Effective theories with symmetric moduli spaces : the landscape and the swampland

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CEA / Saclay

Work in collaboration with
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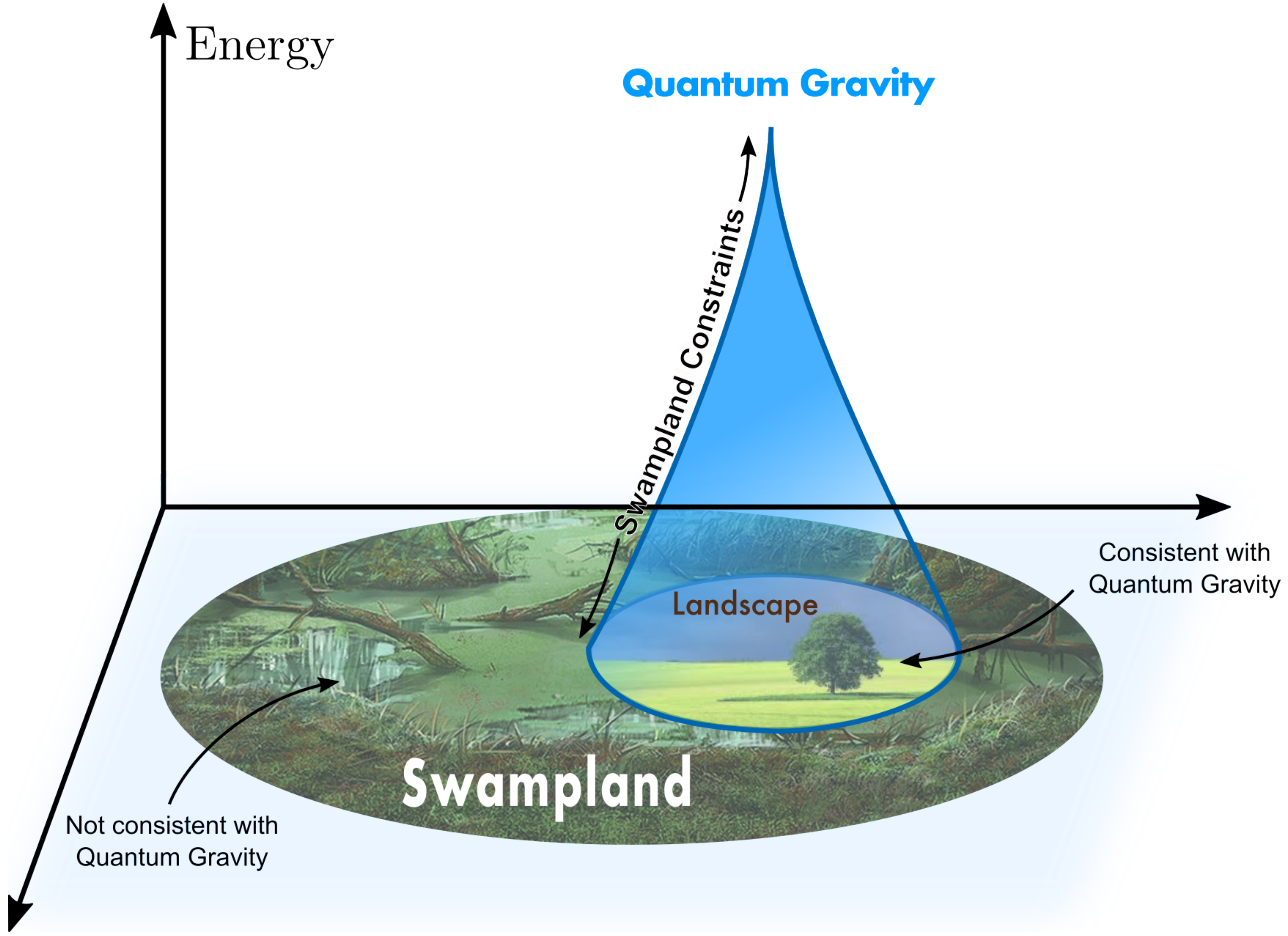
2508.18401

Strings and Geometry — May, 2026

Introduction and spoiler



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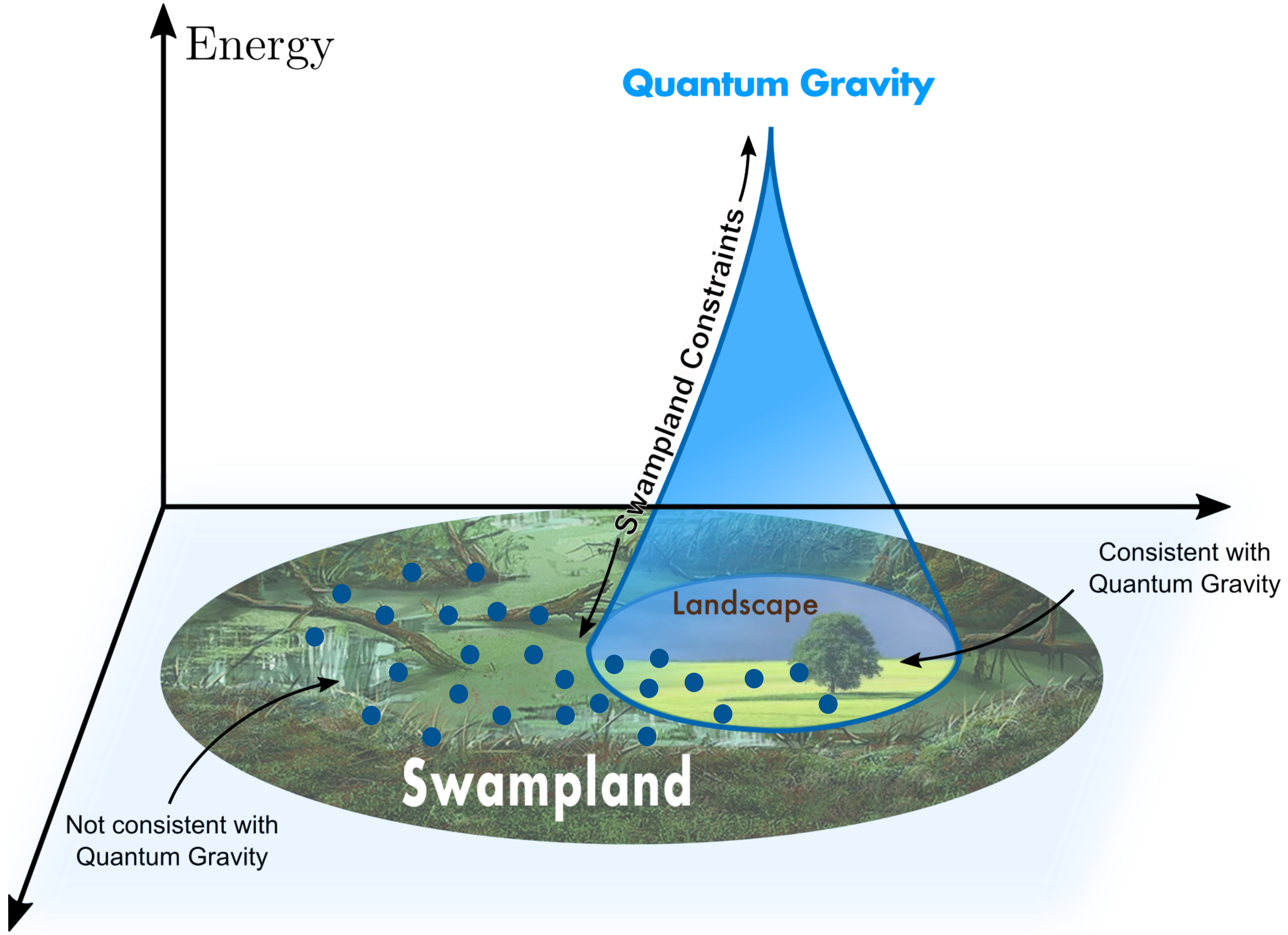

- Theories with symmetric moduli spaces

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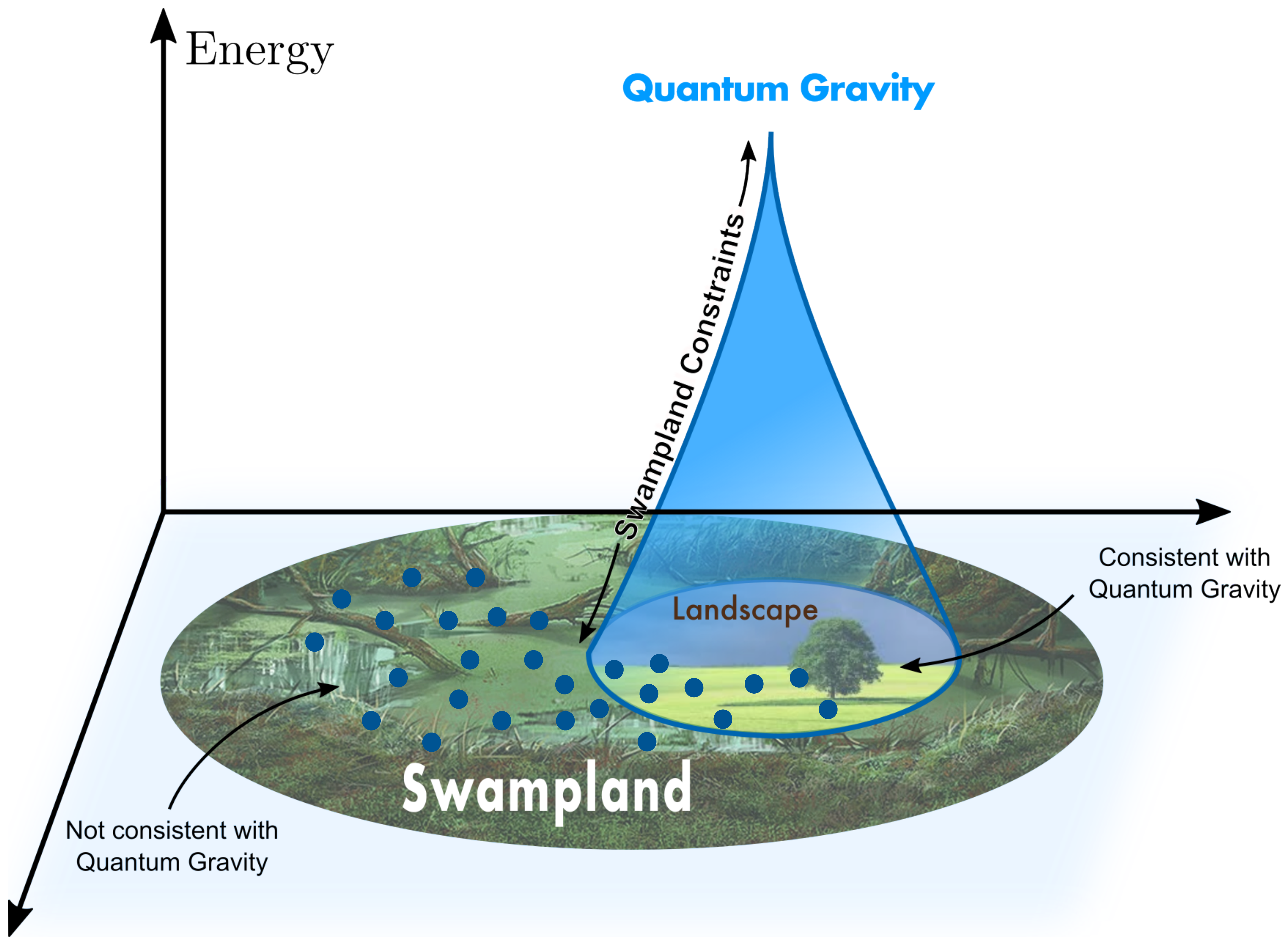

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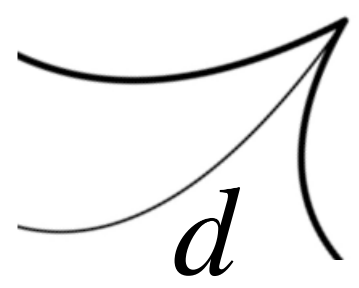
Swampland Distance Conjecture

- $m \sim e^{-\alpha\, d}\, n$



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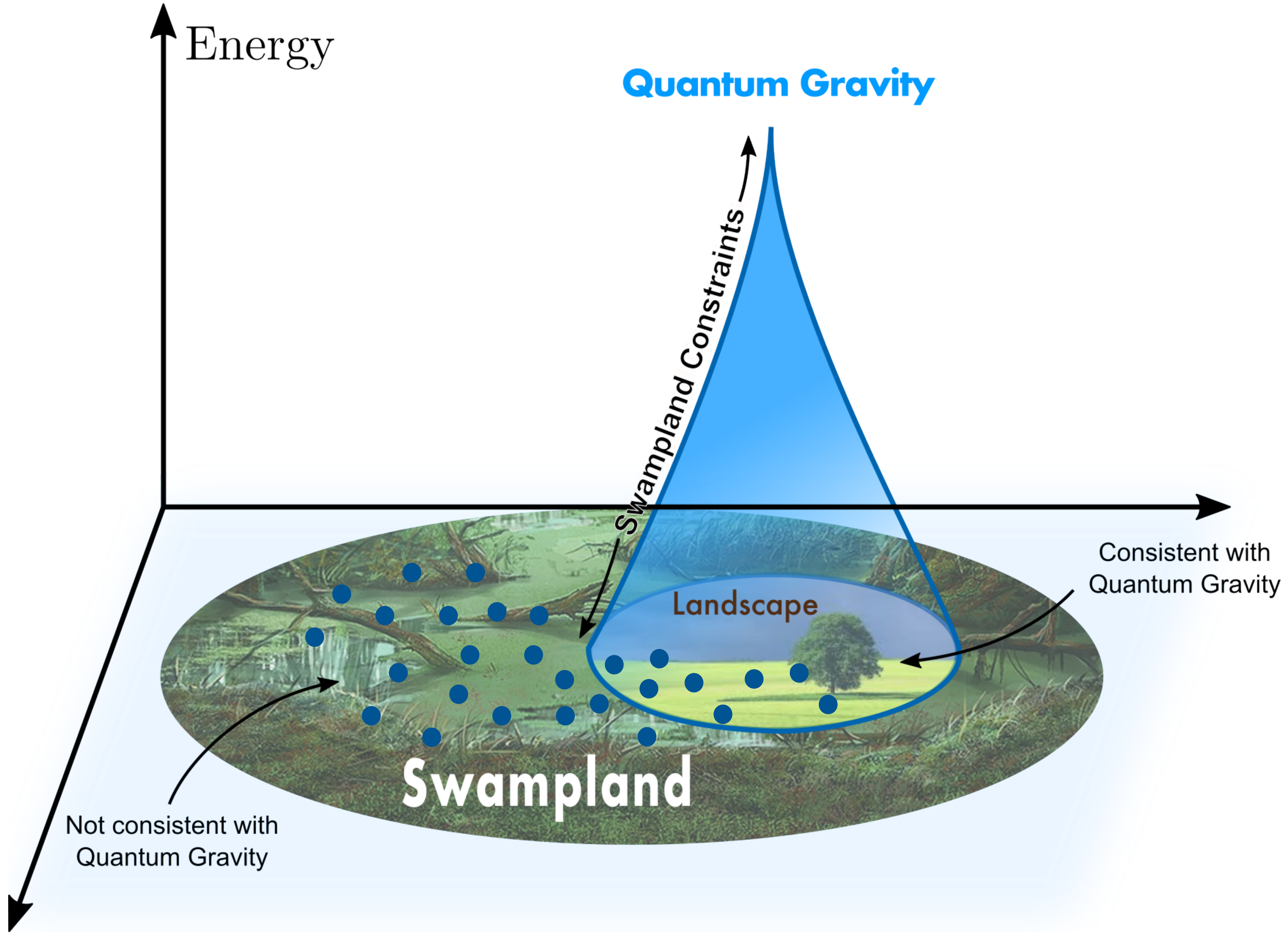

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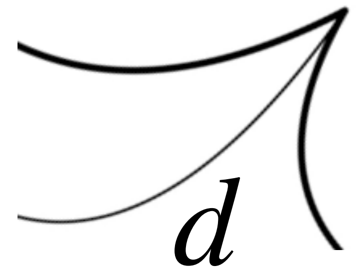

Prove under mild assumptions

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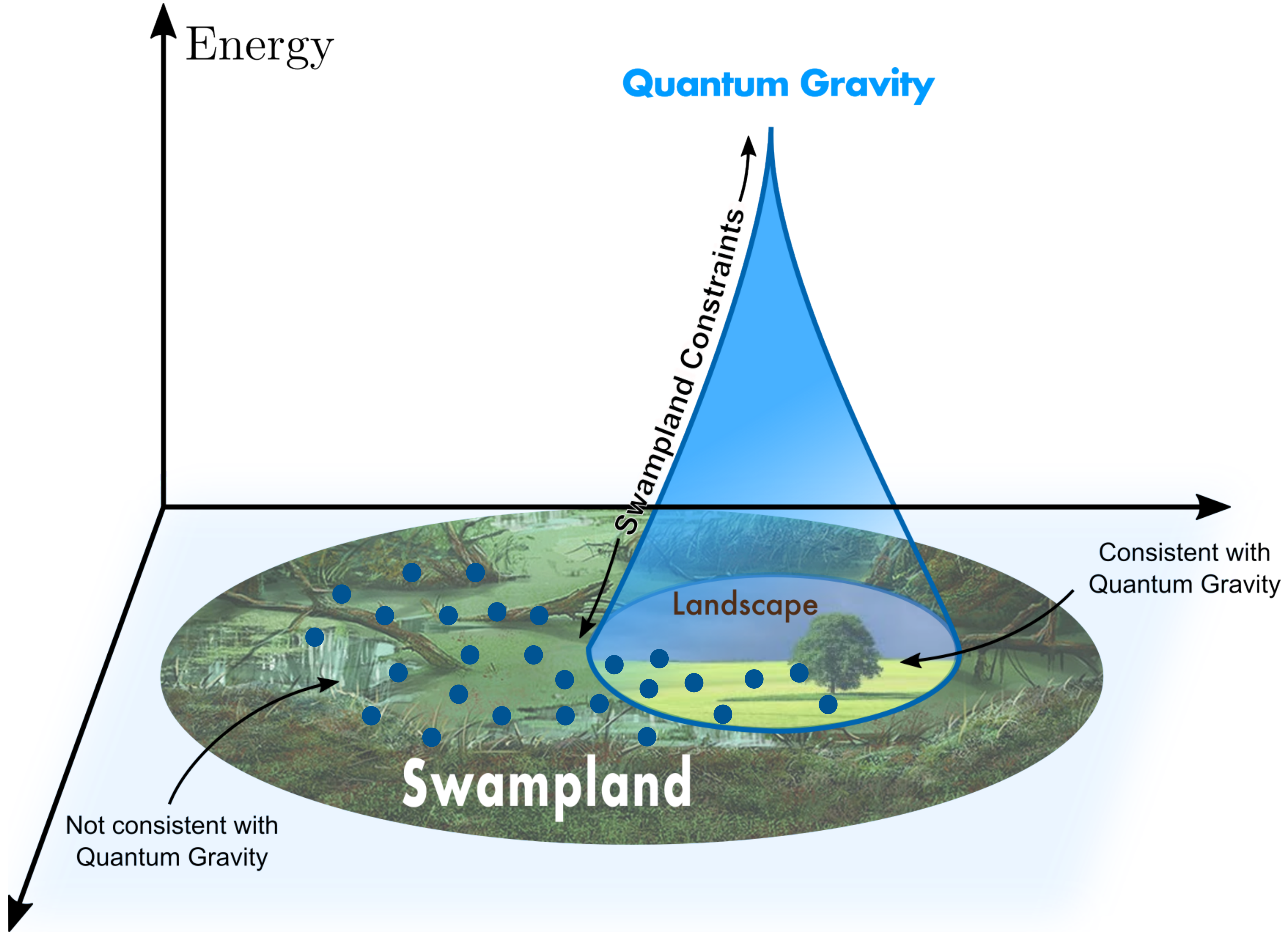

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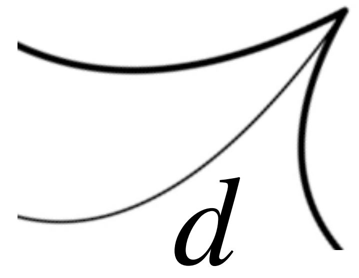

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"Emergent string conjecture rates"

- $\alpha = \alpha_{\text{string}}$ or α_{KK}

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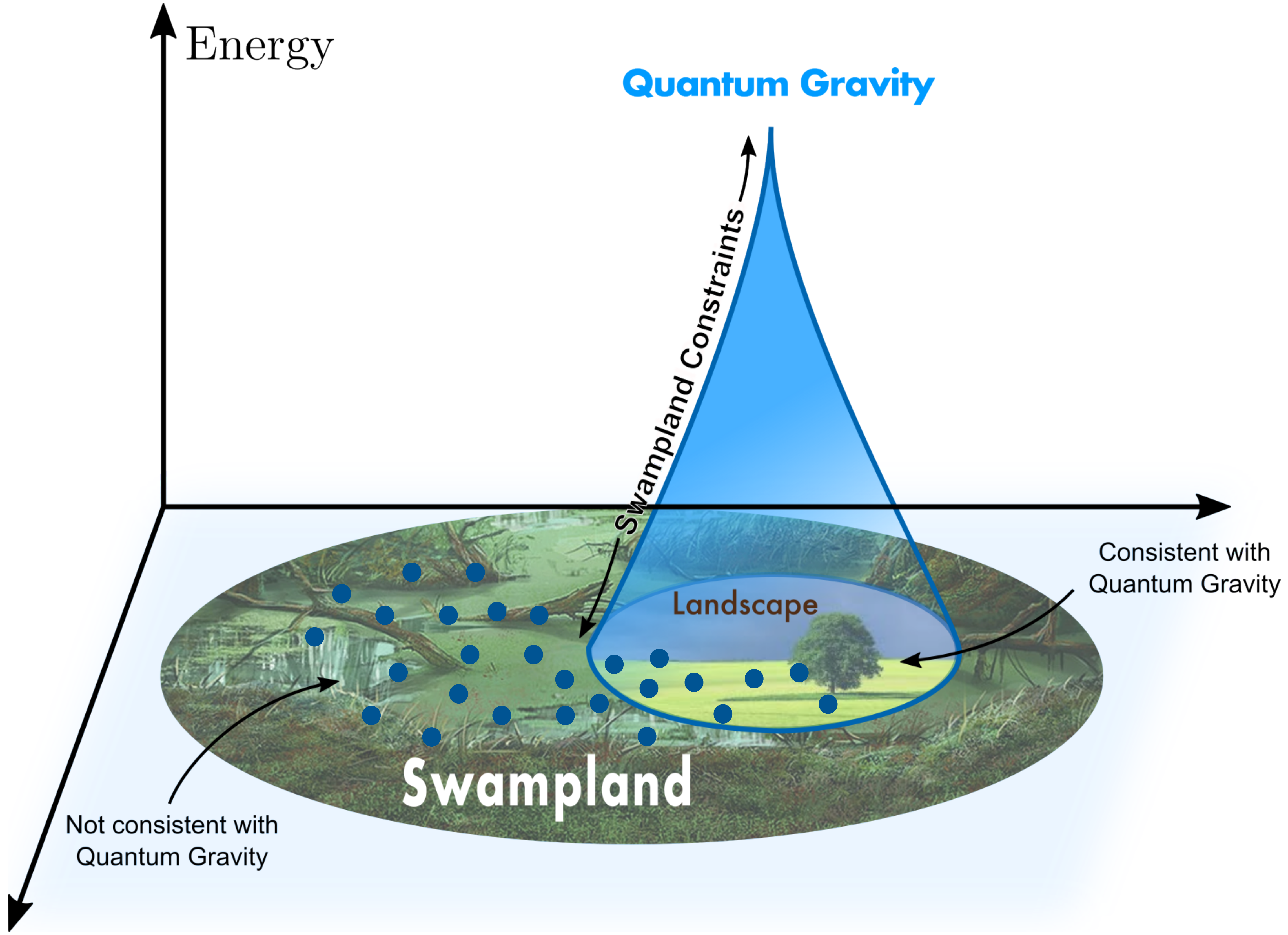

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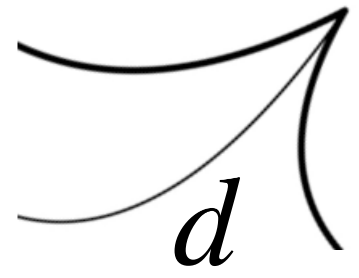

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Finite list of theories! (33)

Non-compact locally symmetric spaces

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(But may be much more general?? Stay tuned!)

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(e.g. $O(k, k, \mathbb{Z}) \backslash \mathbb{R} \times O(k, k, \mathbb{R}) / (O(k, \mathbb{R}) \times O(k, \mathbb{R}))$)

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- Geodesics
- Boundaries

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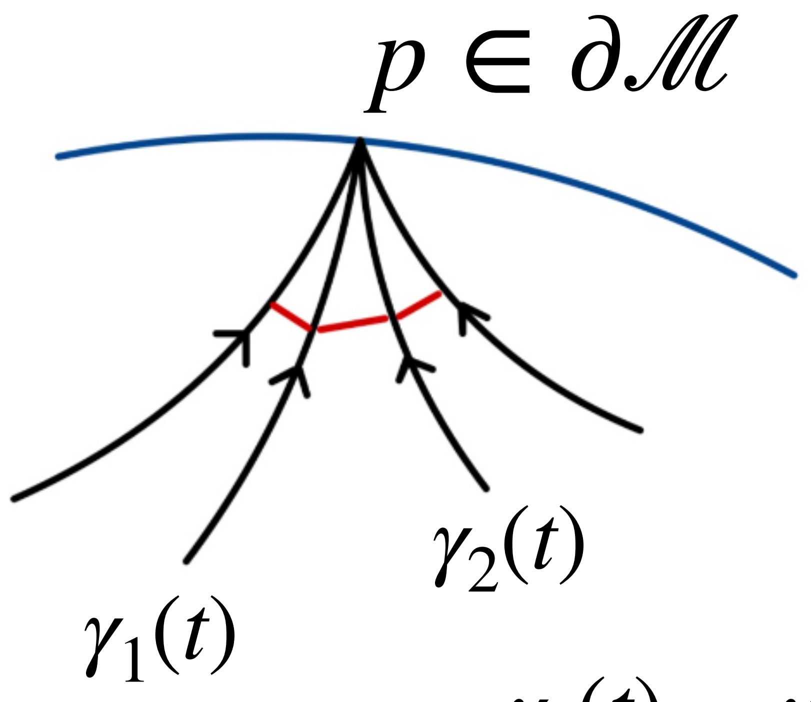

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finite distance for $t \to \infty$

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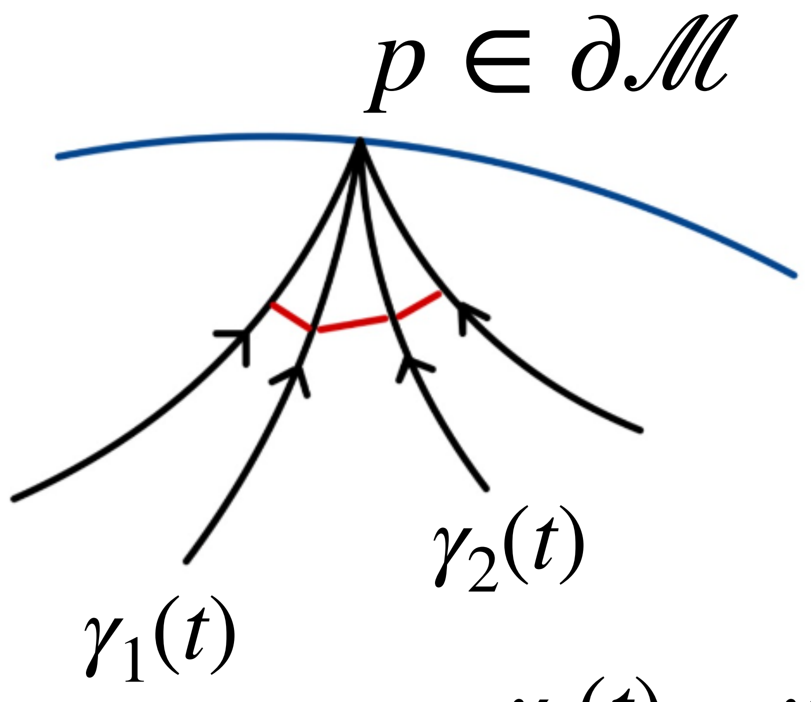

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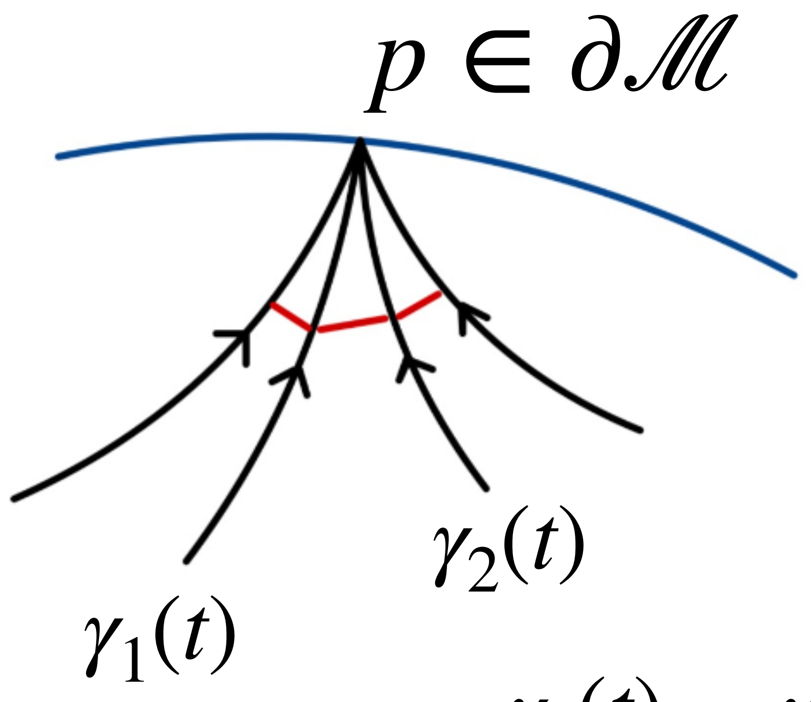

