Swampland and Cone Conjectures in Calabi-Yau Moduli Spaces

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Based on work to appear with Ben Heidenreich and Nicholas Pittman
See also 2108.08309 with Murad Alim and Ben Heidenreich;
2212.10573 with Naomi Gendler, Ben Heidenreich, Liam McAllister, and Jakob Moritz

Outline

- The Landscape and the Swampland [4 slides]
- 5d Supergravity and Calabi-Yau Geometry [8 slides]
- Cone Conjectures [3 slides]
- Results [1 slide]

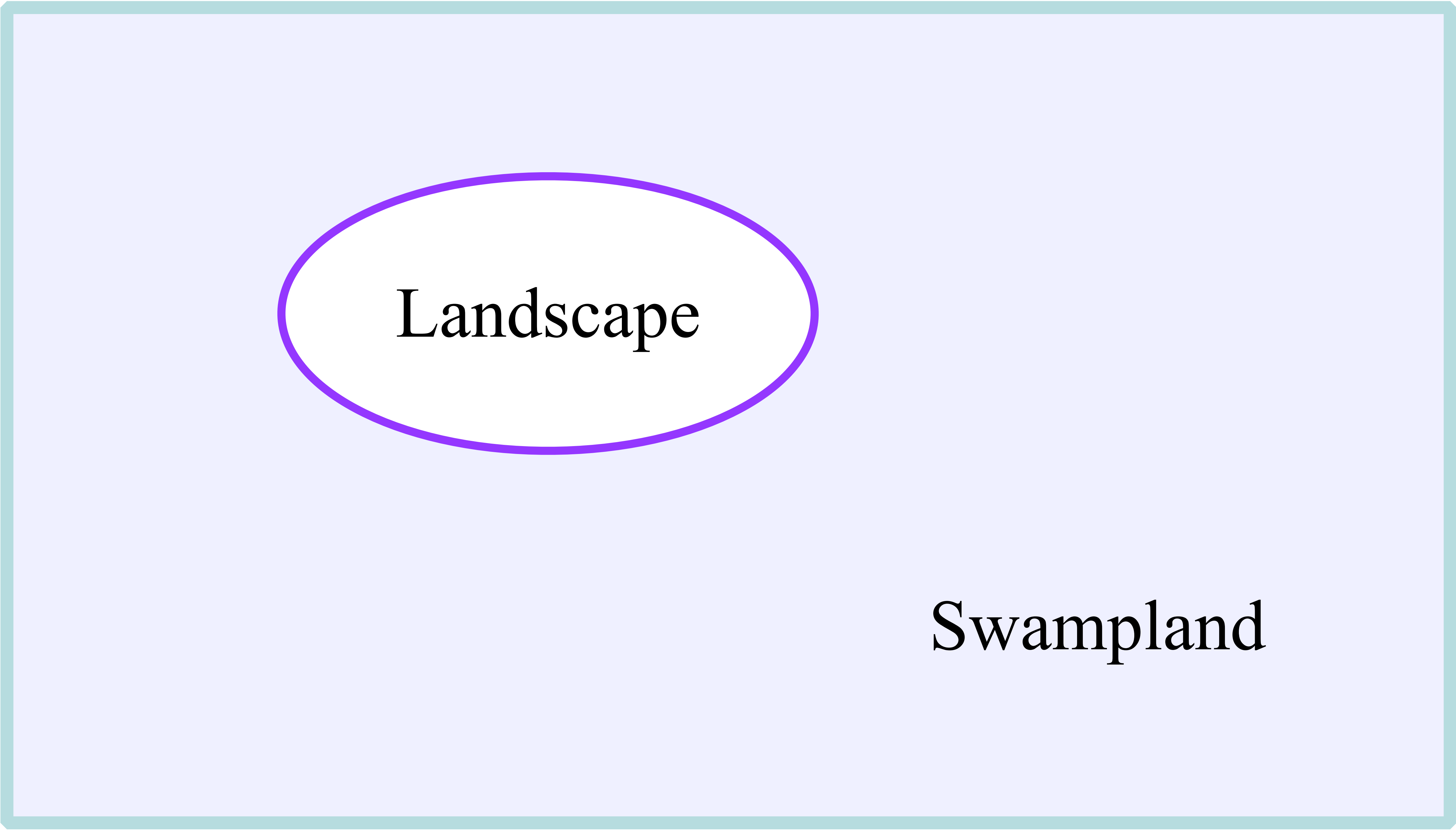
The Landscape and the Swampland

The Landscape

The Swampland

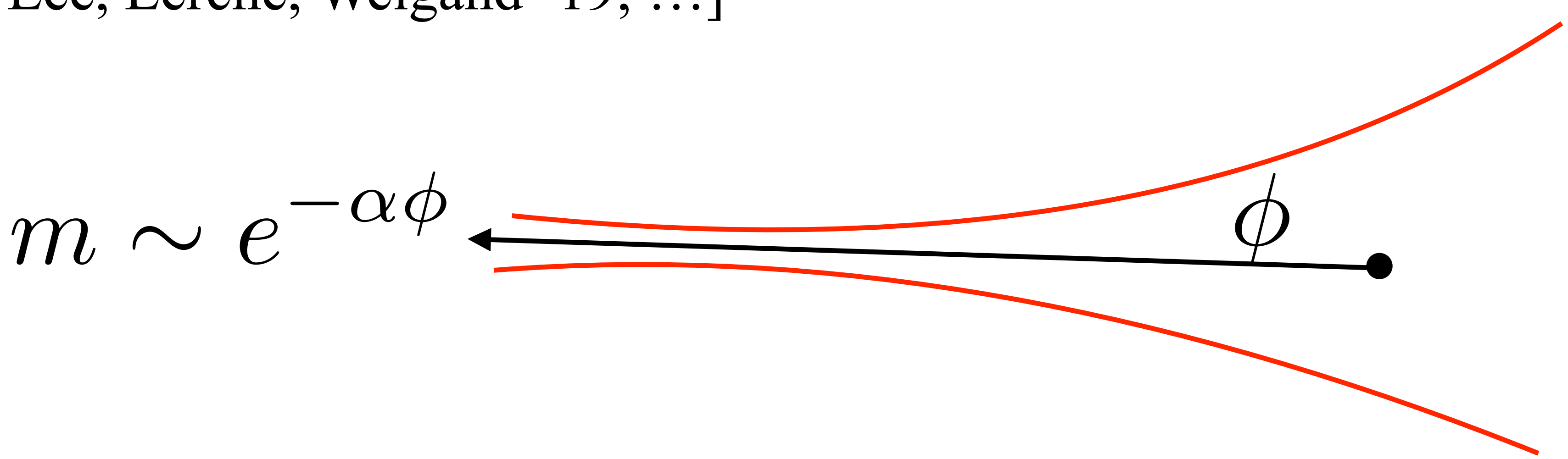
Vafa ’05, Ooguri, Vafa ‘06

Goal: Delineate Boundary



Light States at Infinite-Distance

- Many of the most well-established swampland conjectures deal with infinite-distance limits in scalar field moduli space, which (a) are under good theoretical control and (b) admit universal behavior.
- Various Swampland conjectures (Distance Conjecture, Weak Gravity Conjectures, Emergent String Conjecture) imply that infinite-distance limits must have towers of light particles and/or light strings, whose mass/tension vanish asymptotically in the infinite-distance limit. [Ooguri, Vafa ’06; Arkani-Hamed, Motl, Nicolis, Vafa ’06; Lee, Lerche, Weigand ’19; …]



5d Supergravity and Calabi-Yau Geometry

5d supergravity

- 5d supergravity features two multiplets with massless scalar fields
  - Hypermultiplets
  - Vector multiplets, which also feature a vector boson
- In addition, every 5d supergravity theory features a gravity multiplet, which also has a vector boson
- Thus, there are $N = n_v + 1$ vector bosons, and n_v vector multiplet moduli

BPS Particles

- In theories with 8+ supercharges, charged particles must satisfy BPS bound:

$$m \geq |\zeta_{q_i}(a_i)|$$

- Central charge ζ depends on moduli a_i, charge q_i of particle
- 4d: ζ complex. 5d: ζ real
- Particles that saturate the BPS bound are called BPS particles

