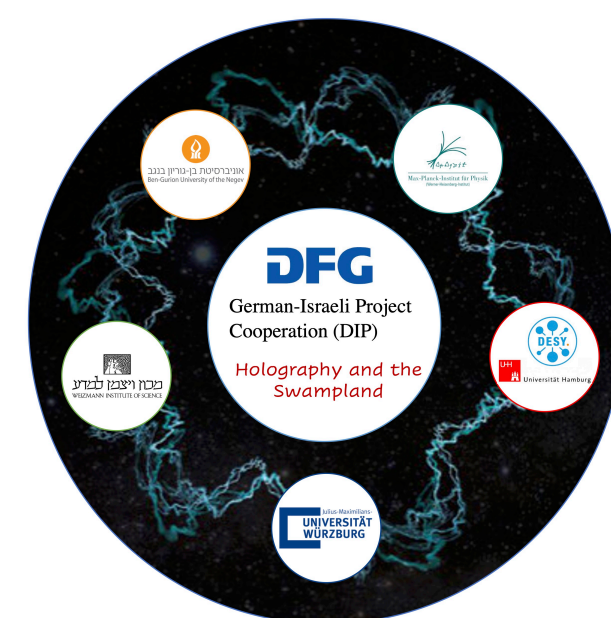


Infinity and Finiteness in Strings and Geometry

- [2603.02304](#) [hep-th] with [Fabio Mantegazza](#), [Enrico Marchetto](#), [Eli Pomoni](#), [Thorben Skrzypek](#)
- [2603.24666](#) [math.AG] with [Antonella Grassi](#), [Rick Miranda](#), [Kapil Paranjape](#), [Vasudevan Srinivas](#)

Timo Weigand, Strings and Geometry, Uppsala, May 21, 2026



**CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE**



Two hallmarks of Quantum Gravity

Infinite gravitational towers
asymptotically in moduli space:

Swampland Distance Conjecture
[Ooguri,Vafa'06]

Swampland Program /
Quantum Gravity

Finiteness of
light, gravitational
degrees of freedom in
interior of moduli space

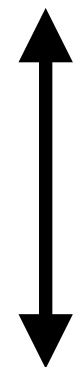
see talk by D. Prieto

Duality [Ooguri,Vafa'06] /
Emergent String Conjecture
[Lee,Lerche,TW'19]

Two hallmarks Quantum Gravity

Infinite gravitational towers
asymptotically in moduli space:

Swampland Distance Conjecture
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Part I:

Nature of towers in AdS:

**Partial weak coupling
limits in 4d N=2**

**Swampland Program /
Quantum Gravity**



Duality [Ooguri,Vafa'06] /
Emergent String Conjecture
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Part II:

Bounds on MW group of
elliptic fibrations:

Math proofs and new results

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Duality [Ooguri, Vafa'06] /
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STRINGS & GEOMETRY

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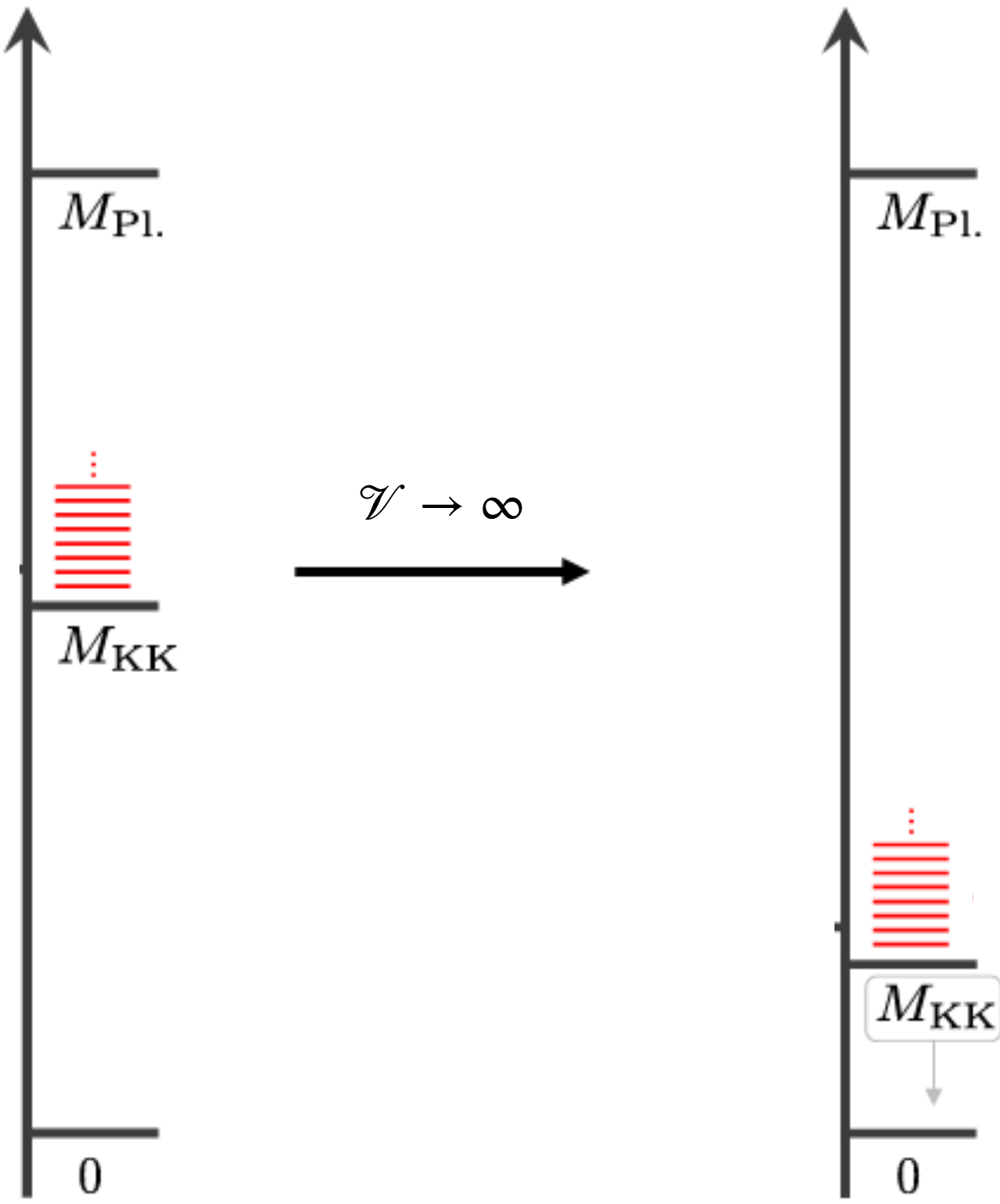
Part I:

String Towers in AdS gravity

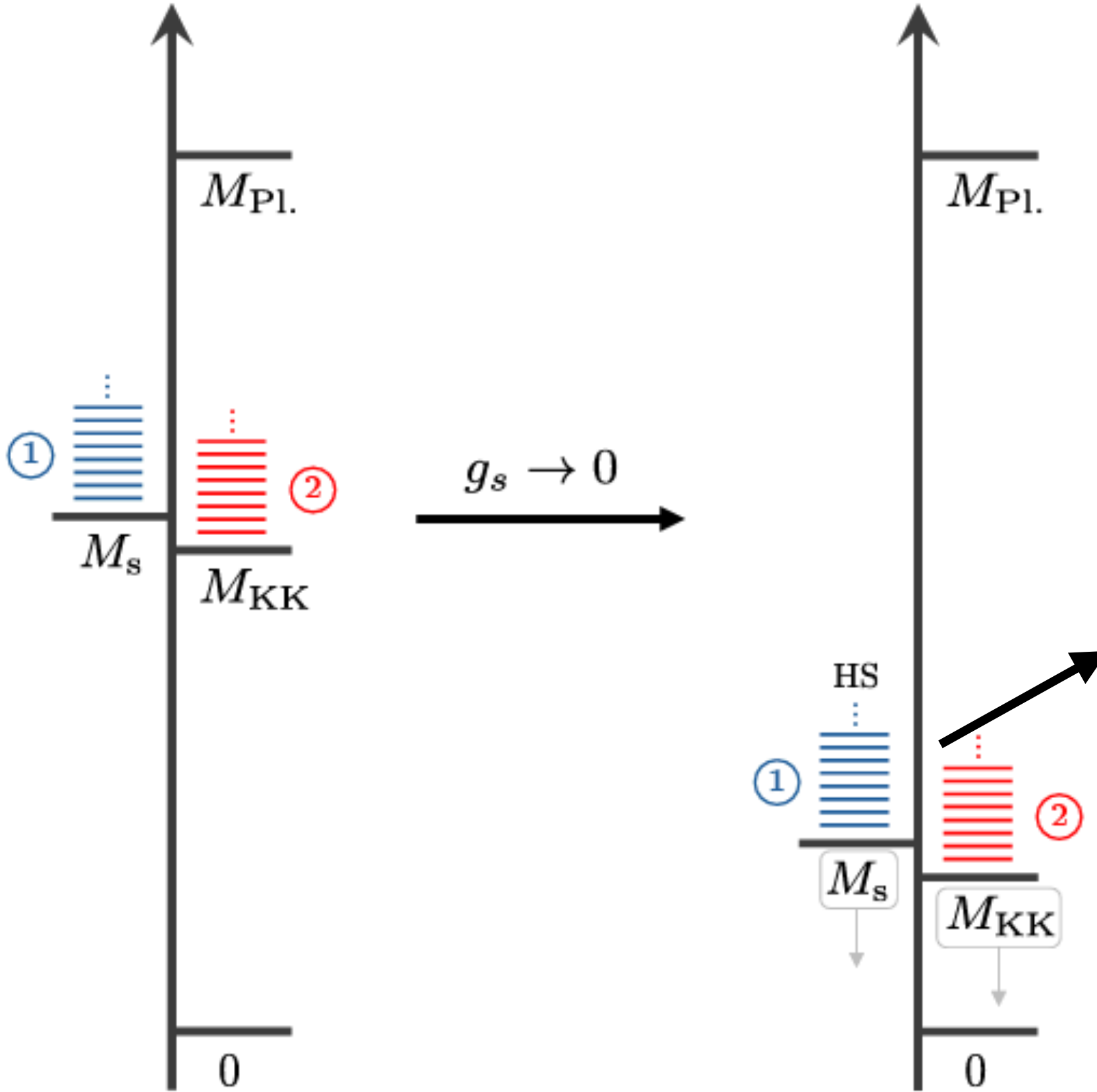
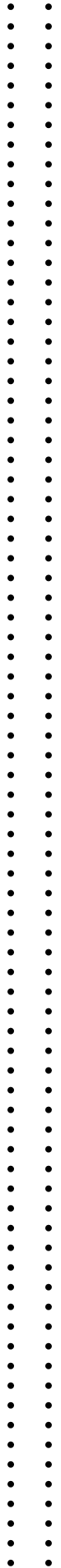
[Mantegazza, Marchetto, Pomoni, Skrzypek, TW'26]

Infinite distance limits in flat space gravity

Decompactification limit



Emergent string coupling



String tower:

- fundamental string
- parametrically at KK scale
- exponential degeneracy:
 $d \sim e^{cm}$

What is the analogue in AdS_{d+1} ?

Infinite distance directions via dual CFT_d (conformal manifold)

CFT Distance Conjecture: [Perlmutter,Rastelli,Vafa,Valenzuela'20] [Baume,Calderon-Infante'20]

$d \geq 3$: Infinite Distance loci are weak coupling and give rise to towers of higher-spin (HS) currents.

