Complexity of Effective Theories in the Quantum Gravity Landscape

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Based on: 2503.15601 & 2601.18863 with Thomas Grimm, Mick van Vliet
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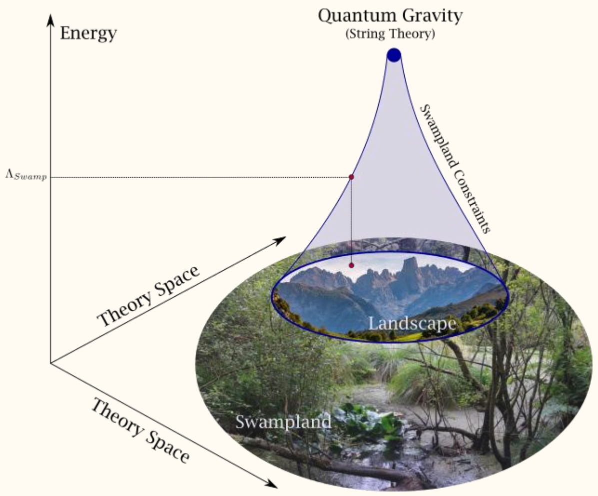
HARVARD

Context

Gravity and information bounds

- **Swampland Program:** Fundamental constraints associated to QG theories
  - No Global Symmetries [Banks, Dixon '98]
  - Distance Conjecture [Ooguri, Vafa '07]
  - ….

  } Entropy bounds and Finiteness arguments (spectrum, distance)



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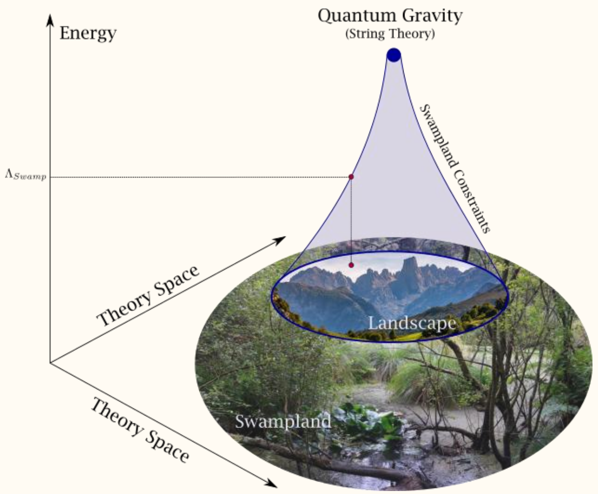
- We look for optimal framework, covering all features in nature but excluding unphysical objects.

- **Our goal:** Unify swampland conditions on EFTs as statements about finiteness of geometric and logical complexity. [Grimm, DP, van Vliet '26]



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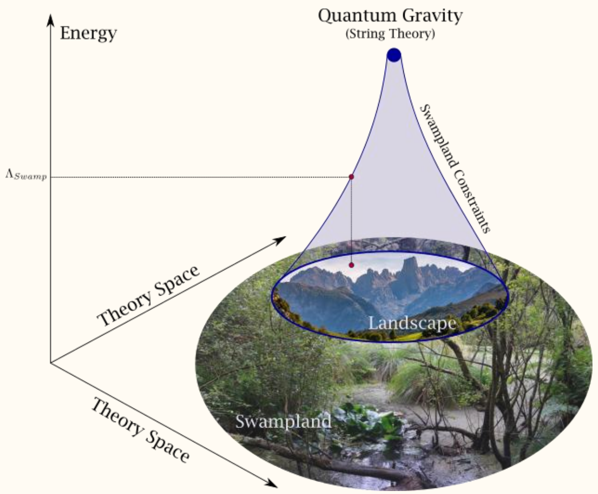
- **Our goal:** Unify swampland conditions on EFTs as statements about finiteness of geometric and logical complexity. [Grimm, DP, van Vliet '26]

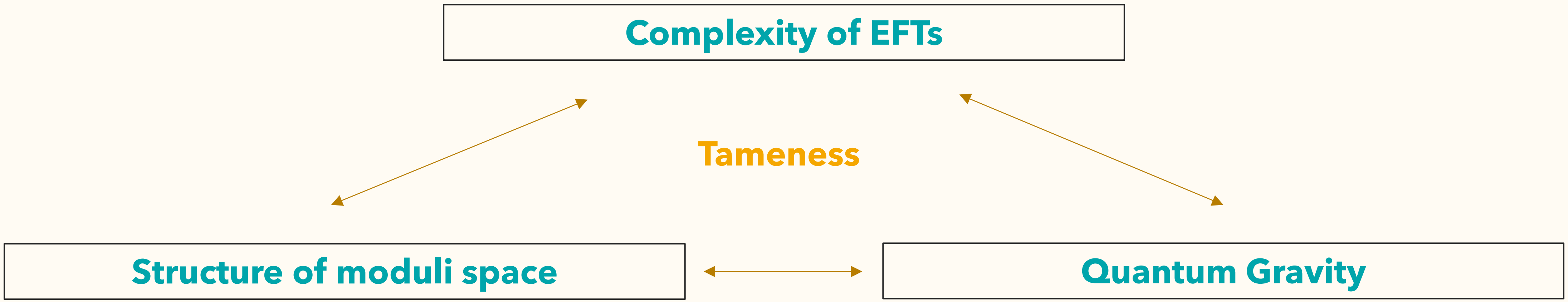
- Need a well-defined notion of complexity:
  - Tame geometry and sharp o-minimality. [Van den Dries '98] [Binyamini, Novikov '22]
  - Quantified in terms of two integers (F,D).

- EFTs that do not accept it → **Swampland**.



Outline

- Tameness framework and Complexity
- Application to EFTs
- Seiberg-Witten Example
- Complexity and Quantum Gravity



Characterizing Complexity

Tameness and O-minimality

- Tameness principle from mathematical logic: o-minimality. [Van den Dries '98]

A **o-minimal structure** is
- Collection of subsets of $\mathbb{R}^n$ with finitely many connected components.
- Contains the collection of zero sets of polynomials.
- Closed under basic set theoretical operations (union, intersection, product, complement, projection…).

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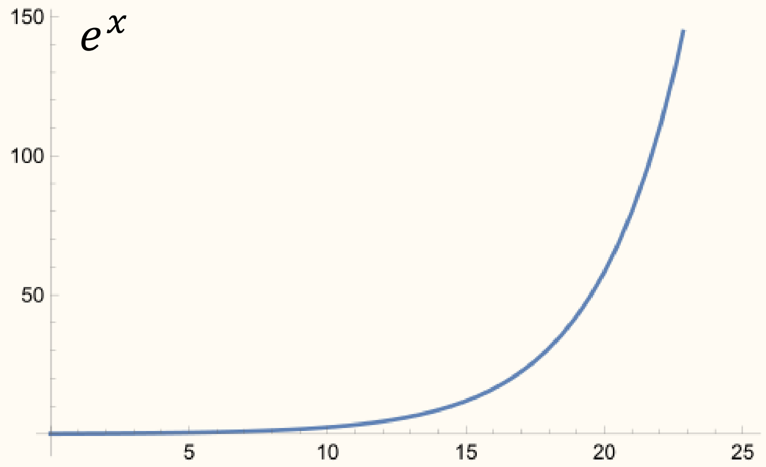
- Applies to functions → Tame graph.

