# Limits in Warpland and CFTs

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# OUTLINE

## 1) Kaluza-Klein (KK) spectrum in warped compactifications

Based on [arXiv: 2504.16984] with **Ignacio Ruiz** and **Salvatore Raucci**

## 2) Update on tensionless string limits in 4d SCFTs

Based on [ongoing] with **José Calderón-Infante** and **Ángel Uranga**

# KK spectrum in warped compactifications

# MOTIVATION

Long standing challenge for strings geometries/compactifications:

Determine the Kaluza-Klein (KK) spectrum in warped compactifications

$$\mathrm{d}s_D^2 = e^{2\rho(y)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{2\sigma(y)} \mathrm{M}_{ij} \mathrm{d}y^i \mathrm{d}y^j$$

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Recent works in string compactifications:

[D. Andriot, P. Marconnet and D. Tsimpis'21] [G.B. De Luca and A. Tomasiello'21] [G.B. De Luca, N. De Ponti, A. Mondino and A. Tomasiello'24] [R. Blumenhagen, A. Gligovic and S. Kaddachi'22] [R. Blumenhagen, M. Brinkmann and A. Makridou'22] [B. Valeixo Bento, D. Chakraborty, S. Parameswaran and I. Zavala'22] [Bonifacio, D. Mazac and S. Pal'23] [M. Etheredge, B. Heidenreich, J. McNamara, T. Rudelius, I. Ruiz and I. Valenzuela'23] [S. Lust, M. Nee and L. Randall'25]

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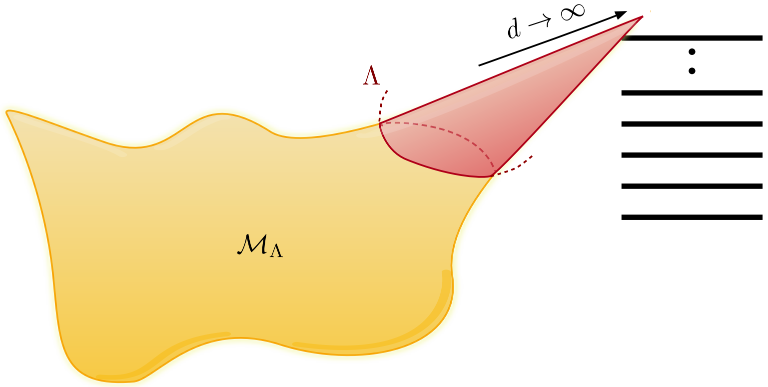
**Our motivation:** Swampland Distance Conjecture [Ooguri-Vafa'05]



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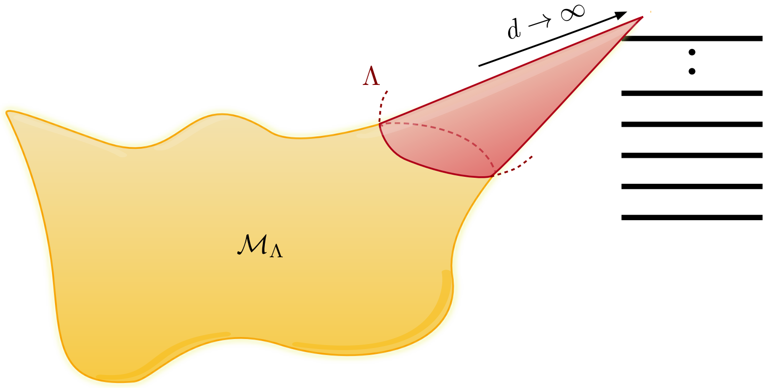

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KK tower

Critical string oscillator modes

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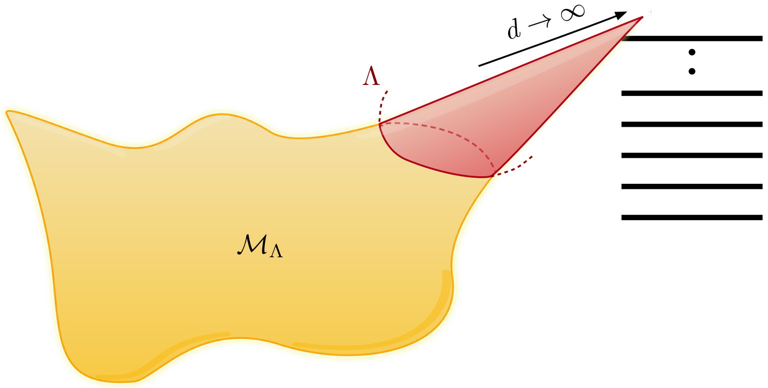

