# Non-toric 5d SCFT’s from Reid’s Pagoda


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# Motivation

- To construct new 5d $\mathcal{N}=1$ SCFT's.

Path : M-theory on $CY_3$ that evade standard classifications

$\rightsquigarrow$ non-toric geometries

Discovery : A new physical mechanism that drives

$$\boxed{\frac{1}{g_{YM}^2} \longrightarrow 0}$$

# Geometric engineering : M-theory / $CY_3$

# Compact 4-cycles $\longleftrightarrow$ rank of theory

= dim C.B.

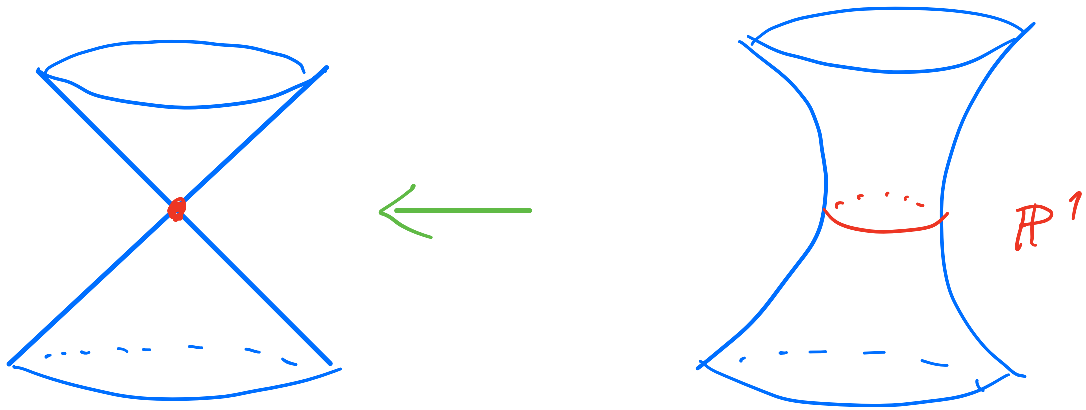
Compact 2-cycles $\longleftrightarrow$ hypers from

wrapped M2 - branes

Kähler moduli of 2 - cycles $\longleftrightarrow$ real masses of hypers.

# Example I : Rank zero

Enter the conifold $\quad uv = z^2 + w^2$




M2 / $\mathbb{P}^1$ $\leftarrow$ 1 free hyper $\quad$ GV inv. = 1.

5d EFT $\leftarrow$ 1 free hypermultiplet

# Spectral curve

M-th / $\mathbb{C}^2/\mathbb{Z}_2$ 3 scalars $(\Phi, \varphi)$ in $\mathrm{adj}_{SU(2)}$

M-theory geometry: $uv = \det(\mathbb{1}_2 z - \langle \Phi \rangle)$

- $\langle \Phi \rangle = 0 \longrightarrow uv = z^2 \simeq \mathbb{C}^2/\mathbb{Z}_2 \times \mathbb{C}_w$

- $\langle \Phi \rangle = \begin{pmatrix} w & 0 \\ 0 & -w \end{pmatrix} \longrightarrow uv = z^2 - w^2 \simeq$ conifold.

$$\downarrow \text{def}$$

? $\longleftarrow uv = z^2 - w^2 + \varepsilon$

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- $\langle \Phi \rangle = \begin{pmatrix} \omega & 0 \\ 0 & -\omega \end{pmatrix} \longrightarrow uv = z^2 - \omega^2 \simeq$ conifold.

$$\Big\downarrow \text{def}$$

$\langle \Phi \rangle = \begin{pmatrix} \omega & 1 \\ \varepsilon & -\omega \end{pmatrix} \longleftarrow uv = z^2 - \omega^2 + \varepsilon$

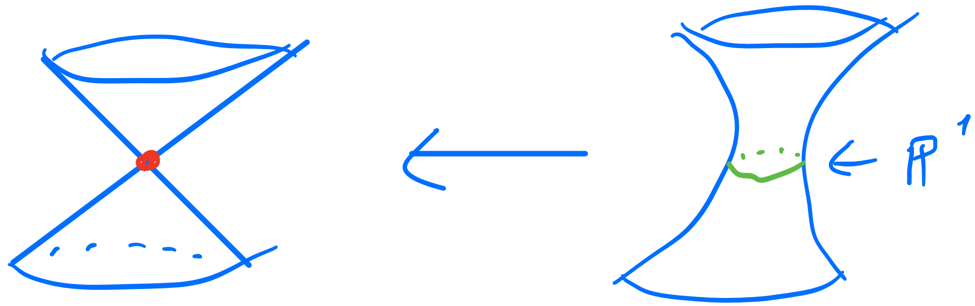
Deformation $\longleftrightarrow$ VEV for hyper

