SymTFTs with Interfaces in Geometric Engineering

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Work to appear with
Daniel Robbins, Subham Roy

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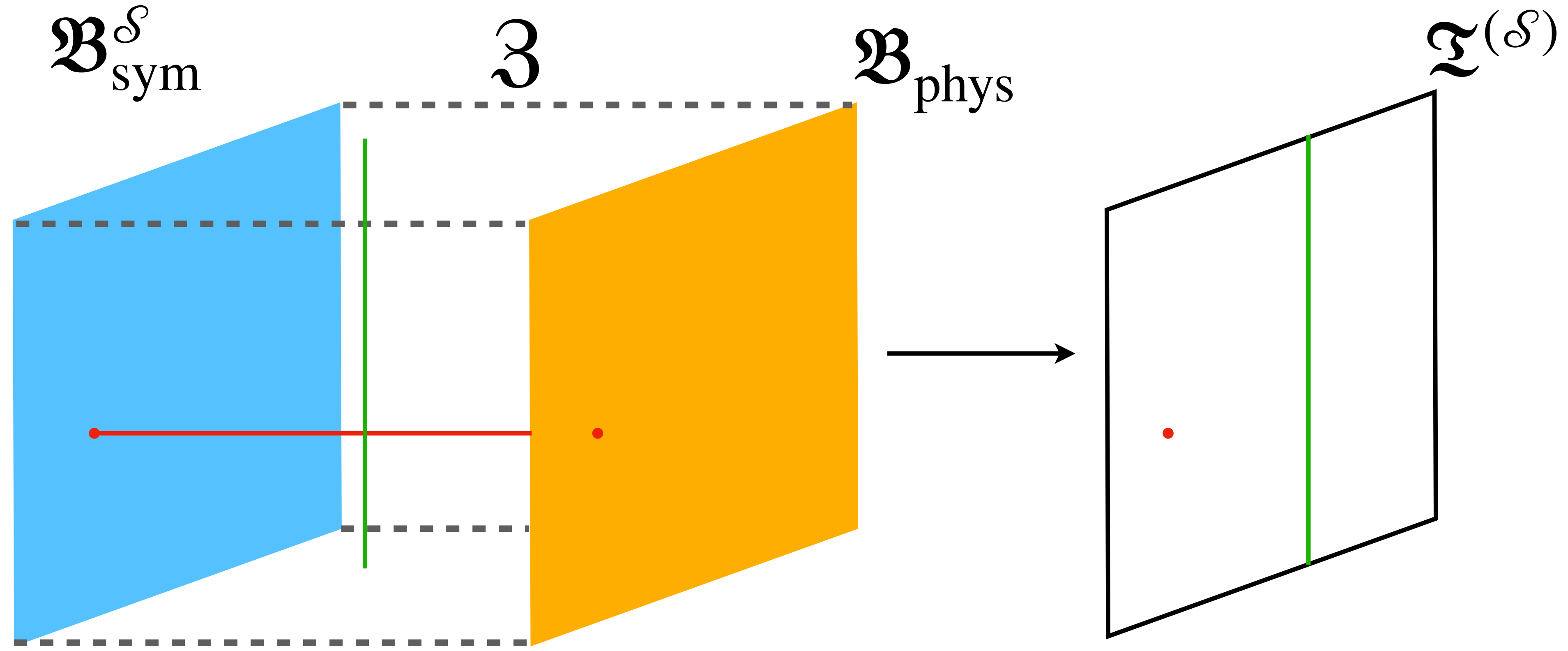
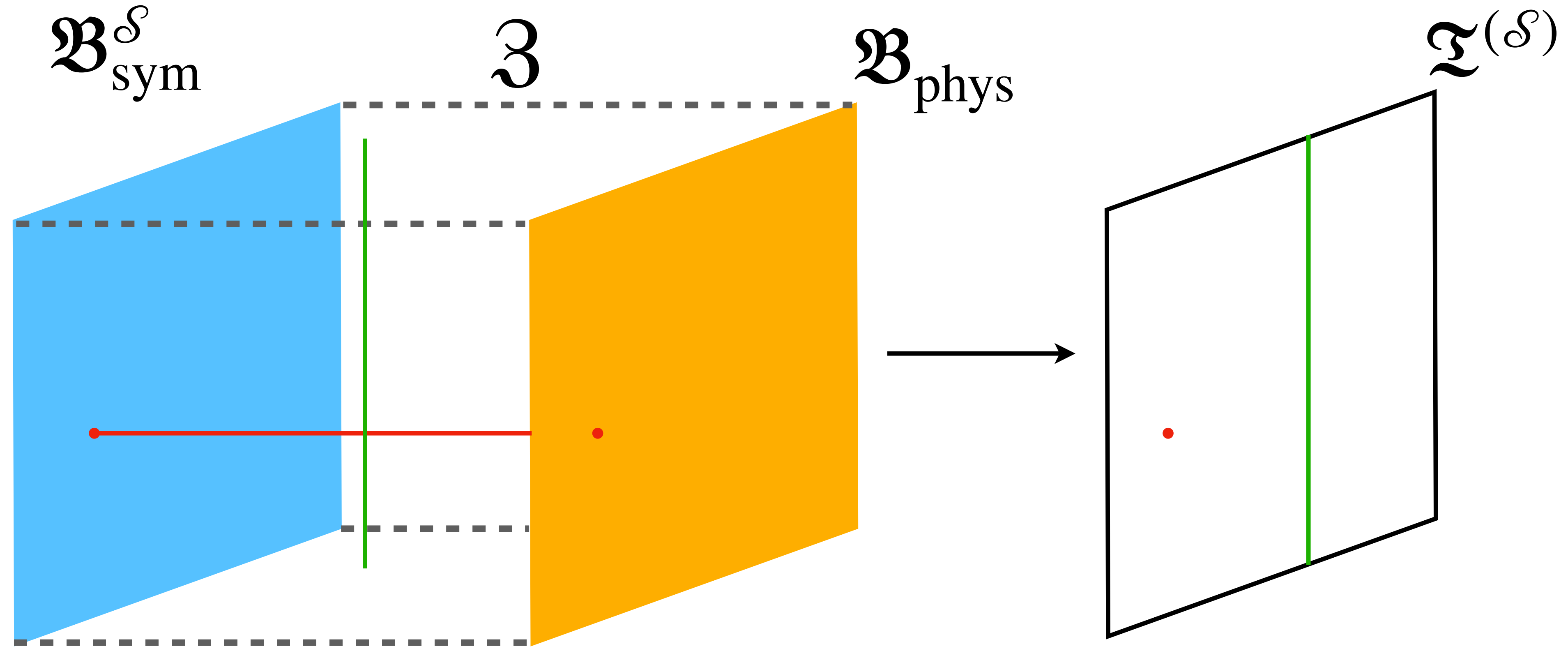


Symmetry TFT with interfaces

- Symmetry operators and charged defects in d dimensions can be encoded in a (d+1)-dim *Symmetry Topological Field Theory (SymTFT)* [Ji/Wen '19, Gaiotto/Kulp '20, Apruzzi/Bonetti/García Etxebarria/Hosseini/Schafer-Nameki '21, Freed/Moore/Teleman '22]



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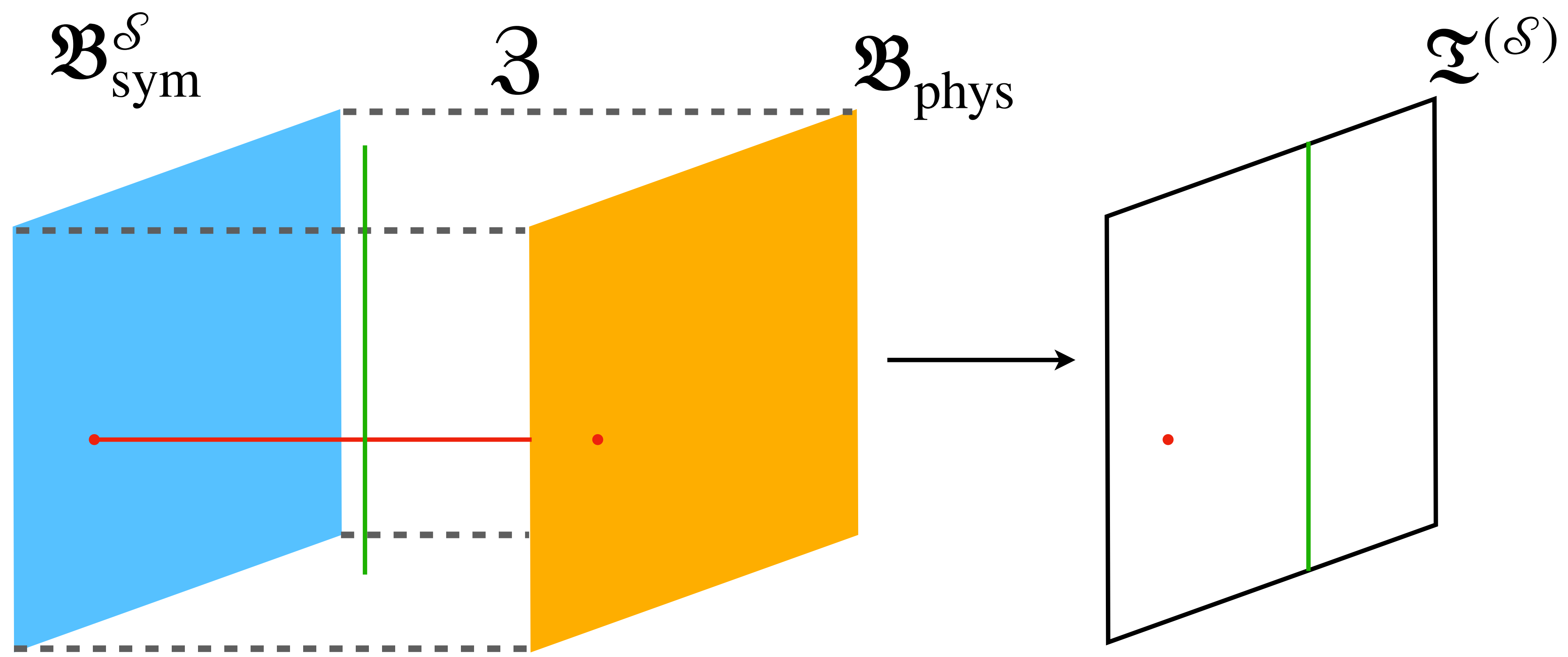

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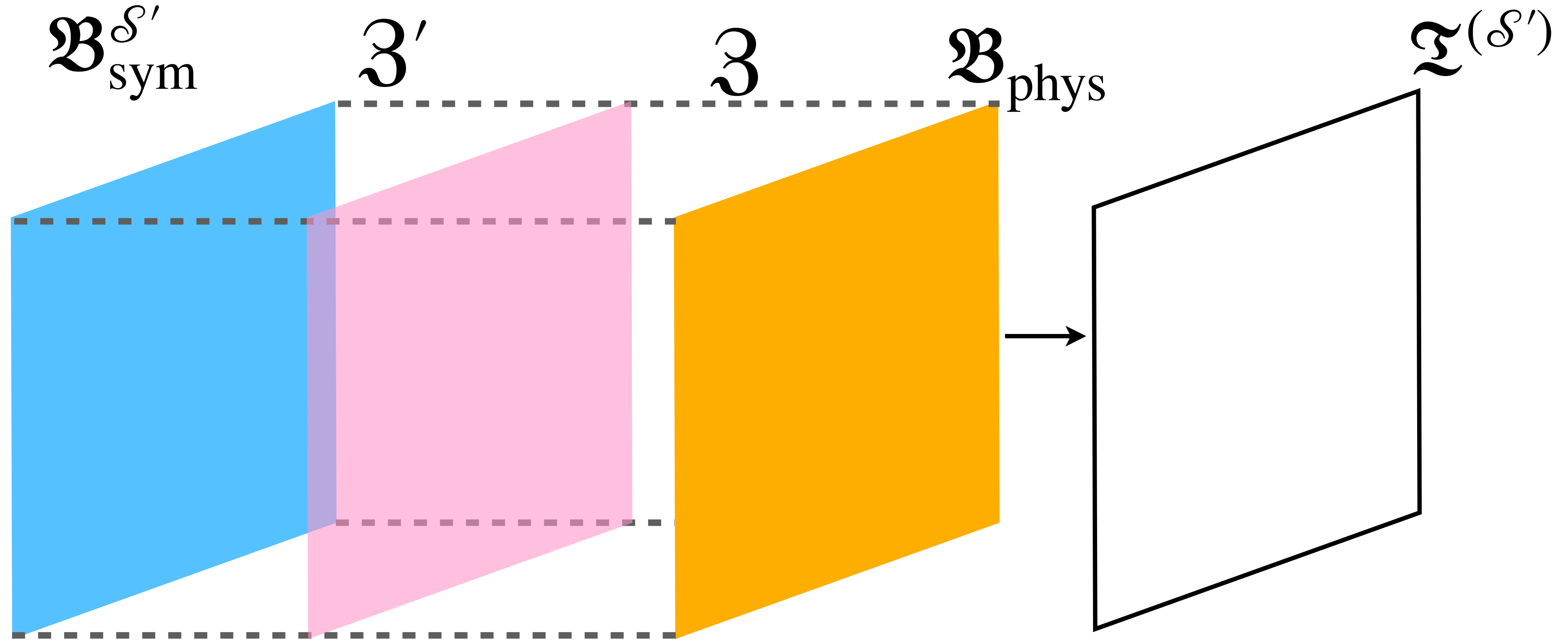

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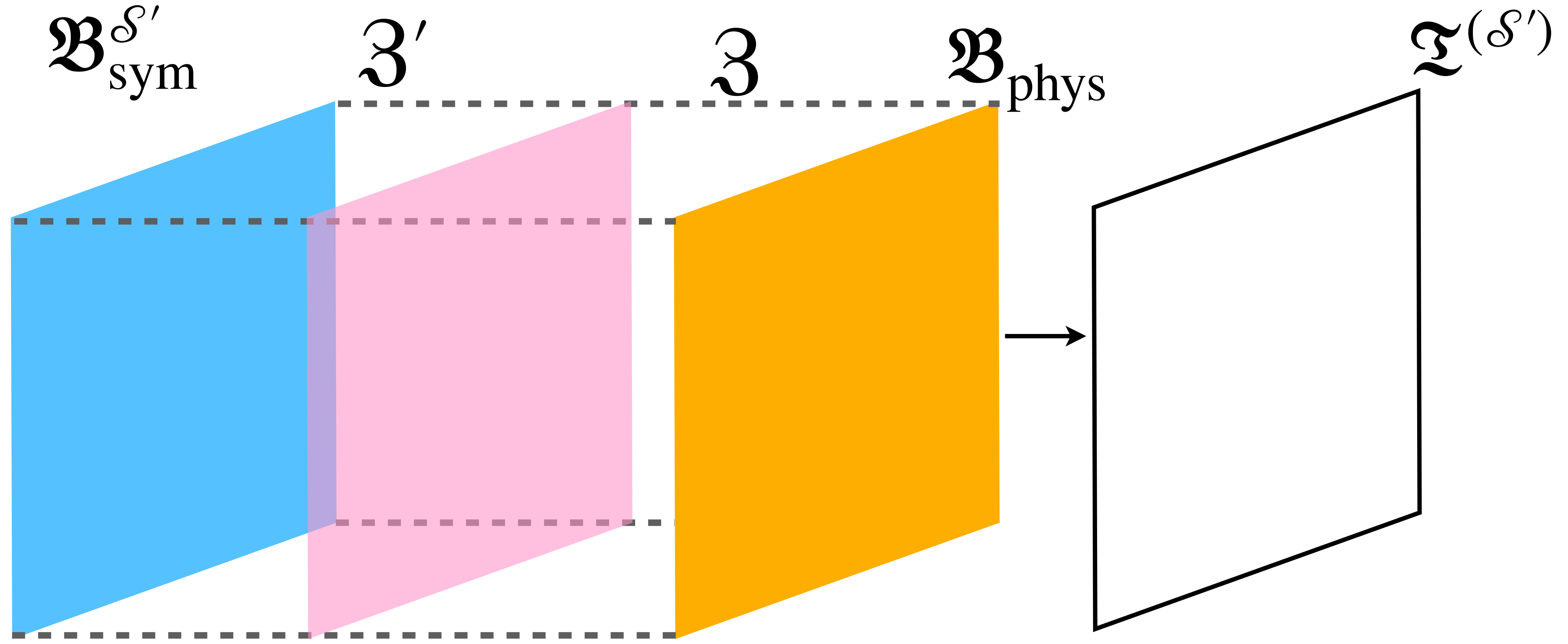

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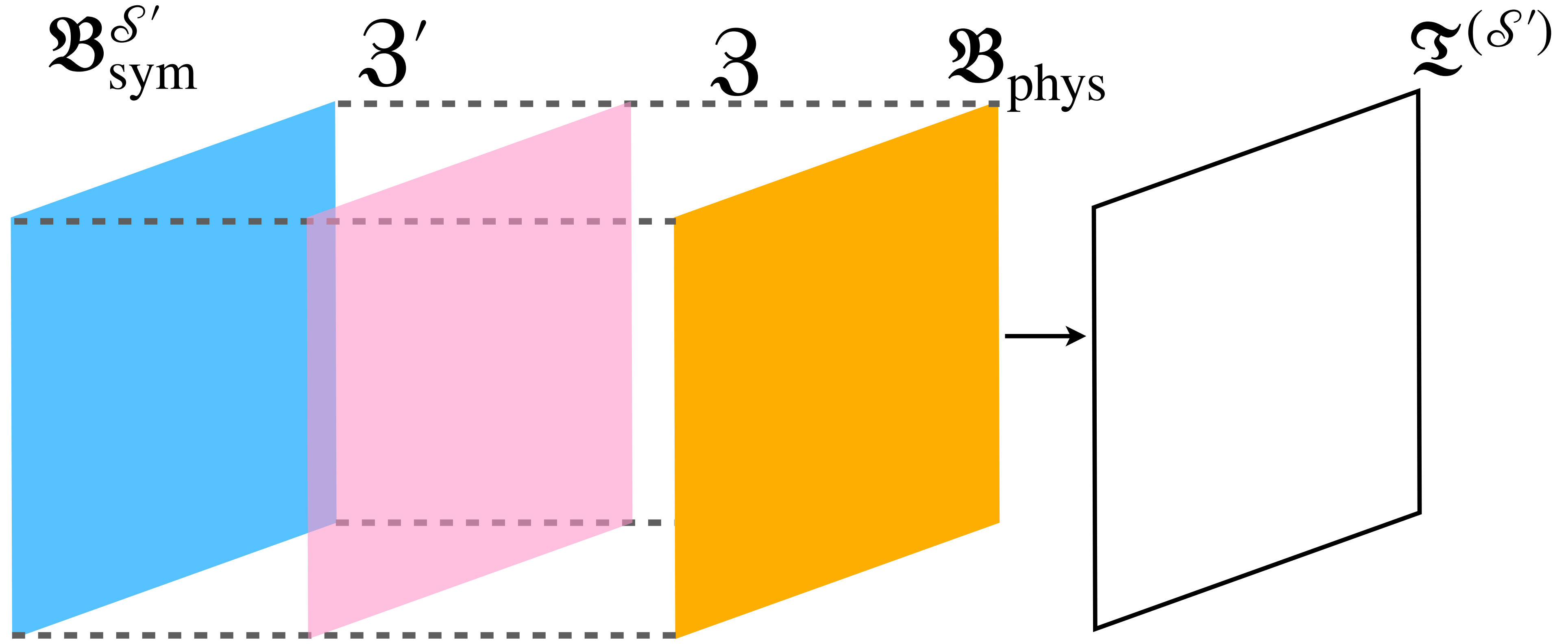

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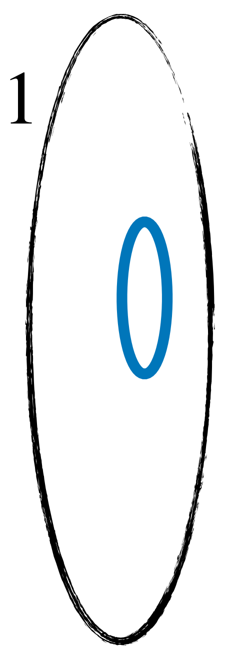
- How are these features geometrically realized in string theory?