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$$\alpha_{str} = \frac{1}{\sqrt{d-2}}$$

$d =$ space-time dimension

$n =$ compact space dimension

suggesting the bounds: $$\frac{1}{\sqrt{d-2}} \leq \alpha \leq \sqrt{\frac{d-1}{d-2}}$$

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1-dim KK tower

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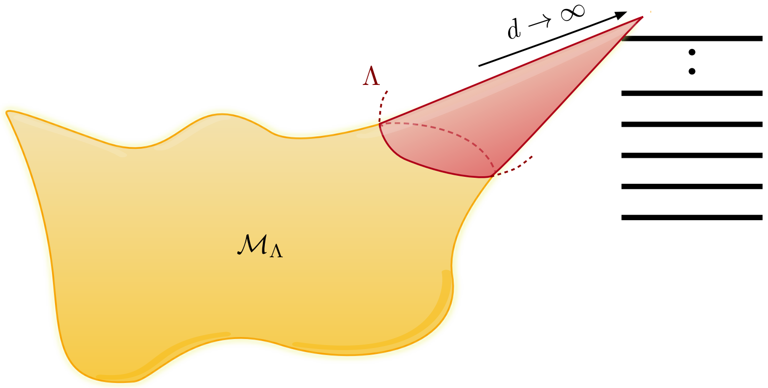

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unwarped result!

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The lower bound is especially important for phenomenological applications

as it yields an upper bound on the geodesic field range that can be accommodated before the EFT breaks down

$$\Delta\phi \leq \frac{1}{\alpha} \log \frac{M_p}{m_{\text{tower}}}$$

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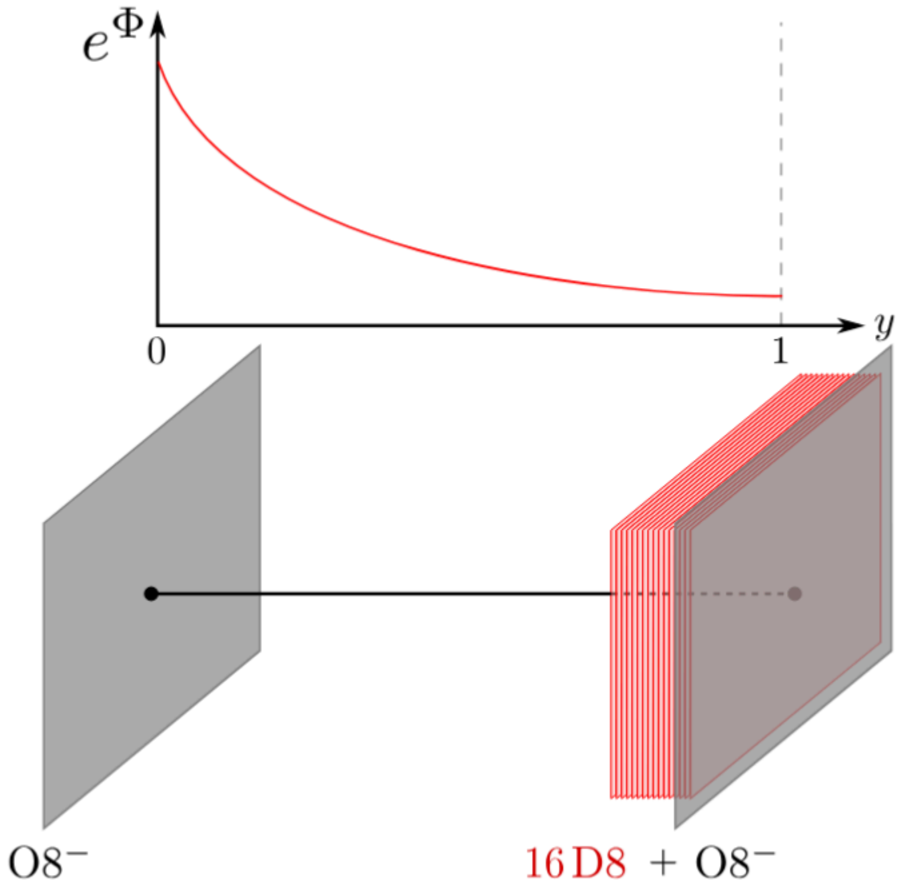
$$G_{MN} = \Omega^2(x^9)\eta_{MN} \qquad e^{\Phi_{\mathrm{I}'}(x^9)} = z(x^9)^{-5/6} \qquad \Omega(x^9) = Cz(x^9)^{-1/6},$$