- Examples:
  - Polynomials
  - Exponentials
  - Analytical functions in compact domains



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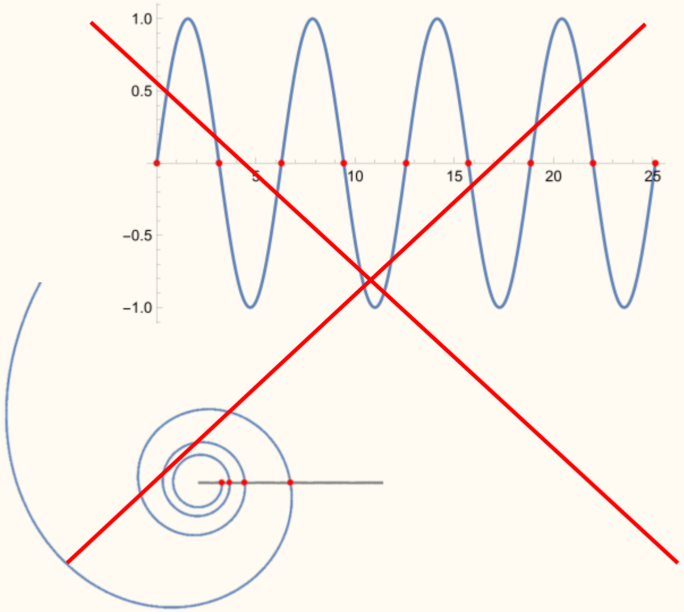
- Counterexamples: The sin(x) in the real line.



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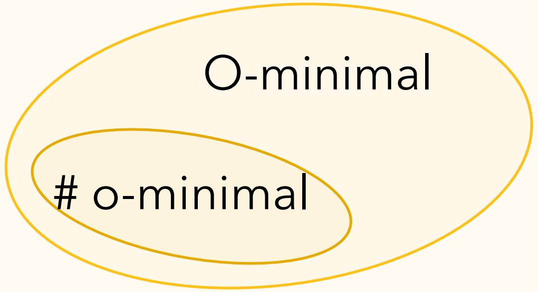
- **Refinement**: **Sharp o-minimality** [Binyamini, Novikov '22]
  - Quantifies information in each set, function with two integers (F,D) → (format, degree)
  - Universal bound on number of connected components $P_F(D)$.

- Examples
  - Polynomials ✓
  - Exponentials ✓
  - Analytical functions in compact domains ✗



Examples

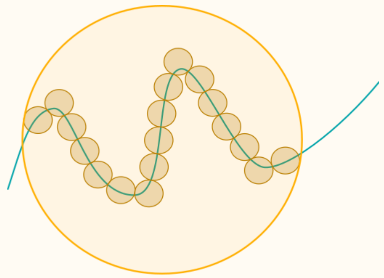
- Complexity for polynomials: Amount of information needed to specify the function.

$$P(x) = a_1x^2 + a_2x + a_3$$

F: number of variables
D: degree

Sufficient to bound:

- Number of zeroes, minima, maxima, number of data points needed to approximate $P(x)$...



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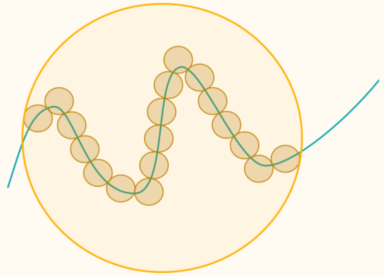

- General analytical functions are not necessarily sharp o-minimal.
- Many satisfy differential equations that generate recursion relations: $\frac{d}{dx}\mathrm{e}^{\mathrm{ax}} = \mathrm{a}\mathrm{e}^{\mathrm{ax}}$

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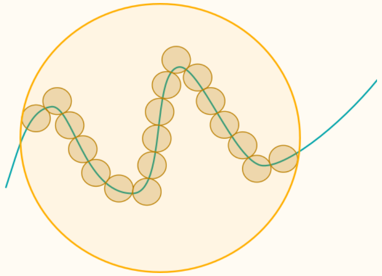

- Many satisfy differential equations that generate recursion relations: $\frac{d}{dx} e^{ax} = a e^{ax}$
- **Sharp o-minimal example: Pfaffian construction** → Polynomials + diff. equations

Pfaffian chain $f_1(x), \dots, f_r(x)$

$$\begin{cases} \partial_x f_1 = P_1(x, f_1) \\ \partial_x f_2 = P_2(x, f_1, f_2) \\ \vdots \\ \partial_x f_r = P_r(x, f_1, \dots, f_r) \end{cases}$$

Pfaffian function $f(x) = P(x, f_1, \dots, f_r)$

- Pfaffian functions are sharp o-minimal:

Degree: $D = \deg(P) + \sum \deg(P_i)$
Format: $F = n + r$

Complexity of EFTs

Complexity and EFT Lagrangians

- A general d-dimensional EFT **Lagrangian** with cutoff Λ

$$\mathcal{L} = \sum_n \Lambda^{d-\Delta_n} c_n \mathcal{O}_n(\Phi, \partial\Phi)$$

Complexity?

- Two main challenges:
  - Need to consider all consistent local operators.
  - There is an infinite-dimensional space $\mathcal{M}$ spanned by Wilson coefficients.

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- For some subspaces of $\mathcal{M}$, the Lagrangian can accept lower complexity descriptions: **emergent simplicity**.

EFT domains

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- Toy example: 0d Lagrangian

$$\mathcal{L}(\phi_1, \dots, \phi_N) = \frac{1}{2}\sum_{j=1}^{N} m_j^2 \phi_j^2 + \frac{\lambda}{4!}\left(\sum_{j=1}^{N} \phi_j^2\right)^2$$

$(N+1)$ parameters

$(\mathcal{F}, \mathcal{D}) = (N, 4)$

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$(N+1)$ parameters
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  - If $\lambda = 0$, $(\mathcal{F}, \mathcal{D}) = (N, 2)$
  - If $m_1^2 = m_2^2$, take polar coordinates $(\mathcal{F}, \mathcal{D}) = (N-1, 4)$

