
A TALE OF TWO M-THEORIES



Miguel Montero
IFT Madrid



Strings and Geometry, Uppsala
May 20 2026

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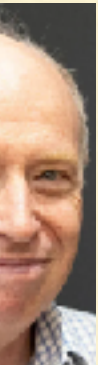
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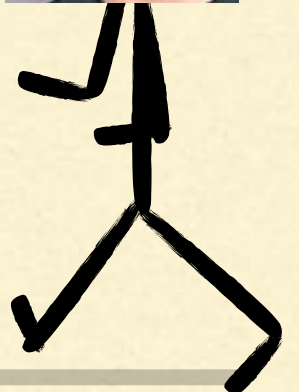
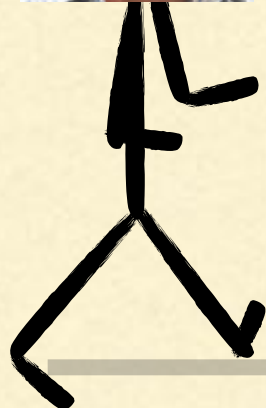
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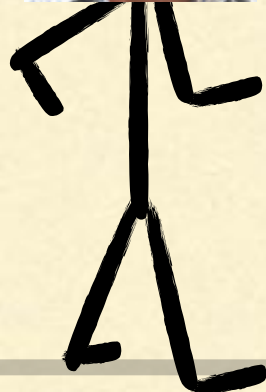


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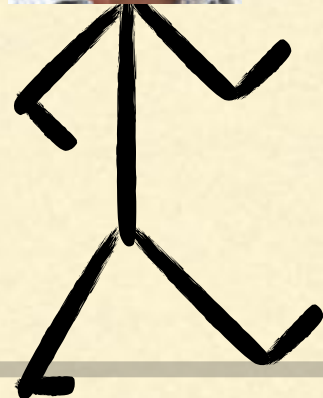


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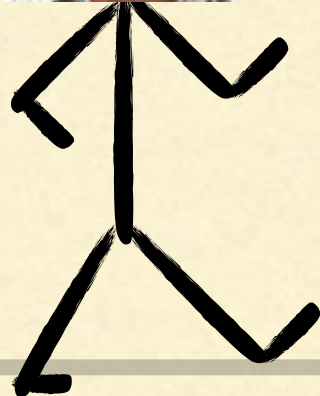
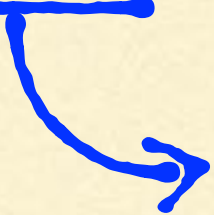
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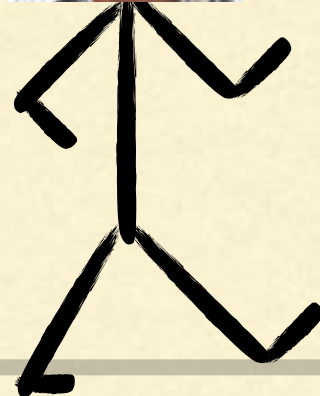
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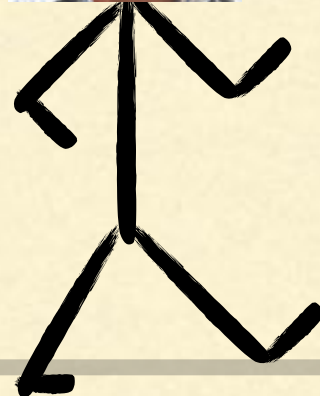
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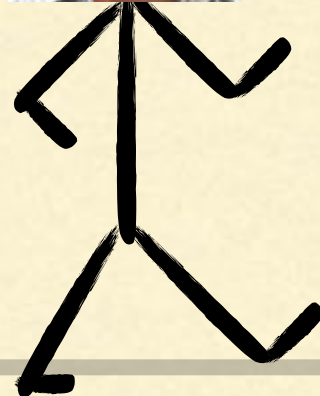


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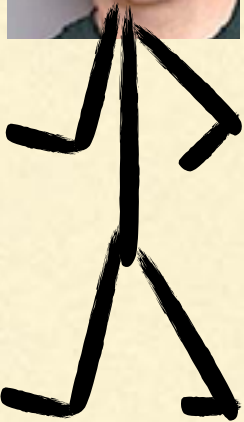
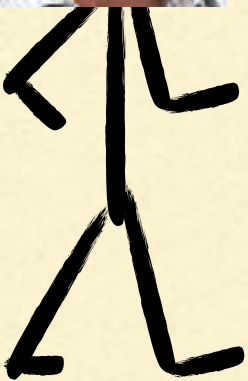
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Please
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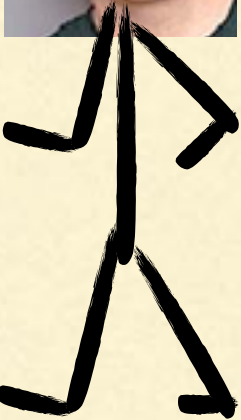
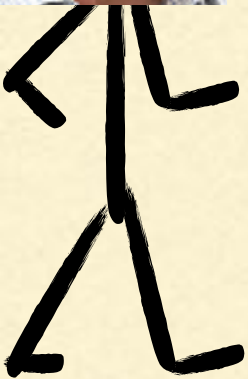


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for several d , using this to prove, after 25 years, that M-theory is **anomaly free**.



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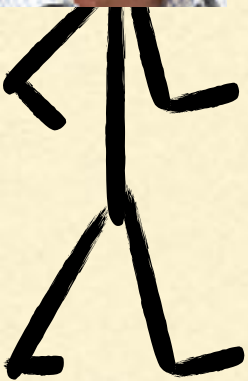
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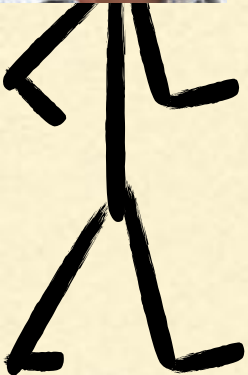
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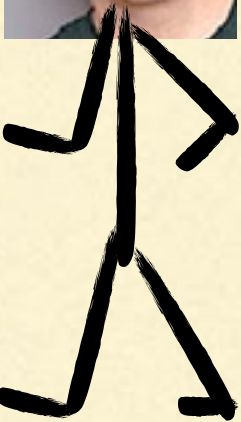
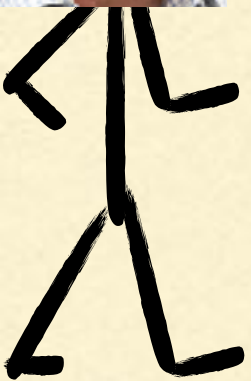
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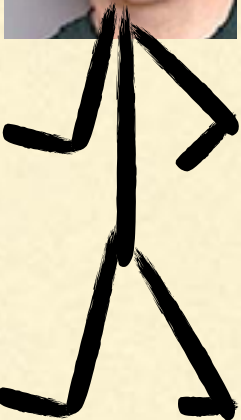
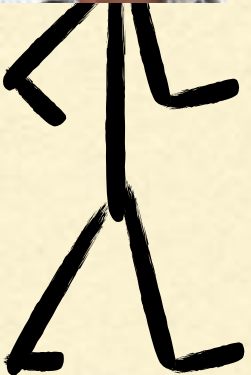
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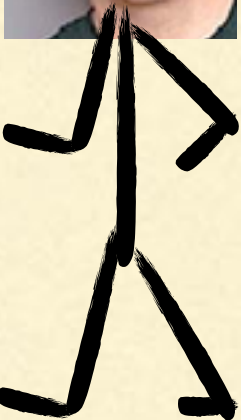
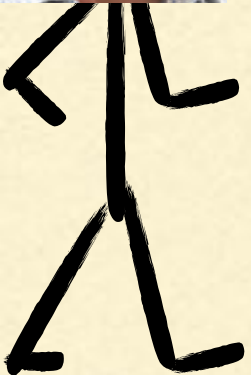
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θ is a **discrete theta angle**, leading to two different 11d SUGRAS and, potentially to two different versions of M-theory.