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- Boundaries

Defined by parabolic subgroups $P \subset G$ fixing the point

The duality group $\Gamma \Rightarrow$ locally symm spaces

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$\mathscr{M}$ locally symmetric and compactifiable [Delgado, van de Heisteeg, Raman, Torres, Vafa '24]

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Bonus: the spectrum transforms in Γ representations $\Longrightarrow$ extend to G representation

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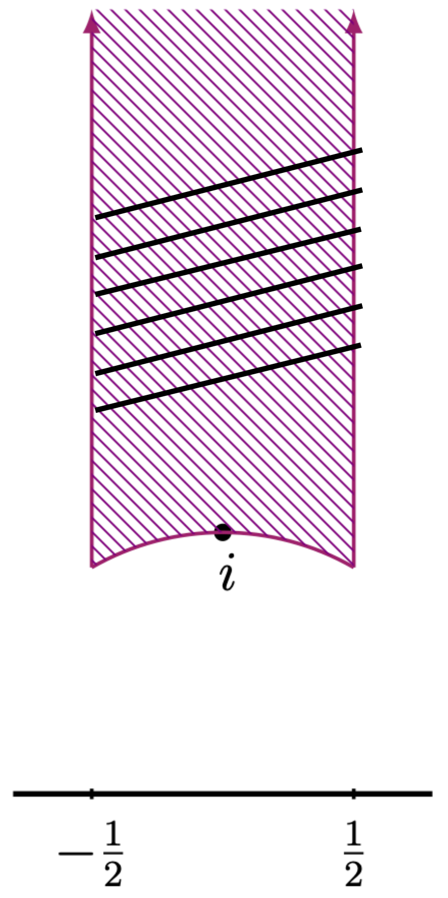
- The relevant parabolic subgroups are rational, $P(\mathbb{Q})$ $\Rightarrow$ e.g. B-field is rational



- Only these geodesics reach the boundary. The others have ergodic motion

Swampland Distance Conjecture

[Ooguri, Vafa '06]

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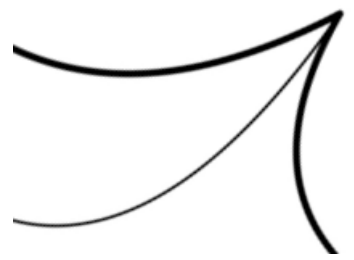

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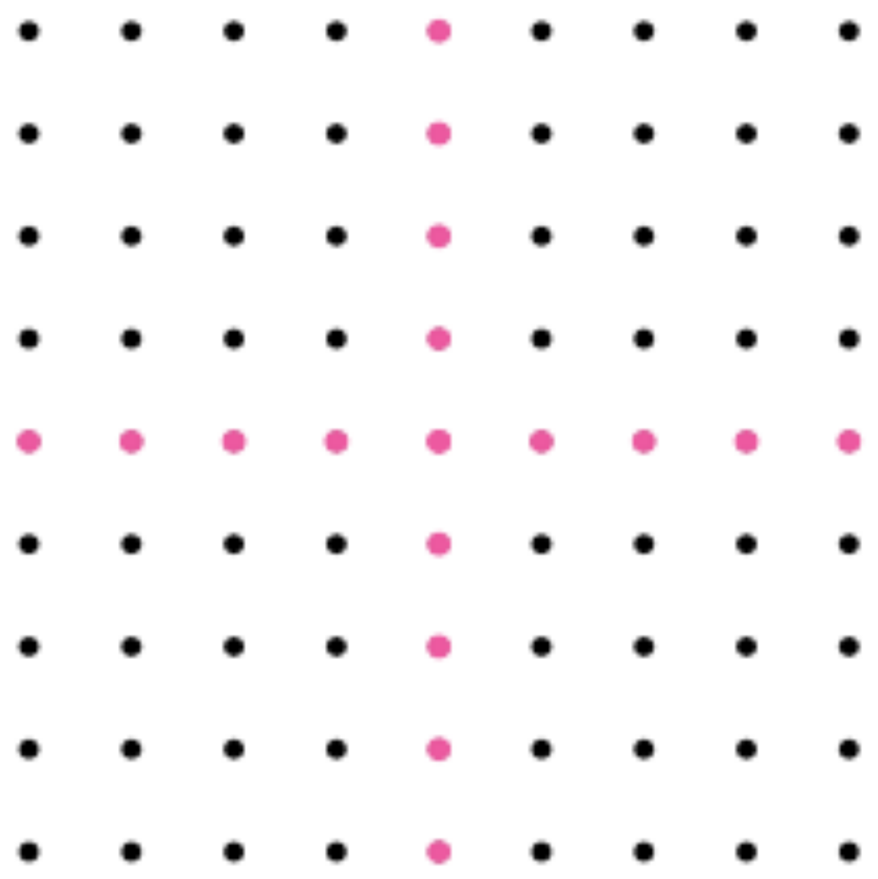
Spectrum space of states in rep ρ of G

rep ρ acts on $\mathbb{R}^N$
lattice $L \simeq \mathbb{Z}^N$
Π invt under Γ



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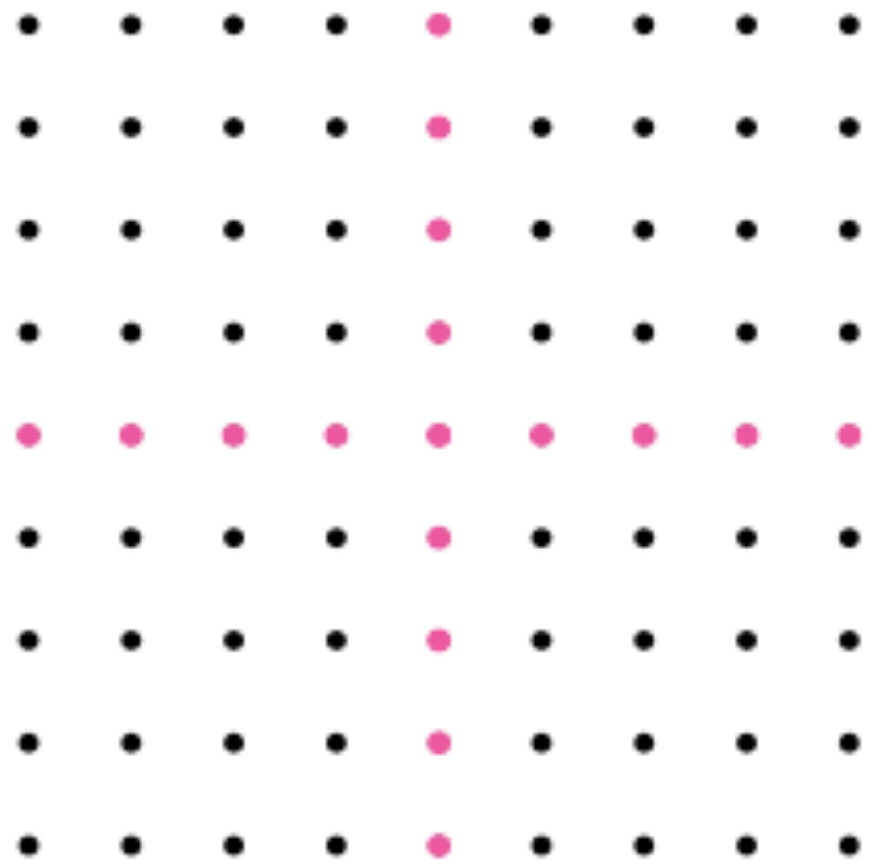

- **Mass** of the states: invariant function of the charges and the moduli (depends on rep ρ)

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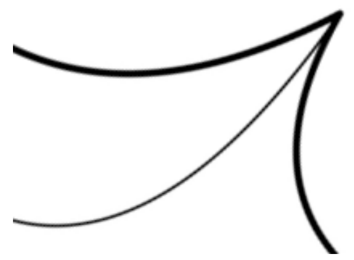

Spectrum

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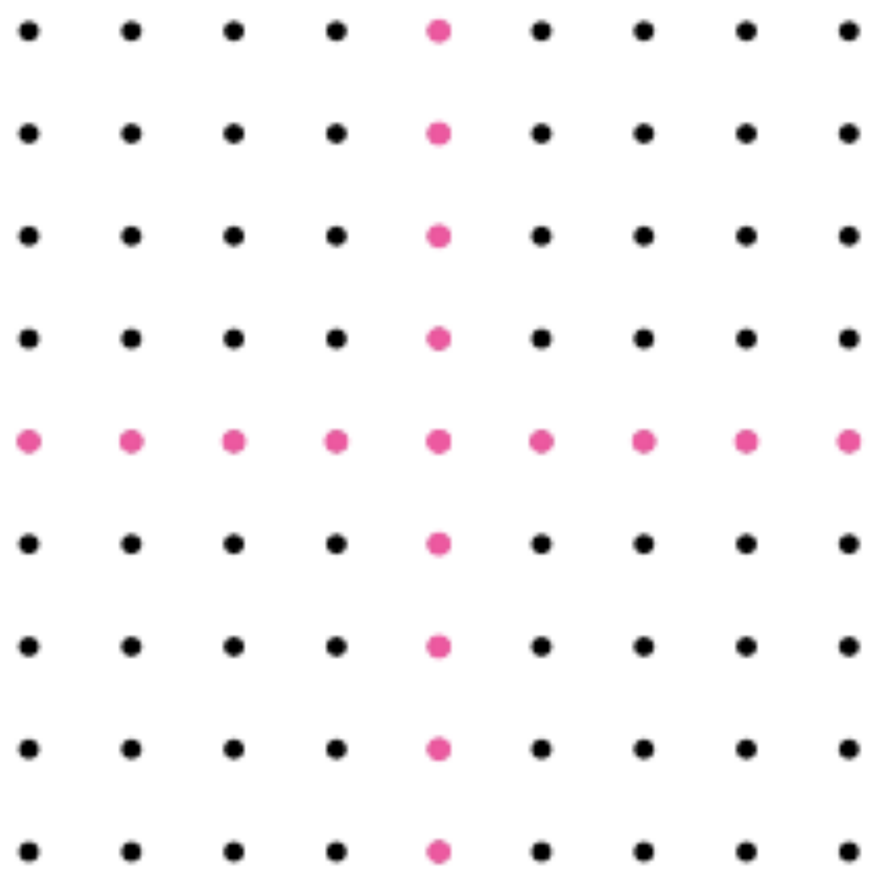

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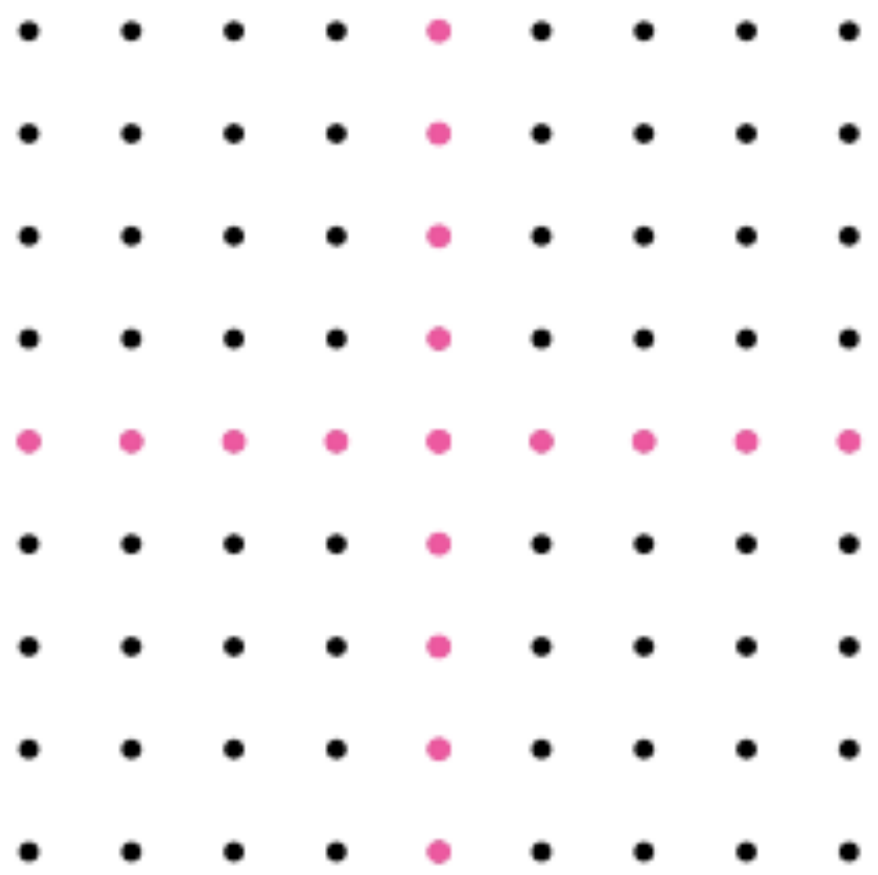

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In general: $m^2(q,\phi) = f(P)$

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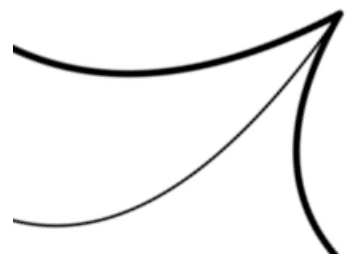

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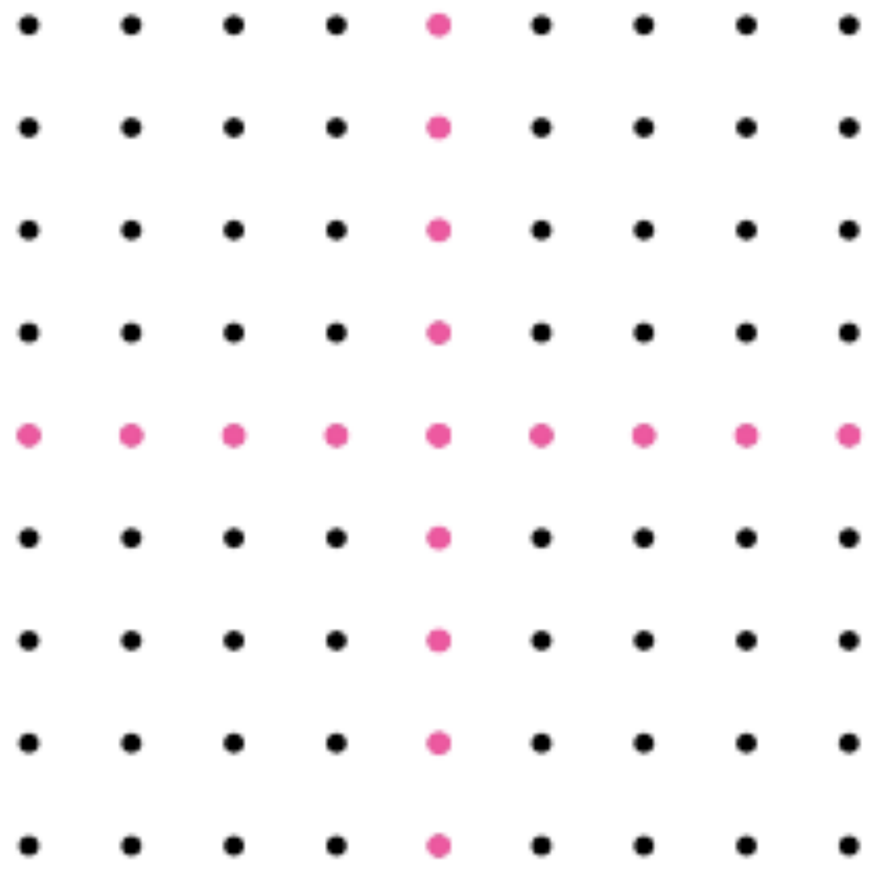

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assuming f analytic

In general: $m^2(q,\phi) = f(P) \approx \text{const} + P^2 + \mathcal{O}(P^4)$

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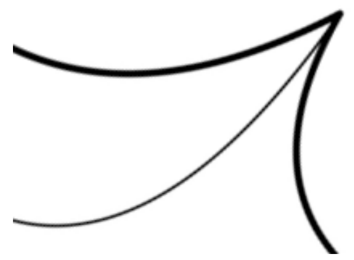

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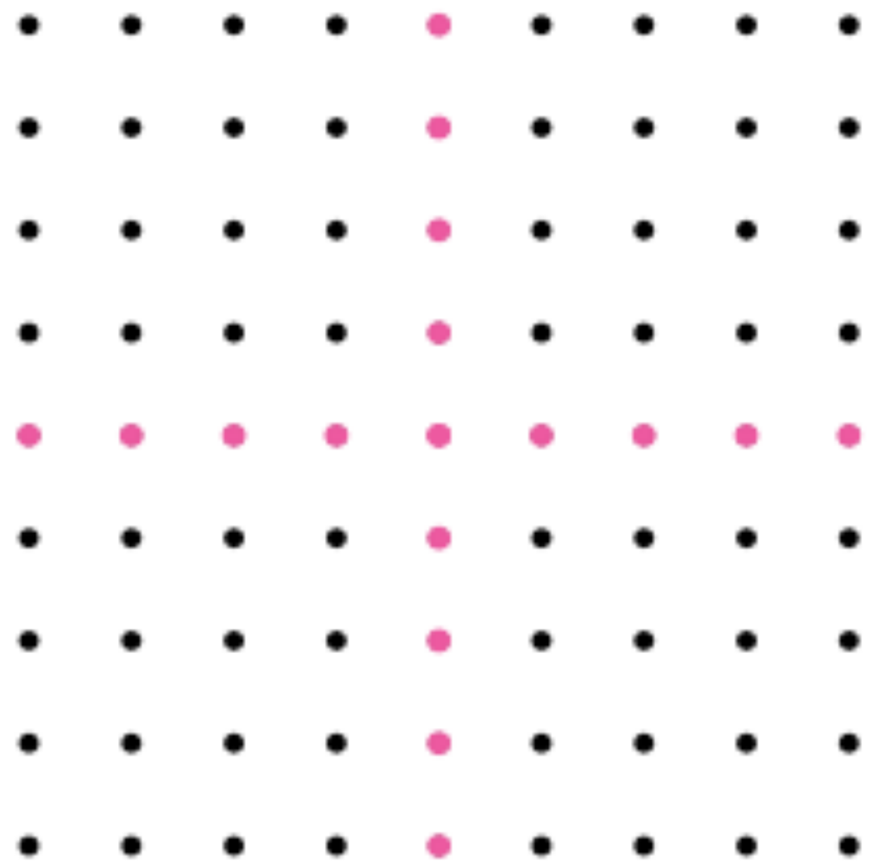

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In general: $m^2(q,\phi) = f(P) \;\approx \cancel{\text{const}} + P^2 + \mathscr{O}(P^4)$

assuming one state becomes massless

Mass

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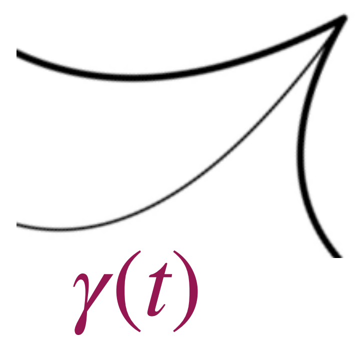

- Along geodesics $\gamma(t) = e^{Ht}$

$$P = \mathscr{E}(t)q$$

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$R_1(t)$

O(k,k)

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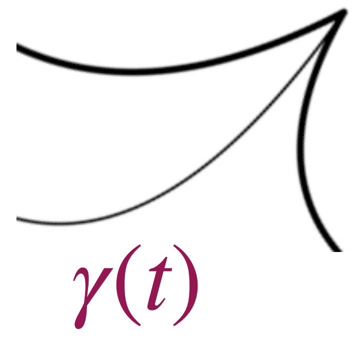

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e.g take

$$H = \text{diag}(-\lambda,0,\ldots,0,\lambda,0,\ldots,0) \qquad \langle H,H\rangle = 1 \Rightarrow \lambda = 1$$

$$q_{\omega_*} = (n_1,0,\ldots,0,0,\ldots,0) \Rightarrow \omega_* = (1,0,\ldots,0)$$

Proof of Swampland distance conjecture

- Mass of the fastest-decaying states

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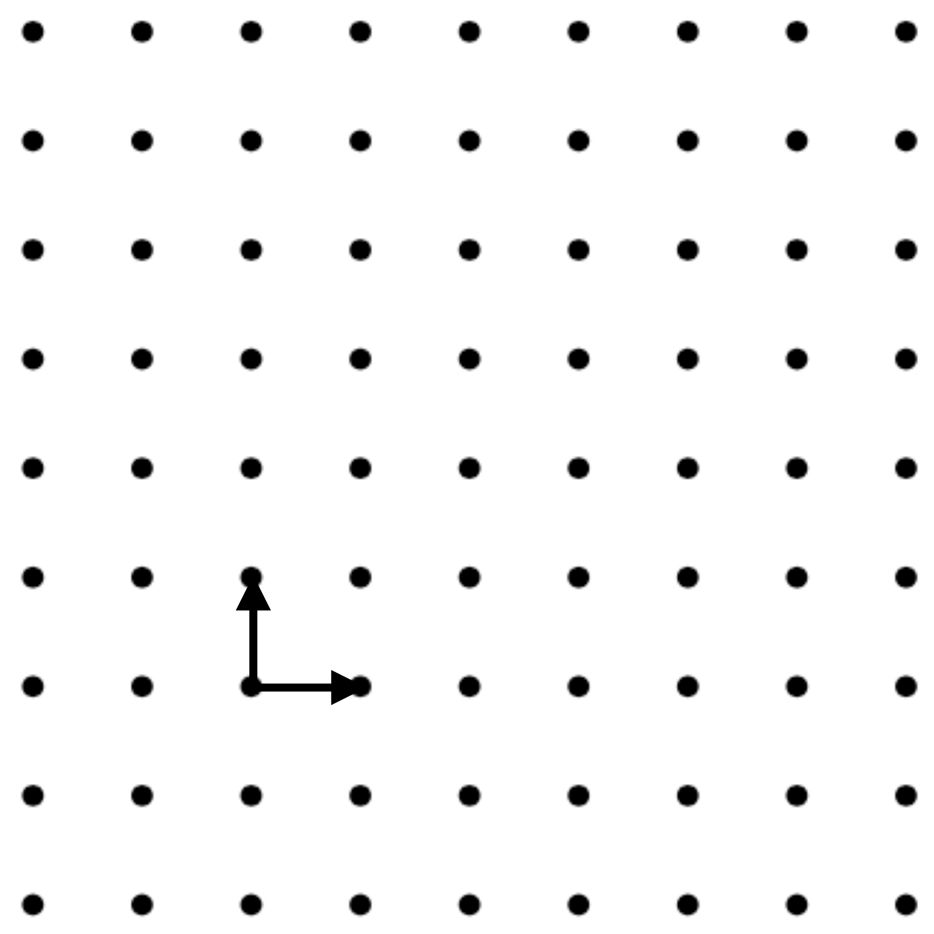

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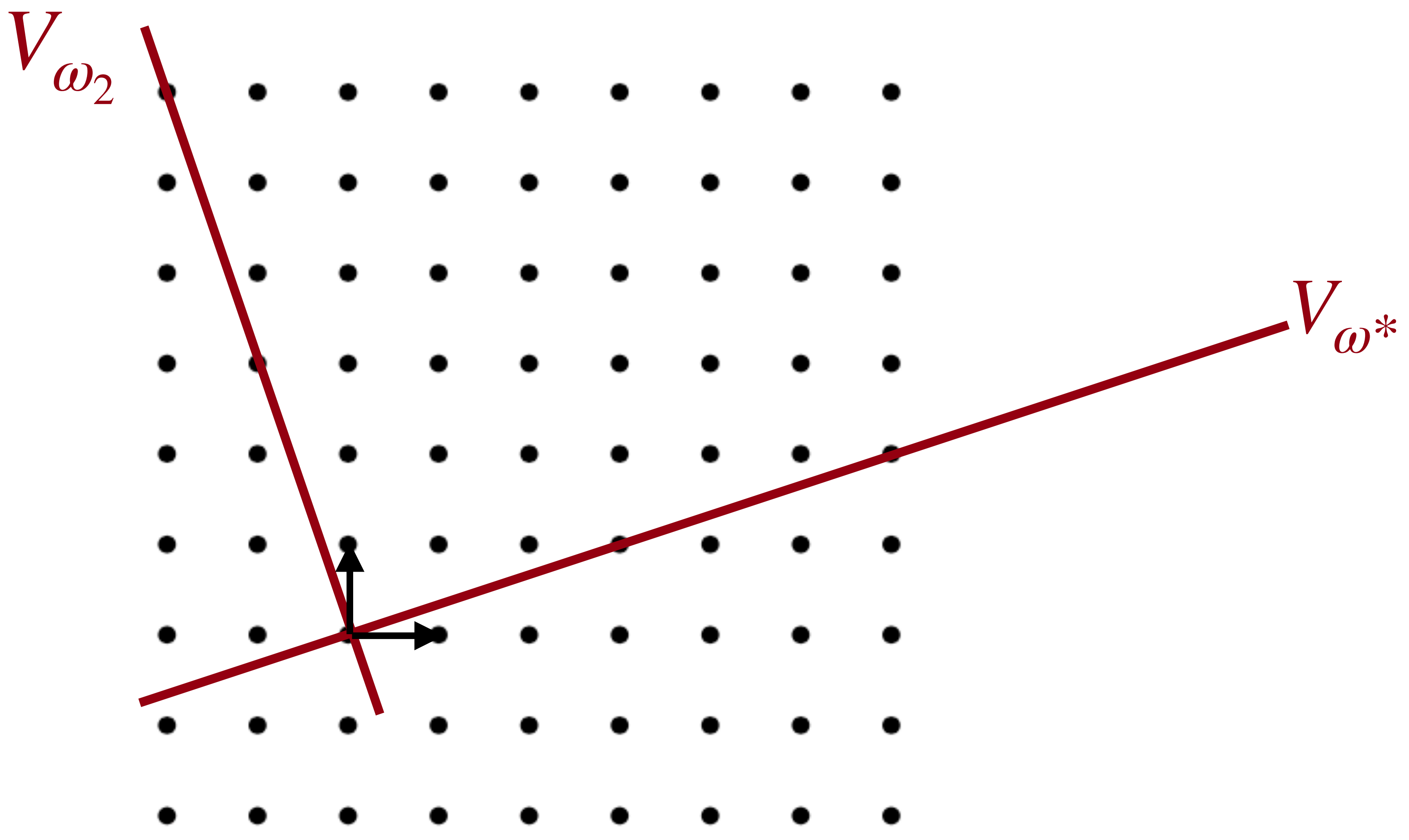

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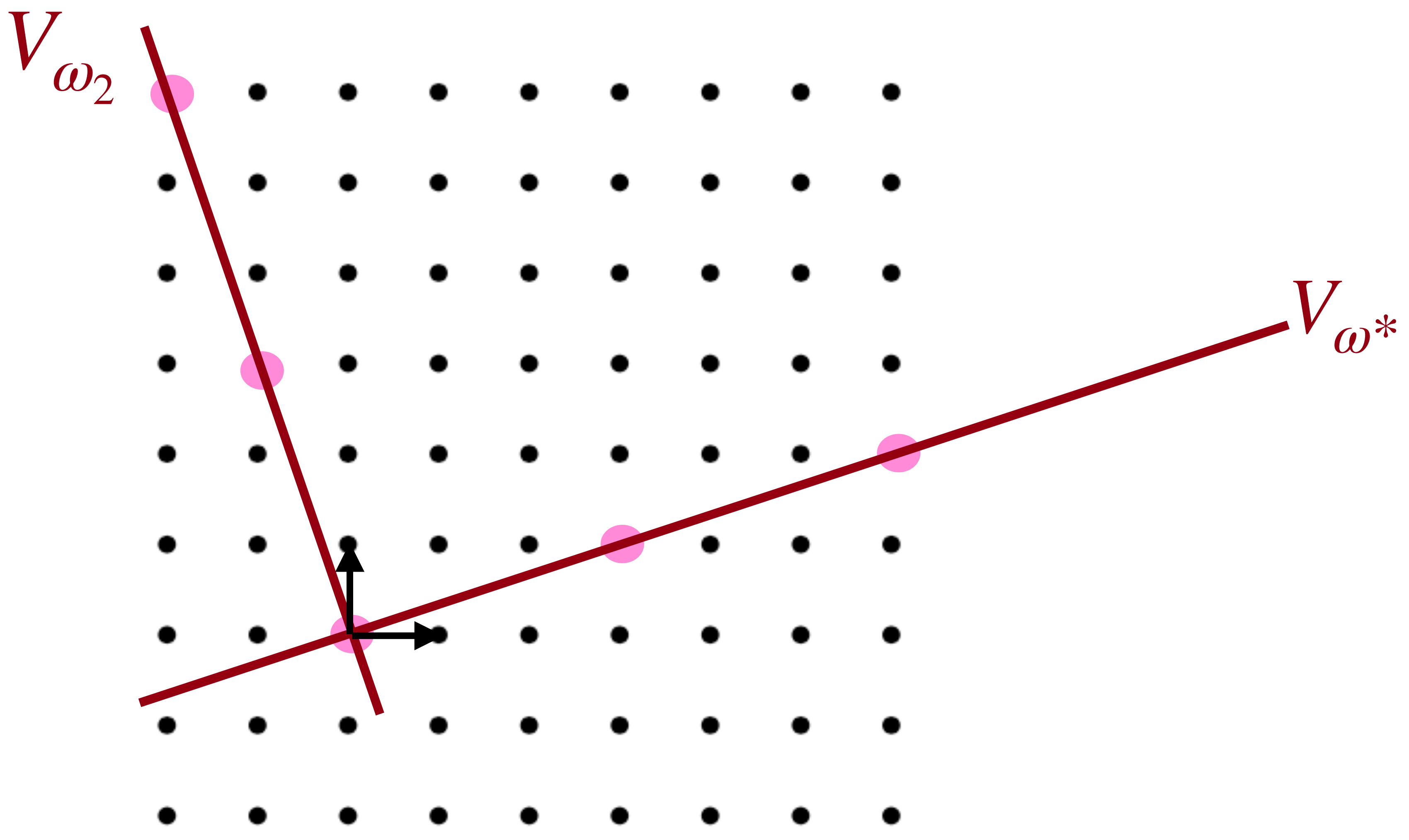