- Proof of HS \rightarrow weak coupling: [Baume, Calderon-Infante'23]
- Detailed study in overall weak coupling limits: [Calderon-Infante,Valenzuela'24] [Calderon-Infante, Mohseni'26]
[Baume,Mantegazza-to appear]

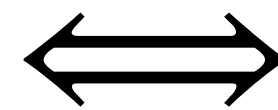
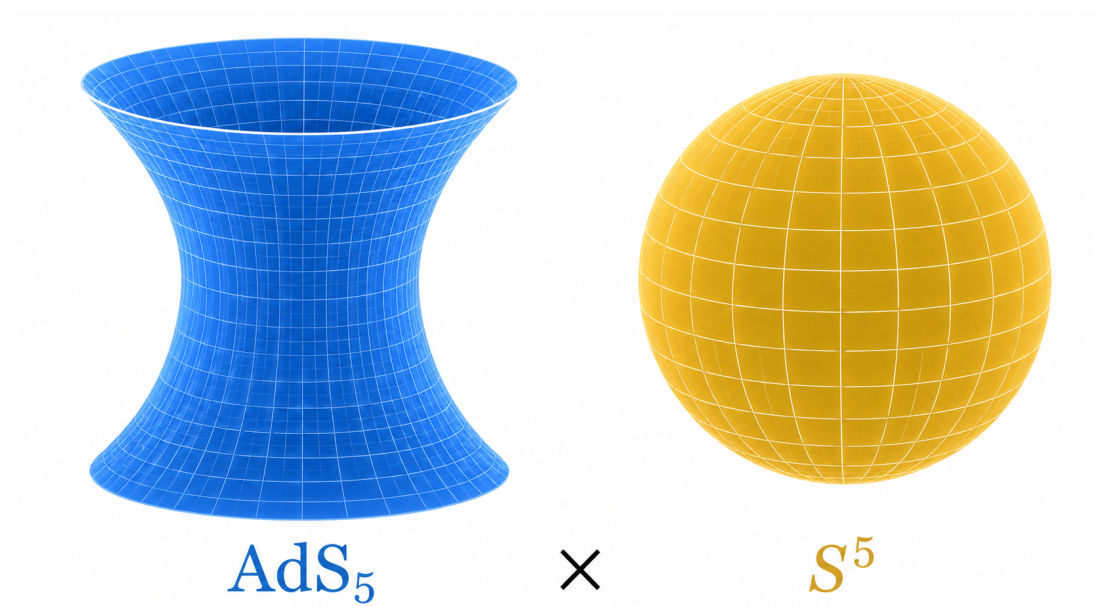
\rightsquigarrow talk by Irene Valenzuela

This talk:

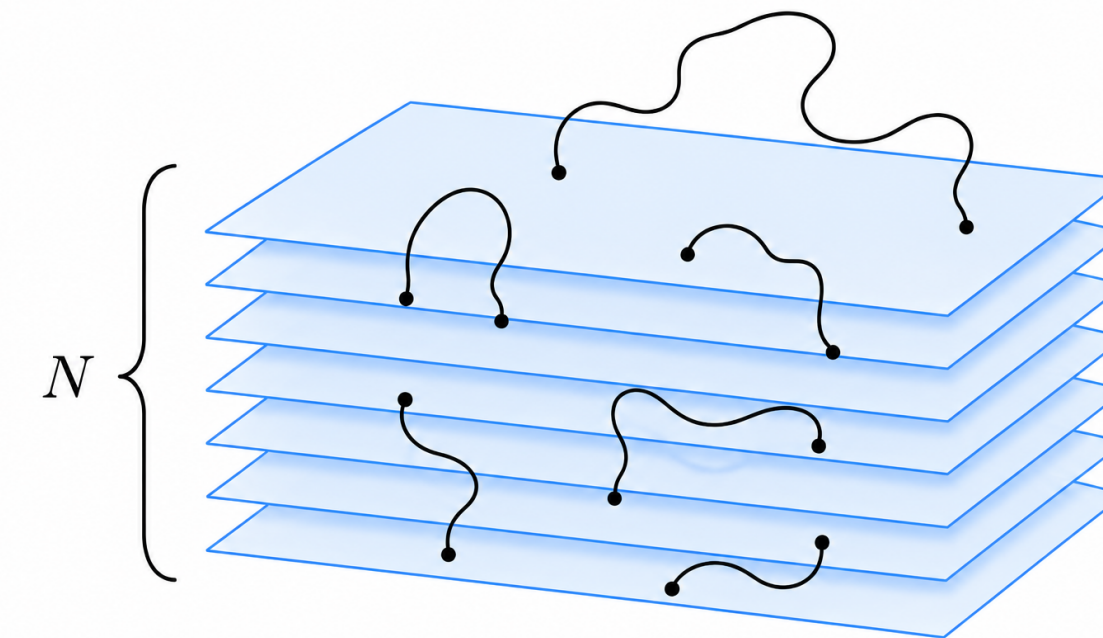
- Mixed weak coupling limits and
- comparison to emergent strings in flat space

Reminder: $AdS_5 \times S^5$

Type IIB on $AdS_5 \times S^5$



$N = 4$ SYM on $\mathbb{R}^{1,3}$



5d Planck scale

$$M_{Pl,5}$$

AdS scale

$$M_{AdS} \sim \ell_{AdS}^{-1}$$

String scale

$$M_s \sim \ell_s^{-1}$$

rank N

modulus: 't Hooft coupling $\lambda = g_{YM}^2 N$

$$\lambda = 4\pi g_s N \sim \frac{M_s^4}{M_{AdS}^4}$$

$$\frac{M_{AdS}^3}{M_{Pl}^3} \sim \frac{1}{N^2}$$

$N \gg 1$, fixed

Reminder: $AdS_5 \times S^5$

$$\frac{M_s^4}{M_{AdS}^4} \sim \lambda \gg 1 \quad \text{effective supergravity regime}$$

States via (strongly coupled) dual CFT:

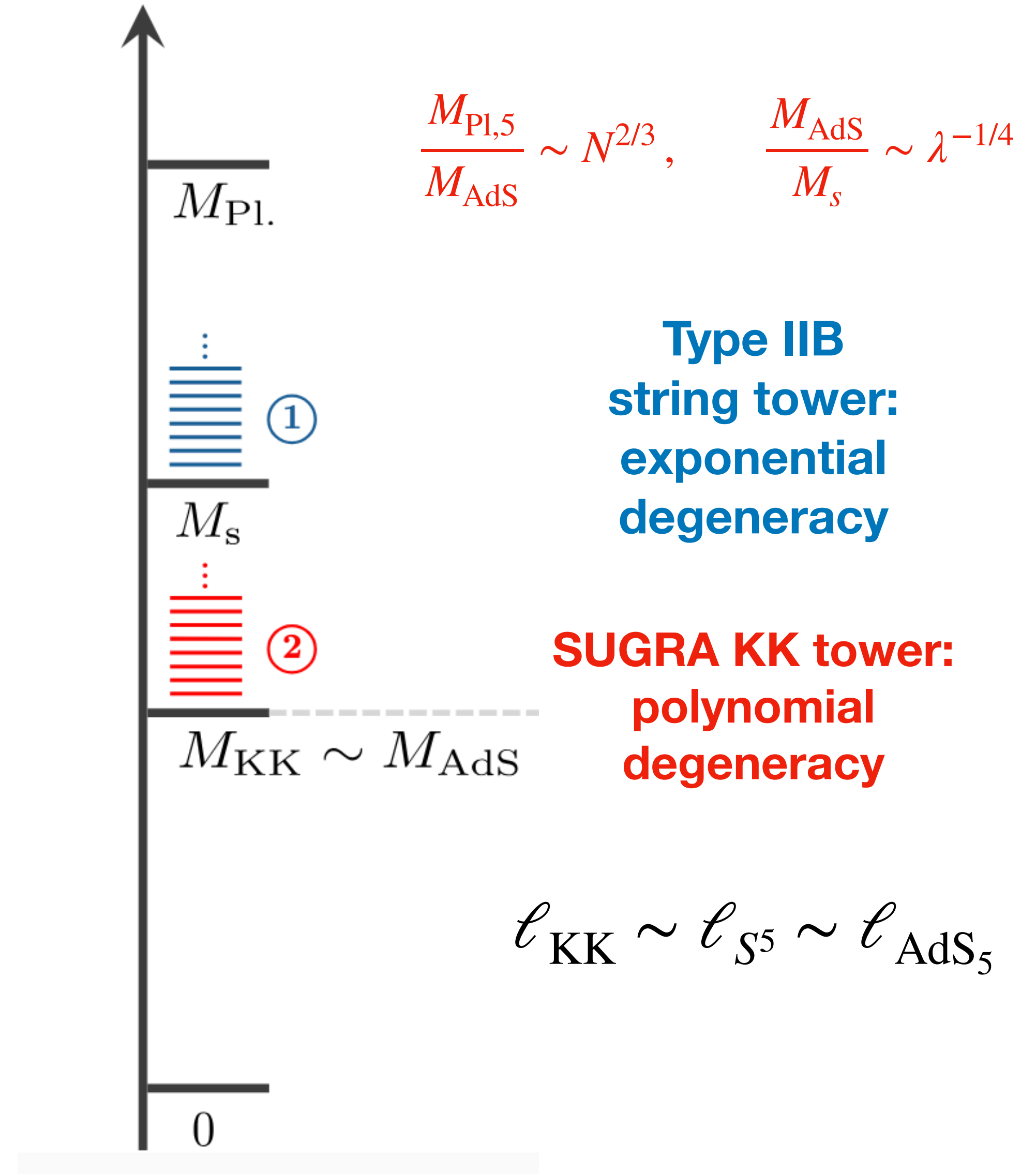
$$\frac{m_{bos}^2}{M_{AdS}^2} = (\Delta - j - \bar{j} - 2)(\Delta + j + \bar{j} - 2)$$

[Metsaev'03]

(j, \bar{j}) : spin of dual SYM operator

Δ : conformal dimension of dual SYM operator

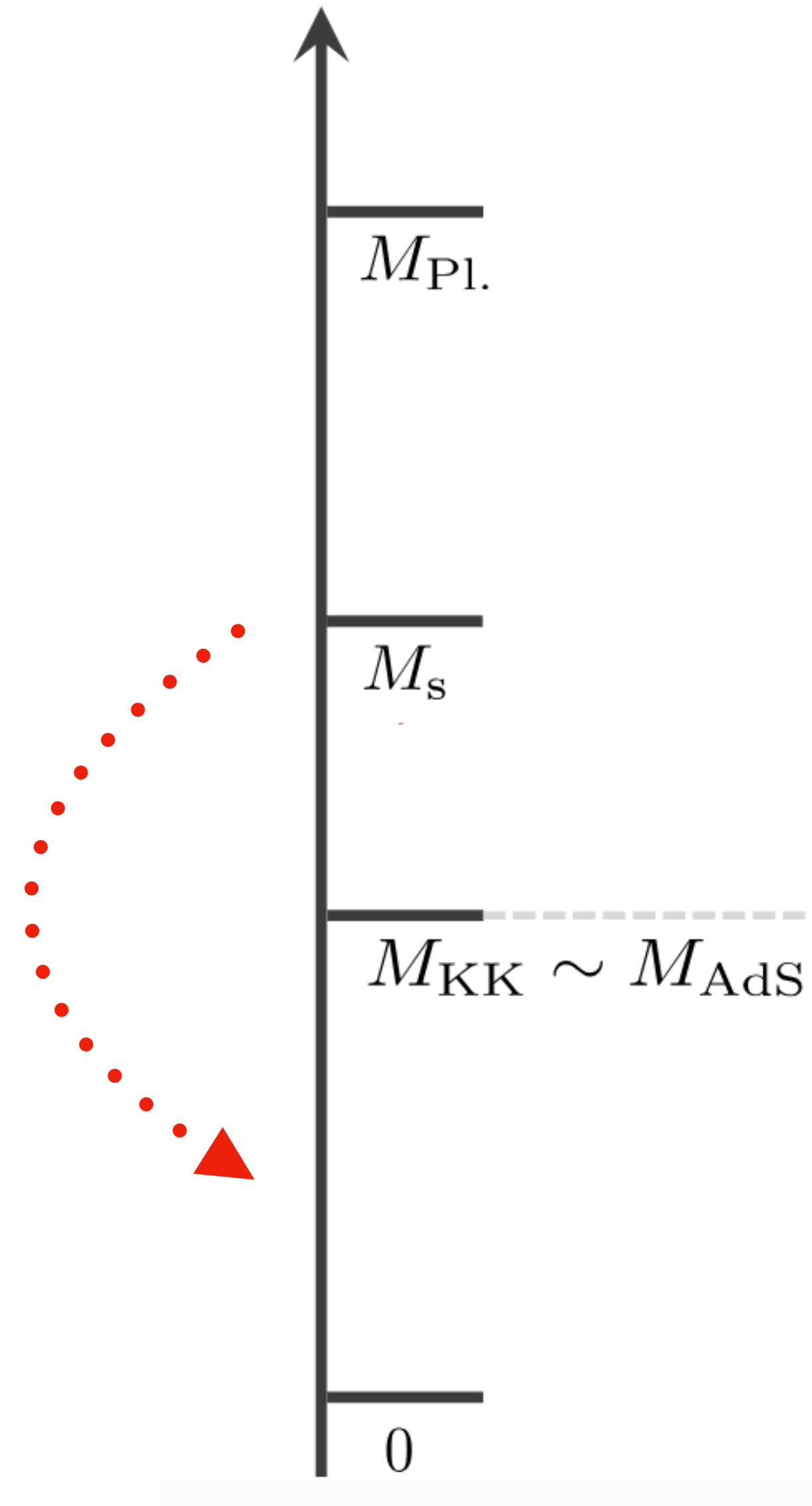
$$\Delta(\lambda) = \Delta_0 + \gamma(\lambda) \quad \gamma(\lambda): \text{anomalous conformal dimension}$$



Weakly coupled string limit in AdS: $N=4$

$$\frac{M_s^4}{M_{\text{AdS}}^4} \sim \lambda \rightarrow 0: \quad \text{substringy regime:}$$