# Reid's Pagoda

$$uv - z^2 + w^{2k} = 0 \qquad \underline{k \geq 1}$$

- **Small resolution**: $\begin{pmatrix} u & z+w^k \\ z-w^k & v \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = 0$

$1 \times$ exceptional $\mathbb{P}^1$




Only $1$ $\mathbb{P}^1$ $\leftarrow$ $1$ free hyper ?

<u>Spectral curve</u> : $uv = \det\left(\mathbb{1}_2\,\omega - \langle\Phi\rangle\right)$

$$\langle\Phi\rangle = \begin{pmatrix} \begin{pmatrix} 0 & 1 & & 0 \\ & 0 & 1 & \\ & & \ddots & 1 \\ z & & & 0 \end{pmatrix} & 0 \\ 0 & \begin{pmatrix} 0 & 1 & & 0 \\ & 0 & 1 & \\ & & \ddots & 1 \\ -z & & & 0 \end{pmatrix} \end{pmatrix}$$

**Spectral curve** : $uv = \det\left(\mathbb{1}_2 w - \langle\Phi\rangle\right)$

$$\langle\Phi\rangle = \begin{pmatrix} \begin{pmatrix} 0 & 1 & & 0 \\ & 0 & 1 & \\ & & \ddots & 1 \\ z & & & 0 \end{pmatrix} & \begin{pmatrix} & & 0 & \\ & & & \\ 1 & \cdots & \cdots & 1 \end{pmatrix} \\ \begin{pmatrix} & & 0 & \\ & & & \\ a_0 & \cdots & & a_{k-1} \end{pmatrix} & \begin{pmatrix} 0 & 1 & & 0 \\ & 0 & 1 & \\ & & \ddots & 1 \\ -z & & & 0 \end{pmatrix} \end{pmatrix}_{2k \times 2k}$$

$k$- hypers (in w/ A. Sangiovanni

**Deformations** : $uv = z^2 - w^{2k} + \underbrace{\sum_{i=0}^{k-1} a_i w^i}_{\text{normalizable}}$

deforming $\longleftrightarrow$ Higgsing matter

$$\text{Pagoda} \overset{?}{\longleftrightarrow} k \times \text{ free hypers}$$

Not quite!

Only $1 \times \mathbb{P}^1$ $\Rightarrow$ only one real mass $m = \int_{\mathbb{P}^1} J$

for all $k$ matter modes.

