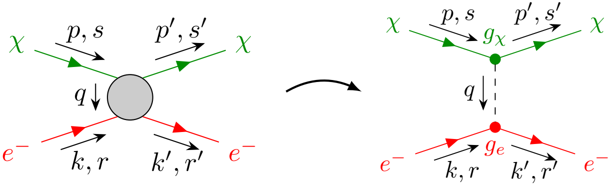

non-relativistic limit
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14 simple operators
in the leading order

- example: scalar coupling

$$\mathcal{M} \simeq \underbrace{-i\,\frac{g_\chi\, g_e}{q^2+M^2}\,4\,m_\chi m_e}_{c_1}\;\underbrace{\delta^{ss'}\delta^{rr'}}_{\mathcal{O}_1}$$

- other examples:

$$\mathcal{O}_4^{rr'ss'} = \frac{\boldsymbol{\sigma}^{rr'}}{2}\cdot\frac{\boldsymbol{\sigma}^{ss'}}{2}\,, \qquad \mathcal{O}_{15}^{rr'ss'} = \left[\left(\frac{\boldsymbol{\sigma}^{rr'}}{2}\times\frac{\boldsymbol{q}}{m_e}\right)\cdot\boldsymbol{v}^\perp\right]\left(\frac{\boldsymbol{\sigma}^{ss'}}{2}\cdot\frac{\boldsymbol{q}}{m_e}\right)$$

Linear response theory

Catena & Spaldin, 2402.06817

- another decomposition:

$$\mathcal{M} = \sum_i c_i\,\mathcal{O}_i$$

$$= \sum_a \underbrace{F_a^{ss'}(\boldsymbol{q}, \boldsymbol{v}_\chi)}_{\text{DM physics}}\ \underbrace{J_a^{rr'}(\boldsymbol{v}_e^\perp)}_{\text{electronic part}}\ , \qquad \boldsymbol{v}_\chi \equiv \frac{\boldsymbol{p}}{m_\chi}\ , \qquad \boldsymbol{v}_e^\perp \equiv \frac{\boldsymbol{k}+\boldsymbol{k}'}{2m_e}$$

- electronic operators:

$$J_0^{rr'} \equiv \delta^{rr'}\ , \quad J_A^{rr'} \equiv \boldsymbol{v}_e^\perp \cdot \sigma^{rr'}\ ,$$

$$\boldsymbol{J}_5^{rr'} \equiv \boldsymbol{\sigma}^{rr'}\ , \quad \boldsymbol{J}_M^{rr'} \equiv \boldsymbol{v}_e^\perp\,\delta^{rr'}\ , \quad \boldsymbol{J}_E^{rr'} \equiv -i\,\boldsymbol{v}_e^\perp \times \boldsymbol{\sigma}^{rr'}\ ,$$

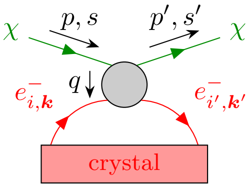
- example: scalar coupling

$$\mathcal{M} \simeq \overbrace{\underbrace{-i\,\frac{g_\chi\,g_e}{q^2+M^2}\,4\,m_\chi m_e}_{c_1}\,\delta^{ss'}}^{F_0}\ \underbrace{\overbrace{\delta^{rr'}}^{J_0}}_{\mathcal{O}_1}\ .$$

Int. rate for bounded electrons & generalized susceptibilities

Catena et al., 1912.08204, Catena & Spaldin, 2402.06817

- electronic states $\neq$ momentum eigenstates



$$\mathcal{M} = \sum_a \overbrace{F_a^{ss'}(\boldsymbol{q}, \boldsymbol{v}_\chi)}^{\text{DM physics}} \; \overbrace{J_a^{rr'}(\boldsymbol{v}_e^\perp)}^{\text{electronic part}}$$

$$\Big\downarrow \; \left| \begin{array}{l} \text{integral over} \\ e^- \text{ wave fun.} \end{array} \right|^2$$

$$|\widetilde{\mathcal{M}}_{\boldsymbol{ik}\to\boldsymbol{i'k'}}|^2 \simeq \sum_{ab} \underbrace{\mathcal{F}_{ab}(\boldsymbol{q}, \boldsymbol{v}_\chi)}_{\text{DM physics}} \times [\text{material response}]$$