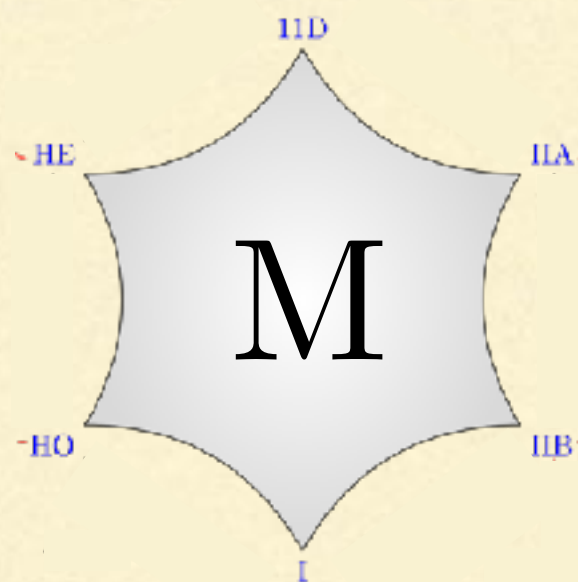
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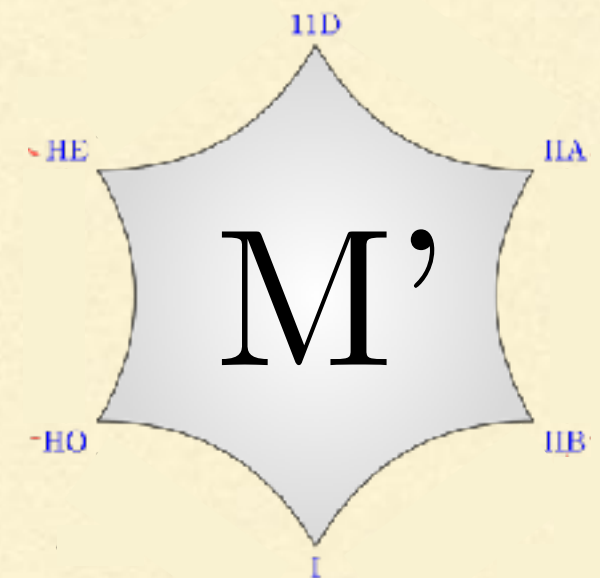
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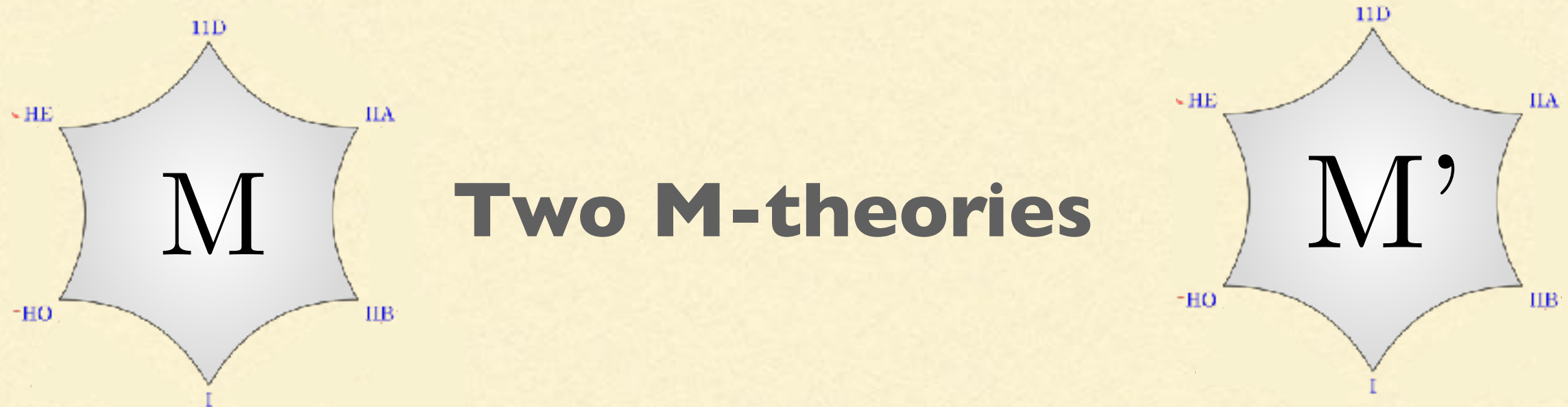


Two M-theories



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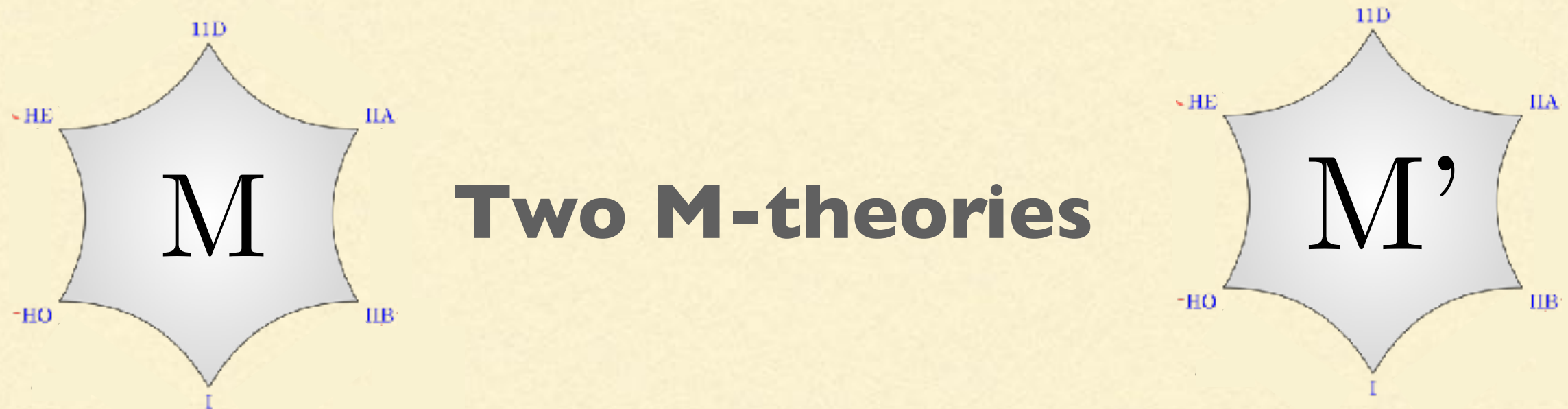
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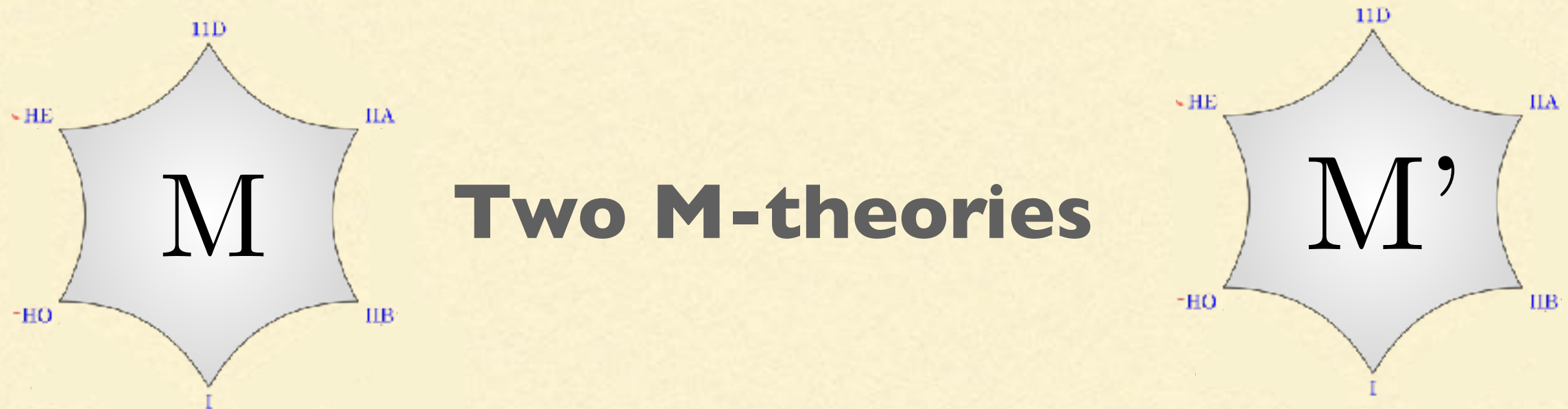