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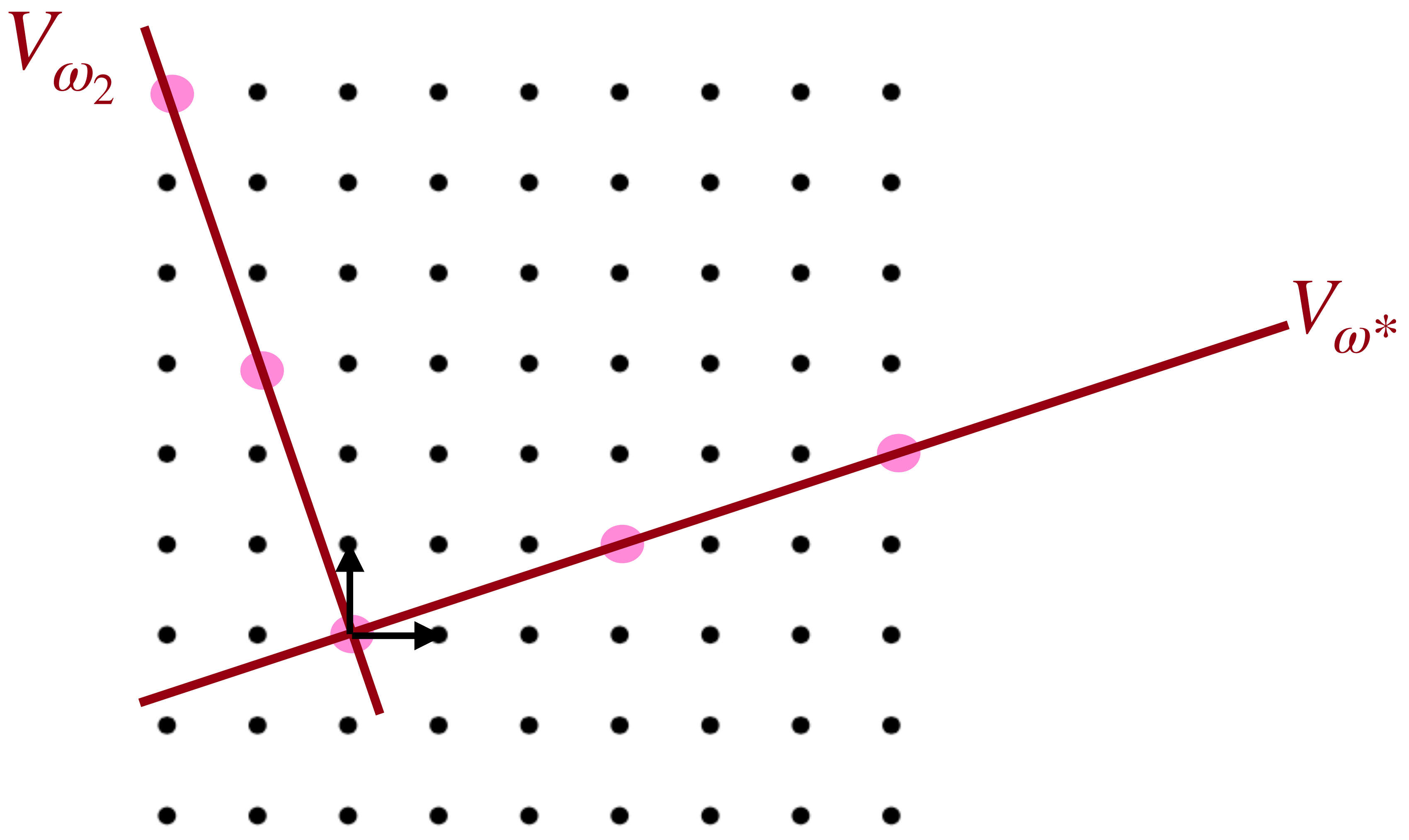

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Convex hull formulation: Set of α define a convex hull   Calderon Infante, Uranga, Valenzuela 20

Convex hull formulation for symmetric spaces

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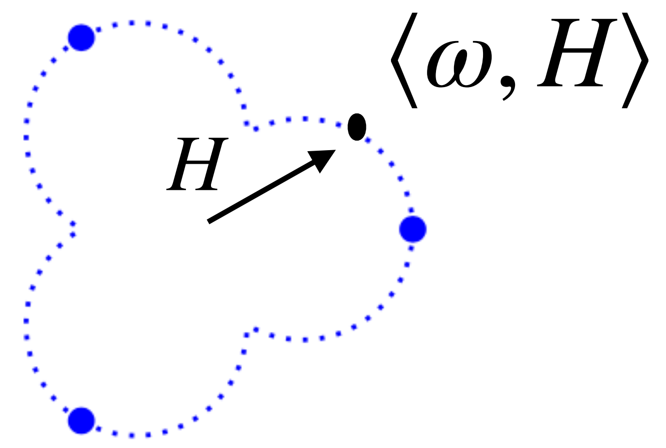

α - Hull

Calderon Infante, Uranga
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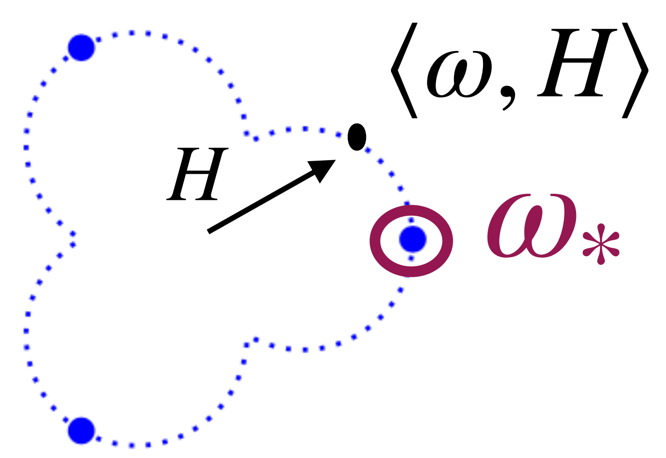

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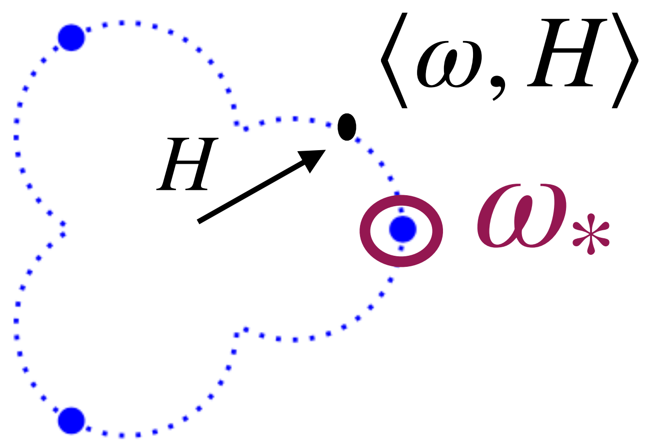

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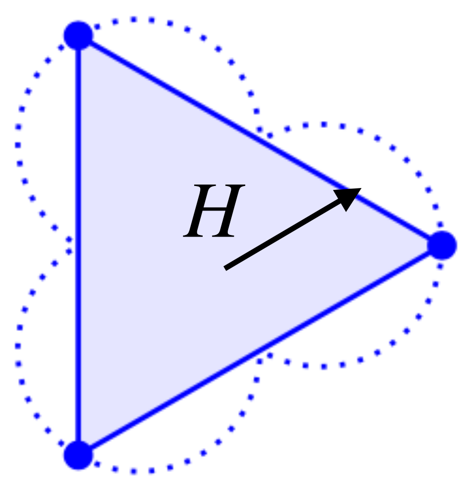

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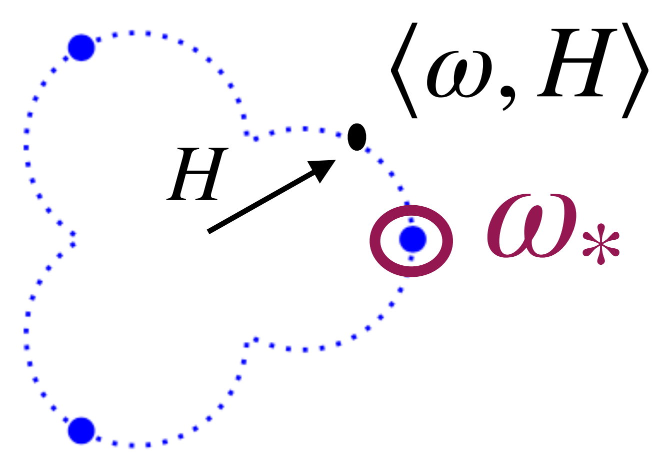

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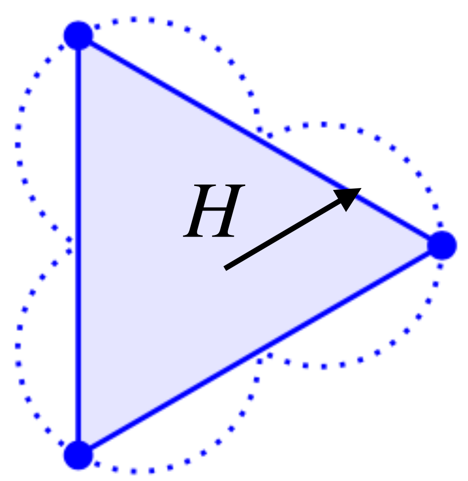

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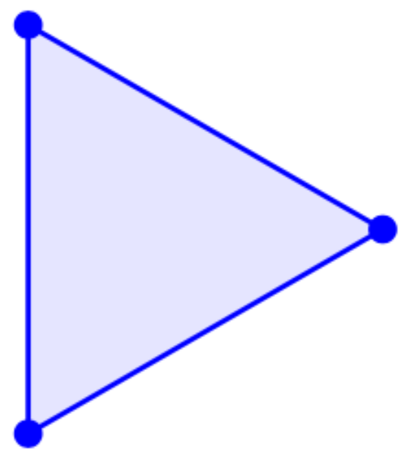

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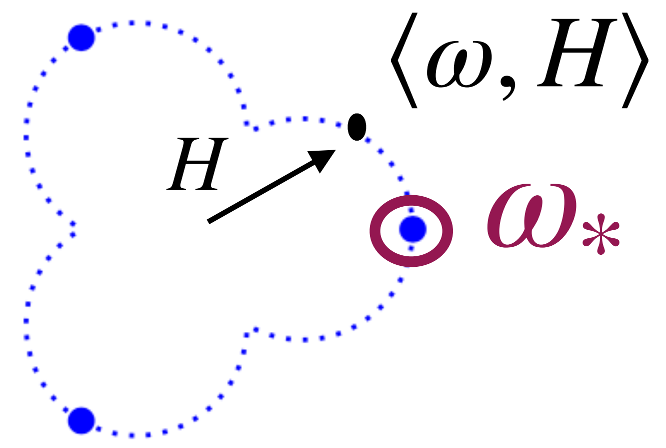

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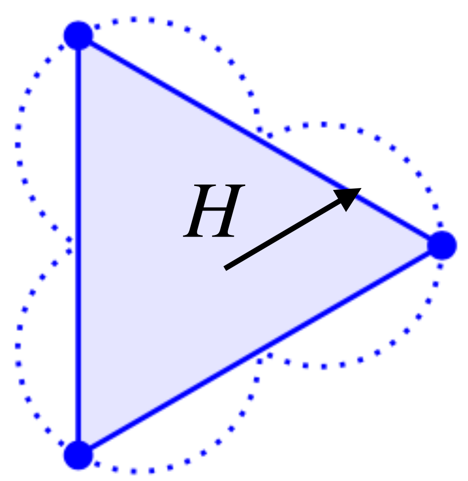

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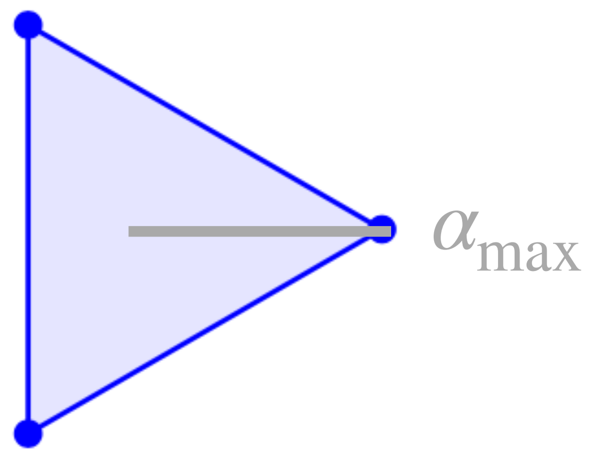

Convex hull: Weyl polytope of the **3**

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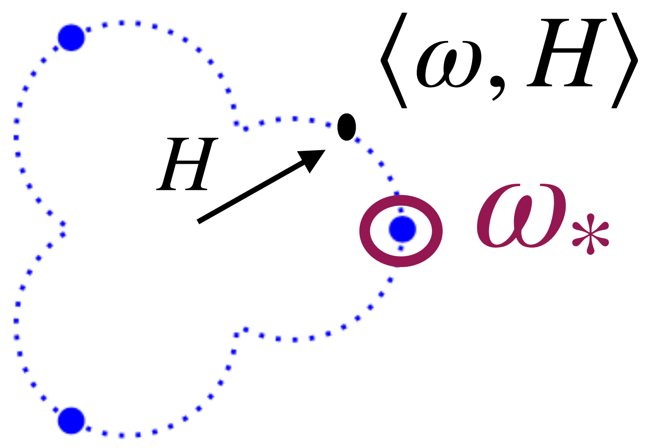

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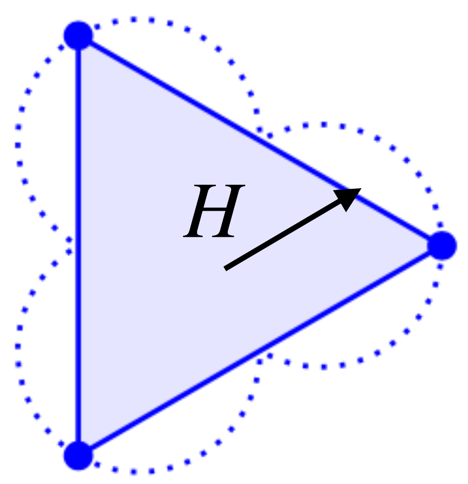

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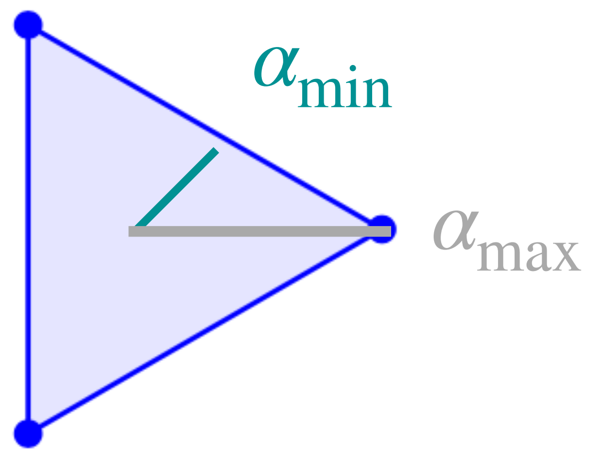

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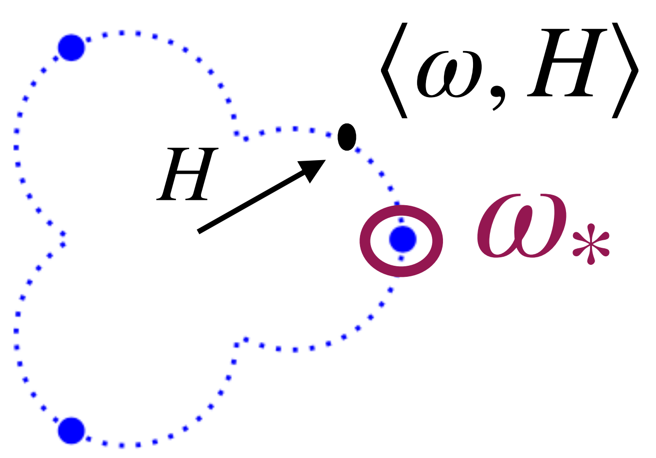

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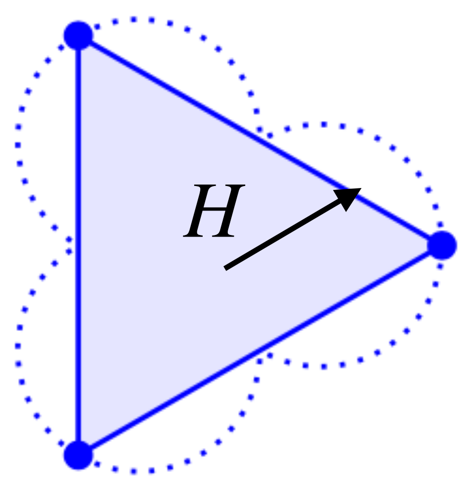

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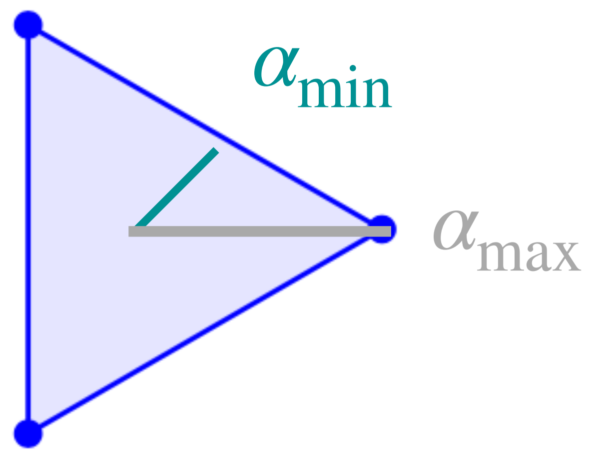

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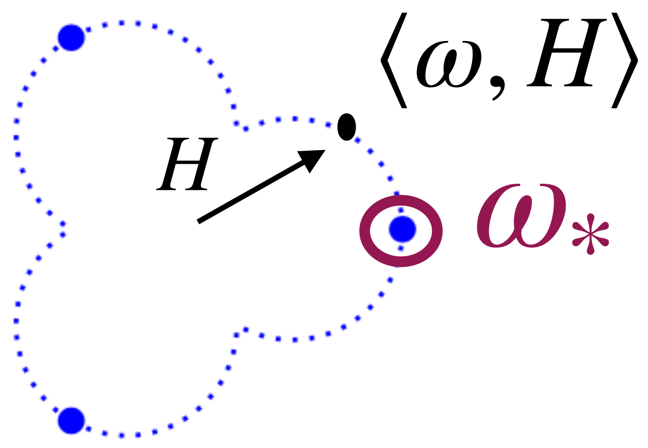

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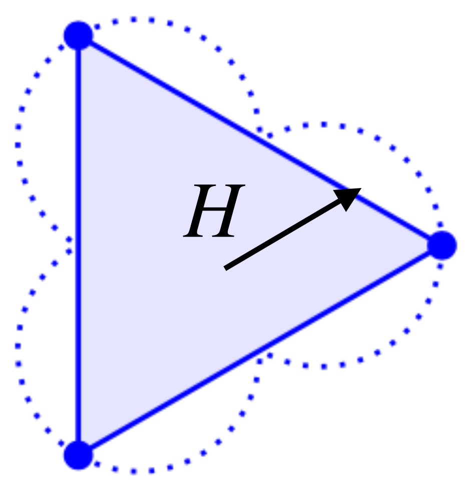

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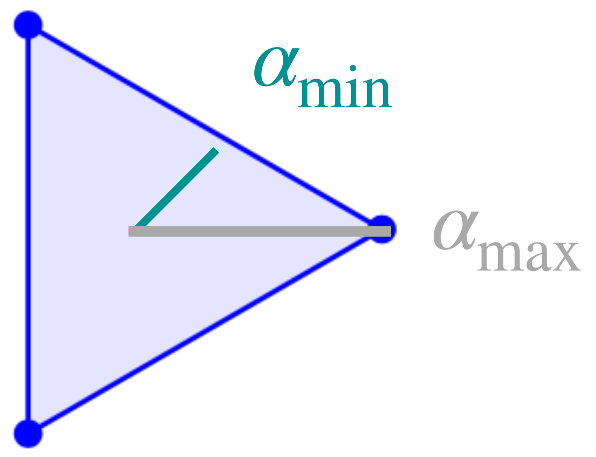

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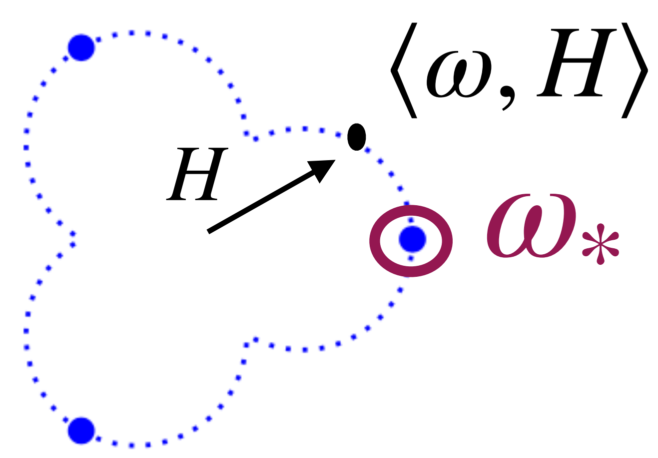

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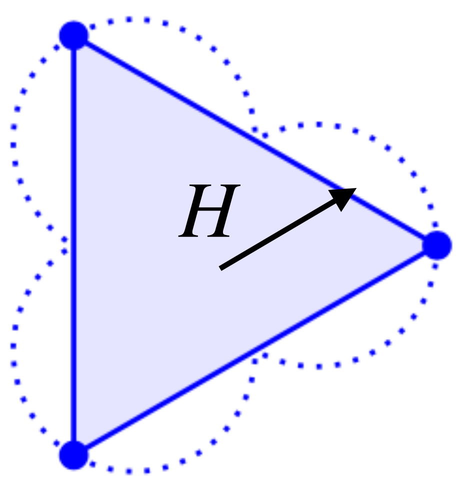

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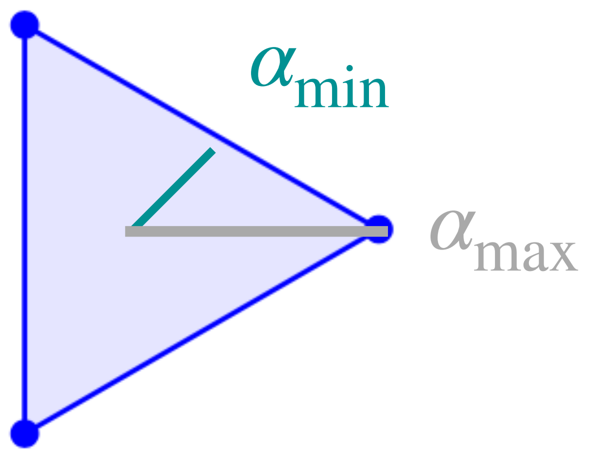

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Convex hull formulation for symmetric spaces

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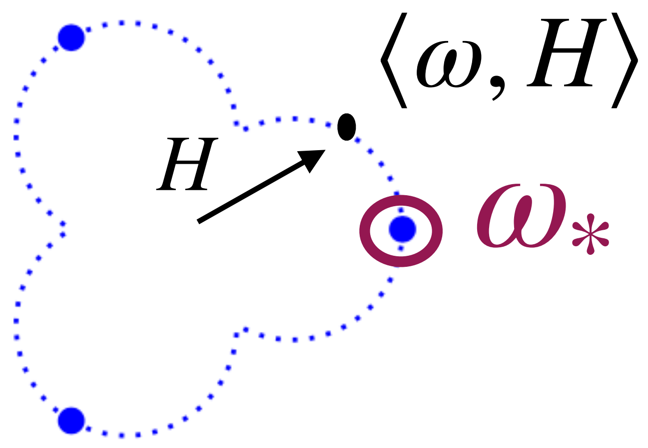

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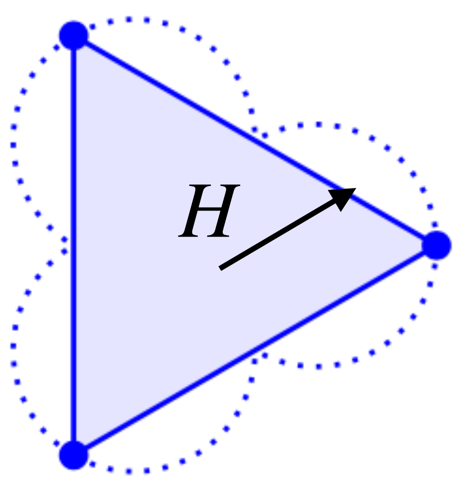

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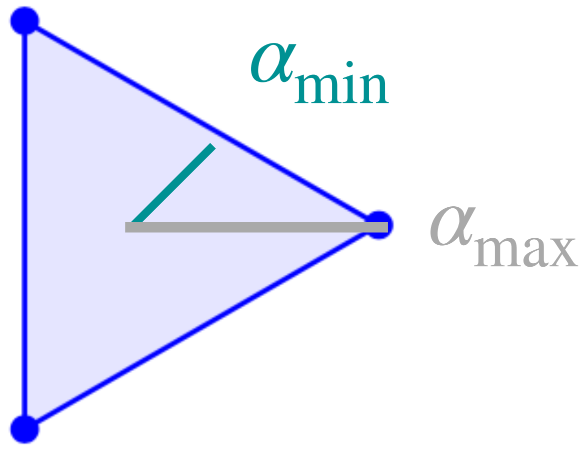

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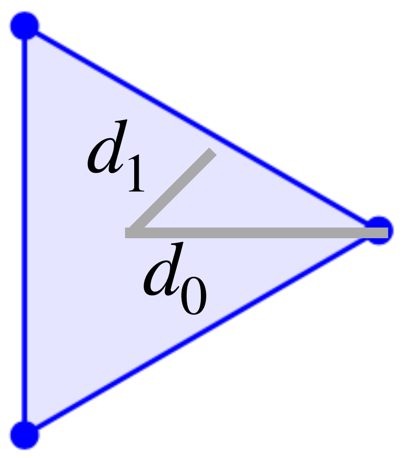
EFTs satisfying emergent string conjecture rates

- Set of eff theories in d dimensions

Such that

decay rates $$d_n = \begin{cases} \alpha_{\mathrm{KK,n+1}} = \sqrt{\frac{1}{d-2} + \frac{1}{n+1}} \\ \alpha_{\mathrm{string}} = \sqrt{\frac{1}{d-2}} \end{cases}$$



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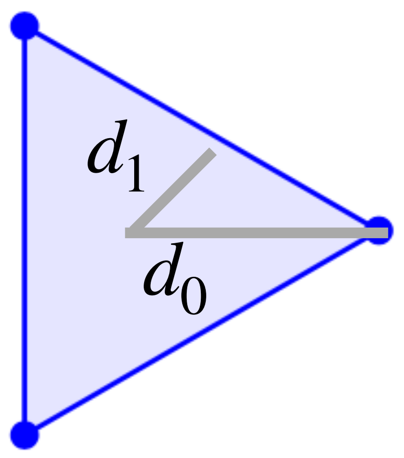

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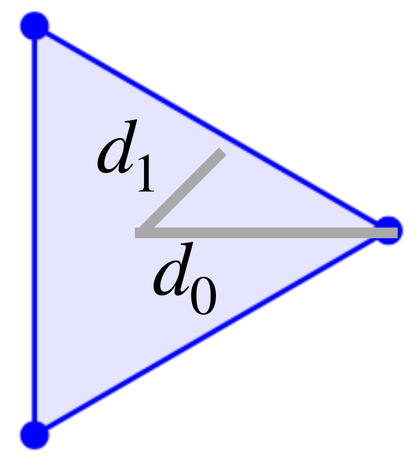