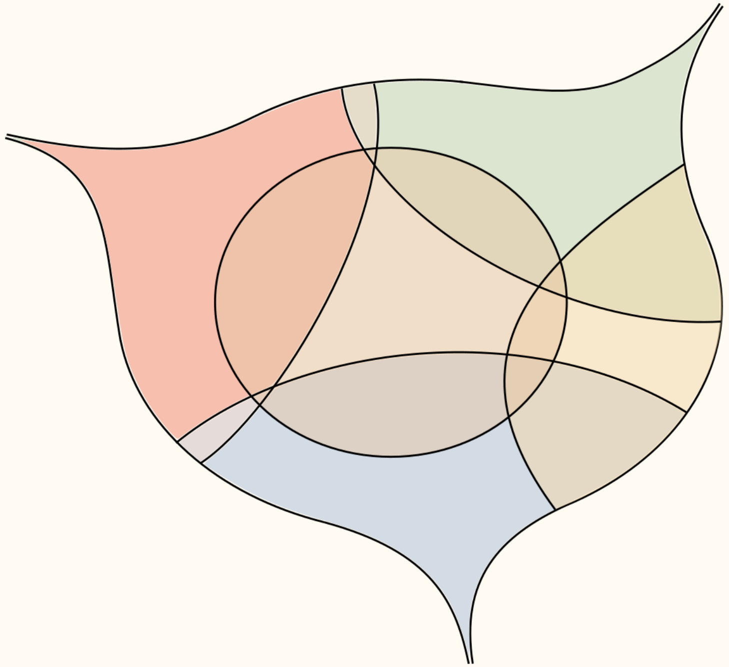
Parameter spaces and EFT domains

- In more generic cases, there might not be a globally valid Lagrangian across $\mathcal{M}$.
- We can view $\mathcal{M}$ as (union of) manifold with its own notion of complexity and decompose it in cells.

  -**EFT domain**: Subset $U \subset \mathcal{M}$ such that the EFTs are described by a single Lagrangian with fixed complexity.

  -**EFT covering**: Set $\{U_i\}$, with $\mathcal{M} = \cup_i U_i$.

- In many practical cases: $\mathcal{M} \rightarrow$ Vevs of moduli.
- What theories admit an EFT covering of finite complexity?



Complexity in Seiberg-Witten Theory

Seiberg-Witten Theory

- We consider $\mathcal{N}=2, SU(2)$ pure gauge theory in $d=4$. [Seiberg, Witten '94]
- Two $\mathcal{N}=1$ chiral hypermultiplets, A and W_α with effective Lagrangian

$$\mathcal{L}=\frac{1}{4\pi}Im\left[\int d^2\theta d^2\bar{\theta}\frac{\partial F(A)}{\partial A}\bar{A}+\int d^2\theta\left(\frac{1}{2}\sum_\alpha\frac{\partial^2 F(A)}{\partial A^2}W^\alpha W_\alpha\right)\right]$$

$$F(a)=\frac{1}{2}\tau_0 a^2+\frac{i}{\pi}a^2\log\frac{a}{\Lambda}+\frac{1}{2\pi i}a^2\sum_{n=1}^{\infty}c_n\left(\frac{\Lambda}{a}\right)^{4n}$$ Prepotential

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- What happens in other regions? → Dual variable $a_D = \frac{\partial F(a)}{\partial a}$, with new $F_D(a_D)$ converging for $a_D \ll \Lambda$.

Electric-magnetic duality

Moduli Space of Seiberg-Witten

- Moduli space of Seiberg-Witten = Moduli space of elliptic curve.

- Elliptic curve: Locus of $\mathbb{P}^2$ given by

$$y^2 = (x + \Lambda^2)(x - \Lambda^2)(x - z)$$

Complex structure (→ z)

$$a(z), a_D(z)$$

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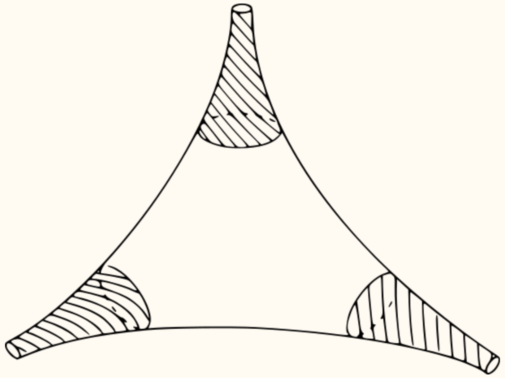
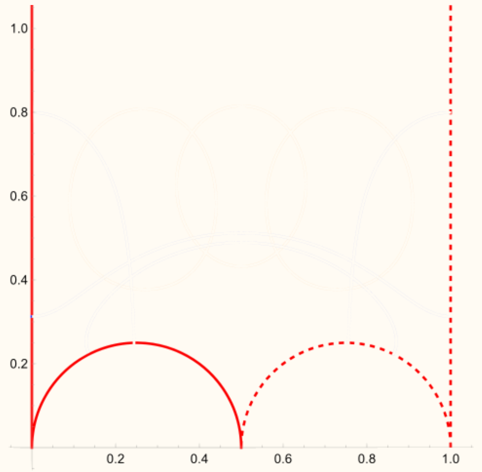
$$\mathcal{M} = \mathbb{P}^1 \setminus \{\Lambda^2, -\Lambda^2, \infty\} \xrightarrow{\tau(z)} \mathbb{F}(\mathbb{H}/\Gamma(2))$$



Period Map = Coupling function

Complexity?



Explicit computation

- **Idea:** Cut moduli space into smaller sets
  Periods → Log-Noetherian Functions → Effective Format [Binyamini '24]
  Combine complexity of individual pieces + the associated map $\tau(z) = \frac{\Pi_1(z)}{\Pi_0(z)}$

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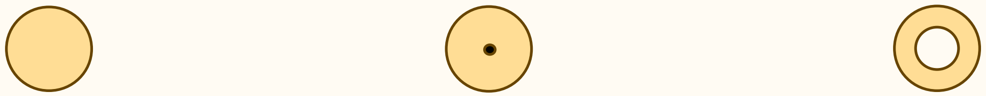
- Complexity of domains $\mathcal{C}$ + functions f on domains
  - Domains



Disc Punctured disc Annulus

  - Describe periods rewriting Picard-Fuchs equations into a Log-Noetherian chain

$$\partial^{\mathcal{C}}_{z^j} f_i = P_{ij}(f_1, \dots, f_n) \qquad \begin{aligned} \partial^{\mathcal{C}}_{z^j} &= \partial_{z^j} \ (disk) \\ \partial^{\mathcal{C}}_{z^j} &= z^j \partial_{z^j} \ (annulus\ or\ punctured\ disk) \end{aligned}$$

LN-function: $f = P(f_1, \dots, f_n)$

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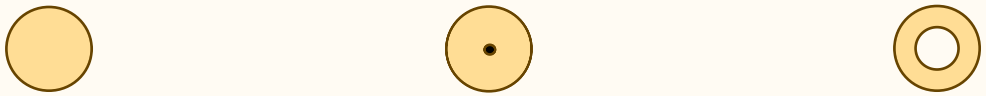