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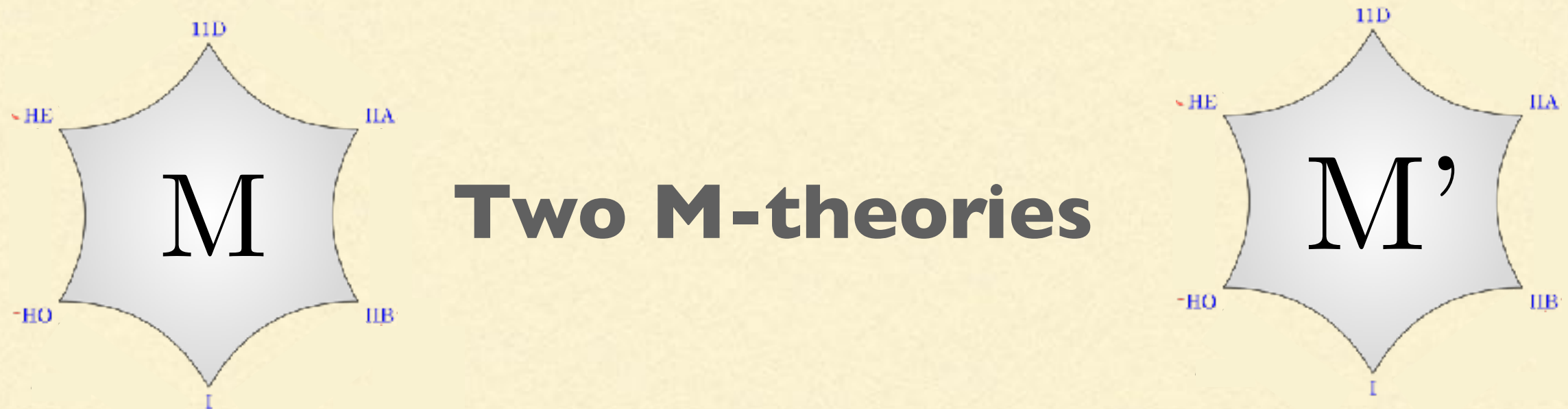
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[Debray-
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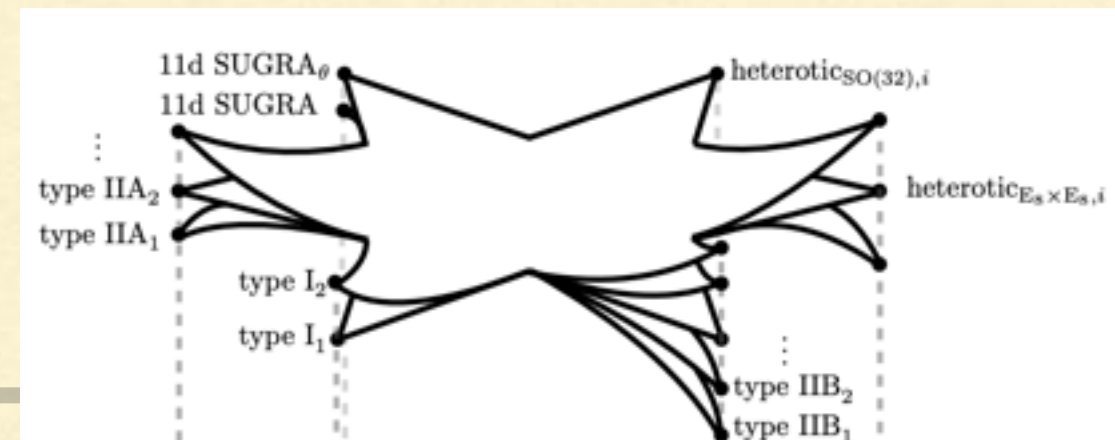
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These would naively lead to an even
crazier multiplicity of theories!



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Meaning the θ is there, but can be shifted by a symmetry transformation:

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So vacua with different θ_{QCD} have identical physics.

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★ $\mathcal{S}$ is spontaneously broken (Imagine $\langle \bar{\psi}\psi \rangle \neq 0$ in QCD analogy)

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Anomaly  Topological GS mechanism

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So, can we add $e^{2\pi i \eta(\mathcal{M})}$ to the action in heterotic?

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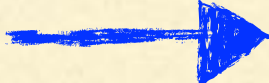
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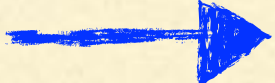
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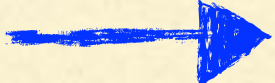
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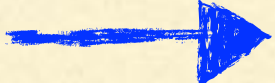
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ξ anomalous
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**Anomaly cancellation
via top. GS mechanism**

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But there's no obstacle in it being a **spontaneously broken** transformation, a field redefinition.

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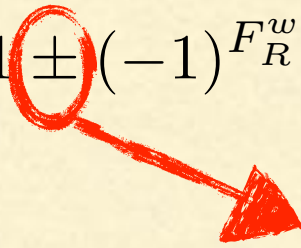
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See Matilda Delgado's talk!

There is a **sign ambiguity!**

It controls the **chirality of spacetime fermions**

There is a worldsheet global parity symmetry (field redefinition)
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[Parra, Kaidi, Tachikawa '19, many, many others...]

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global sym



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Heterotic θ
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[Tachikawa-Yonekuyra '25]

