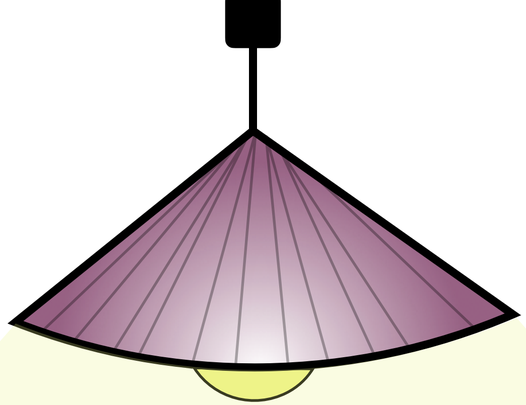
[**M.D.**, L. Eberhardt, M. Tomasevic 'WIP]



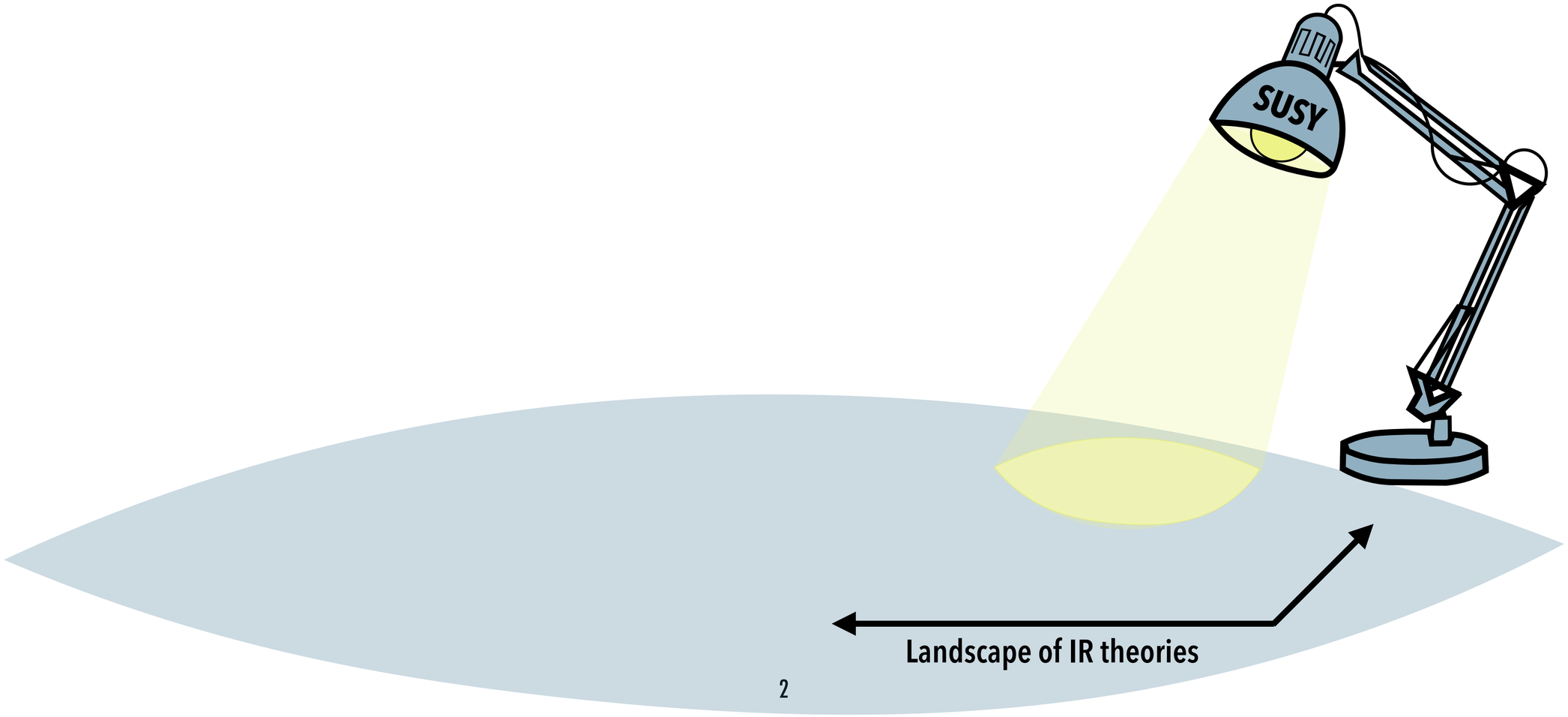
SPACETIME CONSTRAINTS FROM WORLDSHEET ANOMALIES

MATILDA DELGADO

The Supersymmetric Lamppost

Efforts to connect string theory to low energy physics and our universe:

→ focused on supersymmetric setups



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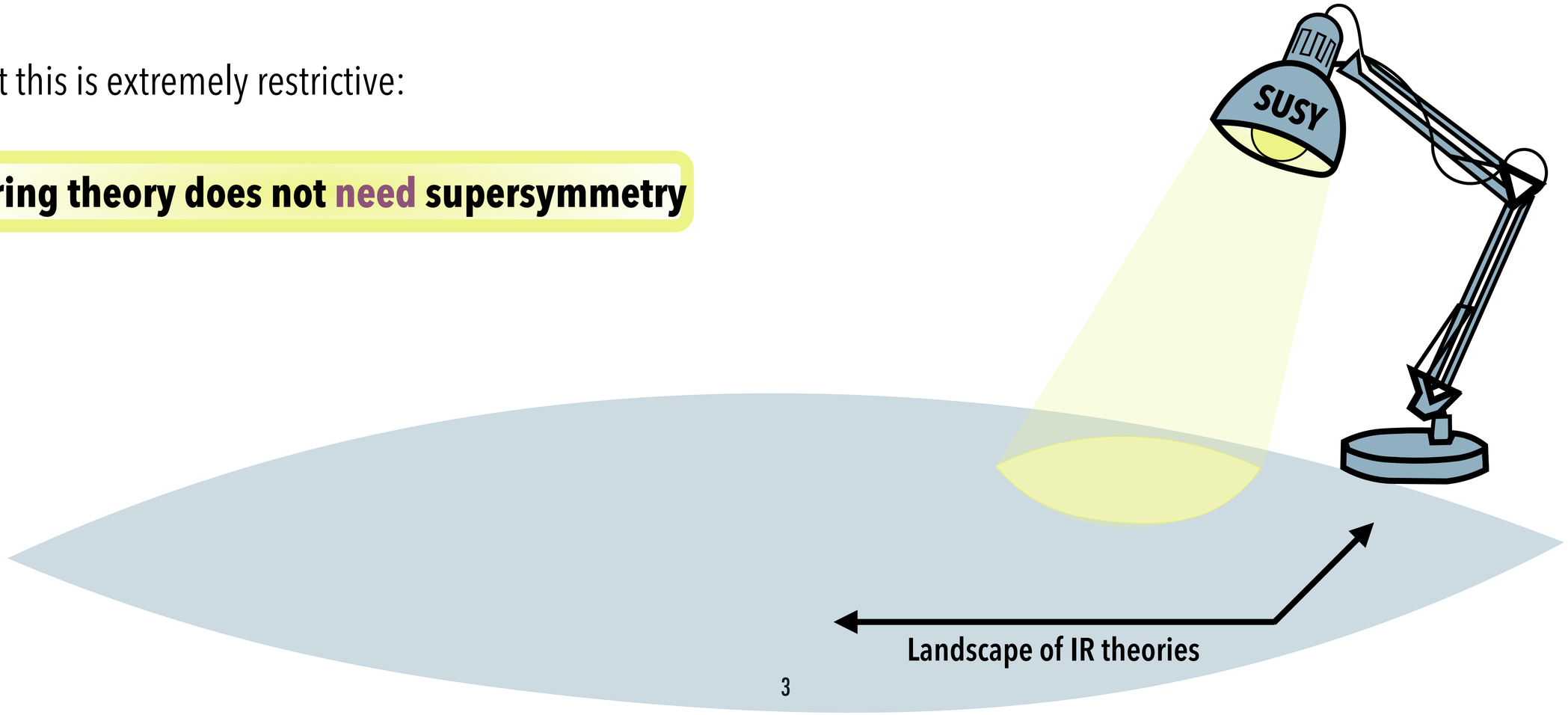
String theory does not need supersymmetry



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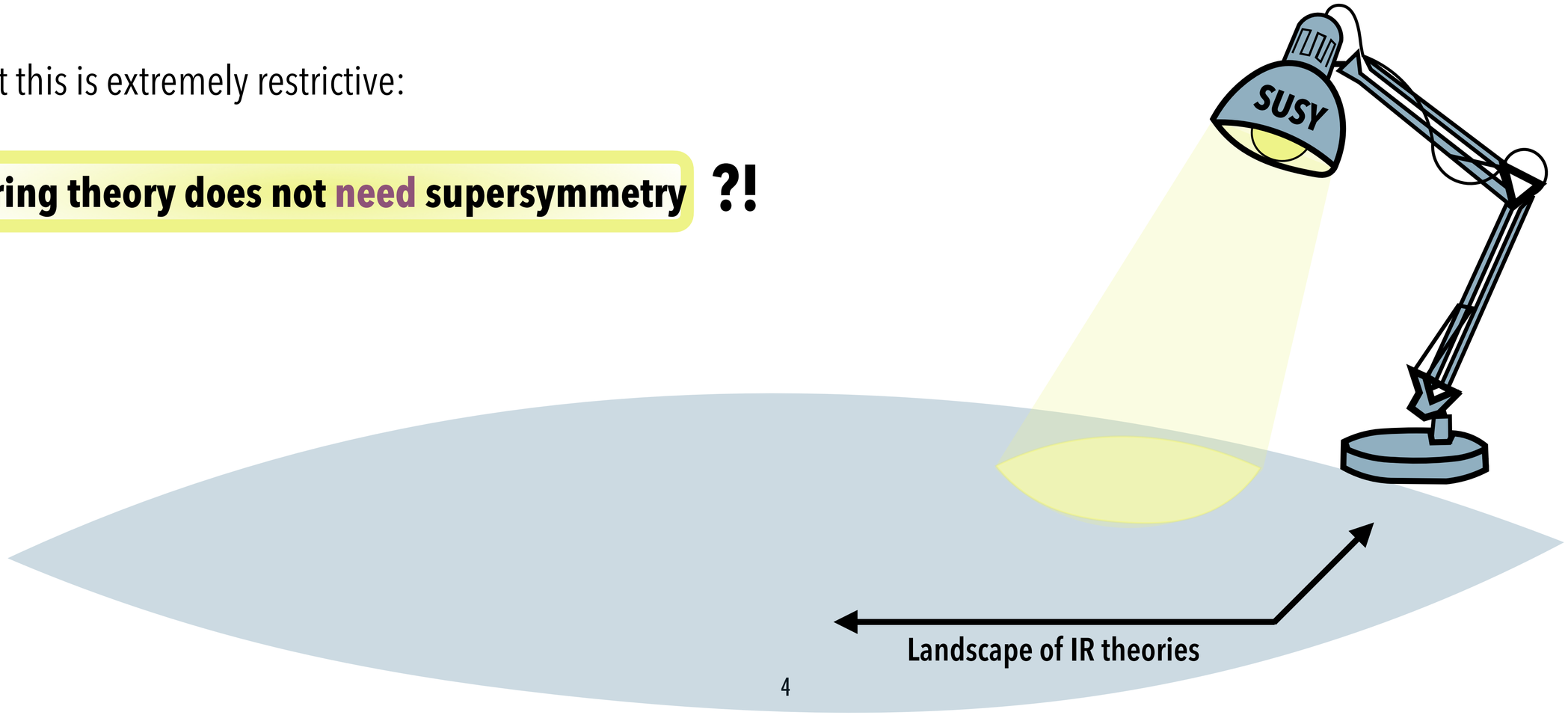

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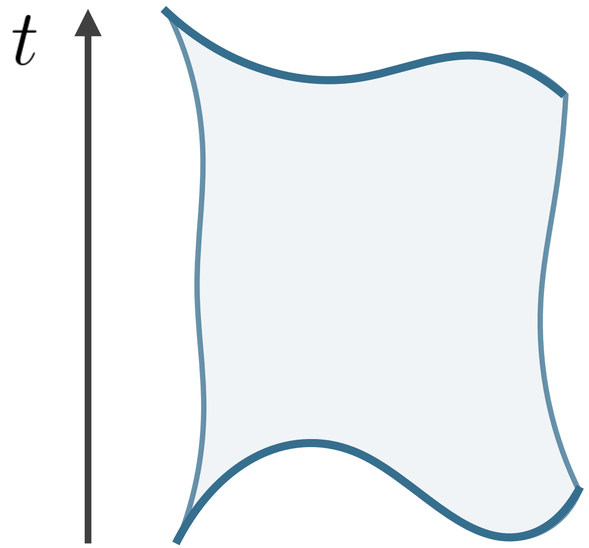
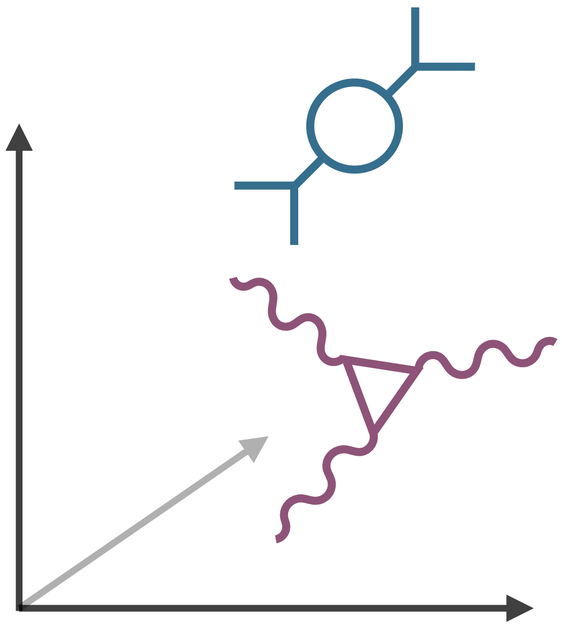