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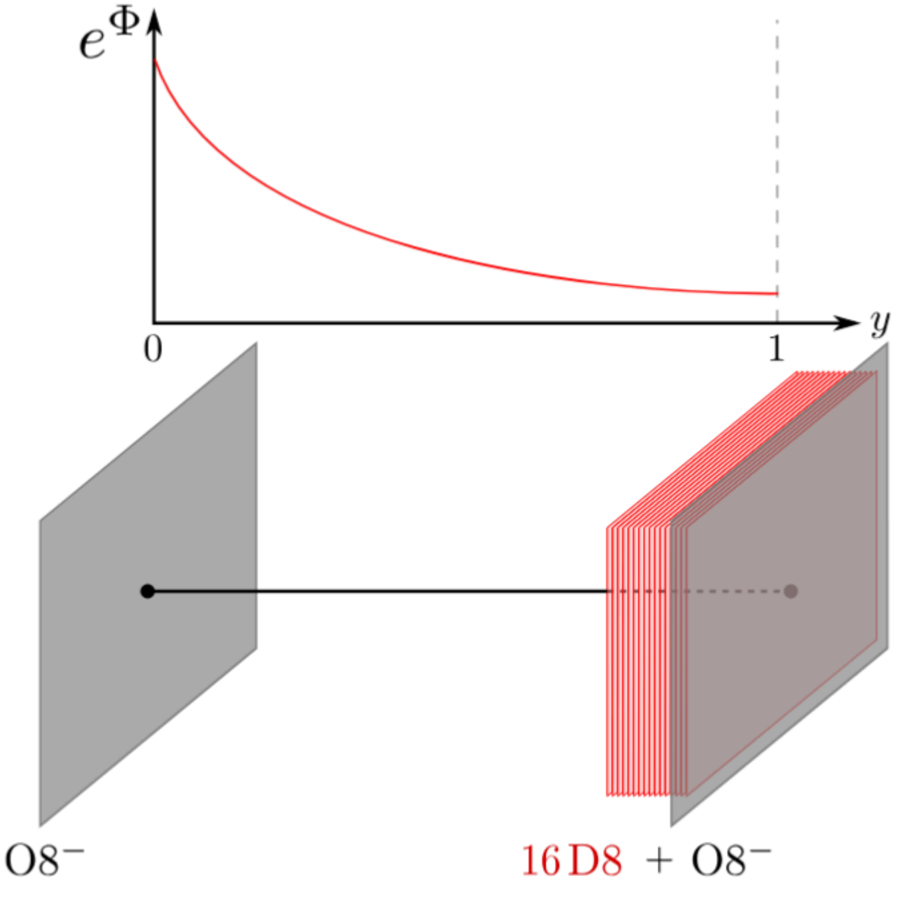
Consider direction in moduli space along which t
interval decompactifies to10D massive Type IIA:



KK exponential rate gets reduced!

$$\alpha_{KK} = \frac{5}{2\sqrt{7}} < \sqrt{\frac{8}{7}} = \sqrt{\frac{d+n-2}{n(d-2)}}$$

[Etheredge,Heidenreich,McNamara,Rudelius,Ruiz,IV'23]

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The lower bound risks to be violated if we have strong warping

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Could we ever get a KK tower whose mass decreases at a slower rate than the expectation for string modes?

Let's check it!

# MINKOWSKI WARPED COMPACTIFICATIONS

For concreteness, we focus on Minkowski warped compactifications

$$\mathrm{d}s^2_D = e^{2\rho(y)}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + e^{2\sigma(y)}\mathrm{M}_{ij}\mathrm{d}y^i\mathrm{d}y^j$$

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We will study the KK spectrum for scalar modes

in highly warped (decompactification) limits

$$\lim_{\Delta\varphi\to\infty} \vec{\nabla}_y \rho(y) \neq 0 \quad \text{and/or} \quad \lim_{\Delta\varphi\to\infty} \vec{\nabla}_y \sigma(y) \neq 0\,,$$