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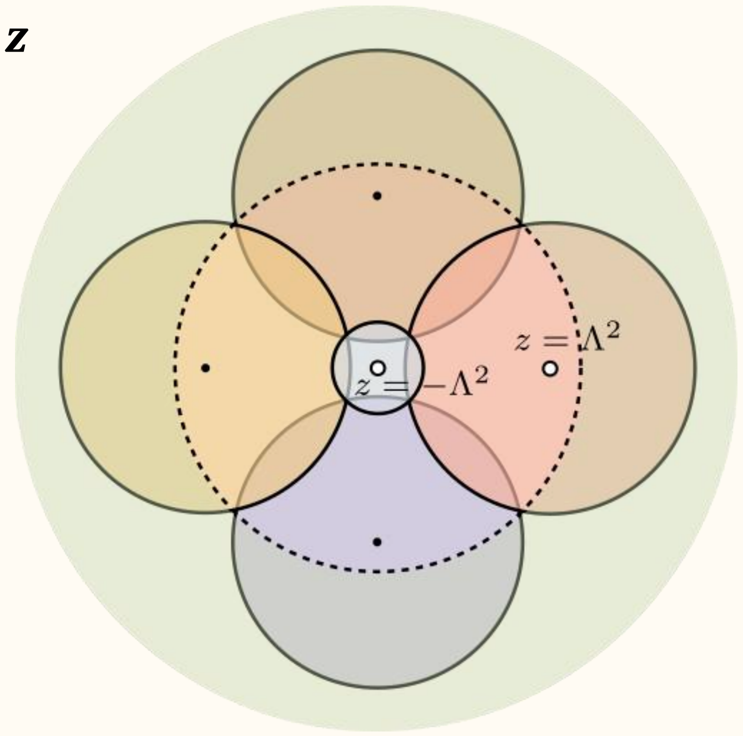
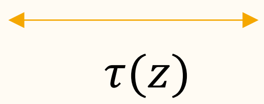
LN-function: $f = P(f_1, \dots, f_n)$

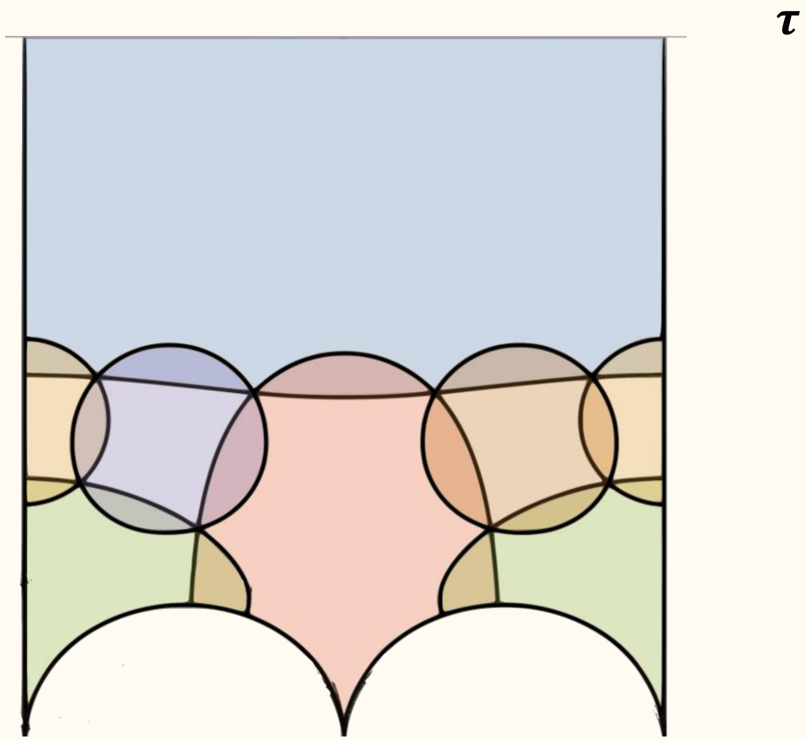
LN-functions are tame and conjectured to be sharply o-minimal

  - Periods are effective o-minimal: $\mathcal{F} \simeq D + F$ [Binyamini '24]

Explicit computation



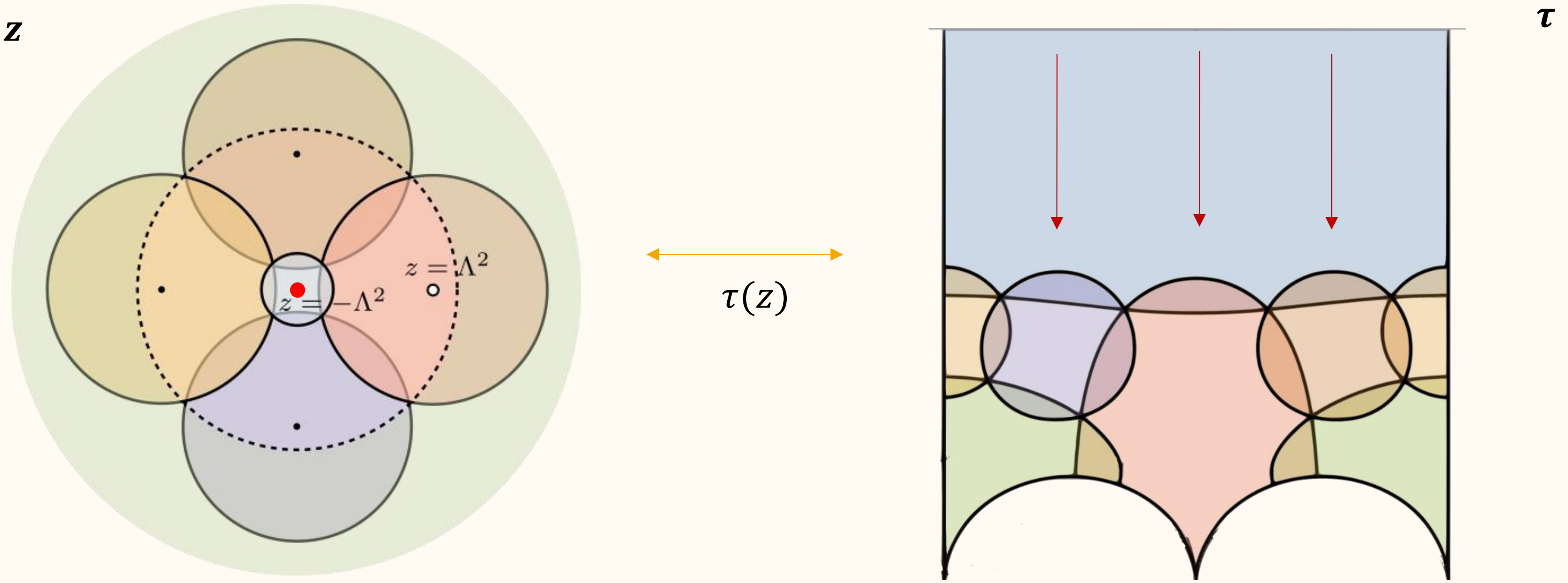




z	$-\Lambda^2$	Λ^2	∞	$-3\Lambda^2$	$\Lambda^2(2i-1)$	$-\Lambda^2(2i+1)$	Total
$\mathcal{F}(\tau)$							

[Carrascal, Ellen, Grimm, DP '25]