- interaction rate per dark particle

$$\Gamma(\boldsymbol{v}_\chi) \sim \int \frac{d^3q}{(2\pi)^3} \underbrace{\overbrace{\sum_{ii'} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3}}^{\text{sum over electronic states}} |\widetilde{\mathcal{M}}_{\boldsymbol{ik}\to\boldsymbol{i'k'}}|^2 \, \delta(\text{cons.})}$$

- total interaction rate

$$\Gamma \equiv \frac{1}{n_\chi V} \frac{dN}{dt} = \int \frac{d^3q}{(2\pi)^3} d^3v \, \rho(\boldsymbol{v}_\chi) \overbrace{\sum_{ab} \mathcal{F}_{ab}(\boldsymbol{q}, \boldsymbol{v}_\chi) (\chi_{a^\dagger b} - \chi^*_{b^\dagger a})(\boldsymbol{q}, \omega_{v,q})}$$

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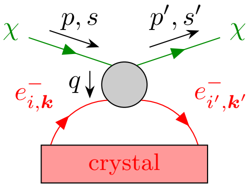

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generalized susceptibilities

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Theoretical upper bounds on the interaction rate

- χ is analytic and causal, so

$$\int_0^\infty \frac{d\omega}{\omega} \Im\left[\frac{4\pi\alpha}{q^2}\chi_{a^\dagger a}(\omega,\boldsymbol{q})\right] = \frac{\pi}{2}\left[\frac{4\pi\alpha}{q^2}\chi_{a^\dagger a}(0,\boldsymbol{q})\right] \leq \frac{\pi}{2}$$

- conclusion: upper bound

material-dependent exact value $\rightarrow$

$$\Gamma_{a^\dagger a} = \int q^4 dq \int_0^\infty d\omega\, f_{aa}(\omega,q)\, \Im\frac{4\pi\alpha}{q^2}\chi_{a^\dagger a}(\boldsymbol{q},\omega_{\boldsymbol{v},\boldsymbol{q}})$$

material-independent upper bound $\rightarrow$

$$\Gamma^{\text{opt}}_{a^\dagger a} = \frac{\pi}{2}\int q^4 dq\, \max_\omega \left[\omega f_{aa}(\omega,q)\right]$$

$$f_{aa}(\omega,q) \equiv \rho^{(0)}_\omega(\omega;q)\,\mathcal{F}^{(0)}_{aa}(\omega,q) + \rho^{(2)}_\omega(\omega;q)\,\mathcal{F}^{(2)}_{aa}(\omega,q)$$
$$a = 0, A, 5_k, M_k, E_k$$

- truncated thermal local distribution of DM $\Rightarrow$ $\rho^{(0)}_\omega(\omega;q)$, $\rho^{(2)}_\omega(\omega;q)$
- effective models of DM-e^- interactions $\Rightarrow$ $\mathcal{F}^{(0)}_{aa}(\omega,q)$, $\mathcal{F}^{(2)}_{aa}(\omega,q)$
- material science $\Rightarrow$ $\Im\frac{4\pi\alpha}{q^2}\chi_{a^\dagger a}(\boldsymbol{q},\omega_{\boldsymbol{v},\boldsymbol{q}})$