2D SCFTs on the string worldsheet

Low-energy limit

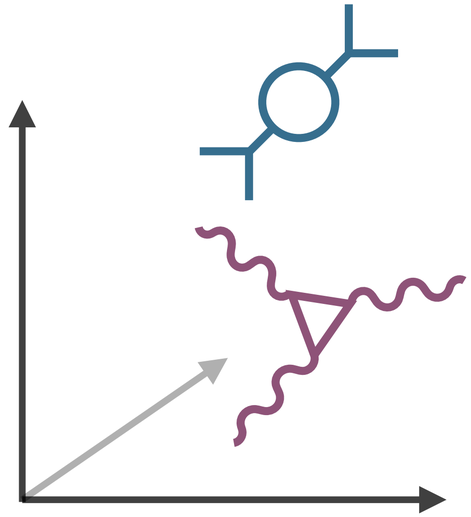


D-dimensional EFT of Einstein Gravity minimally coupled to a QFT

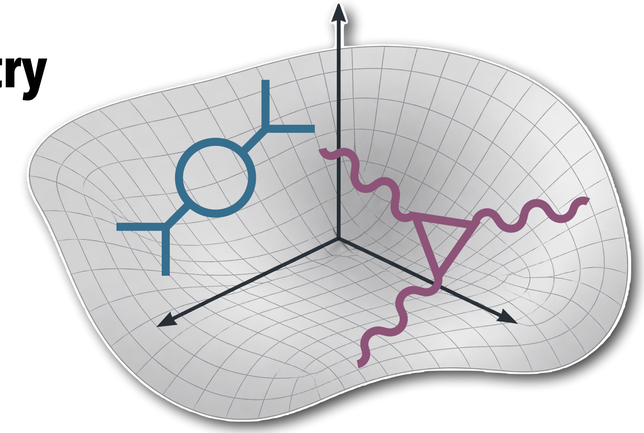
SUPERSYMMETRIC or NOT

String Theory does not need supersymmetry

One can also consider supersymmetric string theories
On supersymmetry breaking backgrounds



Break supersymmetry



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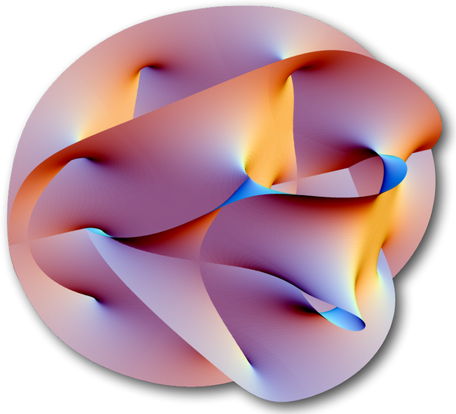

Tori, K3, Calabi-Yau 3/4-folds,
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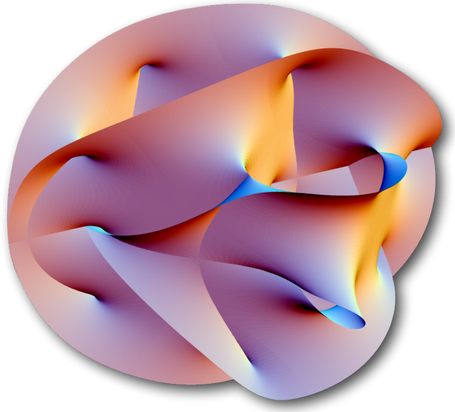

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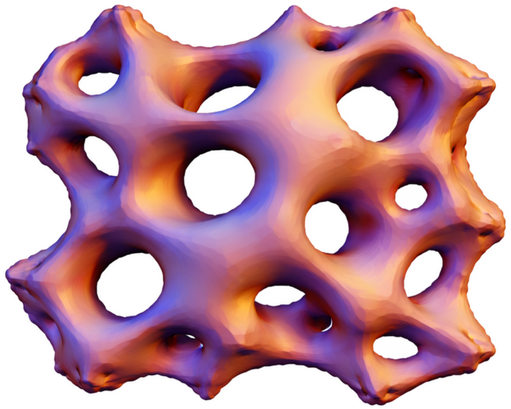

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Pretty much anything else
Example: Riemann surface with g>1,

Most backgrounds break SUSY

String Theory does not need supersymmetry

There are abundant mechanisms for spontaneous supersymmetry breaking

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What's the catch?

String Theory does not need supersymmetry

There are abundant mechanisms for spontaneous supersymmetry breaking

What's the catch?

without supersymmetry, string loop corrections and alpha' corrections are abundant
We lose computational control

Assessing the properties and even ***fundamental consistency*** of these theories is very hard!

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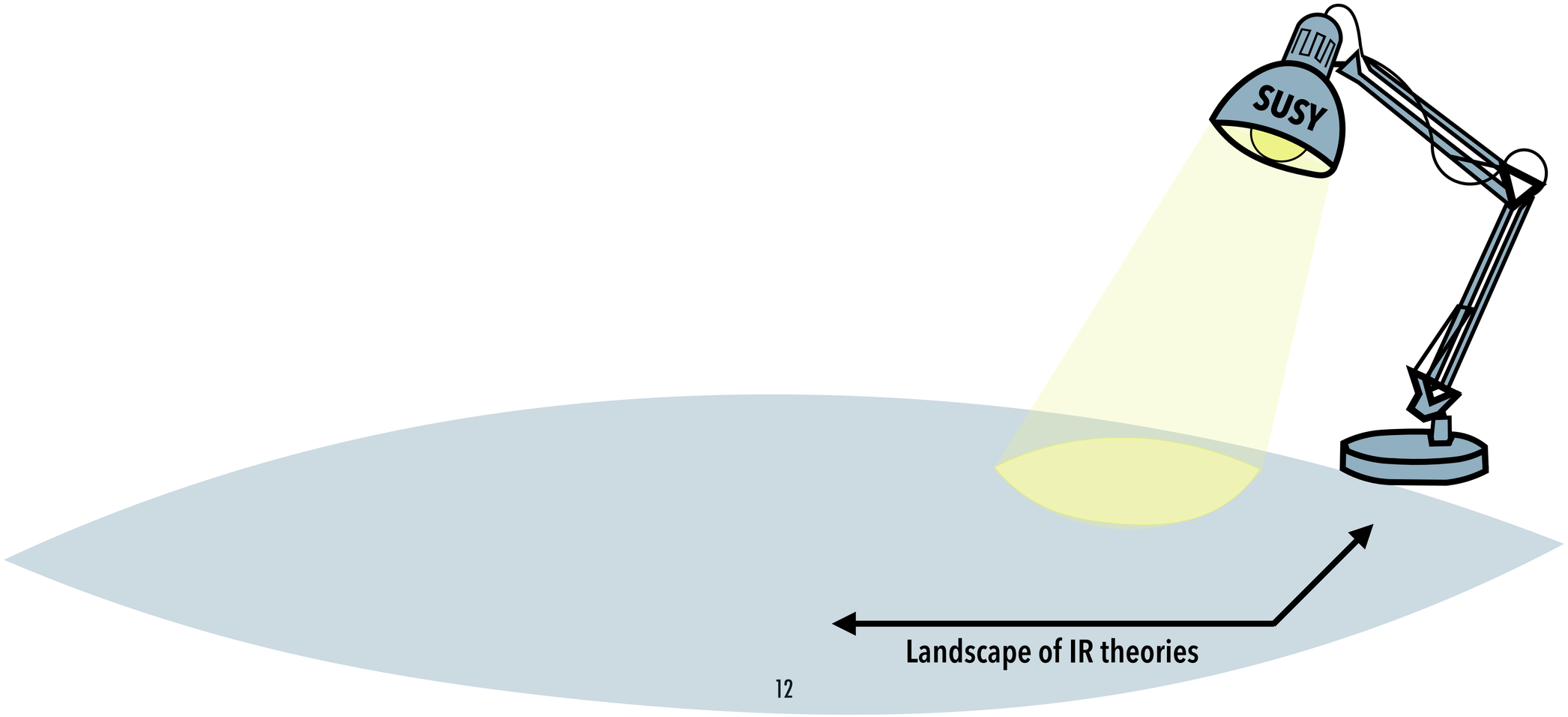

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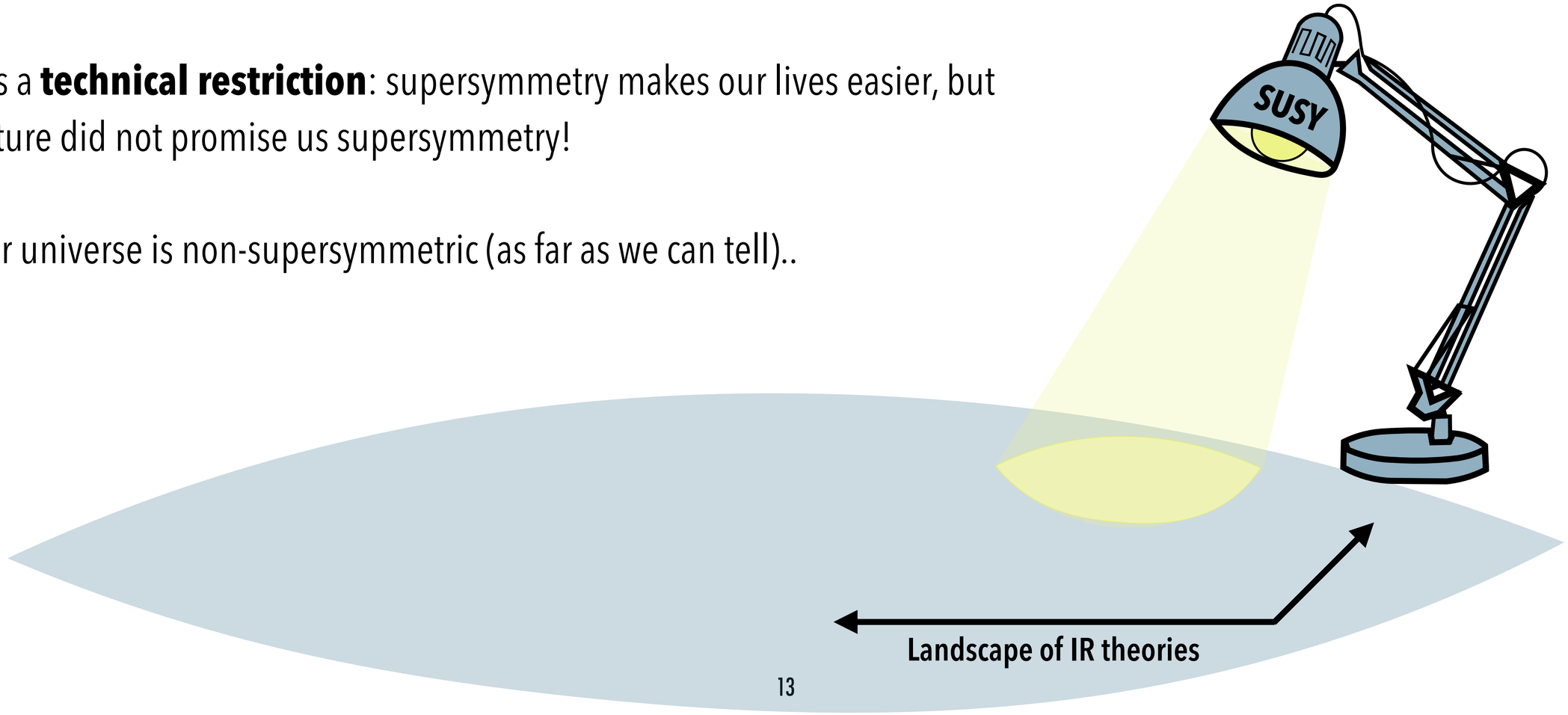
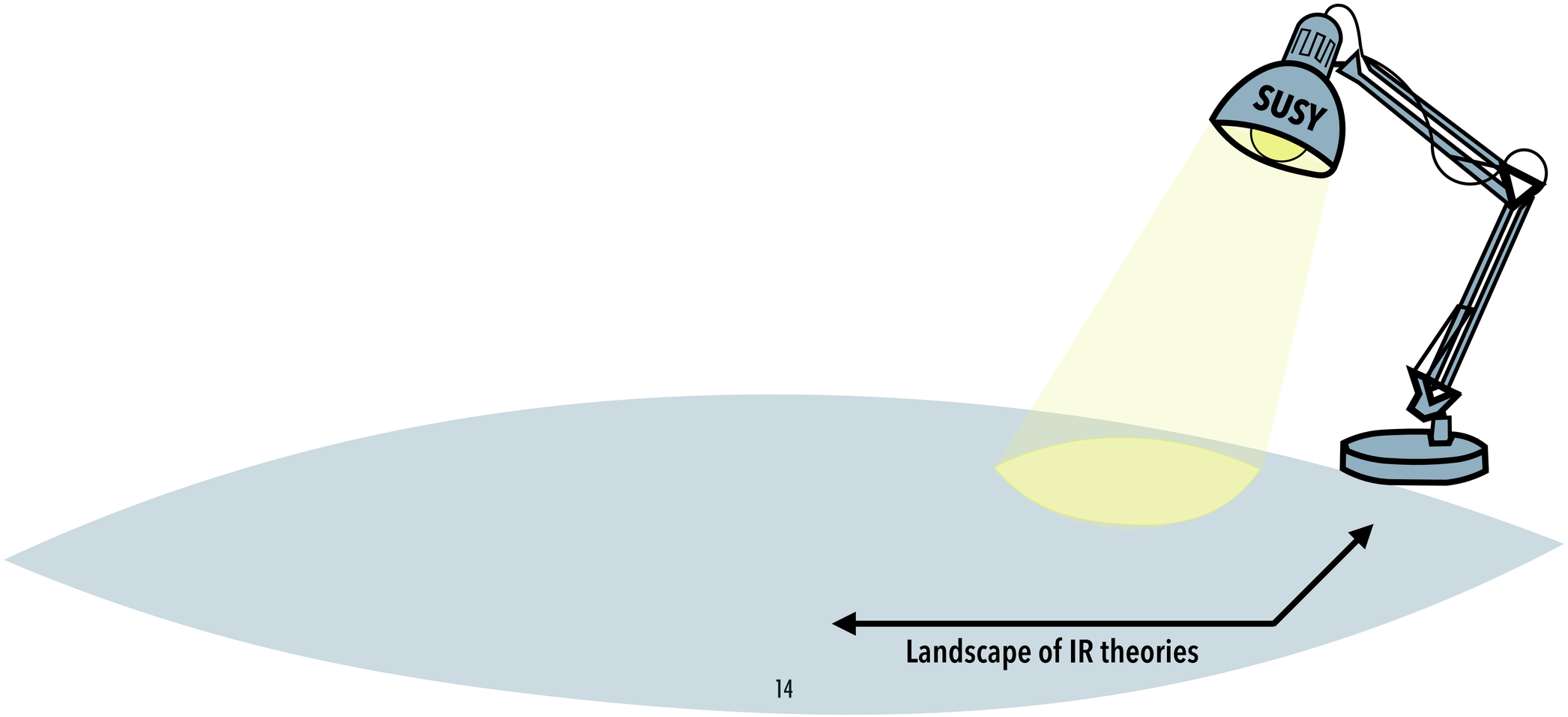
It is a **technical restriction**: supersymmetry makes our lives easier, but nature did not promise us supersymmetry!

Our universe is non-supersymmetric (as far as we can tell)..



Tools for strings without supersymmetry

Faced with control issues, what are the tools at our disposal?



