5d SCFTs on Brane Webs & CY3s

— non toric —

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@ Strings & Geometry 2026, Uppsala

based on:

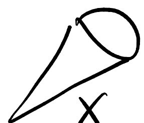
- 2605.03119 w/ A. Bourget, M. De Marco, M. Del Zotto, A. Sangiovanni
- WIP w/ M. De Marco, M. Del Zotto, A. Sangiovanni

(- WIP w/ H. Argüz, A. Bourget, P. Bousseau, S. Schäfer-Nameki)

5d $\mathcal{N}=1$ SCFTs in String Theory

two common constructions:

1) M-theory on local CY3

$\mathbb{R}^{1,4} \times$  X

geometric engineering

→ also Andrés talk next

2) Worldvolume theory on 5-brane web in IIB (w/ 7-branes)

brane construction

→ so we start here

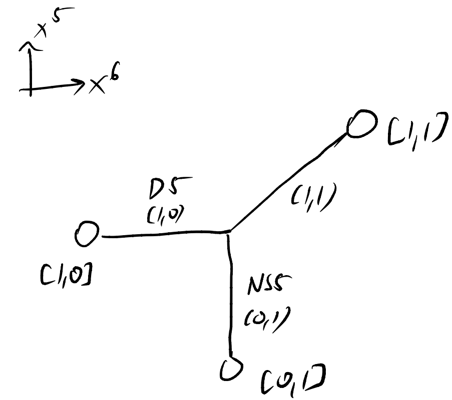
Brane Webs (5d generalisation of 3d Hanany-Witten)

IIB	0	1	2	3	4	5	6	7	8	9
(1,0)5 = D5	x	x	x	x	x		x			
(0,1)5 = NS5	x	x	x	x	x	x				
(p,q) 5	x	x	x	x	x	angle α				
[p,q] 7	x	x	x	x	x			x	x	x

$so(1,4)$ (columns 0–4) $\qquad$ $so(3)_R$ (columns 7–9)

$\tan\alpha = \frac{q}{p}$ for $\tau = i$

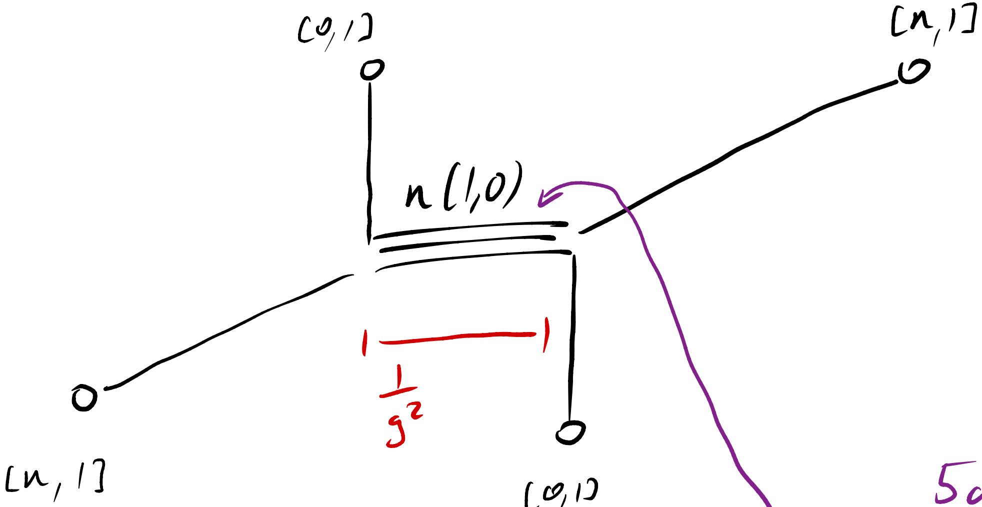
cartoon:



$$M_{[p,q]} = \begin{pmatrix} 1-pq & p^2 \\ -q^2 & 1+pq \end{pmatrix}$$

brane webs ~ tropical curve (+ 7 branes)

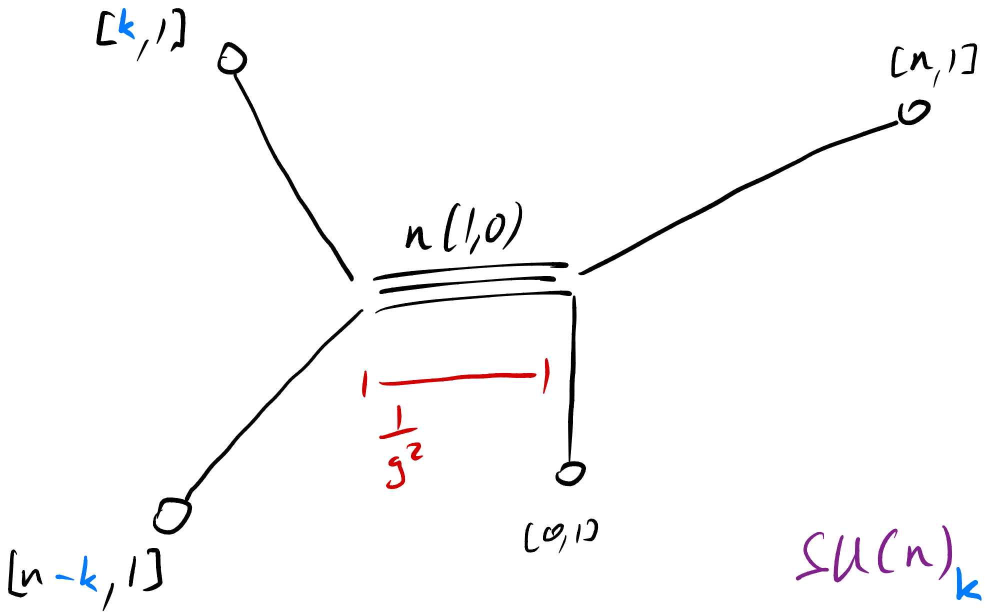
Let's construct some 5d $\mathcal{N}=1$ theories!