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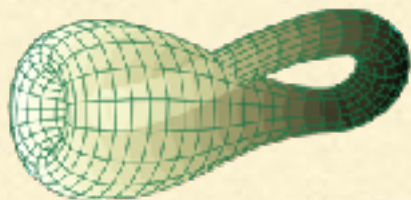
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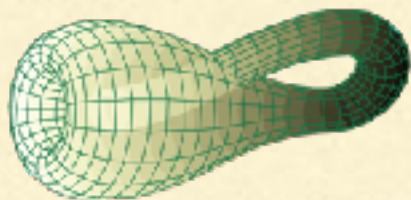
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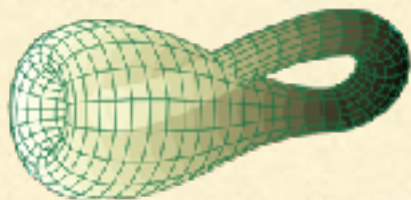
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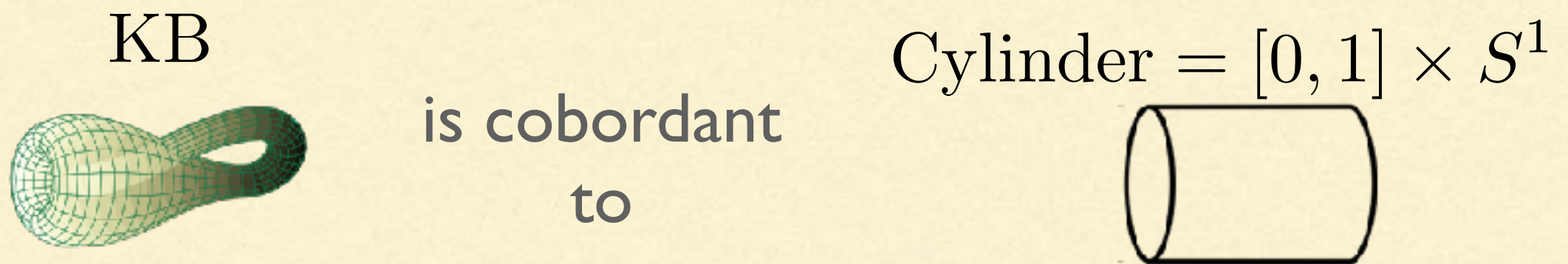
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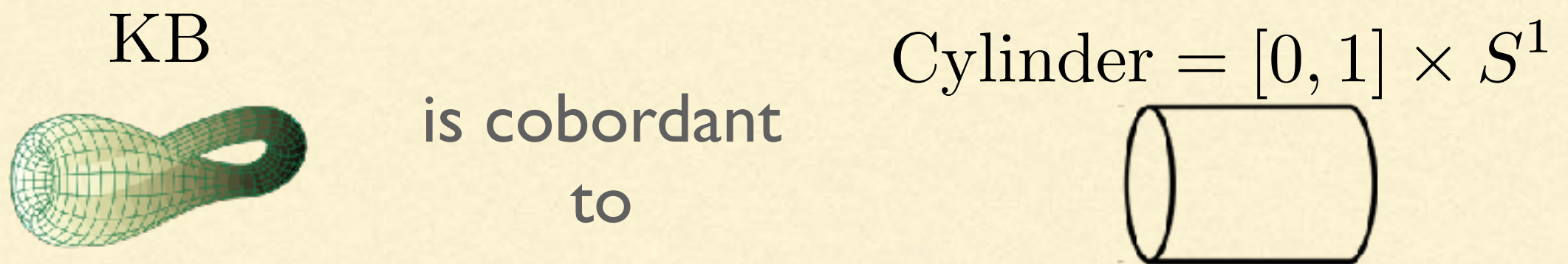
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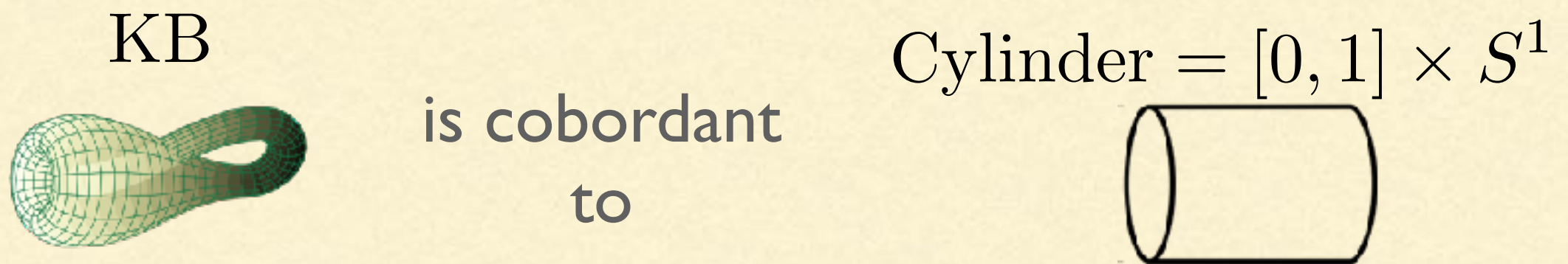
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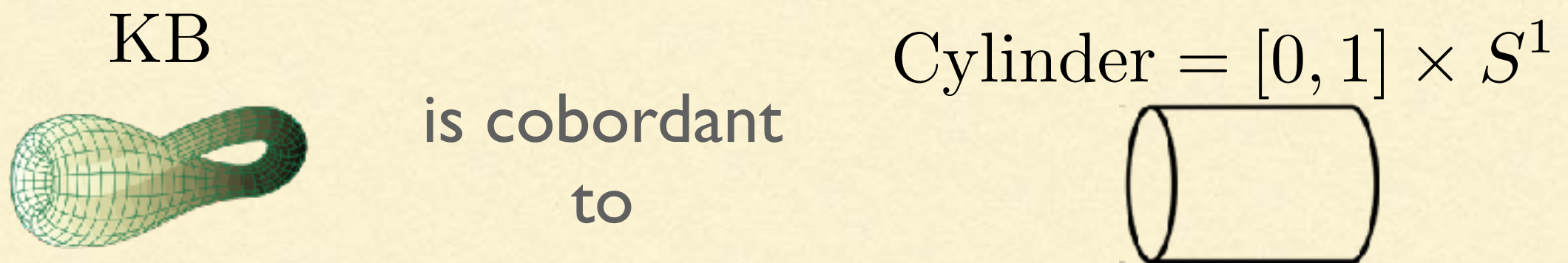
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Hence, the theta angles that detect them are **equivalent**

$$\theta_{\text{M-theory}} = \theta_{\text{Het}} \quad \text{and both unphysical.}$$

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In 10d, this uplifts to a symmetry that flips the product

$$(-1)^{F_L} \cdot \Gamma_{10} = \pm \mathbf{I}$$

on fermion fields.

If you guessed that this time reversal implements a
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It really is there, but its like in QCD with massless quarks.

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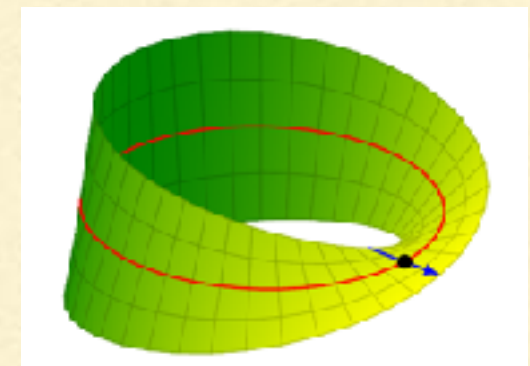
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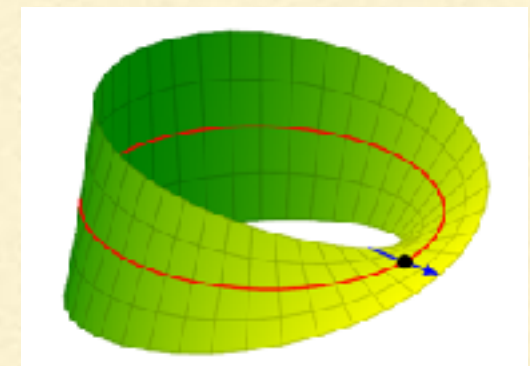