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Two cases:

one internal warped dimension

- Codimension 1 : $ds_D^2 = e^{2\rho(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2\sigma(y)}dy^2$
- Codimension >1 : $\mathrm{d}s_D^2 = e^{2\rho(y)}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + e^{2\sigma(y)}\mathsf{M}_{ij}\mathrm{d}y^i\mathrm{d}y^j$

# CODIMENSION-1 MINKOWSKI WARPED COMPACTIFICATIONS

**Setup**

Consider EFT in D dimensions

$$S = \int \mathrm{d}^D x \sqrt{-G_D} \left\{ \frac{1}{2\kappa_D^2} \left[ \mathcal{R}_D - \partial_M \hat{\varphi} \partial^M \hat{\varphi} \right] - V(\hat{\varphi}) \right\} + S_{\mathrm{loc.}}$$

with $V(\hat{\varphi}) = \kappa_D^{-2} V_0 e^{\gamma \hat{\varphi}}$

compactified to d=(D-1)-dimensions including sources, so that we get a Minkowski solution

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Examples:
- Type I' Polchinski-Witten solution
- Tachyonic-free non-SUSY models and their Dudas–Mourad vacua
- Examples of dynamical cobordism
- 3d vacua from N=1 flux compactifications with EFT membranes

# CODIMENSION-1 MINKOWSKI WARPED COMPACTIFICATIONS

We compute the dimensionally reduced kinetic terms for the moduli

$$\begin{aligned}\mathsf{G}_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b =&(d-1)(d-2)(\partial\theta)^2 - e^{-(d-2)\theta}\int_{X_n}\mathrm{d}^n y\sqrt{\mathsf{M}}\,e^{(d-2)\rho+n\sigma}\,\Big[(d-1)(d-2)(\partial\rho)^2\\ &+n(n-1)(\partial\sigma)^2+2n(d-1)\partial_\mu\rho\partial^\mu\sigma-\hat{\mathsf{G}}_{\hat{a}\hat{b}}\partial_\mu\hat{\varphi}^{\hat{a}}\partial^\mu\hat{\varphi}^{\hat{b}}\Big]\ ,\end{aligned}$$

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We solve for general solutions (with arbitrary exponential potential) to the equations of motion in order to obtain the general internal profiles for the warping and the scalar field

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condition for no acceleration (when V>0)
Strong deSitter conjecture/TCC
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*sharpened bound* for
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⟷

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Strong deSitter conjecture/TCC
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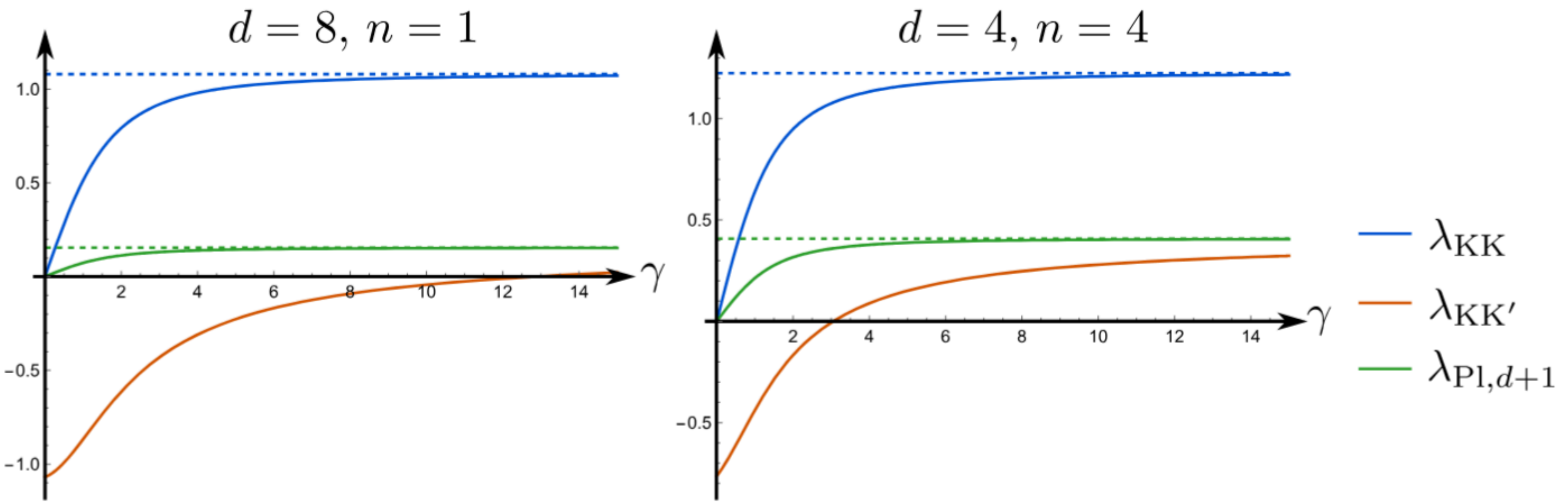
(Bound valid both for V>0 or V<0)