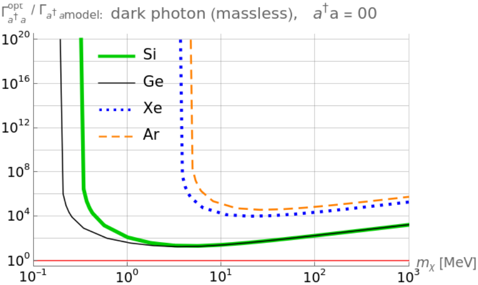
Results: methodology

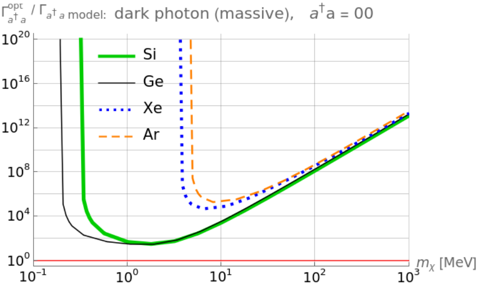
$$f(m_\chi) \equiv \frac{\textbf{theor. upper bound}}{\textbf{true interaction rate}} \begin{array}{l} \leftarrow \text{(material-}\textbf{independent}\text{)} \\ \leftarrow \text{(material-}\textbf{dependent}\text{)} \end{array}$$

- effective models of DM-e^- interactions
  - dark photon
  - anapole
  - magnetic dipole
  - electric dipole
- different materials: which is closest to **saturating the bound** ($f(m_\chi) \to 1$) for a given model?
  - Si, Ge, Xe, Ar
  - numerical data based on Catena et al., 2105.02233, 2210.07305

Results

dark photon model $\mathcal{L}_{\text{int}} = g_x \, \bar{\chi}\gamma^{\mu}\chi \, A'_{\mu} \quad \Rightarrow \quad \mathcal{F}_{00} \propto (m_{A'}^2 + q^2)^{-2}$ (others 0)

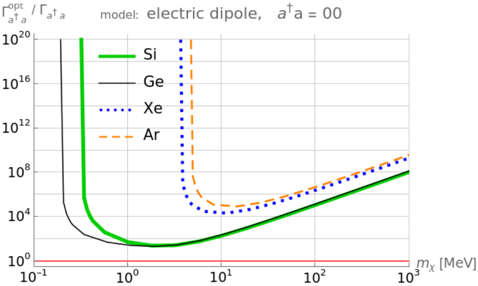




electric dipole model

$$\mathcal{L}_{\text{int}} = \frac{g}{\Lambda} \, i \, \bar{\chi}\sigma^{\mu\nu}\gamma_5\chi \, F_{\mu\nu}$$

$\Rightarrow \quad \mathcal{F}_{00} \propto q^{-2}$ (others 0)



Results

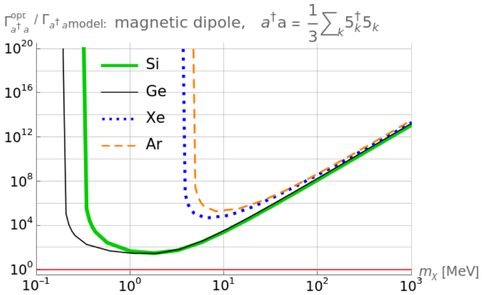
magnetic dipole model

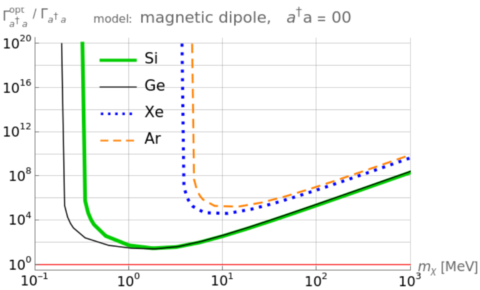
$$\mathcal{L}_{\text{int}} = \frac{g}{\Lambda}\,\bar{\chi}\sigma^{\mu\nu}\chi\, F_{\mu\nu}$$