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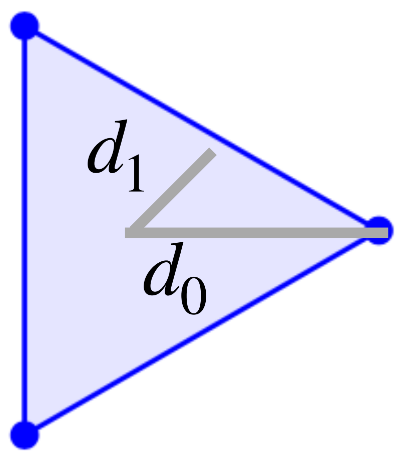

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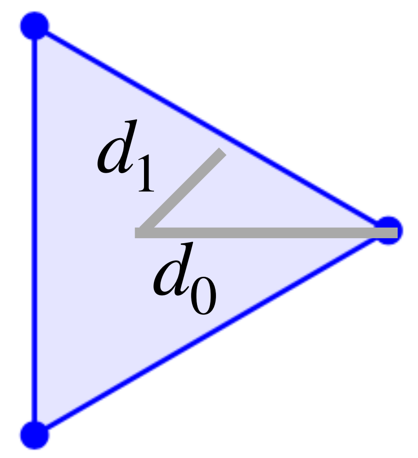

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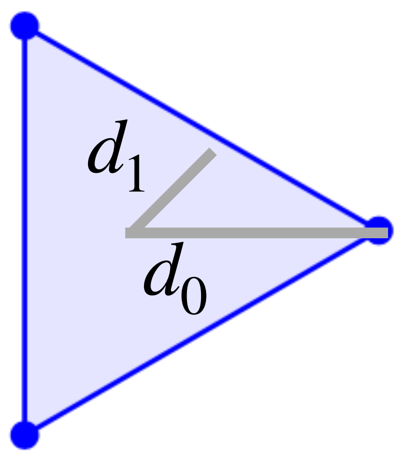
facets: regular polytopes

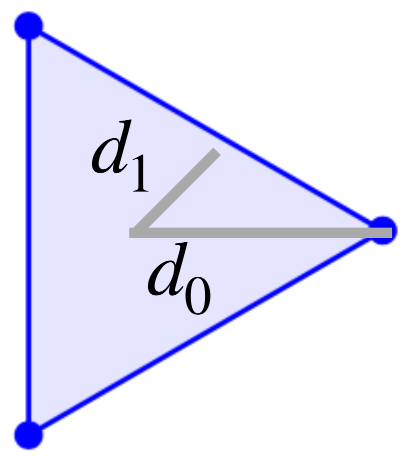
Facets

- $d_n = \alpha_{\mathrm{KK},n+1} = \sqrt{\frac{1}{d-2} + \frac{1}{n+1}}$ $\quad n \leq r - 1$



Facets

- $d_n = \alpha_{\mathrm{KK},n+1} = \sqrt{\frac{1}{d-2} + \frac{1}{n+1}}$ $\qquad n \leq r - 1 \qquad \Rightarrow \; d_n^2 - d_{n+1}^2 = \frac{1}{n} - \frac{1}{n+1}$



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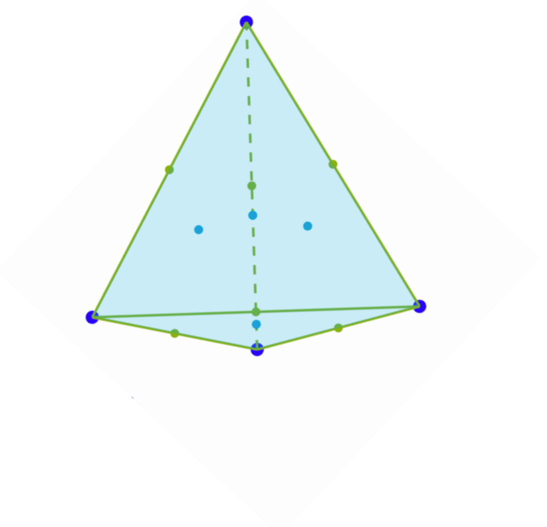
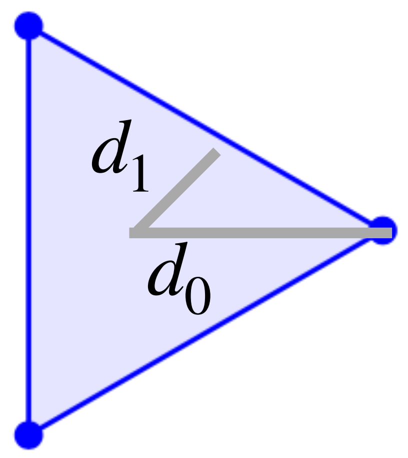


Simplex



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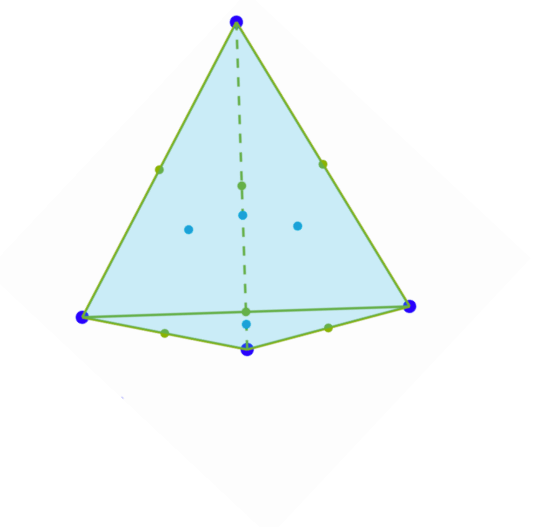

Simplex

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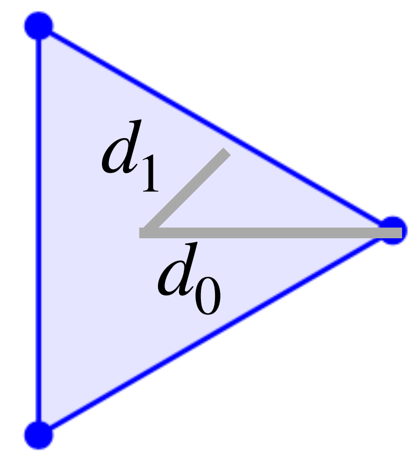
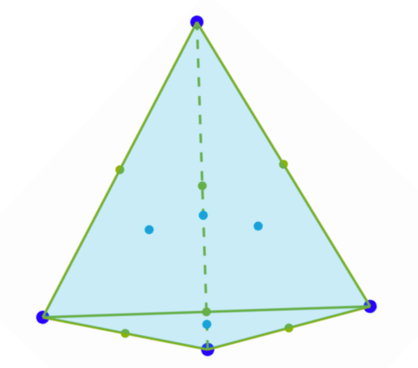
$$d_{r-1} = \alpha_{\mathrm{string}} = \sqrt{\frac{1}{d-2}}$$



Facets

- $d_n = \alpha_{\mathrm{KK,n+1}} = \sqrt{\frac{1}{d-2} + \frac{1}{n+1}}$ $\quad n \leq r-1 \quad \Rightarrow d_n^2 - d_{n+1}^2 = \frac{1}{n} - \frac{1}{n+1}$



Simplex

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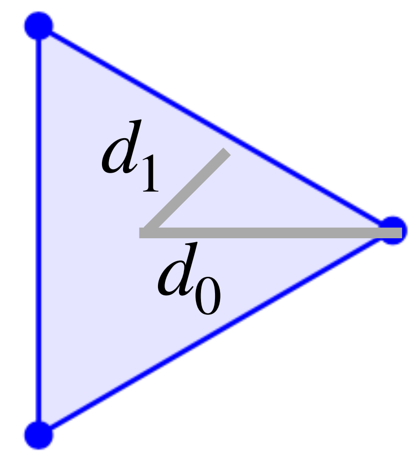

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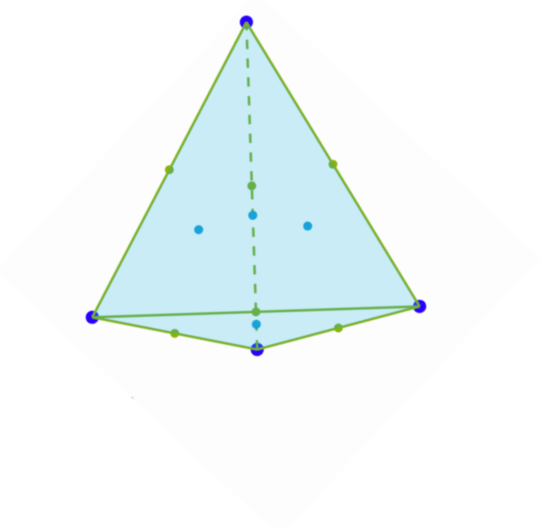

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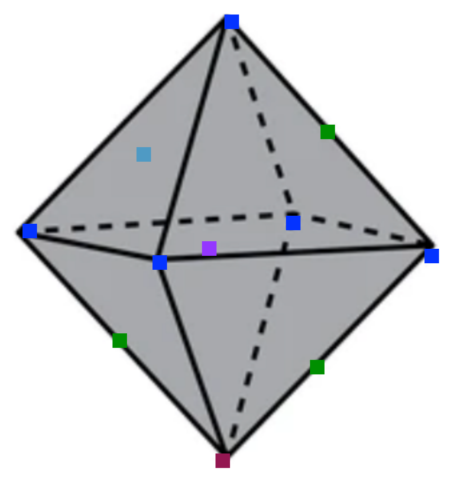
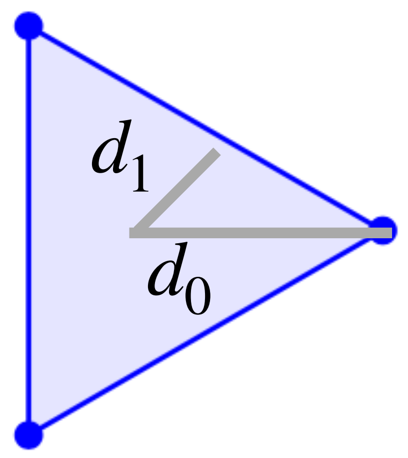


Orthoplex



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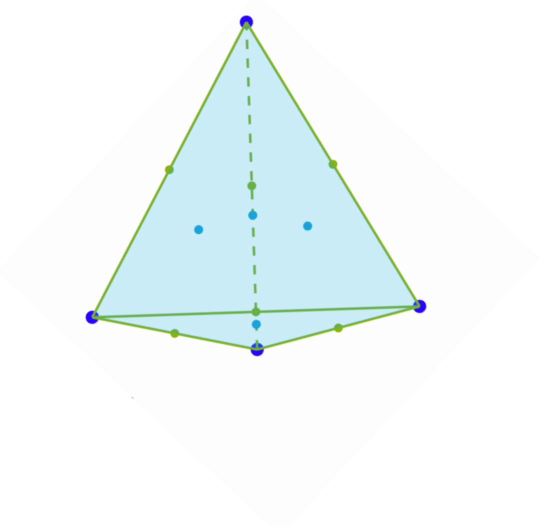

Simplex

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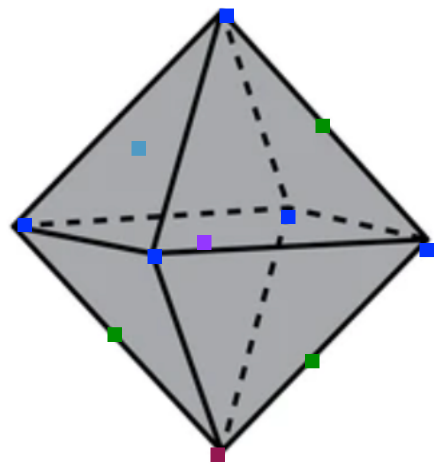
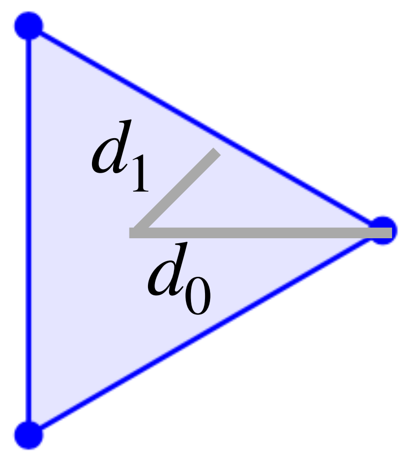


Orthoplex

Full polytope



Facets

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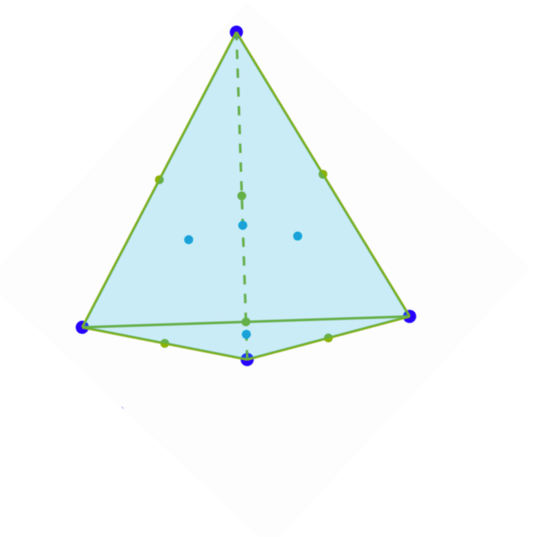

Simplex

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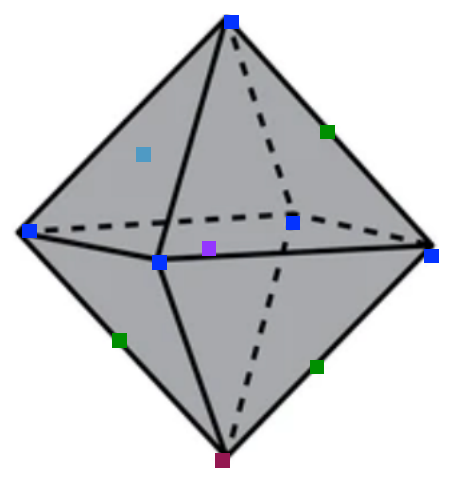
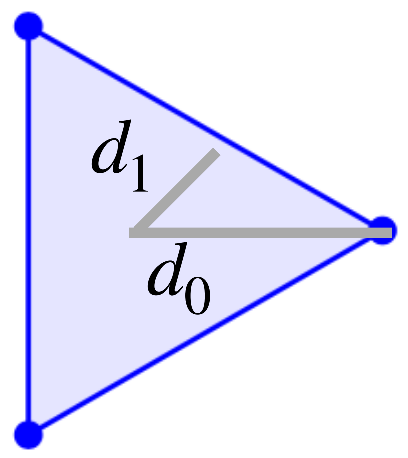


Orthoplex

Full polytope

- $d_{r-1} = \sqrt{\frac{1}{d-2} + \frac{1}{r}}$ or $\sqrt{\frac{1}{d-2}}$



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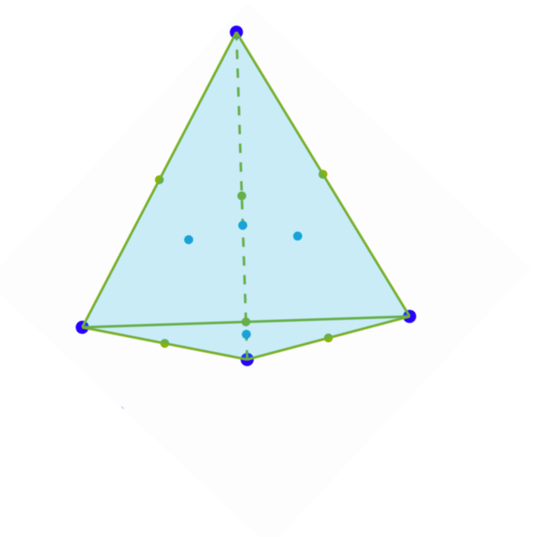

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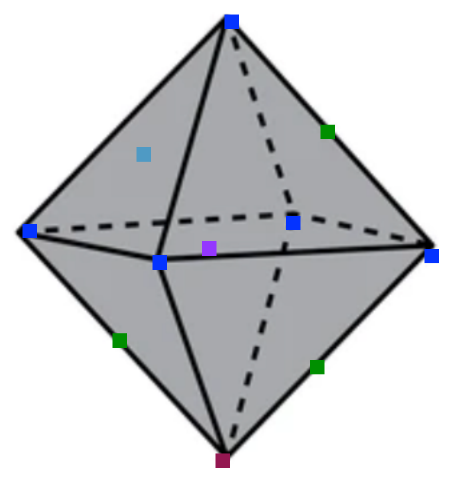

Orthoplex

Full polytope

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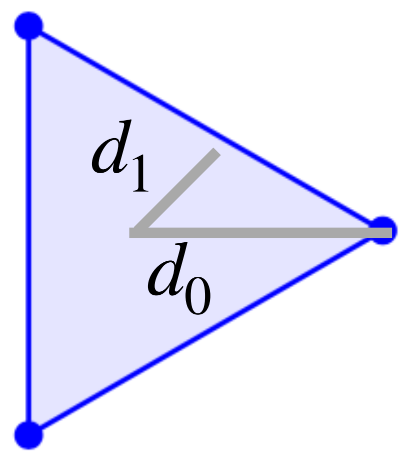

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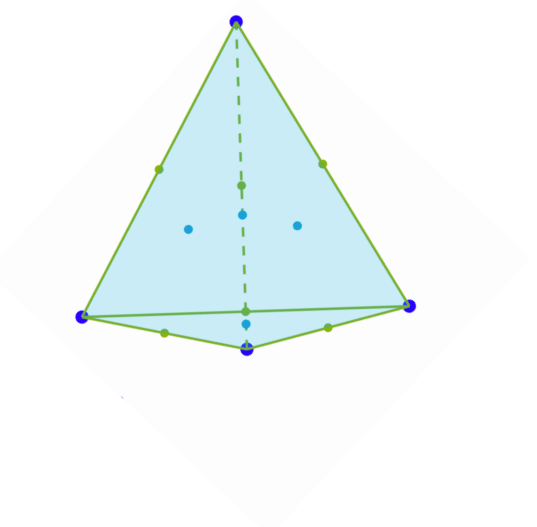

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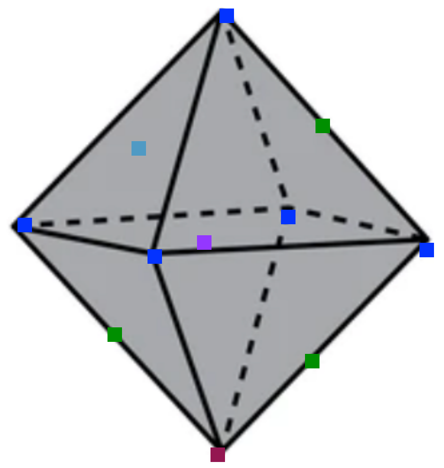

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Full polytope

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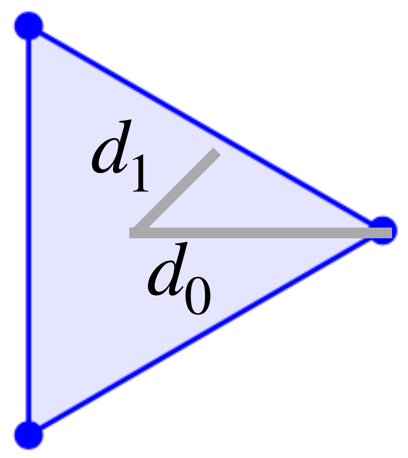
Finite list of polytopes, unique d for each



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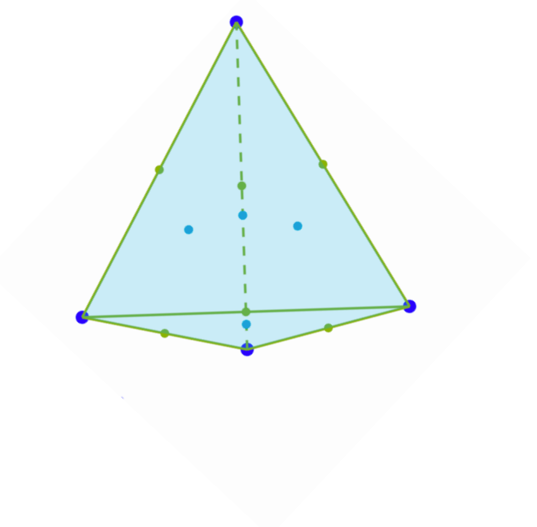

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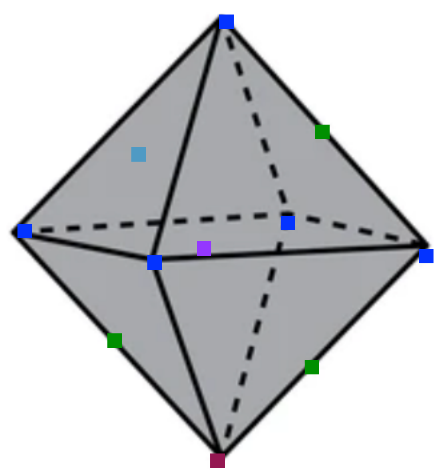

Orthoplex

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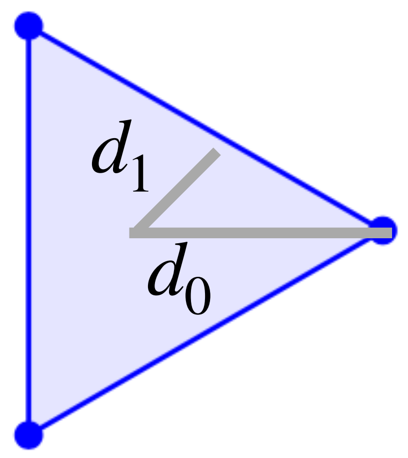
Finite list of polytopes, unique d for each

Rank 2 polytopes agree with [Etheredge, Heidenreich, Rudelius, Ruiz, Valenzuela '24]



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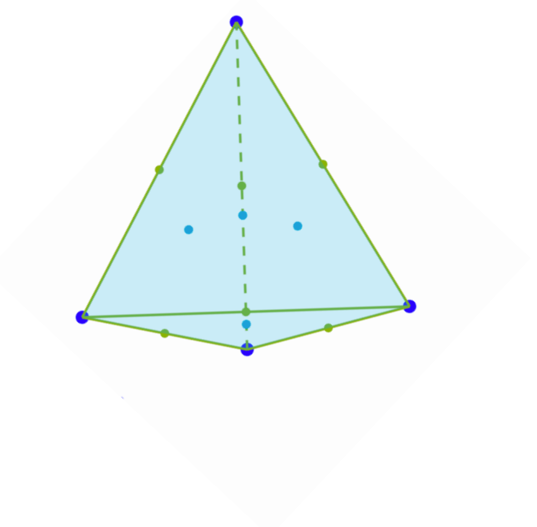

Simplex

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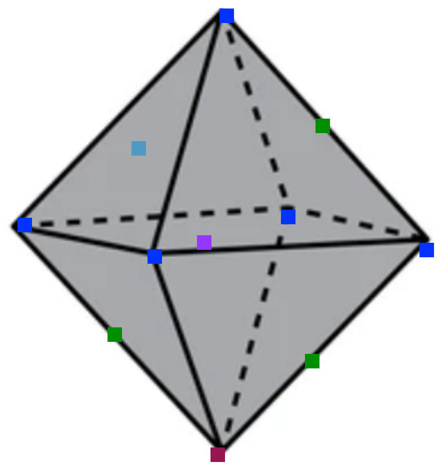

Orthoplex

Full polytope

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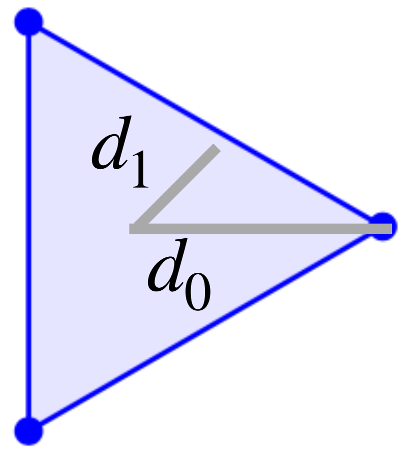
Finite list of polytopes, unique d for each

Rank 2 polytopes agree with [Etheredge, Heidenreich, Rudelius, Ruiz, Valenzuela '24]

All realised at least by one symm mod. space EFT (G, ρ, d) !



EFTs in the landscape

EFTs in the landscape

Restric to

- G_a of split form $(O(k,k))$

EFTs in the landscape

Restric to

- G_a of split form $(O(k,k))$

•All vertices at same distance

EFTs in the landscape

Restric to

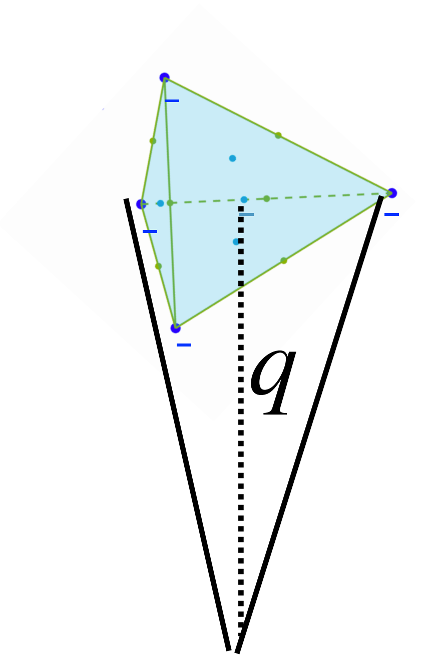
- G_a of split form $(O(k,k))$
- All vertices at same distance

$\Rightarrow$ Particles in a **single irrep** ρ of G, with highest weight λ

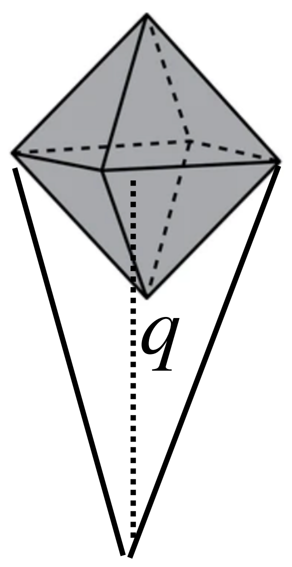
For $G = G' \times \mathbb{R}$ (e.g. $O(k,k) \times \mathbb{R}$ for NSNS dof in T^k compactifications)

$$\lambda = (\lambda', q)$$

highest weight



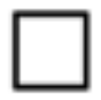
$$q = \sqrt{\frac{1}{d-2} + \frac{1}{k}}$$



$$q = \sqrt{\frac{1}{d-2}}$$

polytope	$\mathfrak{g}$	ρ	q	d
β_k	$\mathfrak{so}(k,k) \oplus \mathbb{R}$ $\mathfrak{so}(k+1,k) \oplus \mathbb{R}$ $\mathfrak{sp}_{2k} \oplus \mathbb{R}$	$\mathbf{2k}$ $\mathbf{2k+1}$ $\mathbf{2k}$	$\sqrt{\frac{1}{d-2}}$	d
Δ_k	$\mathfrak{sl}_k \oplus \mathbb{R}$	$\mathbf{k}$	$\sqrt{\frac{1}{d-2}+\frac{1}{k}}$	d

For G semi-simple

r		polytope
1		
2		
		
		
		
3		
		
		
4		octacube
		tetroctahedric
5		demipenteract
6		V_{27}
7		V_{56}
8		V_{240}

For G semi-simple

r		polytope
1		
2		
		
		
		
3		
		
		
4		octacube
		tetroctahedric
5		demipenteract
6		V_{27}
7		V_{56}
8		V_{240}

r	$\mathfrak{g}$	d	α_s	$D_{\max}$	ρ	polytope
1	$\mathfrak{sl}_2$	d		d	$\mathbf{2}$	
	$\mathfrak{sl}_2$	d	-	$d+1$	$\mathbf{2}$	
2	$\mathfrak{sl}_3$	3	-	5	$\mathbf{8}$	
	$\mathfrak{g}_{2(2)}$				$\mathbf{14}$	
	$\mathfrak{g}_{2(2)}$				$\mathbf{7}$	
	$\mathfrak{so}(2,2)$	3		4	$(\mathbf{2},\mathbf{2})$	
	$\mathfrak{sp}_4$				$\mathbf{4}$	
	$\mathfrak{sp}_4$				$\mathbf{5}$	
	$\mathfrak{sl}_3$	5		6	$\mathbf{3}$	
	$\mathfrak{sl}_2^{\oplus 2}$	4		6	$(\mathbf{2},\mathbf{2})$	
3	$\mathfrak{sl}_2^{\oplus 3}$	4		6	$(\mathbf{2},\mathbf{2},\mathbf{2})$	
	$\mathfrak{sp}_4 \oplus \mathfrak{sl}_2$				$(\mathbf{5},\mathbf{2})$	
	$\mathfrak{sp}_4 \oplus \mathfrak{sl}_2$				$(\mathbf{4},\mathbf{2})$	
	$\mathfrak{so}(4,3)$				$\mathbf{8}$	
	$\mathfrak{sp}_6$				$\mathbf{14}'$	
	$\mathfrak{sp}_6$	3		6	$\mathbf{14}$	
	$\mathfrak{so}(4,3)$				$\mathbf{21}$	
	$\mathfrak{sl}_4$				$\mathbf{15}$	
	$\mathfrak{e}_{3(3)} \cong \mathfrak{sl}_2 \oplus \mathfrak{sl}$	8		11	$(\mathbf{2},\overline{\mathbf{3}})$	
4	$\mathfrak{so}(4,4)$	3		6	$\mathbf{28}$	octacube
	$\mathfrak{so}(5,4)$				$\mathbf{36}$	
	$\mathfrak{sp}_8$				$\mathbf{27}$	
	$\mathfrak{f}_{4(4)}$				$\mathbf{52}$	
	$\mathfrak{f}_{4(4)}$				$\mathbf{26}$	
	$\mathfrak{e}_{4(4)} \cong \mathfrak{sl}_5$	7		11	$\overline{\mathbf{10}}$	tetroctahedric
5	$\mathfrak{e}_{5(5)} \cong \mathfrak{so}(5,5)$	6		11	$\mathbf{16}$	demipenteract
6	$\mathfrak{e}_{6(6)}$	5		11	$\mathbf{27}$	V_{27}
7	$\mathfrak{e}_{7(7)}$	4		11	$\mathbf{56}$	V_{56}
8	$\mathfrak{e}_{8(8)}$	3		11	$\mathbf{248}$	V_{240}