Better described by dual weakly coupled CFT



Weakly coupled string limit in AdS: $N=4$

$\frac{M_s^4}{M_{\text{AdS}}^4} \sim \lambda \rightarrow 0$: **substringy regime:**

Better described by dual weakly coupled CFT

• **Define** an “effective mass”:

$$\frac{m_{\text{eff.}}^2}{M_{\text{AdS}}^2} = (\Delta - j - \bar{j} - 2)(\Delta + j + \bar{j} - 2)$$

$$\Delta(\lambda) = \Delta_0 + \gamma(\lambda) \rightarrow \Delta_0$$

• **Asymptotically massless states:**

$$\Delta_0 = 2 + J, \quad j = \bar{j} = \frac{J}{2}$$

$$\frac{m_{\text{eff.}}^2}{M_{\text{AdS}}^2} \sim 2J\lambda + \mathcal{O}(\lambda^2) \rightarrow 0$$

Higher-spin tower

$$\text{tr}[\phi^i \partial^J \phi_i] + \text{tr}[\bar{\lambda}^I \partial^{J-1} \lambda_I] + \text{tr}[\bar{F} \partial^{J-2} F]$$

as predicted by CFT
Distance Conjecture

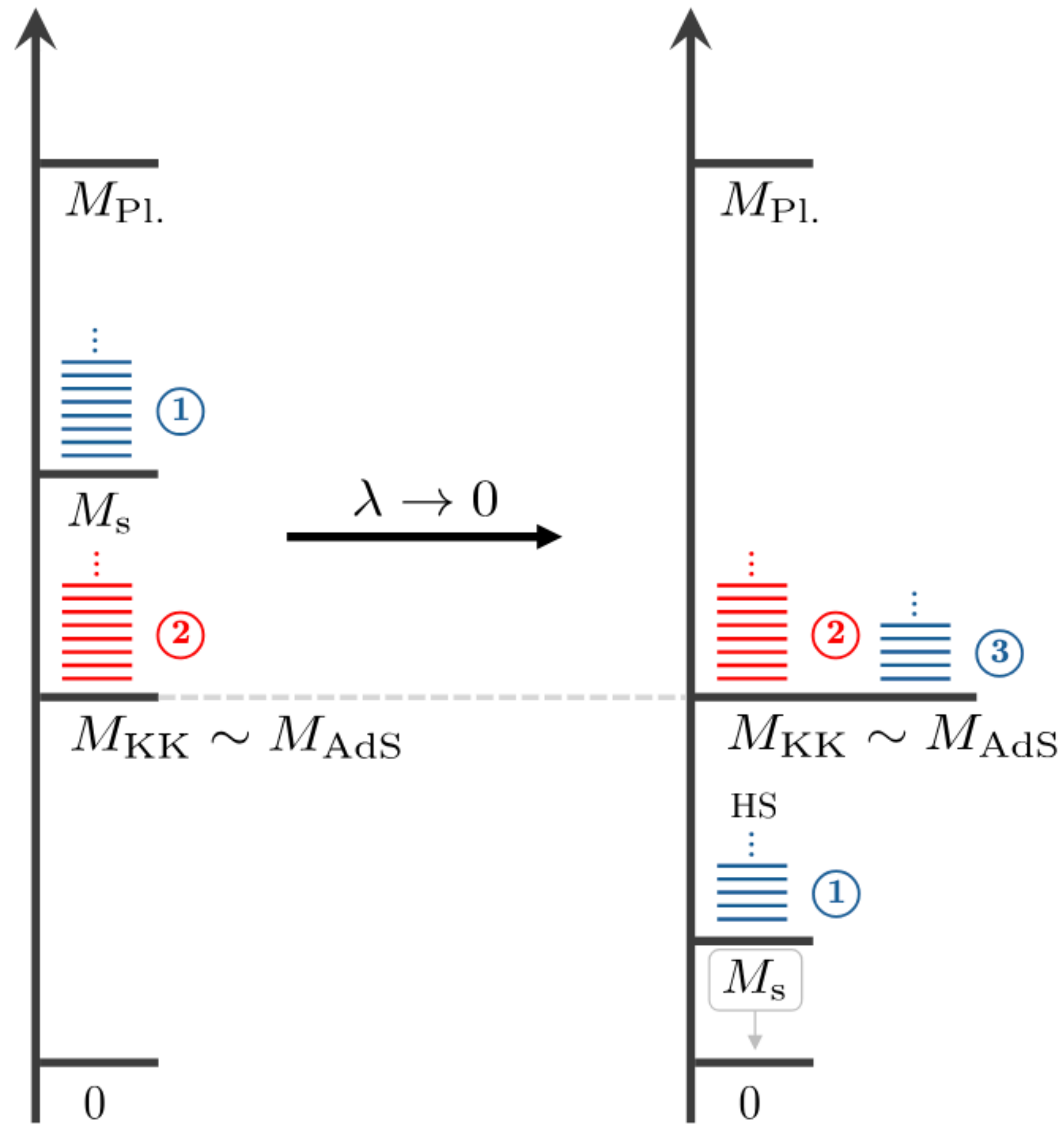
• But: Polynomial rather than exponential degeneracy

$$\text{deg}(m_{\text{eff.}}) \sim J^2$$

[Perlmutter, Rastelli, Vafa, Valenzuela'20]

[Baume, Calderon-Infante'20,'23]

Weakly coupled string limit in AdS: $N=4$



$$\frac{M_s^4}{M_{AdS}^4} \sim \lambda \rightarrow 0: \text{ substringy regime}$$

exponential degeneracy, but stuck at AdS scale due to curvature

$$\frac{m_{eff.}^2}{M_{AdS}^2} = (\Delta - j - \bar{j} - 2)(\Delta + j + \bar{j} - 2)$$

asymptotically massless, but only polynomial degeneracy

③ Remaining string tower of exponential degeneracy

① Higher spin tower (R-charge 0)

General mixed weak coupling limit in $N=2$

String theory on $AdS_5 \times X$
with $N=2$ SUSY



Dual conformal manifold can be
higher-dimensional
with **more general weak coupling limits**

gauge couplings:

$$(g_1, g_2, \dots, g_k)$$

overall free limit:

$$\text{all } g_i \rightarrow 0$$

[Calderon-Infante, Valenzuela'24]

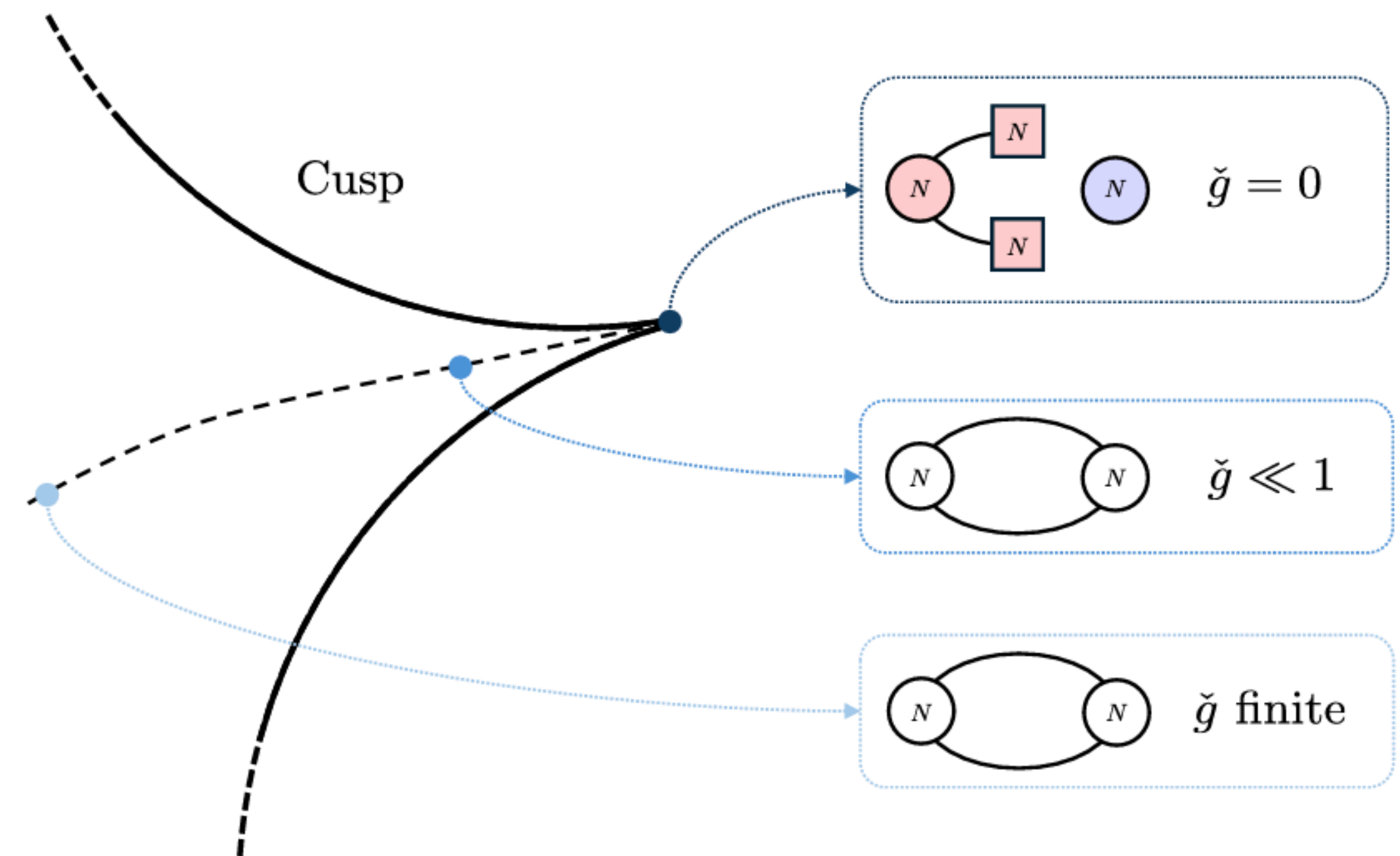
[Calderon-Infante, Mohseni'26]

[Baume, Mantegazza-to appear]

mixed limit:

$$\check{g} := g_k \rightarrow 0, \\ \text{remaining finite}$$

[Mantegazza, Marchetto,
Pomoni, Skrzypek, TW'26]

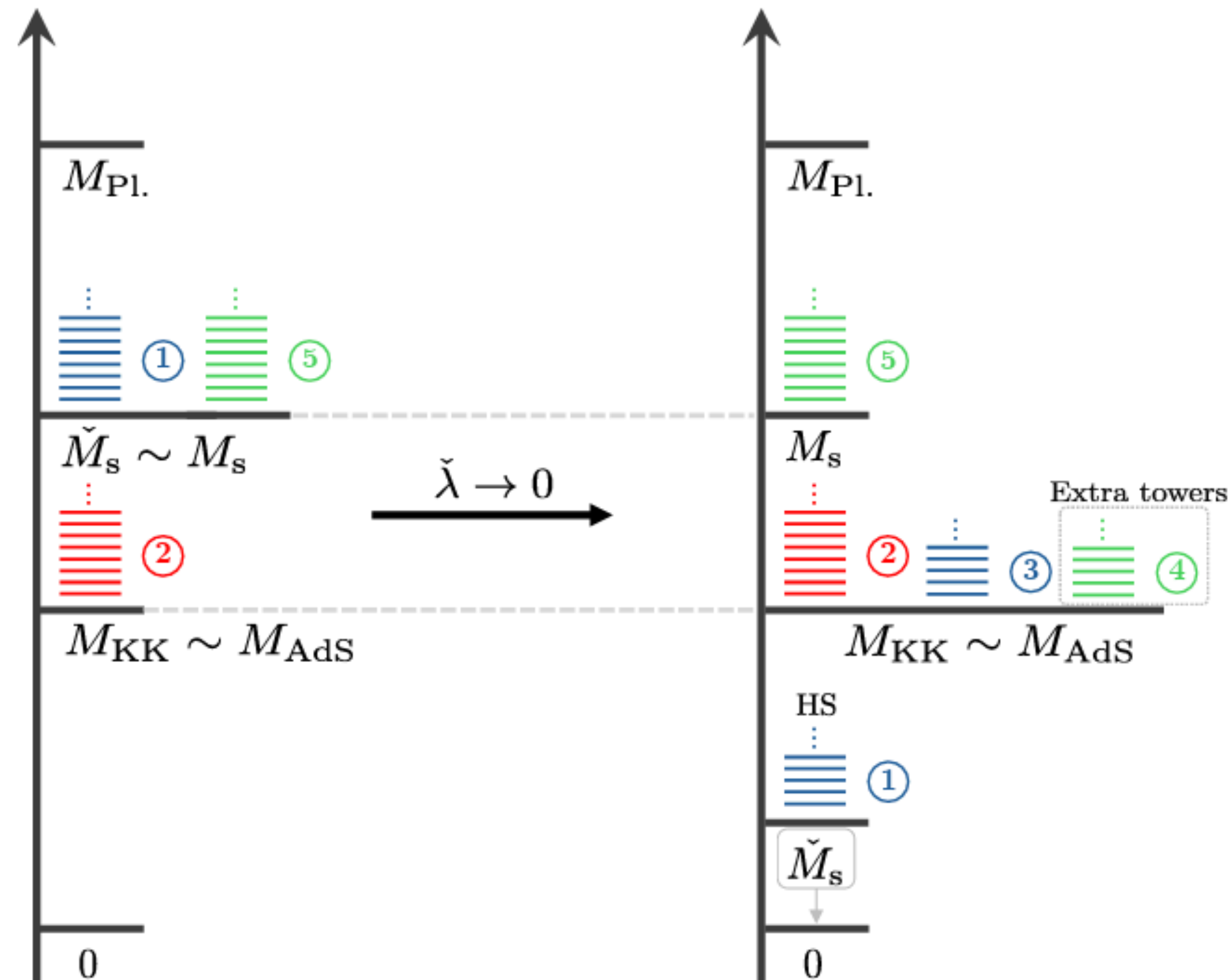


see talk by Irene Valenzuela

General mixed weak coupling limit

Mixed limit: $\check{g} := g_k \rightarrow 0$, remaining finite

[Mantegazza, Marchetto, Pomoni, Skrzypek, TW'26]