Plan of the talk

1. Topological SymTFT-interface in M-theory geometric engineering
2. Interfaces in SymTFT of "frozen" $d \geq 6$ $\mathfrak{sp}(N)$ SYM
3. Summary & Outlook



SymTFT in M-theory geometric engineering

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- Topological operators in $W_{D+1=12-m}$ from “branes at infinity”:
  - M2s on $H_1^{\text{tor}}(\partial X) \rightsquigarrow$ generators for magnetic (D-3)-form symmetry,
  - M5s on $H_{m-3}^{\text{tor}}(\partial X) \rightsquigarrow$ generators for electric 1-form symmetry,
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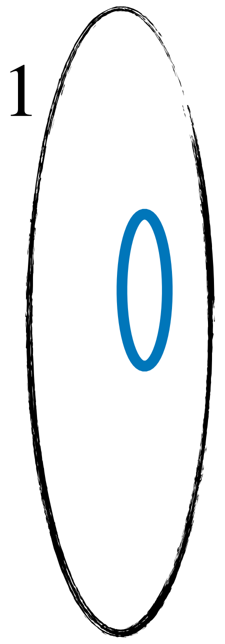

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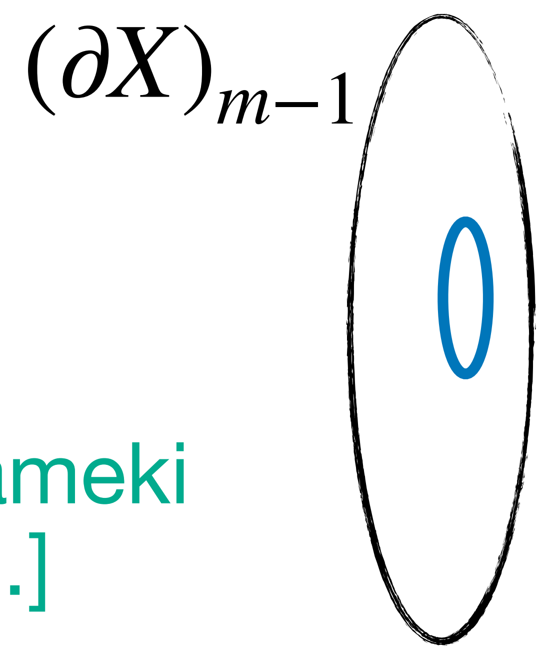

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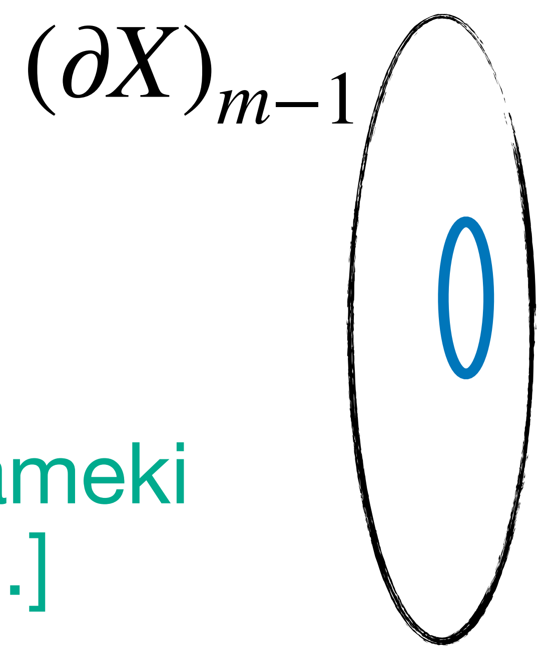

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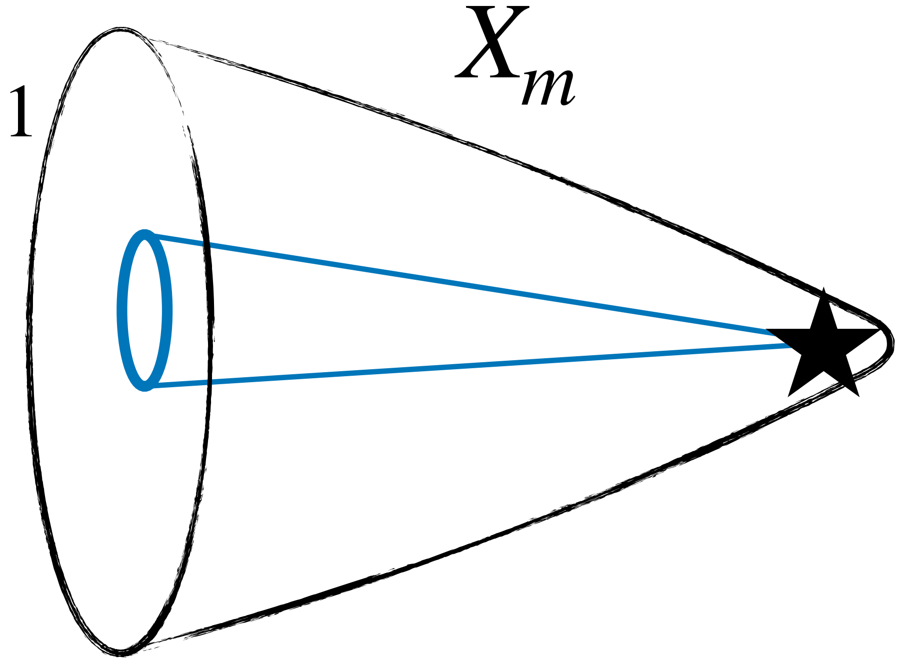
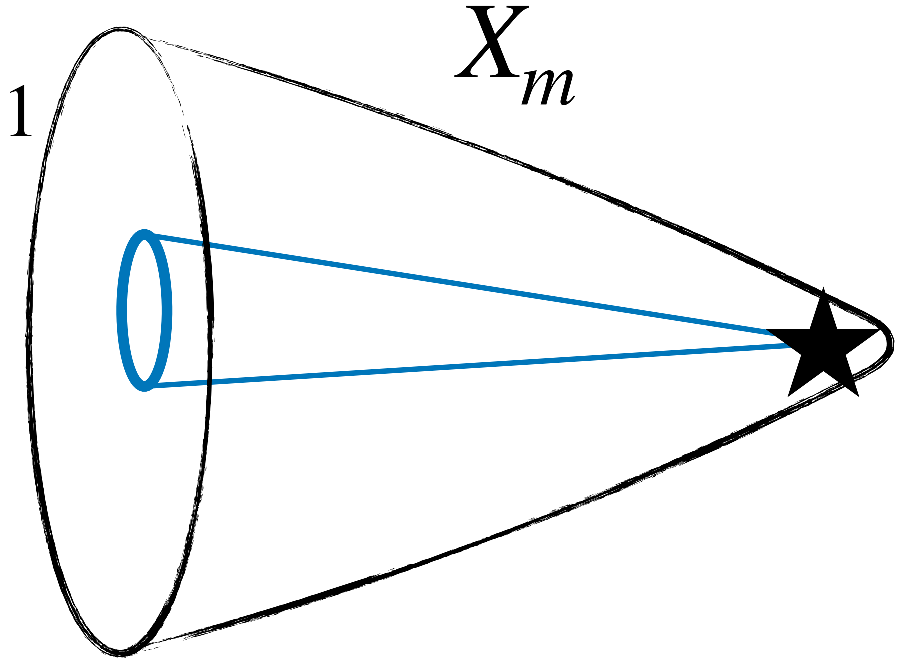
$$\int G_4\, G_7 \rightsquigarrow \ell(e^{(2)}, e^{(m-2)}) \int_{W_{12-m}} B_2\, \delta B_{9-m} = \frac{1}{N} \int_W B_2\, \delta B_{9-m}$$



Coupling to physical boundary…

- Dynamical objects in $\mathfrak{B}_{\text{phys}}$ by wrapping $H_p(X)$ (compact cycles)
- Probe objects from $H_p(X, \partial X)/H_p(X) \equiv H_p(X, \partial X)/\text{im}(\jmath) = \text{coker}(\jmath)$



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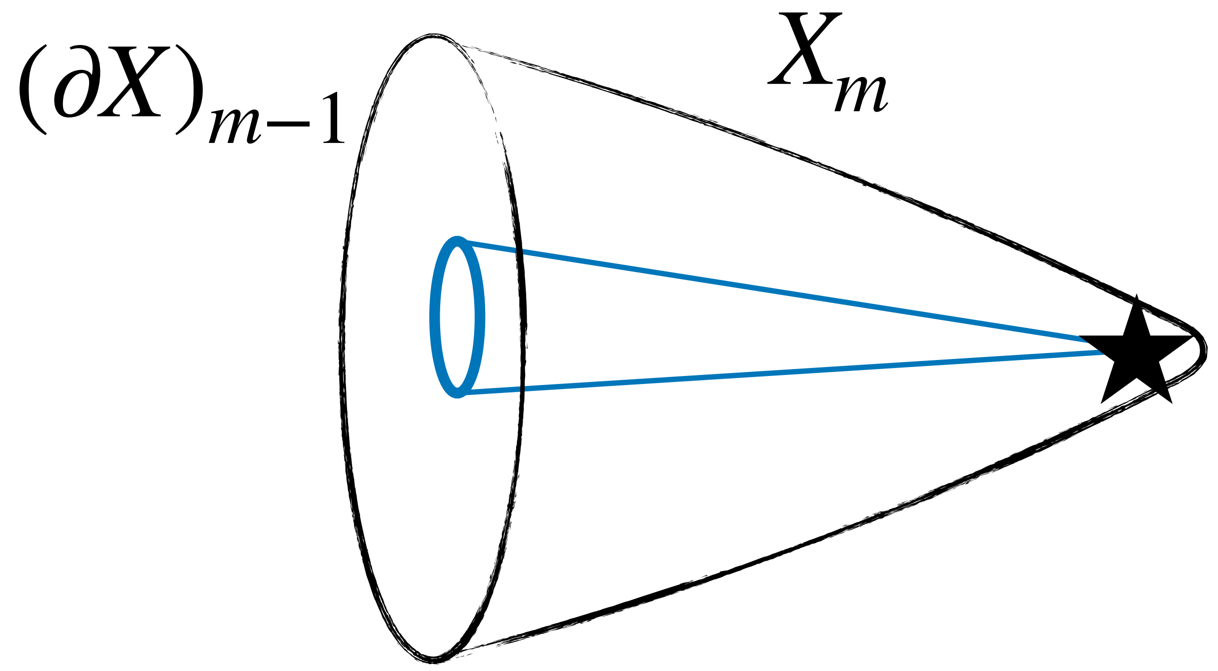

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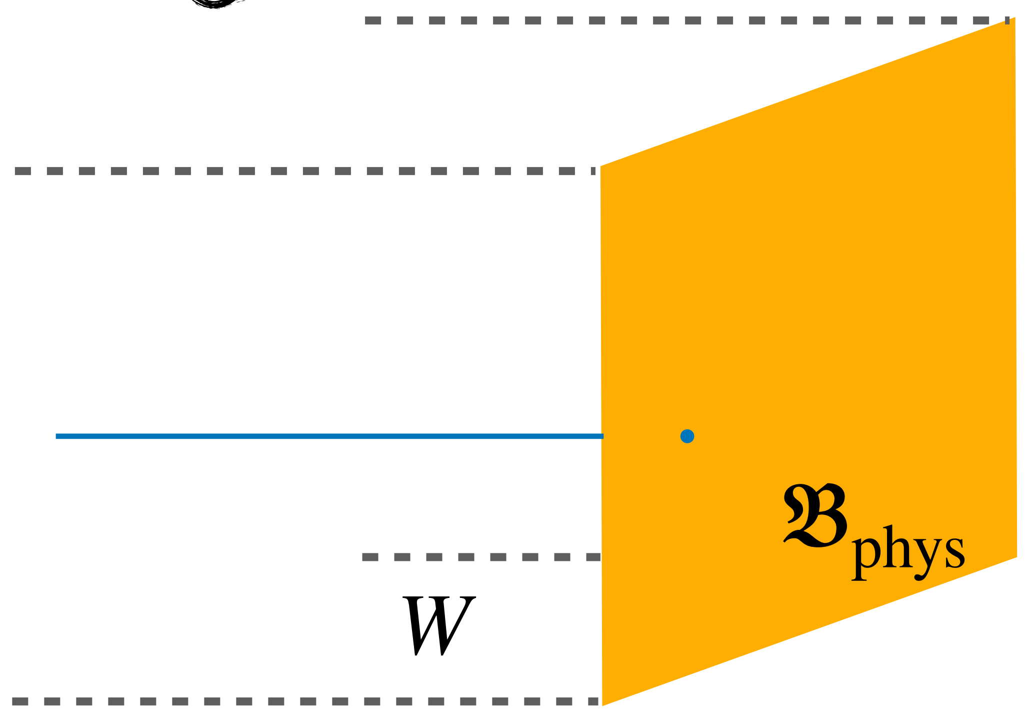

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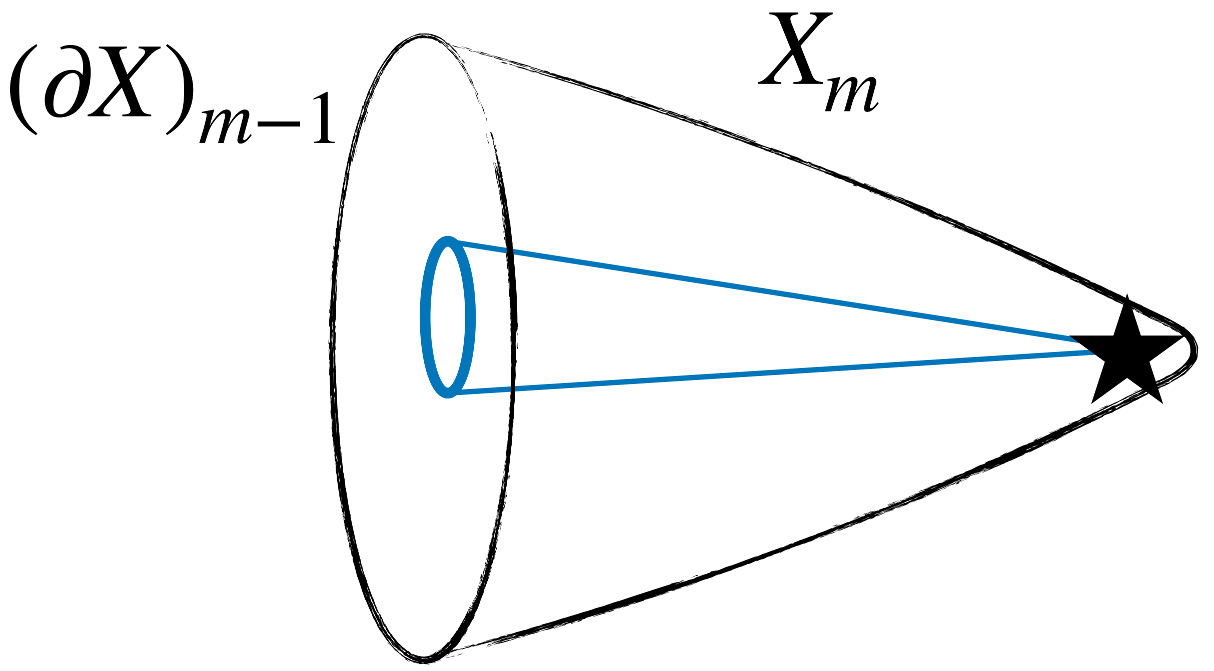