5d $\mathcal{N}=1$

$SU(n)_0$ pure gauge theory

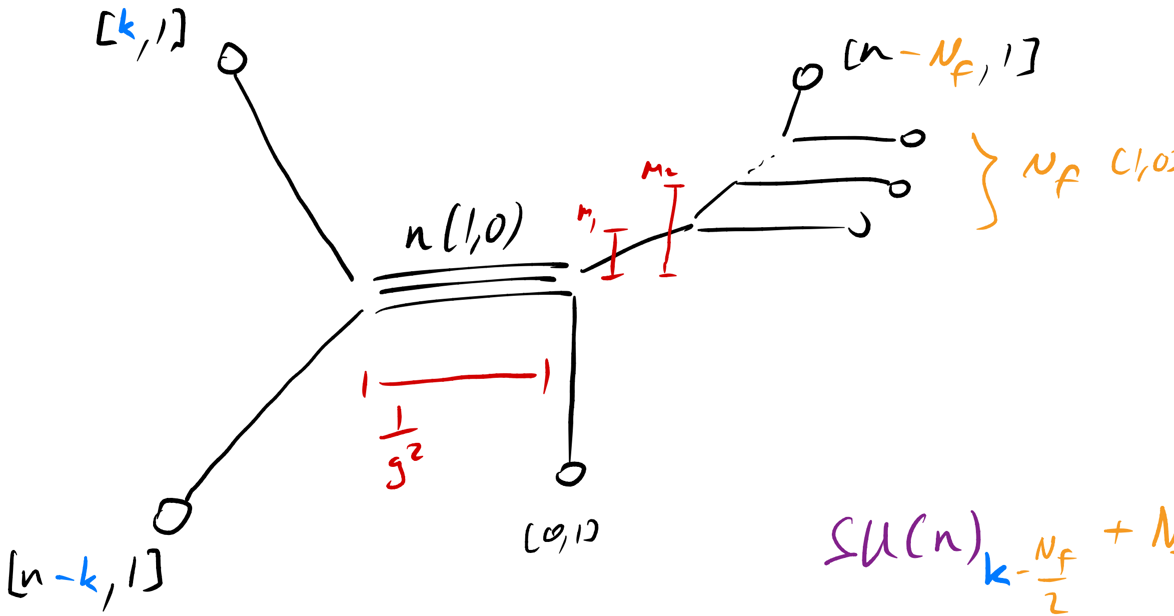
Let's construct some 5d $\mathcal{N}=1$ theories!



$SU(n)_k$ pure gauge theory

CS level

Let's construct some 5d $\mathcal{N}=1$ theories!

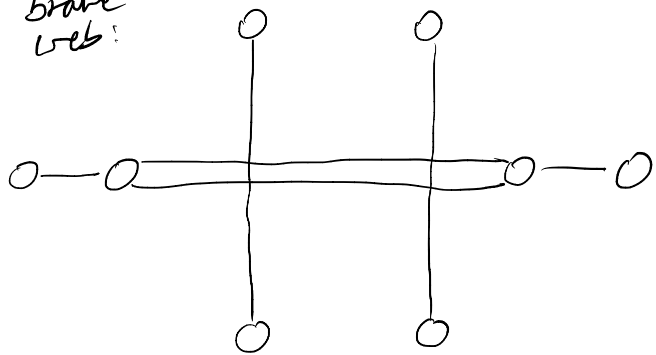


$SU(n)_{k-\frac{N_f}{2}} + N_f$ fund. hypers

SU(2) + 4 F,

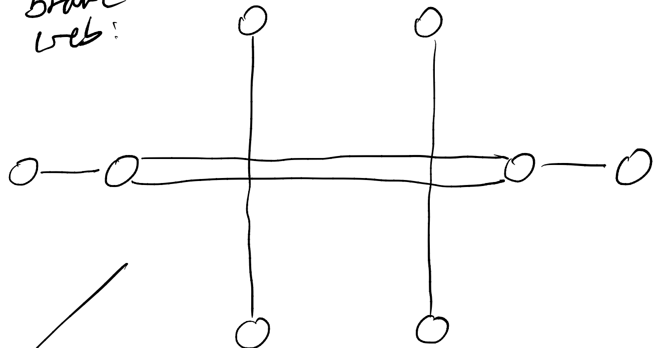
finite coupling

brane web:

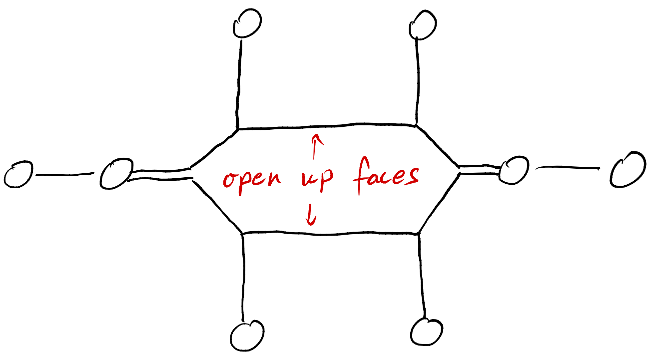


SU(2) + 4 F, finite coupling

brane web:



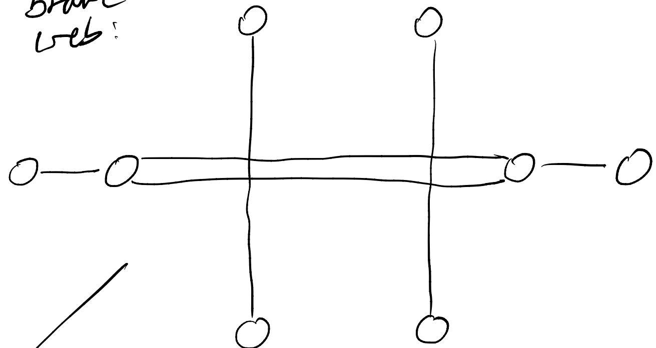
Coulomb branch

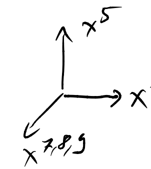


$\Rightarrow$ 1 Coulomb branch modulus

$SU(2) + 4\,F$,
finite coupling

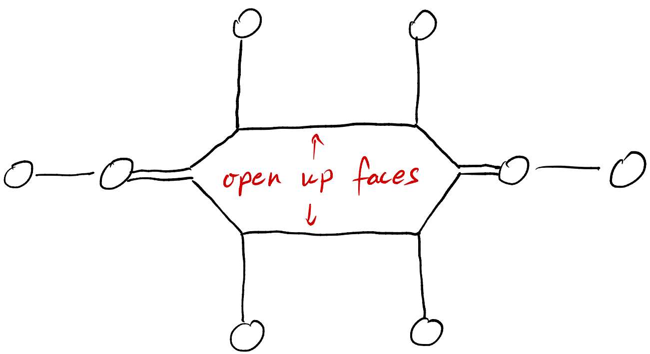
brane web:

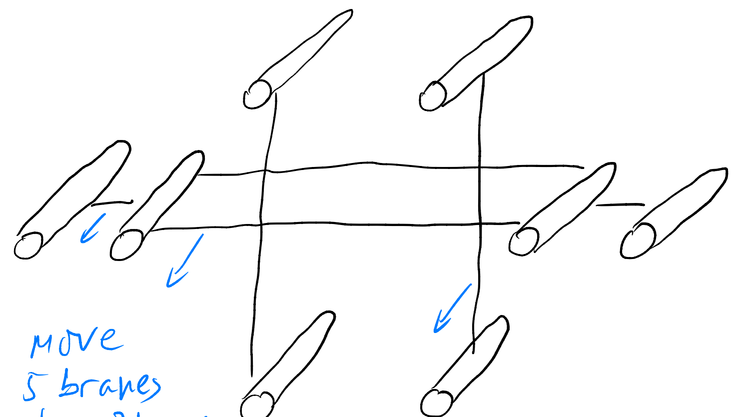




Coulomb branch

Higgs branch





move
5 branes
along 7branes

$\Rightarrow$ 1 Coulomb branch modulus

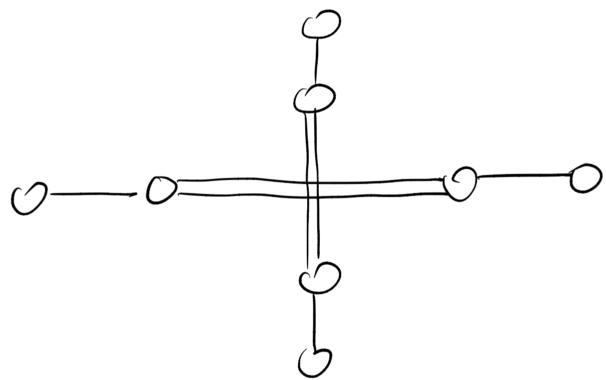
$\Rightarrow$ 5 Higgs branch moduli

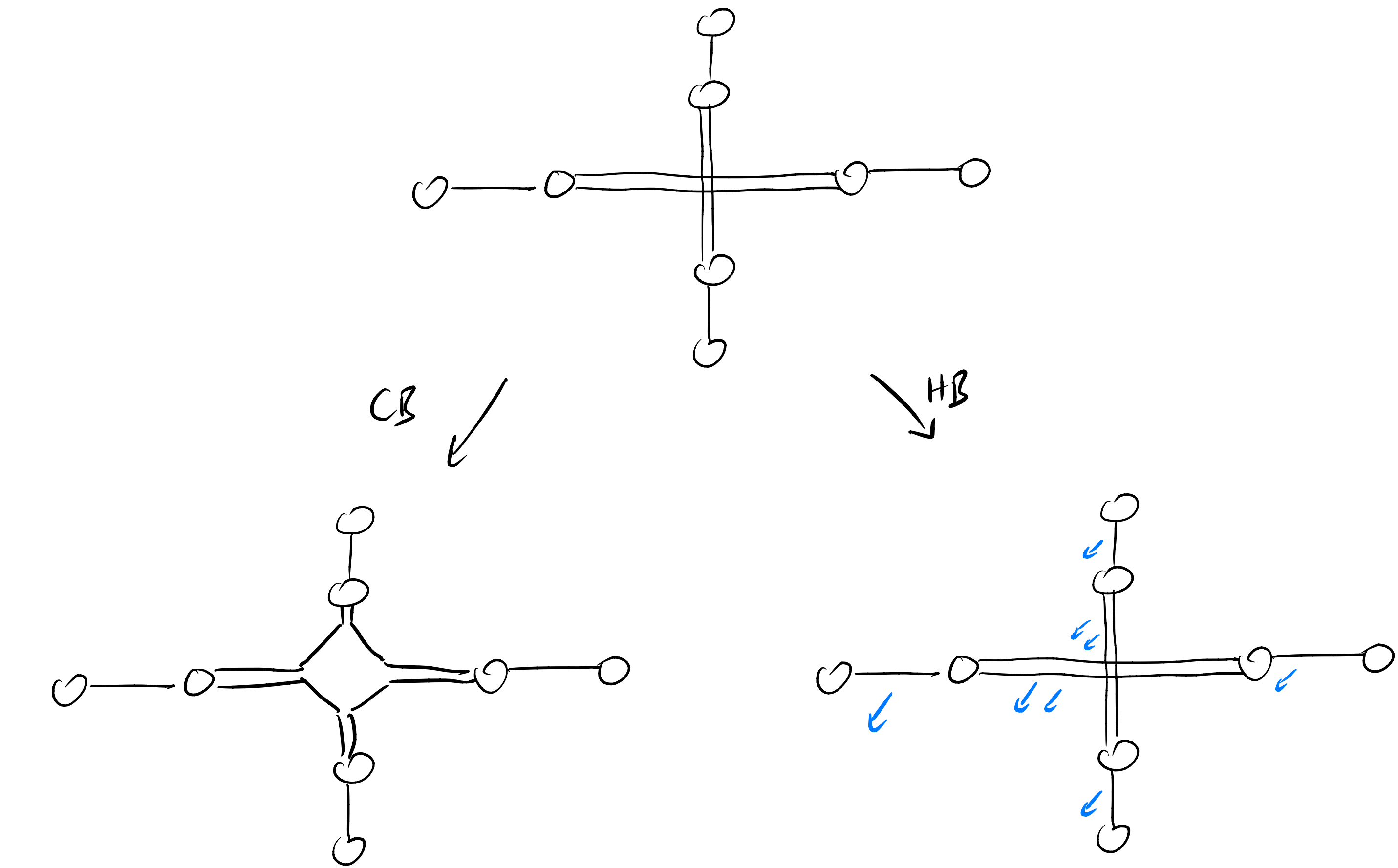
- $SU(2)+4F$ is of course not an SCFT!

- Has UV completion as Seiberg "rank-1 E_5 theory"

- Obtained in BW by sending $\frac{1}{g^2}\to 0$, i.e.

rank-1 E_5 theory
Brane Web:





$\Rightarrow$ still 1 CB modulus

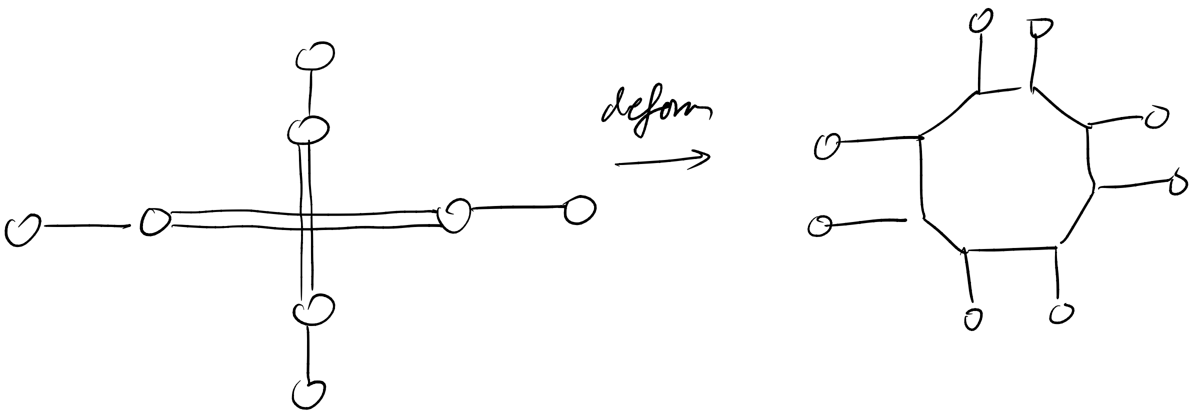
$\Rightarrow$ now 7 Higgs moduli

Geometric engineering

M-th on (local) CY3

Which CY3 constructs the rank-1 E_5 theory?

↳ can dualise the BW!



