# String probes
# and the supergravity landscape

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# String probes and the supergravity landscape

Theories that couple consistently to gravity are generally expected to possess strings and branes charged under the theories' higher-form fields.

The idea advocated in [Kim,Shiu,Vafa]: exploit consistency of probes to constrain the properties of theories that are not in the swampland

→ Novel results in many different contexts e.g. [Lee,Weigand][Kim,Tarazi,Vafa] [Hamada,Vafa][Katz,Kim,Tarazi,Vafa][Angelantonj,Bonnefoy,Condeescu,Dudas][Tarazi,Vafa] [Cheng,Minasian,Theisen][Loges][Hamada,Loges][Angelantonj,Condeescu,Dudas,Leone] [Baume,Oehlmann,Ruhle][Kim,Vafa,Xu][Brady,Tennison,Vandermeulen][Montero,Zapata] [Kaufmann,Monnee,Weigand,Wiesner]

**This talk:** Two new applications of string probes

1. Worldsheet explanation for **no global center 1-form symmetries**
2. Study 6d SUGRA models with no F-theory realization

# 1. No global center 1-form symmetries

# The setup

Focus on QFTs $\mathscr{T}$ in $d \geq 4$ possibly coupled to gravity, with continuous gauge group

$$G = \prod_{\alpha} G_{\alpha}$$

Assumptions:

**1. Higher form fields**:

$\mathscr{T}$ includes a $U(1)$ two-form field $B_2$

$\widetilde{B}_{d-4} =$ magnetic dual

**2. Brane probes**:

Spectrum of $\mathscr{T}$ includes a string $\mathscr{S}$ and magnetic dual brane $\mathscr{D}$ charged under $B_2$, $\widetilde{B}_{d-4}$

# Faithful string probes

We say $\mathcal{S}$ is a **faithful string probe** for $G$ if

- --- There exist CS couplings $b_\alpha \int \widetilde{B}_{d-4} \wedge \mathrm{Tr}\, F_\alpha \wedge F_\alpha$ with $b_\alpha \geq 1$ for all $G_\alpha$

  (also important in [Kaya,Rudelius])

- --- The worldsheet theory that describes $\mathcal{S}$ factors as:

$$\mathcal{T}_{\mathcal{S}} = \mathcal{T}_{c.o.m.} \times \widetilde{\mathcal{T}}_{\mathcal{S}}$$

$\widetilde{\mathcal{T}}_{\mathcal{S}}$ : unitary compact CFT with current algebra $\bigotimes_\alpha \widehat{G}_{\alpha, b_\alpha}$

- Common in supergravity theories in various dimensions
- Not common otherwise (but e.g. E-strings and M-strings in 6d SCFTs are faithful)

# Faithful strings and gauge group topology

Let $G = \widetilde{G}/\Gamma$

$\widetilde{G}$ = universal cover of $G$

$\Gamma \subset \mathcal{Z}(\widetilde{G})$

Associated to $\widetilde{G}$ is the center one-form symmetry $\widetilde{G}^{[1]} \simeq \mathcal{Z}(\widetilde{G})$ acting by discrete shift on $\widetilde{G}$ connection:

$$A \to A + \gamma$$

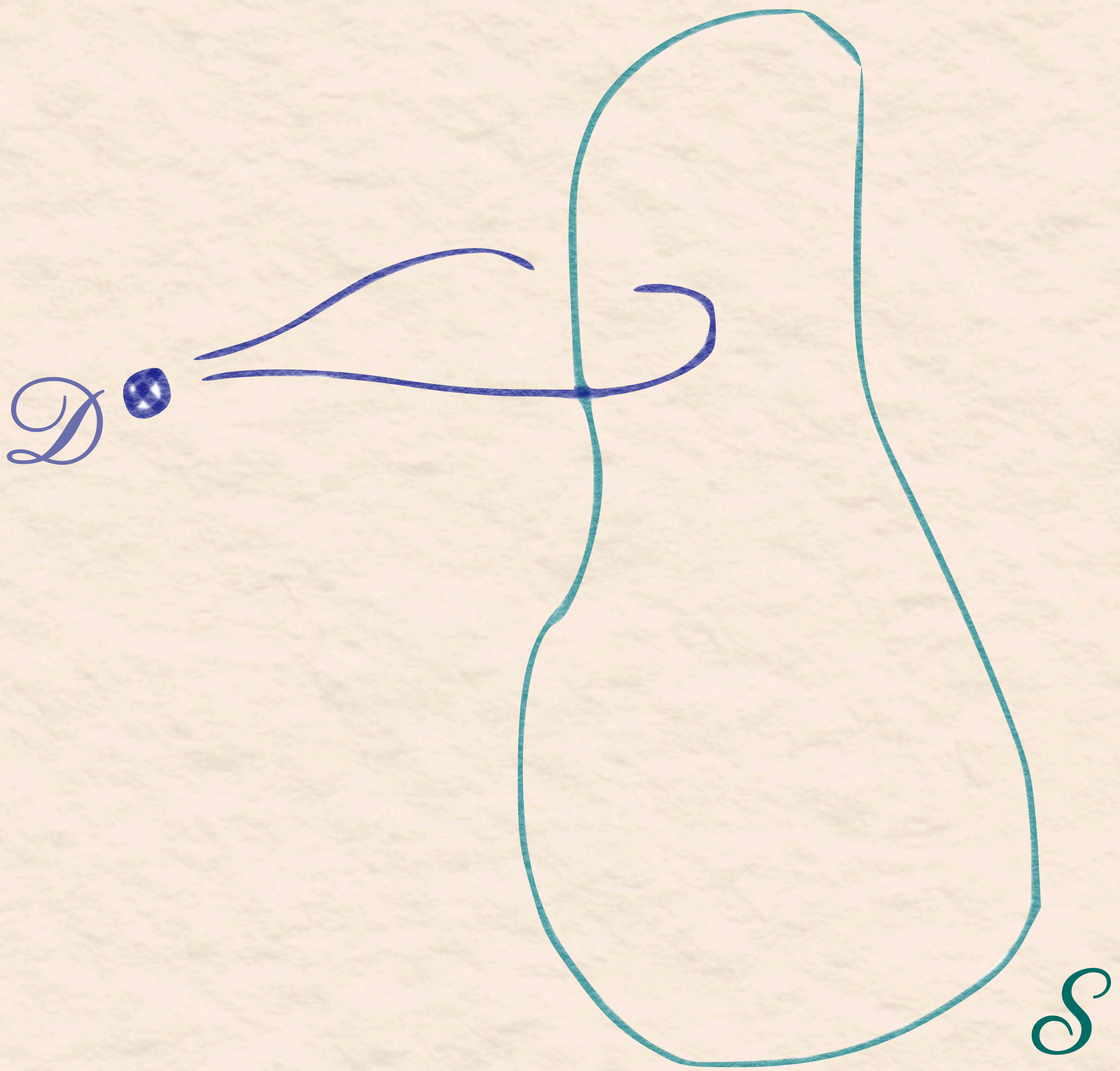
flat b.g. one-form with periods in $\mathcal{Z}(\widetilde{G})$

In principle $\widetilde{G}^{[1]} \simeq \mathcal{Z}(\widetilde{G})$ global, broken, or gauged

Q: How does the string $\mathcal{S}$ resolve the topology of $G$?

# Faithful strings and gauge group topology

Consider the following setup on 4d slice




$$\left(b_\alpha \int_{X^\perp} \widetilde{B}_{d-4}\right) \cdot \int \mathrm{Tr}\, F_\alpha \wedge F_\alpha$$

$$\downarrow$$

$$\left(b_\alpha \int_{X^\perp} \widetilde{B}_{d-4}\right) \cdot \int \mathrm{Tr}\, F_\alpha \wedge F_\alpha$$

$$+$$

$$2\pi i b_\alpha \cdot \int \mathrm{Tr}\, F_\alpha \wedge F_\alpha$$

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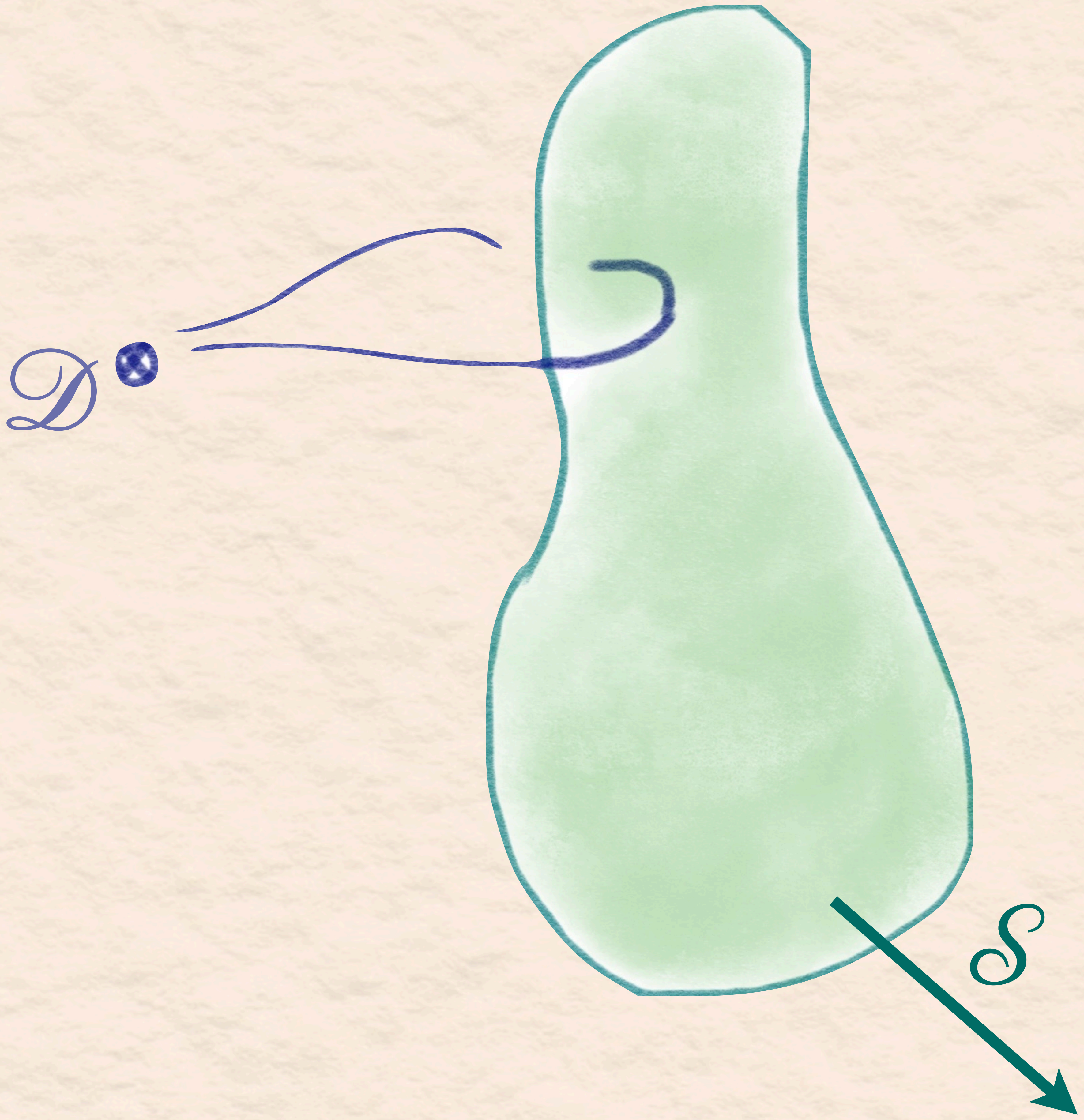