For G semi-simple

r	$\mathfrak{g}$	d	α_s	$D_{\max}$	ρ	polytope
1	$\mathfrak{sl}_2$	d		d	$\mathbf{2}$	
	$\mathfrak{sl}_2$	d	-	$d+1$	$\mathbf{2}$	
2	$\mathfrak{sl}_3$ $\mathfrak{g}_{2(2)}$ $\mathfrak{g}_{2(2)}$	3	-	5	$\mathbf{8}$ $\mathbf{14}$ $\mathbf{7}$	
	$\mathfrak{so}(2,2)$ $\mathfrak{sp}_4$ $\mathfrak{sp}_4$	3		4	$(\mathbf{2},\mathbf{2})$ $\mathbf{4}$ $\mathbf{5}$	
	$\mathfrak{sl}_3$	5		6	$\mathbf{3}$	
	$\mathfrak{sl}_2^{\oplus 2}$	4		6	$(\mathbf{2},\mathbf{2})$	
3	$\mathfrak{sl}_2^{\oplus 3}$ $\mathfrak{sp}_4 \oplus \mathfrak{sl}_2$ $\mathfrak{sp}_4 \oplus \mathfrak{sl}_2$ $\mathfrak{so}(4,3)$ $\mathfrak{sp}_6$	4		6	$(\mathbf{2},\mathbf{2},\mathbf{2})$ $(\mathbf{5},\mathbf{2})$ $(\mathbf{4},\mathbf{2})$ $\mathbf{8}$ $\mathbf{14'}$	
	$\mathfrak{sp}_6$ $\mathfrak{so}(4,3)$ $\mathfrak{sl}_4$	3		6	$\mathbf{14}$ $\mathbf{21}$ $\mathbf{15}$	
	$\mathfrak{e}_{3(3)} \cong \mathfrak{sl}_2 \oplus \mathfrak{sl}_3$	8		11	$(\mathbf{2},\overline{\mathbf{3}})$	
4	$\mathfrak{so}(4,4)$ $\mathfrak{so}(5,4)$ $\mathfrak{sp}_8$ $\mathfrak{f}_{4(4)}$ $\mathfrak{f}_{4(4)}$	3		6	$\mathbf{28}$ $\mathbf{36}$ $\mathbf{27}$ $\mathbf{52}$ $\mathbf{26}$	octacube
	$\mathfrak{e}_{4(4)} \cong \mathfrak{sl}_5$	7		11	$\overline{\mathbf{10}}$	tetroctahedric
5	$\mathfrak{e}_{5(5)} \cong \mathfrak{so}(5,5)$	6		11	$\mathbf{16}$	demipenteract
6	$\mathfrak{e}_{6(6)}$	5		11	$\mathbf{27}$	V_{27}
7	$\mathfrak{e}_{7(7)}$	4		11	$\mathbf{56}$	V_{56}
8	$\mathfrak{e}_{8(8)}$	3		11	$\mathbf{248}$	V_{240}

For G semi-simple

Only 29 theories

r	$\mathfrak{g}$	d	α_s	$D_{\max}$	ρ	polytope
1	$\mathfrak{sl}_2$	d		d	$\mathbf{2}$	
	$\mathfrak{sl}_2$	d	-	$d+1$	$\mathbf{2}$	
2	$\mathfrak{sl}_3$	3	-	5	$\mathbf{8}$	
	$\mathfrak{g}_{2(2)}$				$\mathbf{14}$	
	$\mathfrak{g}_{2(2)}$				$\mathbf{7}$	
	$\mathfrak{so}(2,2)$	3		4	$(\mathbf{2},\mathbf{2})$	
	$\mathfrak{sp}_4$				$\mathbf{4}$	
	$\mathfrak{sp}_4$				$\mathbf{5}$	
	$\mathfrak{sl}_3$	5		6	$\mathbf{3}$	
	$\mathfrak{sl}_2^{\oplus 2}$	4		6	$(\mathbf{2},\mathbf{2})$	
3	$\mathfrak{sl}_2^{\oplus 3}$	4		6	$(\mathbf{2},\mathbf{2},\mathbf{2})$	
	$\mathfrak{sp}_4 \oplus \mathfrak{sl}_2$				$(\mathbf{5},\mathbf{2})$	
	$\mathfrak{sp}_4 \oplus \mathfrak{sl}_2$				$(\mathbf{4},\mathbf{2})$	
	$\mathfrak{so}(4,3)$				$\mathbf{8}$	
	$\mathfrak{sp}_6$				$\mathbf{14'}$	
	$\mathfrak{sp}_6$	3		6	$\mathbf{14}$	
	$\mathfrak{so}(4,3)$				$\mathbf{21}$	
	$\mathfrak{sl}_4$				$\mathbf{15}$	
	$\mathfrak{e}_{3(3)} \cong \mathfrak{sl}_2 \oplus \mathfrak{sl}_3$	8		11	$(\mathbf{2},\overline{\mathbf{3}})$	
4	$\mathfrak{so}(4,4)$	3		6	$\mathbf{28}$	octacube
	$\mathfrak{so}(5,4)$				$\mathbf{36}$	
	$\mathfrak{sp}_8$				$\mathbf{27}$	
	$\mathfrak{f}_{4(4)}$				$\mathbf{52}$	
	$\mathfrak{f}_{4(4)}$				$\mathbf{26}$	
	$\mathfrak{e}_{4(4)} \cong \mathfrak{sl}_5$	7		11	$\overline{\mathbf{10}}$	tetroctahedric
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6	$\mathfrak{e}_{6(6)}$	5		11	$\mathbf{27}$	V_{27}
7	$\mathfrak{e}_{7(7)}$	4		11	$\mathbf{56}$	V_{56}
8	$\mathfrak{e}_{8(8)}$	3		11	$\mathbf{248}$	V_{240}

For G semi-simple

Only 29 theories

$D_{\max} \leq 11!$

Very nice, finite, but…

- A symmetry of the polytope

$$\lambda' = p\,\lambda \qquad \langle\, \cdot\,,\,\cdot\,\rangle' = \frac{1}{p^2}\langle\, \cdot\,,\,\cdot\,\rangle \qquad p \in \mathbb{N}$$

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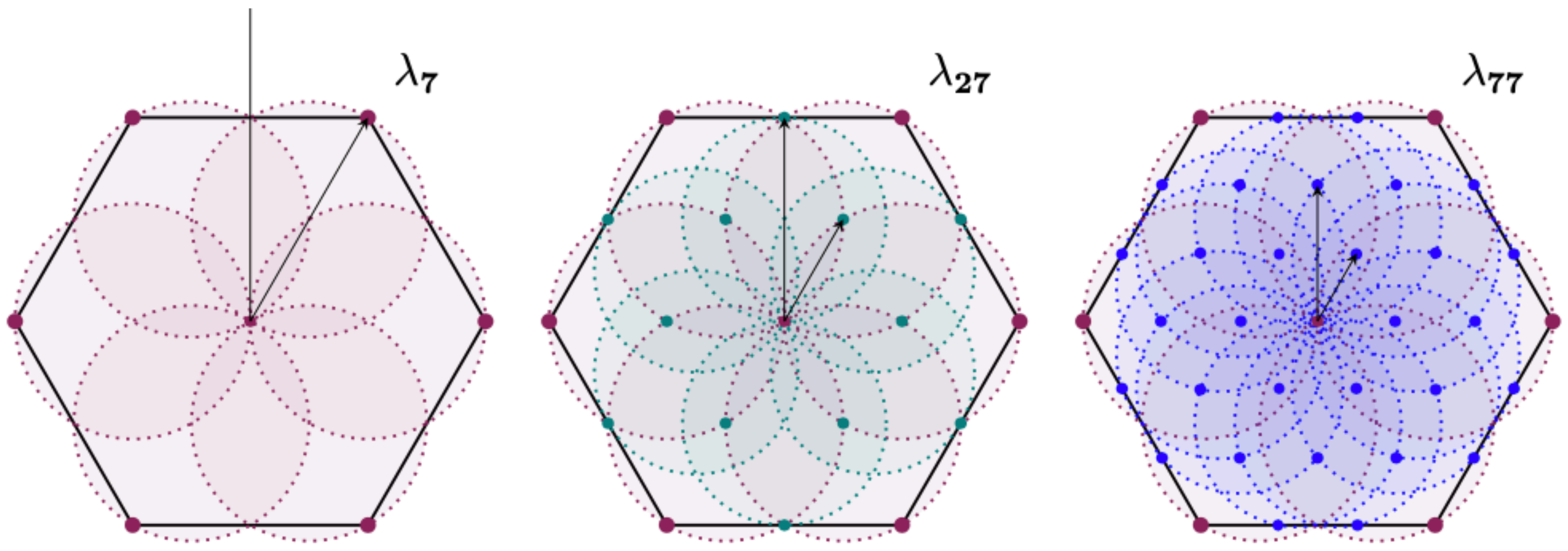
- A symmetry of the polytope

$$\lambda' = p\,\lambda \qquad \langle \cdot\,,\cdot \rangle' = \frac{1}{p^2}\langle \cdot\,,\cdot \rangle \qquad p \in \mathbb{N}$$

$\mathfrak{g}_2$



…

$p = 1$

λ_{min}

$p = 2$

$p = 3$

Very nice, finite, but…

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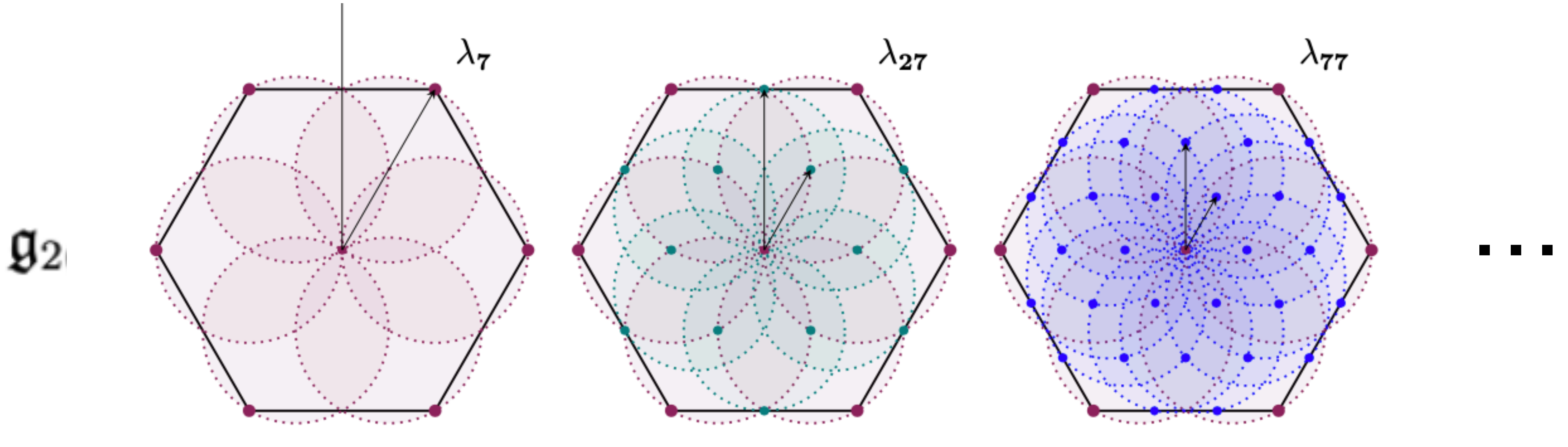

$p = 1$ $\qquad p = 2 \qquad p = 3$

$\lambda_{\min}$

- For each (G, λ, d) in the table there is an infinite family $(G, p\,\lambda, d)$!

Are these theories related?

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Moduli spaces inside moduli spaces $G' \subset G$

(polytopes inside polytopes)

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Two types of slices

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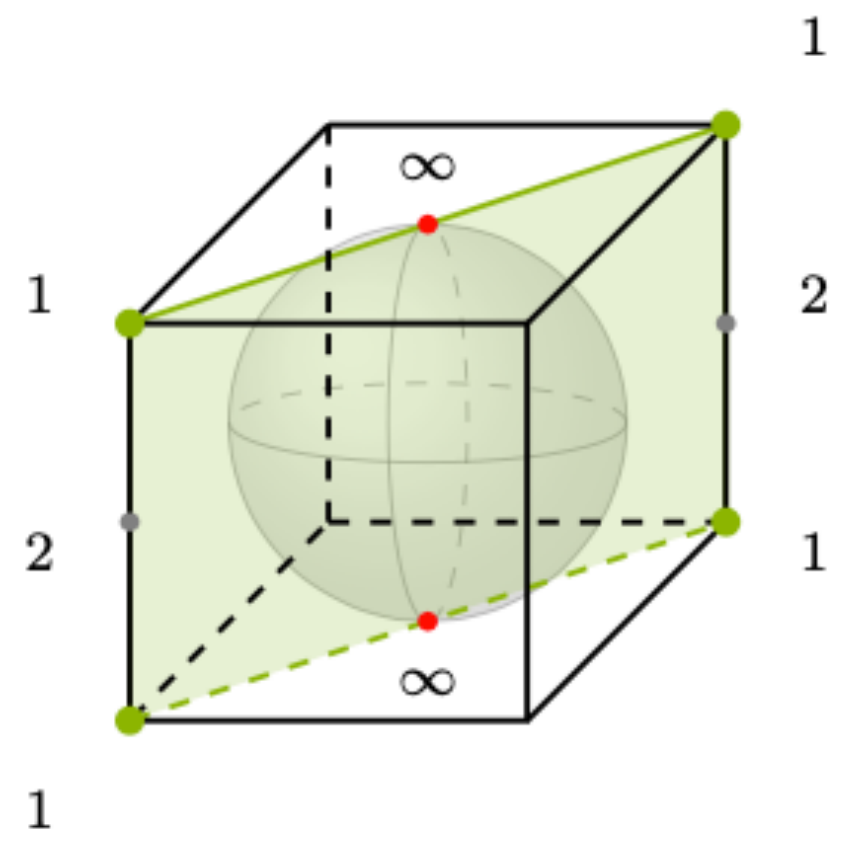

passing through origin

$d' = d$

orbifold

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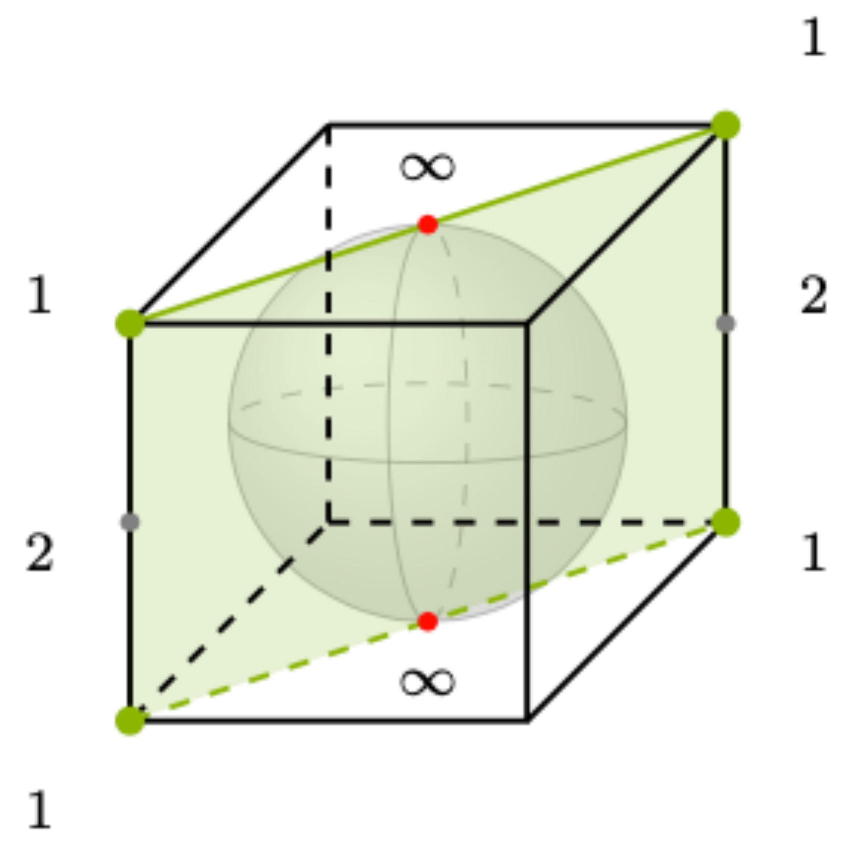

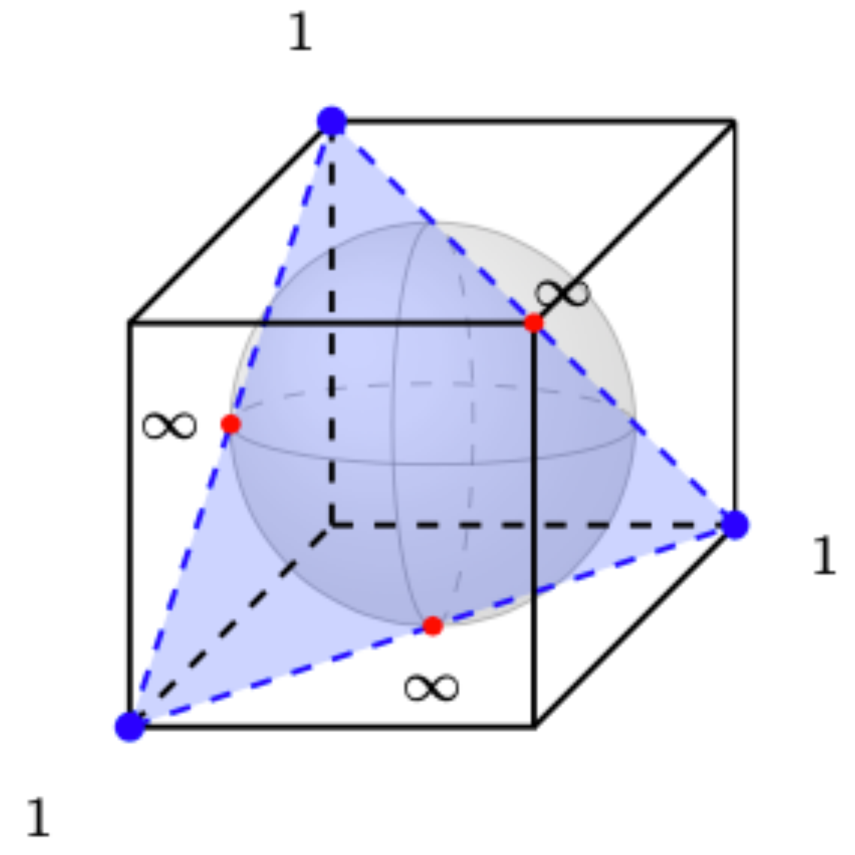

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not passing through the origin, $\perp$ to a weight

decompactification

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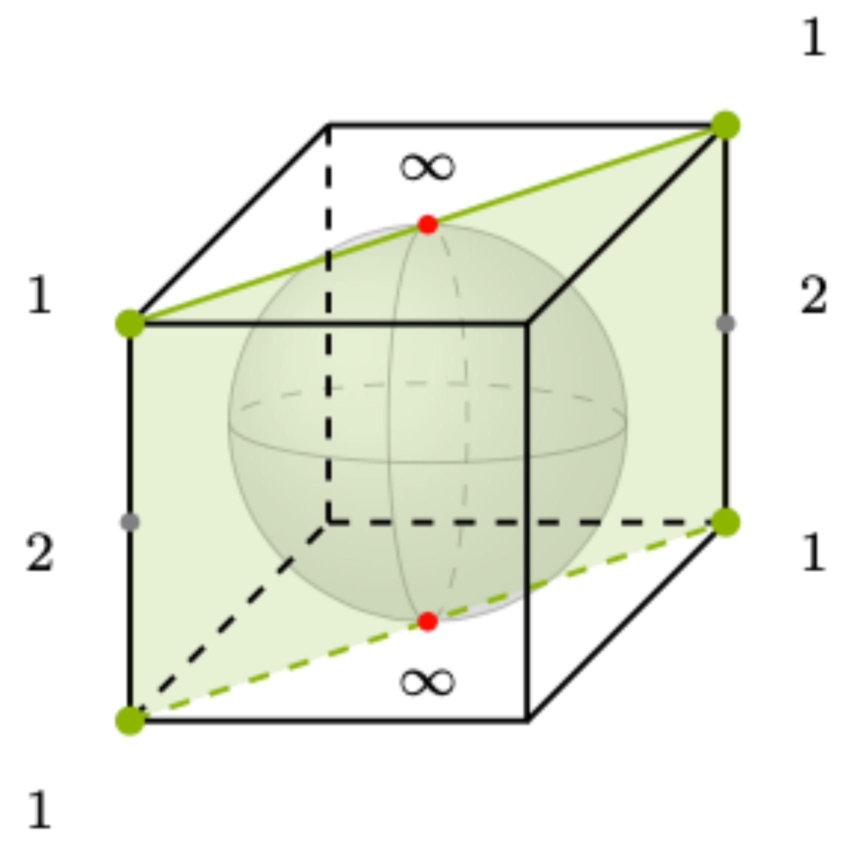
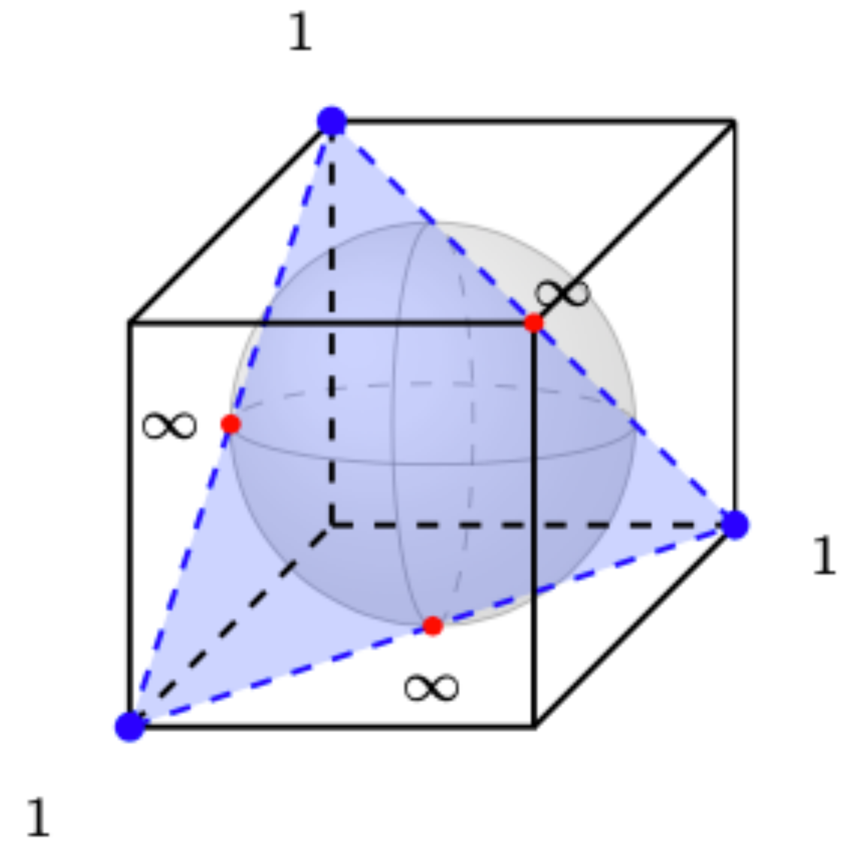


$Sp(6), \mathbf{14}, d = 4$



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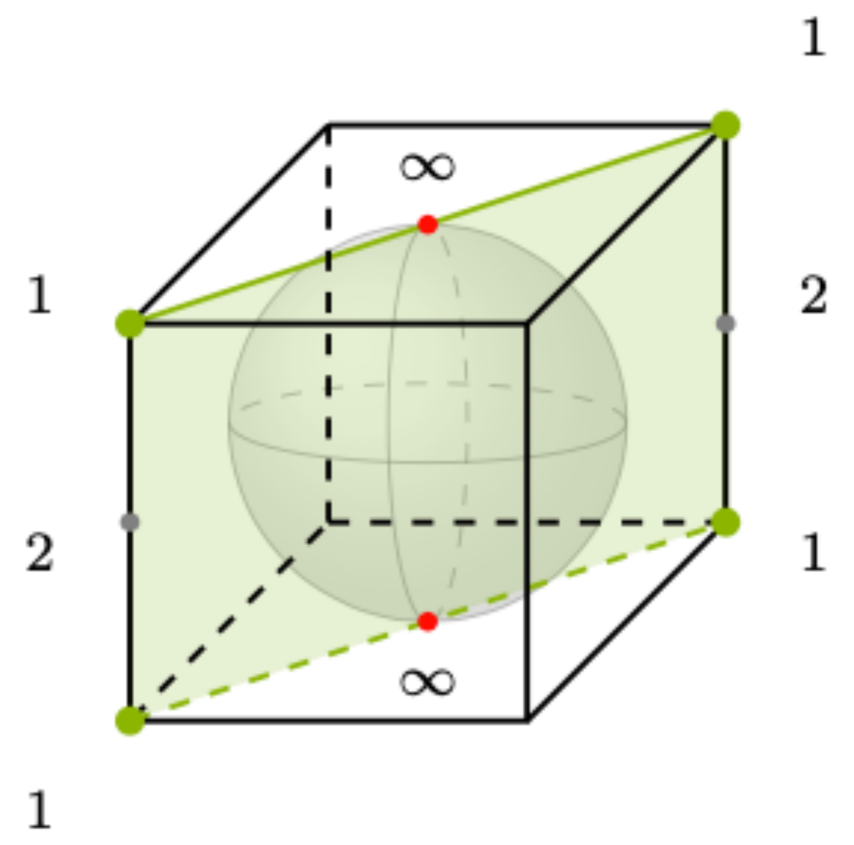

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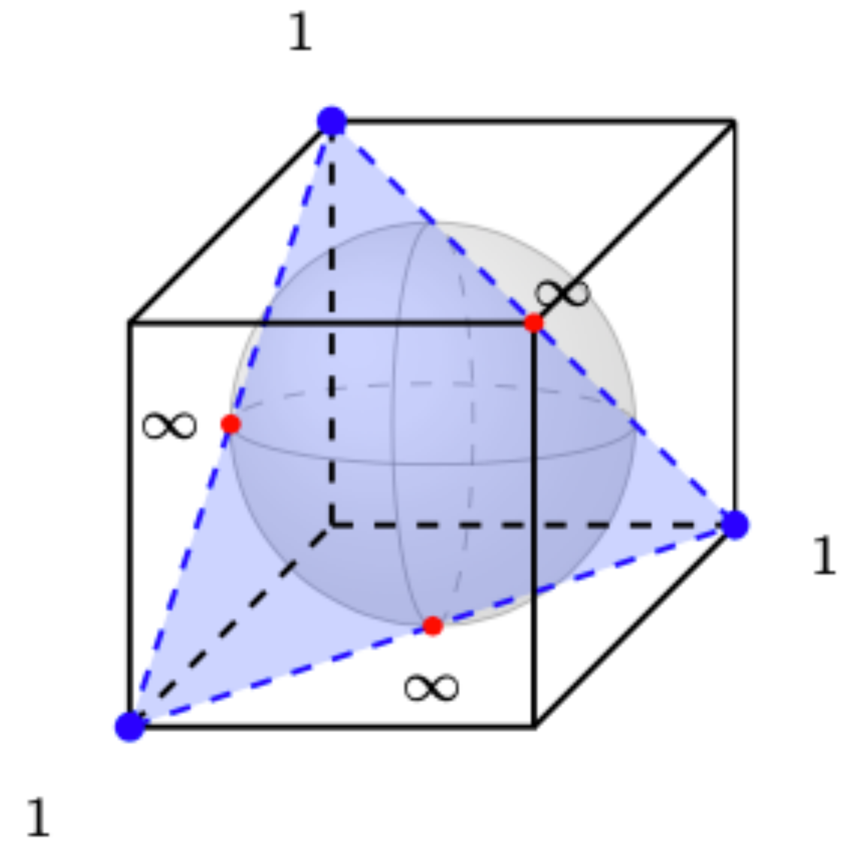