Novelty from interacting sector:

- ④ extra, **interacting** tower at AdS scale, related to HS via **SUSY**, of **exponential** degeneracy

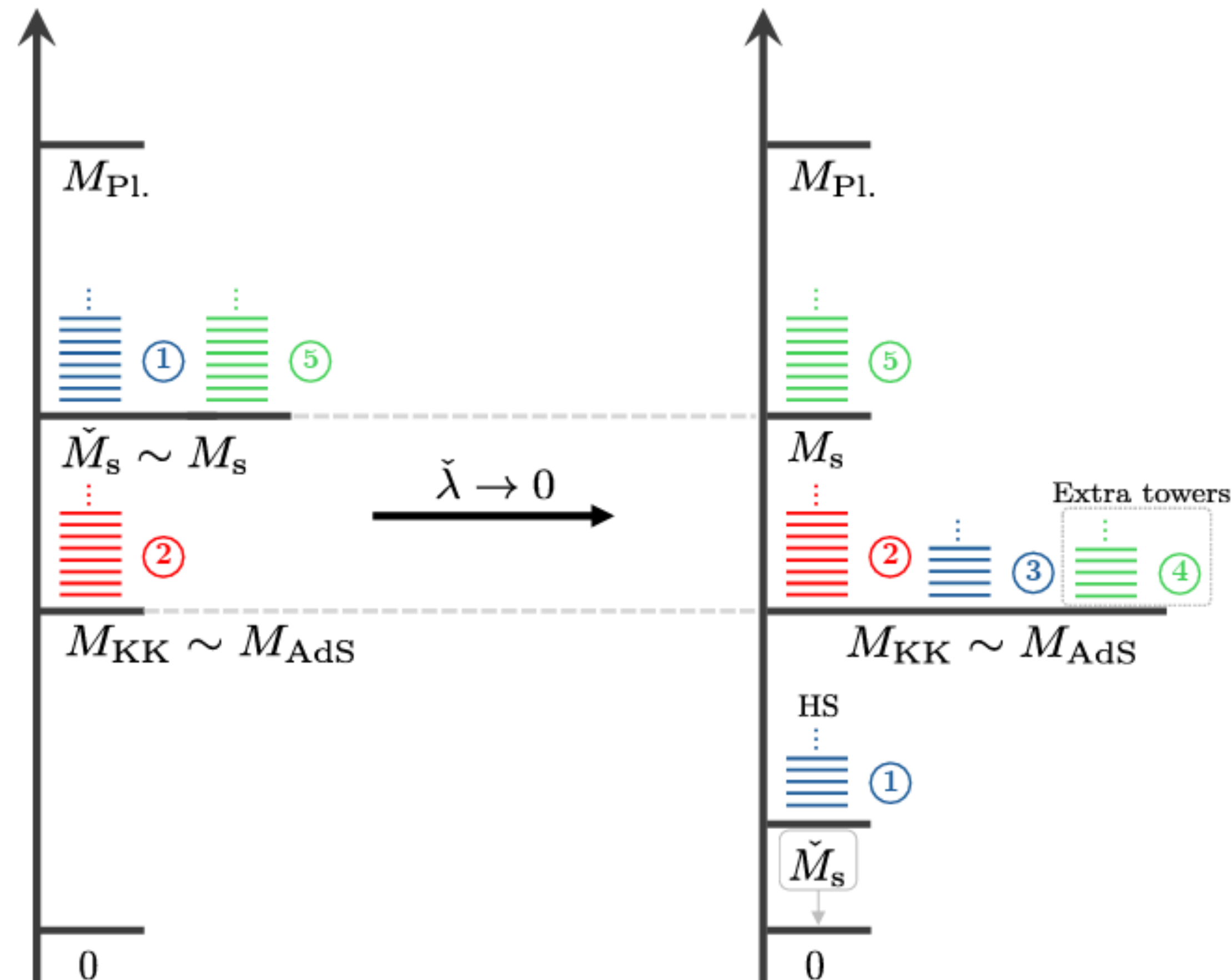
From asymptotically free sector:

- ① free massless HS tower (polynomial)
- ③ accompanied by free exponential tower at AdS scale

General mixed weak coupling limit

Mixed limit: $\check{g} := g_k \rightarrow 0$, remaining finite

[Mantegazza, Marchetto,
Pomoni, Skrzypek, TW'26]



Novelty from interacting sector:

- ④ extra, **interacting** tower at AdS scale,
related to HS via SUSY, of **exponential** degeneracy

Key point:

This is a consequence of representation theory only!

Extra tower from recombination rule

Mixed limit: $\check{g} := g_k \rightarrow 0$, remaining finite in 4d N=2 SCFT

Long superconformal multiplet: $\mathcal{A}_{R,r(j,\bar{j})}^\Delta$ R-symmetry: $SU(2)_R \times U(1)_r$

Suppose a multiplet hits
extremality bound

$$\check{g} \rightarrow 0 : \quad \Delta \rightarrow 2 + 2j + 2R + r \quad \text{and/or} \quad \Delta \rightarrow 2 + 2\bar{j} + 2R - r$$

Long multiplet $\xrightarrow{\check{g} \rightarrow 0}$ **Short BPS multiplets** **Recombination rules**
[Dolan,Osborn'02] [Gadde,Pomoni,Rastelli'09]

E.g. $\mathcal{A}_{R, \bar{j}-j(j,\bar{j})}^{2R+j+\bar{j}+2} \cong \hat{\mathcal{C}}_{R(j,\bar{j})} \oplus \hat{\mathcal{C}}_{R+\frac{1}{2}(j-\frac{1}{2},\bar{j})} \oplus \hat{\mathcal{C}}_{R+\frac{1}{2}(j,\bar{j}-\frac{1}{2})} \oplus \hat{\mathcal{C}}_{R+1(j-\frac{1}{2},\bar{j}-\frac{1}{2})}$

for $\Delta \rightarrow 2 + 2j + 2R + r$ and $\Delta \rightarrow 2 + 2\bar{j} + 2R - r$

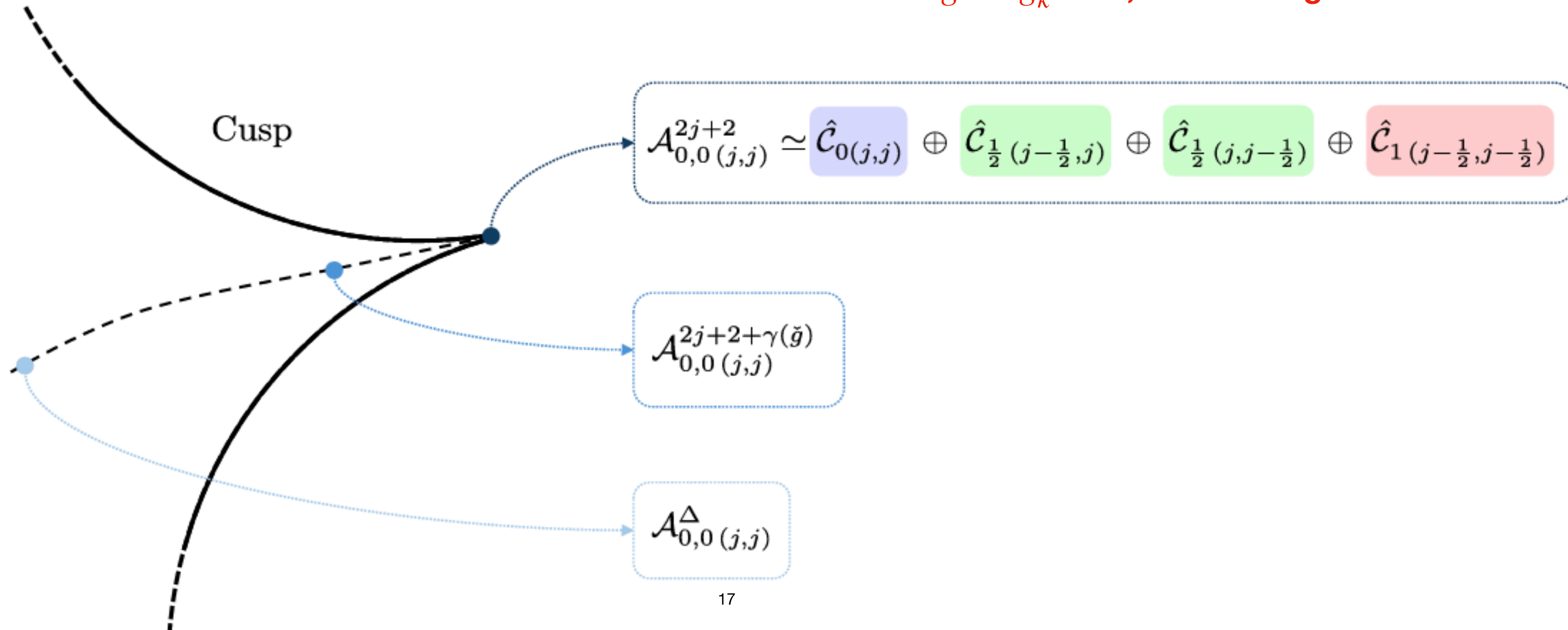
↓
short multiplet whose primary has
 $r = j - \bar{j}, \quad \Delta = 2 + 2R + j + \bar{j}$

Extra tower from recombination rule

Consider this recombination for $\mathcal{A}_{0,0}^{\Delta}(j,j)$:

Long multiplet with $R = 0 = r$, $j = \bar{j}$

in mixed limit: $\check{g} := g_k \rightarrow 0$, remaining finite



Extra tower from recombination rule

$$\text{tr}[\check{\phi}\partial^J\check{\phi}] + \text{tr}[\check{\lambda}^I\partial^{J-1}\check{\lambda}_I] + \text{tr}[\check{F}\partial^{J-2}\check{F}]$$



$$\mathcal{A}_{0,0}^{2j+2}(j,j)$$

\simeq

$$\hat{\mathcal{C}}_{0(j,j)} \oplus \hat{\mathcal{C}}_{\frac{1}{2}(j-\frac{1}{2},j)} \oplus \hat{\mathcal{C}}_{\frac{1}{2}(j,j-\frac{1}{2})} \oplus \hat{\mathcal{C}}_{1(j-\frac{1}{2},j-\frac{1}{2})}$$

\Downarrow
 HS currents \Downarrow
 Extra states

[Dolan,Osborn'02]

[Gadde,Pomoni,Rastelli'09]