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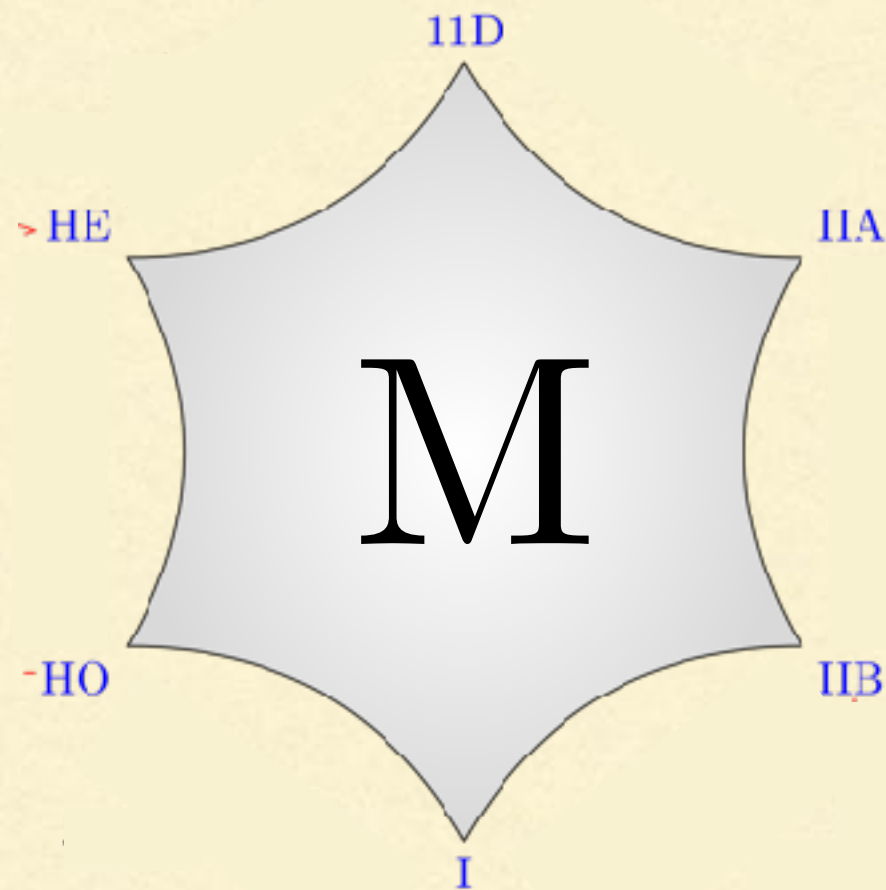
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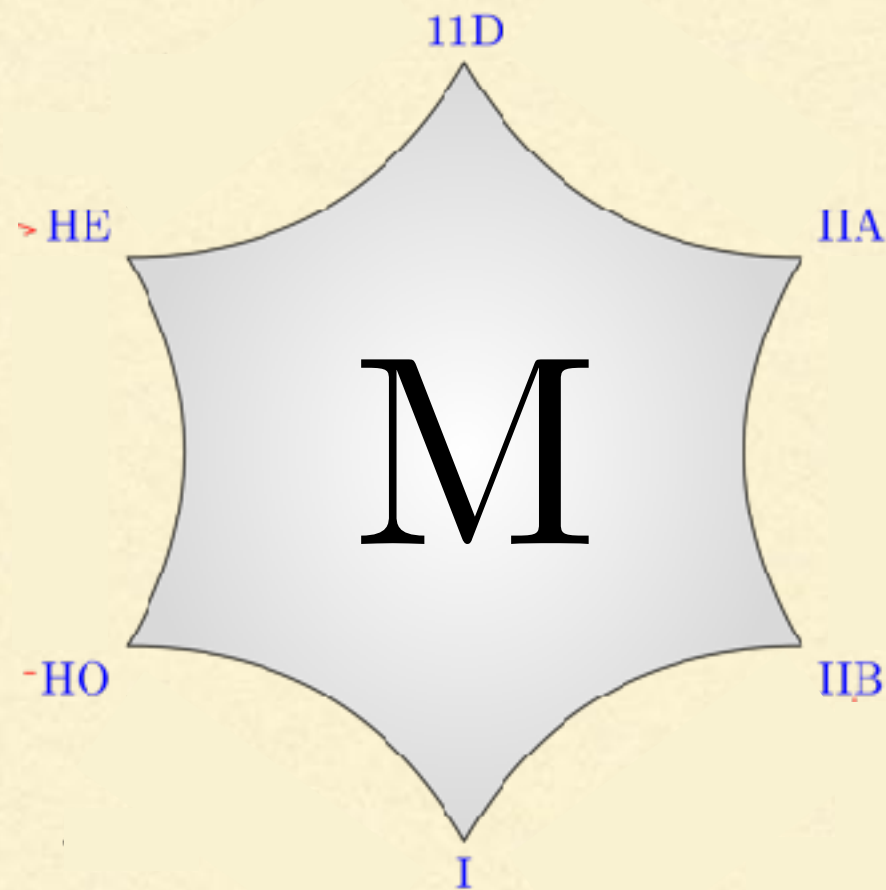
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- ★ The transformations flipping θ can also be seen in holography, where they correspond to field redefinitions in the dual CFT