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$$+$$

$$2\pi i b_\alpha \cdot \int \mathrm{Tr}\, F_\alpha \wedge F_\alpha$$

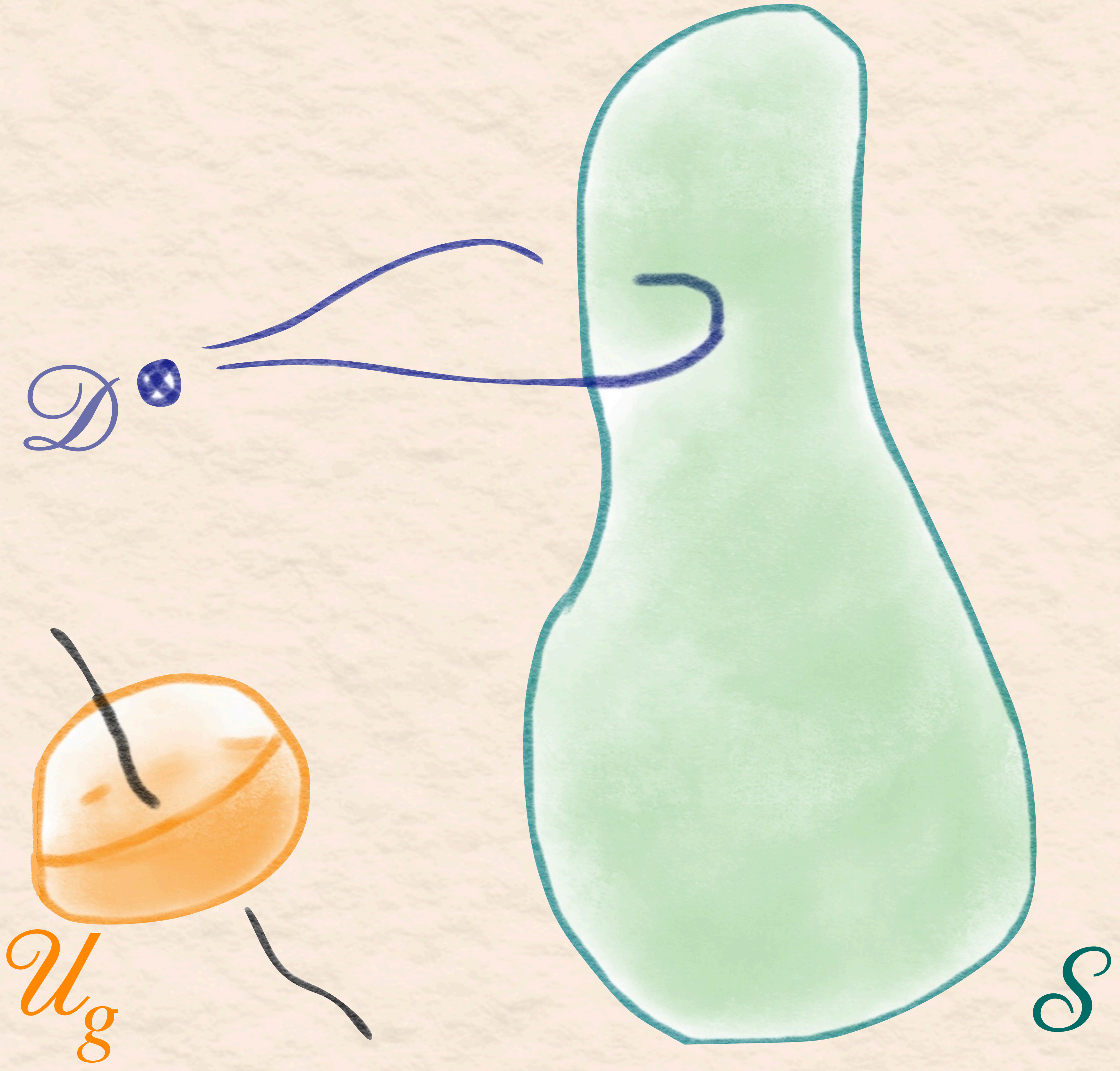
Interface $\supset$ $G_\alpha$ CS theory at level $b_\alpha$