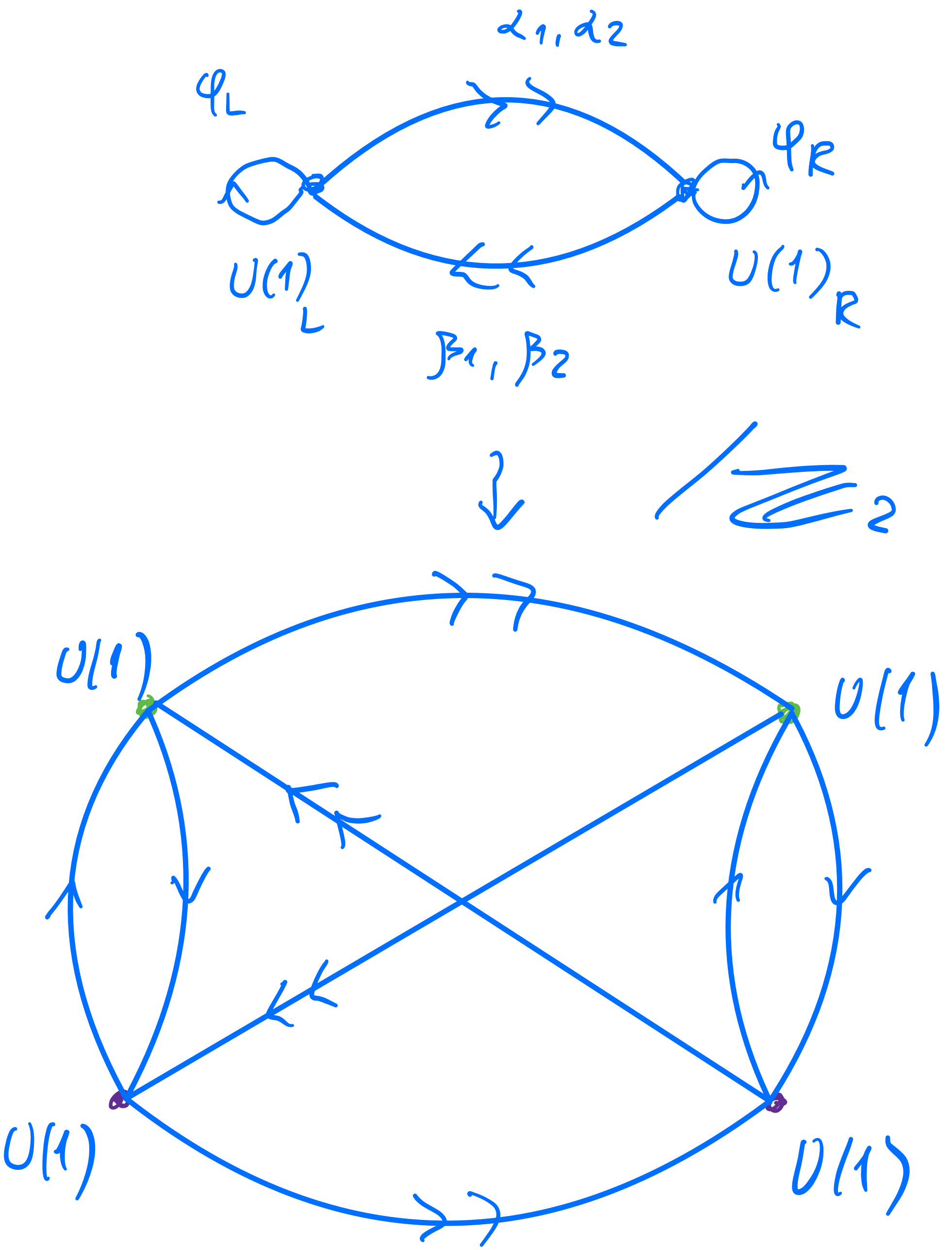
As if we had gauged a discrete $S_k$ (or $\mathbb{Z}_k$)

Curious, but not that interesting $\leftarrow$ rank-0.

Can turn this into rank-N theory.

How? Orbifolding!

# Orbifold the Pagoda




PAGODA

$\downarrow$

PAGODA$/\mathbb{Z}_2$

# Projective geometry

$$\mathbb{C}^4 : \begin{array}{cccc} (u, & v, & z, & w) \\ 1 & 1 & 1 & 1 \end{array} \supseteq \quad uv = z^2 - w^{2k}$$

$$\Big\downarrow \mathbb{Z}_2$$

$$\widehat{\mathbb{C}^4/\mathbb{Z}_2} \qquad \begin{array}{c|ccccc} & u & v & z & w & p \\ \hline \mathbb{C}^* & 1 & 1 & 1 & 1 & -2 \end{array} \supseteq \quad uv = z^2 - w^{2k} p^{k-1}$$

Exceptional locus : $p = 0 \longrightarrow uv = z^2 \subseteq \mathbb{P}^3$

Singular Hirzebruch $\mathbb{F}_2 \longleftarrow$ rank - 1 theory !

# Resolve $\mathbb{F}_2$

$\widehat{\mathbb{C}^4/\mathbb{Z}_2}$

| | $u$ | $v$ | $z$ | $w$ | $t$ | $\sigma$ |
|---|---|---|---|---|---|---|
| $\mathbb{C}^*_1$ | 1 | 1 | 1 | 1 | $-2$ | 0 |
| $\mathbb{C}^*_2$ | 1 | 0 | 1 | 0 | 0 | $-1$ |