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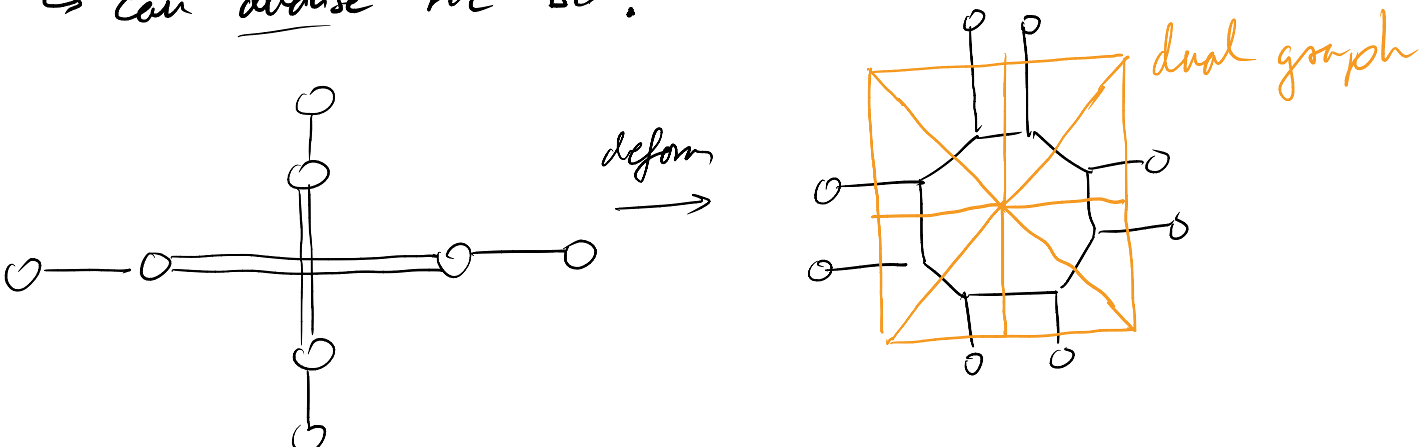
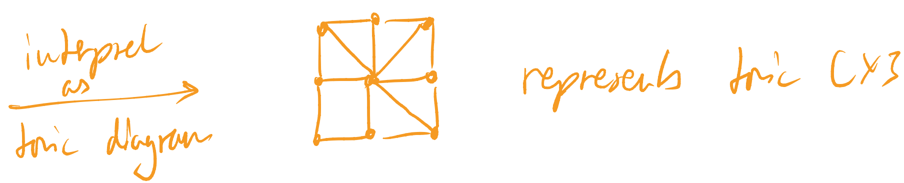
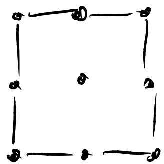




toric diagram:

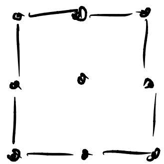


describes a complete intersection

$$\left\{ \begin{array}{l} xy = z^2 \\ uv = z^2 \end{array} \right\}$$

local CY3 singularity

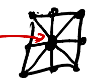
toric diagram:



describes a complete intersection

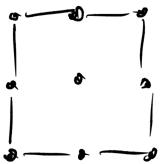
$$\begin{Bmatrix} xy = z^2 \\ uv = z^2 \end{Bmatrix}$$

local CY3 singularity

* Have 1 compact divisor in blow up 

  → 1 dim CB

toric diagram:



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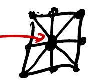
* Have 1 compact divisor in blow up 

$\rightarrow$ 1 dim CB

* Using [Collinucci, Valandro] can obtain deformations

$$\left\{ \begin{array}{l} xy = z^2 + \alpha_5 u + \alpha_6 v + \alpha_7 z \\ uv = z^2 + \alpha_1 x + \alpha_2 y + \alpha_3 z + \alpha_4 \end{array} \right\}$$

$\alpha_i \rightarrow$ def params

7 dim Higgs branch

can add constant term in either equation, no difference

generic $\alpha_i \rightarrow$ get smooth CY3

Can do further checks!

$\longrightarrow$ CY3 constructs the E_5 theory.

- The Brane Web & the CY3 which construct the same theory are easily related, because they are toric.

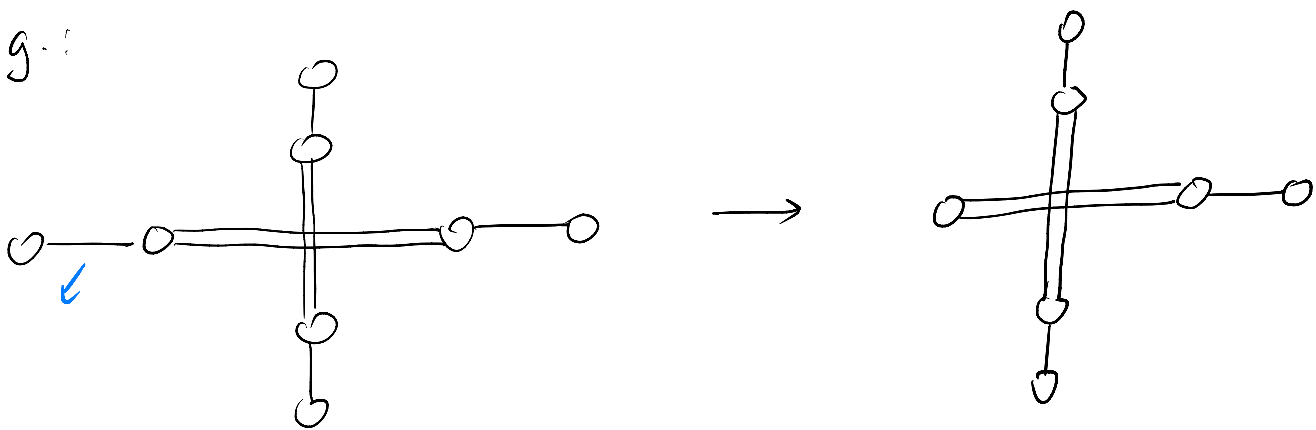
- What about non-toric case?

What does this even mean for a Brane Web?