$\widehat{G}_\alpha$ KMA on bdry $\mathcal{S}$

# Faithful strings and gauge group topology

Consider the following setup on 4d slice

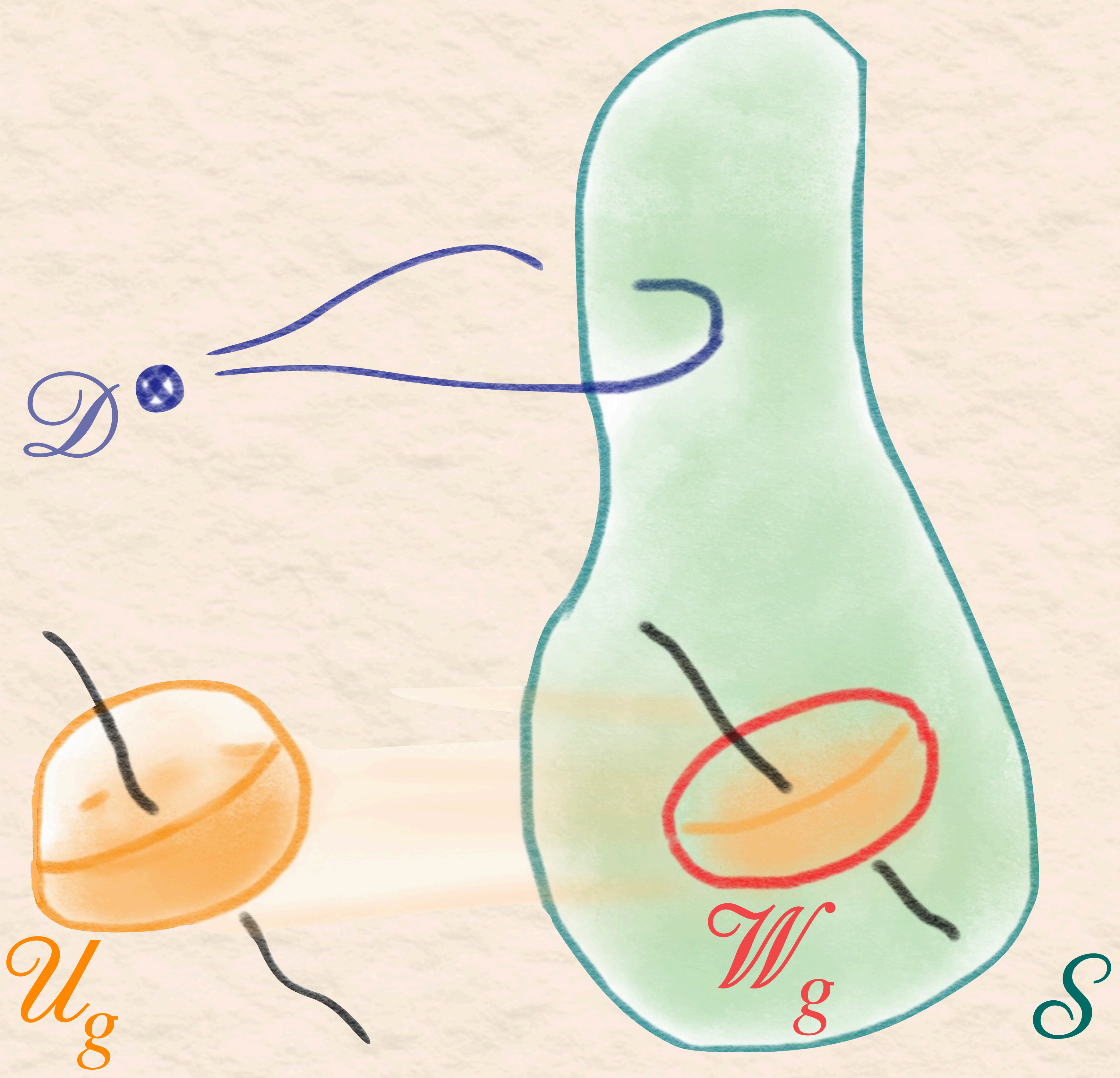
Gukov-Witten type operators $\mathcal{U}_g$ in bulk