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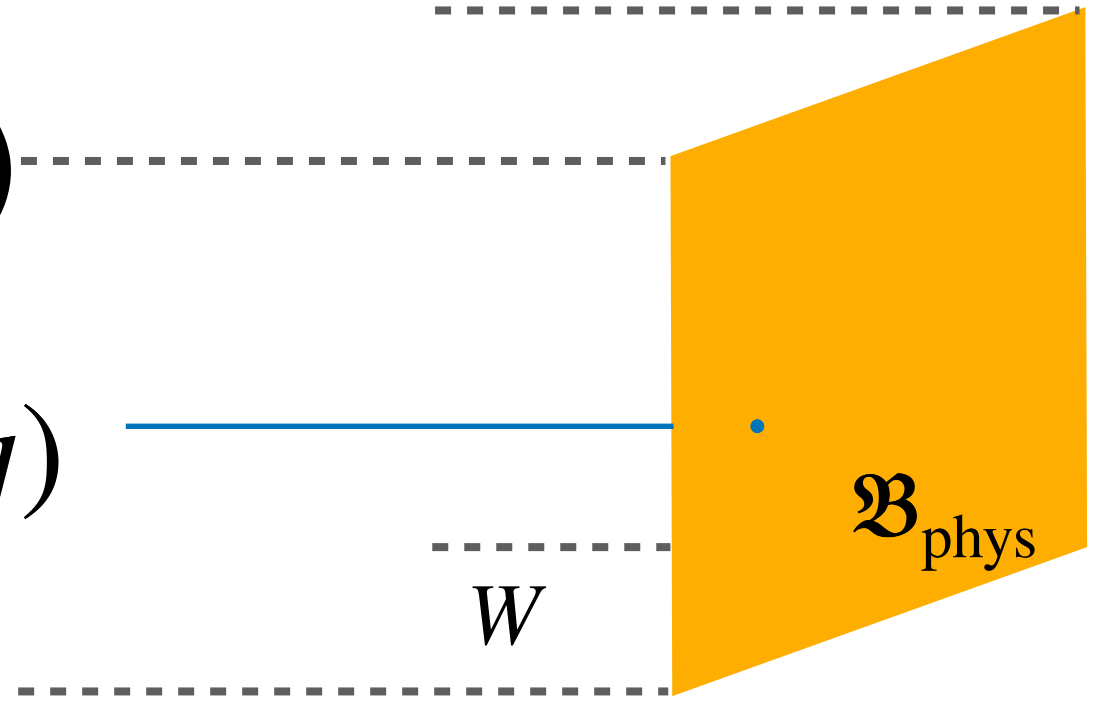

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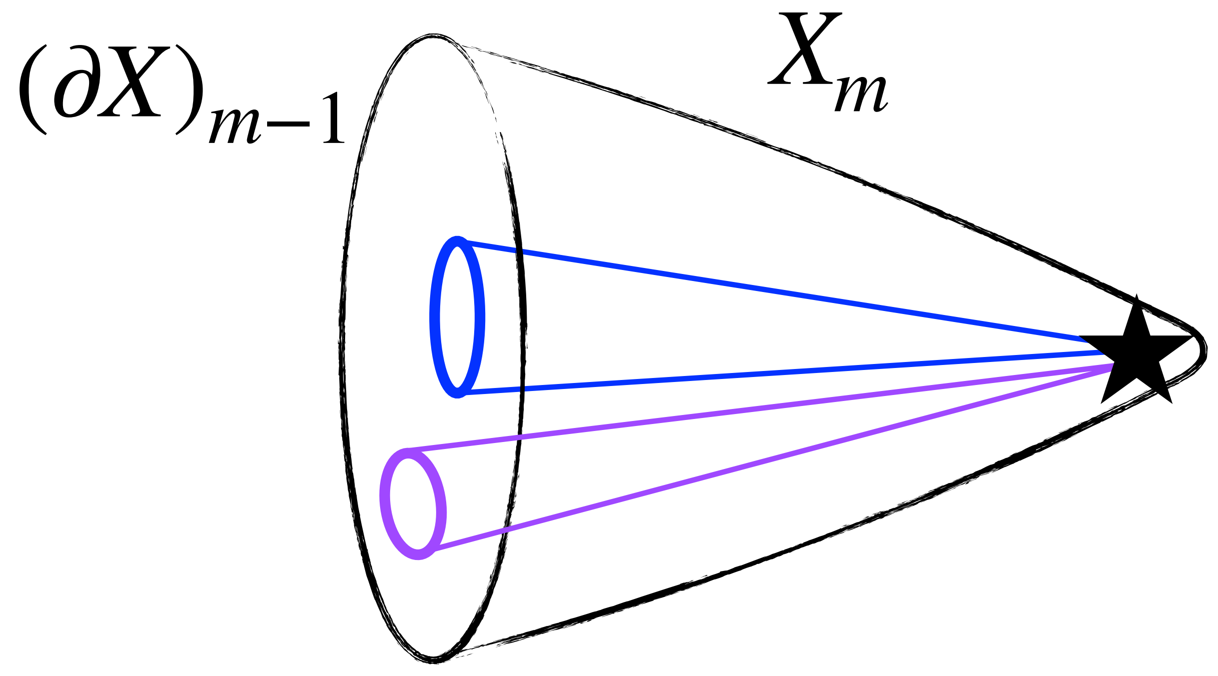
- In cohomology: $H^2(X) \ni \eta \overset{\partial^*}{\mapsto} e \in H^2(\partial X)$, with $G_4 = B_2\, e^{(2)}$ resp. $G_4 = E_2\, \eta^{(2)}$, then $E_2 = B_2|_{\mathfrak{B}}$ has Dirichlet b.c.

Coupling to physical boundary… without torsion



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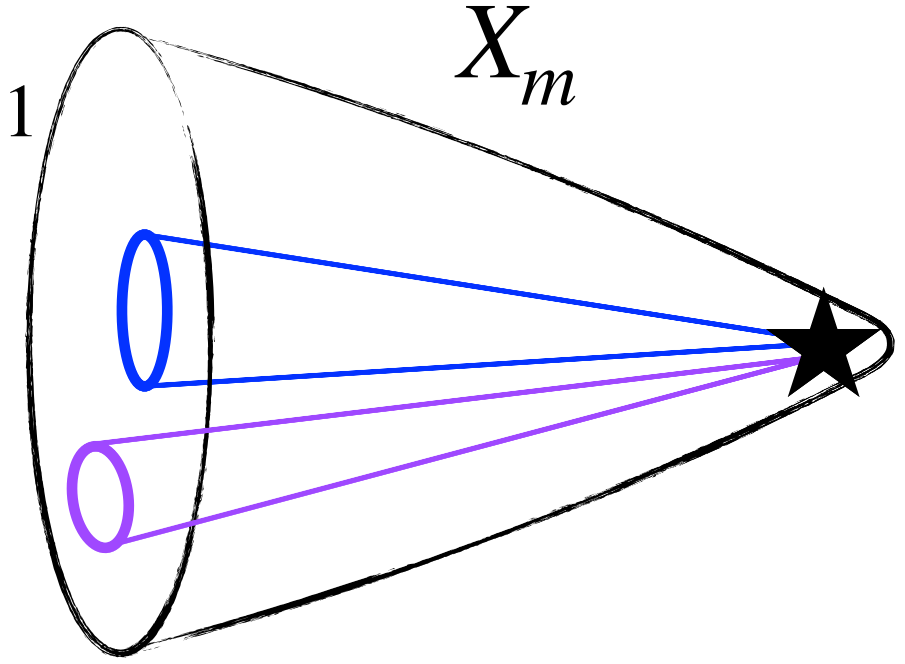

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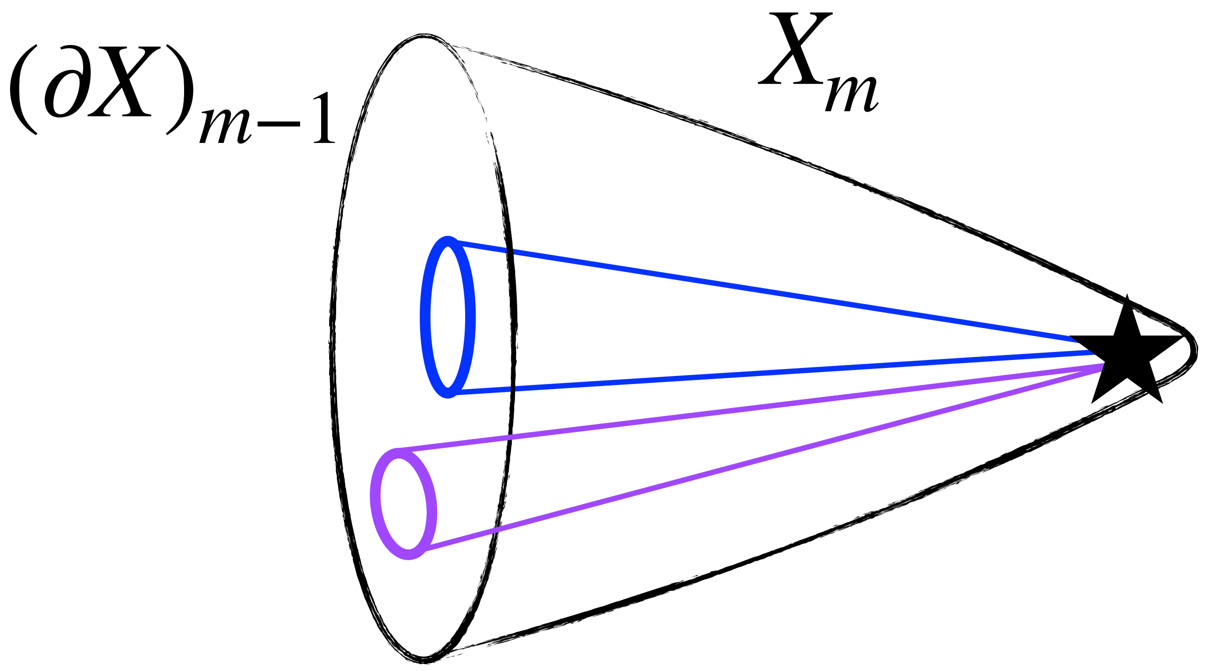

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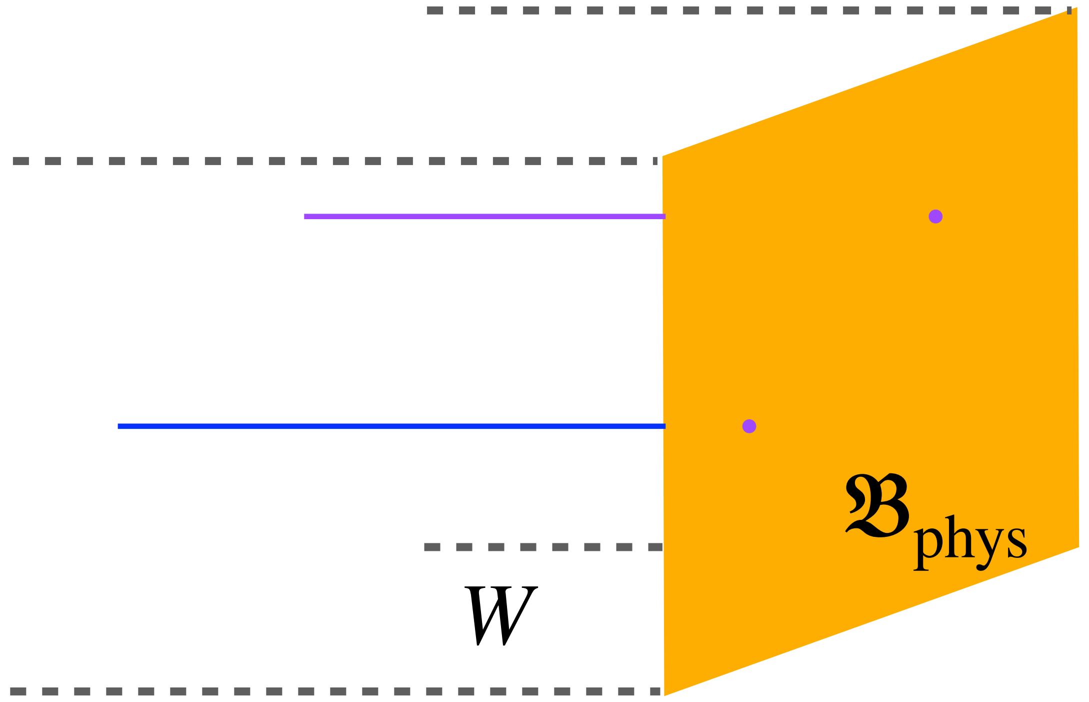

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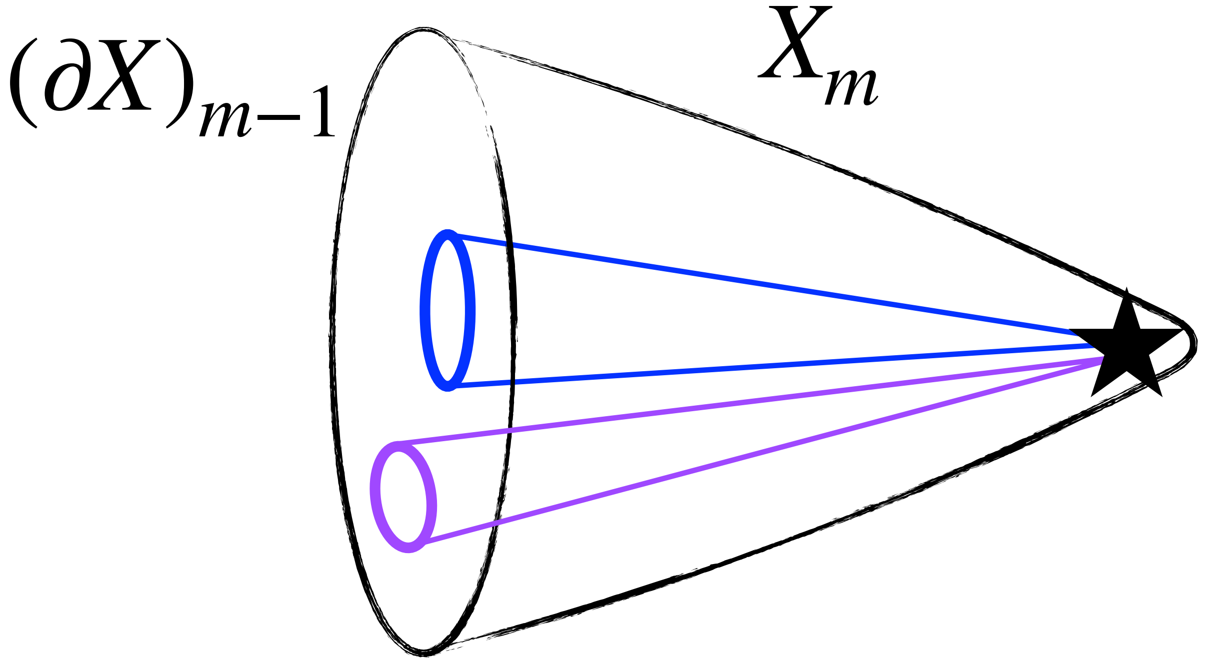

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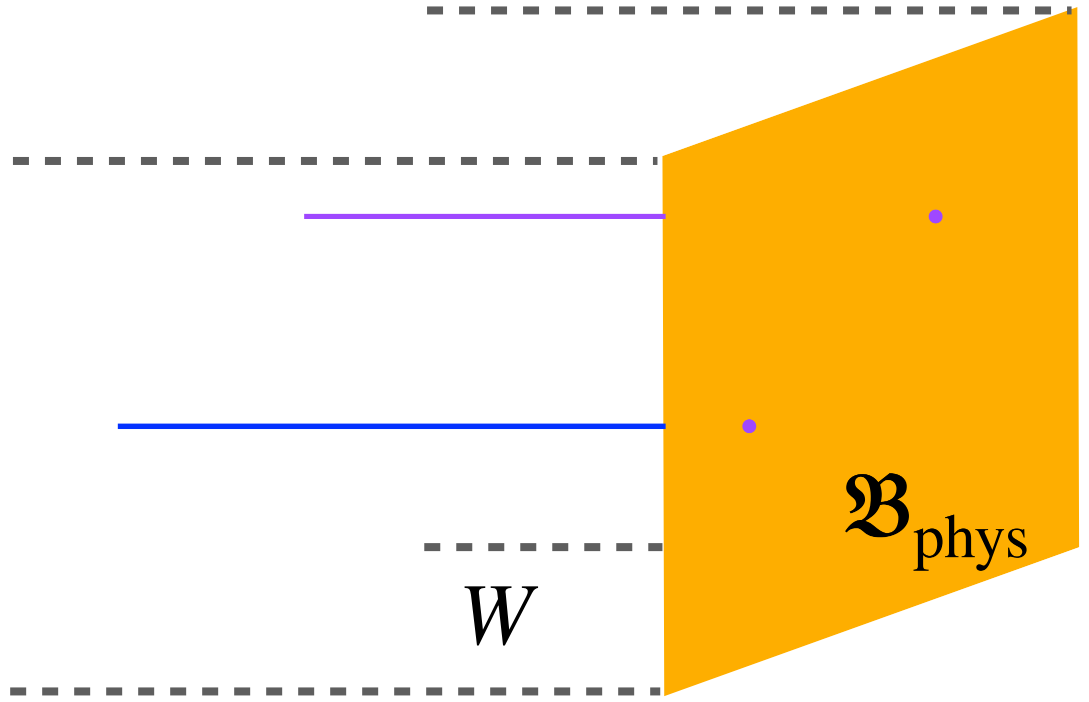

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- Can show in cohomology: $B_p|_{\mathfrak{B}} = \frac{1}{N} f_p = \frac{1}{N} da_{p-1}$ as eom;
  so defects are (non-top.) Wilson/'t Hooft operators of SYM sector.