Explicit computation



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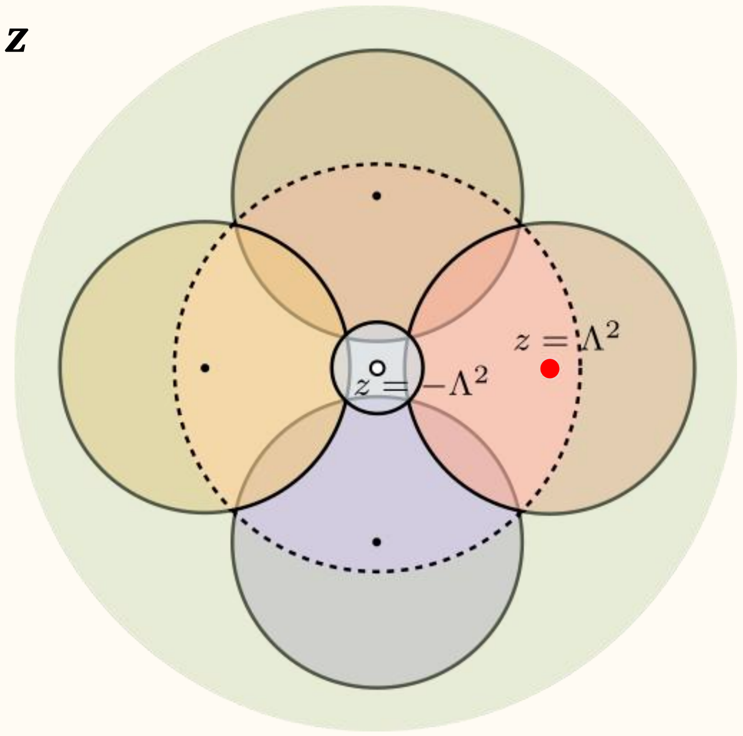

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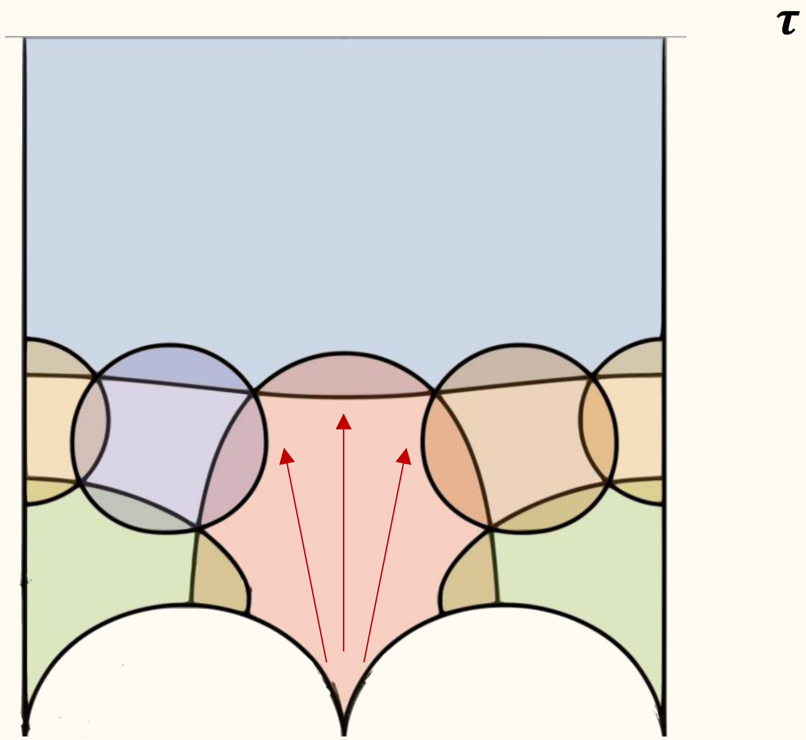

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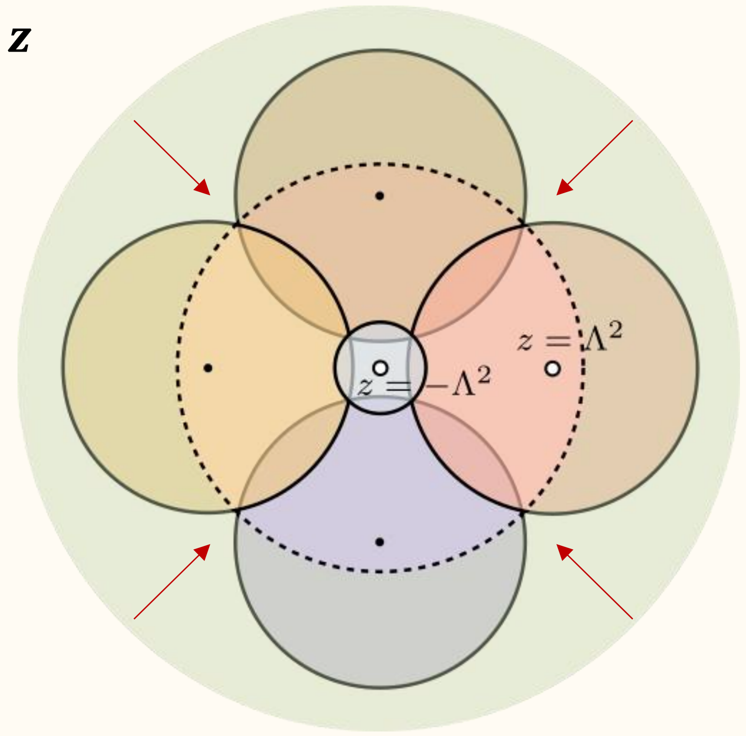

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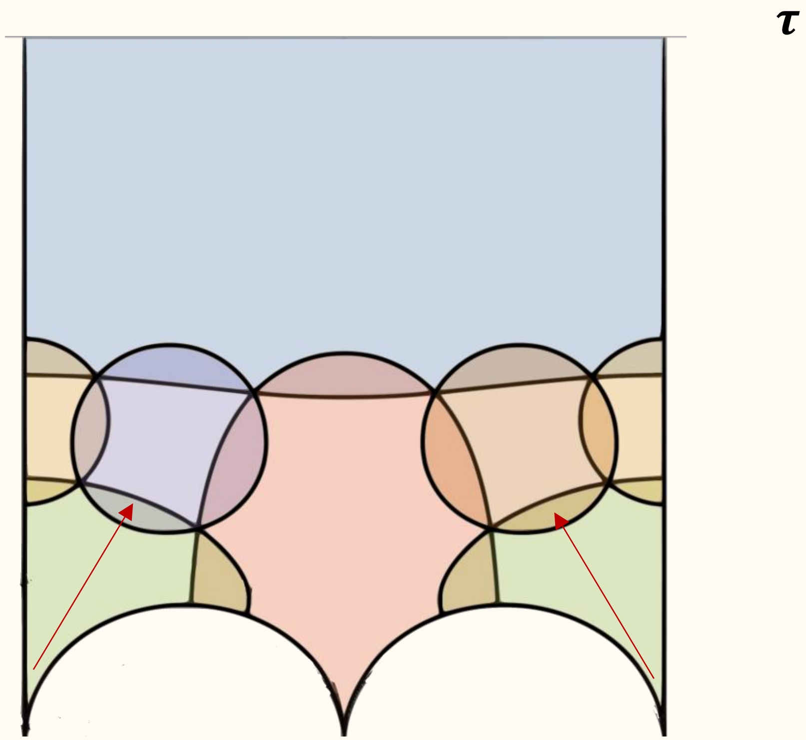