# CODIMENSION-1 MINKOWSKI WARPED COMPACTIFICATIONS

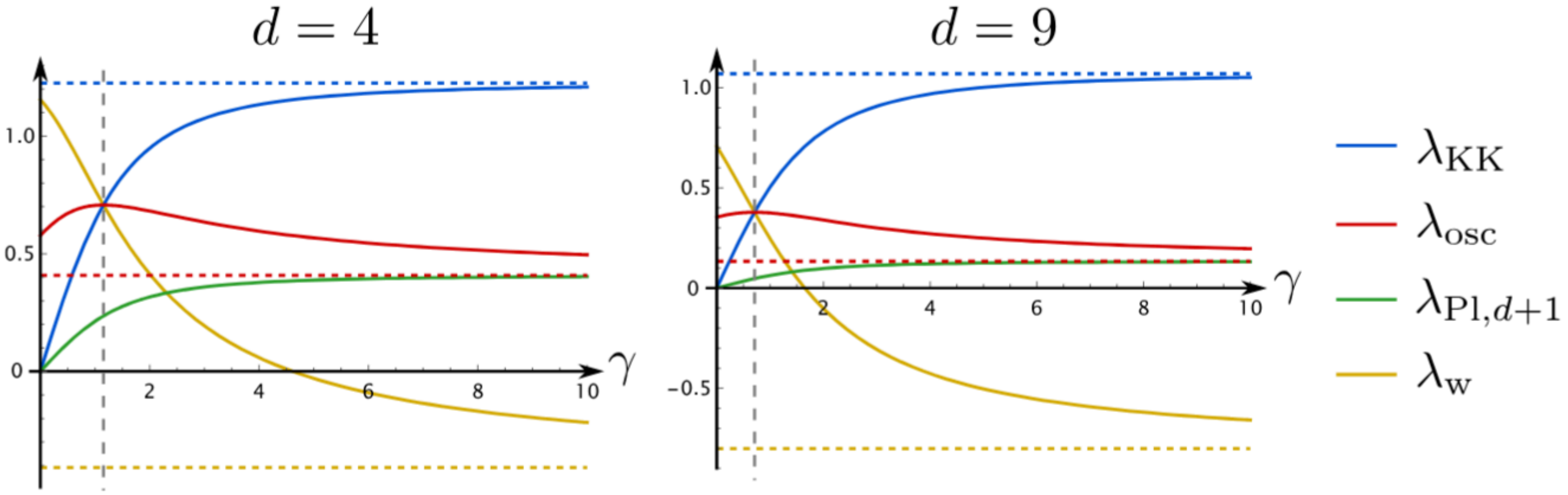
## Exponential rate of Planck scale

$$\alpha_{\mathrm{Pl,d+1}} = \frac{1}{\sqrt{(d-1)(d-2)}}\left(1+\frac{4(d-2)}{(d-1)\gamma^2}\right)^{-\frac{1}{2}}$$



## Exponential rate of string scale

$$\alpha_{\mathrm{osc}} = \frac{1}{\sqrt{(d-1)(d-2)}}\,\frac{2(d-2)+\sqrt{d-1}\,\gamma}{\sqrt{4(d-2)+(d-1)\gamma^2}}$$



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Within this landscape, we do not find highly warped limits (the warping factors seem to be always diluted at infinite distance)

$$\lim_{\Delta\varphi\to\infty} \vec{\nabla}_y \rho(y) \to 0 \quad \text{and} \quad \lim_{\Delta\varphi\to\infty} \vec{\nabla}_y \sigma(y) \to 0\,,$$

so the asymptotic value of the KK exponential rate does not get modified due to the warping

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The Emergent String Conjecture and the associated bounds for the towers hold in all known top-down Minkowski compactifications

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How else can we challenge them?

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How else can we challenge them?