Coupling to physical boundary… with torsion

[see also Cvetič/Heckman/Hübner/Torres ’23, Gould/LL/Sabag ’23, Cvetič/Dierigl/LL/Torres/Zhang ’24]

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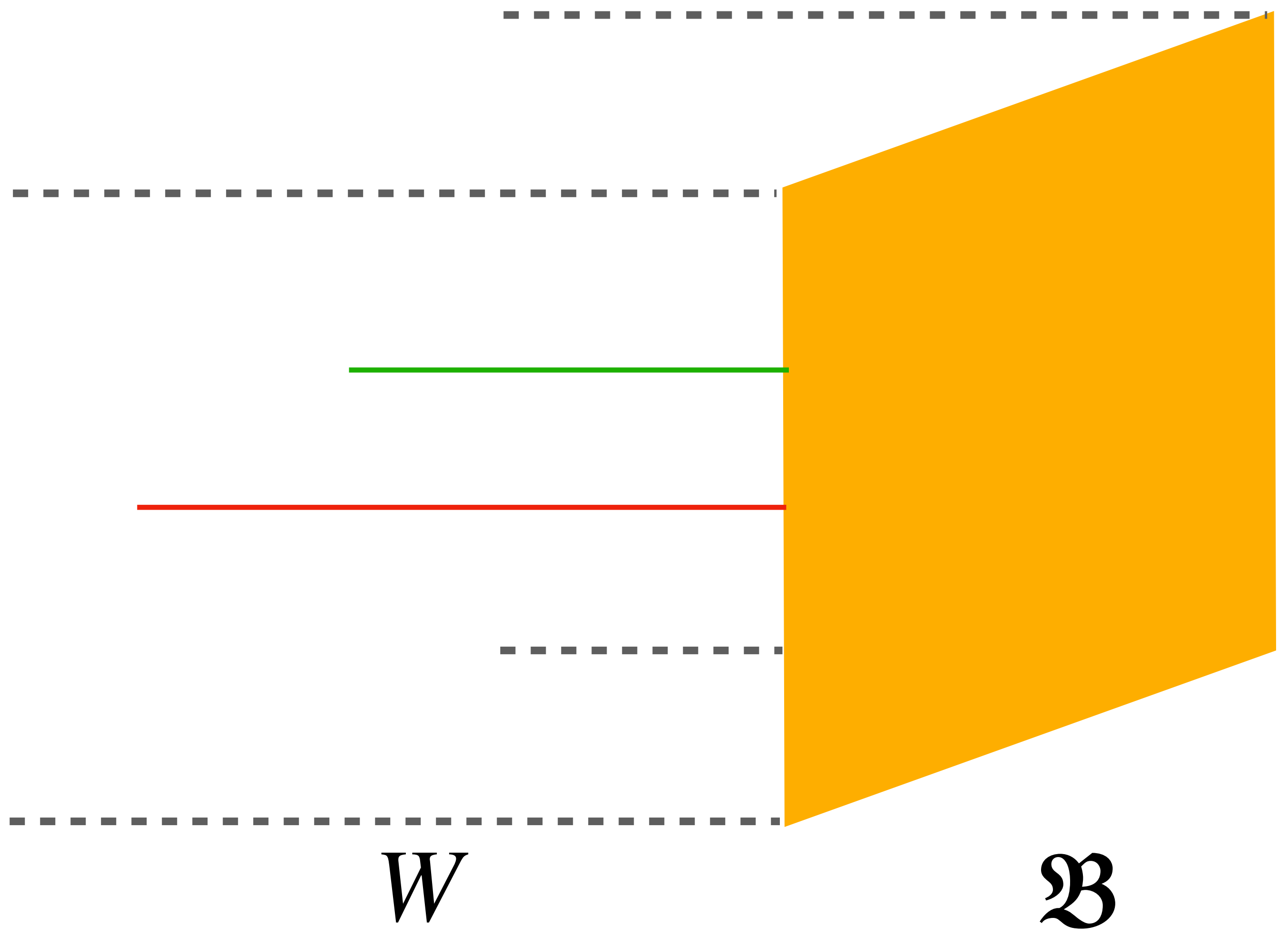

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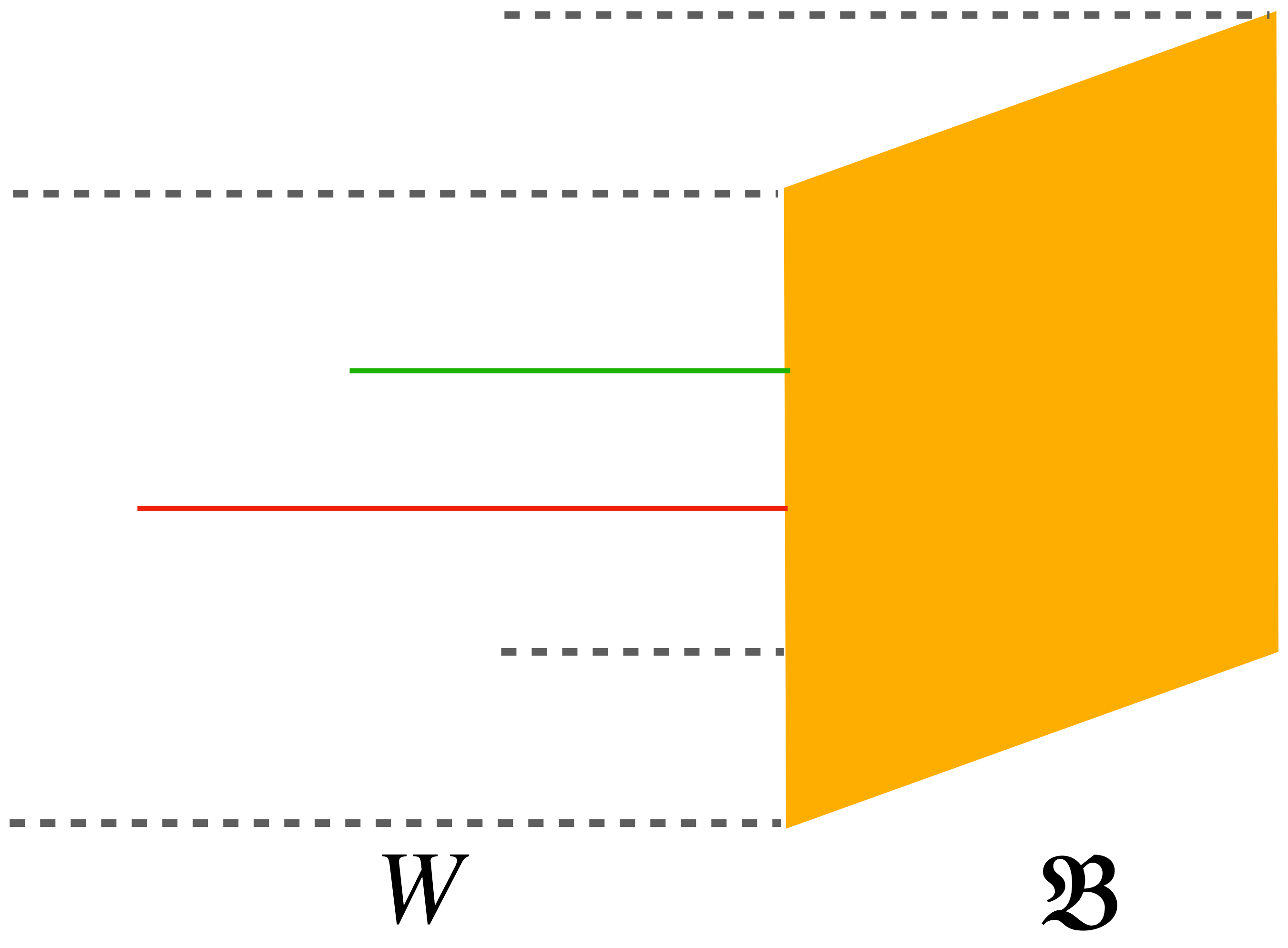

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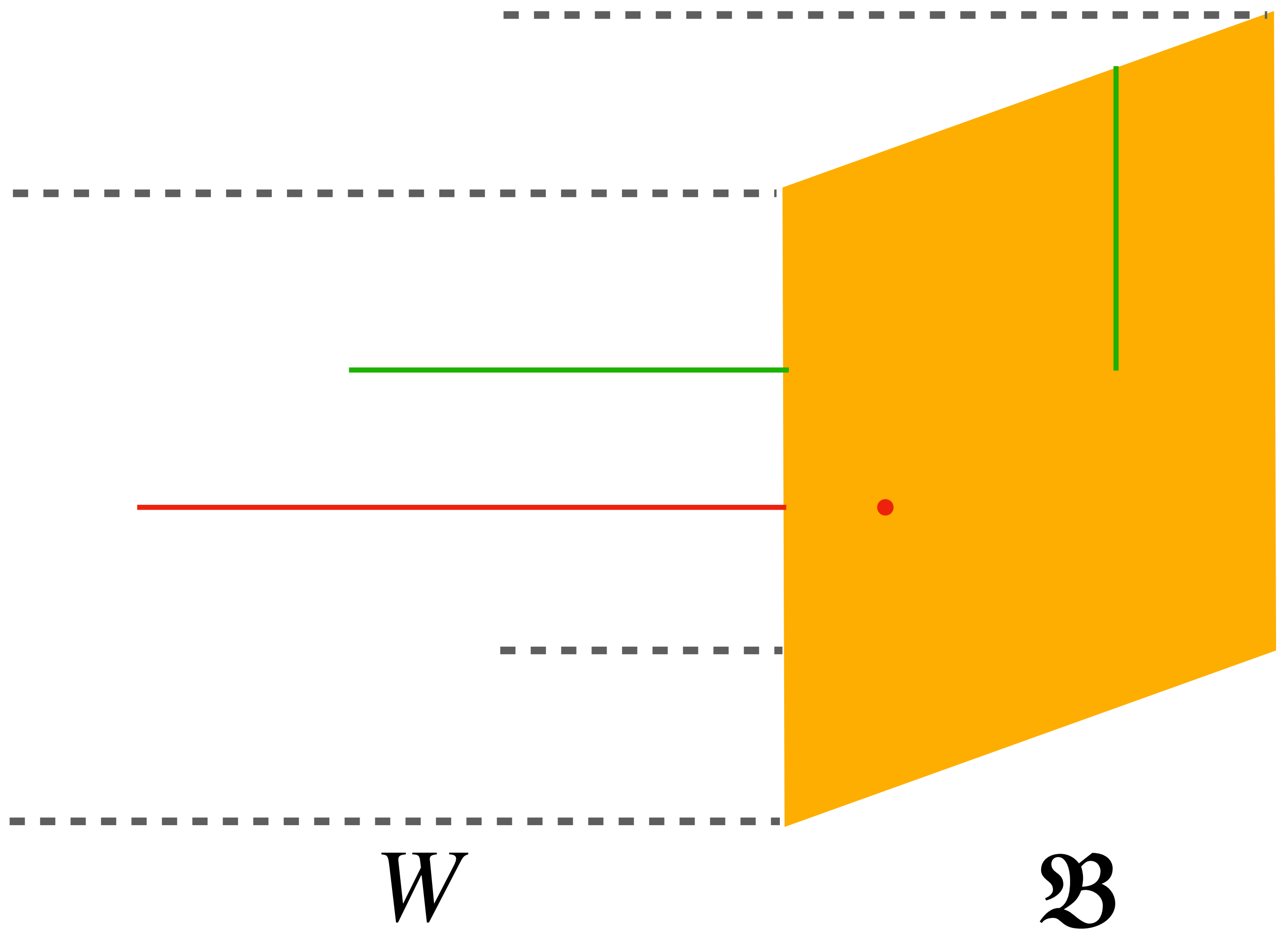
- E.g., $0 \neq \tau_1 \in H_1^{\mathrm{tor}}(X)$, $\Sigma_{m-2} \in H^{\mathrm{tor}}_{m-2}(X, \partial X)$ s.t. $\partial(\Sigma_{m-2}) = s_{m-3}\,, \quad \iota(t_1) = \tau_1$ with $(\tau_1, \Sigma_{m-2}) = \ell(t_1, s_{m-3})$.

Coupling to physical boundary… with torsion

[see also Cvetič/Heckman/Hübner/Torres '23, Gould/LL/Sabag '23, Cvetič/Dierigl/LL/Torres/Zhang '24]



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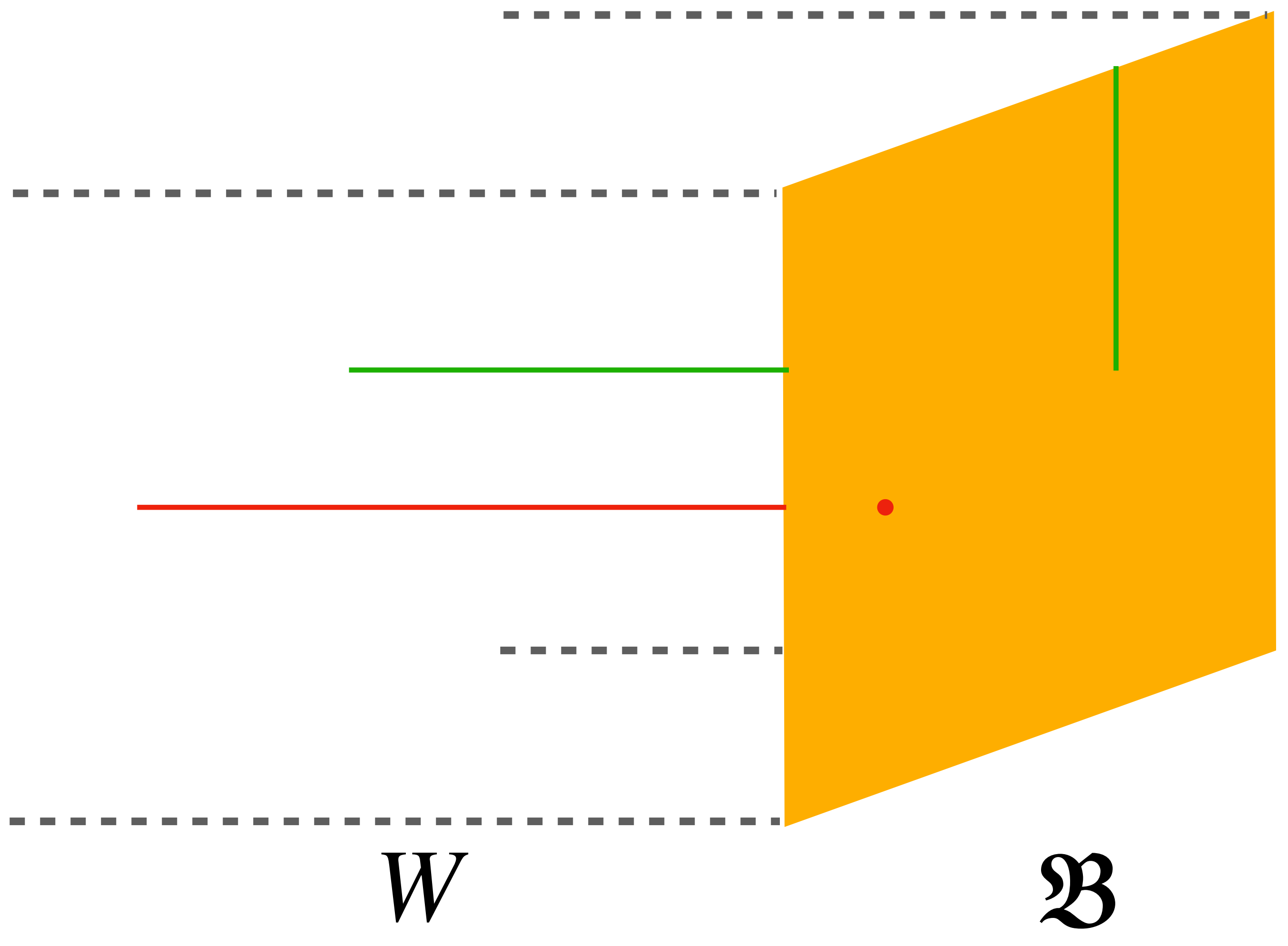

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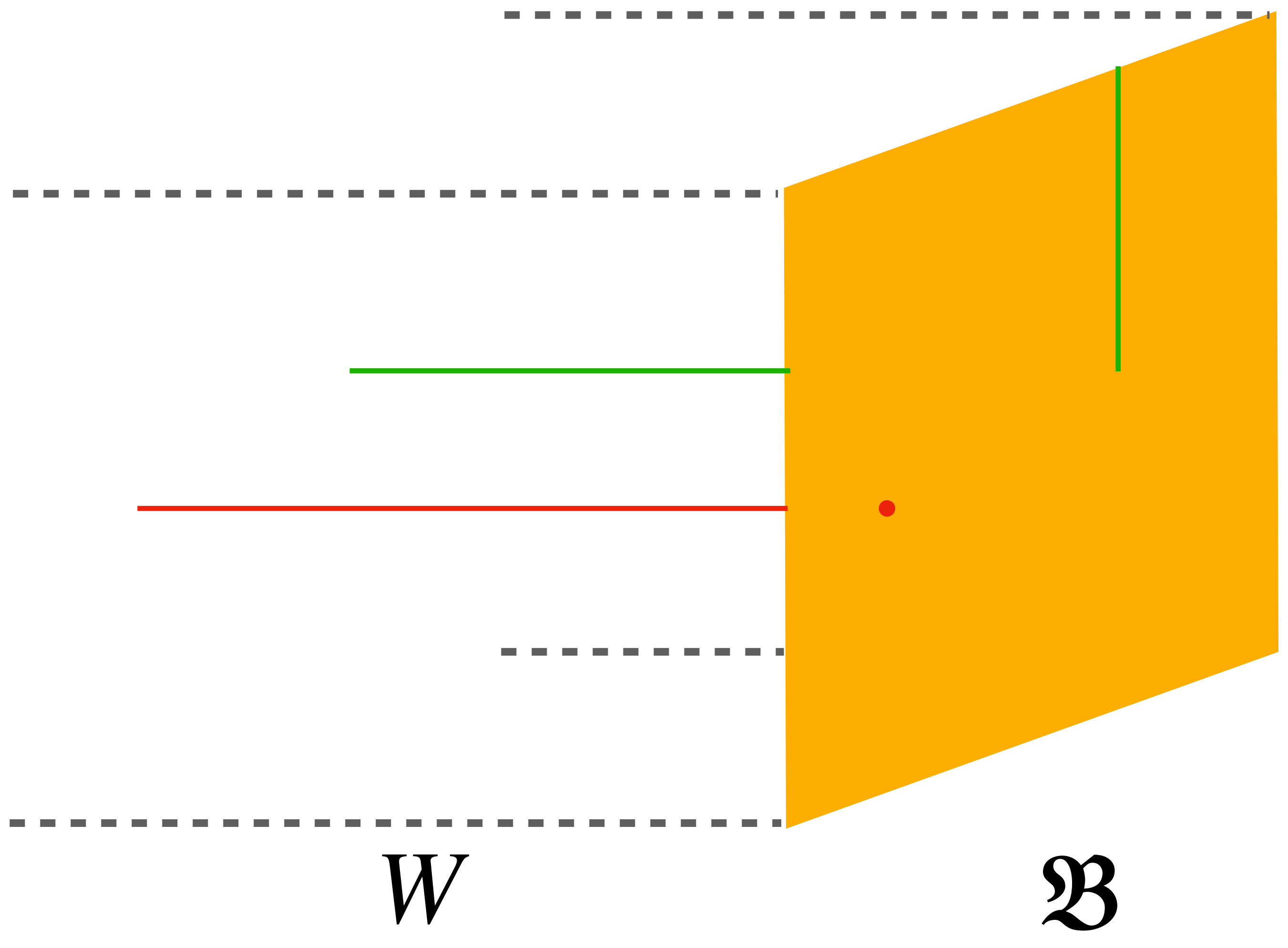
- From cohomology: $G_7 = c_{8-m}\,\omega^{(m-1)}$ with $\omega^{(m-1)} \in H^{m-1}_{\text{tor}}(X,\partial X) = H^{\text{tor}}_{m-2}(X,\partial X)$ $\rightsquigarrow B_{9-m}|_{\mathfrak{B}} = \delta c_{8-m}$ (D), but $B_2|_{\mathfrak{B}}$ free (N) [for $S_W \supset \frac{1}{N}\int_W B_2\,\delta B_{9-m}$]