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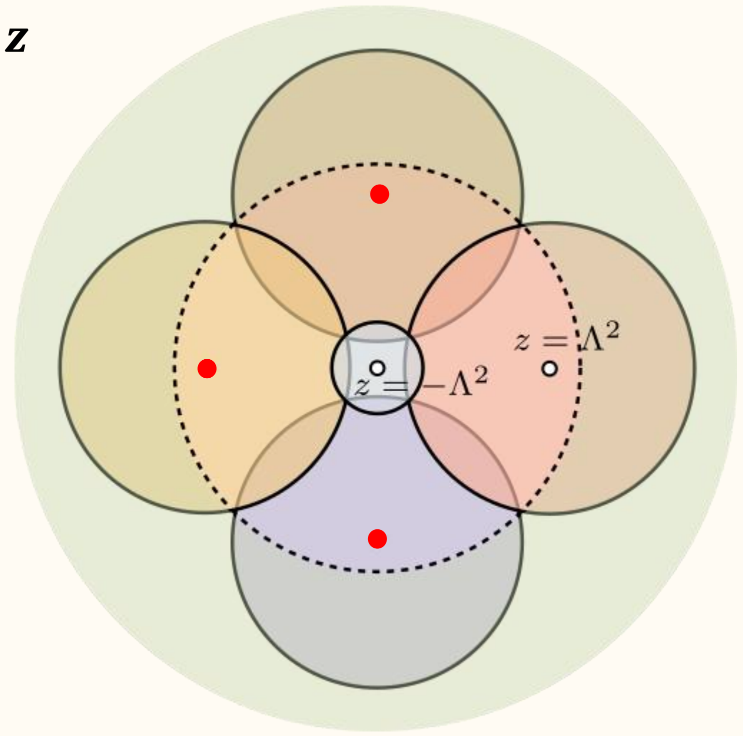

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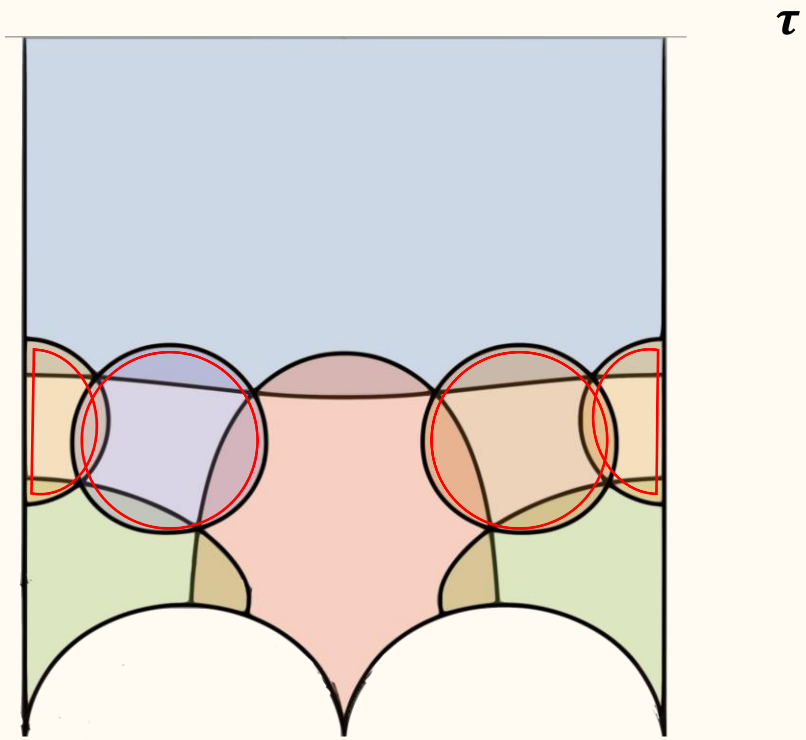

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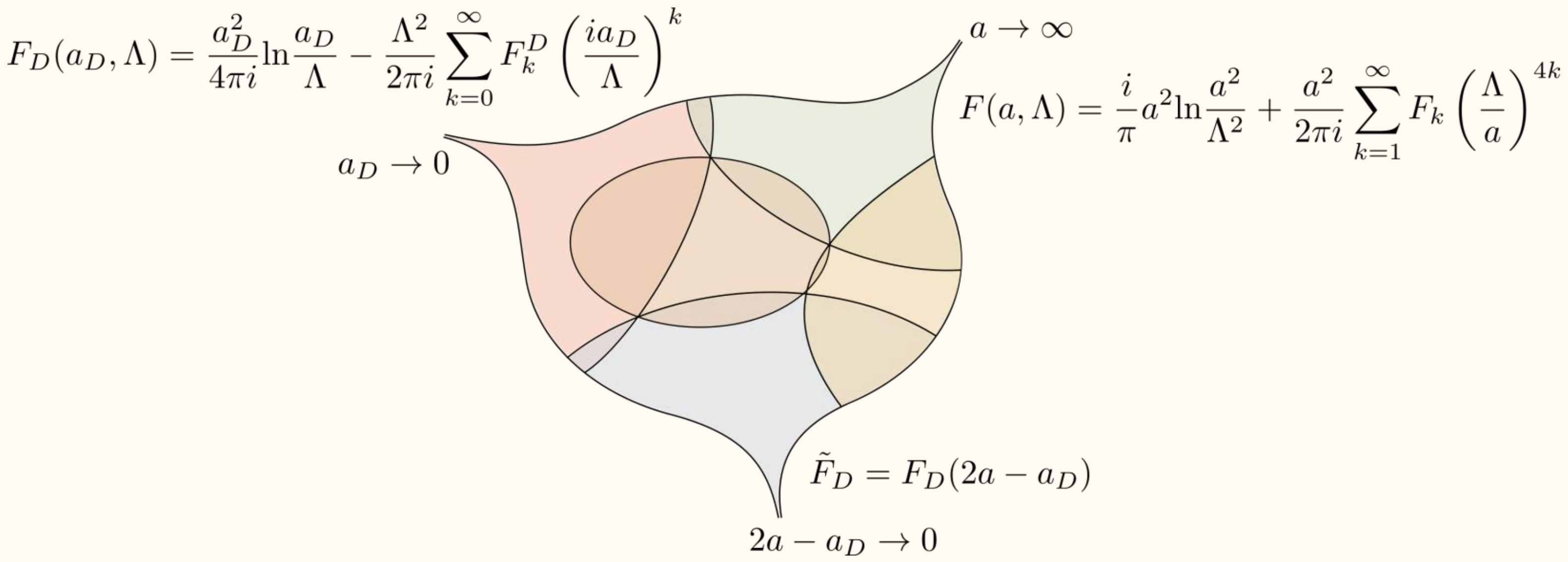
[Carrascal, Ellen, Grimm, DP '25]

Key Findings

- Need at least 6 domains to cover $\mathcal{M}$.
- Main complexity comes from singular points.
- If we extend one domain to the next singularity local complexity diverges → need new representation to keep complexity finite.
- This change of representation is precisely the Electric-Magnetic duality in SW theory.