BPS Bound in 5d

- BPS particles saturate the bound:

$$m(q_I) \geq (2\pi^2)^{1/6} |q_I Y^I|$$

- BPS strings saturate the bound:

$$T(\tilde{q}^I) \geq \frac{1}{2}(2\pi^2)^{-1/6} |\tilde{q}^I \mathcal{F}_I|$$

5d Supergravity from M-theory

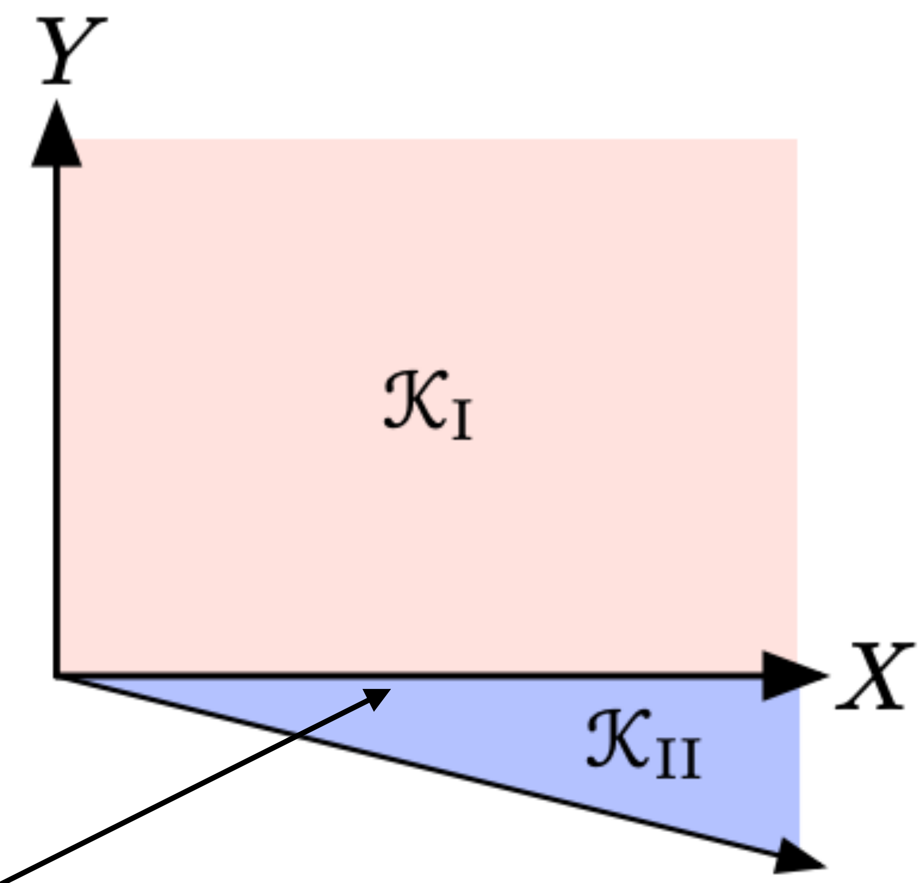
- The standard way to construct a UV-complete 5d supergravity theory in the landscape is to compactify M-theory on a Calabi-Yau threefold.
- Physical properties of the theory are then identified with geometric properties of the Calabi-Yau threefold.
- In particular, the vector multiplet moduli space is identified with the Kähler moduli space of the Calabi-Yau manifold.