From torsion to interface



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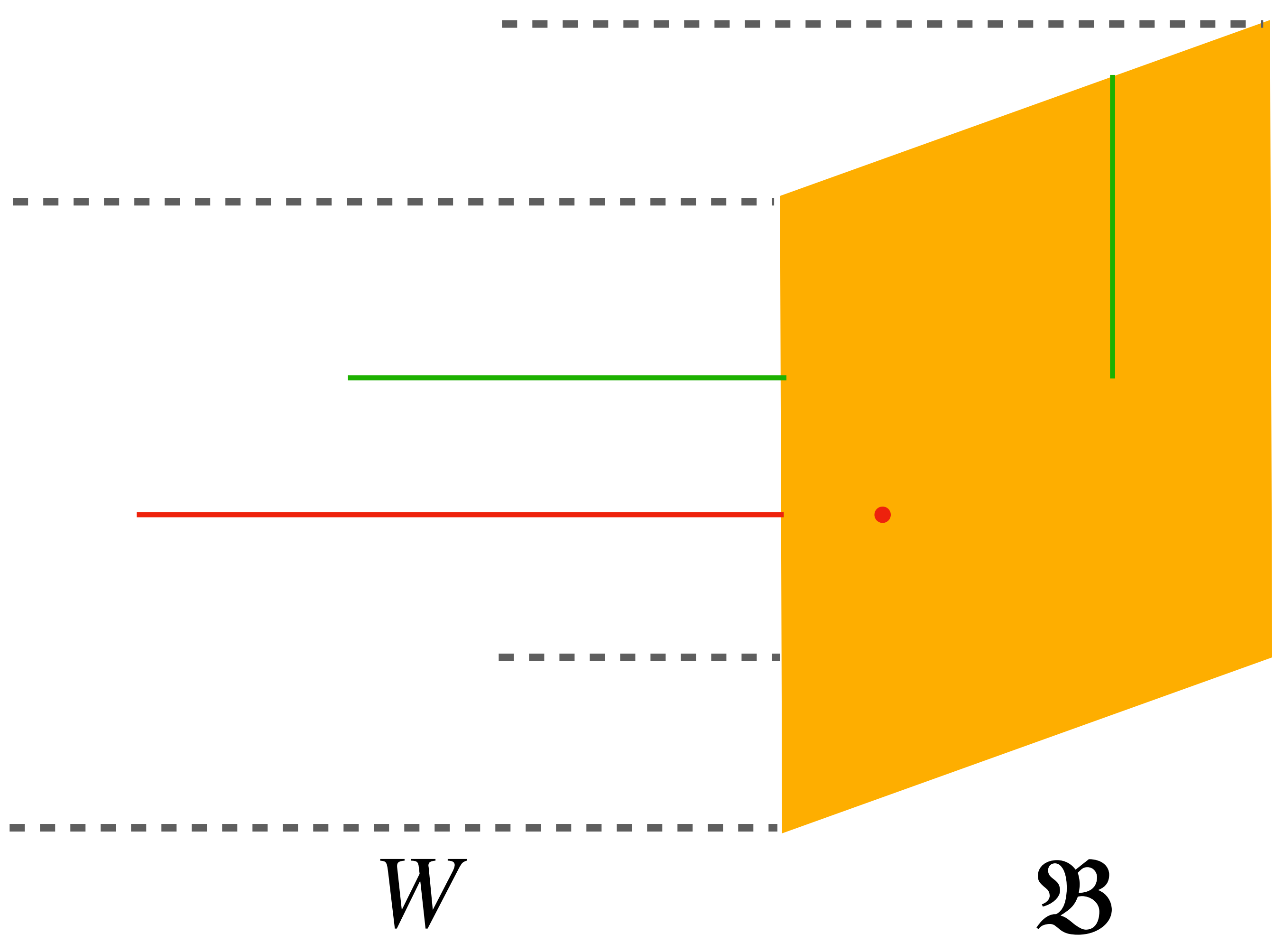

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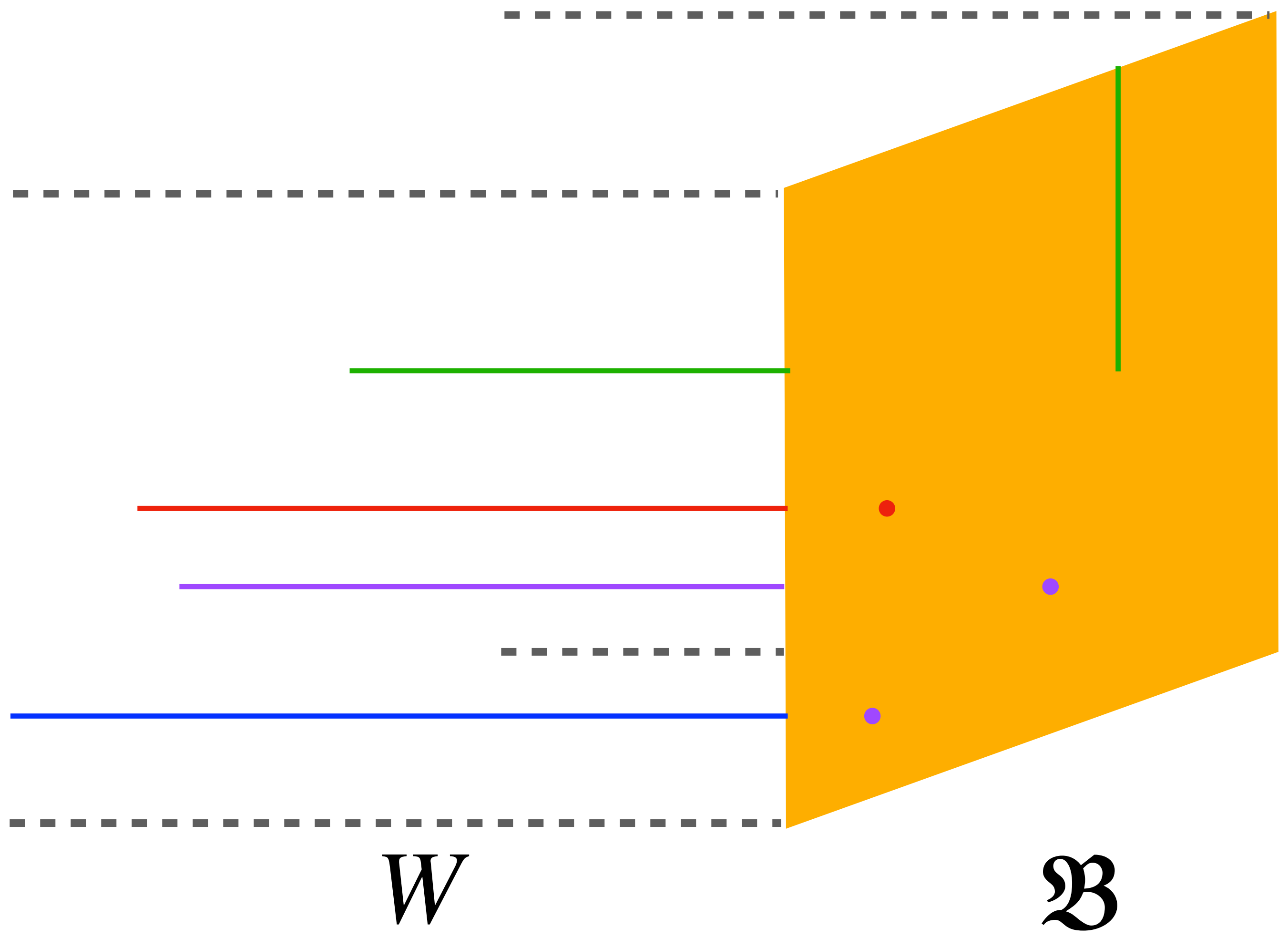

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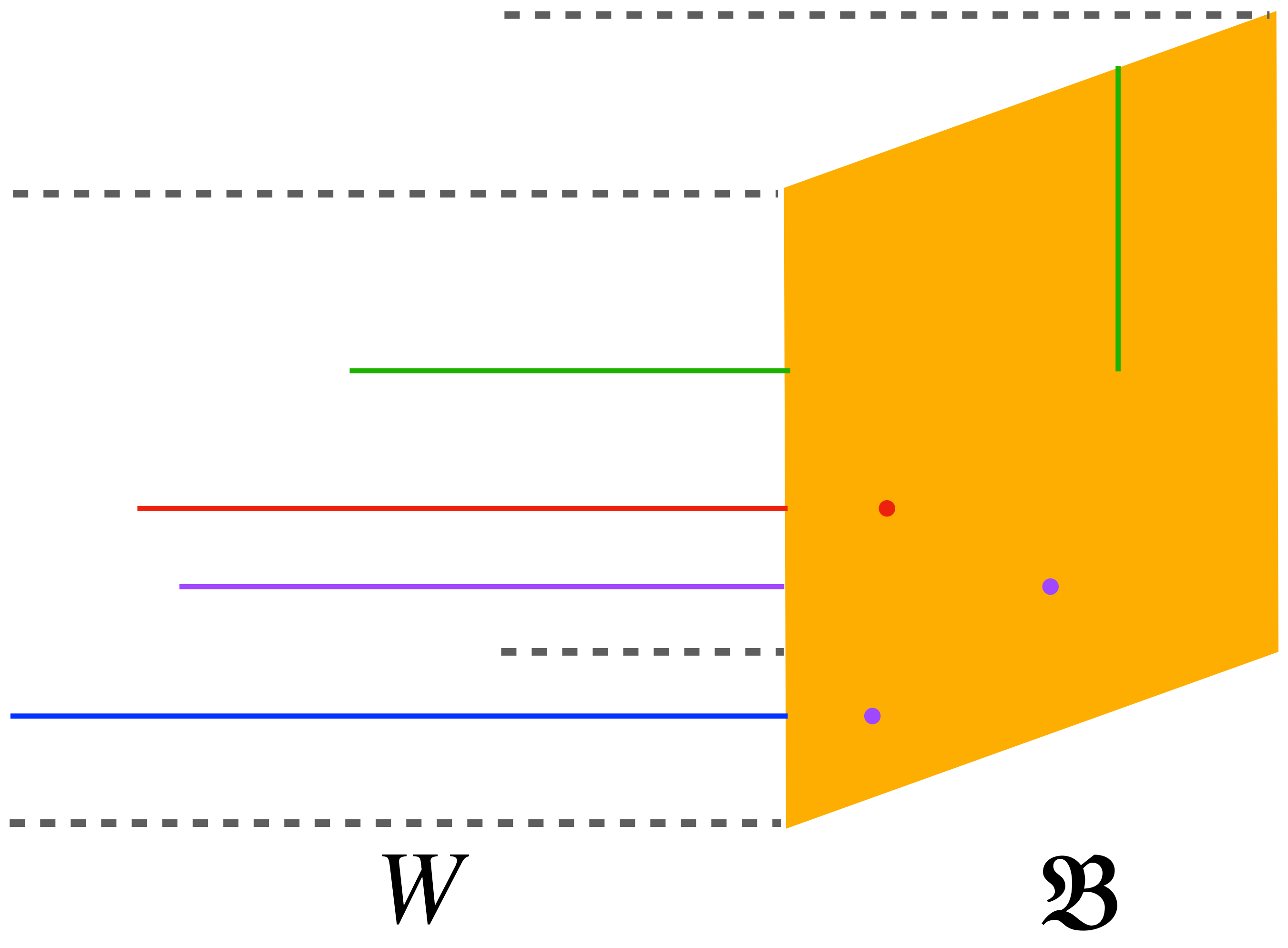

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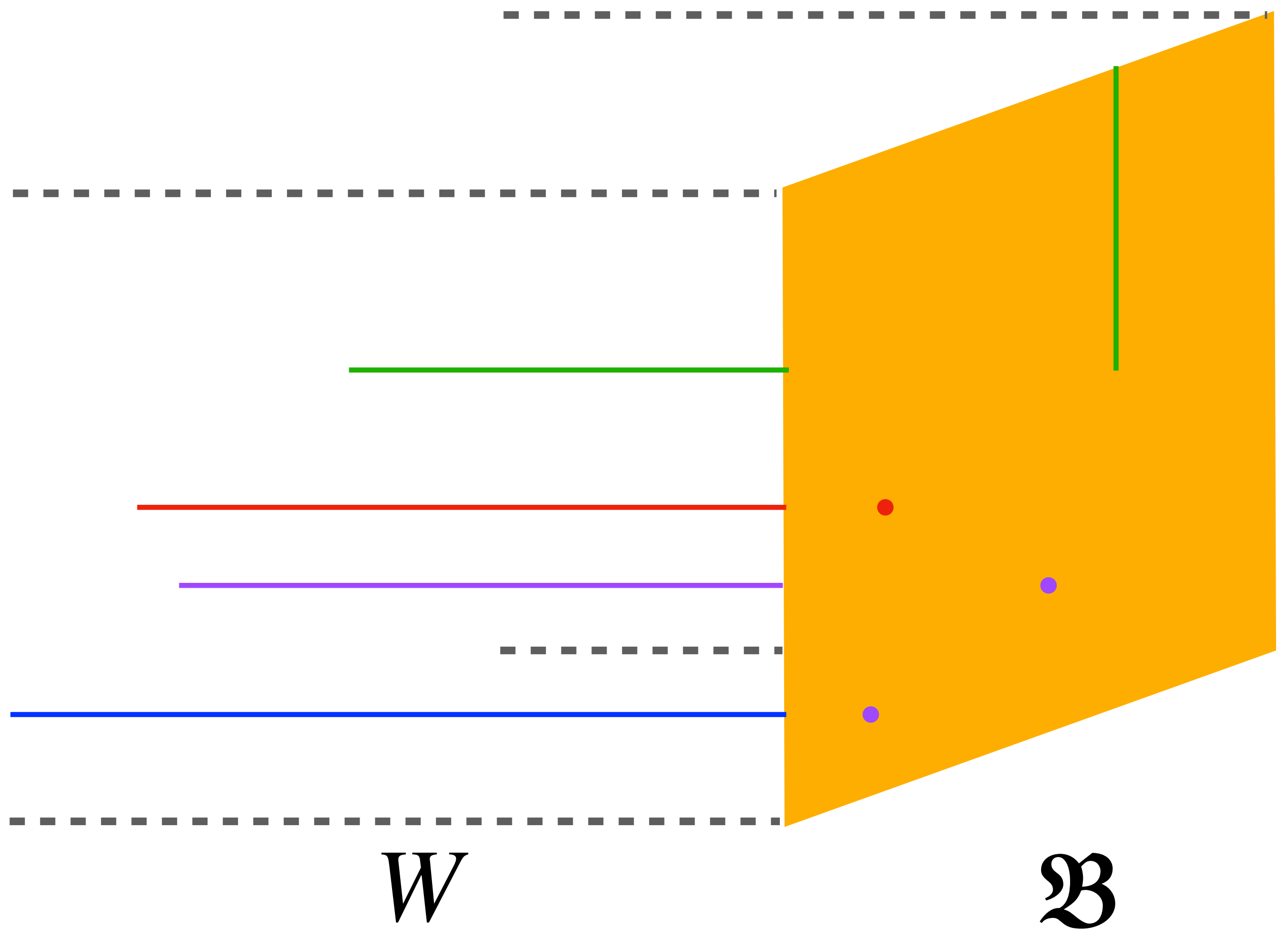