The story is different in AdS…

# Update on Tensionless String Limits in 4d SCFTs

# CFT DISTANCE CONJECTURE

Consider infinite distance limits in moduli spaces of AdS compactifications

In $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$

Bulk moduli space ⟷ Conformal manifold (space of exactly marginal couplings)

field metric ⟷ Zamolodchikov metric $|x-y|^{2d}\langle O_i(x)O_j(y)\rangle = g_{ij}(t^i)$

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**CFT Distance conjecture:**

For d>2: [Perlmutter,Rastelli,Vafa,IV'21] [Baume,Calderon-Infante'21]
For d=2 [Kontsevich-Soibelman'00, Archarya-Douglas'06]

Consider a local unitary CFT with a conformal manifold:

$\exists$ **infinite tower of primary operators** saturating the unitarity bound at **every infinite distance limit** measured by Zamolodchikov metric, such that

$$\gamma_J \sim e^{-\alpha d(\tau,\tau')} \quad \text{as} \quad d(\tau,\tau') \to \infty$$

anomalous dimension

$\gamma_J = \Delta - \Delta_{\text{unitarity}}$

distance measured by Zamolodchikov metric

# CLASSIFICATION OF LIMITS IN 4d SCFTs

(Strings&Geometry 2024 talk)

**Towards a classification program of infinite distance limits in 4d SCFTs:**

(with any level of SUSY) [Calderon-Infante,IV'24]

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$$g_{YM} \to 0 \quad \Rightarrow \quad \mathrm{CFT}_{\mathrm{free}} \times \mathrm{CFT}'$$

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(infinite tower of HS operators saturating unitarity bound)

$$\gamma_J \equiv \Delta_J - (d-2+J) = 0$$

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Infinite distance ← Free point ⇄ Higher Spin symmetry

(infinite tower of HS operators
saturating unitarity bound)

$$\gamma_J \equiv \Delta_J - (d-2+J) = 0$$

$$\tau = \frac{4\pi i}{g_{YM}^2} + \frac{\theta}{2\pi} \qquad \mathcal{O}_\tau = \mathrm{Tr}(F^2 + \dots)$$

$$\text{as} \quad \mathrm{Im}\tau \to \infty \qquad ds^2 = \beta^2 \frac{d\tau d\bar{\tau}}{(\mathrm{Im}\tau)^2}$$

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$$\gamma_J \sim f(J) g_{YM}^2 \sim f(J) \exp\left(-\frac{d(\tau,\tau')}{\beta}\right)$$

$$\text{as} \quad \mathrm{Im}\tau \to \infty \qquad ds^2 = \beta^2 \frac{d\tau d\bar{\tau}}{(\mathrm{Im}\tau)^2}$$

$$\beta^2 = 24\ \mathrm{dim}G$$

gauge group getting free

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If there is a weakly coupled AdS dual (large N), it implies: [Perlmutter,Rastelli,Vafa,IV'21]

$\exists$ Tower of higher spin fields with an exponential rate:

central charge

$$\boxed{\alpha = \sqrt{\frac{2c}{\mathrm{dim}G}}}$$

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Some bulk string becoming tensionless (in Planck units)?

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Anything different than the critical string?