$\hat{\mathcal{C}}_{R(j,\bar{j})}$: short BPS multiplet
 $r = j - \bar{j}, \quad \Delta = 2 + 2R + j + \bar{j}$

$$\frac{m^2}{M_{\text{AdS}}^2} = 0$$

$$\frac{m^2}{M_{\text{AdS}}^2} = \mathcal{O}(1)$$

free

mixed

interacting

SUSY transformation

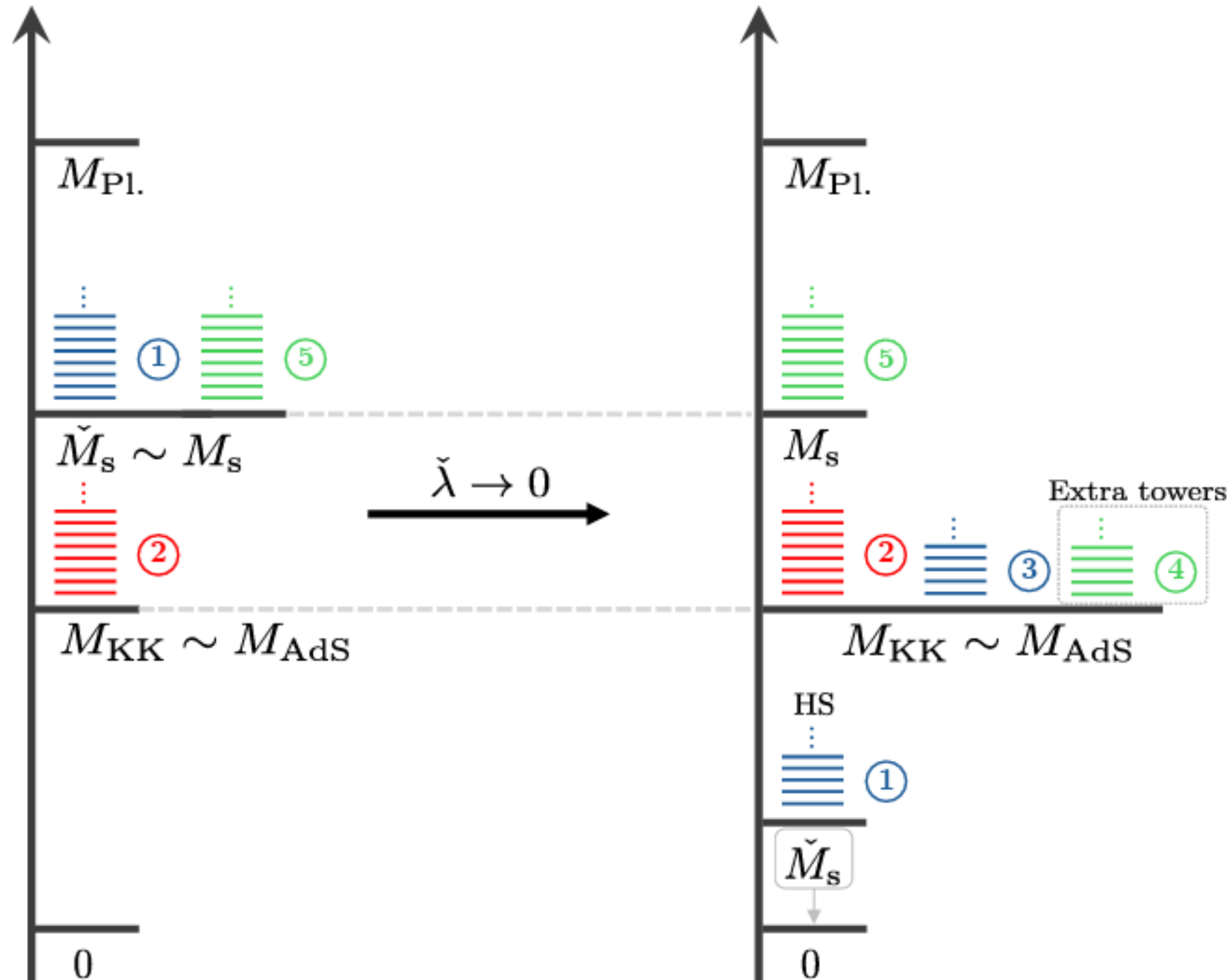
$$Q(\check{g}) = Q_0 + \check{g}Q_1 + \dots$$

$$\check{\lambda}\check{\lambda} \xrightarrow{Q_1} \check{\lambda}\check{\lambda}\check{\lambda} + \check{\lambda}\check{Q}Q + \dots \xrightarrow{Q_1} \check{\lambda}\check{\lambda}\check{\lambda}\check{\lambda} + \check{\lambda}\check{\lambda}\check{Q}Q + \check{Q}Q\check{Q}Q + \dots$$

$\gamma = 0$ due to BPS protection - though strongly coupled

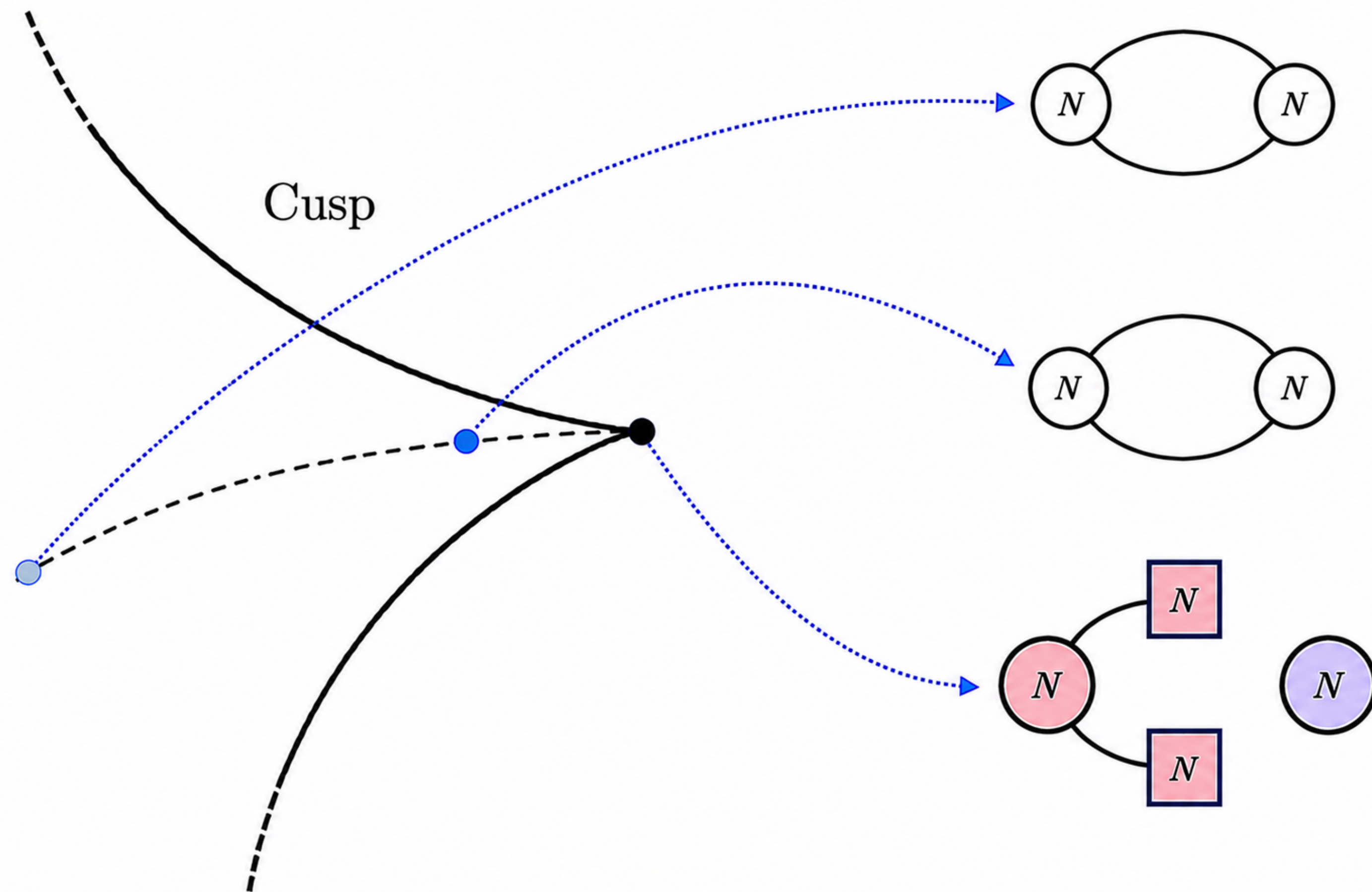
massive due to non-zero R-charge!

Extra tower from recombination rule



[Mantegazza, Marchetto,
Pomoni, Skrzypek, TW'26]

Example: Two-node quiver gauge theory



Orbifold theory

$$g = \check{g}$$

Interpolating theory

$$g \text{ fixed, } \hat{g} \rightarrow 0$$

SCQCD

$$\check{g} = 0, \\ g \text{ fixed}$$

$$\text{AdS}_5 \times S^5/\mathbb{Z}_2 =$$

$$\text{AdS}_5 \times (S^1 \times S^3/\mathbb{Z}_2) \times I$$

$$\begin{array}{ccc} & \nearrow & \nwarrow \\ & U(1)_r & SU(2)_R \end{array}$$

Part of spectrum inherited from orbifold theory:

only R-singlets

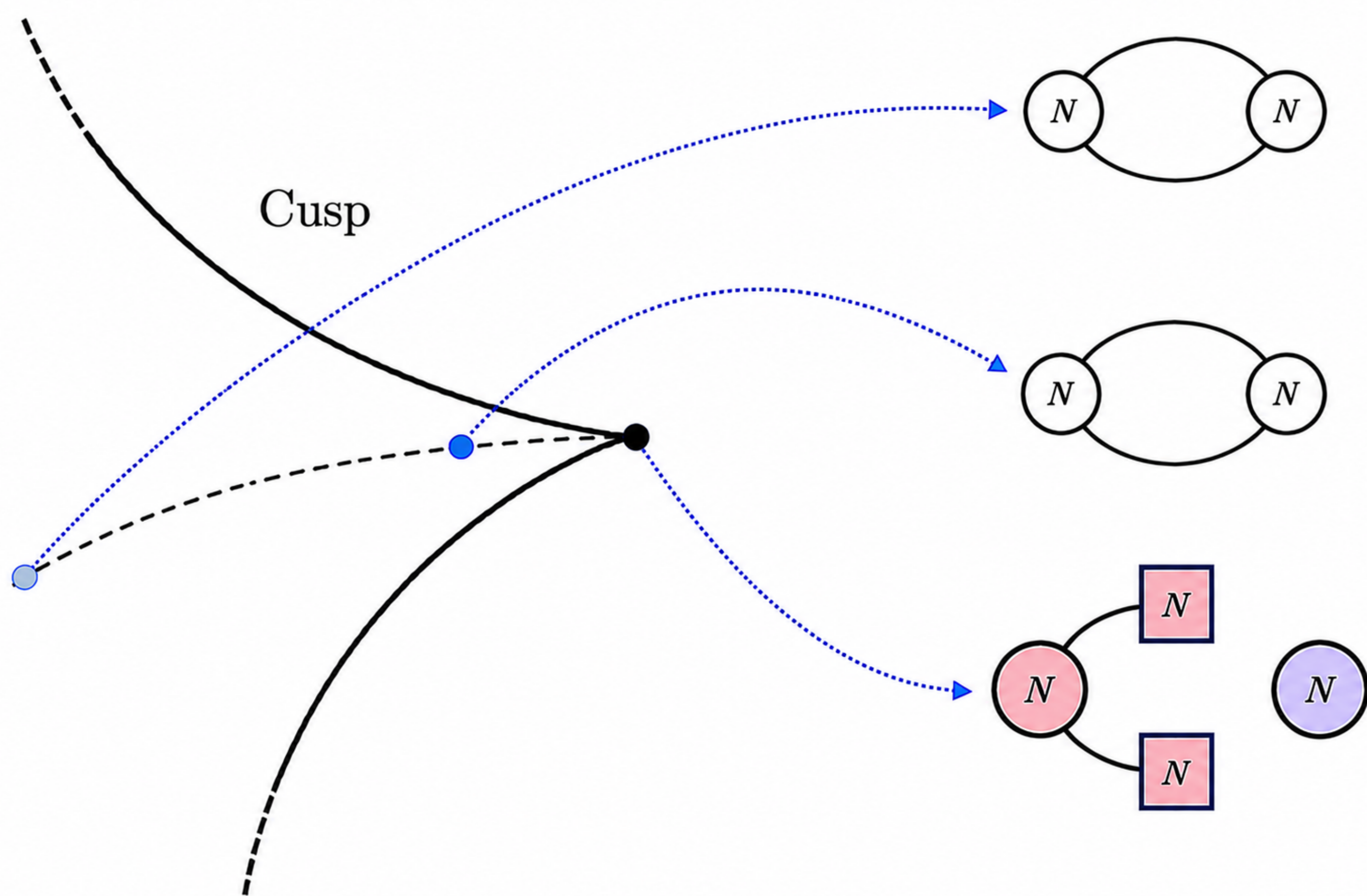
“loss” of S^3 (non-critical string) but still rooted in IIB superstring

[Gadde,Pomoni,Rastelli'09]

Example: Two-node quiver gauge theory

Supersymmetric index:

$$\mathcal{I} := \mathcal{I}^L(t, y, v) = \text{Tr}(-1)^F t^{2(\Delta+j)} y^{2\bar{j}} v^{r-R}$$



$\mathcal{I}_{\text{orbi}}$ known

$\mathcal{I}_{\text{inter}} = \mathcal{I}_{\text{orbi}}$

$\mathcal{I}_{\text{SQCD}}$ known

$\mathcal{I}_{\text{inheri}} = \mathcal{I}_{\text{orbi}}|_{\text{SQCD}}$

Origin of $\Delta\mathcal{I} \neq 0$:

**Additional short multiplets
at $\check{g} \neq 0$
absent for $\check{g} > 0$**

$$\Delta\mathcal{I} = \mathcal{I}_{\text{SCQCD}} - \mathcal{I}_{\text{inheri}}$$

$$\Delta\mathcal{I} \neq 0$$

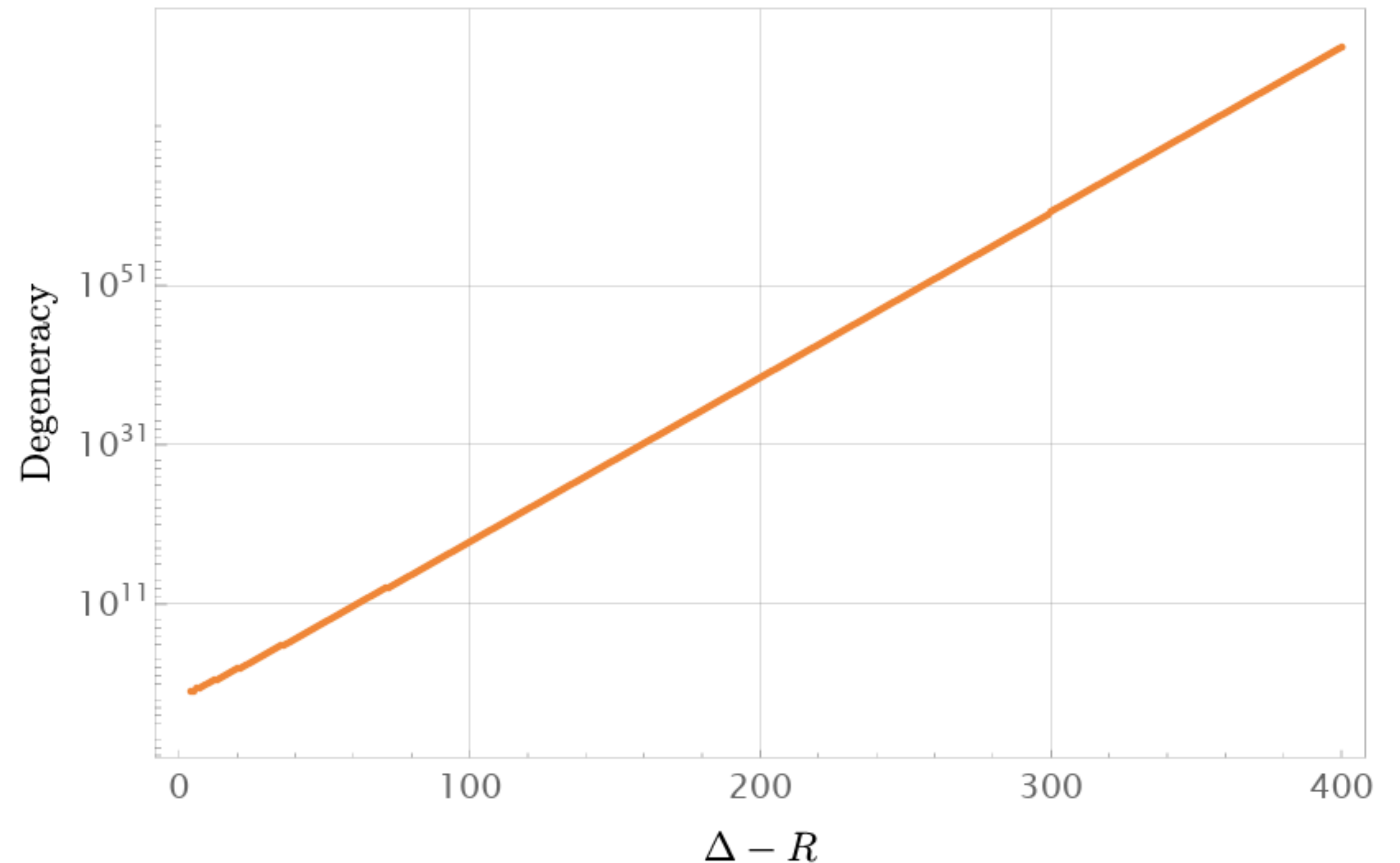
[Gadde,Pomoni,Rastelli'09]

General mixed weak coupling limit

$$\mathcal{J} := \mathcal{J}^L(t, y, v) = \text{Tr}(-1)^F t^{2(\Delta+j)} y^{2\bar{j}} v^{r-R}$$

[Mantegazza, Marchetto, Pomoni, Skrzypek, TW'26]

$$\Delta \mathcal{J} = \mathcal{J}_{\text{SCQCD}} - \mathcal{J}_{\text{inheri}} \neq 0$$



Observe:

Exponential degeneracy for extra tower

**Explicit identification
of states possible via Hamiltonian -
see paper**

Part II:

Boundedness theorems for Mordell-Weil groups

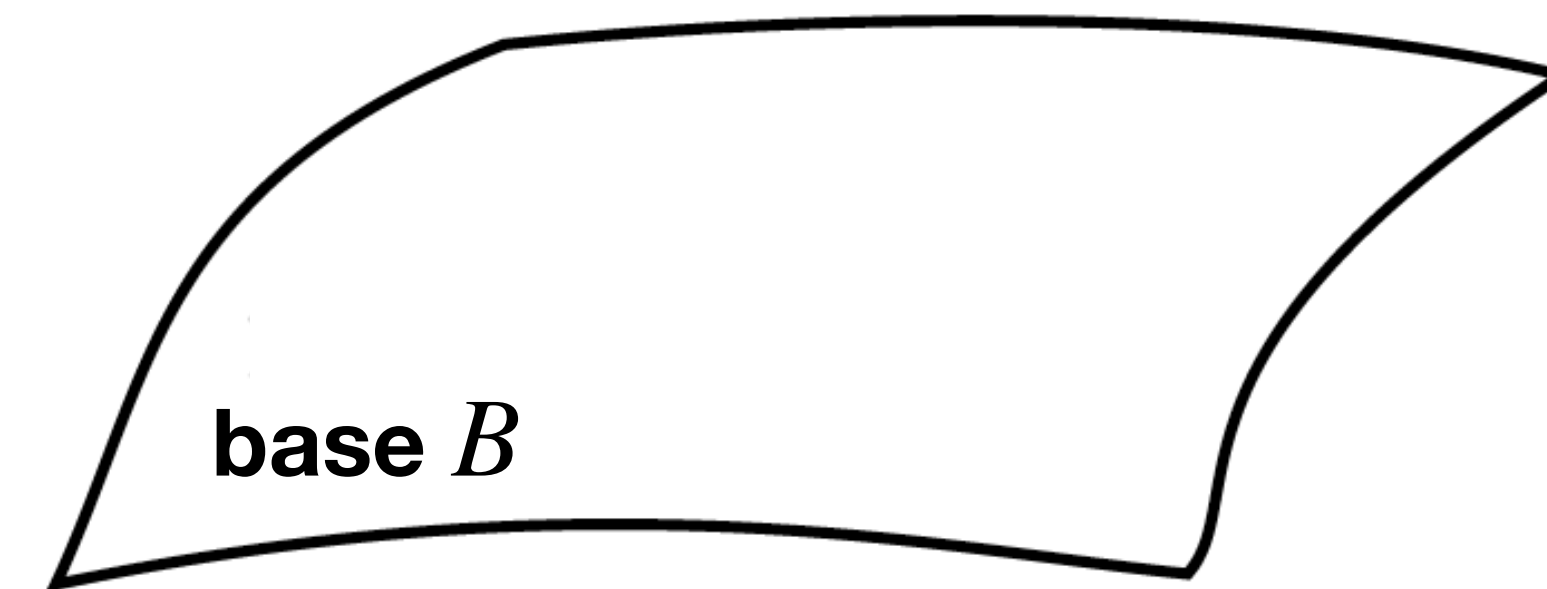
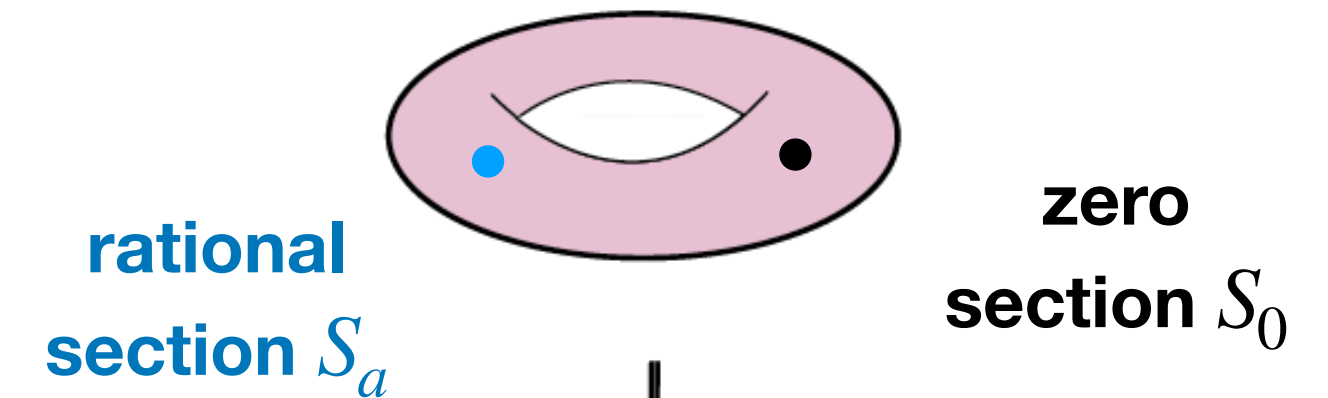
[Grassi, Paranjape, Miranda, Srinivas, TW'26]

Abelian gauge groups and the Mordell-Weil group

Abelian part of the gauge group in F-theory on elliptic CY X



(Free part of) Mordell-Weil group of rational sections of X



Abelian gauge groups and the Mordell-Weil group

Abelian part of the gauge group in F-theory on elliptic CY X



(Free part of) Mordell-Weil group of rational sections of X

Physics motivated bounds on abelian rank via probe strings

of type introduced in [Kim,Shiu,Vafa'19]

- F-theory on elliptic CY3 (6d N=1): [Lee,TW'19]

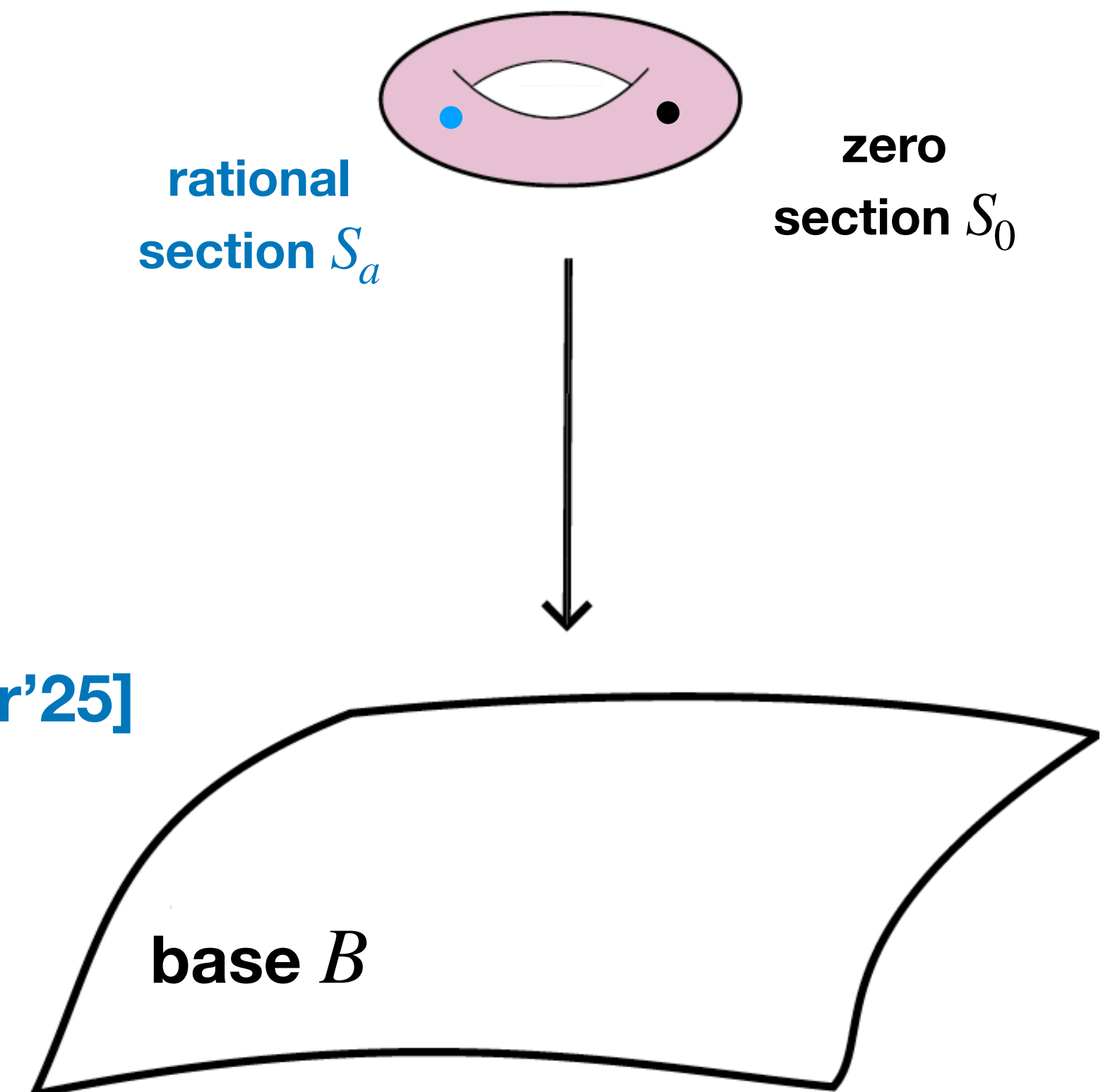
$$\text{rk}(\text{MW}) \leq 32 \quad \text{if } B = \mathbb{P}^2$$

$$\text{rk}(\text{MW}) \leq 20 \quad \text{otherwise (improved to 18 in [Lee,Oehlmann'22])}$$

see also [Kim,Vafa,Xu'24] [Lee,Birkar'25]

- F-theory on elliptic CY4 (4d N=1): [Martucci,Risso,TW'22]

$$\text{rk}(\text{MW}) \leq 10 C \cdot (-K_B) - 2 \quad \text{if rational curve } C \text{ movable on } B$$



Abelian gauge groups and the Mordell-Weil group

Physics motivated bounds on Mordell-Weil group rank via probe strings



...