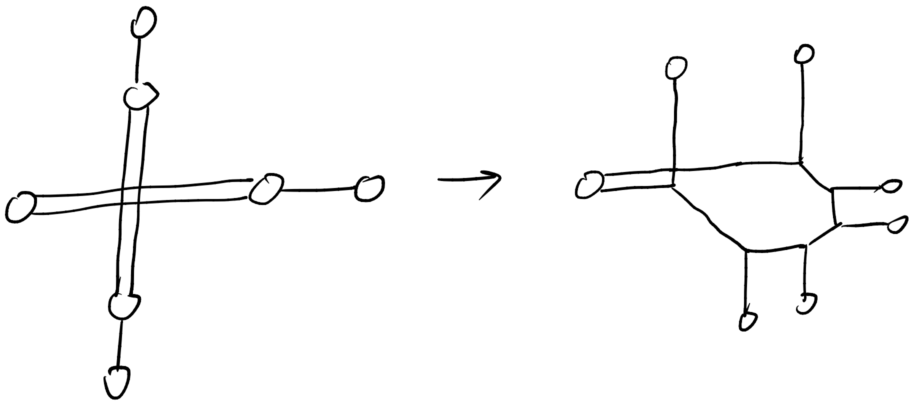
It means there is no dual graph which is a toric diagram... obviously...

We encounter such non-toric Brane Webs very naturally!
When we start with a toric BW, and send 5-branes away (go on Higgs Branch).

E.g.:

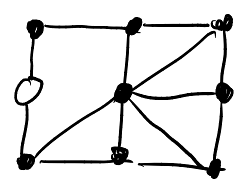


deform this:

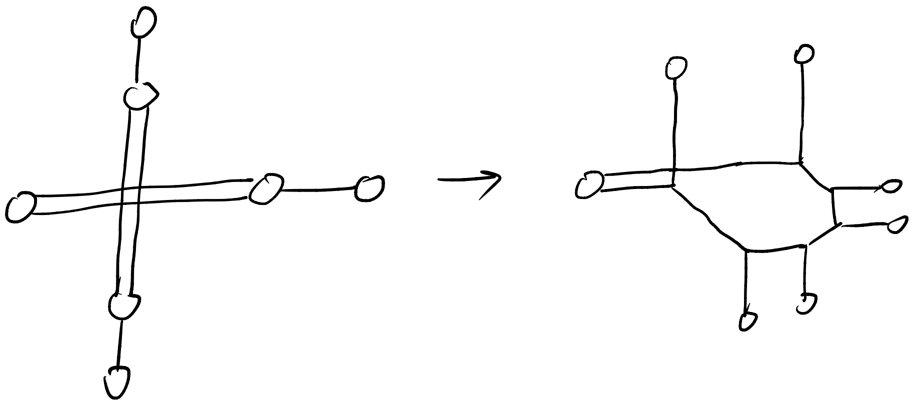


cannot deform further!

dual graph is so called "dot diagram" or "generalised toric polygon" (GTP)



deform this:



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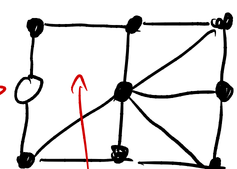


"white dot"

not a triangle

How to interpret?

↳ sending brane away is akin to deformation.

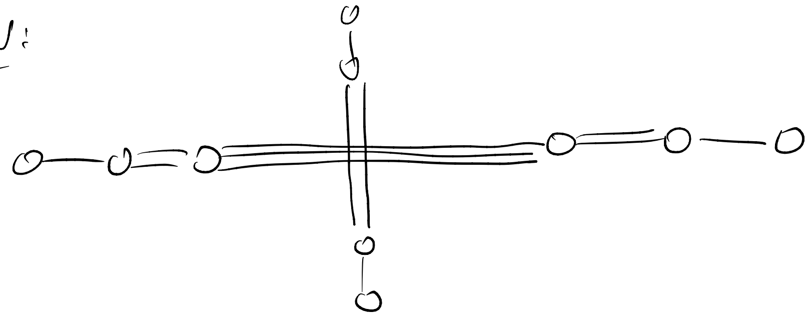
→ Look at example with interesting Higgs branch to show this

More interesting example:

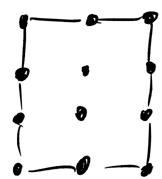
$\overbrace{SU(3)+6F}$:

use this notation for ∞ coupling

BW:



toric diagram:



CY3:

$$\begin{Bmatrix} xy = z^2 \\ uv = z^3 \end{Bmatrix}$$

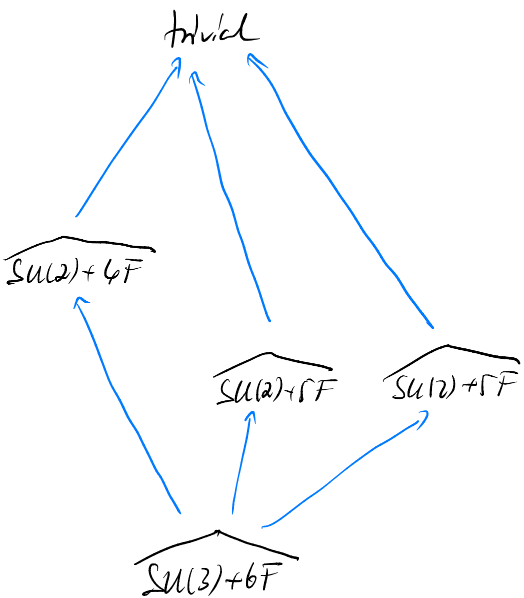
Here the defs are:

$$\left\{ \begin{array}{l} xy = z^2 + \alpha_{10} u + \alpha_{11} v + \alpha_{12} z \\ uv = z^3 + \alpha_1 x^2 + \alpha_2 xz + \alpha_3 y^2 + \alpha_4 yz + \alpha_5 z^2 \\ \qquad + \alpha_6 x + \alpha_7 y + \alpha_8 z + \alpha_9 \end{array} \right\}$$