$$\mathcal{U}_{g_1}\mathcal{U}_{g_2} = \mathcal{U}_{g_1 g_2}$$

# Faithful strings and gauge group topology

Consider the following setup on 4d slice




$\mathcal{U}_{g_1}\mathcal{U}_{g_2} = \mathcal{U}_{g_1 g_2}$

Gukov-Witten type operators $\mathcal{U}_g$ in bulk

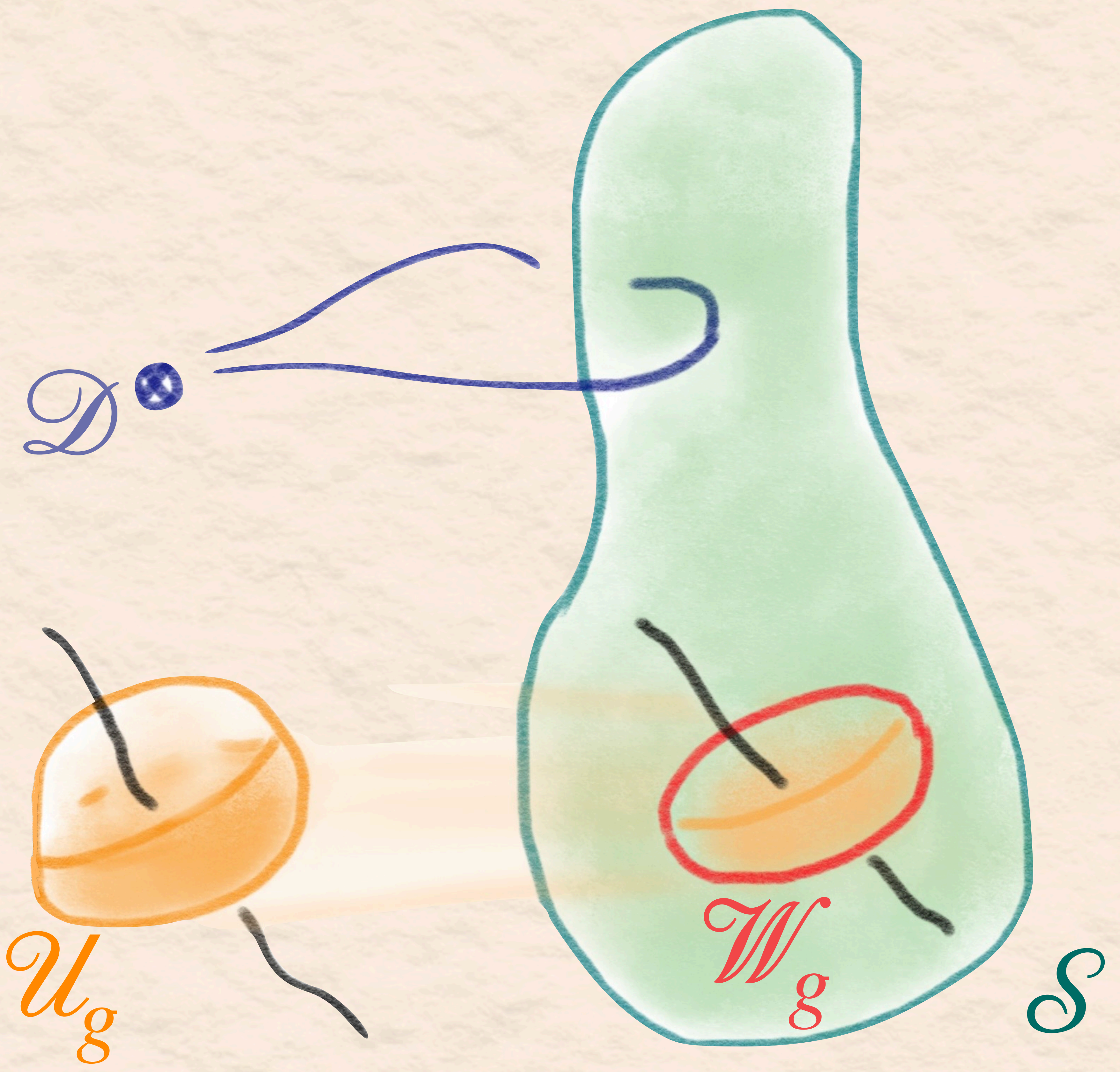
↓

GW operators $\mathscr{W}_g$ on interface

[Hsin,Lam,Seiberg]

# Faithful strings and gauge group topology

Consider the following setup on 4d slice




$\mathscr{U}_{g_1}\mathscr{U}_{g_2} = \mathscr{U}_{g_1 g_2}$

Gukov-Witten type operators $\mathscr{U}_g$ in bulk

↓

GW operators $\mathscr{W}_g$ on interface

[Hsin,Lam,Seiberg]

↓

Simple current operators $\mathscr{J}_g$ on $\mathscr{S}$

[Moore,Seiberg]

# Simple current basics

Let $\mathscr{R}$ denote a rational sector on chiral half of a 2d CFT $\qquad T = T_{\mathscr{R}} + T_{rest}$

**Simple currents** are primaries that obey

$$[\mathscr{J}] \times [\phi_i] = [\phi_{J(i)}]$$

[Schellekens,Yankielowicz]

OPE with other primaries $\phi_i$

Simple currents satisfy $\mathscr{J}^n = 1$ for some $n := \mathrm{ord}(\mathscr{J})$.