(−2)-curve of the $\mathbb{F}_2$

$$uv = z^2 - w^{2k} t^{k-1}$$

$$\downarrow$$

$$uv = z^2 \sigma - w^{2k} t^{k-1}$$

$$\mathrm{Vol}\left(\mathbb{P}^1_{(-2)}\right) \sim \frac{1}{g_{YM}^2}$$

$\widehat{\mathbb{C}^4/\mathbb{Z}_2}$

| | $u$ | $v$ | $z$ | $w$ | $p$ | $\sigma$ |
|---|---|---|---|---|---|---|
| $\mathbb{C}^*_1$ | 1 | 1 | 1 | 1 | $-2$ | 0 |
| $\mathbb{C}^*_2$ | 1 | 0 | 1 | 0 | 0 | $-1$ |

$$uv = z^2\sigma - w^{2k} p^{k-1}$$

<u>Affine patch</u>: $w \neq 0$, set it to $w=1$

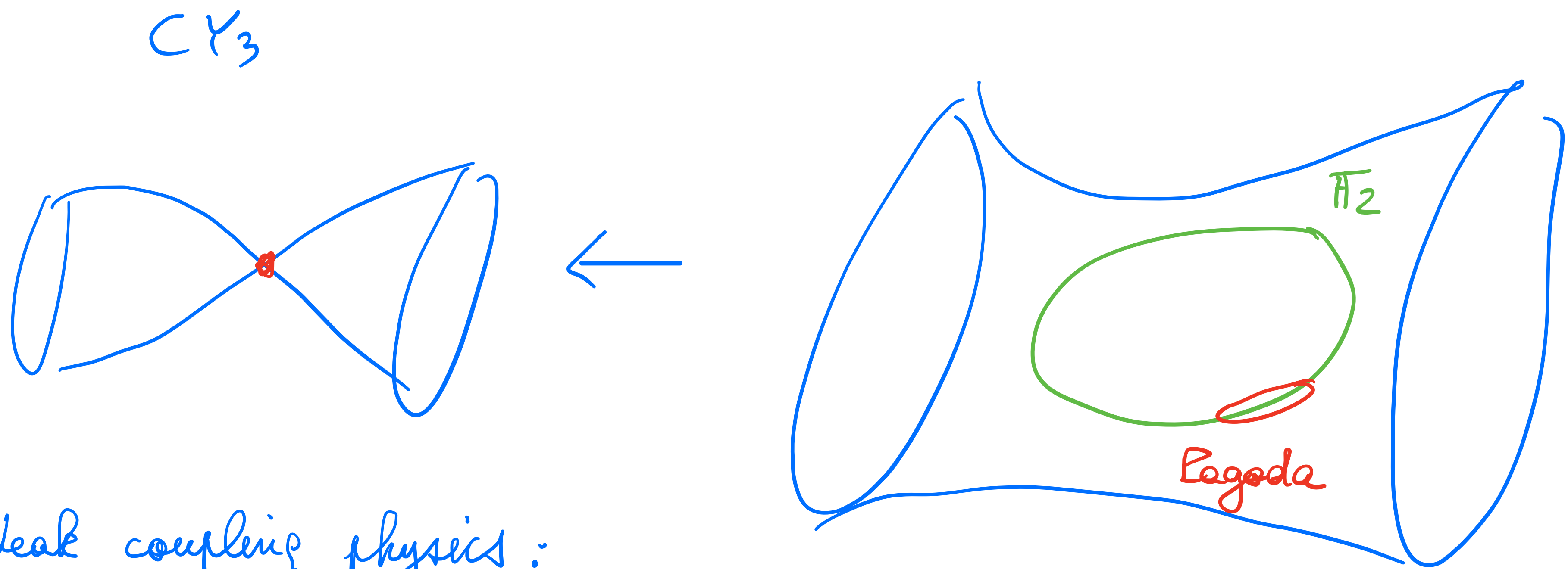
$$\hookrightarrow \quad uv = z^2\sigma - p^{k-1} \simeq \frac{(k-1)}{2}\text{-Pagoda}$$

(odd-$k$) !

So we have a rank-1 theory coupled to

"Pagoda matter"

# Cartoon




Weak coupling physics :

$SU(2)$ SYM + "Pagoda matter"

↗ uncharged

# Higgsing Pagoda matter

$\mathbb{C}^4/\mathbb{Z}_2$

| | $u$ | $v$ | $z$ | $w$ | $p$ |
|---|---|---|---|---|---|
| $\mathbb{C}^*$ | 1 | 1 | 1 | 1 | $-2$ |

$uv = z^2 - w^{2k} p^{k-1}$

$$uv = z^2 - w^{2k} p^{k-1} + \sum_{i=0}^{\frac{k-3}{2}} c_i\, w^{2i+2} p^i$$

switch on, for instance $c_1 w^4 p$

$\underline{w=1}$ patch: $uv = z^2 - p^{k-1} + c_1 p \leftarrow$ smooth !