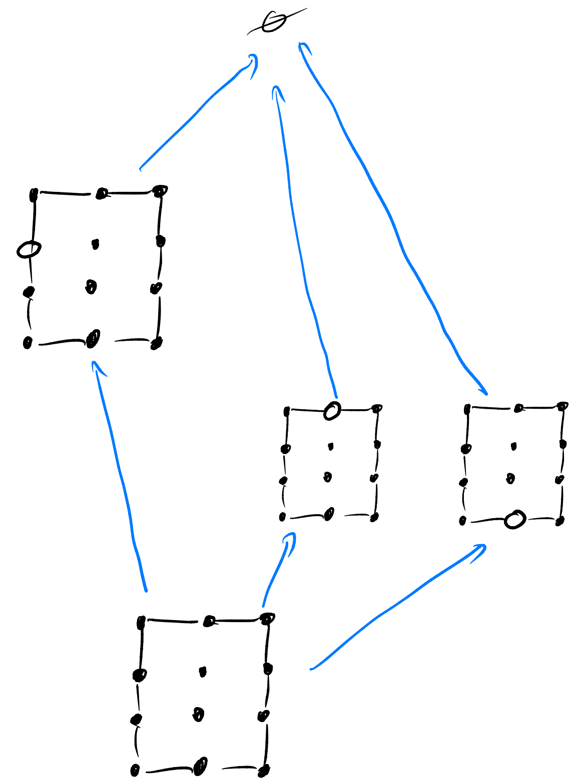
Getting complicated, but the deformation group nicely according to which partial Higgsing they trigger!

partial Higgsings are captured in a Hasse diagram!

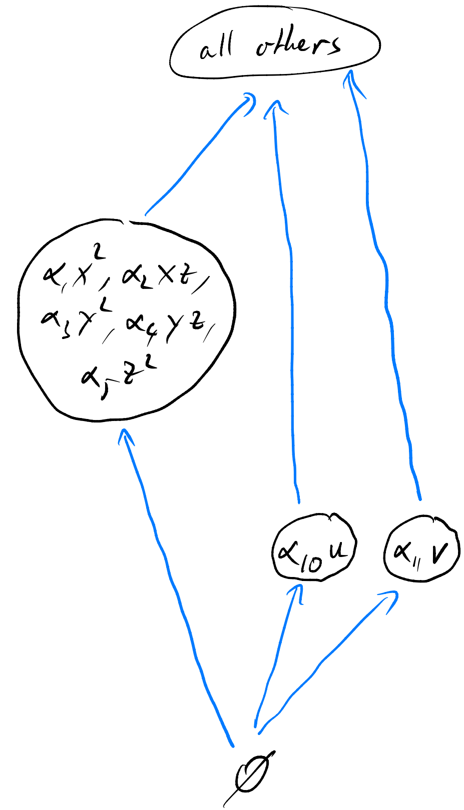
Partial Higgsings:



corresponding GTPs



deformations



→ Can match deformed CY3 with GTP

→ done explicitly in upcoming work
w/ De Marco, Del Zotto, Sangiovanni
for all rectangular GTPs.