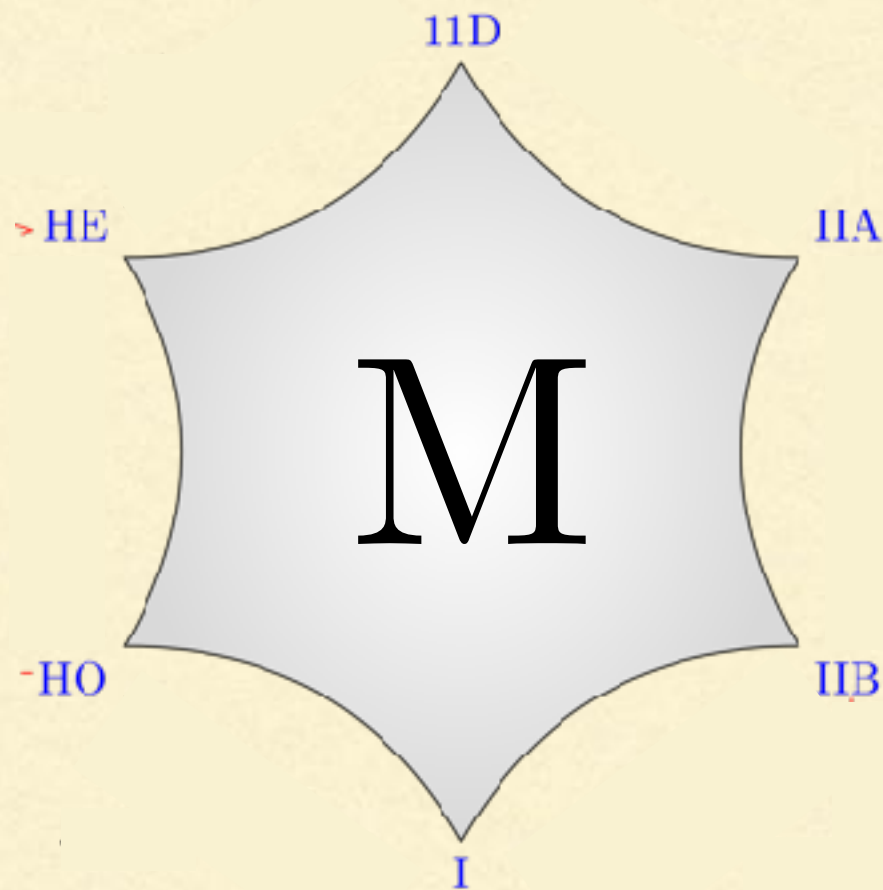


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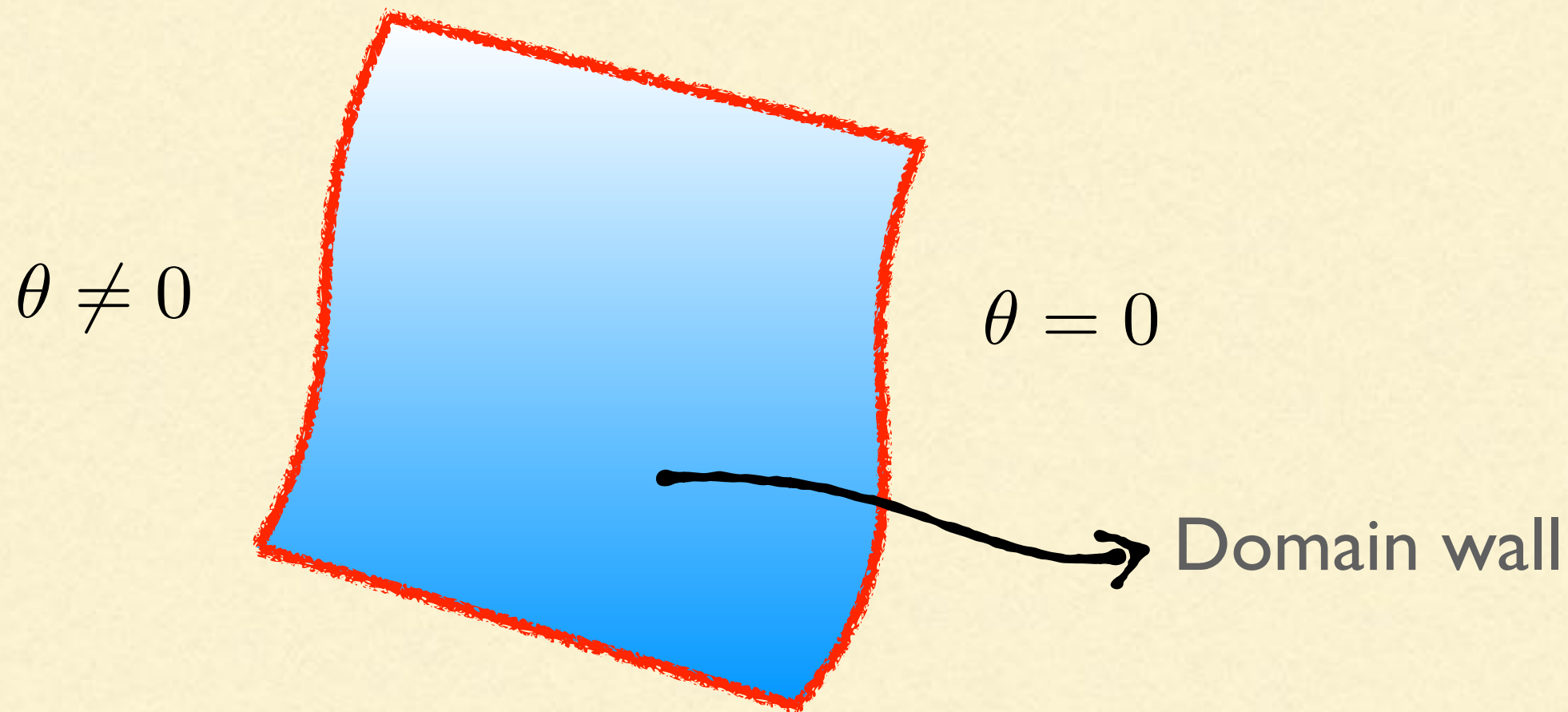
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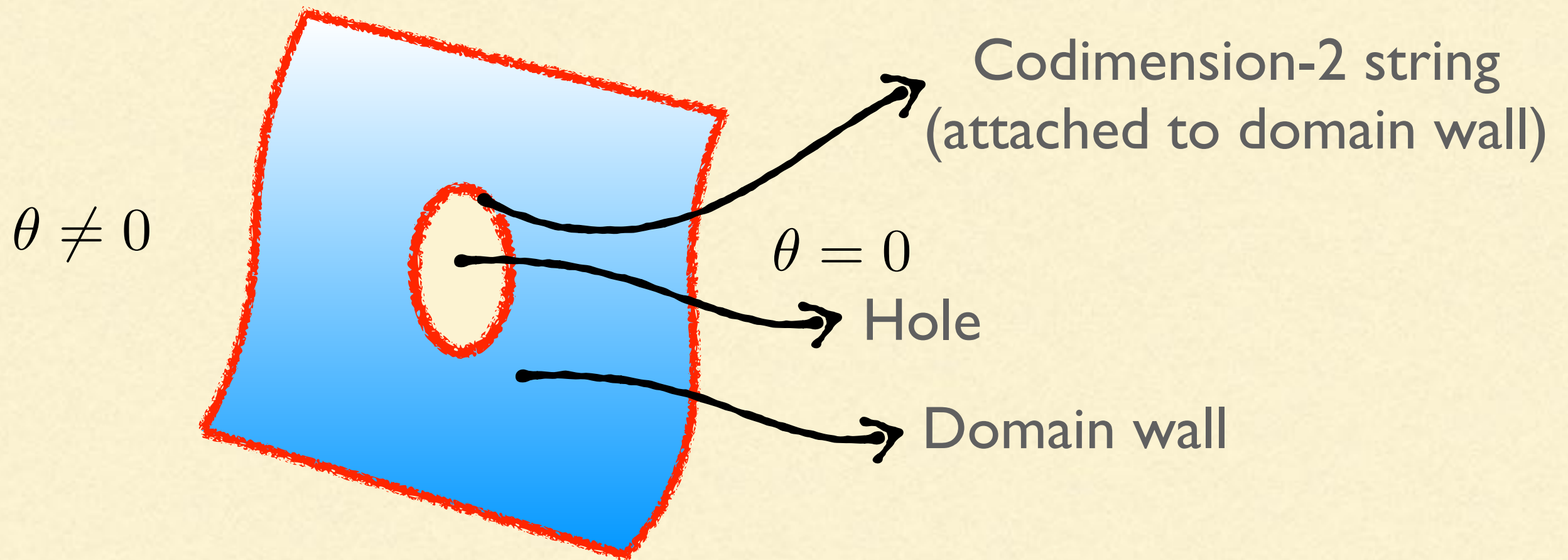
To make things more interesting, let me
end with a possibly **controversial**
remark that follows from applying the
Cobordism Conjecture to these ideas.



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However, when the two vacua are physically equivalent, the domain wall must be able to nucleate **holes**

(Think Dvali-Kaloper-Sorbo domain walls)

This means that the domain walls interpolating

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Here's the controversial claim:

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String Theory,

**[Witten '98, Sethi '12, Bergman '15, Parra de Freitas-
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Thus, Cobordism Conjecture suggests that

**Real K-theory charge is not conserved
in type I string theory**



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Miguel has gone full crackpot”**