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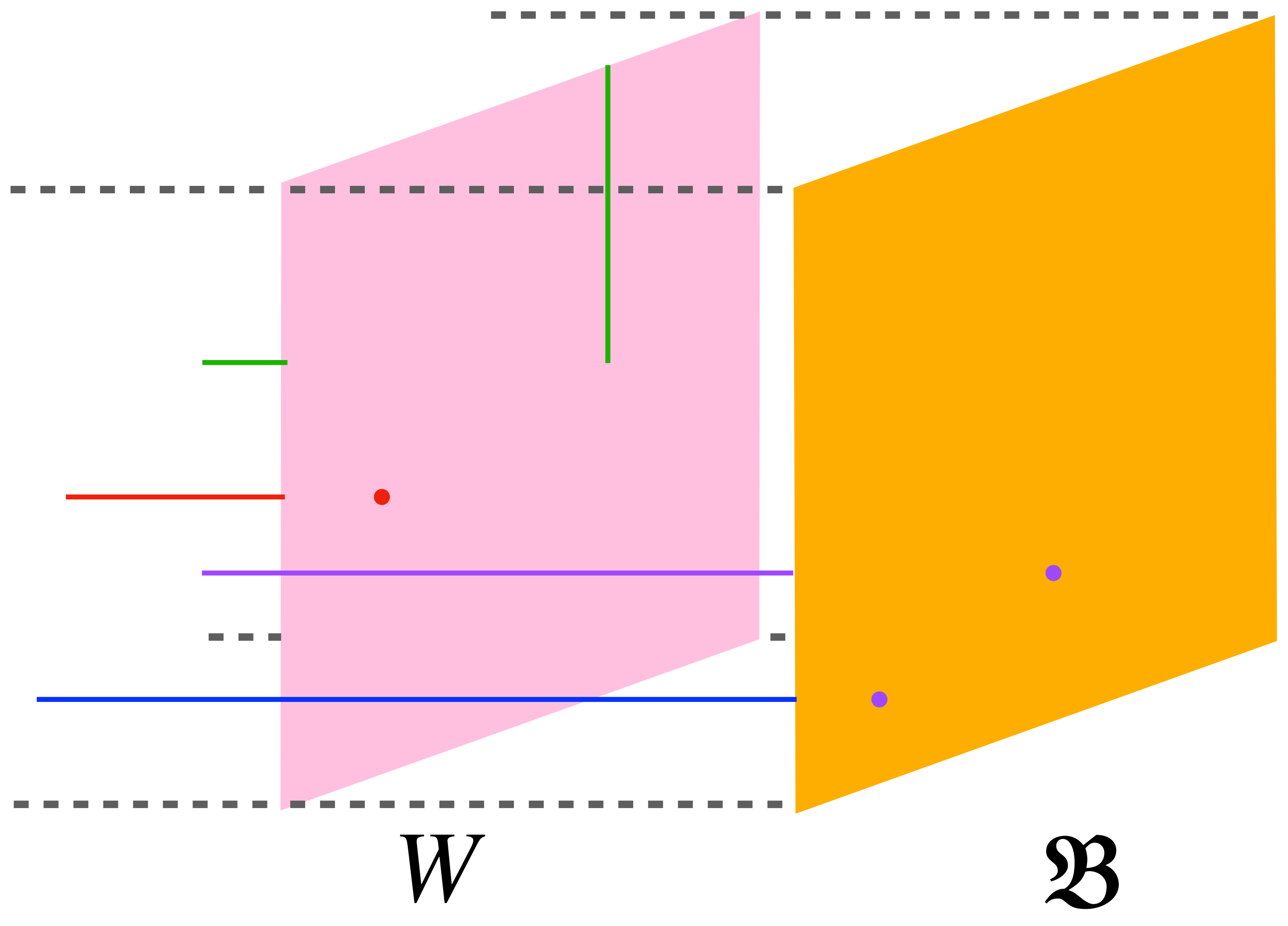

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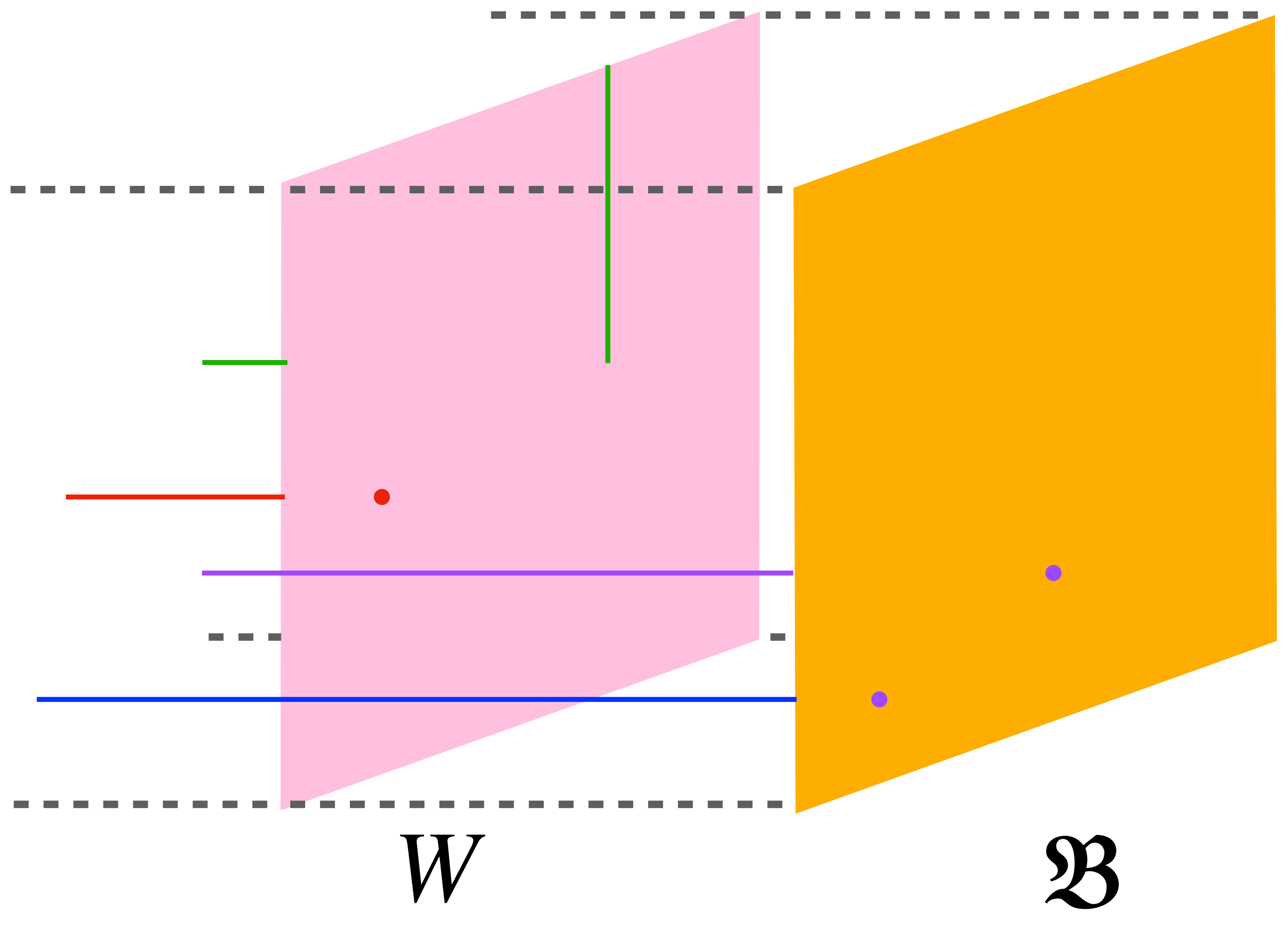

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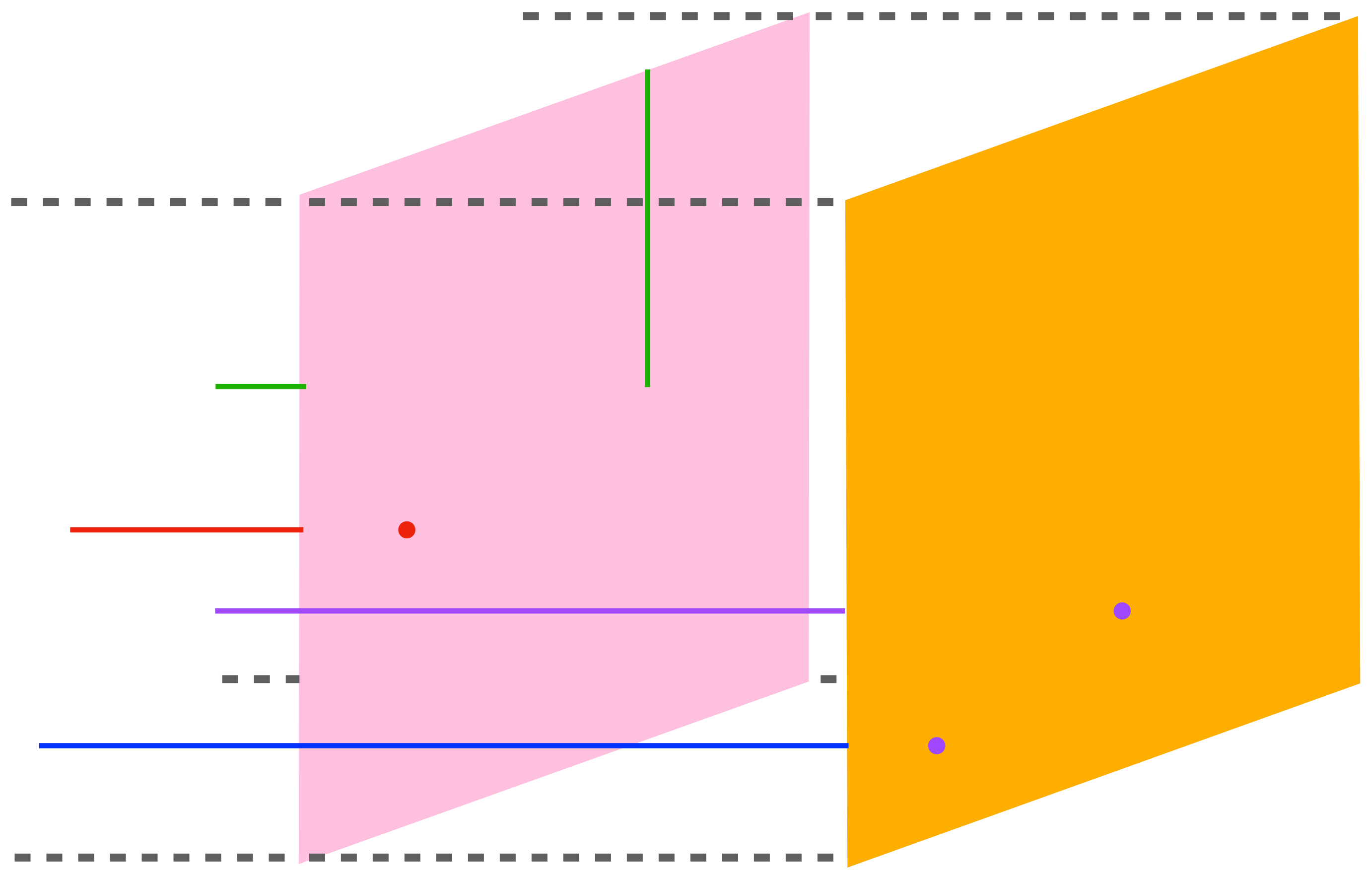

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[non-topological interfaces in stringy SymTFT see [Baume/Heckman/Hübner/Torres/Turner/Yu ’23, Cvetič/Donagi/Heckman/Hübner/Torres ’24, García Etxebarria/Huertas/Uranga ’24, …]]

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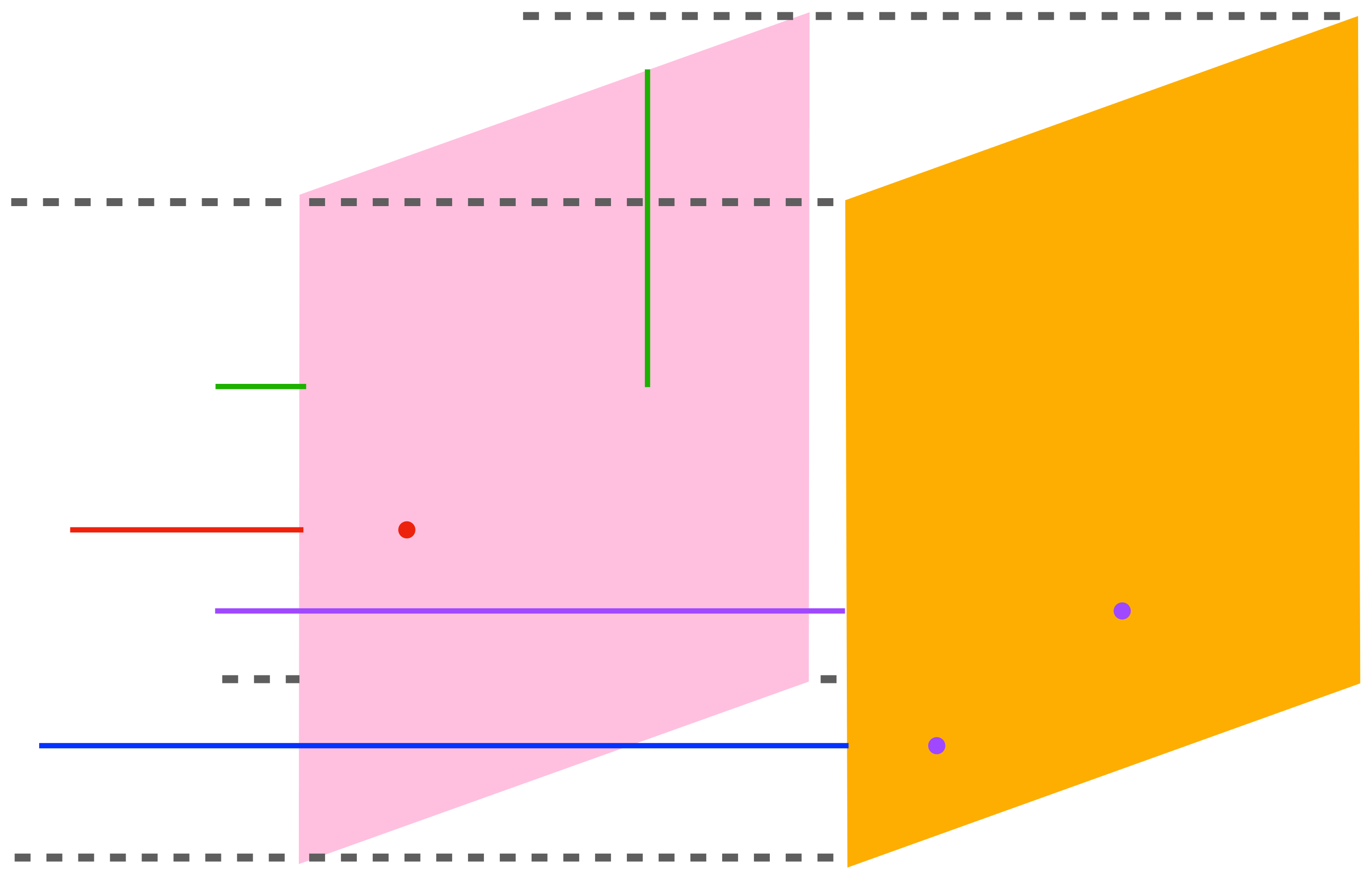

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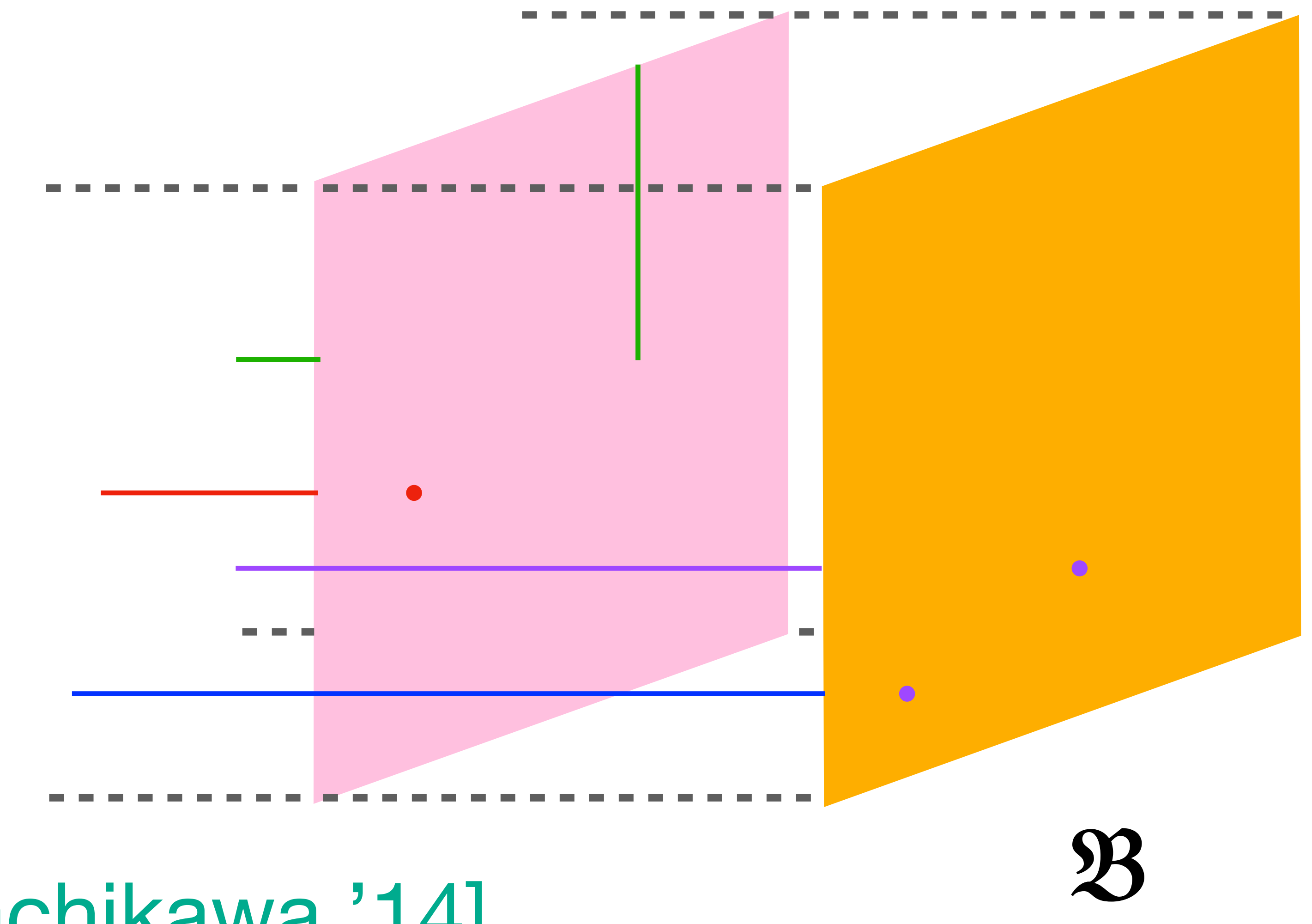
- Note: geometry $\rightsquigarrow$ interface on top of $\mathfrak{B}$