$\rightsquigarrow$ $\boxed{\text{Gauge theory phase obstructed}}$

# Recap

$\mathbb{F}_2$ + Pagodina curve

rank - 1

deform

$\wr$

VEV's to Pagoda matter

$$\frac{1}{g_{YM}^2} \longrightarrow 0 \quad !$$

# Outlook

- What is this Pagoda matter ?
- Symmetries interpretation
- BPS spectrum

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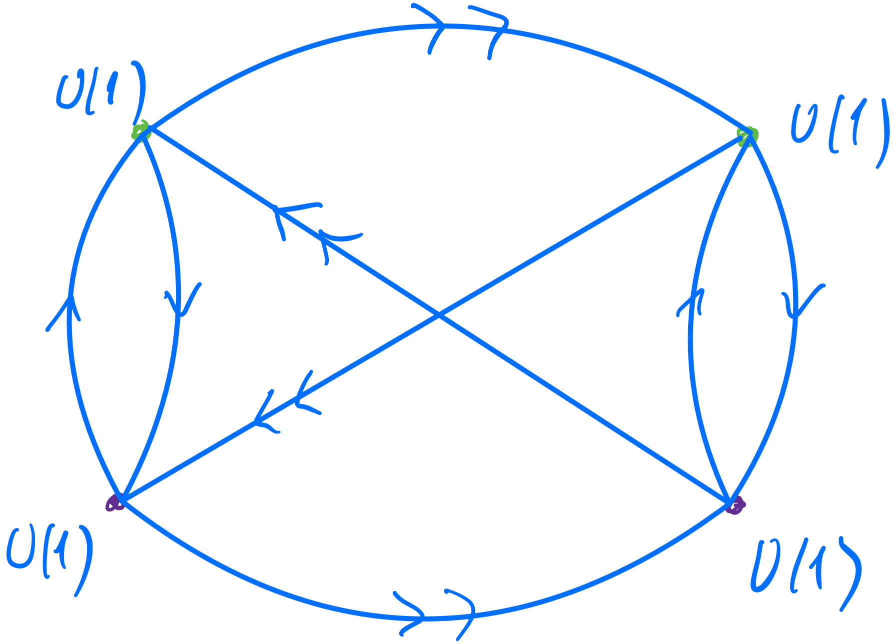
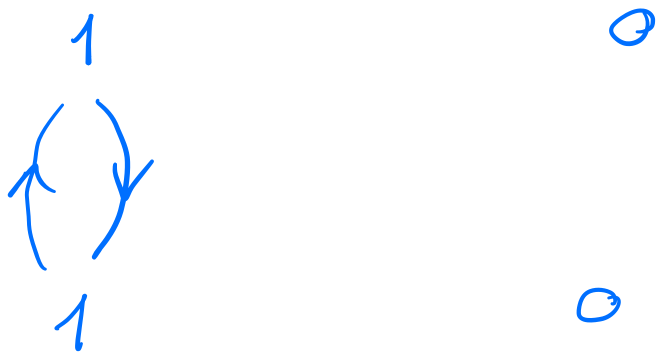
Thank you

# Backup



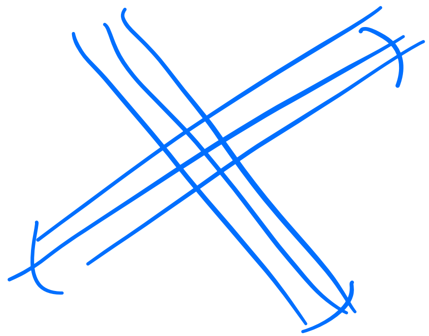

Rep 


Pagoda matter $\langle R_{Pm}, * \rangle = 0$

# Symmetries viewpoint

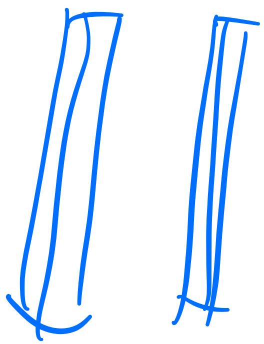
Pagoda as spectral 3-fold $\longrightarrow$