The SC generate the **center of the RCFT** $\mathscr{R}$

$$\mathscr{Z}(\mathscr{R}) = \mathbb{Z}_{n_1} \times \ldots \times \mathbb{Z}_{n_d}$$

E.g. for $\mathscr{R}$ = current algebra $\neq \widehat{E}_{8,2}$, $\qquad \mathscr{Z}(\mathscr{R}) = \mathscr{Z}(G)$ [Fuchs]

# Simple current extensions

[Schellekens,Yankielowicz] [Intriligator]

If conformal weight $h_{\mathcal{J}} \in \mathbb{Z}$, can extend the chiral algebra of $\mathcal{R}$ by $\mathcal{J}$.

Project to operators which are local wrt $\mathcal{J}$ → trivial **monodromy charge**

$$Q_{\mathcal{J}}(\phi_i) := h_{\mathcal{J}} + h_{\phi_i} - h_{\phi_{\mathcal{J}(i)}} = 0 \qquad 1 \ \text{mod}$$

Spectrum organizes into extended conformal families with characters

$$\mathcal{X}_i(\tau) = \sum_{n=1}^{\text{ord}\,(\mathcal{J})} \chi_{\phi_{\mathcal{J}^n(i)}}(\tau)$$

Extension of $\mathcal{R}$ ↔ gauging of one-form symmetry in corresponding 3d TQFT

[Moore,Seiberg]

# Simple current extensions and faithful strings

**Key ingredients for faithful strings:**

**unitarity** + **modularity** of partition function

$$Z_{\mathcal{S}}(\tau,\bar{\tau}) = \sum_{ij} M_{ij}\, \underset{\mathcal{R}}{\underset{\cup}{\chi_i(\tau)}}\; \widetilde{\chi}_j(\tau,\bar{\tau})$$

from residual sector in $\widetilde{\mathcal{T}}_{\mathcal{S}}$

**Thm:** For a given $\mathcal{J} \in \mathcal{Z}(\mathcal{R})$

Either: $Q_{\mathcal{J}}(\phi_i) \neq 0$ for some $\phi_i$

($\mathcal{J}$ broken)

Or: $\mathcal{R}$ is extended by $\mathcal{J}$

(gauged)

Interpretation: this corresponds to breaking/gauging of bulk one-form symmetries

$Q_{\mathcal{J}}(\phi_j) = 0 \Longleftrightarrow$ no obstruction to gauging bulk 1-form sym

In 6d: [Apruzzi,Dierigl,Lin]
In 8d: [Cvetič,Dierigl,Lin,Zhang]

# Examples in higher dimensions

- 10D : $\widetilde{G} = Spin(32)$ heterotic string has spinor $\mathbf{s}$ in its spectrum

  This is a simple current: $h_{\mathbf{s}} = 2, \quad \mathbf{s}^2 = \text{id} \implies$ chiral algebra is extended

  $G = Spin(32)/\mathbb{Z}_2$ $\qquad\qquad$ $\mathcal{X}_1(\tau) = \chi_1(\tau) + \chi_{\mathbf{s}}(\tau)$

- 9D : CHL string has 9 points of maximal enhancement

  - $SU(2)_2 \times E_{8,2}$
  - $SU(2)_2 \times SU(3)_2 \times SU(7)_2$
  - $SU(10)_2$
  - $SU(3)_2 \times E_{7,2}$
  - $SU(5)_2 \times Spin(10)_2$
  - $Spin(18)_2$
  - $SU(4)_2 \times E_{6,2}$
  - $SU(5)_2 \times SU(6)_2$
  - $SU(2)_2 \times SU(9)_2$

[Font,Fraiman,Graña, Nuñez, Parra de Freitas]

$h_{\mathcal{J}} \notin \mathbb{Z}$: no holo SC

Holo $\mathbb{Z}_3$ Kac-Moody SC, but not in CHL spectrum

→ All gauge groups simply connected

# Examples in higher dimensions

- 8D : Focus on CHL model with $\widetilde{G} = SU(2) \times E_7 \times SU(3)$, all CS levels $b_\alpha = 2$

The $(\mathbf{3}, \mathbf{1463}, \mathbf{1})$ KM primary in CHL spectrum is a SC of order 2

$$\Longrightarrow \qquad G = \frac{SU(2) \times E_7}{\mathbb{Z}_2} \times SU(3)$$

consistent with
[Cvetič-Dierigl-Lin-Zhang]
[Font,Fraiman,Graña, Nuñez, Parra de Freitas]

Check: F-theory dual corresponds to elliptic K3 identified by [Hamada,Vafa],
has $\mathbb{Z}_2$ MW torsion.