Kähler Moduli Space

Kähler cone

$$\mathcal{K} = \cup_A \mathcal{K}_A$$

Extended Kähler cone



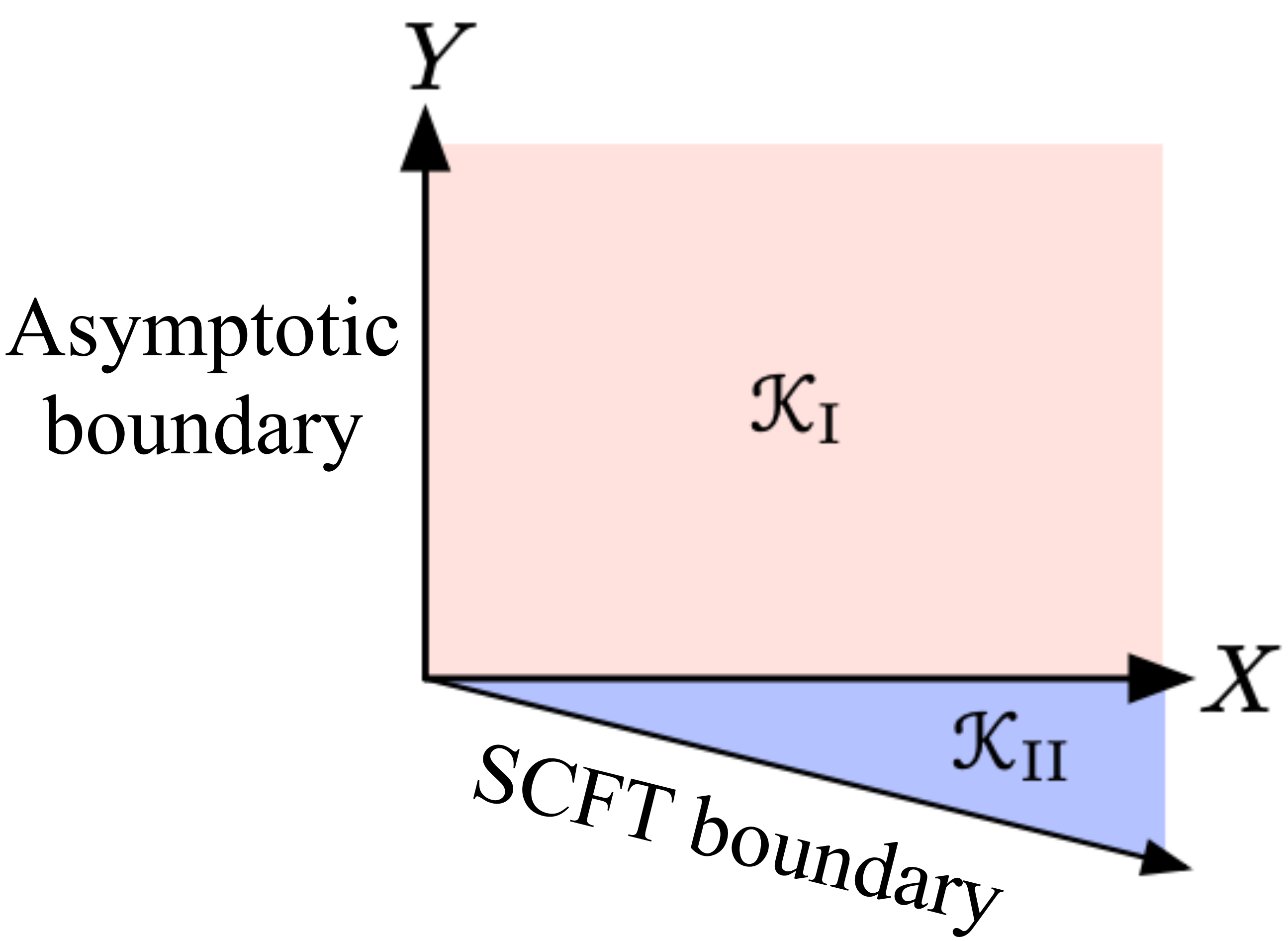
Flop transition

Boundaries of (Extended) Kähler Cone

- **Flop boundaries** involve hypermultiplets becoming massless.
  - Flop boundaries are boundaries of $\mathscr{K}_A$, not $\mathscr{K}$.
- **Weyl reflection boundaries** involve vector multiplets becoming massless and U(1) →SU(2) enhancement.
- **CFT boundaries** are strongly coupled boundaries with interacting SCFTs.
- **Asymptotic boundaries** represent infinite-distance limits in moduli space.
- **Periodic boundaries** represent accumulation points for the action of an infinite-order duality group. Said differently, they are infinite-distance limits of the marked moduli space [Raman, Vafa '24] that are not infinite-distance limits of the moduli space itself.