$$\langle \Phi \rangle = \begin{pmatrix} \begin{pmatrix} 0 & 1 & & 0 \\ & 0 & 1 & \\ & & \ddots & 1 \\ z & & & 0 \end{pmatrix} & 0 \\ 0 & \begin{pmatrix} 0 & 1 & & 0 \\ & 0 & 1 & \\ & & \ddots & 1 \\ -z & & & 0 \end{pmatrix} \end{pmatrix}$$



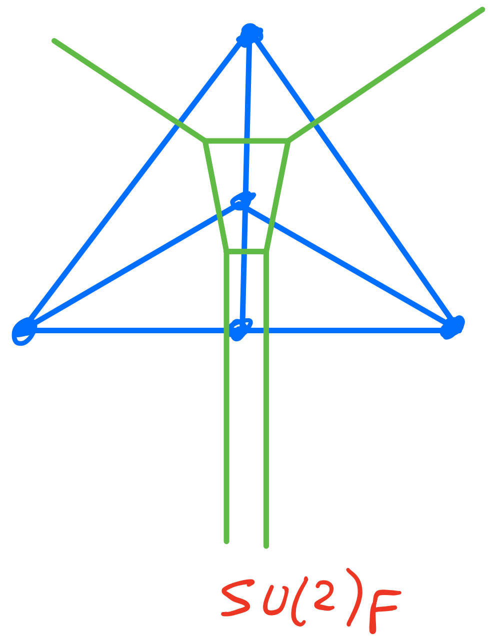
at $z = 0$

$uv = w^{2k}$



flavor $SU(2)_F$

# Local $\mathbb{F}_2$

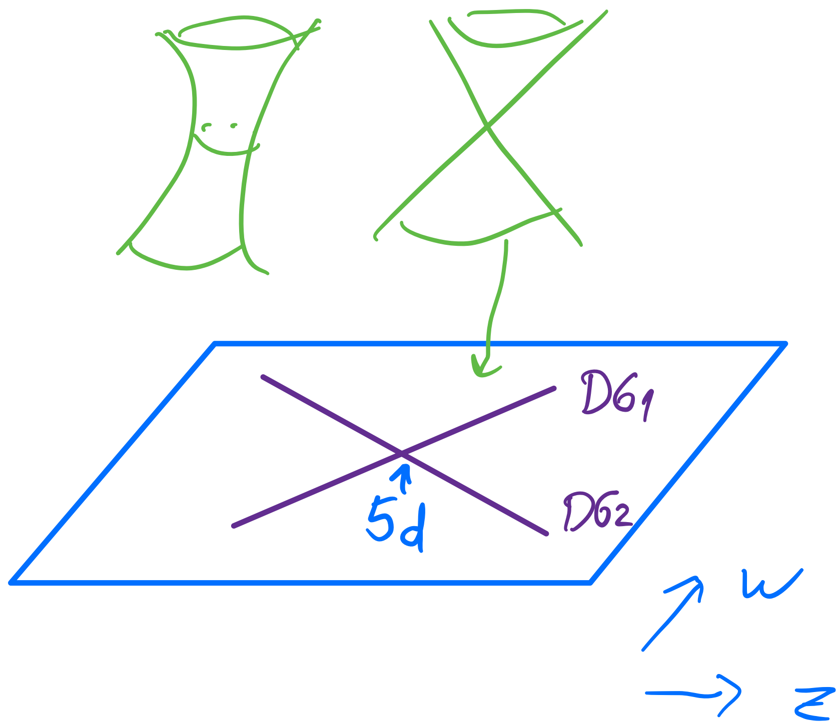



$\langle \Phi \rangle \sim$ position-dependent VEV for background $SU(2)_F$

2) M-theory $\xrightarrow{S^1}$ IIA

$$\underbrace{uv}_{} = z^2 - w^2$$

$\mathbb{C}^*$ - fibration over $\mathbb{C}^2_{\langle z, w \rangle}$ - surface




$(Q, \tilde{Q})$

$1 \quad -1$

2 intersecting D6 - branes $\leftarrow$ 1 free hyper.