Quantum Gravity and Finiteness

Information Bounds for the Landscape

Finite Complexity Conjecture [Grimm, DP, van Vliet '26]

Let $\mathcal{M}_{QG;\Lambda}$ be the set of d-dimensional QG consistent EFTs with cutoff at least Λ

1. The set $\mathcal{M}_{QG;\Lambda}$ is definable in a sharply o-minimal structure with finite complexity $(\mathcal{F}_\Lambda, \mathcal{D}_\Lambda)$.
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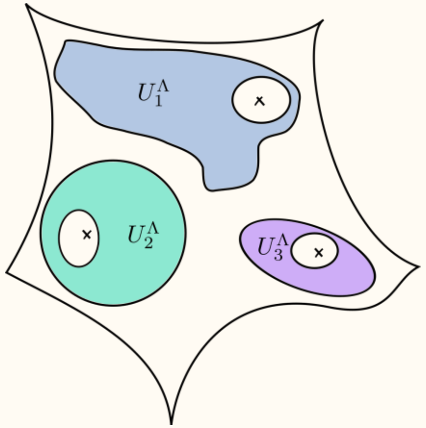

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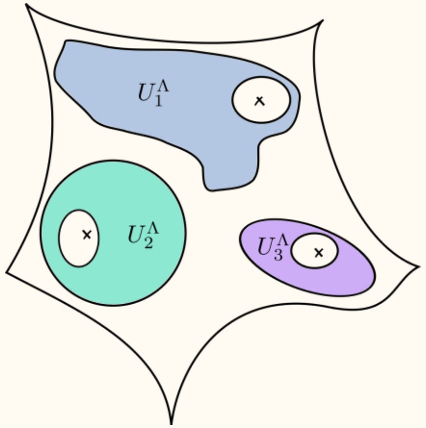

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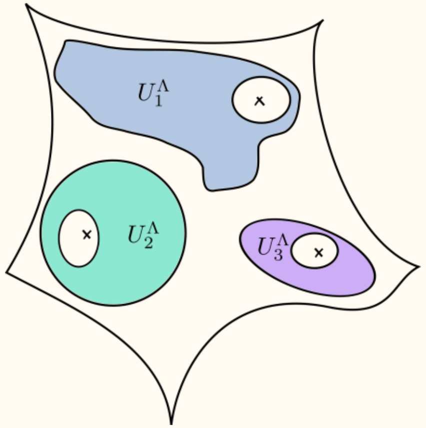
- Combining both statements: $\mathcal{M}_{QG;\Lambda}$ would have a finite EFT covering.
  - Finiteness of CY compactifications.
  - Finiteness of flux vacua.
- We can test the conjecture by looking at slices of $\mathcal{M}_{QG;\Lambda}$:
  - Moduli spaces of high-SUSY settings $\mathcal{M} = \Gamma\backslash G/K$ ✓
  - Complex structure moduli spaces of Calabi-Yau manifolds ~✓ [Bakker, Klingler, Tsimerman '20] [Binyamini '24]



Embeddings and Volumes

- How to assign complexities to $\mathcal{M}_{QG;\Lambda}$?
- Current approach: Tame isometric embedding of $\mathcal{M}_{QG;\Lambda}$ into Euclidean space. [Grimm, DP, van Vliet '25]

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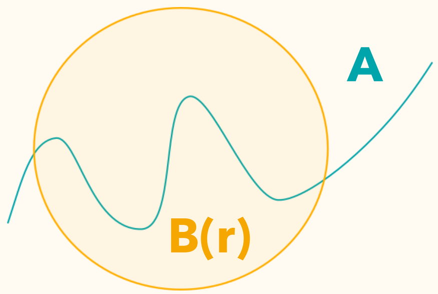
- This has important implications: The volume growth of a set is bounded by its complexity.

Theorem: $A \subset \mathbb{R}^n$ be an l-dimensional tame set with finite complexity (F, D). Then for any ball $B^n(r)$

$$Vol\big(A \cap B^n(r)\big) \leq C(F, D) r^l$$



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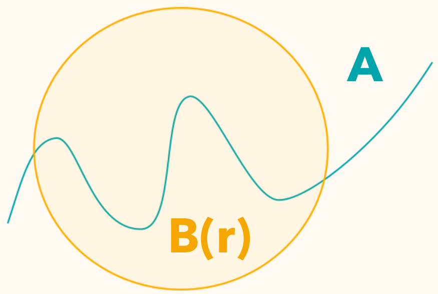
- Recover compactifiability condition



Geodesic balls in moduli space grow at most like Euclidean space at large distances. [Delgado,Heisteeg,Raman,Torres,Vafa,Xu '24]

Connection with **dualities** like $SL(2,\mathbb{Z})$ in Type IIB.
- $\mathbb{H} \rightarrow$ no tame iso. embedding
- $\mathbb{H}/SL(2,\mathbb{Z}) \rightarrow$ admits tame iso. embedding

Conclusions

- **Sharp o-minimality framework:** EFTs compatible with QG have bounded geometric/logical complexity.
- Many Swampland features can be understood in these terms:
    - Finiteness of the string landscape.
    - Volume growth of moduli spaces.
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Thanks!

Complementary Slides

Applications and Examples

- **Arithmetic quotients:**

  Moduli spaces of high-SUSY settings

  $$\mathcal{M} = \Gamma\backslash G/K$$

  definable in $\mathbb{R}_{alg}$ (e.g. $\mathbb{H}/SL(2,\mathbb{Z})$).

- **Calabi-Yau compactifications:**

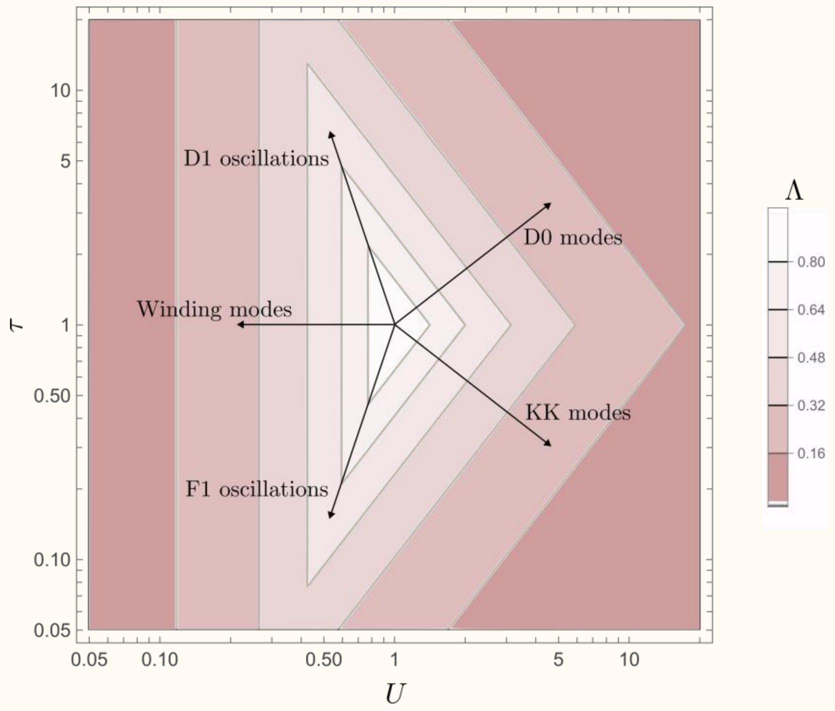
  Complex structure parametrized by period map