Some more examples

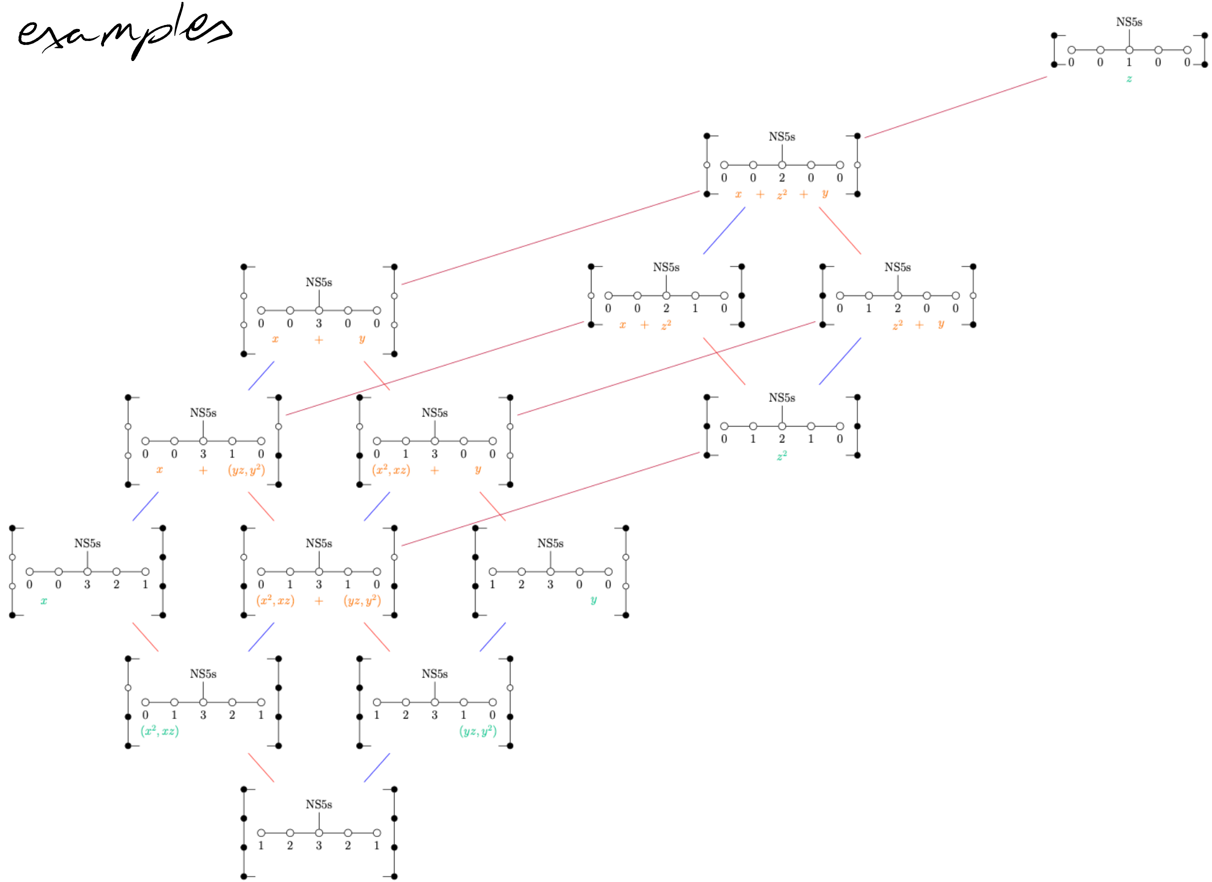


Figure 2: $xy = z^n$ with $n \geq 6$ and $uv = z^3 + f(x, y, z)$.

Some more examples

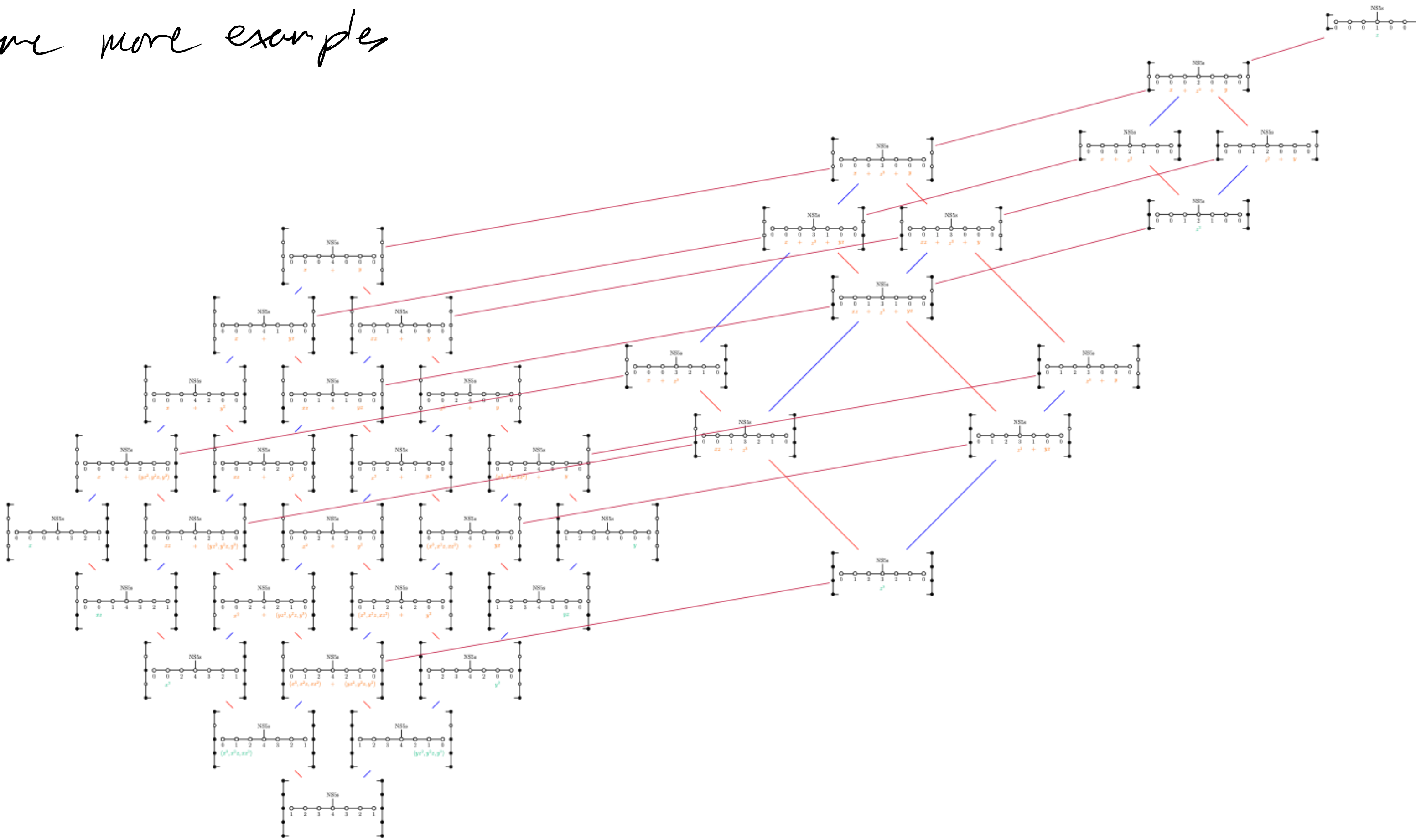


Figure 3: $xy = z^n$ with $n \geq 8$ and $uv = z^4 + f(x, y, z)$.

Outlook:

- How to extract CY3 equations from any GTP?
- Describe divisors & intersections in resolved GTP (upcoming!)