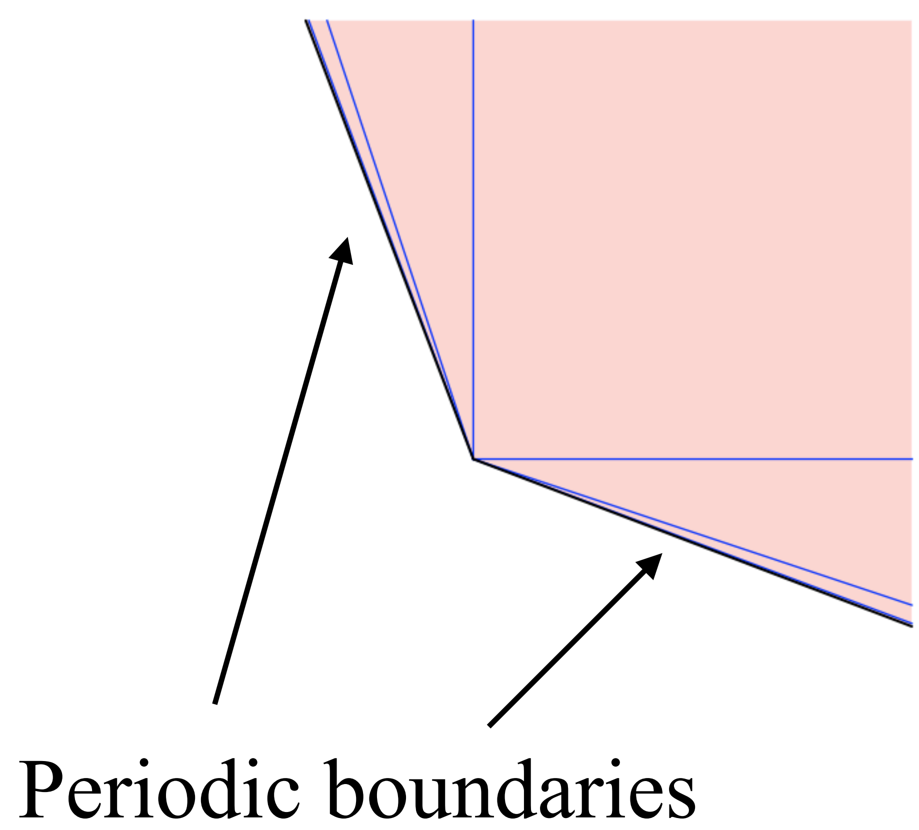
Examples

GMSV Geometry



Greene, Morrison, Strominger ’95;
Greene, Morrison, Vafa ’96;
Alim, Heidenreich, TR ‘21

BCLR Geometry



Brodie, Constantin, Lukas, Ruehle ’21;
Alim, Heidenreich, TR ‘21

Boundaries and Light States

- Various Swampland conjectures (Emergent String Conjecture, tower WGC, WGC for strings) require the presence of light particles and/or strings at infinite-distance limits in moduli space.
- In the GMSV geometry, there exists a tower of BPS particles of charge kq_I, BPS string of charge $\tilde{q}^I$, whose mass/tension vanish at the asymptotic boundary.
- In the BCLR geometry, there exist particle, string charges $q_I, \tilde{q}^I$ whose central charges vanish. However, these charges are **irrational** $\Rightarrow$ no light particles/strings (as expected for periodic boundary).
- Problem: how do we know that this is true in full generality?