## 2. String probes and 6d non-geometric SUGRAs

# The 6d supergravity landscape with no tensors

Focus on 6d $\mathcal{N} = (1{,}0)$ supergravities with no tensor multiplets

- F-theory models described by elliptic fibration over $\mathbb{P}^2$
- No heterotic string in the spectrum
- "***L*-string**" from wrapping D3 brane on the line $L$ in $\mathbb{P}^2$, with (0,4) w.s. SUSY

If all CS couplings $b_\alpha > 1$, $L$-string is a faithful string.

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Partial classification of anomaly free SUGRA models [Hamada,Loges]

- Many do not admit F-theory realization (e.g. violate geometric constraints)
- $L$-string provides a worldsheet way to study them and test consistency

# Search for rational models

L-string has $c_L = 32$. Can we saturate it by Sugawara central charge for $G$?

Almost! For 5 models $31 < c_{Sug} < 32$!

- Residual central charge:
  $c_{res} = 32 - c_{Sug} =$ c(Virasoro minimal model)

| Gauge algebra | Matter content | $c_{res}$ | $\mathcal{R}_{res}$ |
|---|---|---|---|
| $\mathfrak{e}_{6,8}$ | $\mathbf{351'}$ | 4/5 | Potts |
| $\mathfrak{a}_{7,8}$ | $\mathbf{336}$ | 1/2 | Ising |
| $\mathfrak{a}_{5,6} + \mathfrak{d}_{4,6}$ | $(\mathbf{56}, \mathbf{1}) + (\frac{1}{2}\mathbf{20}, \mathbf{28})$ | 1/2 | Ising |
| $\mathfrak{a}_{3,6} + \mathfrak{a}_{3,6} + \mathfrak{e}_{7,2}$ | $(\mathbf{10}, \mathbf{10}, \mathbf{1}) + (\mathbf{6}, \mathbf{1}, \frac{1}{2}\mathbf{56}) + (\mathbf{1}, \mathbf{6}, \frac{1}{2}\mathbf{56})$ | 7/10 | Tri. Ising |
| $\mathfrak{a}_{3,10} + \mathfrak{a}_{11,2}$ | $(\mathbf{35}, \mathbf{1}) + (\mathbf{6}, \mathbf{66})$ | 6/7 | Tri. Potts |

- Left-moving sector is rational, fully known
- No large volume realization in F-theory:
  no universal hyper, violate Kodaira condition

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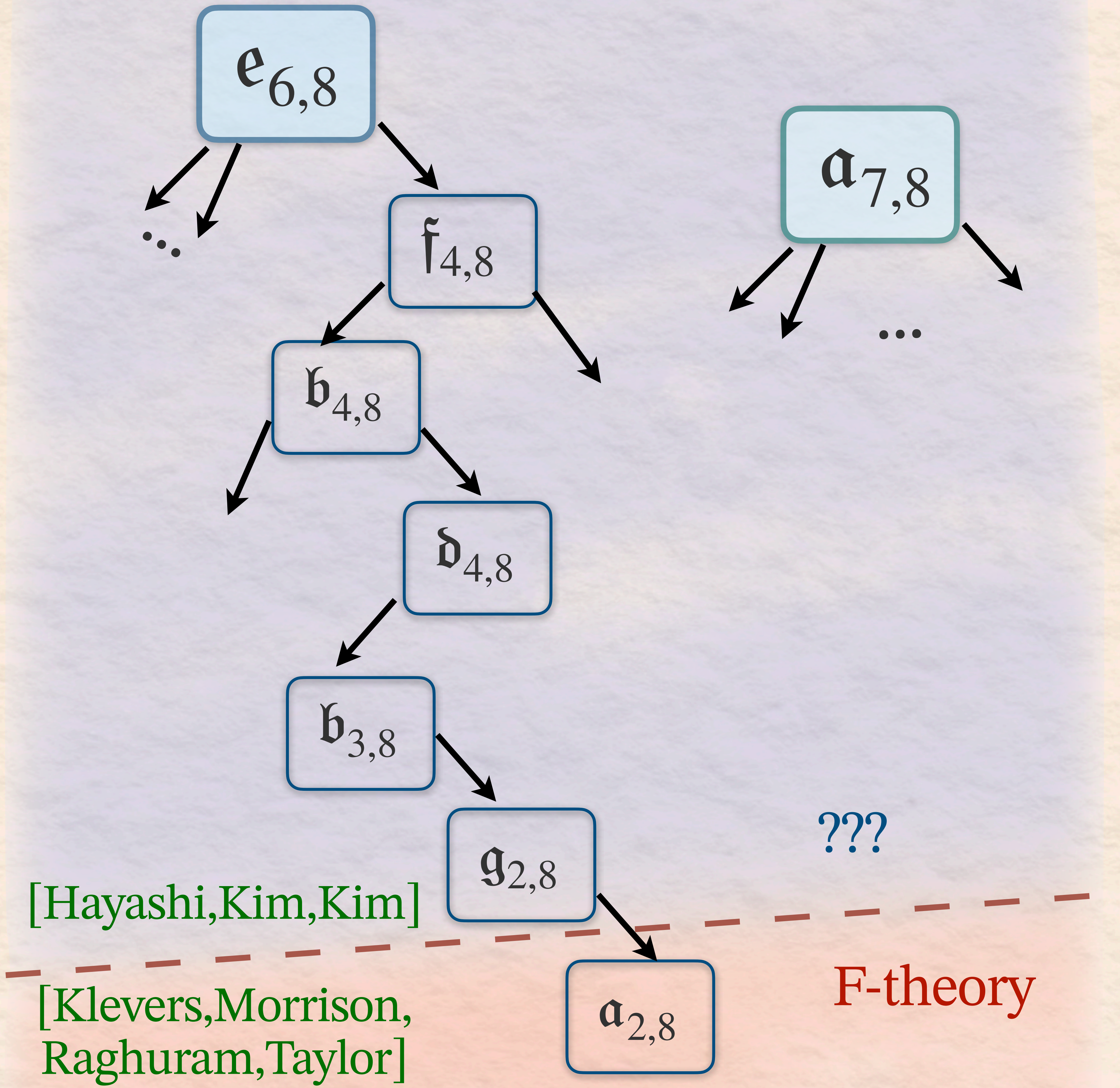
## Higgsing trees