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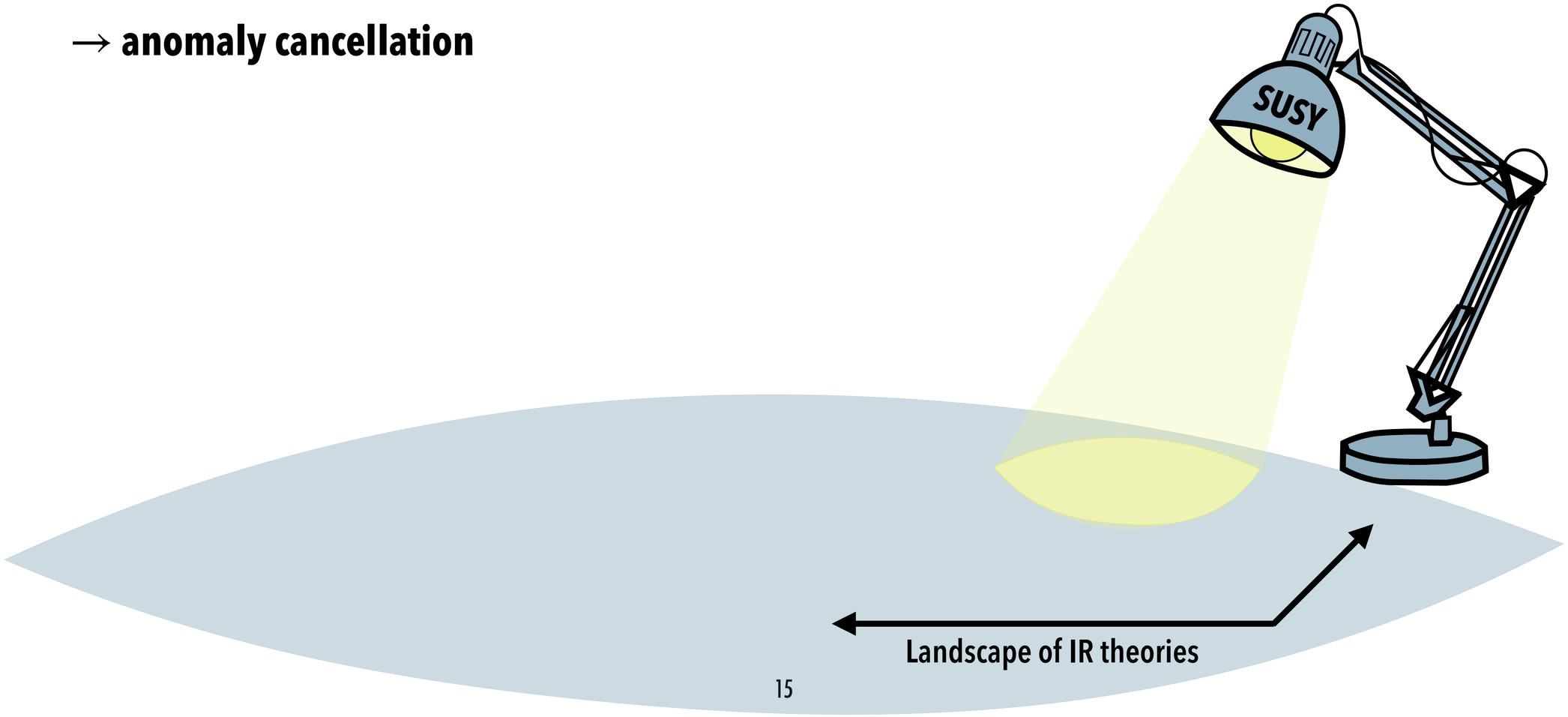

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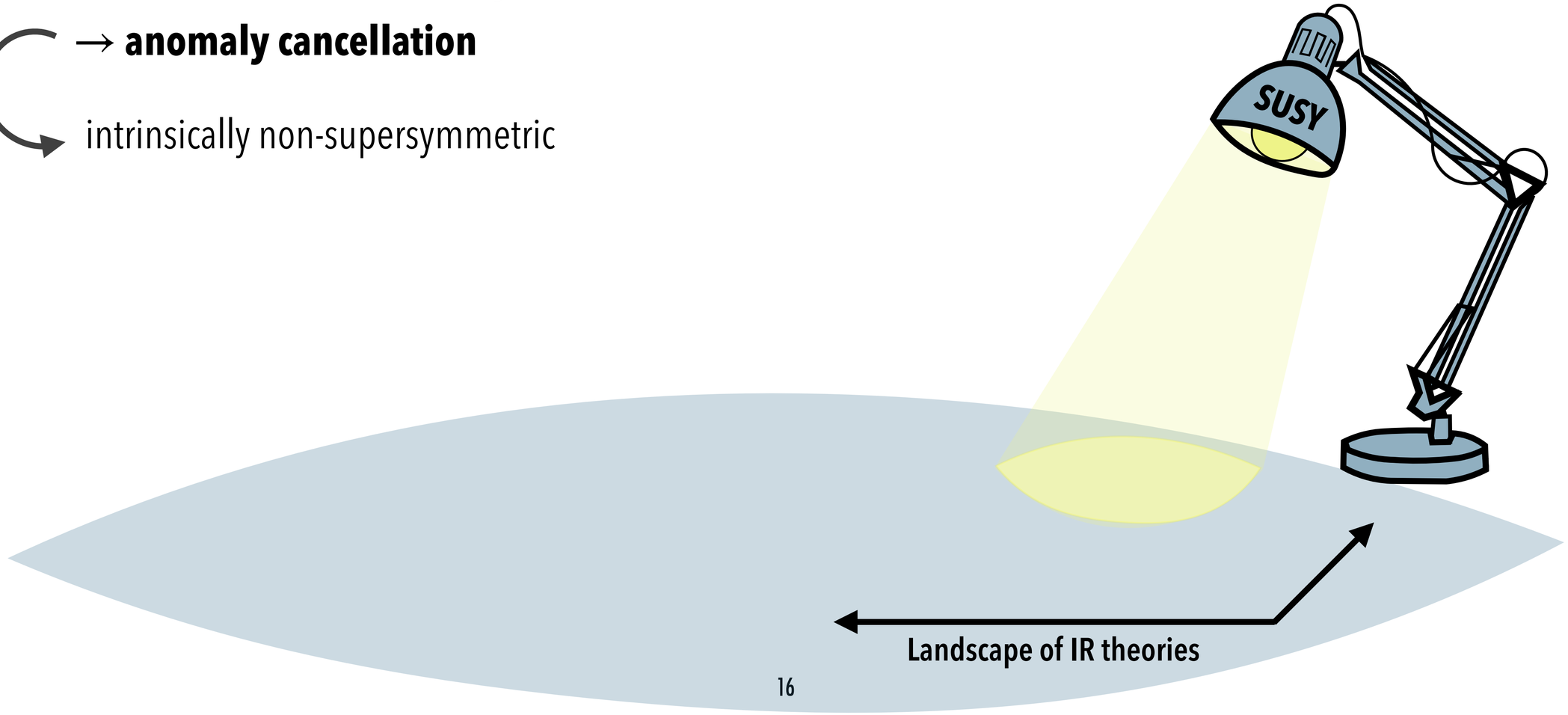

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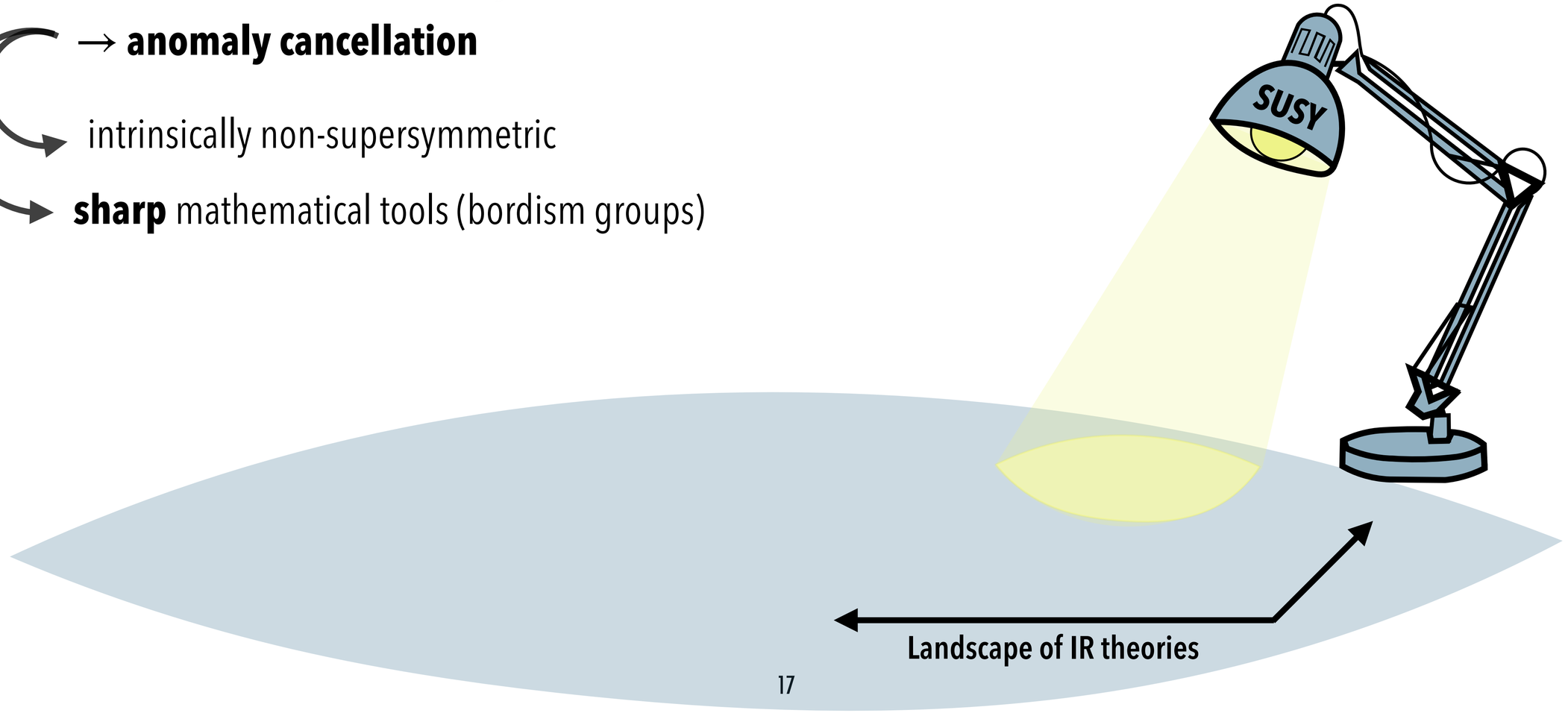
sharp mathematical tools (bordism groups)



TODAY

Use **gauge and gravitational anomaly cancellation**,

To gain traction on:

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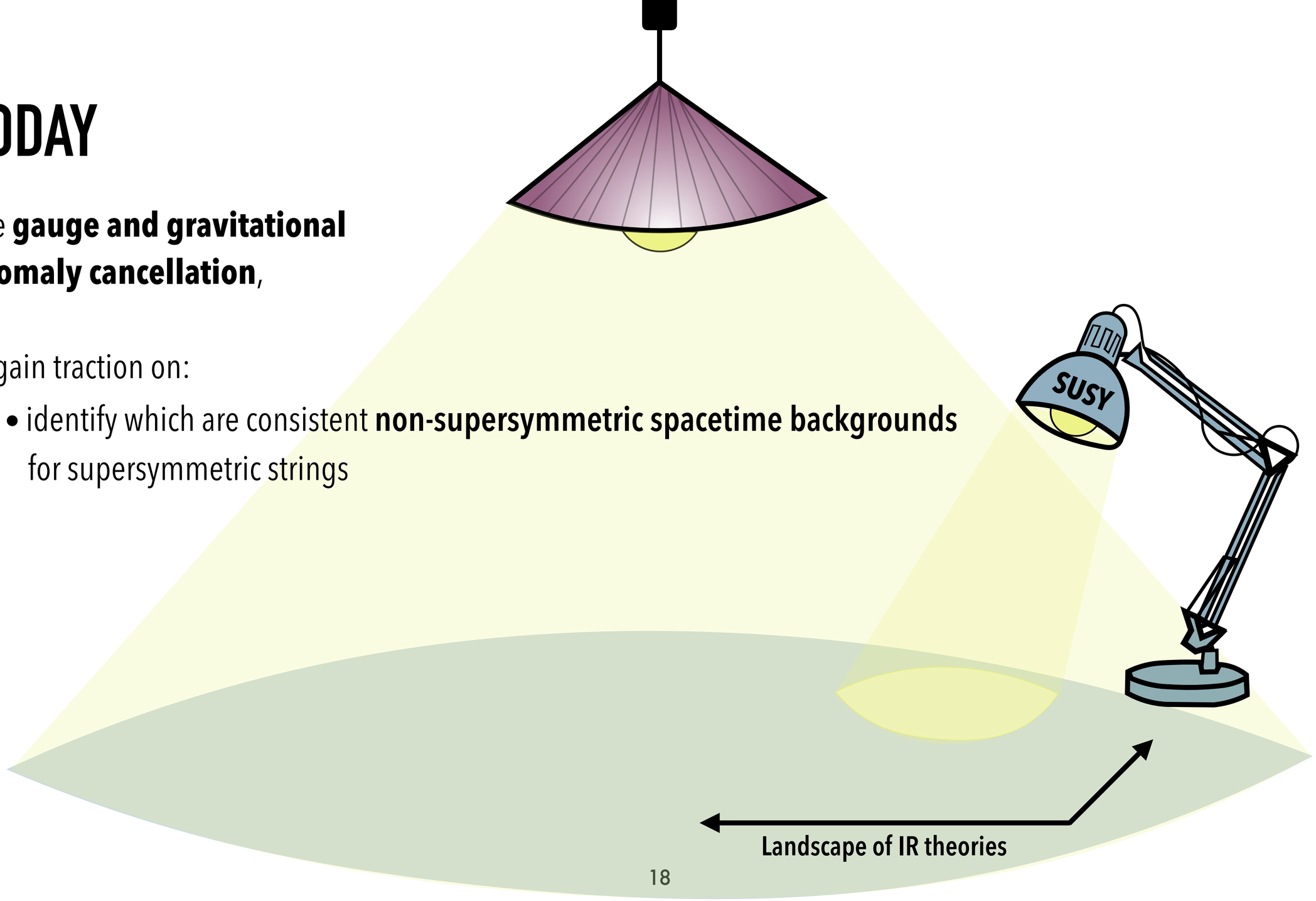