Cone Conjectures

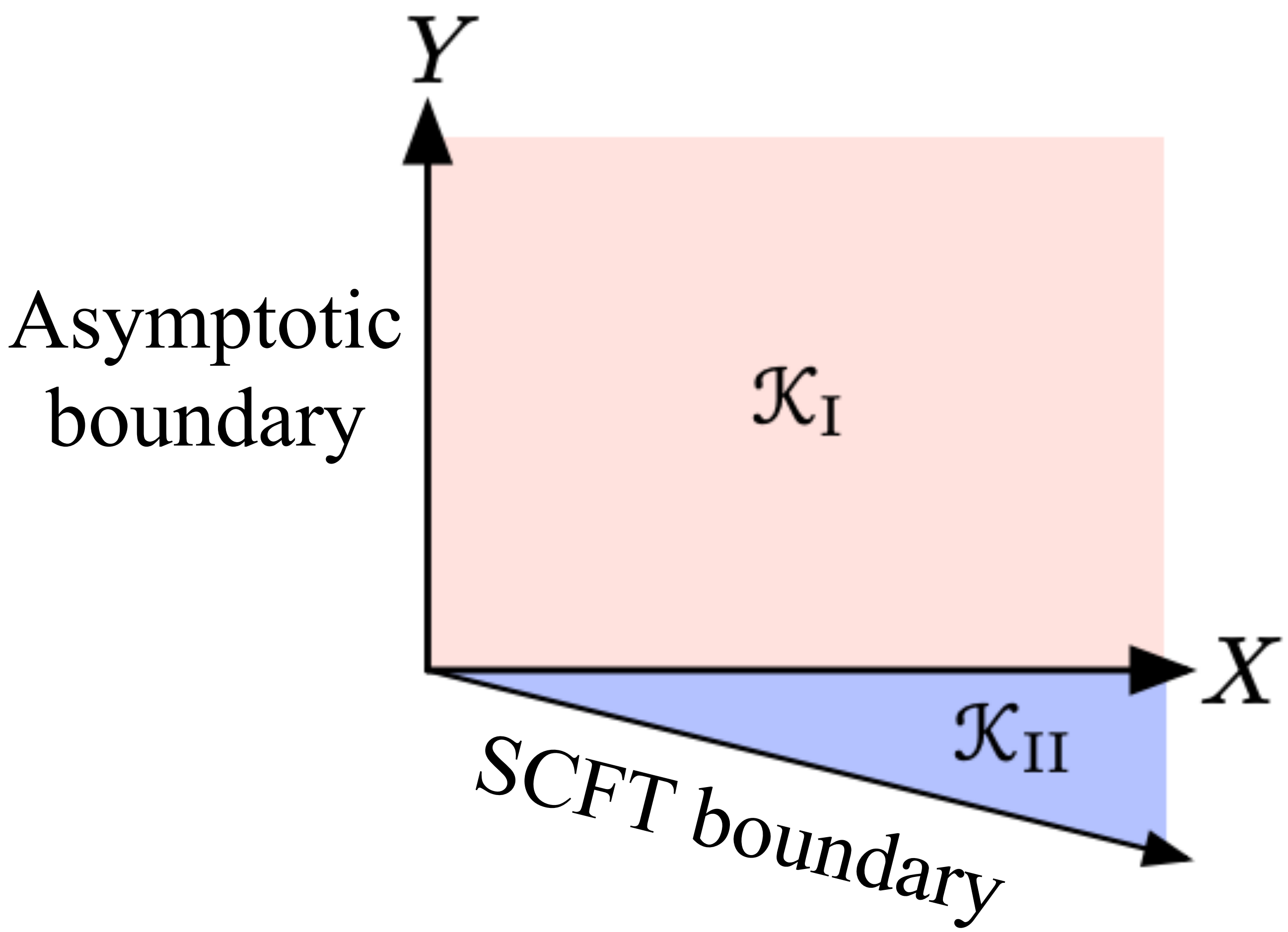
Rational Closure

- The *rational closure* $\mathscr{C}^+$ of a cone $\mathscr{C}$ is the union of all rational polyhedral subcones of $\overline{\mathscr{C}}$.
- Note that the rational closure $\mathscr{C}^+$ contains the interior of $\mathscr{C}$, so it lies somewhere between its interior and closure:

$$\mathscr{C}^\circ \subseteq \mathscr{C}^+ \subseteq \overline{\mathscr{C}}.$$

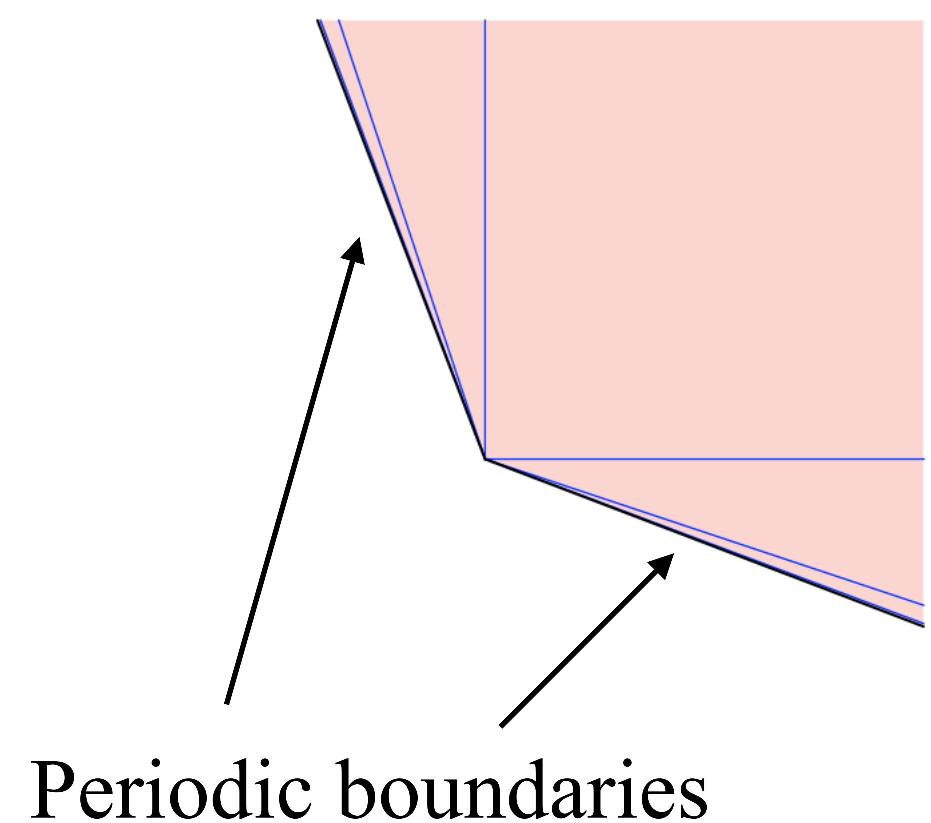
Examples

GMSV Geometry



$$\mathcal{K}^+ = \overline{\mathcal{K}}$$

BCLR Geometry

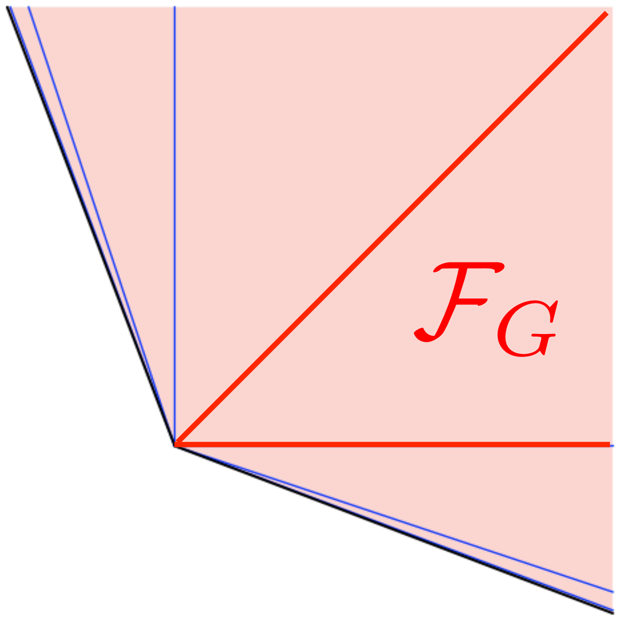


$$\mathcal{K}^\circ = \mathcal{K}^+$$

Cone Conjectures

- **Cone Conjecture** [Morrison '93]. There exists a rational polyhedral subcone $\mathscr{F}_{G_A} \subset \mathscr{K}_A^+$ that is a fundamental domain for the action of the duality group G_A on $\mathscr{K}_A^+$.
- **Birational Cone Conjecture** [Morrison '94]. There exists a rational polyhedral subcone $\mathscr{F}_G \subset \mathscr{K}^+$ that is a fundamental domain for the action of the duality group G on $\mathscr{K}^+$.

BCLR Geometry



$G = \mathbb{Z} \rtimes \mathbb{Z}_2$

Results

Results

- Infinite-distance boundary points in $\mathscr{K}^+$ represent asymptotic boundaries.
- Infinite-distance boundary points of $\overline{\mathscr{K}} \backslash \mathscr{K}^+$ represent periodic boundaries.
- (Birational) cone conjecture ⇒ at every asymptotic boundary of $\mathscr{K}_A^+$ ($\mathscr{K}^+$), there exists a *rational* string charge $\tilde{q}^I$ whose central charge vanishes, (i.e., there exists a tensionless BPS string, as required by the Emergent String Conjecture, WGC for strings).
- Similar argument ⇒ massless particles at asymptotic boundaries assuming novel “dual coordinate birational cone conjecture.”
- Birational cone conjecture ⇒ Any boundary located behind an infinite sequence of flop transitions is a periodic boundary rather than an asymptotic boundary (hence it is not a genuine infinite-distance limit, swampland conjectures do not apply).
- **Main point: beautiful interplay between swampland and cone conjectures!**

Thank you!