Mathematical proof?

Find universal bound for CY 4-folds?

Extend beyond 4-folds?

Extend beyond CY?

Bounds on the Mordell-Weil group

X : elliptic fibration, $\Lambda \neq 0$: discriminant divisor

[Grassi,Paranjape,Miranda,
Srinivas,TW'26]

Theorem 1:

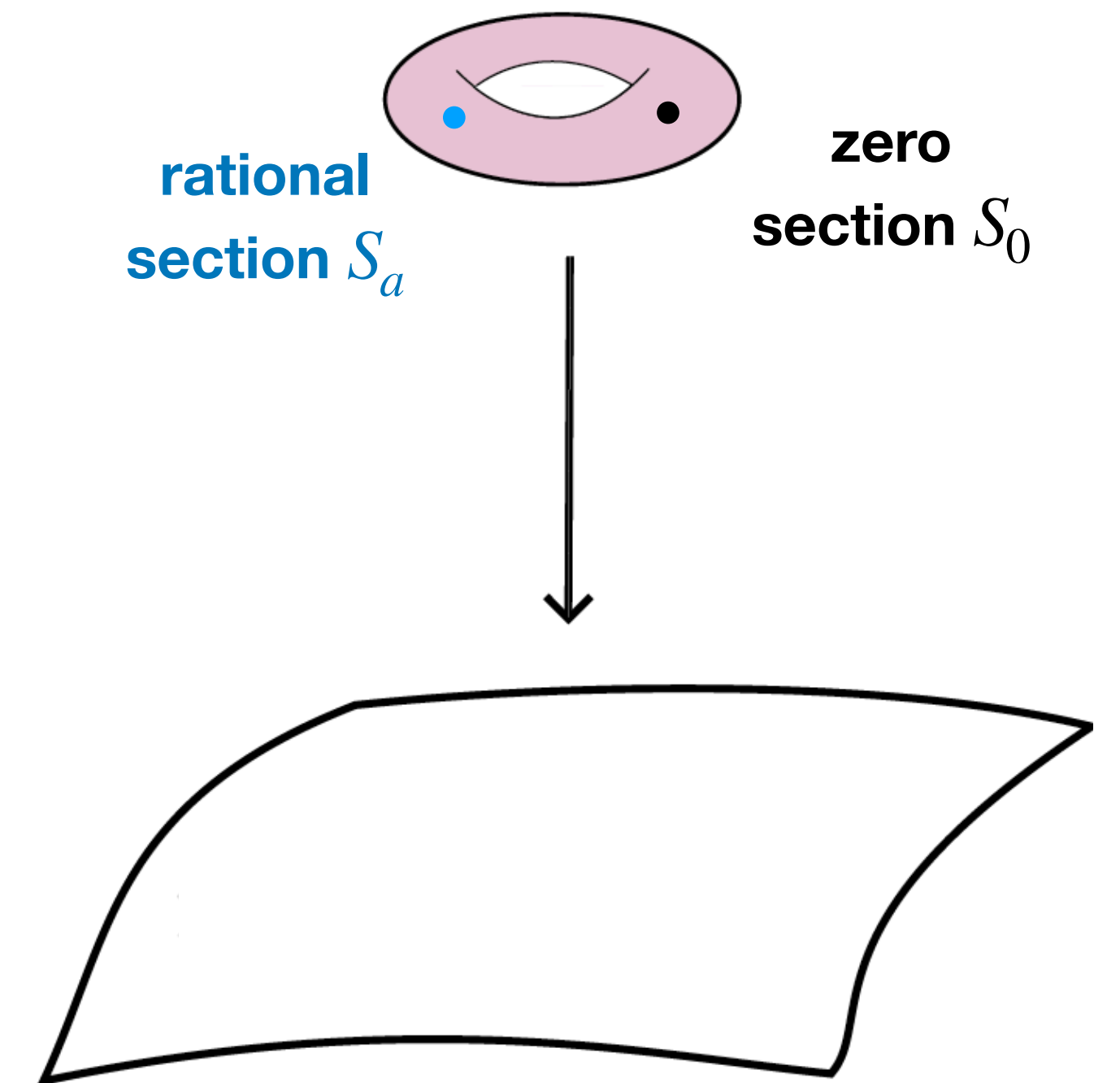
X CY 3-fold: $\text{rk}(\text{MW}(X)) \leq 28$.

Similar bounds for certain classes of non-CY 3-folds, including those not covered by birational boundedness.

Theorem 2:

X CY 4-fold: $\text{rk}(\text{MW}(X)) \leq 38$ if B smooth (after birational trafo).

Conjecture: X : CY n -fold: $\text{rk}(\text{MW}(X)) \leq 10n - 2$



Bounds on the Mordell-Weil group

X : elliptic fibration in any dimension, $\Lambda \neq 0$: discriminant \mathbb{Q} -divisor

[Grassi,Paranjape,Miranda,
Srinivas,TW'26]

1) Bound MW group of X by MW group of surface $Z = \pi^{-1}(C)$

see also [Lee,Oehlmann'22]

✓ possible in any dimension for suitable C

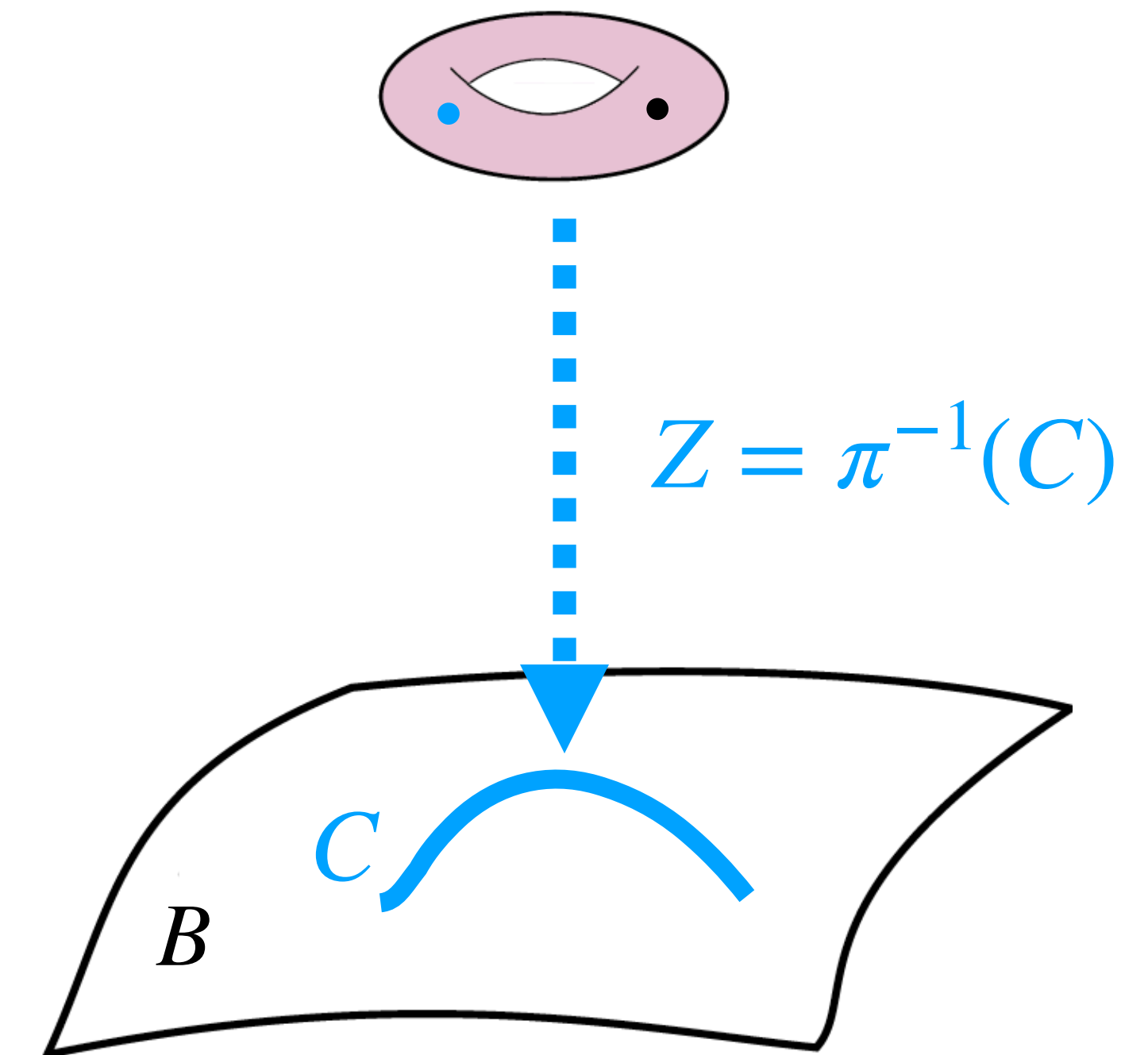
✓ no restrictions on K_X



formalises probe string idea in physics:

Unitarity on string from M5-brane on Z bounds d.o.f.

[Kim,Shiu,Vafa'19] [Lee,TW'19] [Martucci,Risso,TW'22]



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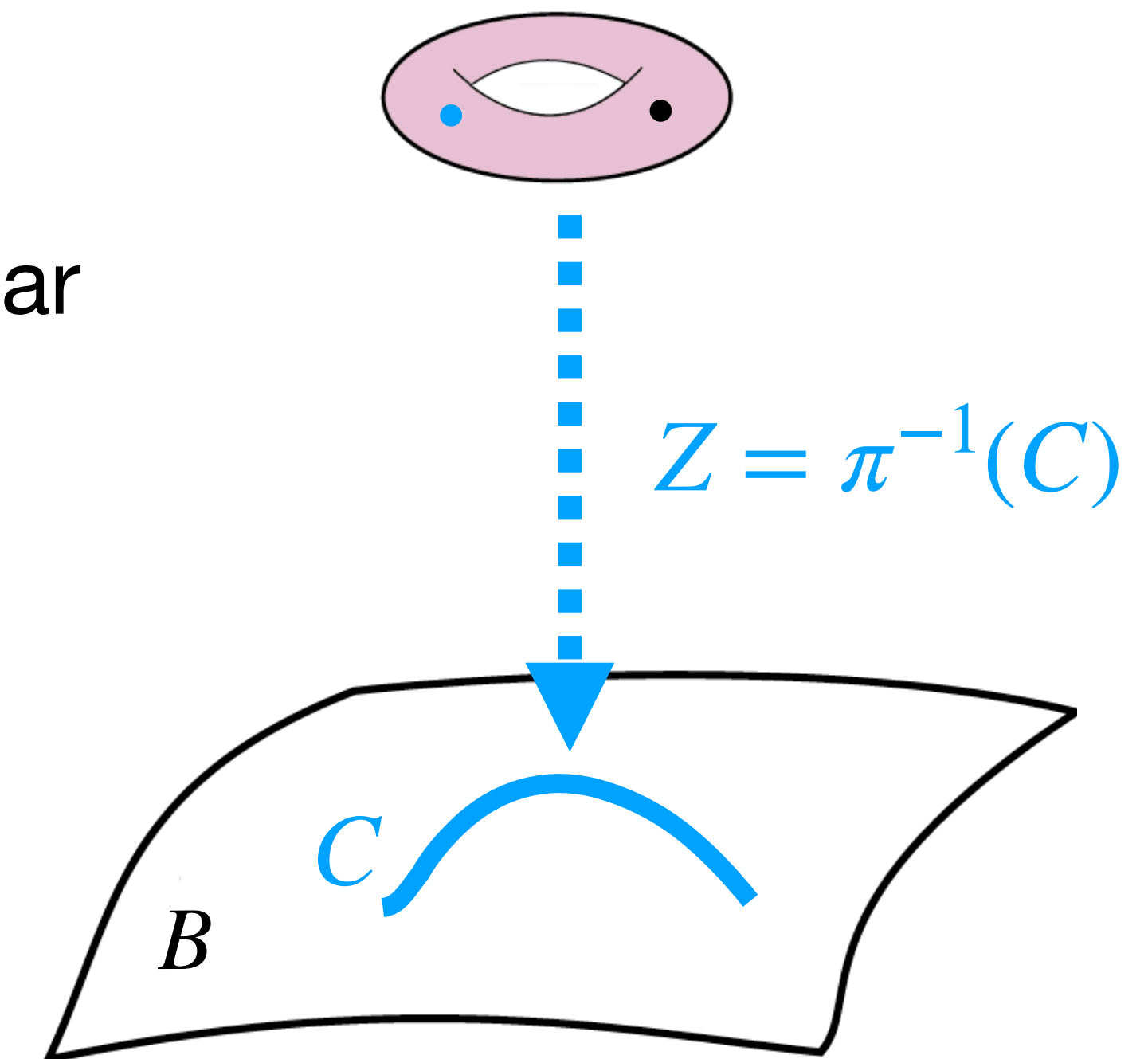
see also [Lee,Oehlmann'22]