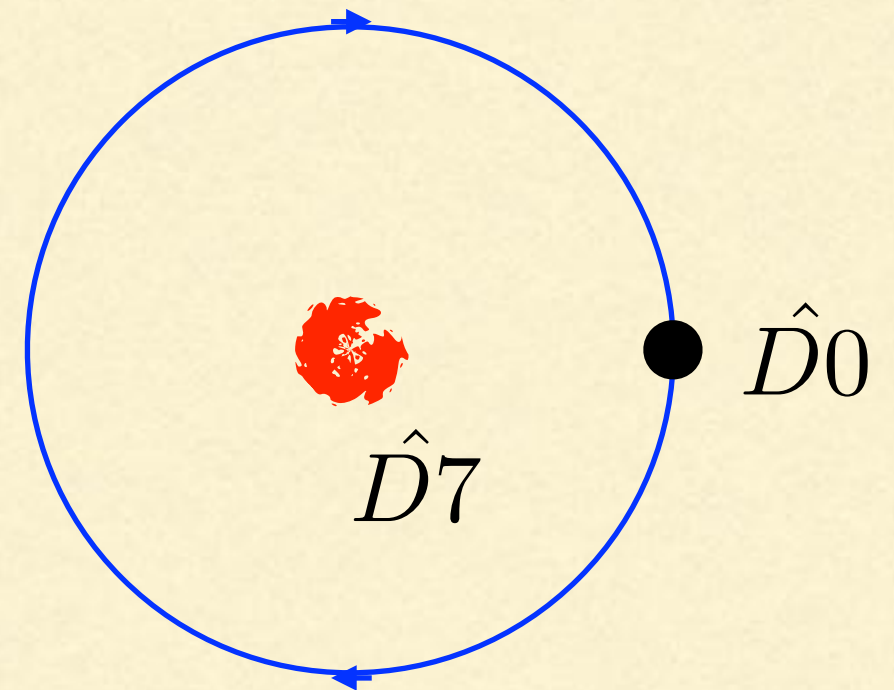


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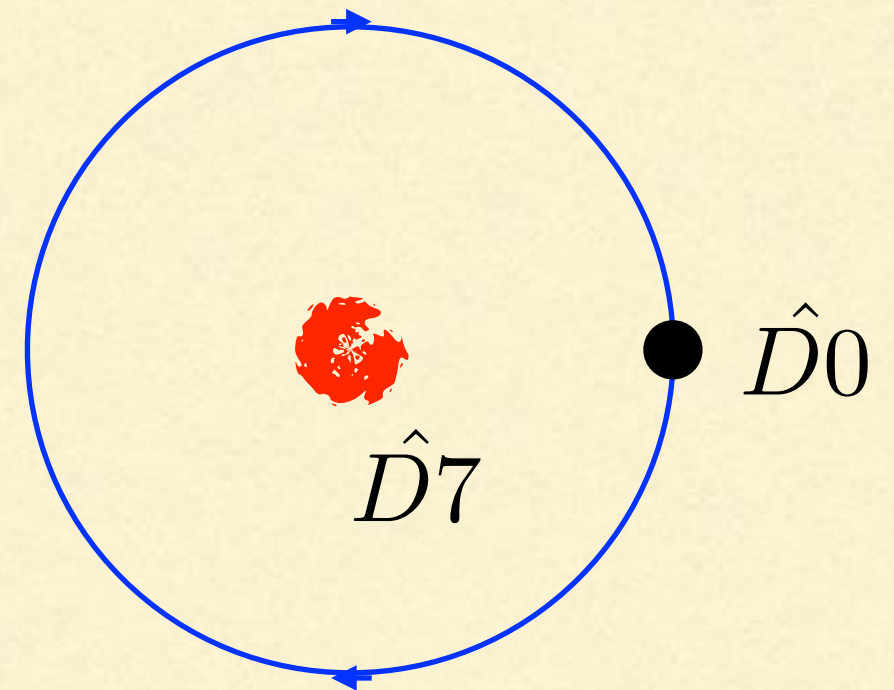


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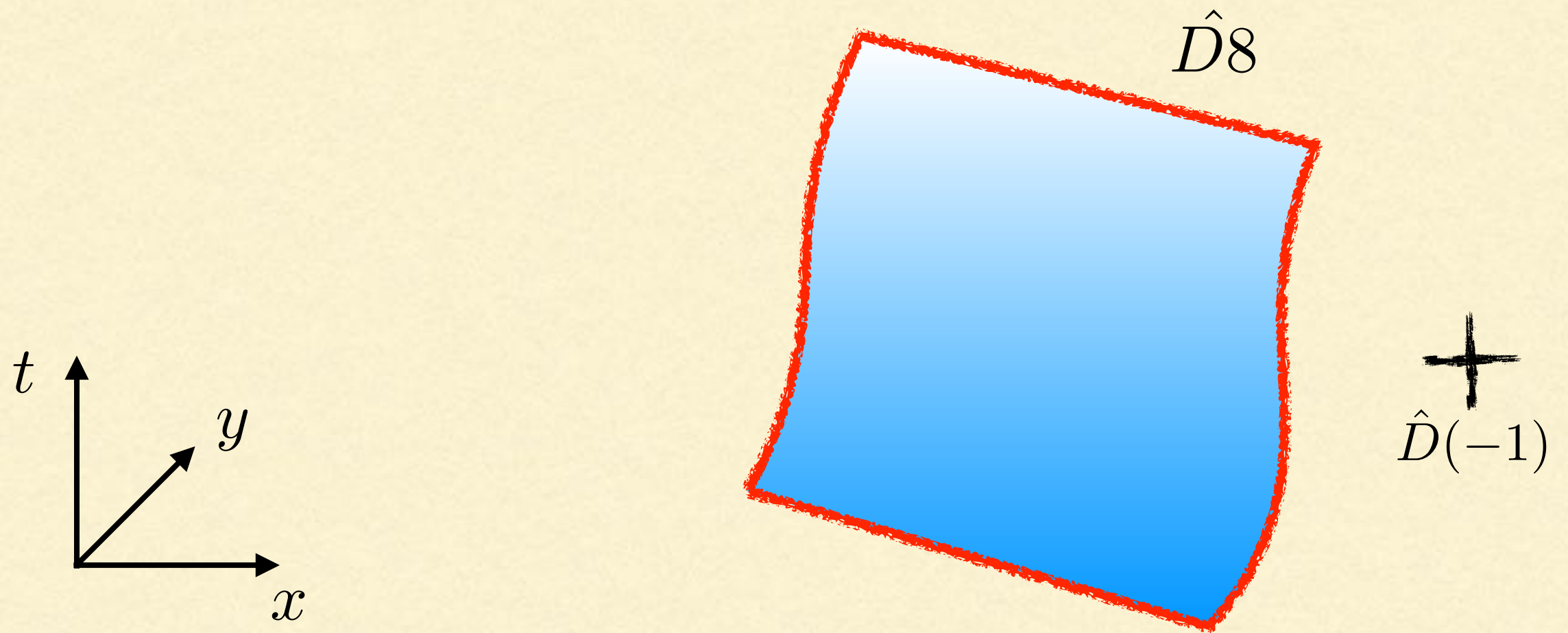
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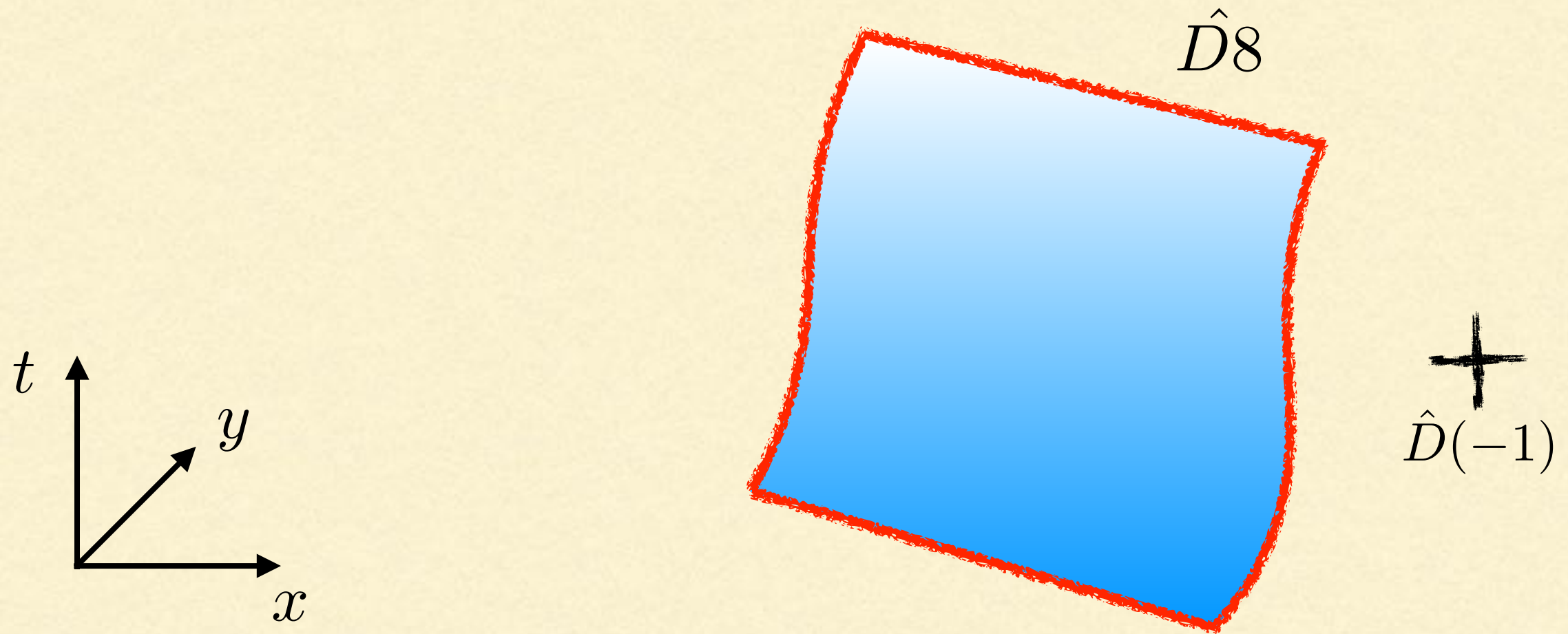


I **agree** with that argument, of course! The KO story is unchanged for any brane of codimension > 1 .