  $$\Phi\colon \mathcal{M}_{cs} \to \Gamma\backslash G/K$$

  definable in $\mathbb{R}_{LN,exp}$.

  [Bakker, Klingler, Tsimerman '20]

- Example: M-theory on T^2
  - Moduli space with two unbounded directions.
  - Cutoff: First state of the tower.
  - EFT domains are of the form:



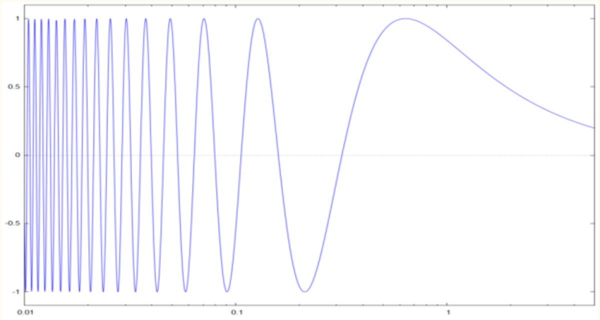
Inside the region, there is a Lagrangian with bounded complexity.

Tame embeddability in action

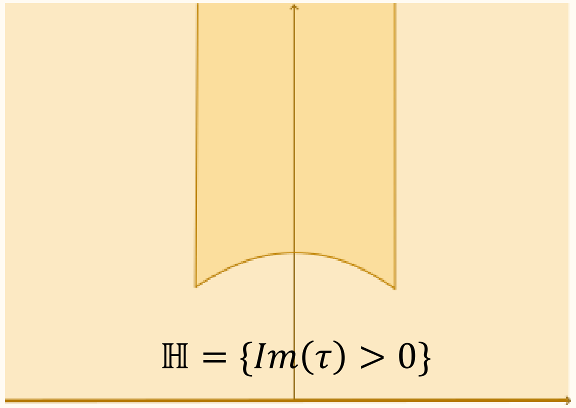
- Consider the complex structure moduli space of type IIB in 10 dimensions.

  $(\mathbb{H}, g_{\mathbb{H}})$ has no tame isometric embedding into Euclidean $\mathbb{R}^n$ for any n.

  Isometric embedding needs at least $\mathbb{R}^6$. $Vol\big(B_{\mathbb{H}}(r)\big) \sim e^r$



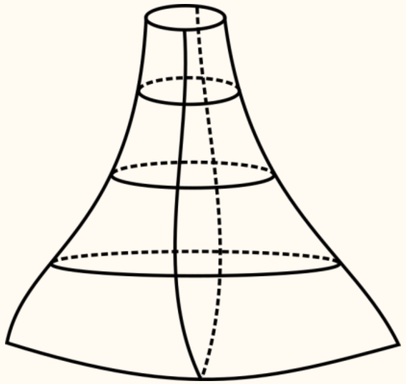
Highly oscillating function to account for asymptotic negative curvature at infinite distances.



$$ds = \frac{d\tau d\bar{\tau}}{Im(\tau)^2}$$

- $\mathbb{H}/SL(2,\mathbb{Z})$ admits a tame embedding in $\mathbb{R}^3$.

$$\mathbb{H}/SL(2,\mathbb{Z}) \leftrightarrow \mathbb{P}^1 \setminus \{0,1,\infty\} \qquad Vol\big(\mathbb{H}/SL(2,\mathbb{Z})\big) = \frac{\pi}{3}$$



- Predicts the existence of dualities.

- Likely generalizes to cases with flux potential. [Baykara, Tomasiello, Vafa '25]

Exact renormalization and Complexity

- For $d \geq 1$, we can consider Wilsonian renormalization.

$$S_\Lambda[\phi] = -\log\left[\int D\chi e^{-S_{\Lambda_0}[\phi+\chi]}\right], \qquad \Lambda_0 > \Lambda$$

- Usual methods take perturbative expansions.

- Under some assumptions there are exact RG flow results → Local Potential Approximation
→ Project out higher derivatives.

- One loop contribution becomes exact

$$\Lambda\frac{\partial}{\partial\Lambda}V_\Lambda(\phi) = -\Lambda^d \log\left(\Lambda^2 + \frac{\partial^2}{\partial\phi^2}V_\Lambda(\phi)\right)$$

[Wegner, Houghton '73]

Similar to defining equations of Log-Noetherian functions. Not known o-minimal structure.

Zero dimensional example

- In some cases, relations between coefficients may allow for resummation.

$$\mathcal{L}(\phi,\chi) = \frac{1}{2}\, m^2\phi^2 + \frac{1}{2}\, M^2\chi^2 + \frac{\lambda}{4!}(\phi^2+\chi^2)^2$$

Integrate out χ

$$\mathcal{L}_{eff}\,(\phi) = \sum_{n\geq 0} c_{2n}\phi^{2n} = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 - \log\left[\int d\chi e^{-\frac{1}{2}\mu^2(\phi)\chi^2 + \frac{\lambda}{4!}\chi^2}\right] \qquad \mu^2 = M^2 + \frac{1}{6}\lambda\phi^2$$

Tame?

Resummation

$$\mathcal{L}_{eff}\,(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 - \log\left[\sqrt{\frac{3}{\lambda}\mu^2(\phi)}\,\exp\left(\frac{3\mu^4(\phi)}{4\lambda}\right) K_{1/4}\left(\frac{3\mu^4(\phi)}{4\lambda}\right)\right]$$

Tame! [Grimm, Ravazzini, van Vliet '24]

- Higher dimensions?

Higher derivative expansion?

- A general EFT Lagrangian contains an infinite series of higher-derivative interactions.
- In the o-minimal framework: field → variable.
  We need to introduce auxiliary variables for each spacetime derivative. $X_\mu = \partial_\mu \phi$, $X_{\mu\nu} = \partial_\mu \partial_\nu \phi$ …
- Infinitely many variables? Challenging for tameness.
- Alternative option: Work in momentum space $\partial_\mu \to p_\mu$ and study $\mathcal{L}(\tilde{\Phi}, p_\mu)$.
- In the following, we focus on **two-derivative sector of position-space** EFT Lagrangians.