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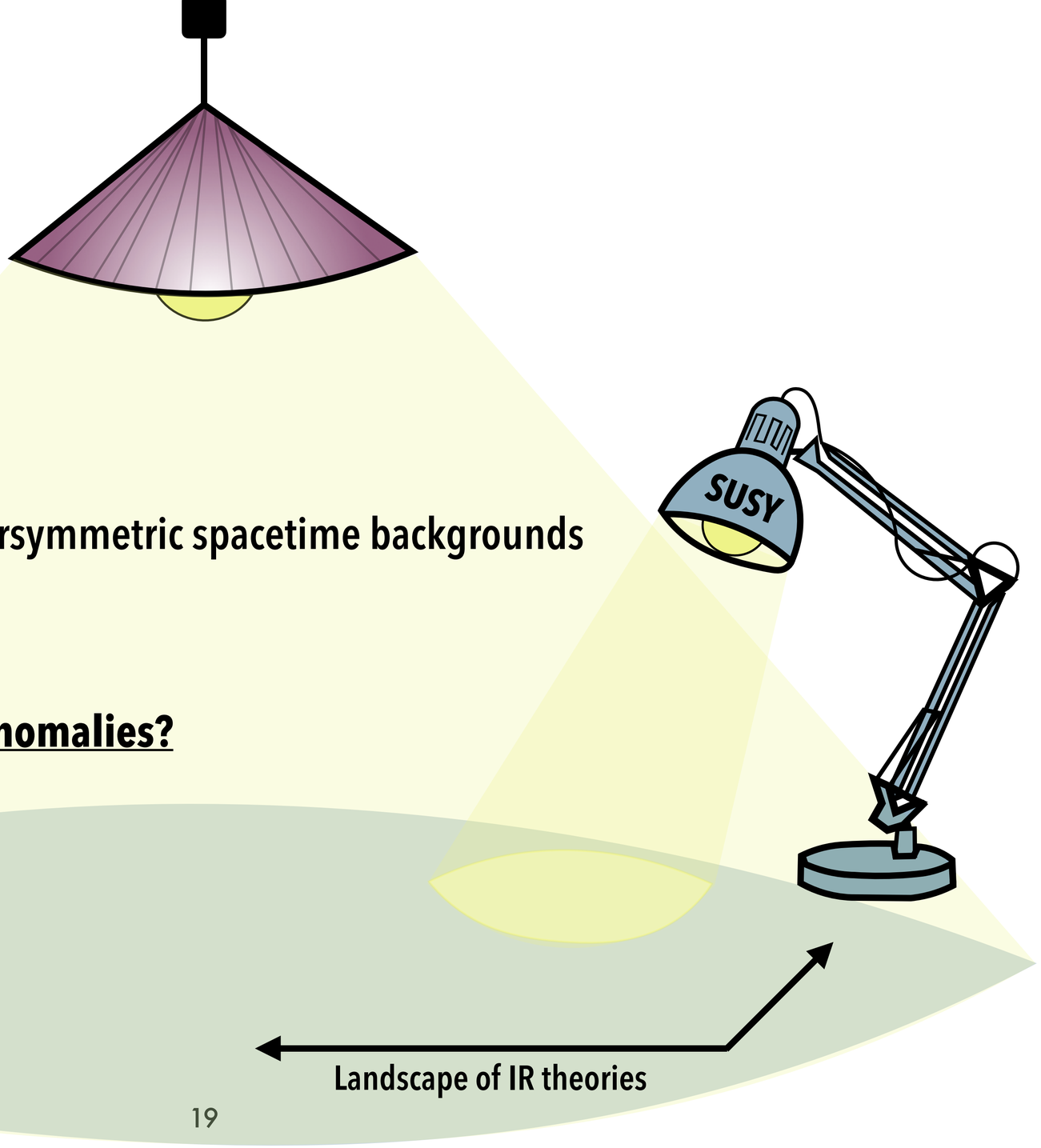
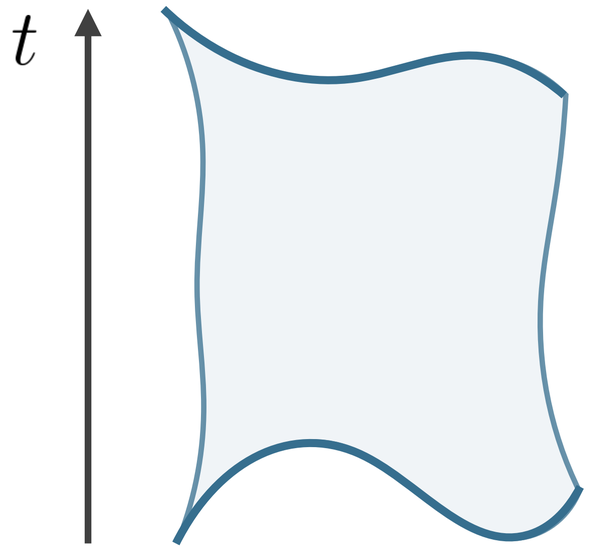


STRING THEORY

Two perspectives:



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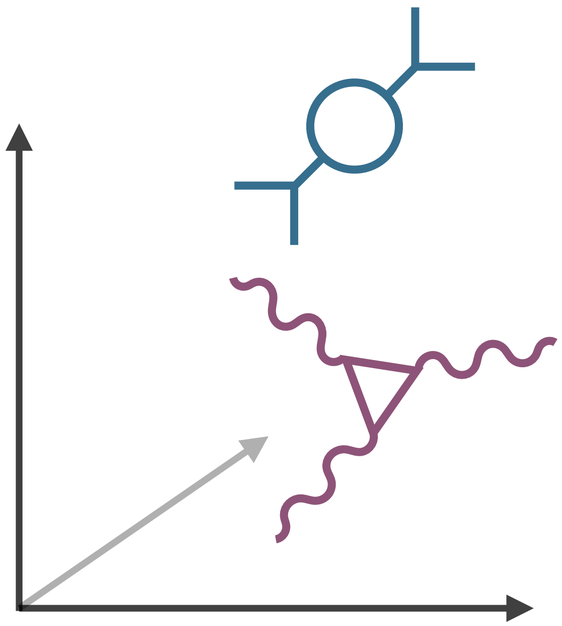

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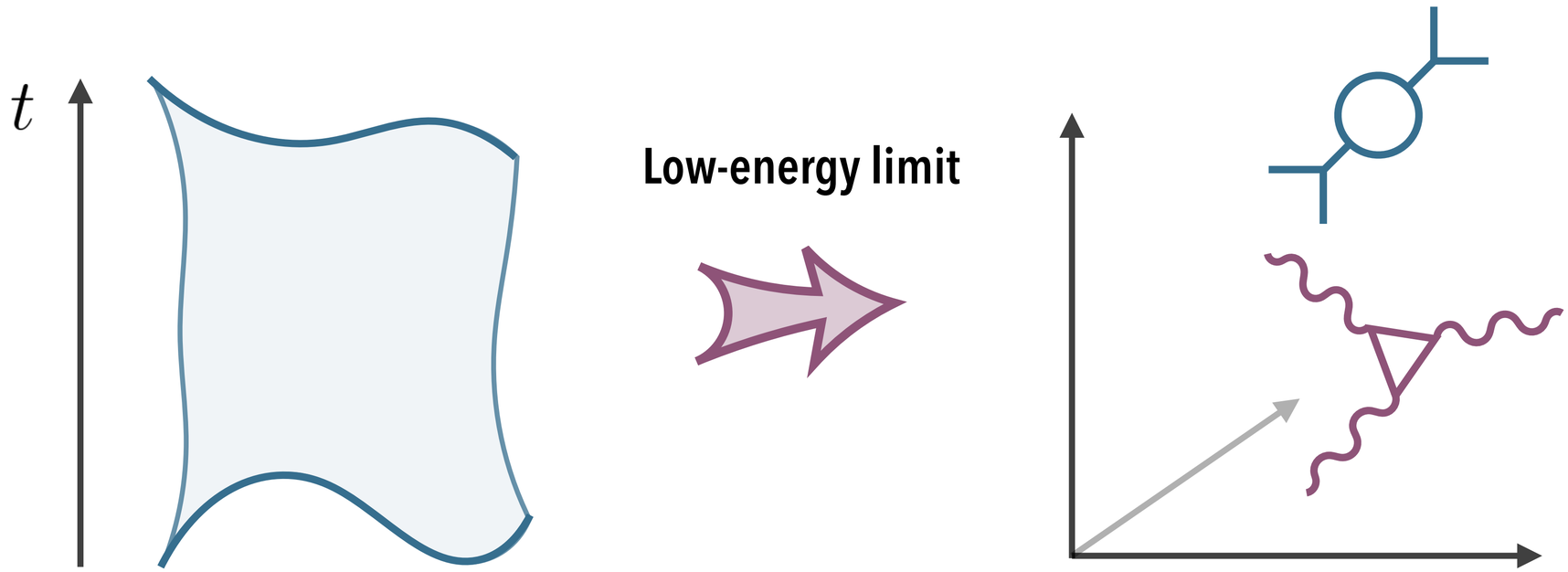

both theories have **chiral matter**

→ both can have anomalies!

STRING THEORY

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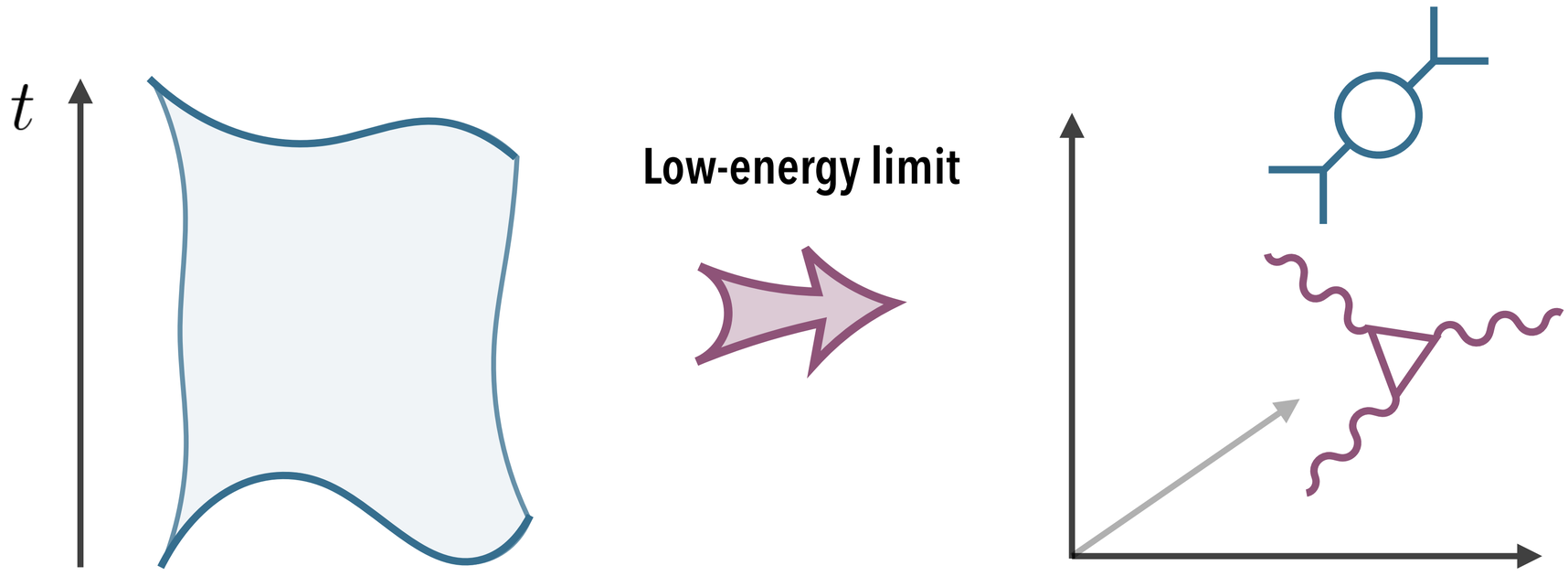
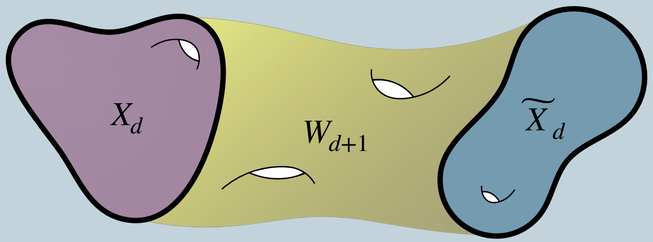


TODAY: study **anomaly cancellation** on the worldsheet → restrictions on spacetime

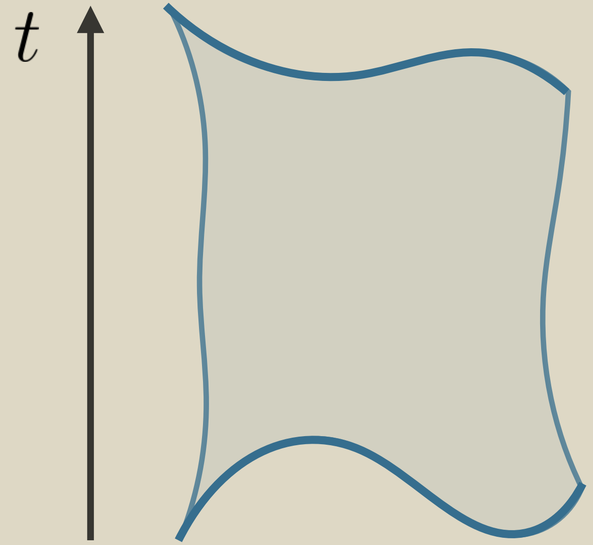
THE PLAN

Anomalies and Bordisms
A crash course



$$\Omega_d$$

Worldsheet anomalies for Type II strings on non-supersymmetric backgrounds



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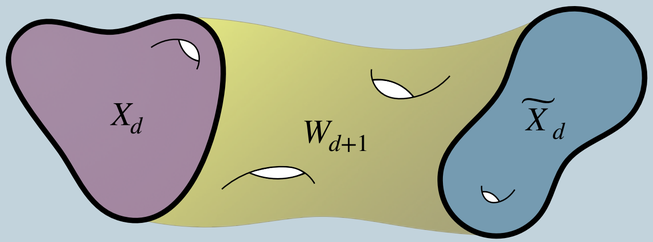

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  Think triangle (n-gon) diagrams ✅✅✅✅

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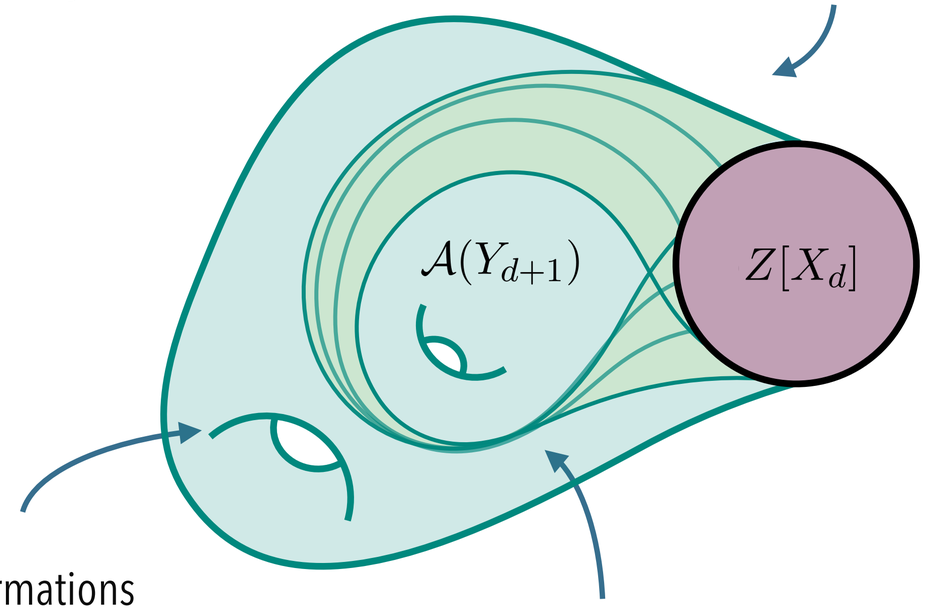