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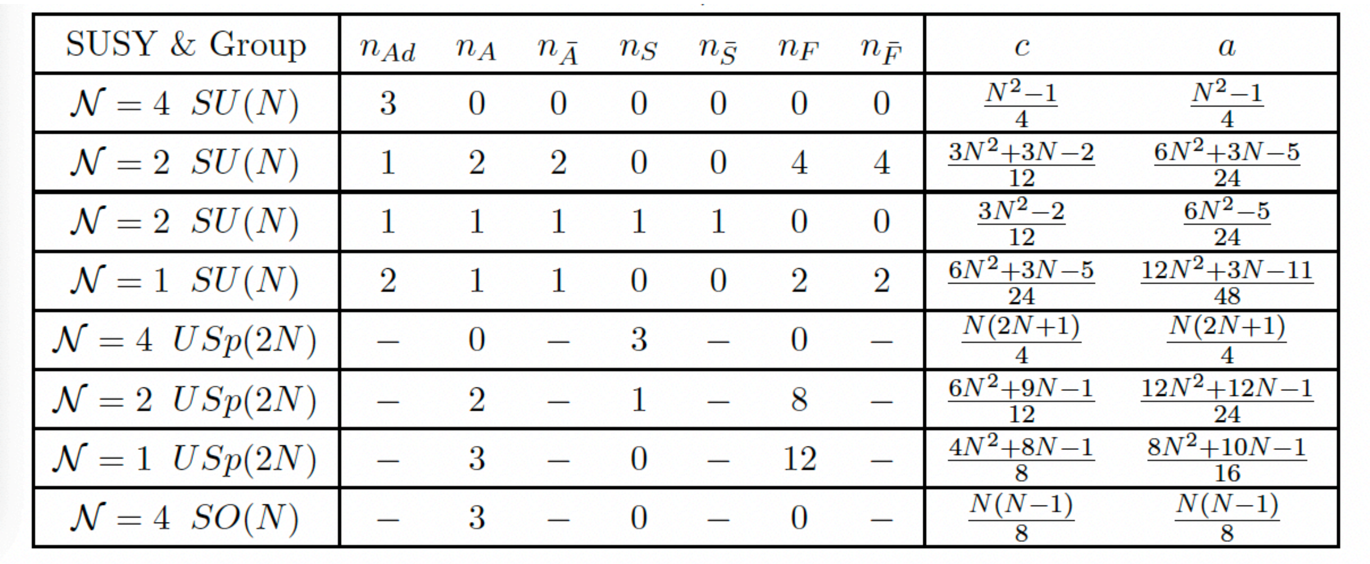
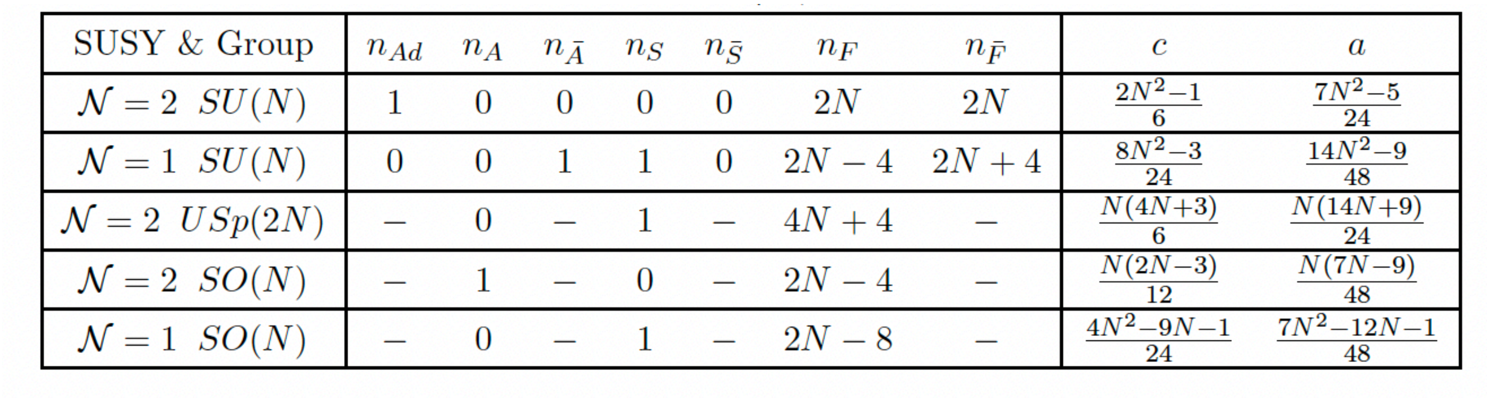


| SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | $c$ | $a$ |
|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{N}=4$ $SU(N)$ | 3 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{N^2-1}{4}$ | $\frac{N^2-1}{4}$ |
| $\mathcal{N}=2$ $SU(N)$ | 1 | 2 | 2 | 0 | 0 | 4 | 4 | $\frac{3N^2+3N-2}{12}$ | $\frac{6N^2+3N-5}{24}$ |
| $\mathcal{N}=2$ $SU(N)$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | $\frac{3N^2-2}{12}$ | $\frac{6N^2-5}{24}$ |
| $\mathcal{N}=1$ $SU(N)$ | 2 | 1 | 1 | 0 | 0 | 2 | 2 | $\frac{6N^2+3N-5}{24}$ | $\frac{12N^2+3N-11}{48}$ |
| $\mathcal{N}=4$ $USp(2N)$ | – | 0 | – | 3 | – | 0 | – | $\frac{N(2N+1)}{4}$ | $\frac{N(2N+1)}{4}$ |
| $\mathcal{N}=2$ $USp(2N)$ | – | 2 | – | 1 | – | 8 | – | $\frac{6N^2+9N-1}{12}$ | $\frac{12N^2+12N-1}{24}$ |
| $\mathcal{N}=1$ $USp(2N)$ | – | 3 | – | 0 | – | 12 | – | $\frac{4N^2+8N-1}{8}$ | $\frac{8N^2+10N-1}{16}$ |
| $\mathcal{N}=4$ $SO(N)$ | – | 3 | – | 0 | – | 0 | – | $\frac{N(N-1)}{8}$ | $\frac{N(N-1)}{8}$ |