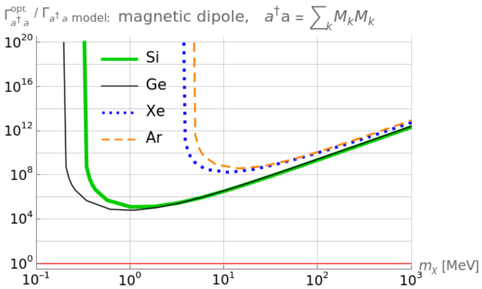
$$\Rightarrow \qquad \mathcal{F}_{00} \propto 1 - 4\,v_q^2\,\frac{m_\chi^2}{q^2} + 4\,v^2\,\frac{m_\chi^2}{q^2}\,,$$

$$\mathrm{Tr}\mathcal{F}_{MM} \propto 8\,\frac{m_\chi^2}{q^2}\,, \qquad \frac{1}{3}\mathrm{Tr}\mathcal{F}_{55} \propto 2\,\frac{m_\chi^2}{m_e^2}$$

(others vanish)







Results

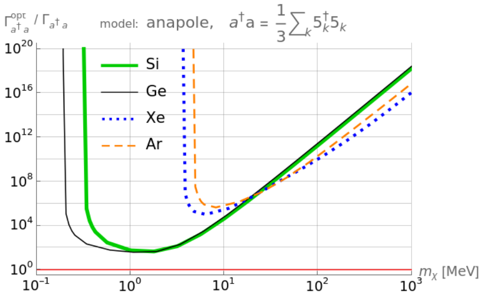
anapole model

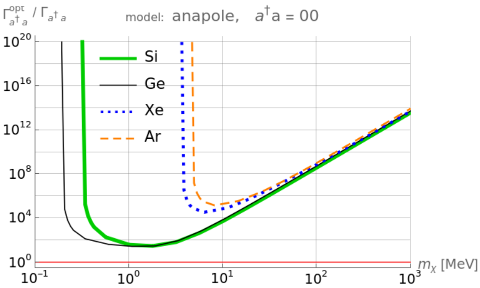
$$\mathcal{L}_{\text{int}} = \frac{g}{2\,\Lambda^2}\,\bar{\chi}\gamma^\mu\gamma_5\chi\,\partial^\nu F_{\mu\nu}$$

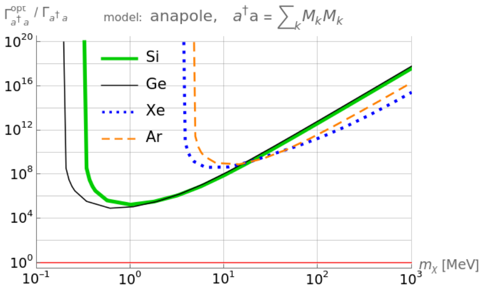
$$\Rightarrow \quad \mathcal{F}_{00} \propto \frac{q}{4\,m_\chi}\left(v_q - \frac{q}{4\,m_\chi}\right) + \frac{1}{4}\,v^2\,,$$

$$\mathrm{Tr}\mathcal{F}_{MM} \propto \frac{3}{4}\,, \qquad \frac{1}{3}\mathrm{Tr}\mathcal{F}_{55} \propto \frac{q^2}{24\,m_e^2}$$

(others vanish)







Summary

- effective approach to non-relativistic DM-e^- interactions
  - small set of operators in the leading order
- linear response theory

$$[\text{interaction rate}] = \int [\text{DM model}] \times [\text{material response of the detector}]$$

- material response $\rightarrow$ generalized susceptibilities $\chi_{a^\dagger b}(\omega, q)$
- Kramers-Kronig relations

$$f: \text{ causal, analytic} \qquad \Rightarrow \qquad \int_0^\infty \frac{d\omega}{\omega} \Im f(\omega) = \frac{\pi}{2} f(0)$$

- material-independent theoretical upper bound on the interaction rate
- results: solids typically better than nobles (excl. $m_\chi \gtrsim 20$ MeV in the anapole model), but all of them **far from the theoretical bound**
- outlook: new materials?

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