# Evidence for rational models

- Anomaly free [Hamada,Loges]
- Flavored elliptic genus? Search for modular invariant combination of characters

$$\mathbb{E}(\vec{m}_G,\tau) \stackrel{?}{=} \sum_{\vec{i},j} C_{\vec{i},j} \left( \prod_\alpha \chi_\alpha^{\hat{\mathfrak{g}}}(\vec{m}_{G_\alpha},\tau) \right) \chi_j^{res}(\tau)$$

consistent with expected low lying spectrum of operators:

$$\mathbb{E}(\vec{m}_G,\tau) \sim 3\,q^{-4/3} + (2\mathcal{I}(\vec{m}_G) - 12)\,q^{-1/3} + \mathcal{O}(q^{2/3})$$

| Gauge algebra | # terms |
| --- | --- |
| $\mathfrak{e}_{6,8}$ | 3720 |
| $\mathfrak{a}_{7,8}$ | 19305 |
| $\mathfrak{a}_{5,6} + \mathfrak{d}_{4,6}$ | 180180 |
| $\mathfrak{a}_{3,6} + \mathfrak{a}_{3,6} + \mathfrak{e}_{7,2}$ | 254016 |
| $\mathfrak{a}_{3,10} + \mathfrak{a}_{11,2}$ | 334620 |

**The outcome**:

- In all cases, we find **exactly one** modular invariant.
- All coefficients $C_{\vec{i},j} \in \mathbb{Z}$ !

# Rational models: the global form of $G$

Flavored elliptic genus organizes into extended characters

→ we can infer $G = \widetilde{G}/\Gamma$ for all models!

→ results extend to entire Higgsing trees

| Gauge algebra | $G$ |
|---|---|
| $\mathfrak{e}_{6,8}$ | $E_6$ |
| $\mathfrak{a}_{7,8}$ | $SU(8)/\mathbb{Z}_4$ |
| $\mathfrak{a}_{5,6} + \mathfrak{d}_{4,6}$ | $SU(6)/\mathbb{Z}_3 \times Spin(8)/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ |
| $\mathfrak{a}_{3,6} + \mathfrak{a}_{3,6} + \mathfrak{e}_{7,2}$ | $(SU(4)/\mathbb{Z}_2 \times SU(4)/\mathbb{Z}_2 \times E_7)/\mathbb{Z}_2$ |
| $\mathfrak{a}_{3,10} + \mathfrak{a}_{11,2}$ | $(SU(4)/\mathbb{Z}_2 \times SU(12)/\mathbb{Z}_2)/\mathbb{Z}_2$ |

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**Bonus result:** 6d simple currents give rise to BPS particles in 5d upon compactification, which modify the 5d Kähler cone.

Perfect match with expectations in [Kim,Vafa]

# Mixed simple current extensions

From w.s. perspective, Kac-Moody SC on equal footing with **mixed** simple currents

$$\mathcal{J} = \mathcal{J}_{KM} \times \mathcal{J}_{min}$$

**All 5 rational models** extended by mixed simple currents of this kind...

E.g. the $E_6 \times$ **Potts** model is extended by a $\mathbb{Z}_3$ mixed simple current with $h_{\mathcal{J}} = 6$

$$\mathcal{J} = (\mathbf{5895396})_{E_{6,8}} \times (\psi_1)_{Potts}$$

**What about the spacetime interpretation?**

*Hint:* some 6d asymmetric orbifold model display similar mixed extension by $\mathbb{Z}_2$ SC in $E_{8,2} \times$ **Ising**, remnant of $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ outer auto of CHL string

[Forgacs,Horvath,Palla,Vecsernyes] [Collazuol,Melnikov]

# Some open questions

## Simple current extensions

--- Bulk interpretation of mixed simple current extensions?

--- What can one say about discrete gauge groups?

--- Applications to lower dimensions/non-supersymmetric models?

## *L*-strings

--- Further evidence for existence of rational models? Other RCFTs with $c_{res} \geq 1$?

--- Full description of rational string worldsheet including right-movers?

--- Consequences of rationality? Connections with arithmetic geometry?