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$\rightarrow$To each anomalous d.o.f. in $Z[X_d]$,
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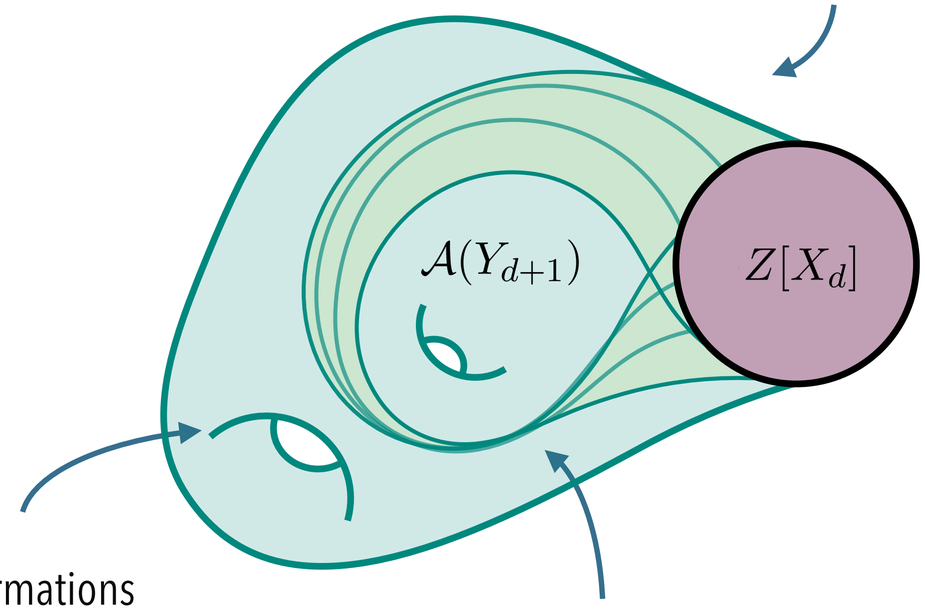

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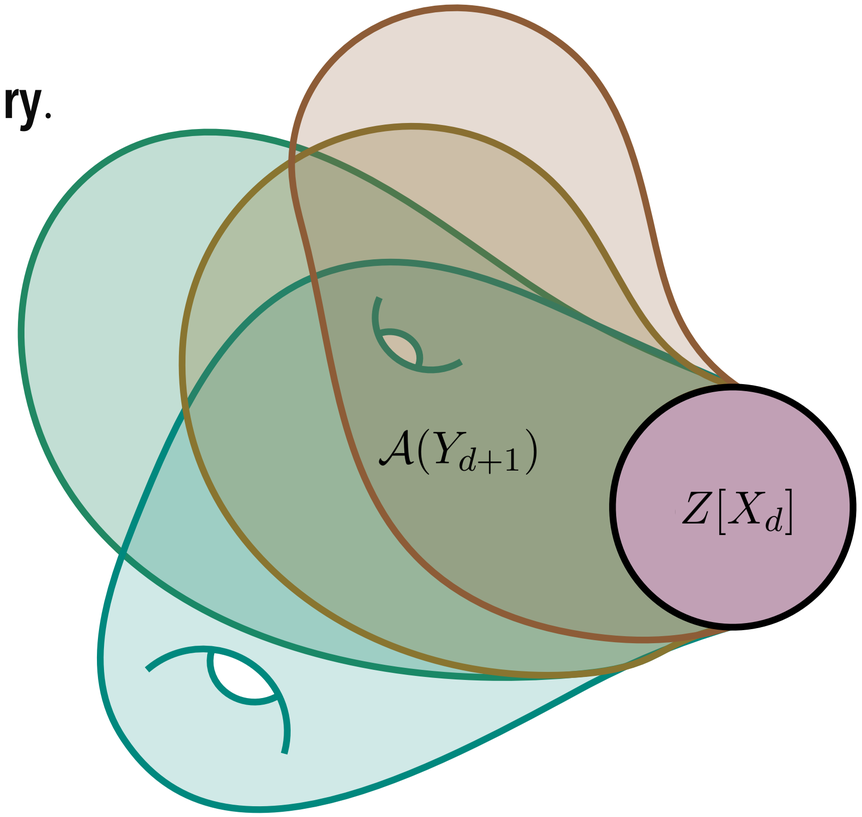

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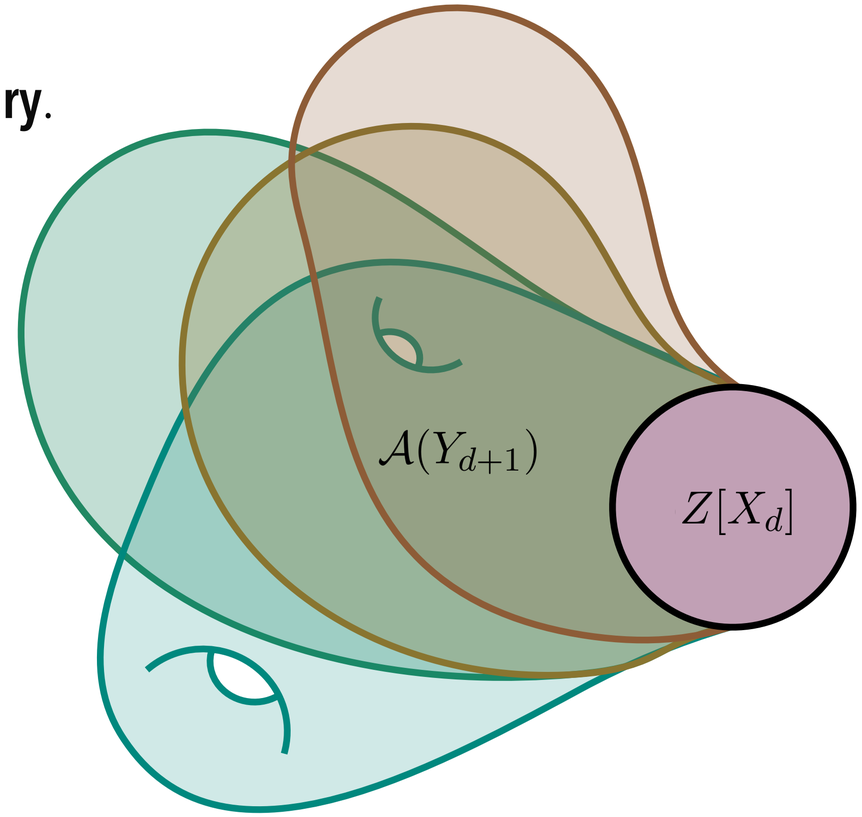

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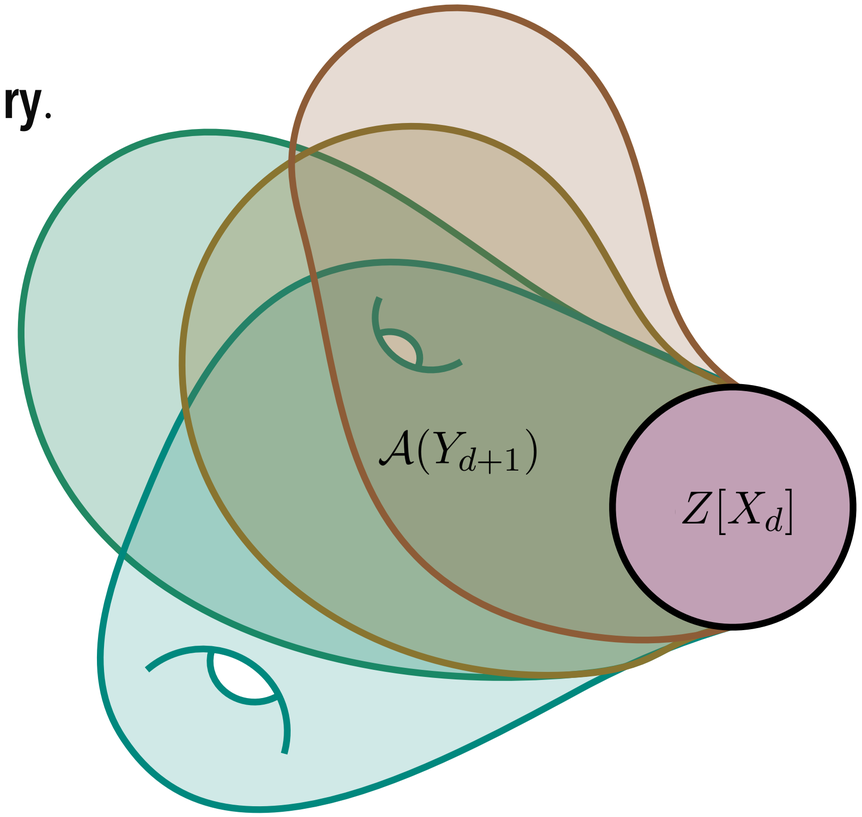

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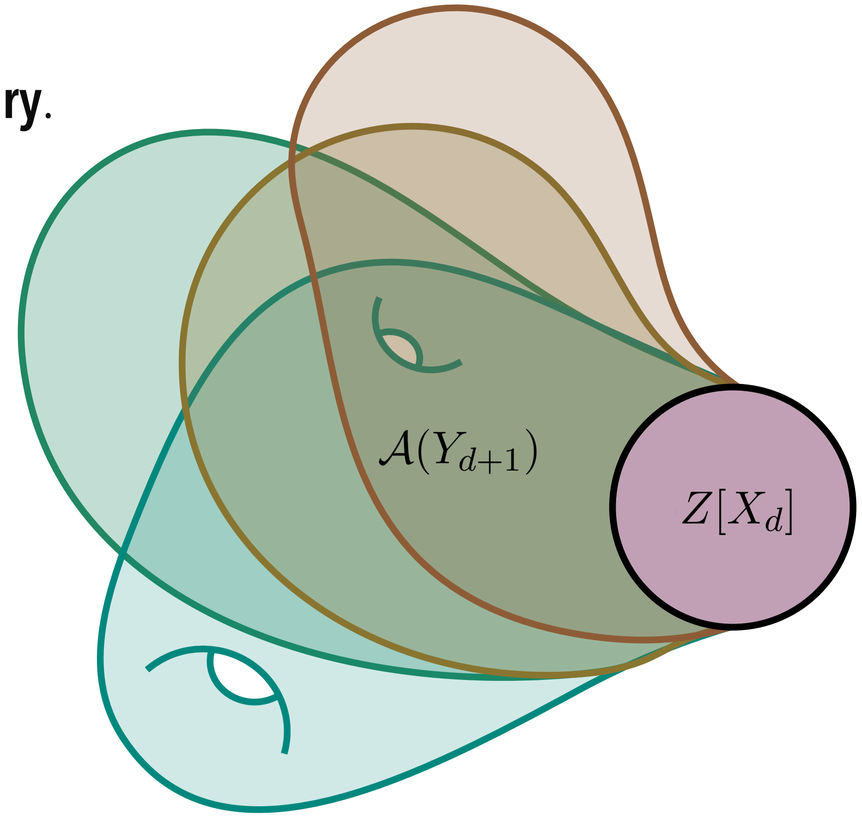

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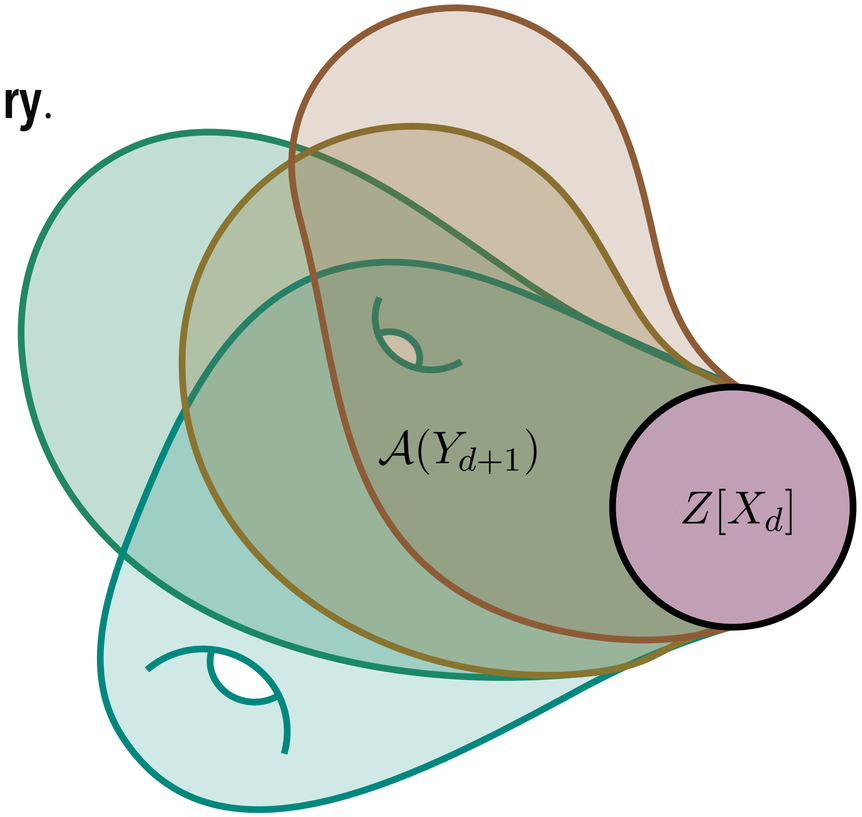
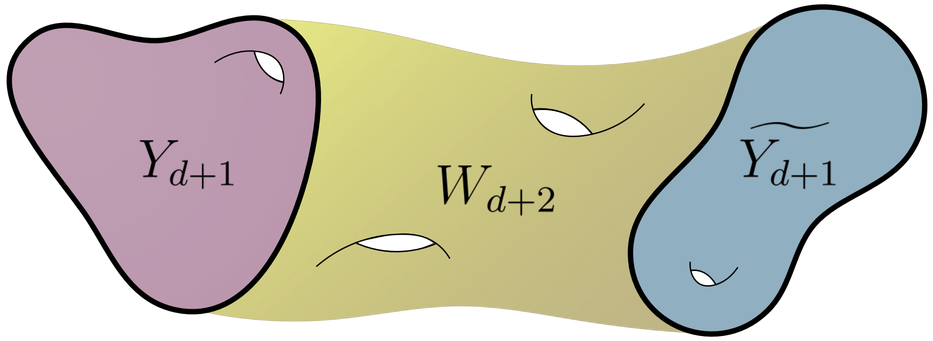


Bordism Groups

Bordism Groups: objects of algebraic topology

↪ classify which manifolds can be deformed into one another and preserve some structure (orientation, spin ..)