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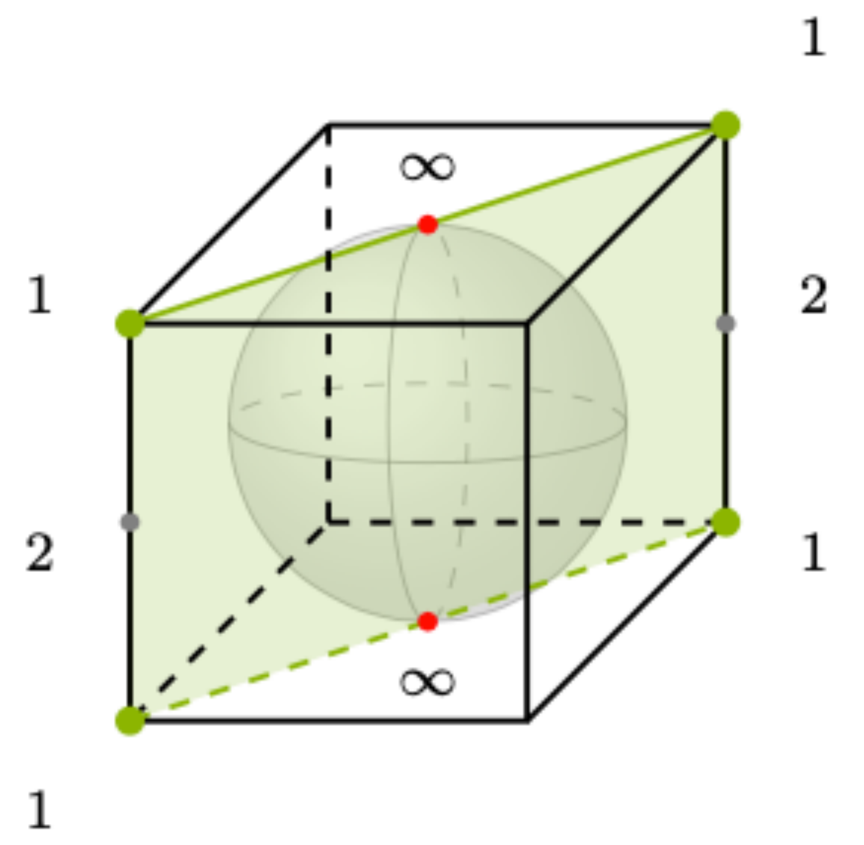

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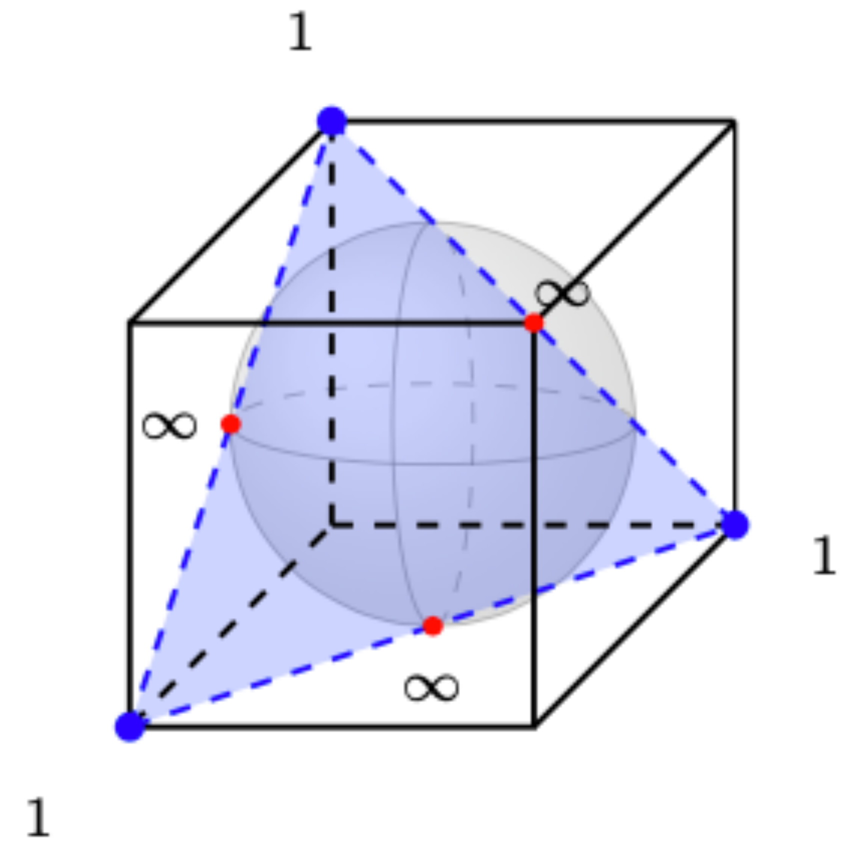

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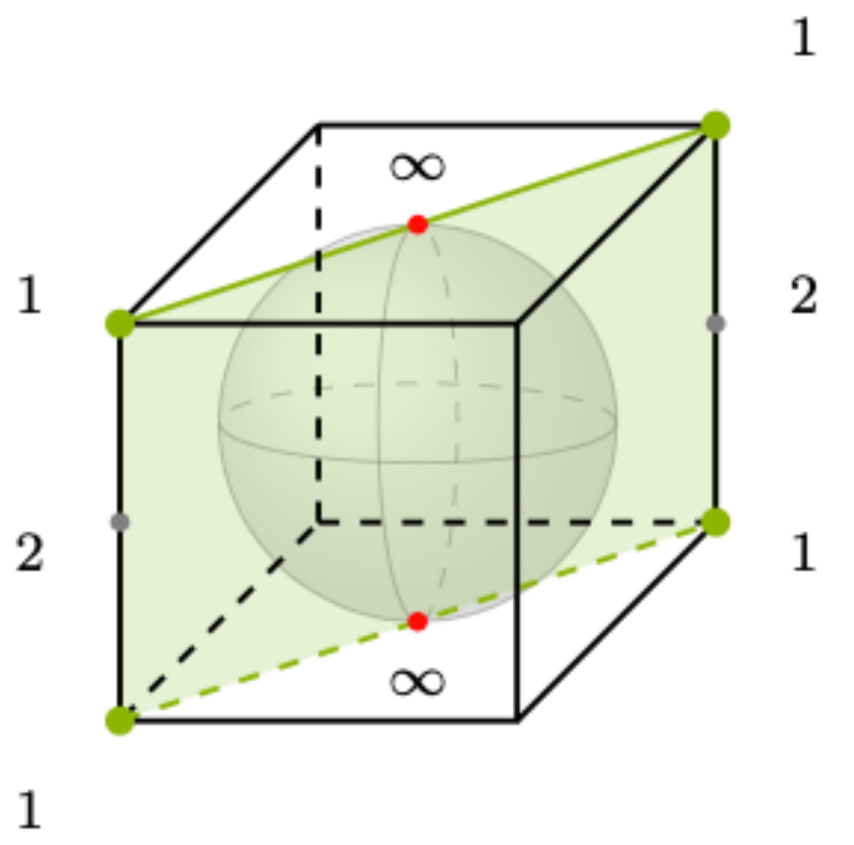

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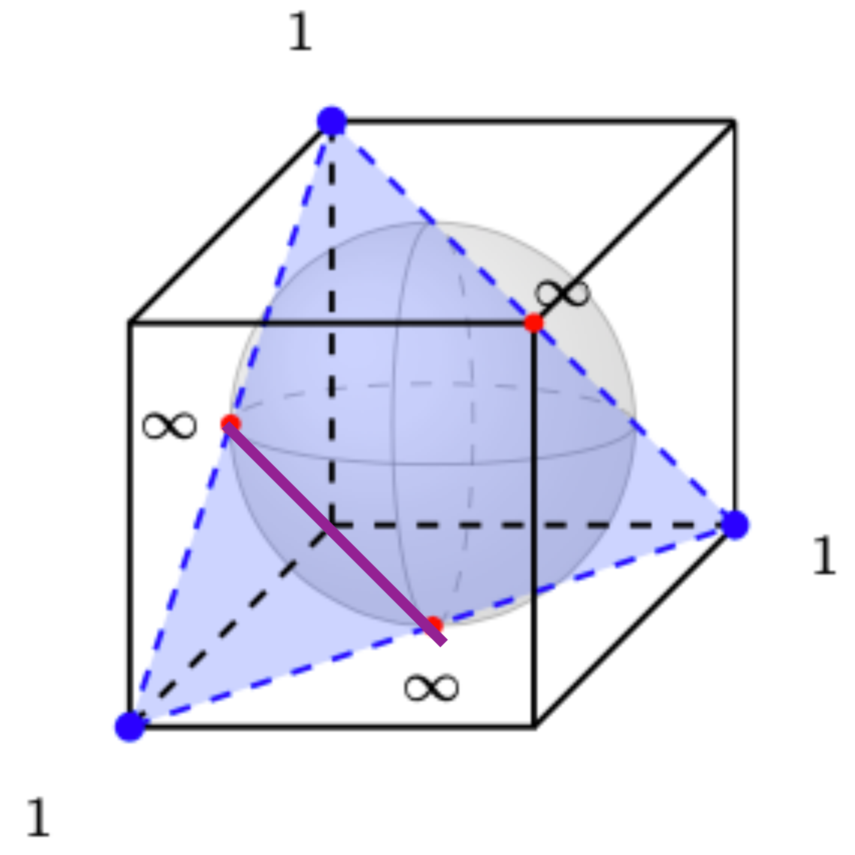

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$SL(3), \mathbf{3}, d = 5$

$SL(2), \mathbf{2}, d = 6$

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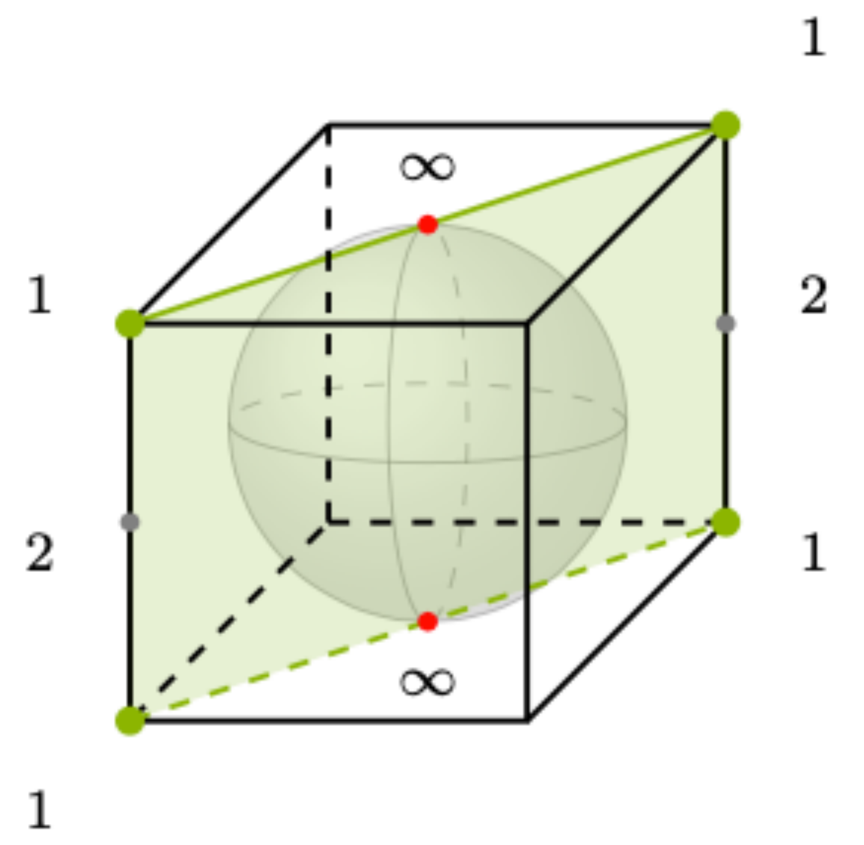

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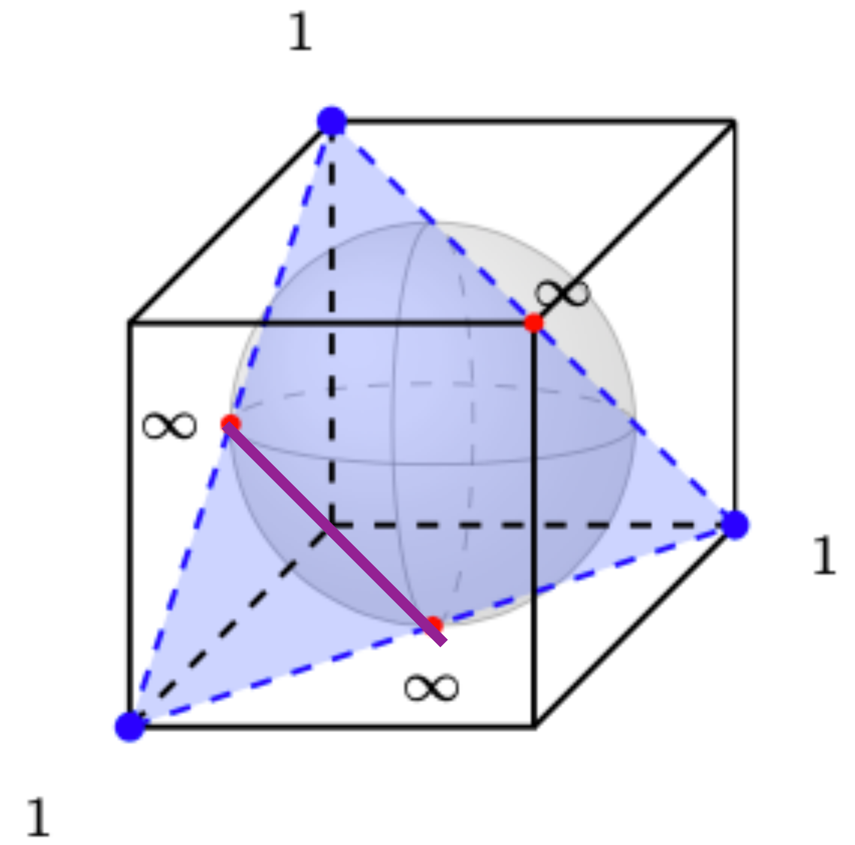

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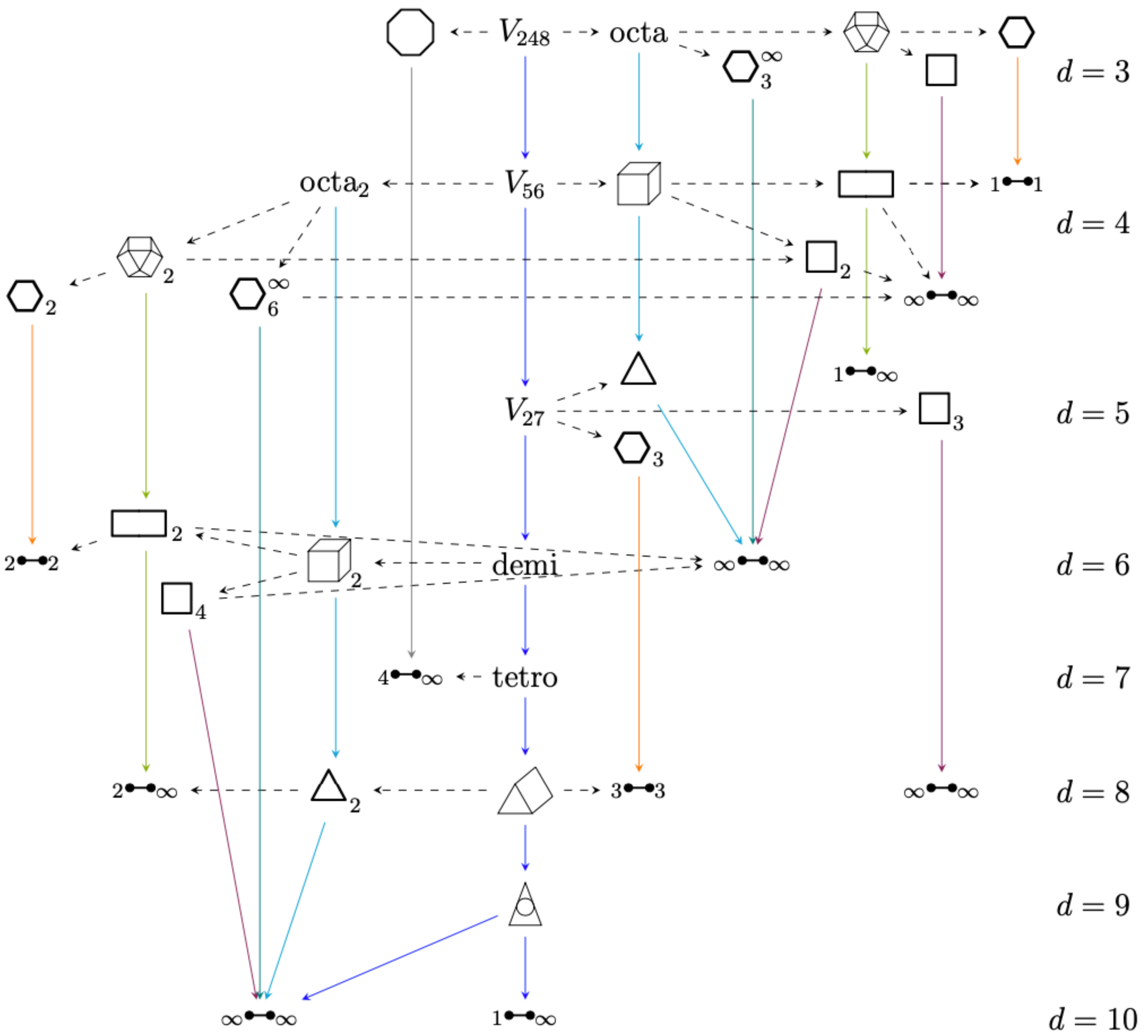
$d' > d$

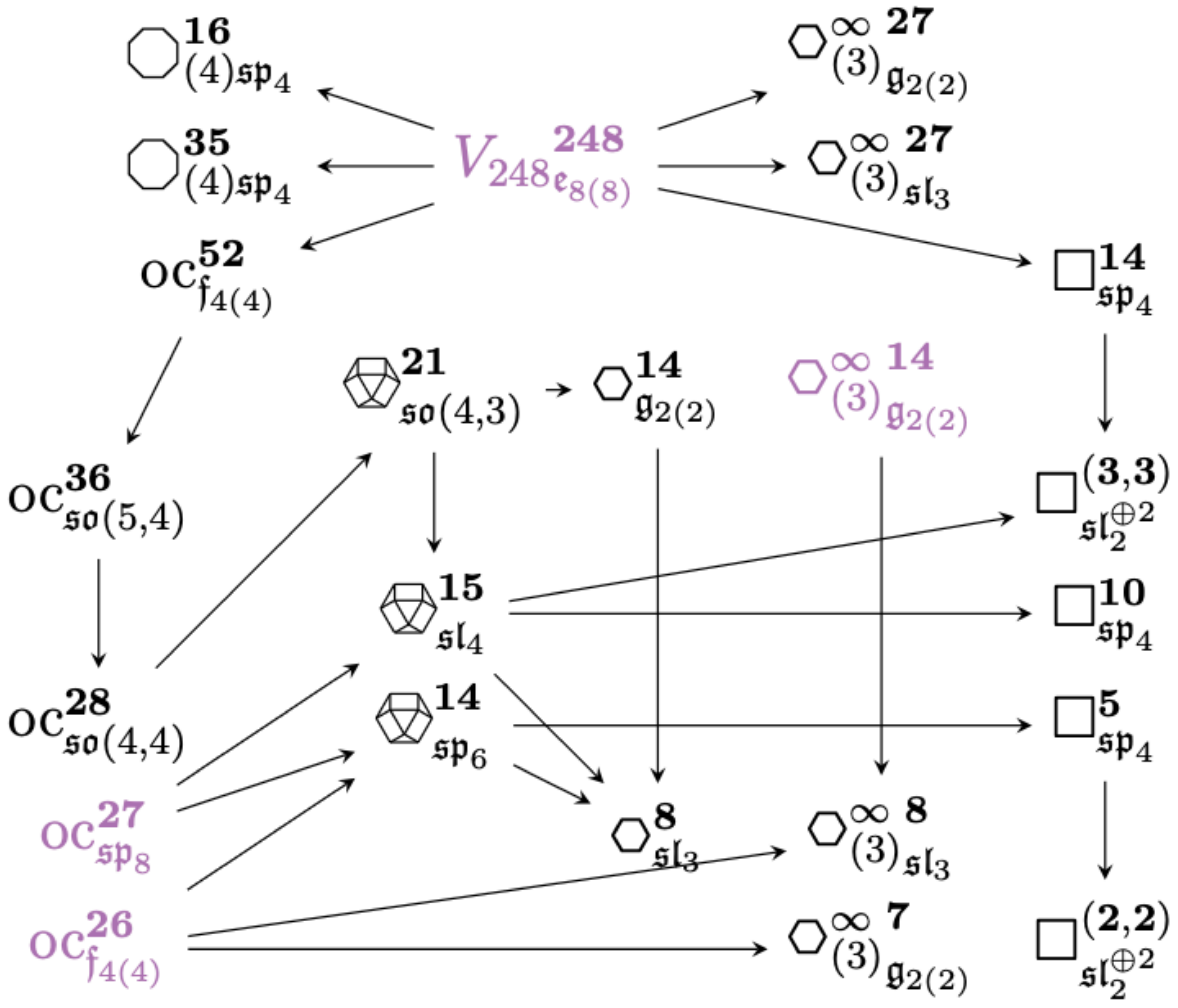
$SL(3), \mathbf{3}, d = 5$

$SL(2), \mathbf{2}, d = 6$ string island

Polytopes inside polytopes



All moduli spaces and (min) rep of particles in $d = 3$



Seed

Summary of results / conclusion

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- Rates α of towers given by distances to facets of the convex hull polytope

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  (they are the Weyl polytope of an irrep)

Summary of results / conclusion

- Assuming split form (O(k,k))

Full set of theories (G, λ, d)

29 theories for semi-simple G

Plus $[sl_k\,, so(k,k)\,, so(k+1,k)\,, sp_{2k}] \oplus \mathbb{R}$

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- Actually, every solution comes in a countably infinite series

  of highest weight: $(G, p\lambda, d) \quad \forall p \in \mathbb{N}$

  some math or swampland argument $\Rightarrow p = 1$??

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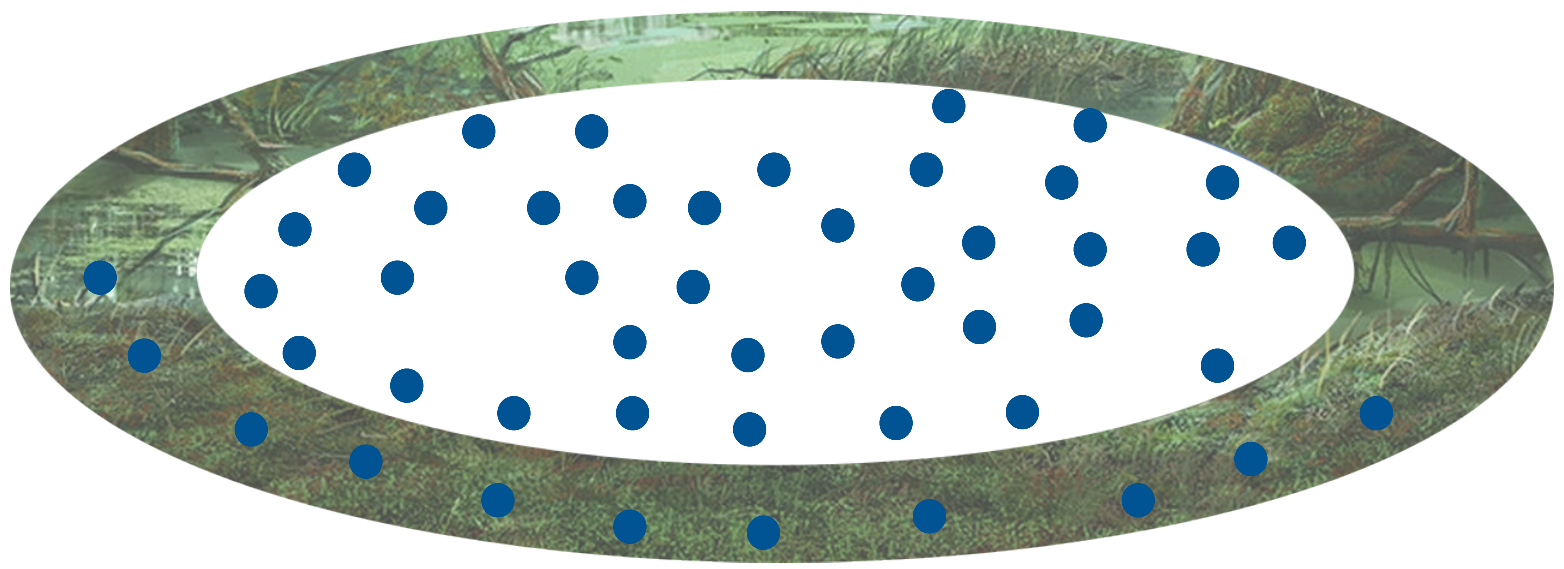
- Three of the families do not come from E_8

$$(Sp(8), \mathbf{27}, d=3) \quad (F_4, \mathbf{26}, d=3) \quad (G_2, \mathbf{14}, d=3)_{n_v=3}$$

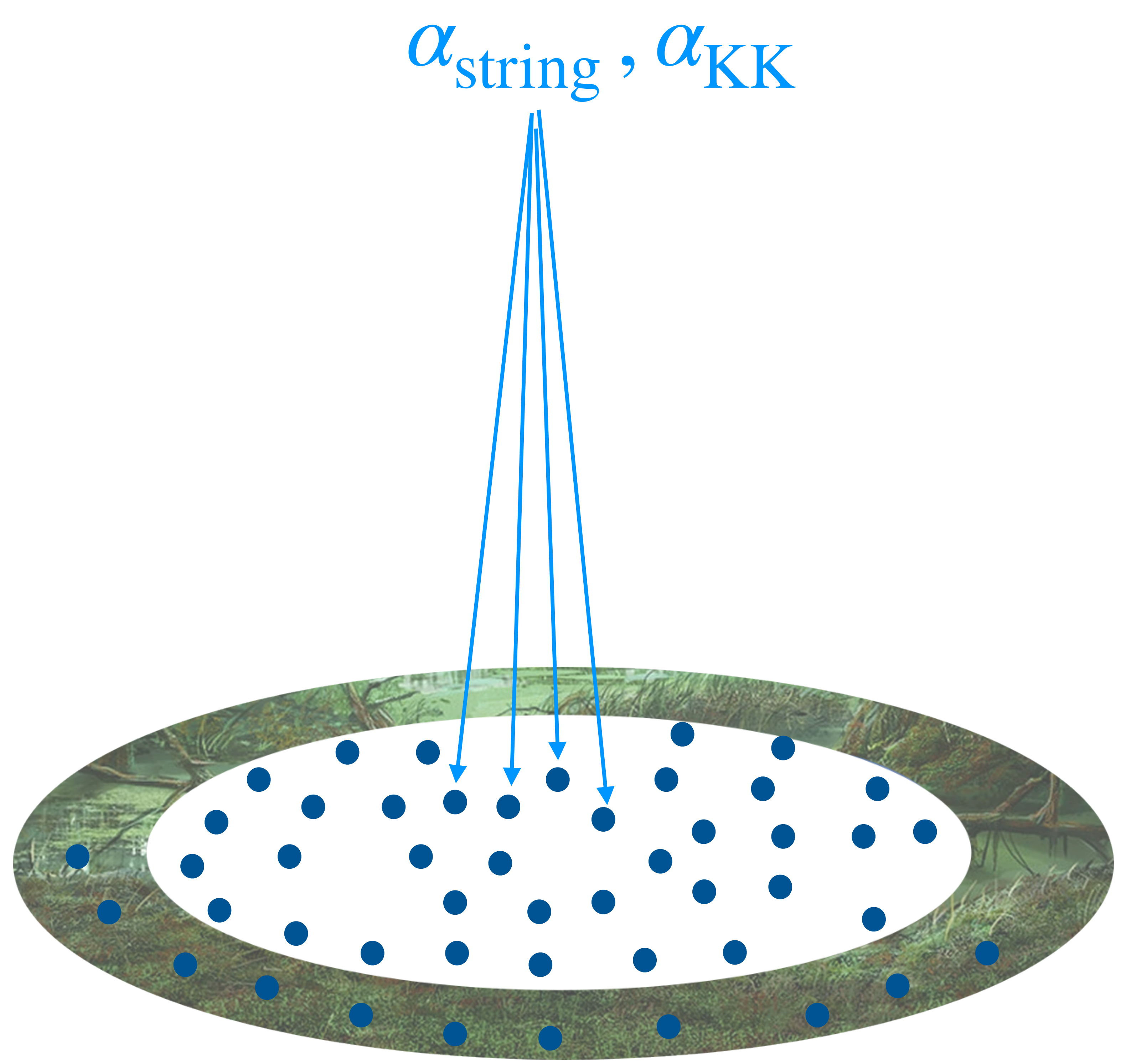
Summary of results / conclusion

$\alpha_{\rm string}\,,\,\alpha_{\rm KK}$



- Theories with symmetric moduli spaces

Summary of results / conclusion



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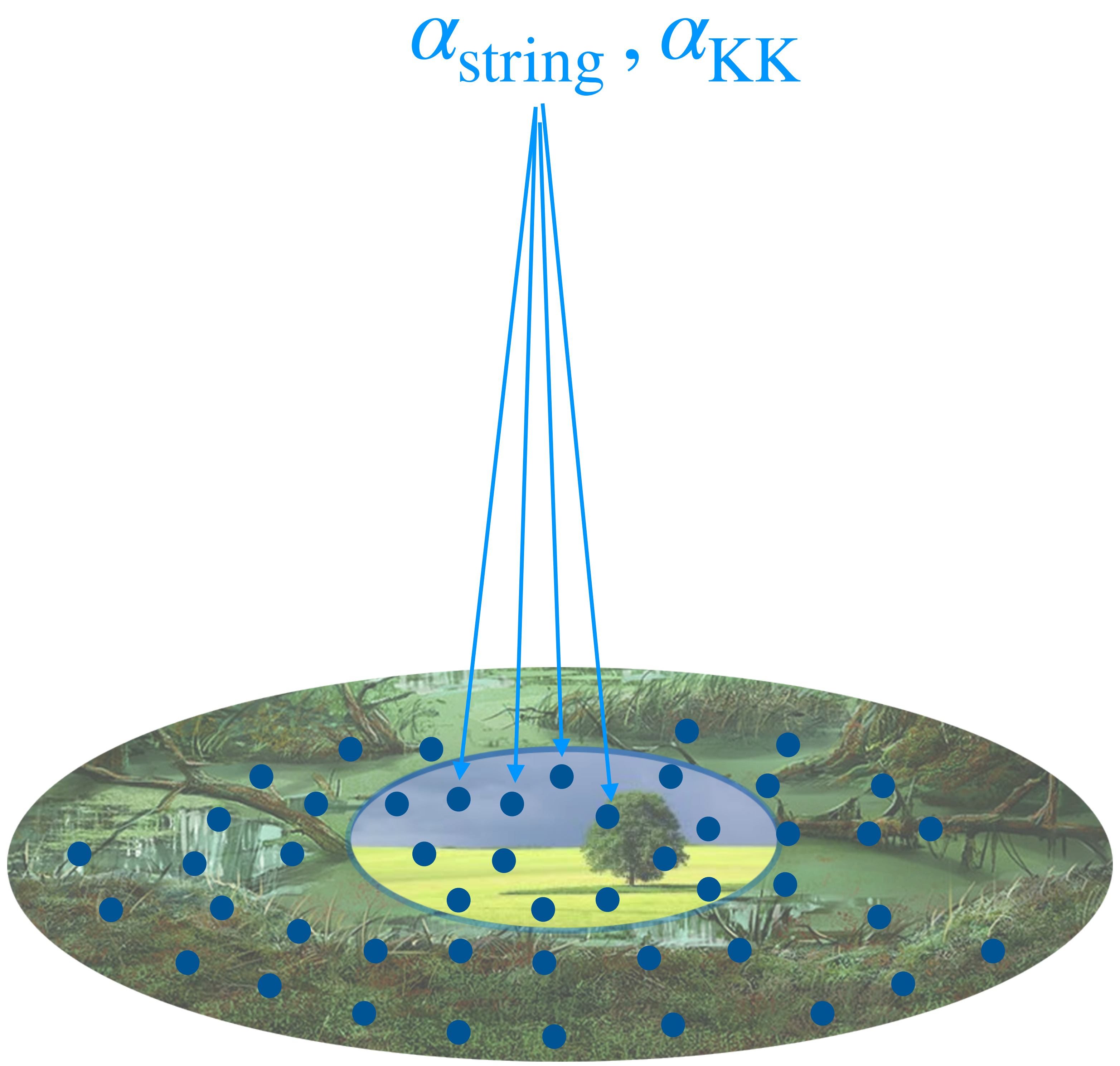