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This is not a physical object you can move around.

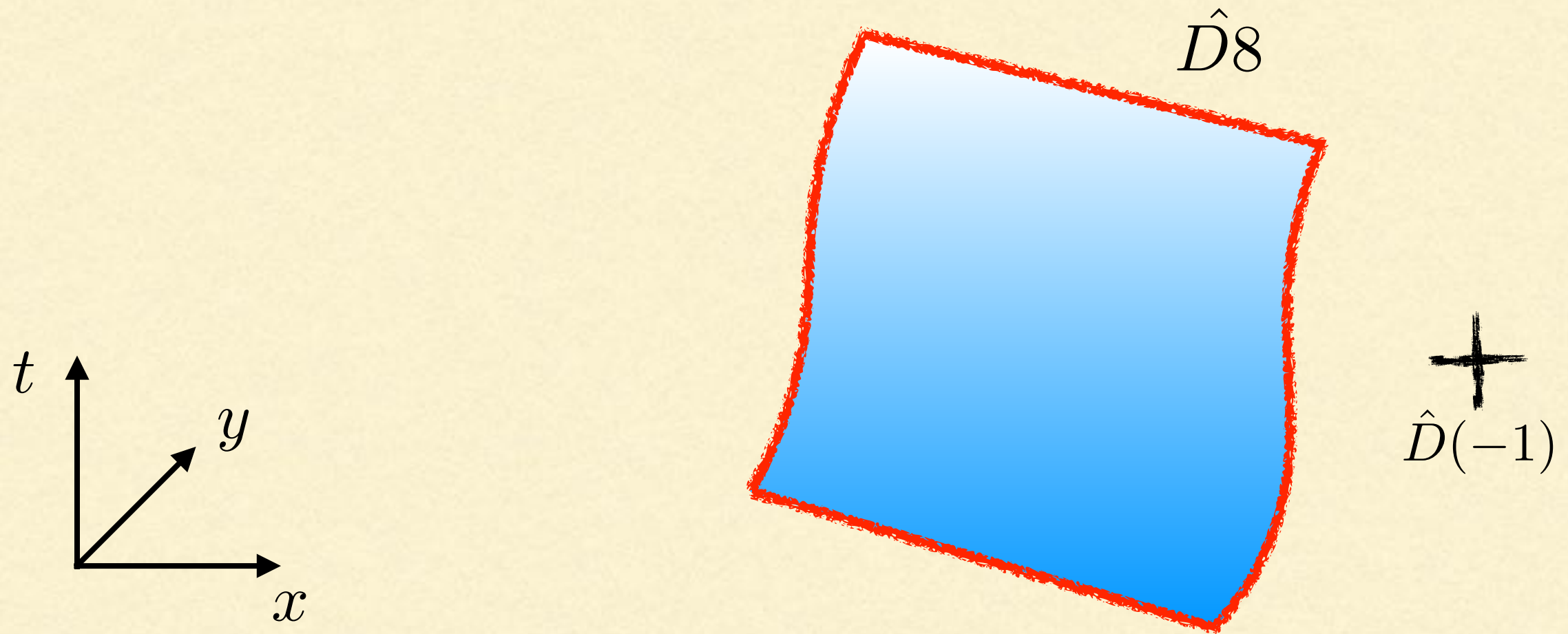


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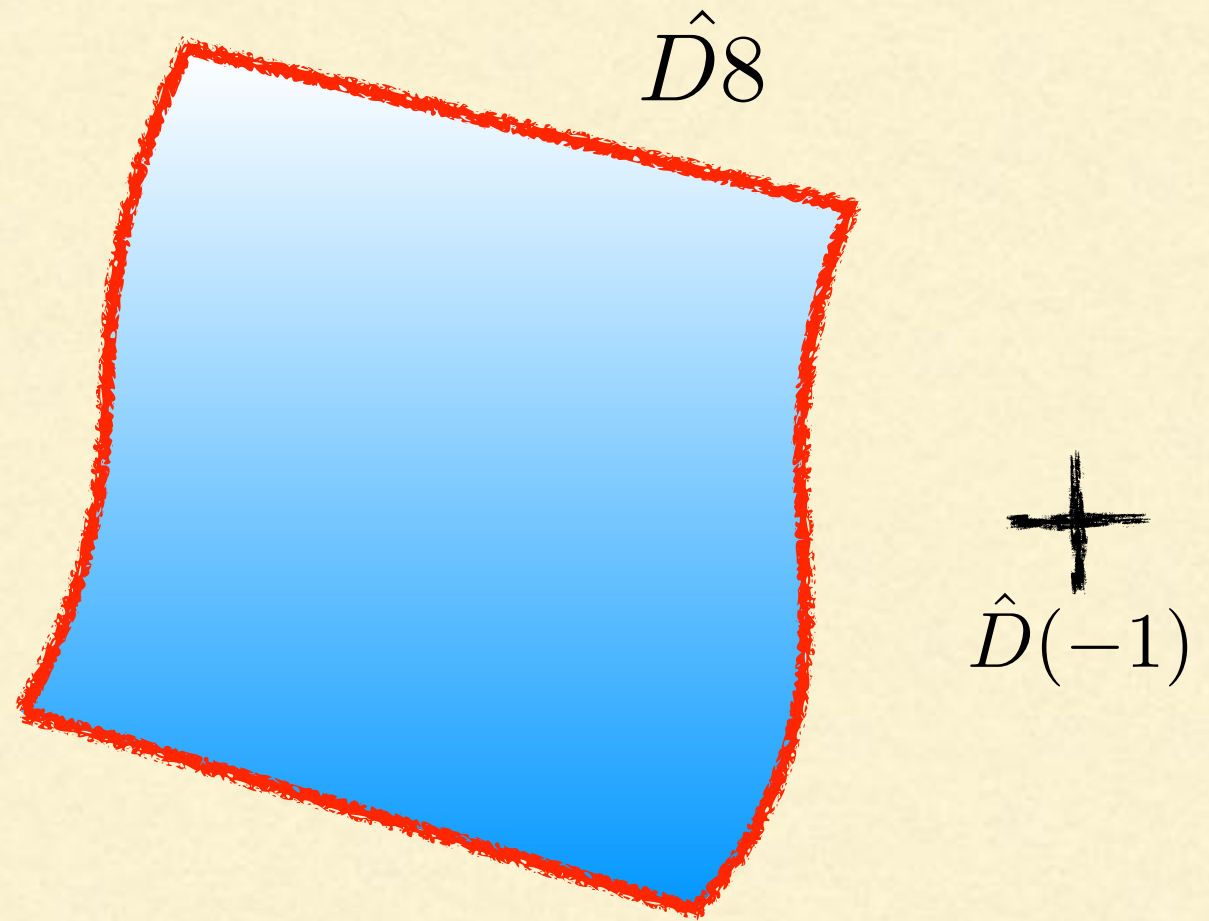
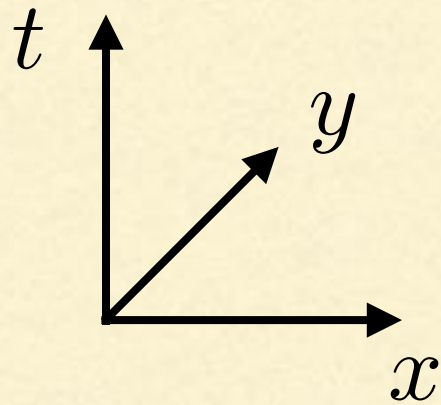
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E. Torres
Swampland
Seminar
(2025)
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...and beyond?

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