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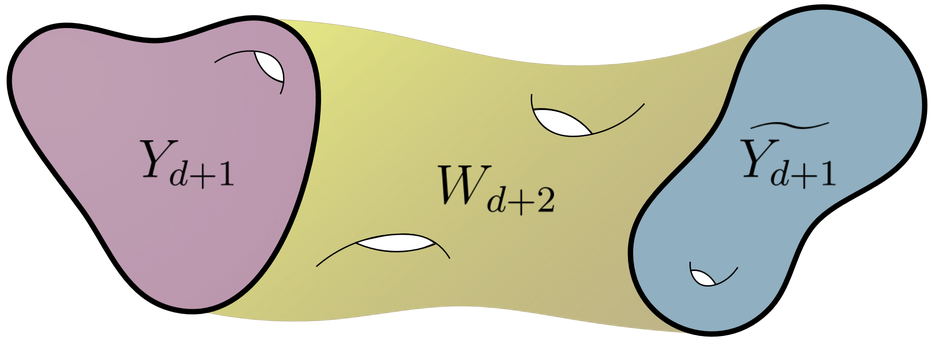

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Gauge/gravitational anomalies

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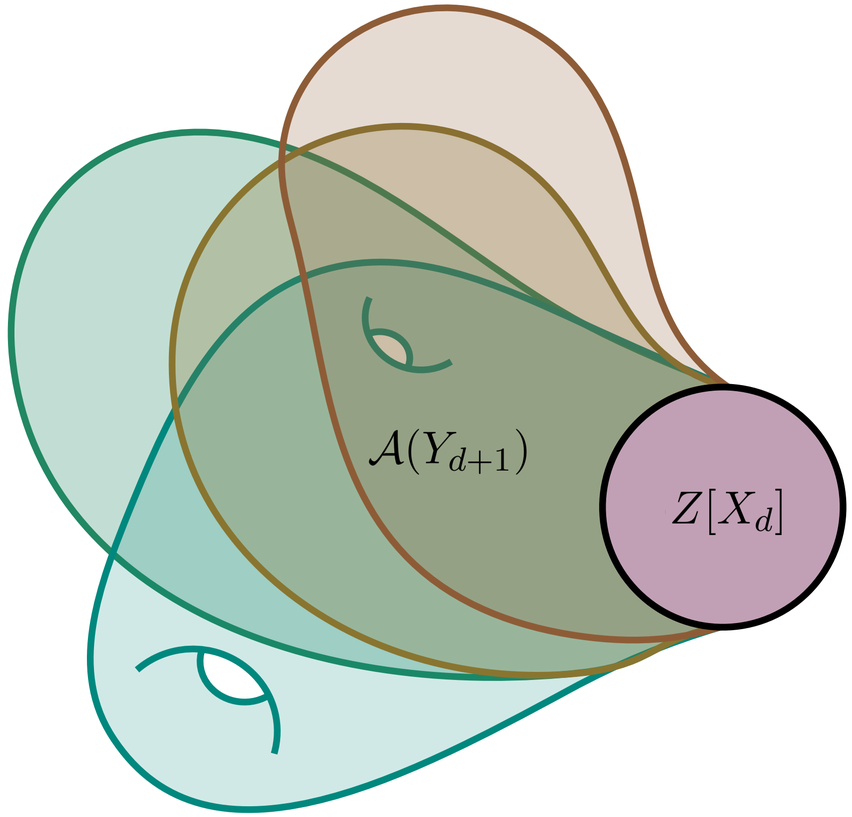

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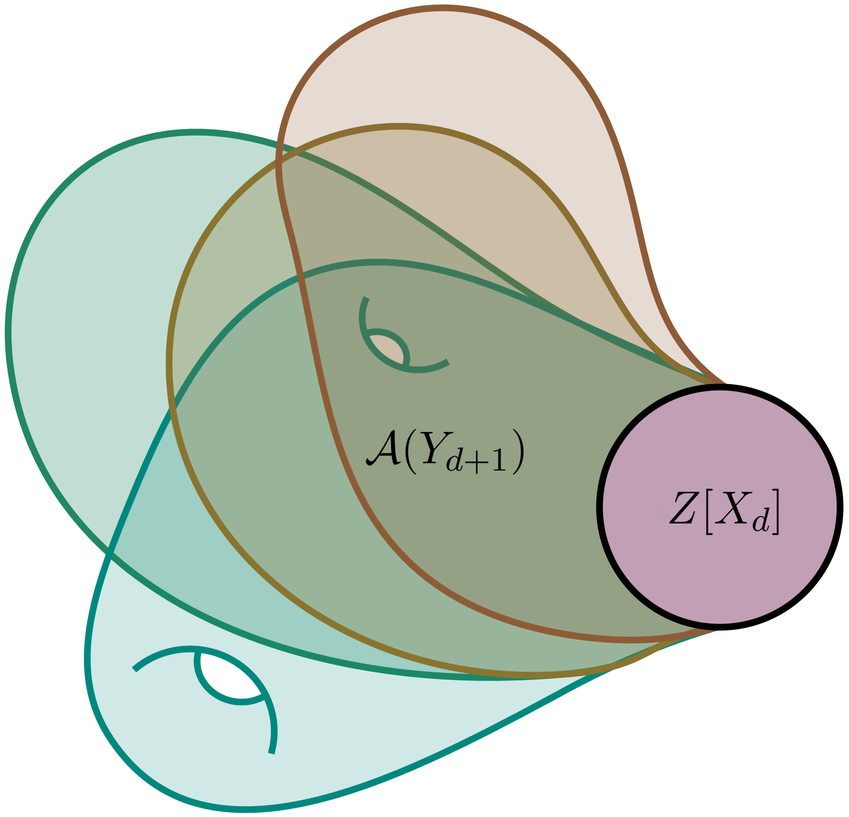

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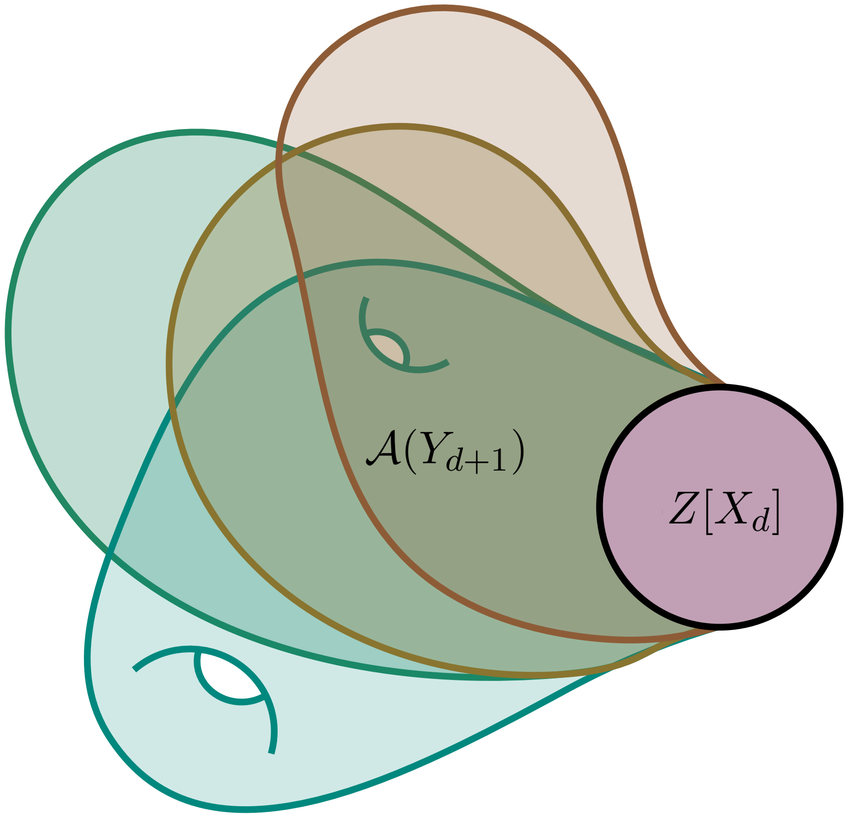
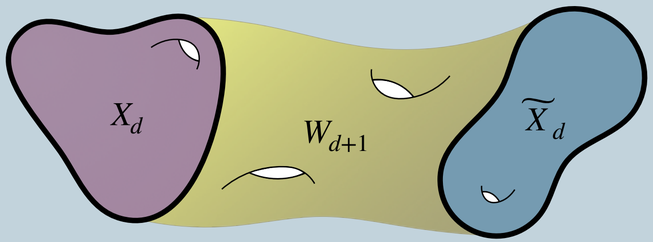
→ **If it is trivial:** you are done! **There are no global anomalies.**

$$\Omega_{d+1} = 0$$



THE PLAN

Anomalies and Bordisms
A crash course



$$\Omega_d$$

Worldsheet anomalies for Type II strings on non-supersymmetric backgrounds

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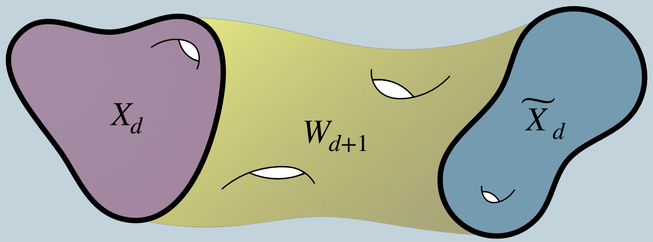

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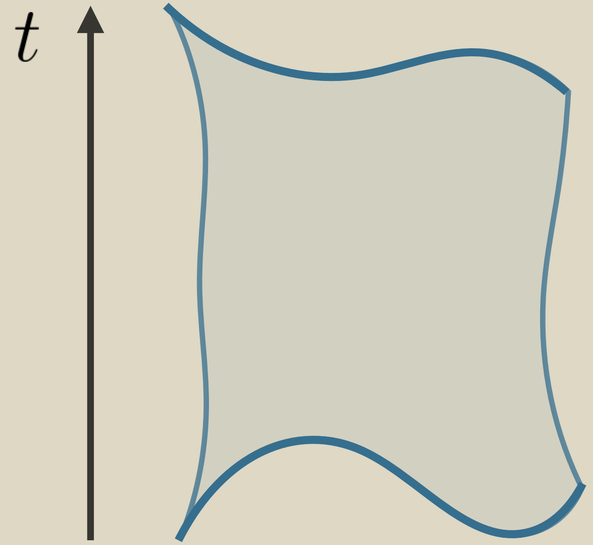
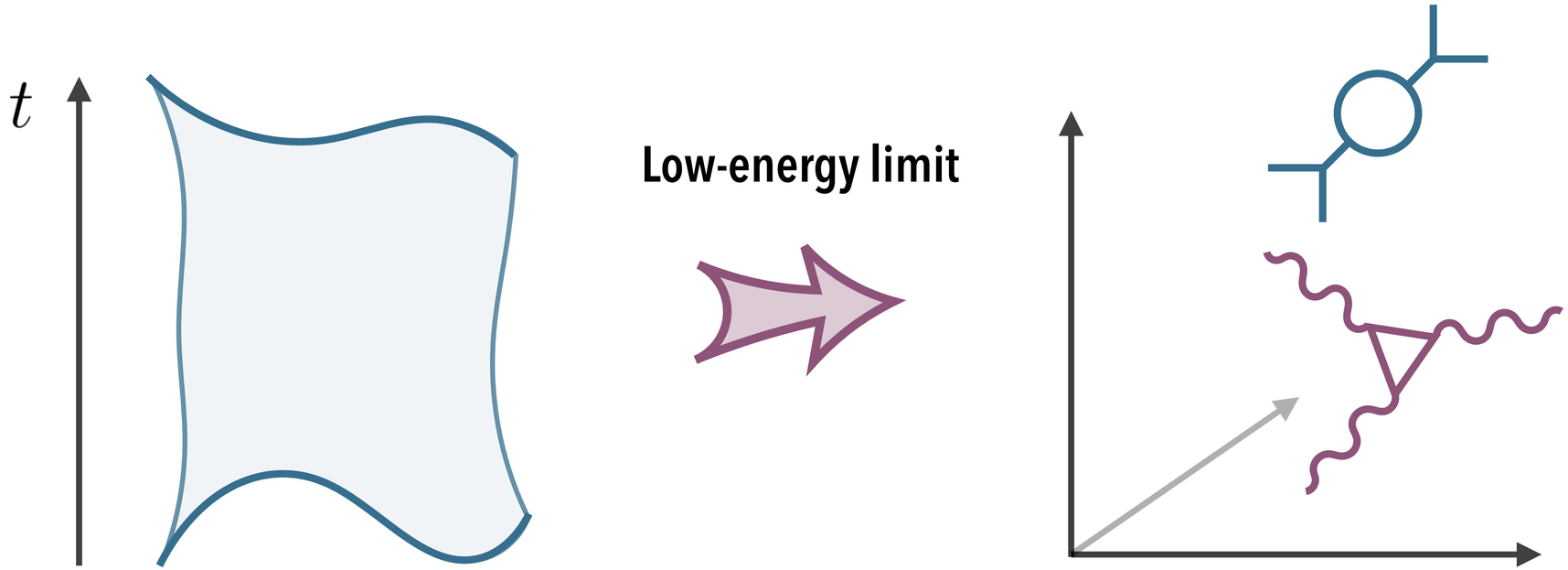


WORLDSHEET ANOMALIES AND SPACETIME



The worldsheet dictates the properties of spacetime in the low energy limit

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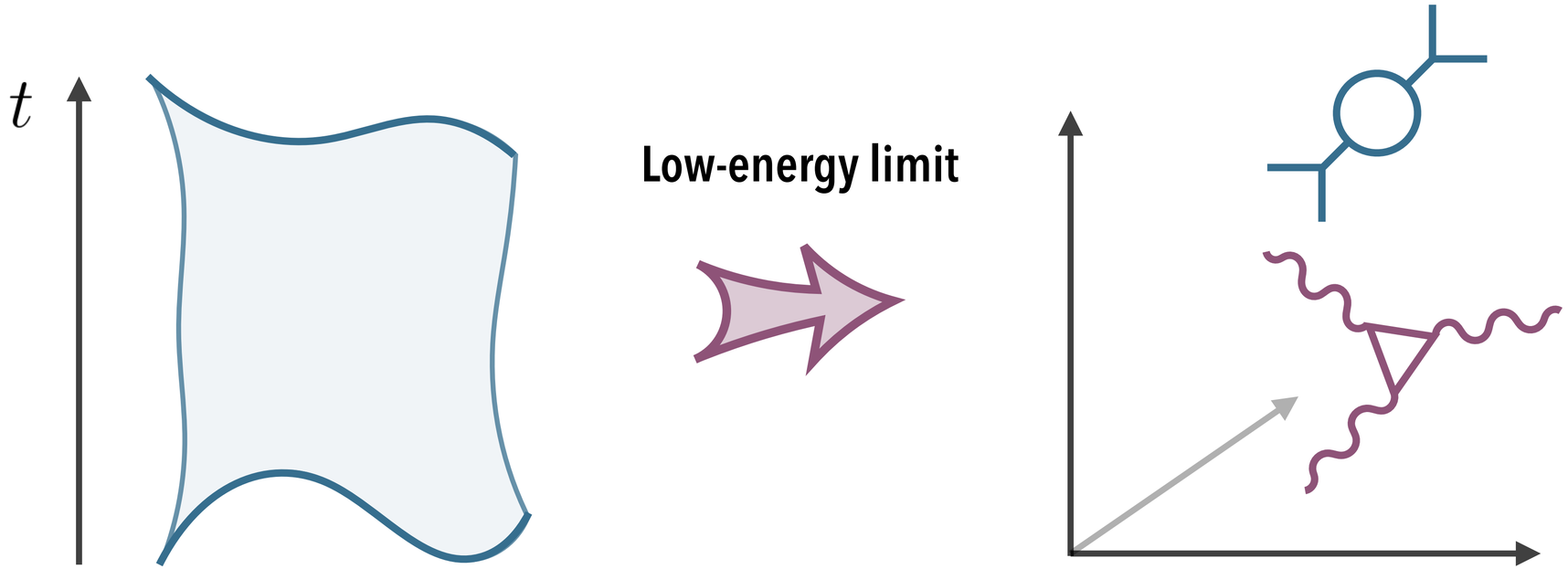

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Are all spacetimes allowed?