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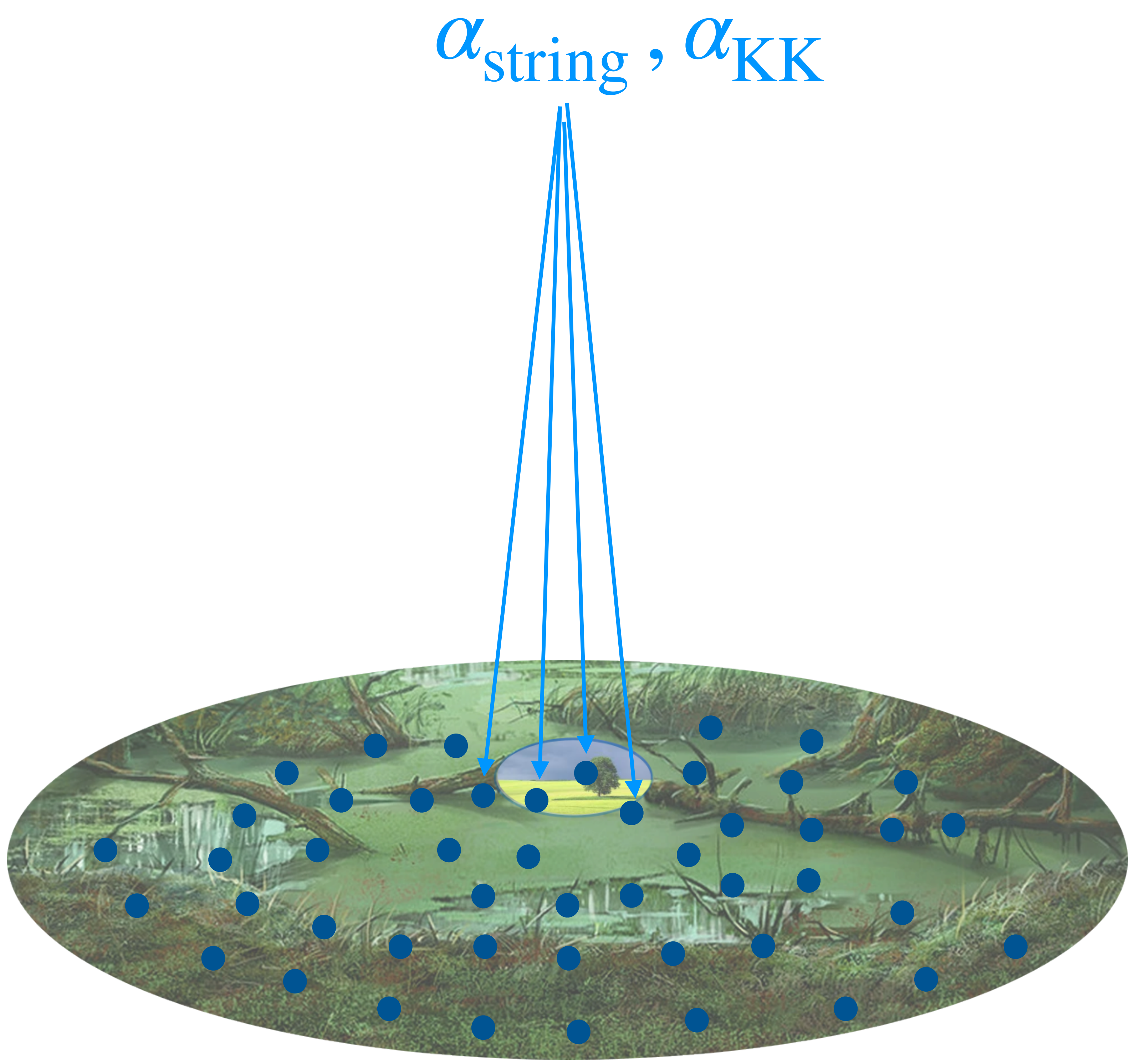

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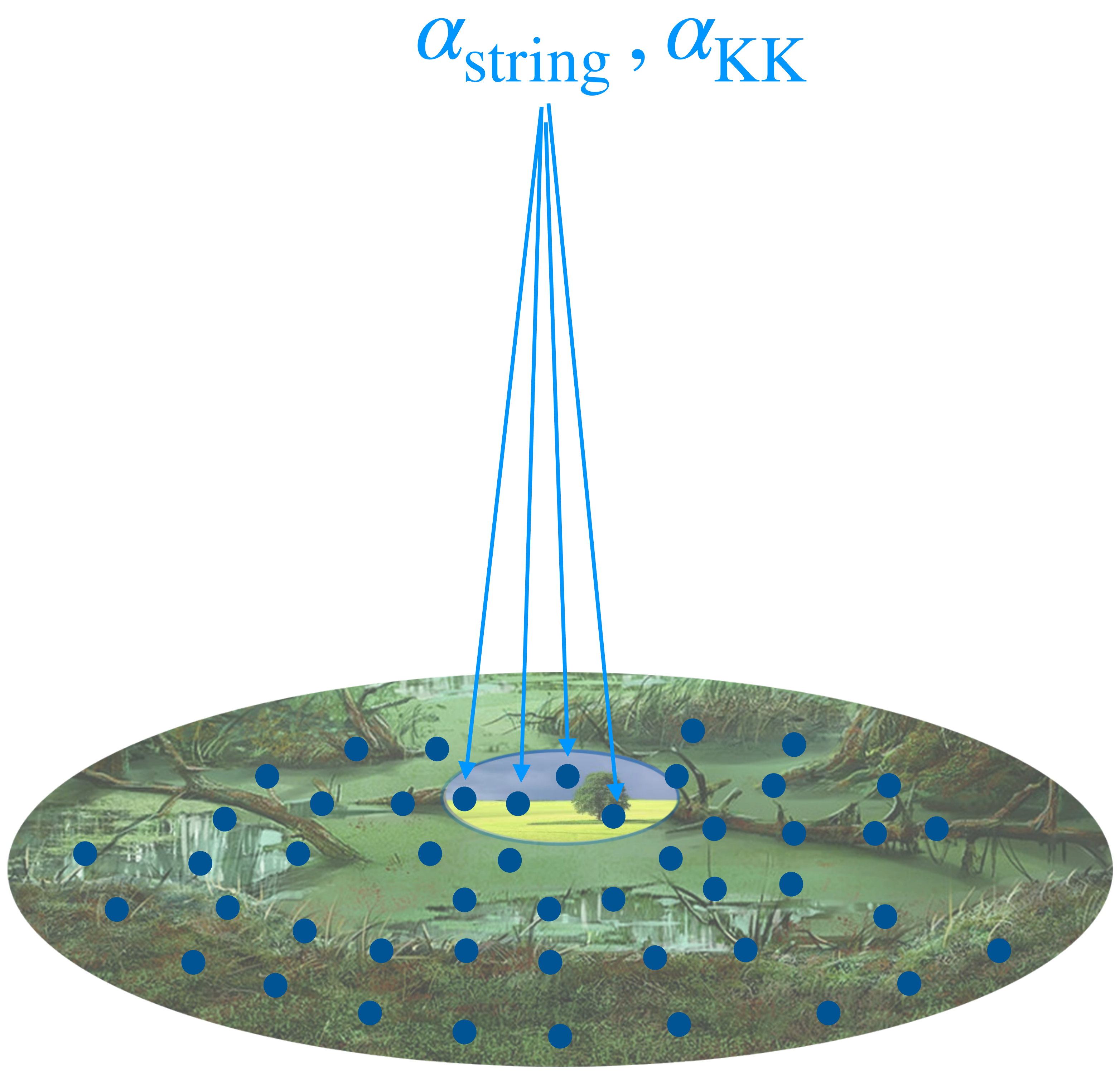

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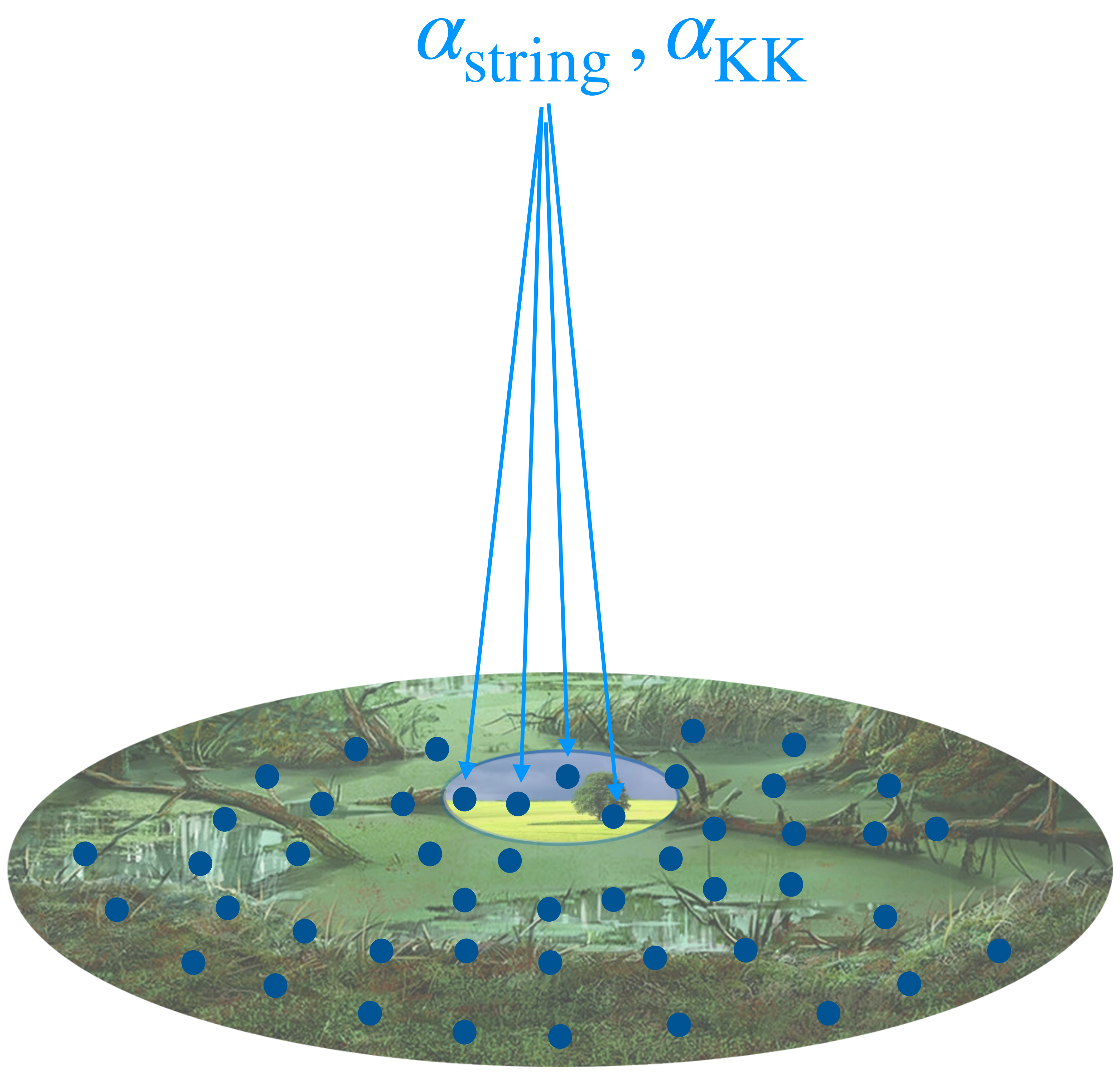

?

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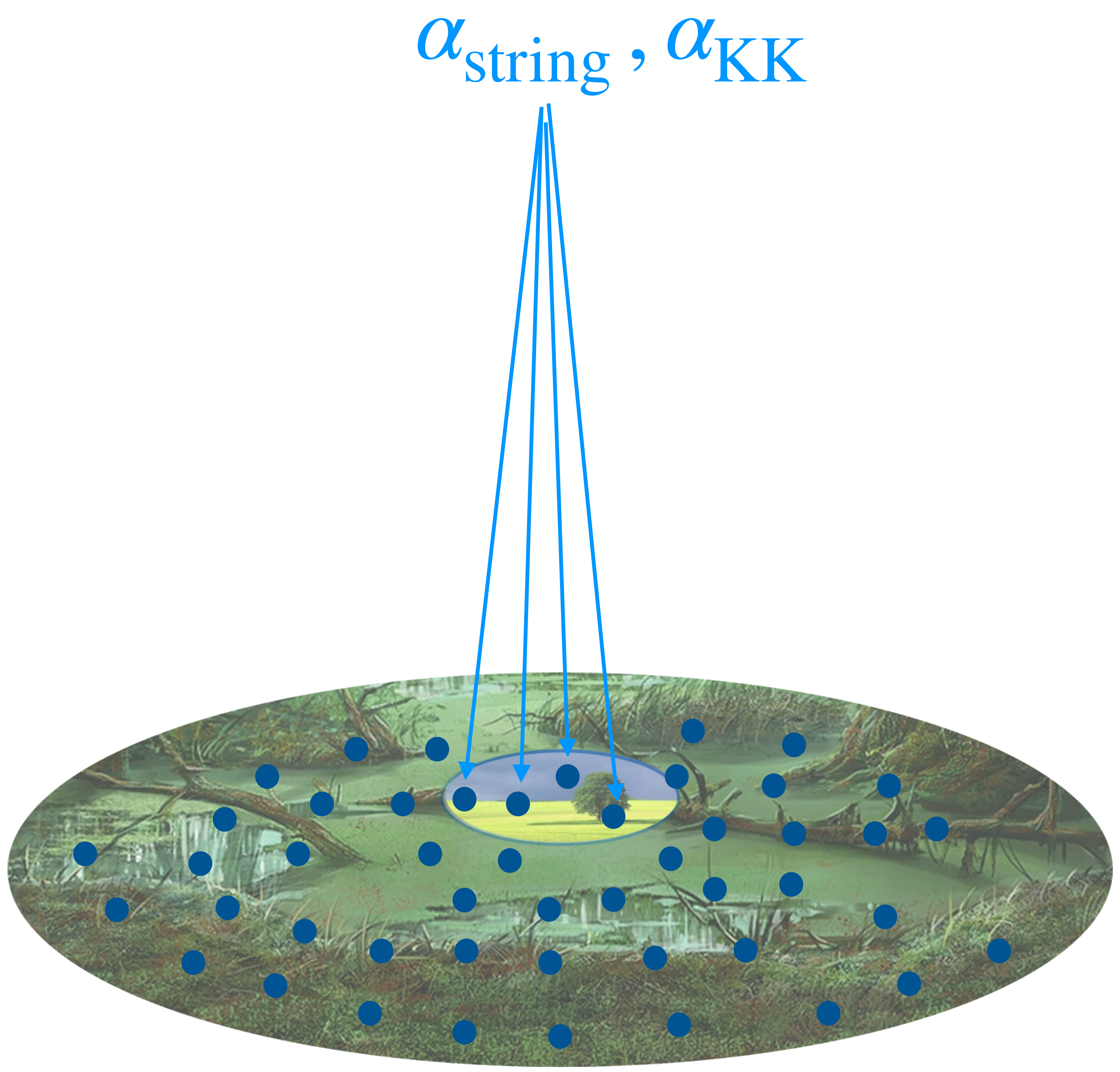

?

Thank you!

Backup slides

A generalisation

$$d_n = \begin{cases} \alpha_{\text{string}} = \sqrt{\dfrac{1}{d-2}} & \text{co-dim 1 facet} \\ \alpha_{\text{KK,n}} = \sqrt{\dfrac{1}{d-2} + \dfrac{1}{n}} & n = \text{dim of facet} + 1 \end{cases}$$

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co-dim 1 facet

$n =$ dim of facet +1

$\downarrow$ generalised to

$n =$ dim of facet + n_{V}

$n_{\text{V}} > 1 \Rightarrow$ no KK tower of a single dimension decompactifying. E.g. T^2 with $\frac{R_1}{R_2}$ fixed

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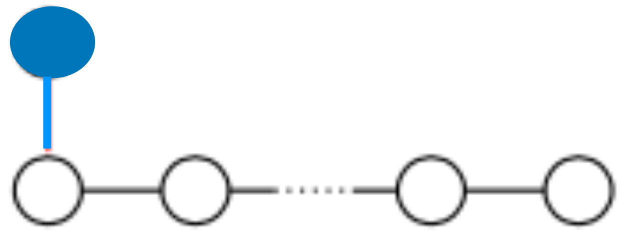
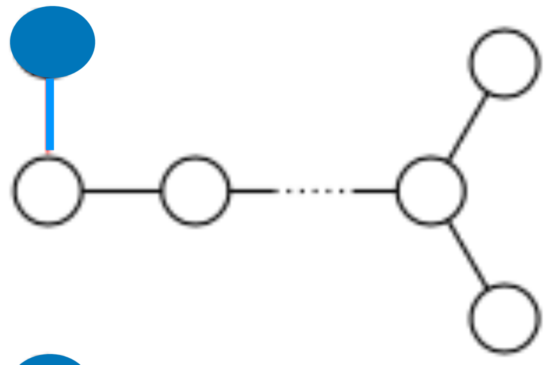
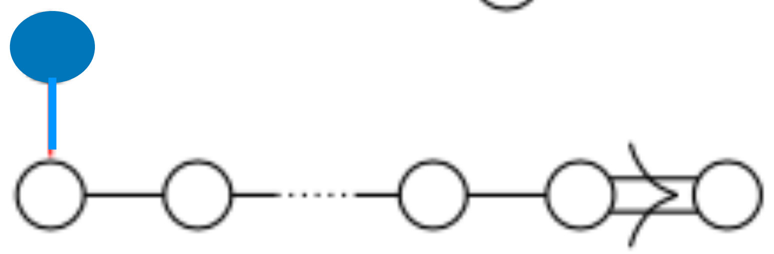
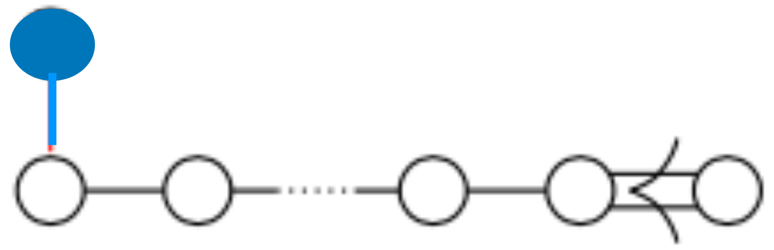
r	$\mathfrak{g}$	$\frac{r_{\text{long}}}{\sqrt{2}} \times \lambda$	d	n_V	$D_{\max}$	$\rho_{\min}$	$\lvert r_{\text{long}}\rvert^2$	polytope
2	$\mathfrak{sl}_3$	$\sqrt{\frac{2}{3}}(1,1)$	3	3	6	**8**	$\frac{4}{3}$	
	$\mathfrak{g}_{2(2)}$	$\sqrt{\frac{2}{3}}(0,1)$				**14**	$\frac{4}{3}$	
	$\mathfrak{g}_{2(2)}$	$\sqrt{2}(1,0)$				**7**	4	
3	$\mathfrak{sp}_4$	$\frac{1}{\sqrt{2}}(1,1)$	3	4	11	**16**	1	
	$\mathfrak{sp}_4$	$\frac{1}{2}(2,1)$				**35**	$\frac{1}{2}$	

Can use λ Extended Dynkin Diagrams

- Facets: simplices and orthoplexes

simplex Δ_k: $\mathfrak{sl}(k+1,\mathbb{R})$ 

orthoplex β_k:

- $\mathfrak{so}(k,k,\mathbb{R}))$ 
- $\mathfrak{so}(k,k+1,\mathbb{R})$ 
- $\mathfrak{sp}(2k,\mathbb{R})$ 

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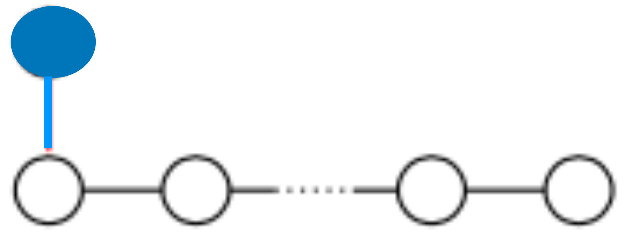
simplex Δ_k: $\mathfrak{sl}(k+1,\mathbb{R})$ 

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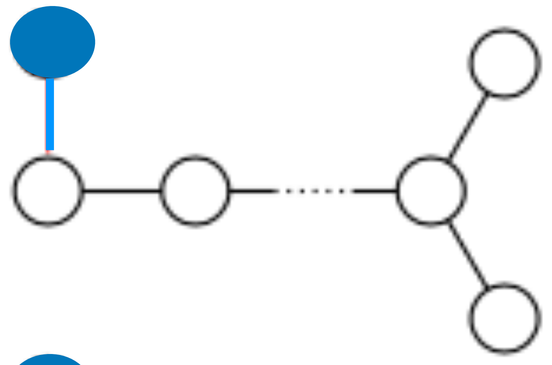
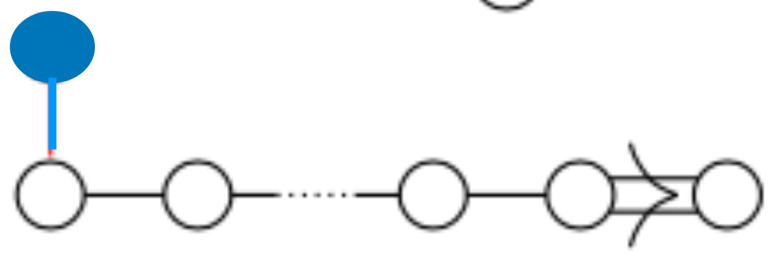
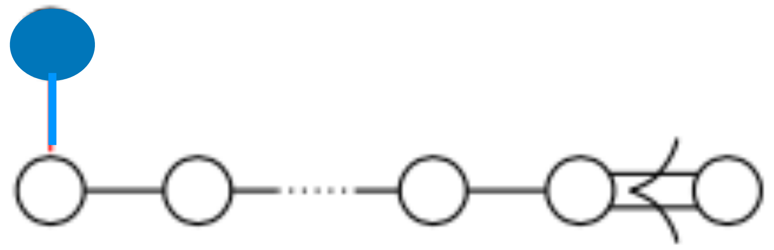
$\mathfrak{so}(k,k,\mathbb{R}))$ 

$\mathfrak{so}(k,k+1,\mathbb{R})$ 

$\mathfrak{sp}(2k,\mathbb{R})$ 

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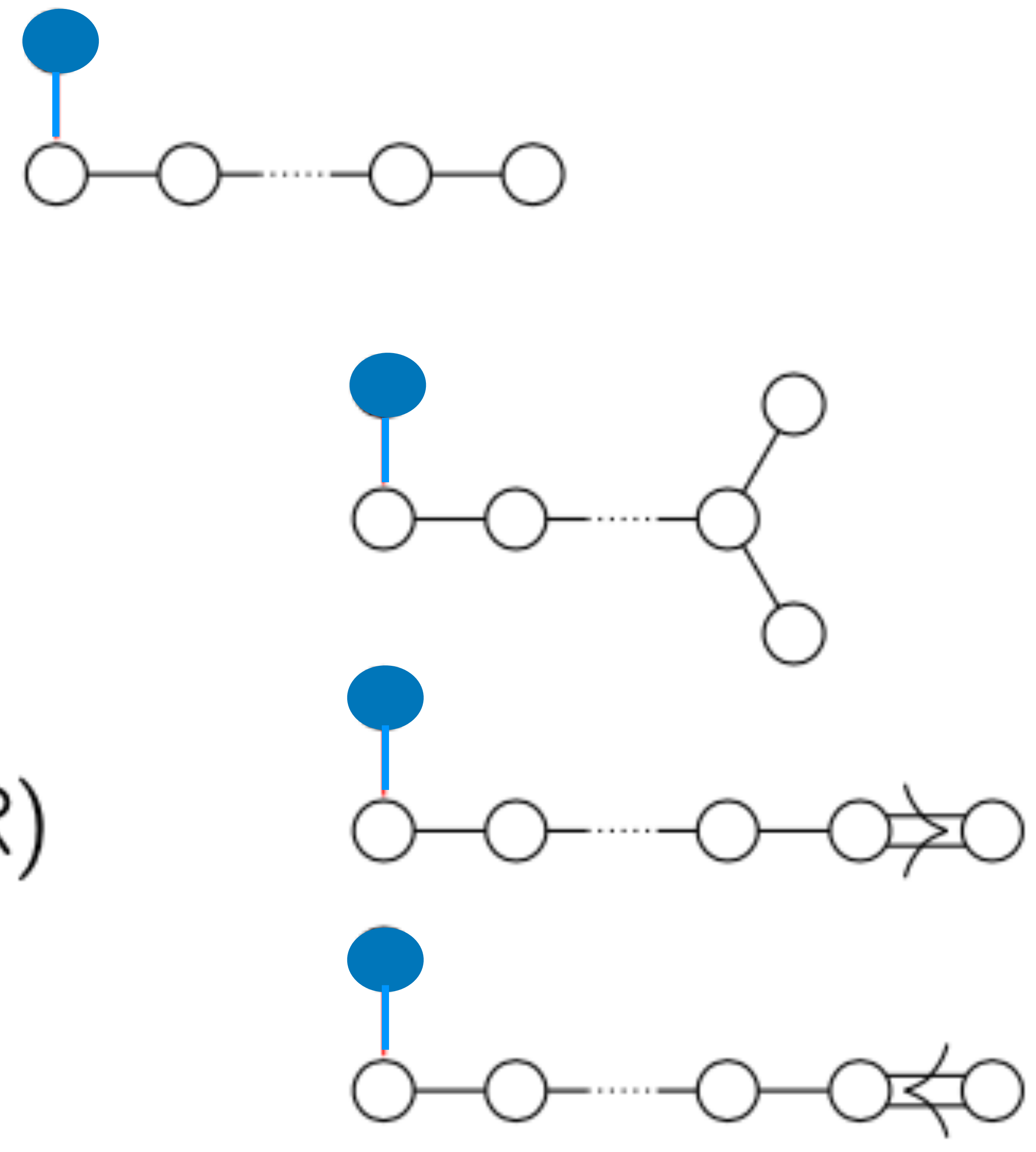

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\- if $G = \mathbb{R} \times G' \rightarrow$ cone over this with appropriate $q \Rightarrow$ fundamental vector rep

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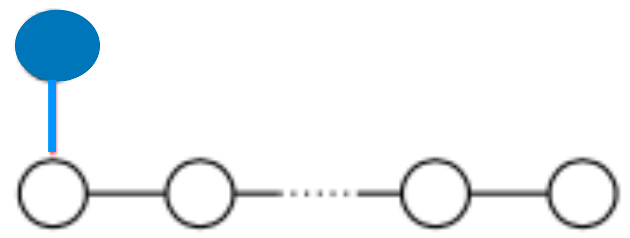
simplex Δ_k: $\mathfrak{sl}(k+1,\mathbb{R})$ 