Plan of the talk

1. Topological SymTFT-interface in M-theory geometric engineering
2. Interfaces in SymTFT of “frozen” $d \geq 6$ $\mathfrak{sp}(N)$ SYM
3. Summary & Outlook

SymTFT for $\mathfrak{sp}(N)$ SYM in d≥6



- M-theory on $X_{m=5} = (Y^{\text{ell}}_{I_{2N}} \times S^1)/\mathbb{Z}_2 \rightsquigarrow$ 6d $\mathfrak{sp}(N)$ SYM;
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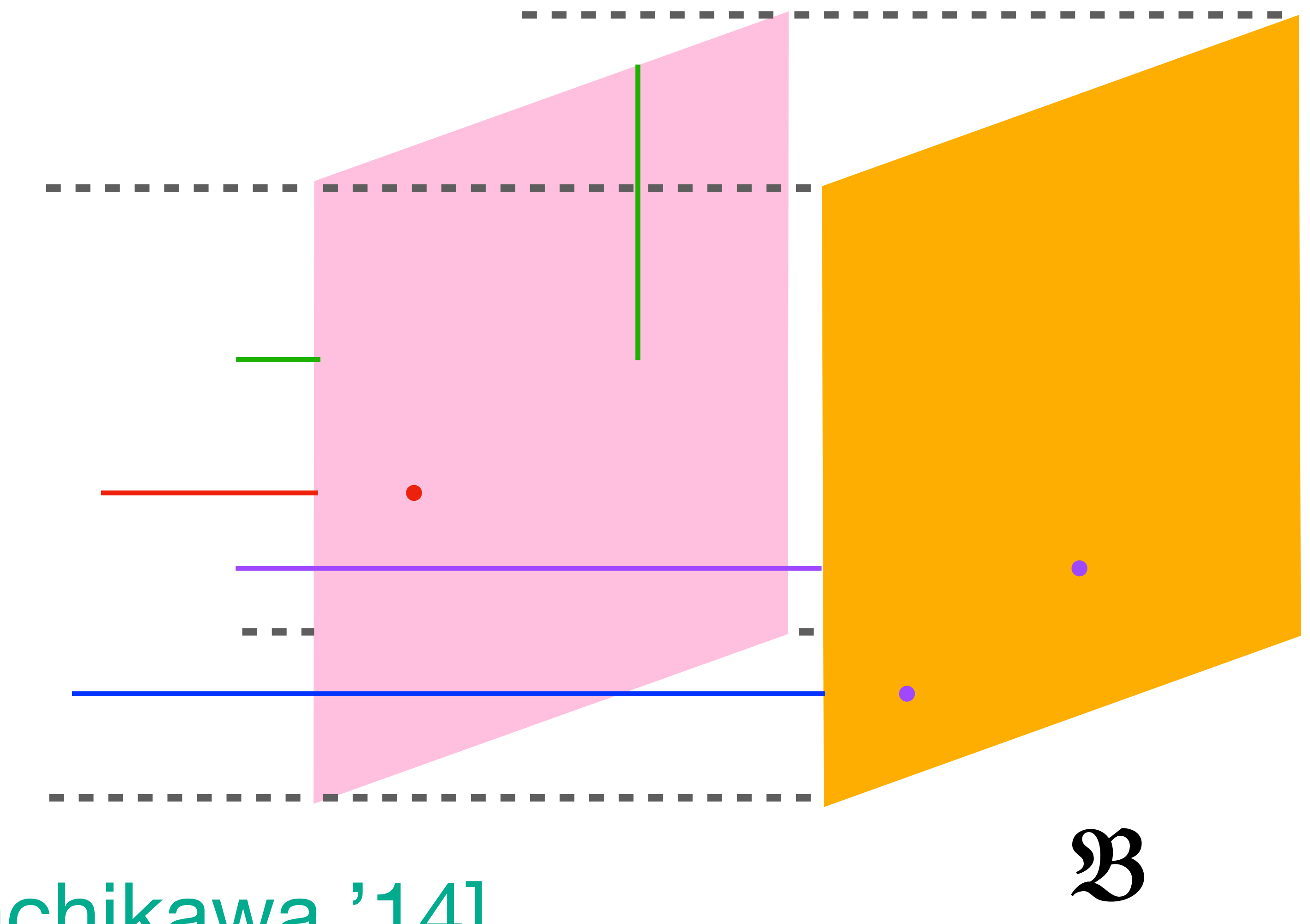

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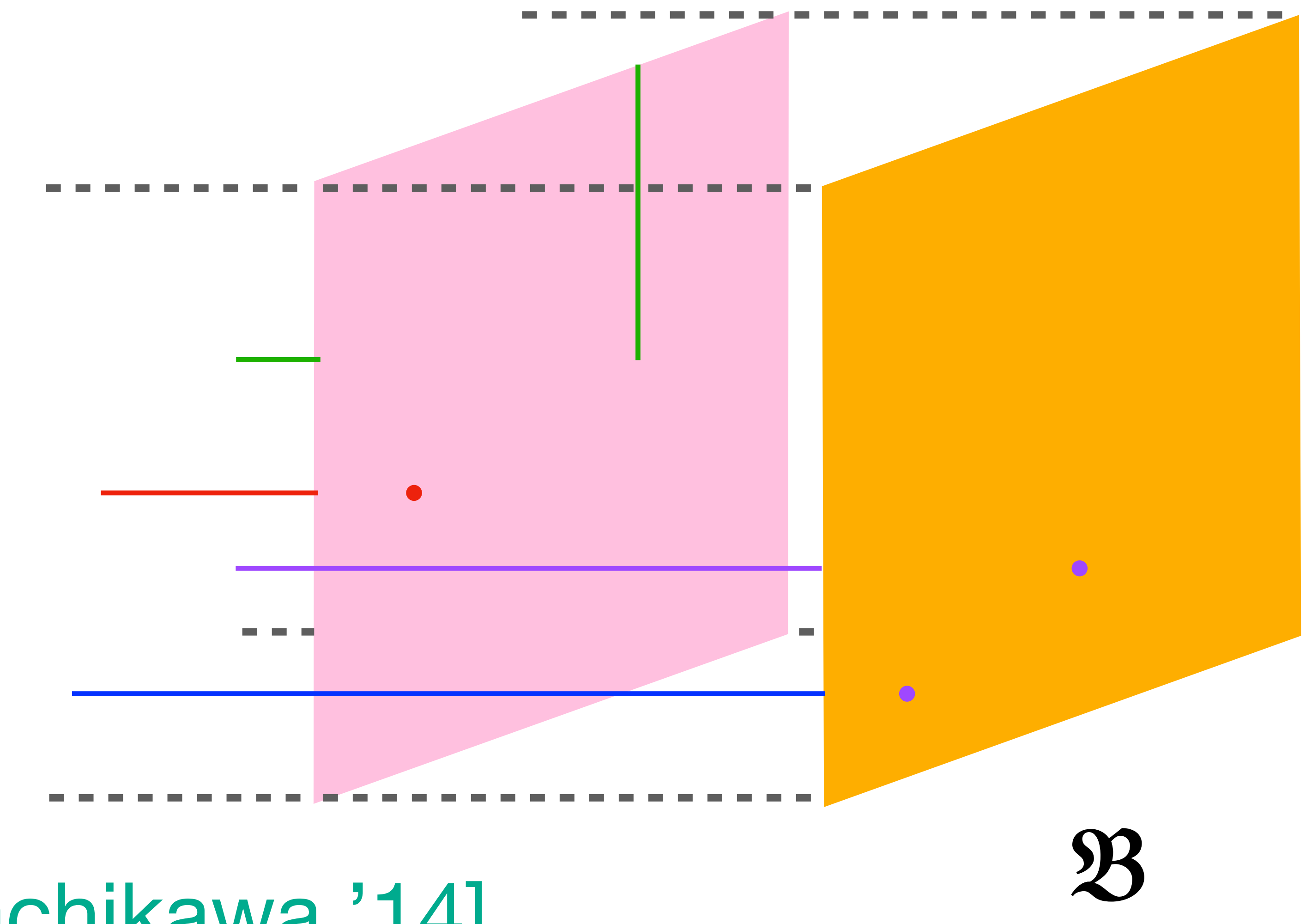

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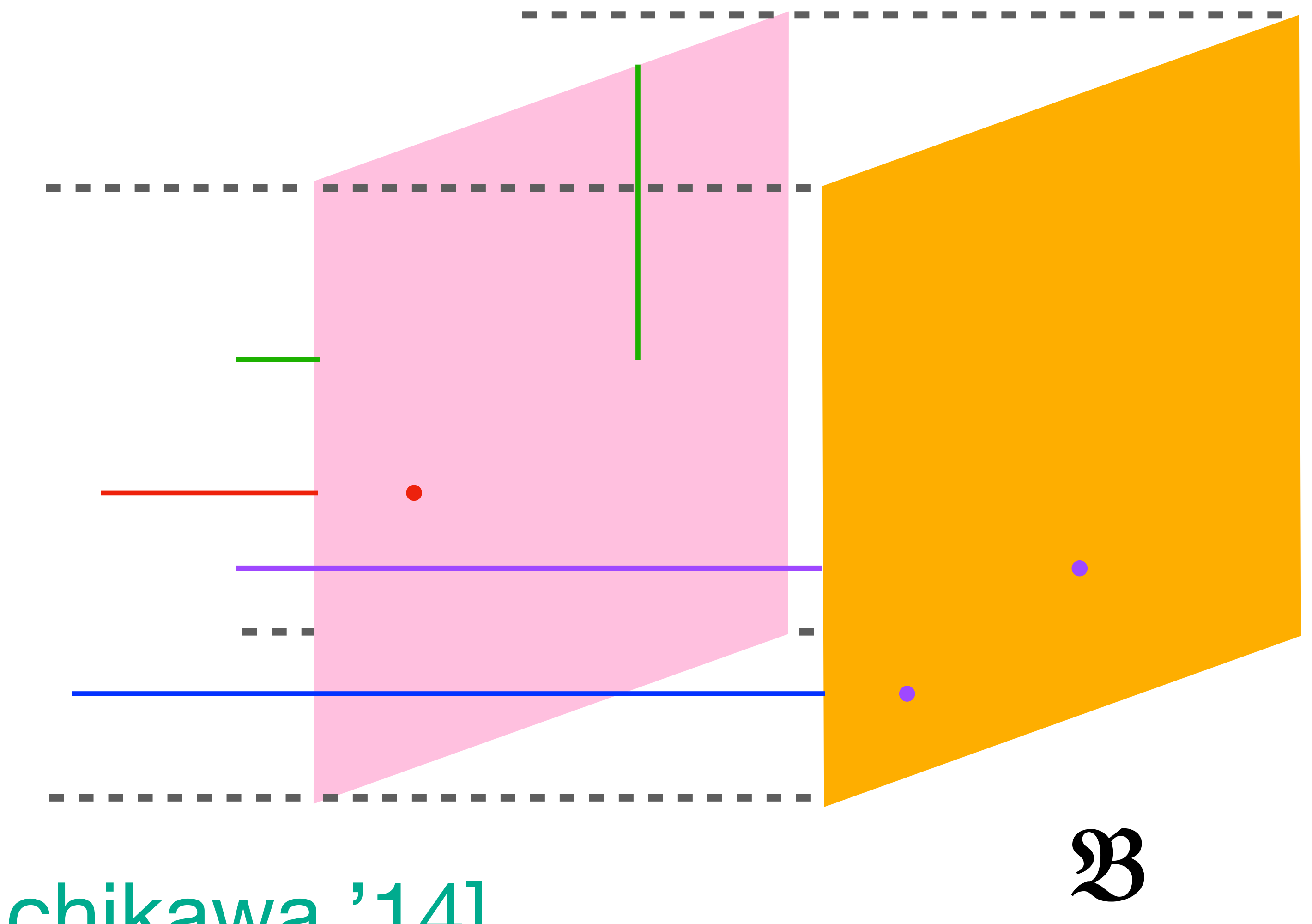

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- Stack $\mathscr{I}$ & $\mathfrak{B} \rightsquigarrow$ charged defects for $\mathfrak{sp}$-center ($\mathscr{W}^{(1)}/\mathscr{H}^{(d-3)}$) and $\mathbb{Z}_2^{(d-3)}$ only, agrees with 7d/8d results [Cvetič/Dierigl/LL/Zhang '22, Cvetič/Dierigl/LL/Torres/Zhang '24].

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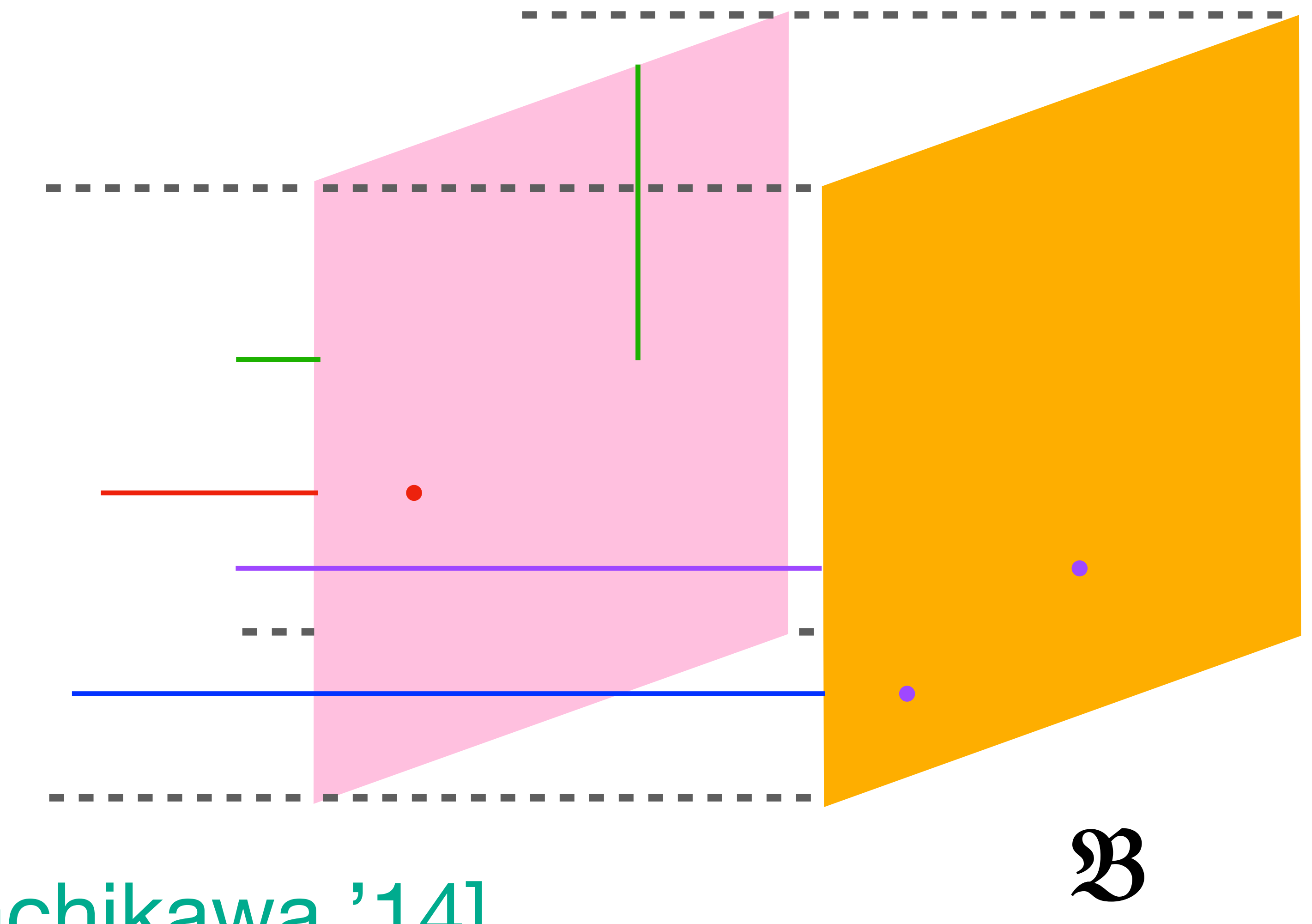

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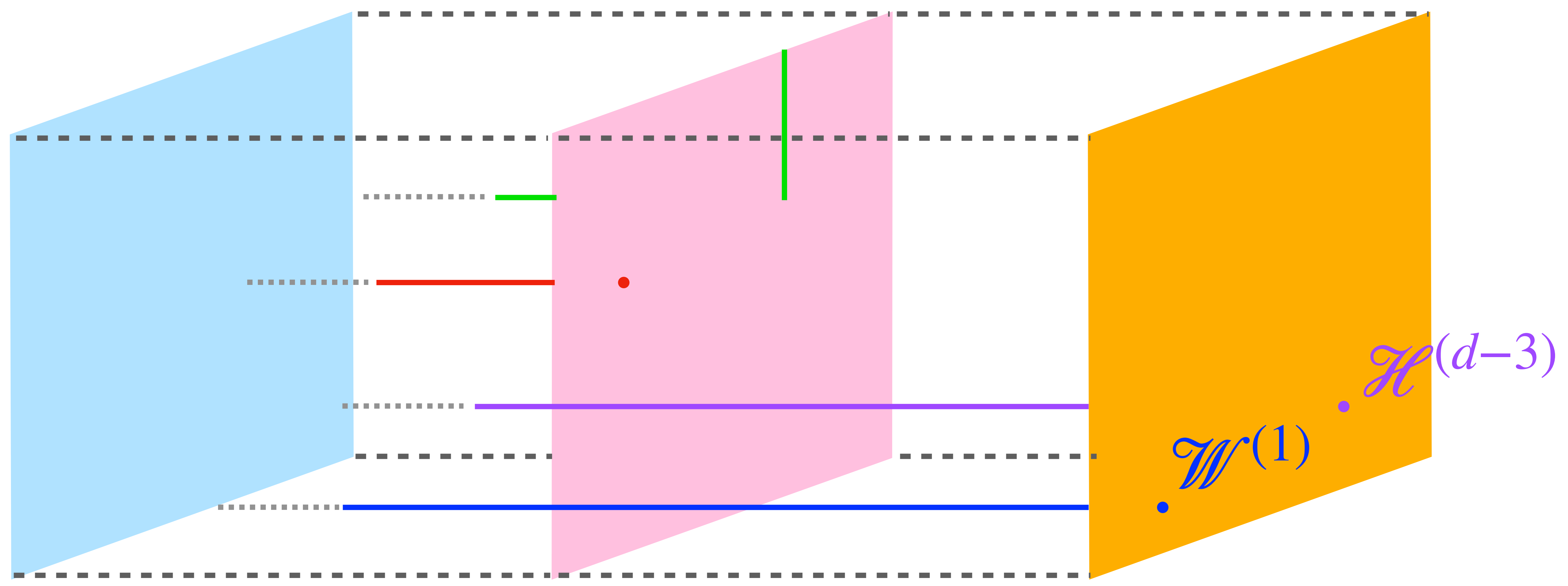
Global forms for $\mathfrak{sp}(N)$ SYM in d≥6