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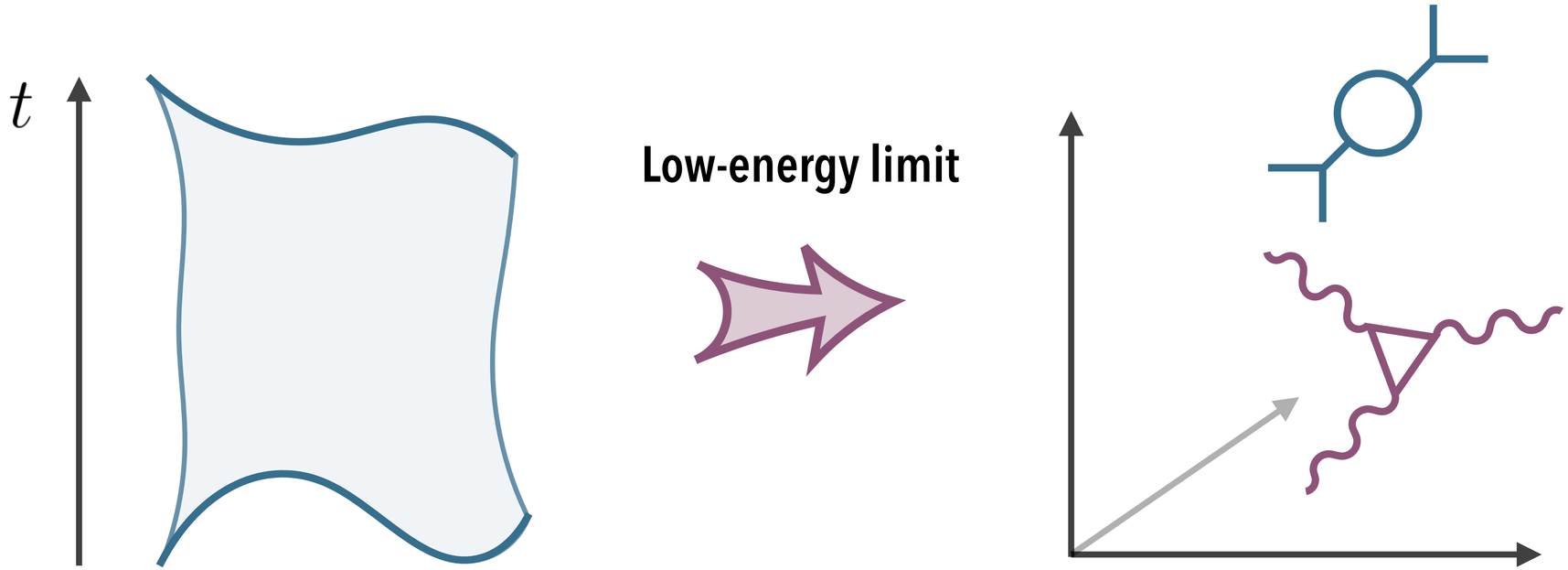

The worldsheet dictates the properties of spacetime in the low energy limit

Are all spacetimes allowed?

NO! Some lead to anomalies on the worldsheet

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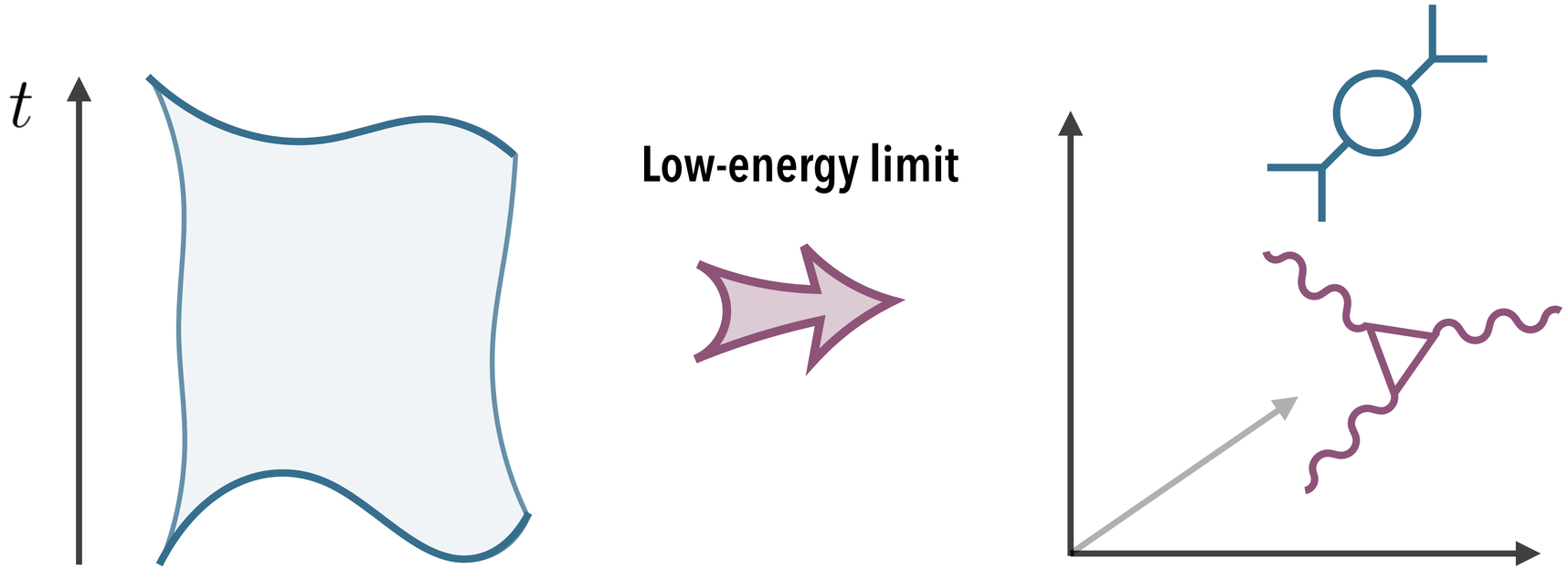
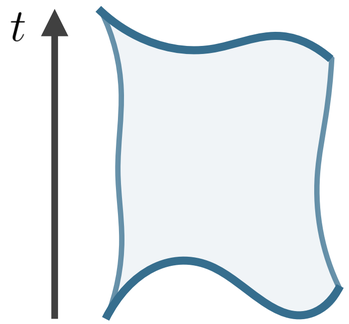


Even small modifications of 10d Minkowski spacetime can lead to inconsistencies!

Let us start by briefly reviewing the worldsheet of Type II strings in flat space

Construction of TYPE II String theory



(1,1) SCFT

Free chiral bosons $\{X_L, X_R\}$

Free chiral fermions $\{\psi_L, \psi_R\}$

SO(8) global symmetry

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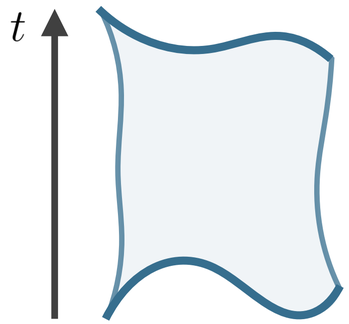

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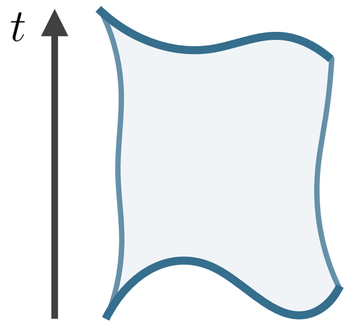

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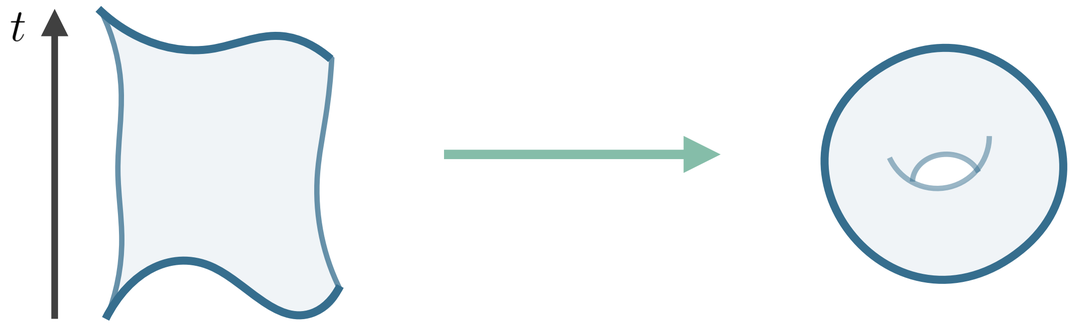

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The worldsheet must be

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(Invariant under re-parametrizations of the torus)

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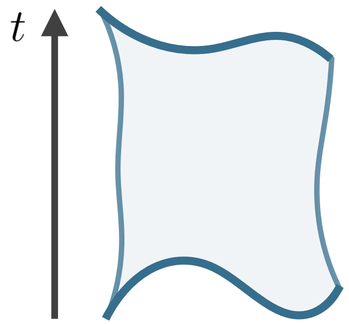

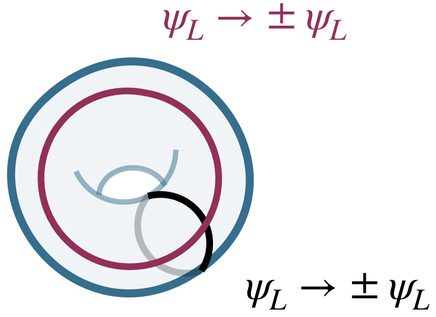

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They must be summed over in a modular invariant way.

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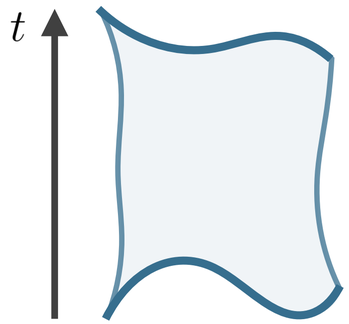

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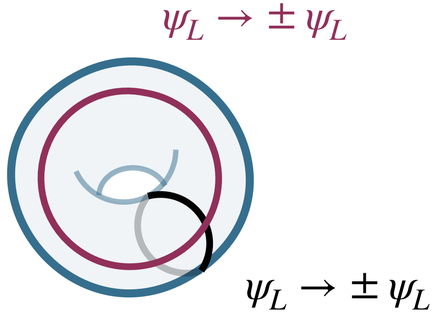

There are different **spin structures** on the torus.

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Gauging the fermion parity symmetry $(-1)^{F_L}$

"GSO projection"

[Seiberg Witten '86]

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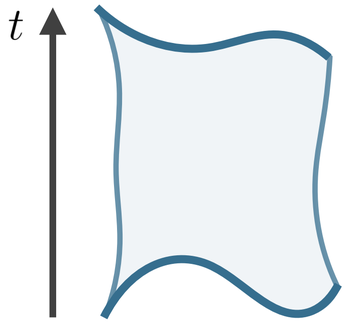

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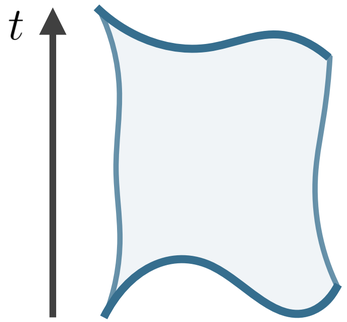

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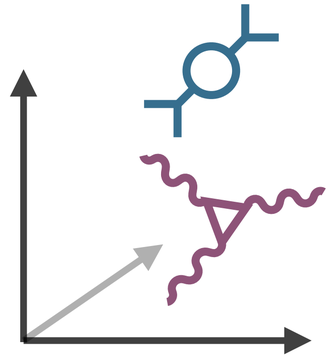

10-dimensional EFT

10d Minkowski

massless particles in SO(8) little group

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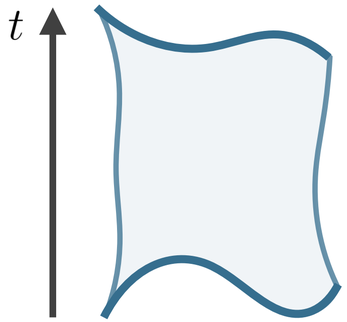

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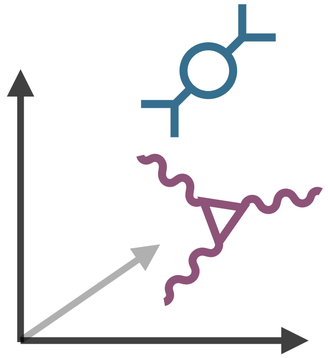

10-dimensional EFT

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This is the end of the story for 10d flat space.
What about less trivial backgrounds?

TYPE II ORBIFOLDS

[**M.D.**, L. Eberhardt, M. Tomasevic 'WIP]

Let us consider a specific type of background of Type II strings:

flat space orbifolds

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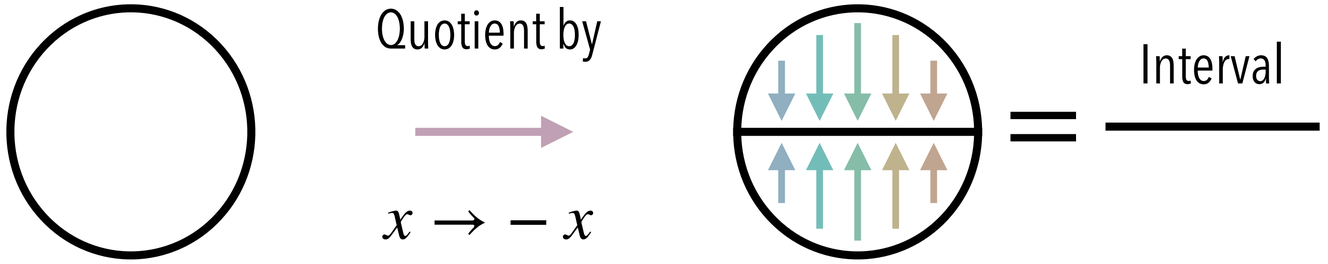
Let us consider a specific type of background of Type II strings:

flat space orbifolds

What's a flat space orbifold?