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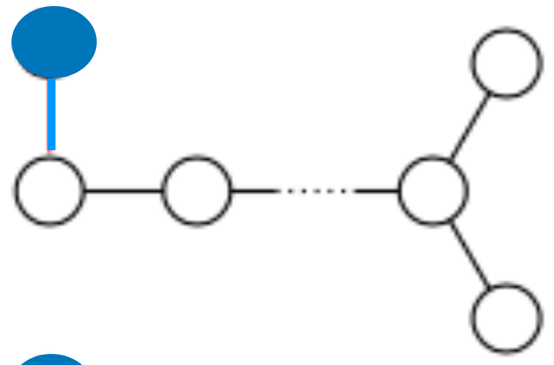

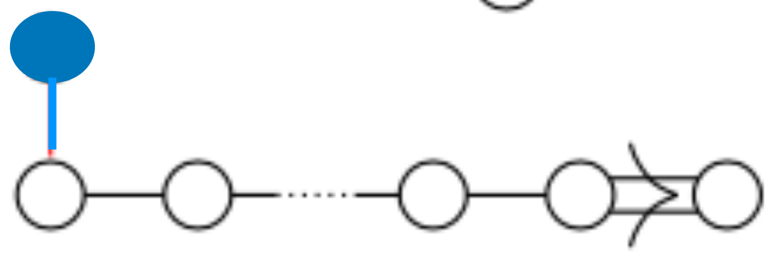

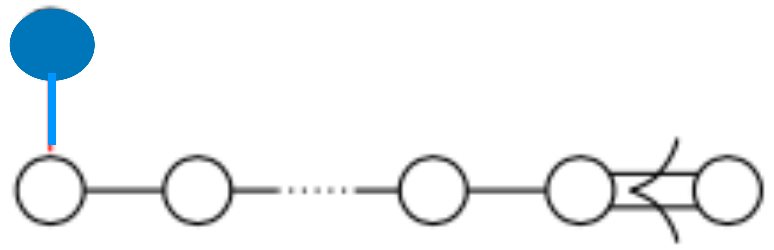

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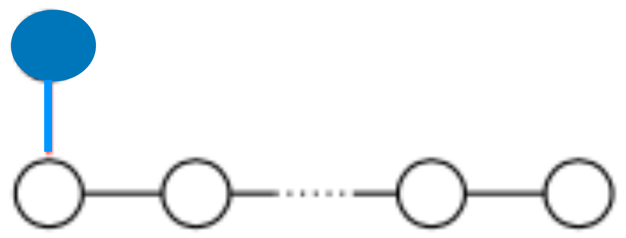
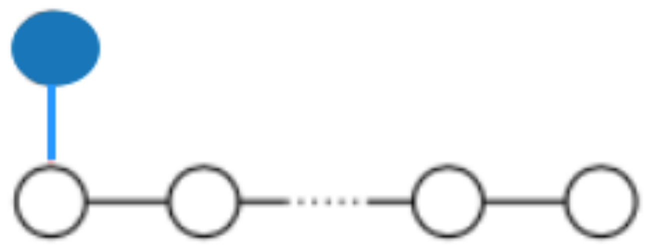
- if G semi-simple $\rightarrow$ last node such that all subdiagrams give simplices or orthoplexes

Can use λ Extended Dynkin Diagrams

•Facets: simplices and orthoplexes

simplex Δ_k: $\mathfrak{sl}(k+1, \mathbb{R})$  $\longrightarrow$ 

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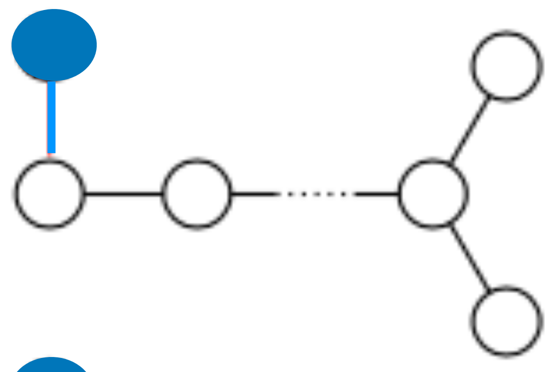

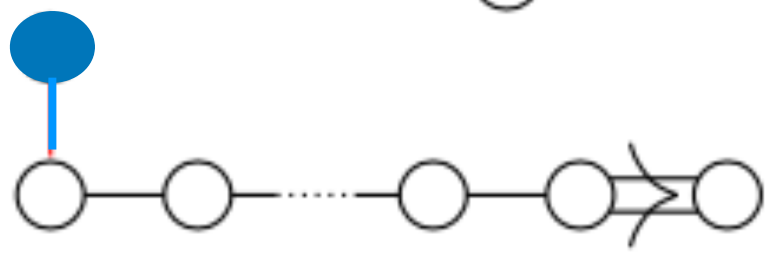

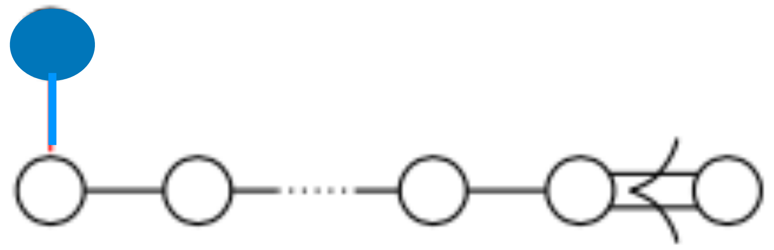

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Can use λ Extended Dynkin Diagrams

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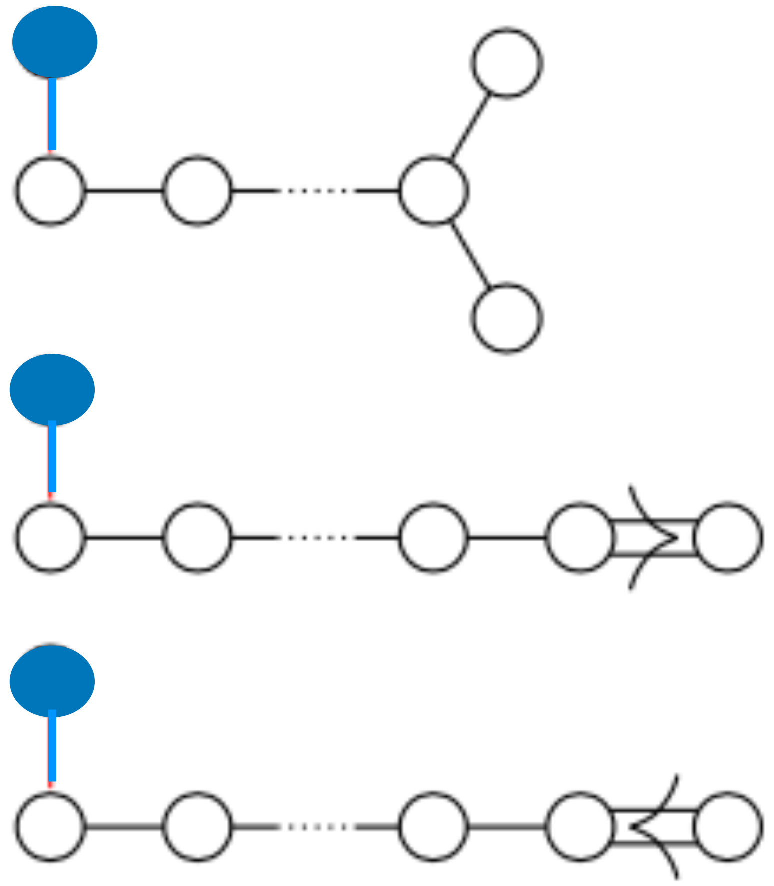

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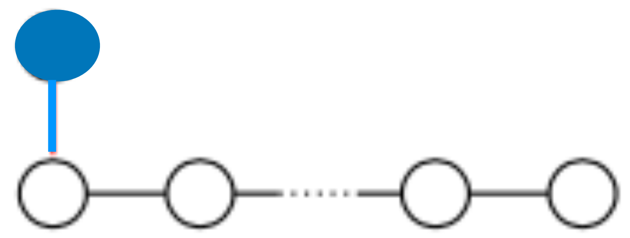
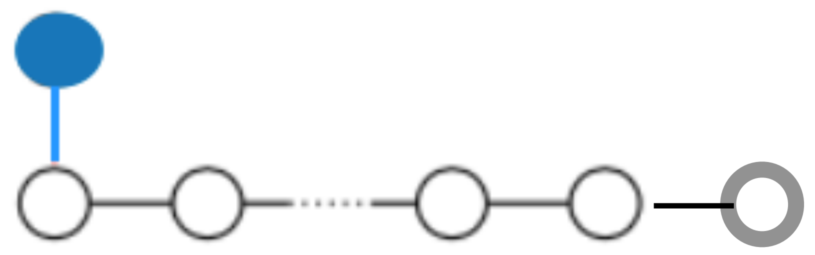


$\rightarrow$



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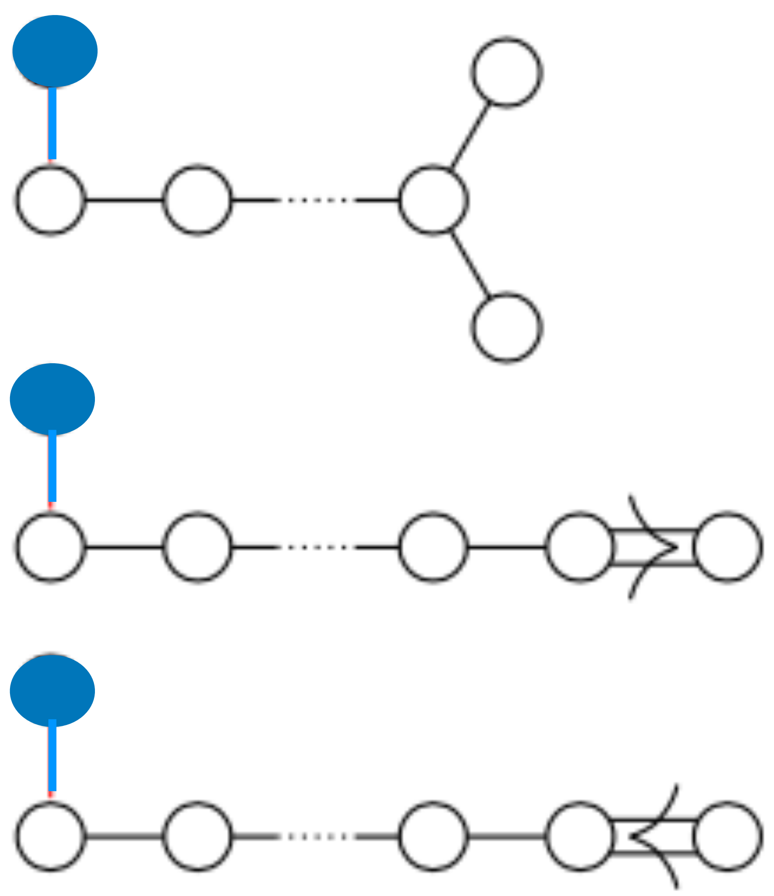

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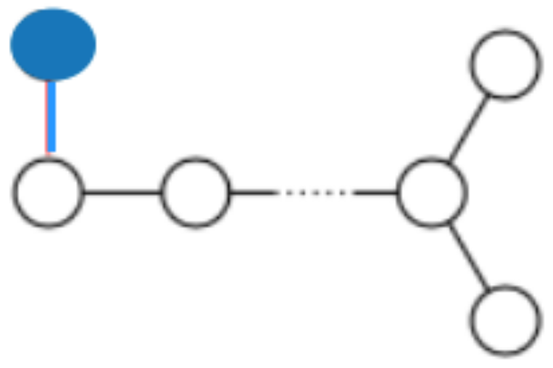

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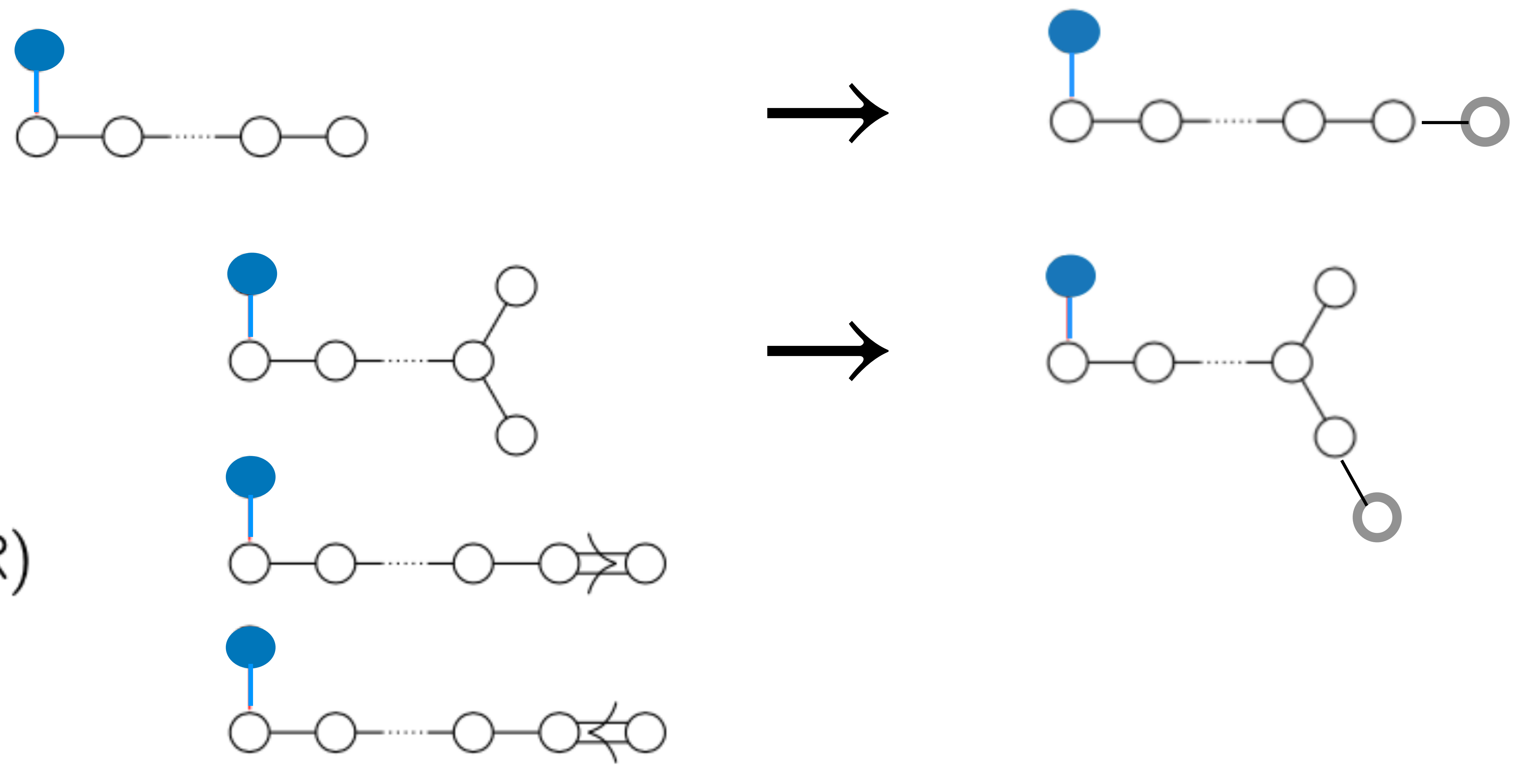

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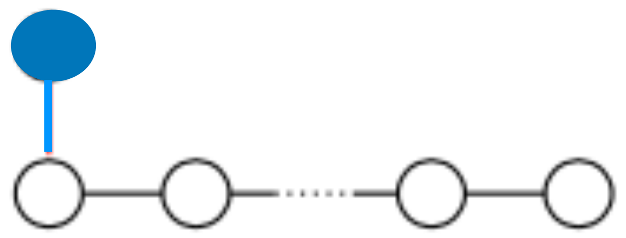
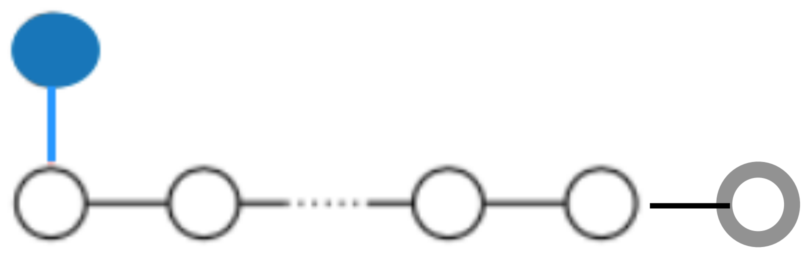
simplex Δ_k: $\mathfrak{sl}(k+1,\mathbb{R})$  $\longrightarrow$ 

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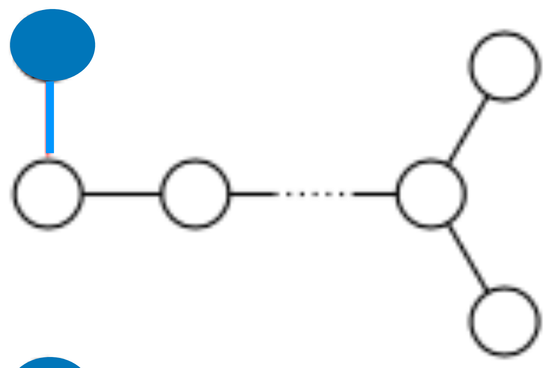
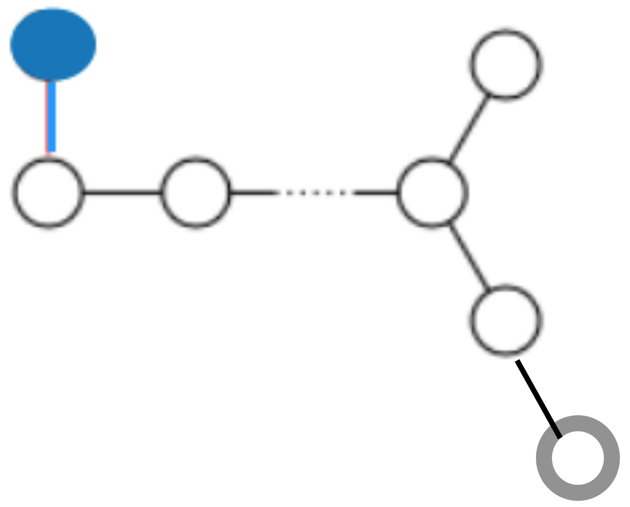
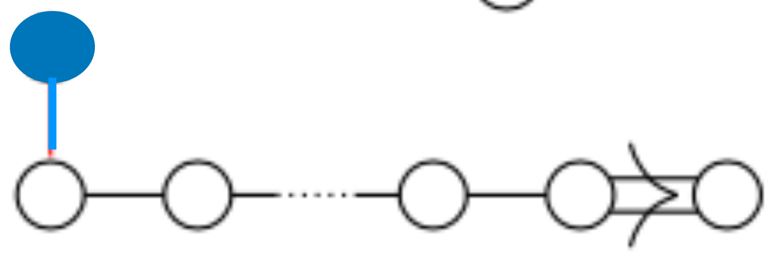
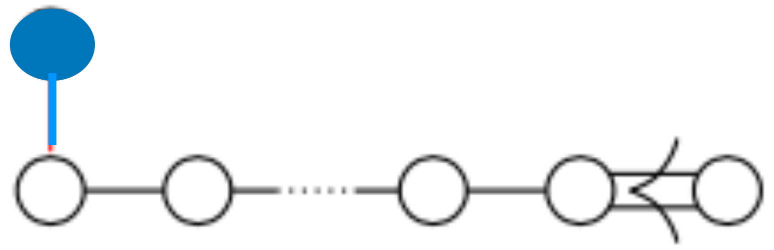
 $\longrightarrow$ 





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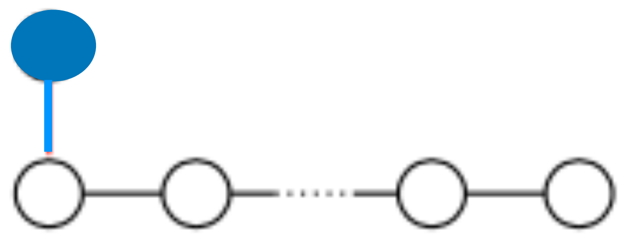
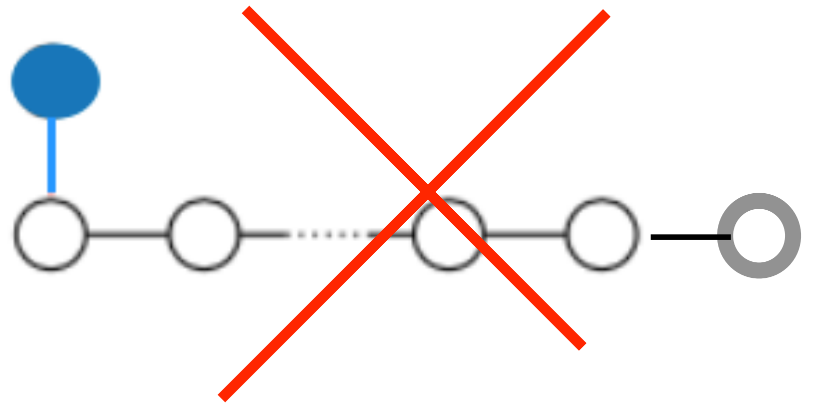
simplex Δ_k: $\mathfrak{sl}(k+1,\mathbb{R})$  $\longrightarrow$  $d \notin \mathbb{N}$

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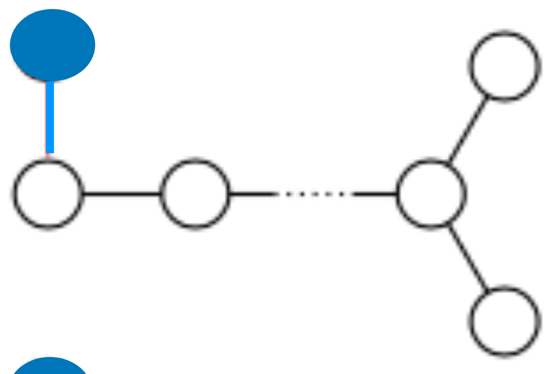
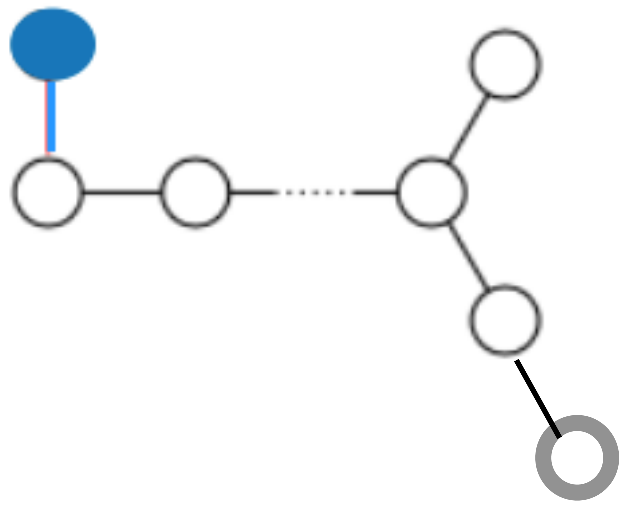
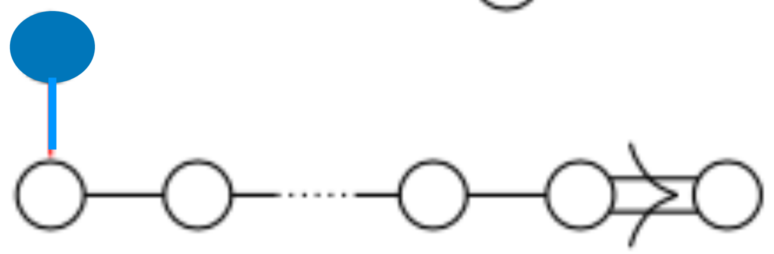
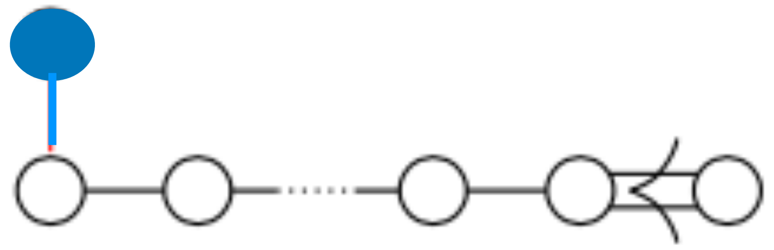
 $\longrightarrow$ 





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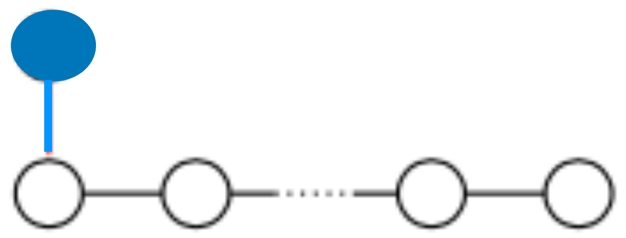

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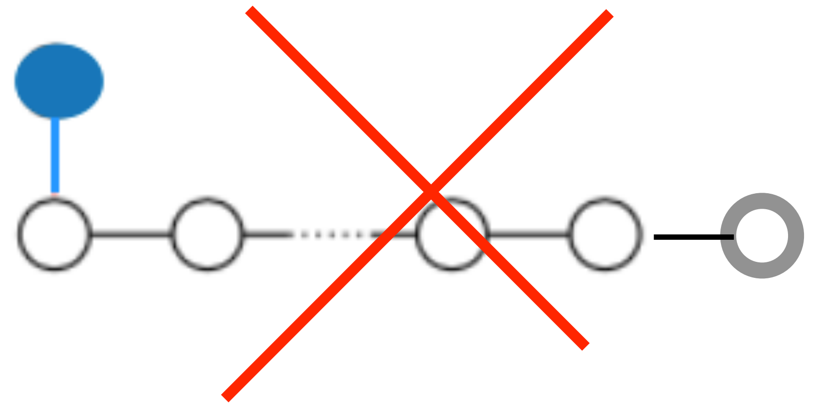

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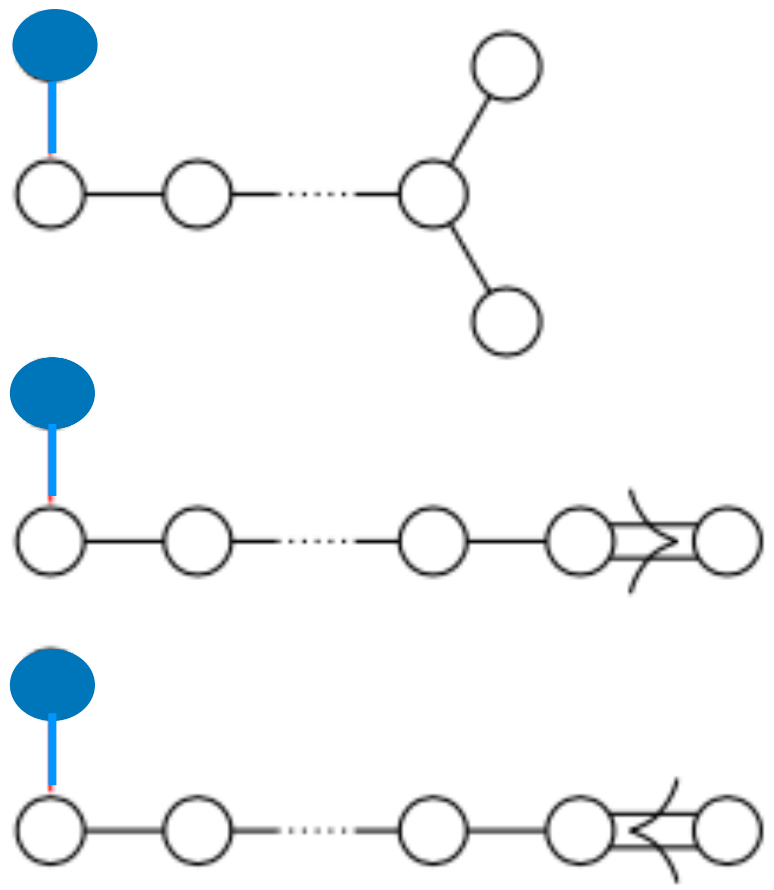

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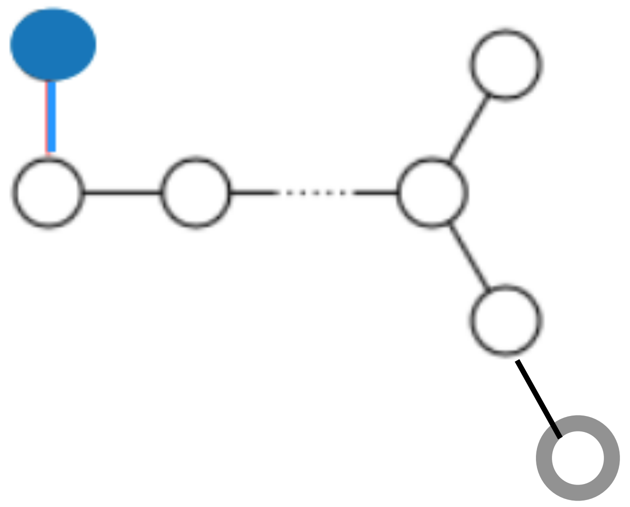

$\boldsymbol{e}_{k+1(k+1)}$

$d = 11 - (k+1)$

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