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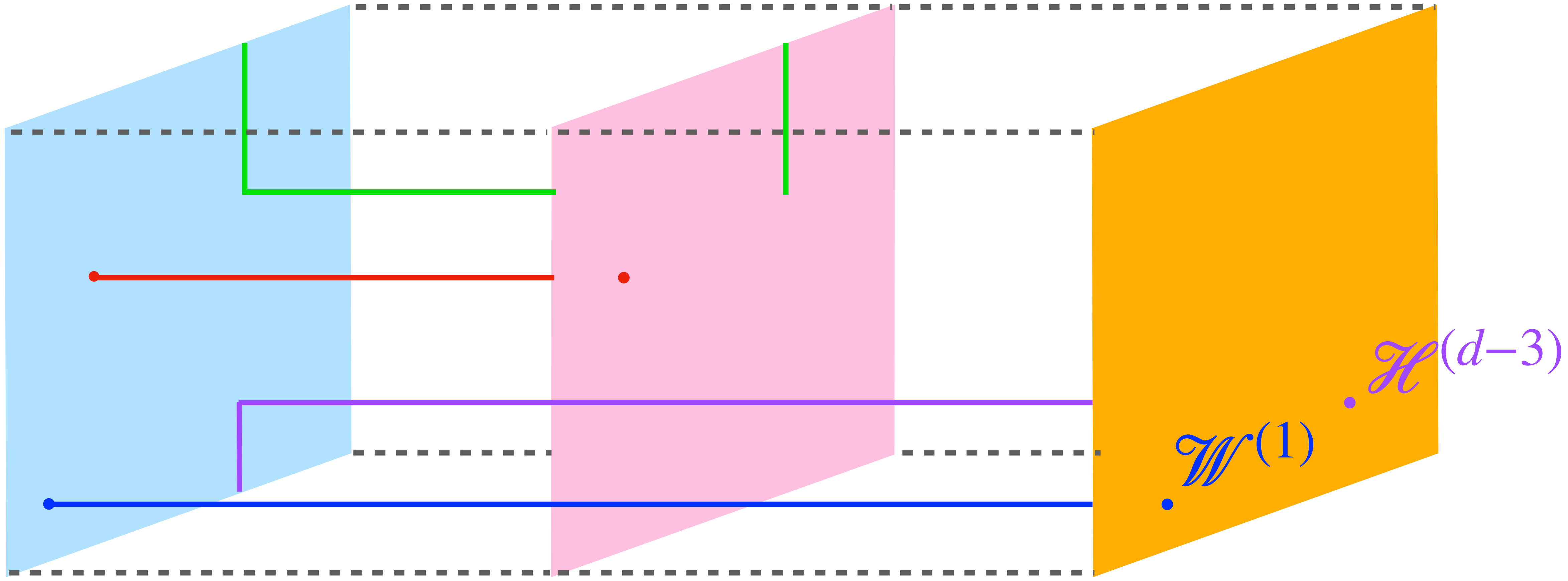

$Sp(N)$ SYM + faithful $\mathbb{Z}_2^{(d-3)}$ symmetry

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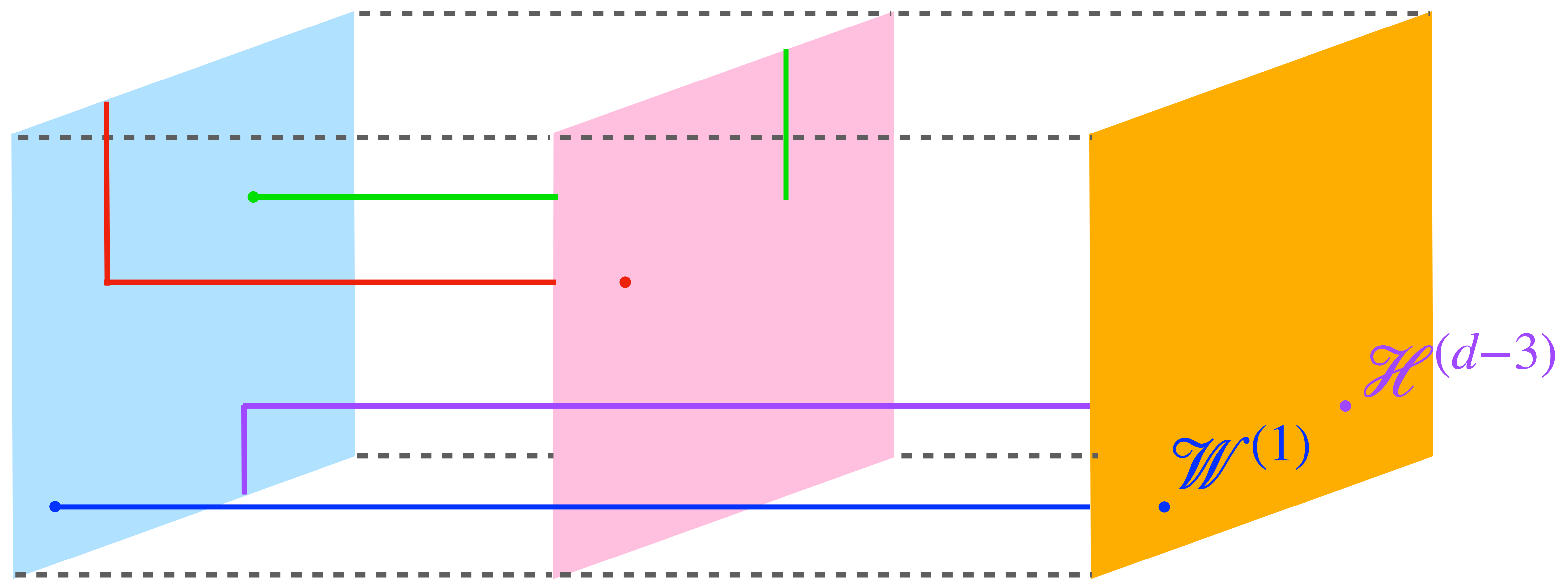

$Sp(N)$ SYM + trivially acting $\mathbb{Z}_2^{(1)}$ symmetry

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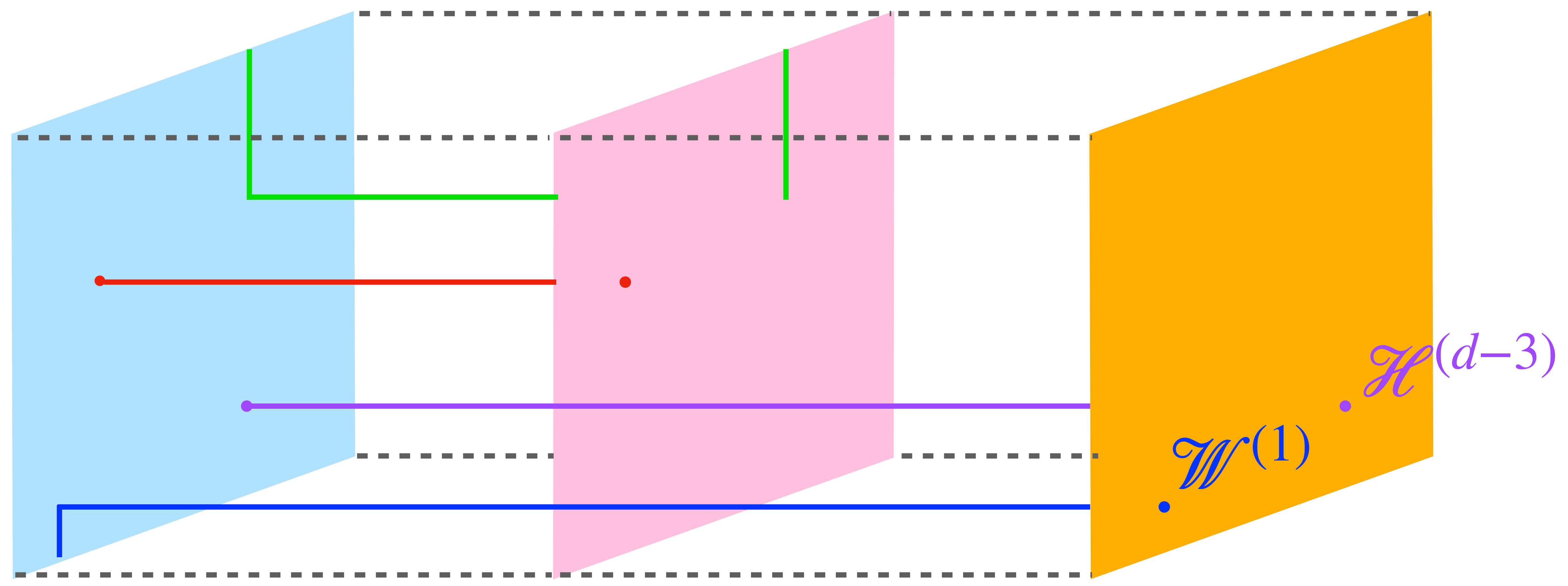

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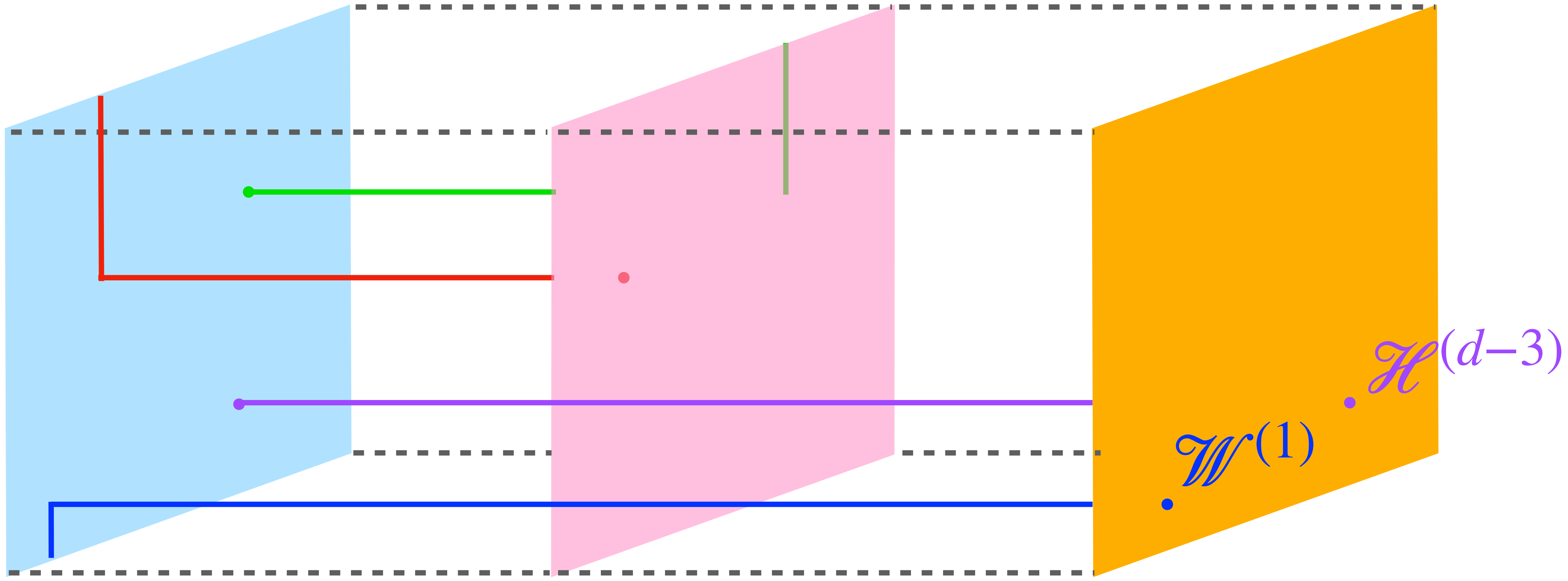

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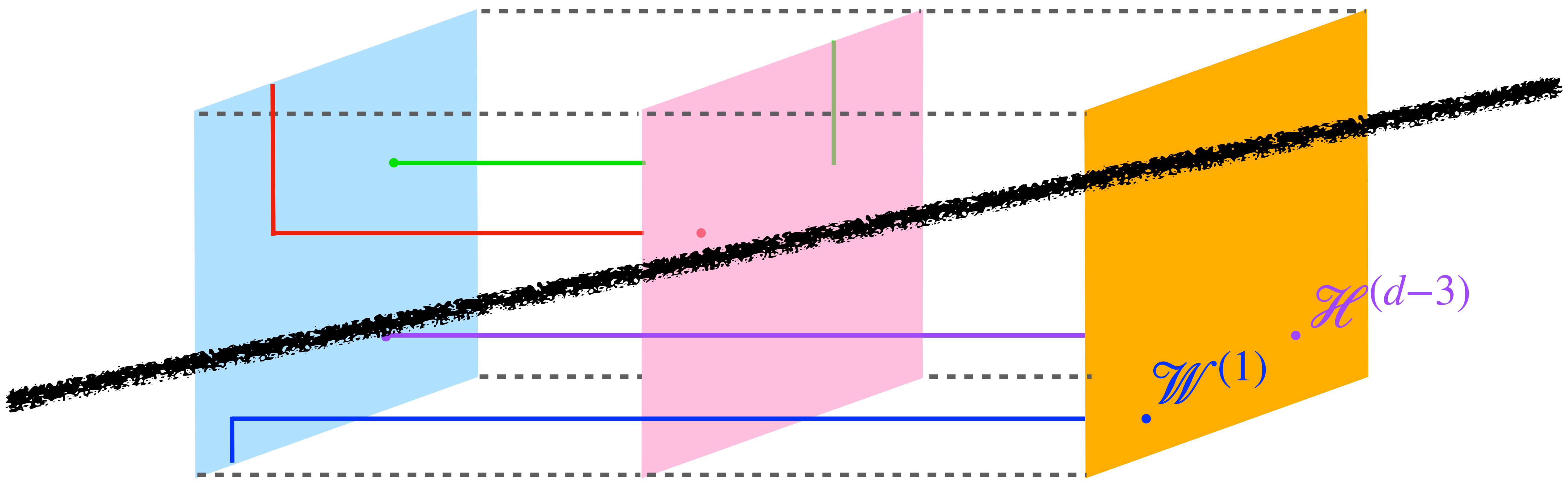

No $Sp(N)/\mathbb{Z}_2$ SYM + trivially acting $\mathbb{Z}_2^{(1)}$ symmetry!

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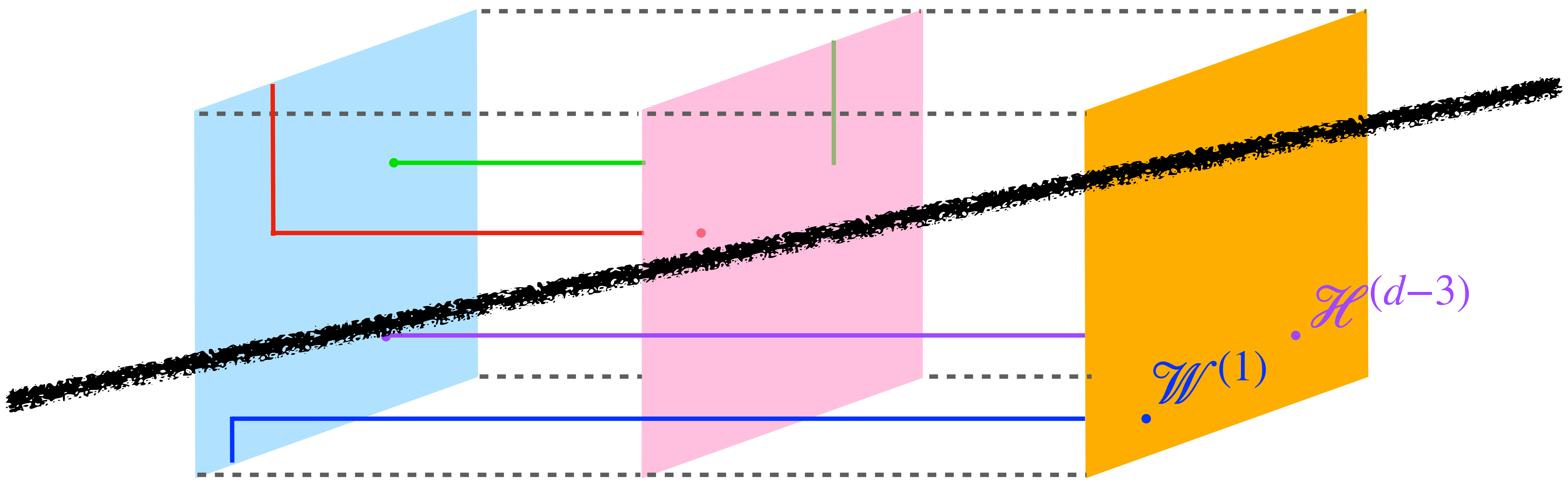

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Constraints on IR phases

- Anomaly-term in SymTFT distinguishes frozen $\mathfrak{sp}(N)$ SYM from “gauge sector + decoupled defects”.

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➡ Should hold with SUSY breaking but topology preserving deformations of geometric engineering setup!

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Thank you!