It's a background obtained by taking flat spacetime and quotienting by a discrete symmetry group

For example: an interval is an orbifold of the circle by a reflection



More generally, you can quotient 10D flat space by any discrete subgroup of SO(8): $\mathcal{M} = \mathbb{R}^{1,1} \times \mathbb{R}^8/G$

TYPE II ORBIFOLDS

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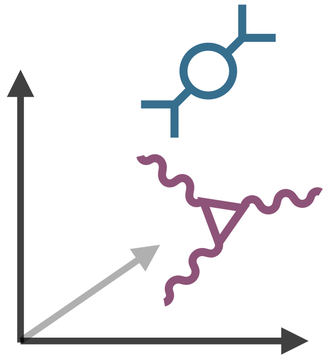

10-dimensional EFT

Flat space orbifolds by $G \subset SO(8)$

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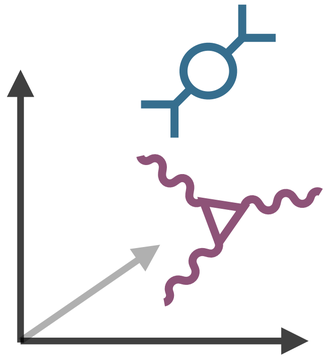

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What does this discrete quotient look Like on the worldsheet?

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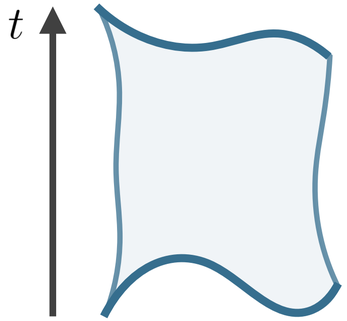

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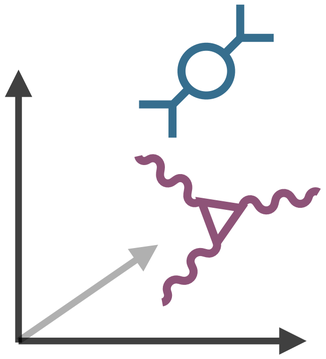

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~~SO(8) global symmetry~~

Gauging the fermion parity symmetry $(-1)^{F_L}$ → Gauging the corresponding G subgroup of $SO(8)$

Flat space orbifolds by $G \subset SO(8)$

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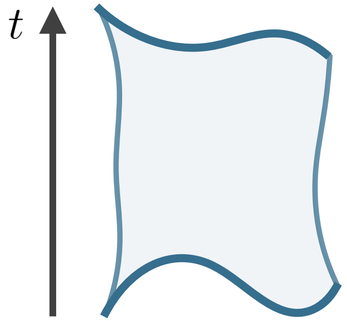

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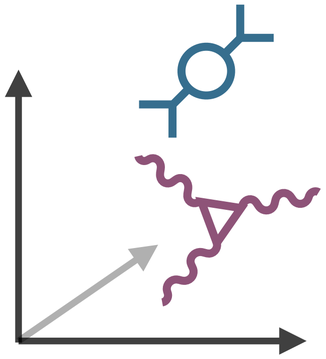

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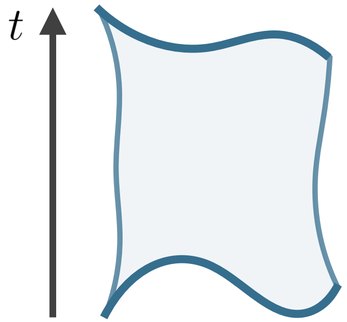

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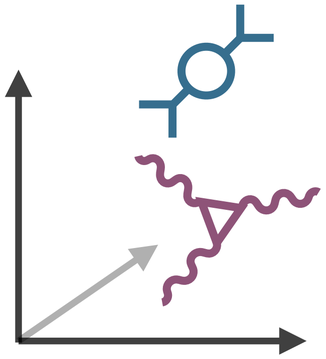

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Flat space orbifolds by $G \subset SO(8)$

$$\mathcal{M} = \mathbb{R}^{1,1} \times \mathbb{R}^8/G$$

Sometimes both of these symmetries cannot Be gauged at the same time!

↪ there is a mixed anomaly!

Type II strings are inconsistent on this orbifold background!

MIXED ANOMALIES

[**M.D.**, L. Eberhardt, M. Tomasevic 'WIP]

Two questions:

For what $G \subset SO(8)$ is there an anomaly?

What does that mean for the spacetime background?

What are the allowed flat space orbifolds?

$$\mathcal{M} = \mathbb{R}^{1,1} \times \mathbb{R}^8/G$$

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$$\Omega_3^{spin}(B\mathbb{Z}_2 \times BG)_{mixed} =$$

$(-1)^{F_L}$ line bundle

G gauge bundle

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$$(a,b) \in H^1(BG,\mathbb{Z}_2) \times H^2(BG,\mathbb{Z}_2)$$

$$(a,b) + (a',b') = (a+a', b+b'+a\cup a)$$

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We claim that this anomaly class if precisely:

Stiefel Whitney classes $(w_1(\mathcal{M}), w_2(\mathcal{M}))$

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$=0$ → $\mathcal{M}$ admits an orientation

$=0$ → $\mathcal{M}$ admits a spin structure

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Bordism invariants $\leftrightarrow$ fermion anomaly theory

Spacetime pulled back on the worldsheet

$$\psi_{\mathrm{L}} \in \Gamma(S \otimes X^*T\mathcal{M} \otimes L_a)$$

$$\psi_{\mathrm{R}} \in \Gamma(S \otimes X^*T\mathcal{M})$$

Wordsheet spin structure

$(-1)^{F_L}$ line bundle

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We learn that in order to be consistent backgrounds

For type II strings,

Flat space obifolds have to:

- be **orientable**
- **admit a spin structure**

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Note that if $G = \mathbb{Z}_n$ with n odd

Then the anomaly vanishes trivially!

All such orbifolds are consistent backgrounds

What does that mean for the spacetime background?

What are the allowed flat space orbifolds?

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Any such orbifold is consistent!

(Painful to check explicitly for large n)

What does that mean for the spacetime background?

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We learn that in order to be consistent backgrounds

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CONCLUSIONS & OUTLOOK

Global anomalies on the worldsheet → constraints on the structure of target space

$$\Omega_3^{spin}(B\mathbb{Z}_2 \times BG)_{mixed} = (H^1(BG, \mathbb{Z}_2), H^2(BG, \mathbb{Z}_2))$$

→ Flat space obifolds have to: - be orientable - admit a spin structure

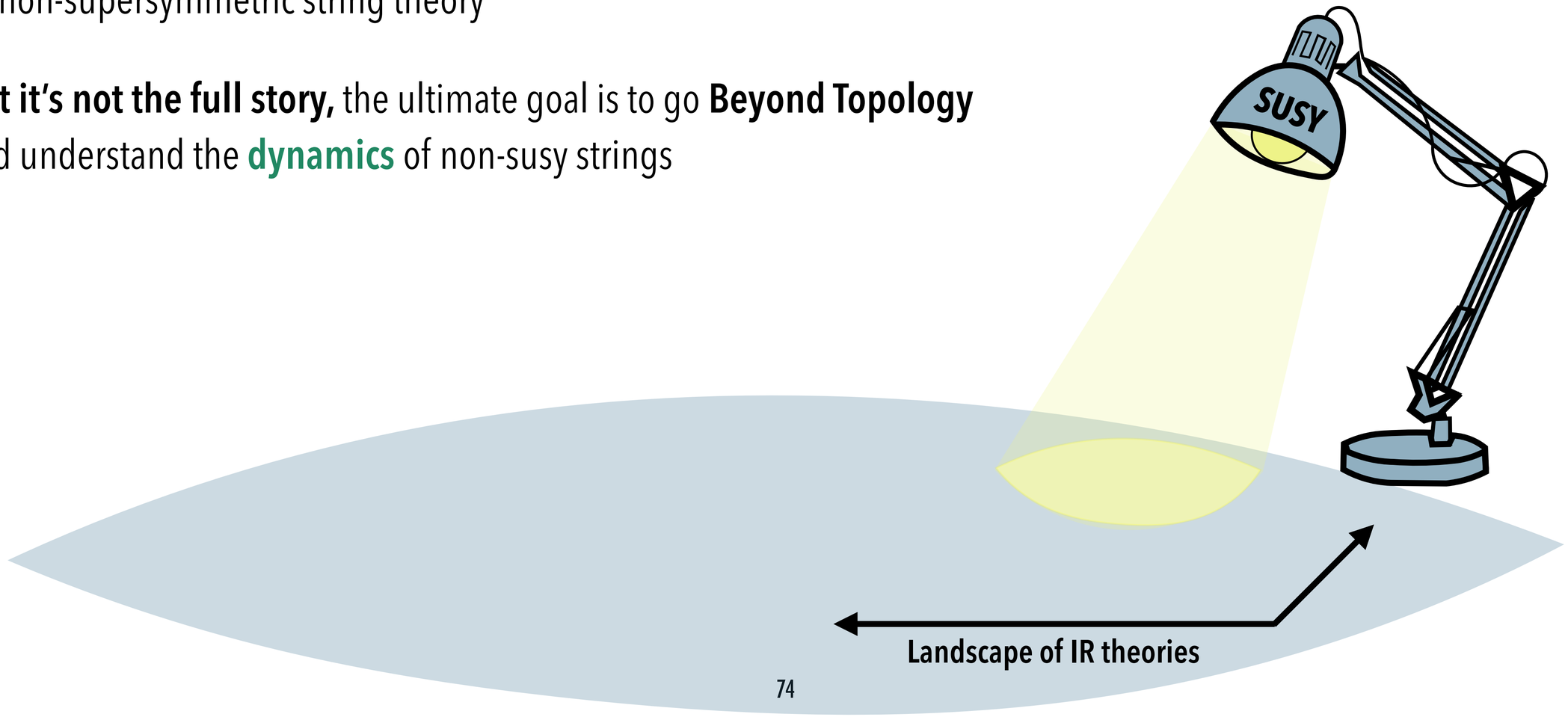
No supersymmetry, some backgrounds are tachyonic (see condition for tachyons in the paper)

Generalisations to more complicated backgrounds? Orientifolds? […]

CONCLUSIONS & OUTLOOK

Anomaly cancellation: Sharp & lets us uncover **the fundamental structures** of non-supersymmetric string theory

But it's not the full story, the ultimate goal is to go **Beyond Topology** and understand the **dynamics** of non-susy strings



CONCLUSIONS & OUTLOOK

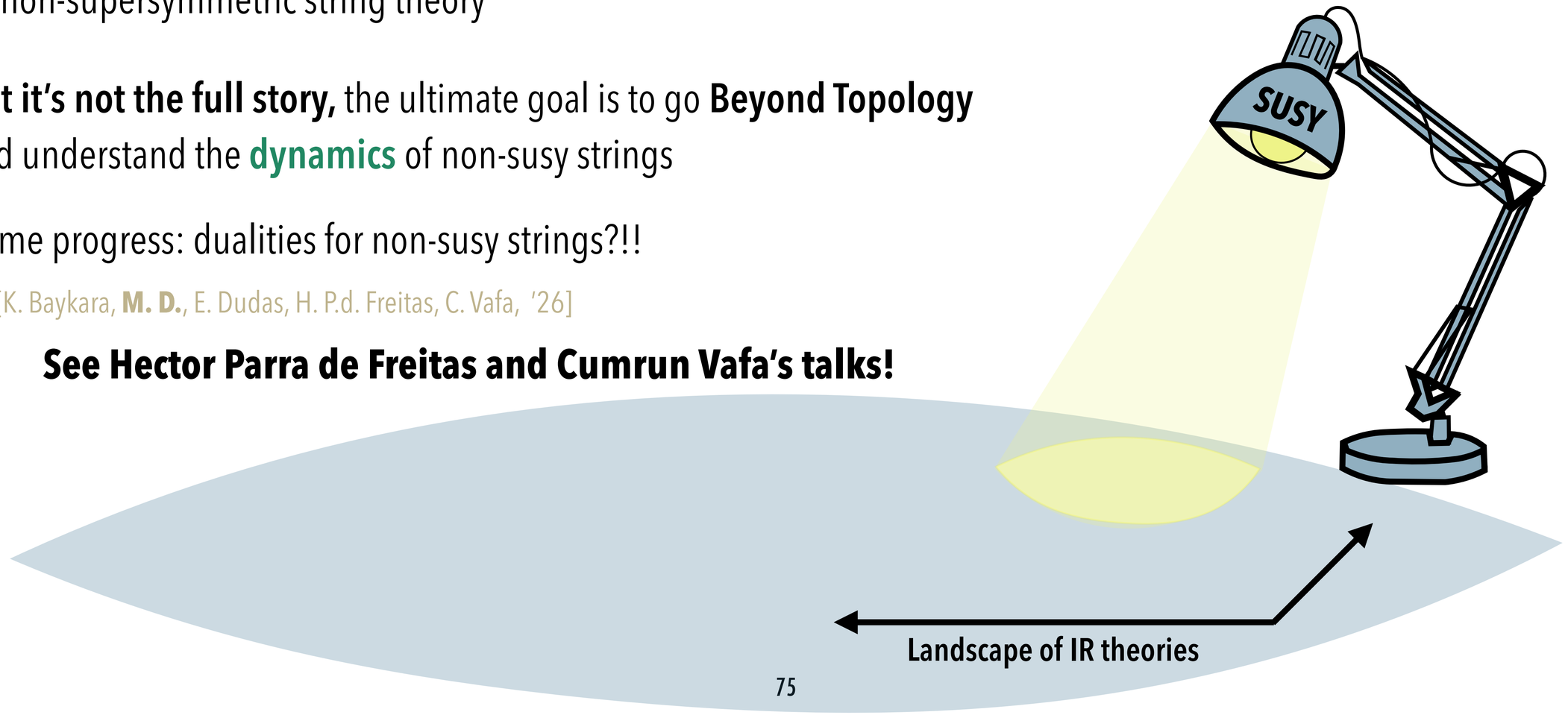
Anomaly cancellation: Sharp & lets us uncover **the fundamental structures** of non-supersymmetric string theory

But it's not the full story, the ultimate goal is to go **Beyond Topology** and understand the **dynamics** of non-susy strings

Some progress: dualities for non-susy strings?!!

[K. Baykara, **M. D.**, E. Dudas, H. P.d. Freitas, C. Vafa, '26]

See Hector Parra de Freitas and Cumrun Vafa's talks!



Thank you!