✓ possible in any dimension for suitable C - even for Z singular

✓ no restrictions on K_X

2) Establish universal bounds by finding suitable C / Z

✓ practical restriction to CY and certain generalizations
for which structure of B and curves $C \subset B$ are known



Bounds on the Mordell-Weil group

X : elliptic fibration in any dimension, not only CY, $\Lambda \neq 0$: discriminant \mathbb{Q} -divisor

1) **Bound MW group of X by MW group of surface $Z = \pi^{-1}(C)$**

[Grassi, Paranjape, Miranda, Srinivas, TW'26]

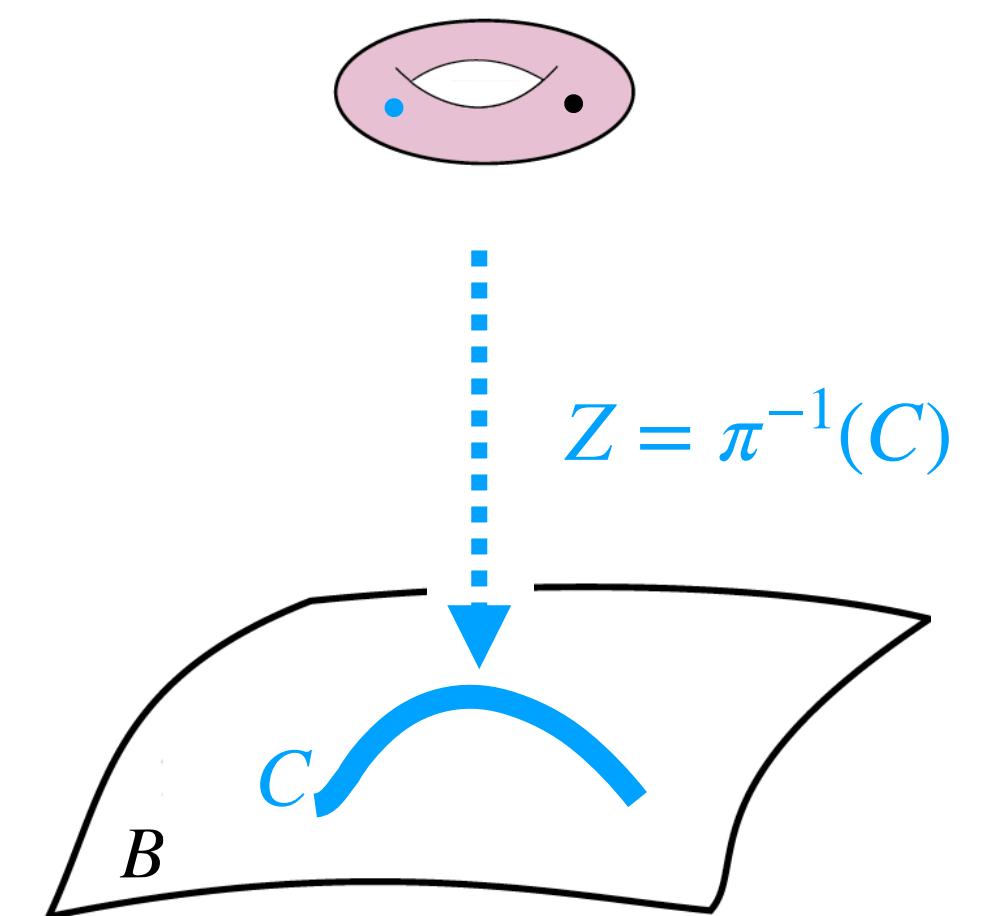
$C \subset B$: movable, smooth general curve with $\Lambda \cdot C > 0$

$C \subset B$: for $|\text{Pic}(B) = 1|$ $C \not\subset \Lambda$ smooth, general*

For $\pi^{-1}(C) = Z$, possibly singular:

$$\text{MW}(X/B) \subseteq \text{MW}(Z/C)$$

$$\implies \text{rk}(\text{MW}) \leq 10 C \cdot (-K_B) + 2g(C) - 2$$



Bounds on the Mordell-Weil group

2) Establish universal bounds by finding suitable C/Z

[Grassi, Paranjape, Miranda, Srinivas, TW'26]

For $X \rightarrow B$ CY in any dimension*, $\Lambda \neq 0$

B is (birationally) a Mori fiber space: $f: B \rightarrow U$, general fiber: Fano variety

Calabi-Yau 3-fold:

General fiber of B is

- smooth \mathbb{P}^1 $\text{rk}(\text{MW}) \leq 18$
- smooth del Pezzo $\text{rk}(\text{MW}) \leq 28$

\updownarrow
for $B = \mathbb{P}^2$

Calabi-Yau 4-fold:

General fiber of B is

- smooth \mathbb{P}^1 $\text{rk}(\text{MW}) \leq 18$
 - smooth del Pezzo $\text{rk}(\text{MW}) \leq 28$
 - Fano 3-fold, $|\text{Pic}| = 1$ $\text{rk}(\text{MW}) \leq 38$
- if smooth: **17 possibilities**

\updownarrow
for $B = \mathbb{P}^3$

Bounds on the Mordell-Weil group

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[Grassi,Paranjape,Miranda,
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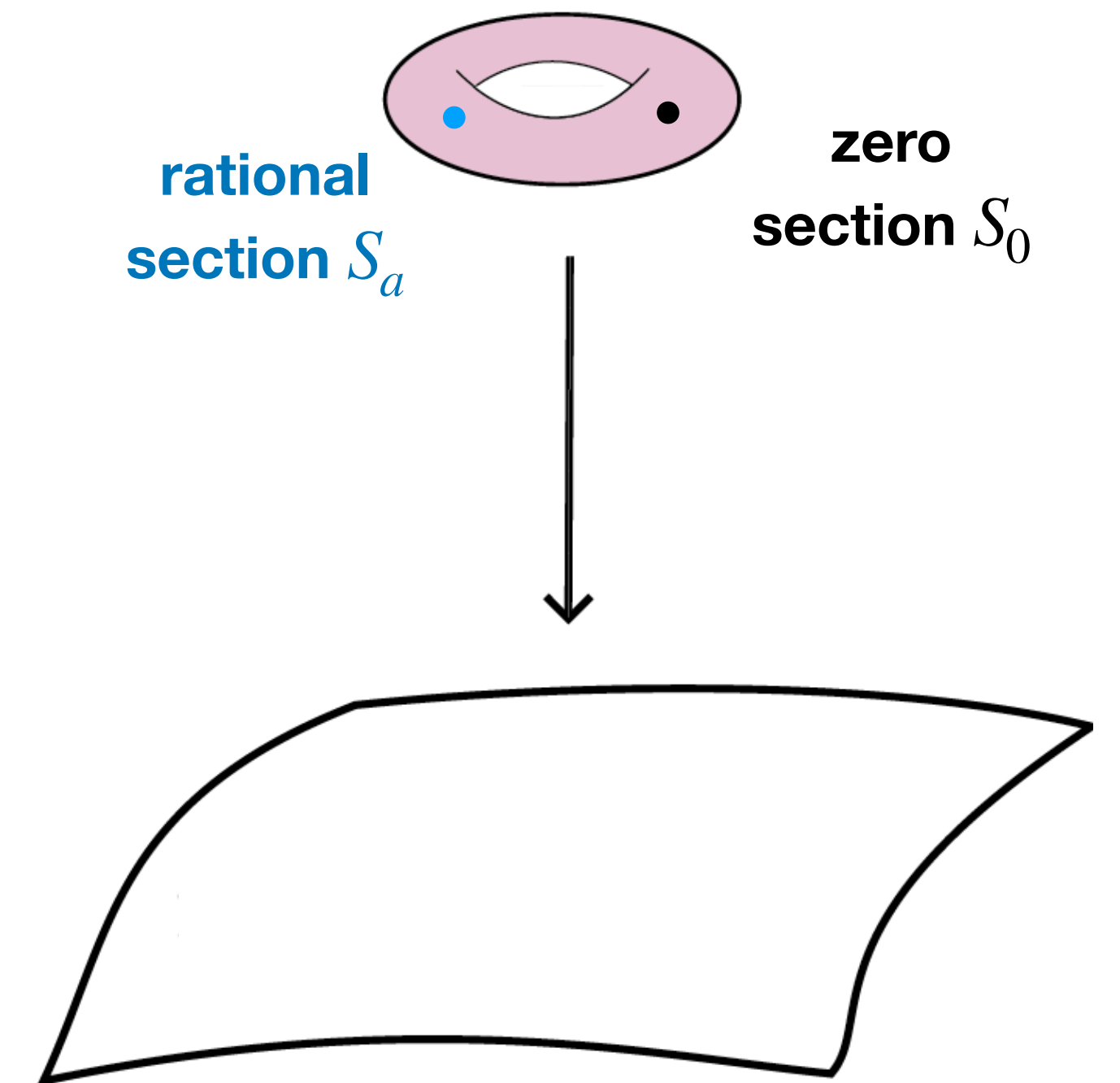
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Summary

I) Partial weak coupling limits in AdS:

- ✓ Protected, strongly coupled exponential tower at AdS scale related to higher-spin tower via SUSY in N=2
- ✓ Recombination allows to trace back to critical 10 Type IIB superstring, though some of the dimensions seem to decouple in limit
 - non-critical, but gravitational / fundamental string

Next step: Generalisation to N=1 supersymmetry — work in progress

II) Geometric proofs for bounds on MW ranks:

- ✓ New bounds for CY 4-folds: $\text{rk}(\text{MW}) \leq 38$ for smooth base
- ✓ Generalizations beyond CY and - conjectured - to any dimension

Summer School of the CRC 1624

August 24—September 11, 2026

Algebra and Geometry meets Quantum Fields and Strings

**University of Hamburg
Geomatikum
Bundesstraße 55
20146 Hamburg**

Organizing Committee:

Vicente Cortés, Craig Lawrie, David Reutter,
Jörg Teschner, Timo Weigand

Information and
Registration:



Speakers:

Aug 24—28:

Lecture series by

Sabin Cautis (The University of British Columbia)
Dustin Clausen (Institut des Hautes Études Scientifiques)
Pavel Etingof (Massachusetts Institute of Technology)
Melissa Liu (Columbia University)

Colloquium talk by

Pierrick Bousseau (University of Oxford)

Aug 31—Sep 4:

Lecture series by

Lara Anderson (Virginia Tech)
Tristan Collins (University of Toronto)
Clay Cordova (University of Chicago)
Michele Del Zotto (Uppsala University)
Nick Rozenblyum (University of Toronto)

Colloquium talk by

Ron Donagi (University of Pennsylvania)

Sep 7—11:

Lecture series by

Laura Fredrickson (University of Oregon)
Marco Gualtieri (University of Toronto/UPC in Barcelona)
Boris Pioline (Laboratoire de Physique Théorique et Hautes
Energies)
Ben Webster (University of Waterloo)

Colloquium talks by

Tomoyuki Arakawa (Kyoto University) and
John Francis (Northwestern University)