II)



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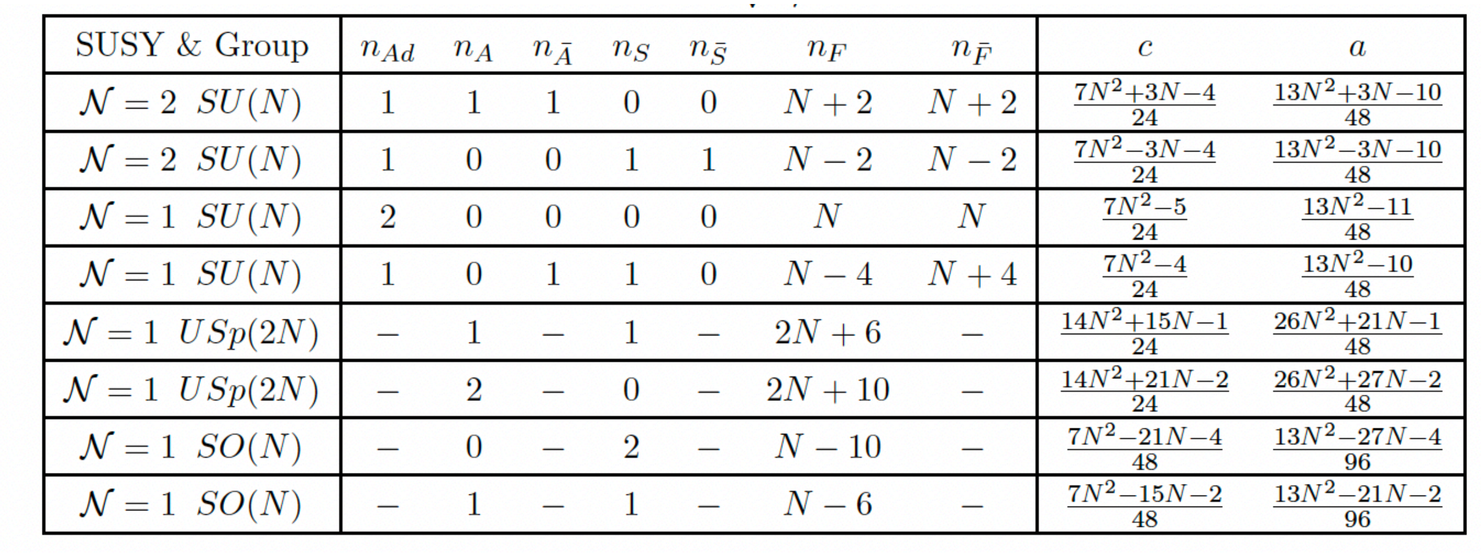
|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{N}=2$ $SU(N)$ | 1 | 0 | 0 | 0 | 0 | $2N$ | $2N$ | $\frac{2N^2-1}{6}$ | $\frac{7N^2-5}{24}$ |
| $\mathcal{N}=1$ $SU(N)$ | 0 | 0 | 1 | 1 | 0 | $2N-4$ | $2N+4$ | $\frac{8N^2-3}{24}$ | $\frac{14N^2-9}{48}$ |
| $\mathcal{N}=2$ $USp(2N)$ | – | 0 | – | 1 | – | $4N+4$ | – | $\frac{N(4N+3)}{6}$ | $\frac{N(14N+9)}{24}$ |
| $\mathcal{N}=2$ $SO(N)$ | – | 1 | – | 0 | – | $2N-4$ | – | $\frac{N(2N-3)}{12}$ | $\frac{N(7N-9)}{48}$ |
| $\mathcal{N}=1$ $SO(N)$ | – | 0 | – | 1 | – | $2N-8$ | – | $\frac{4N^2-9N-1}{24}$ | $\frac{7N^2-12N-1}{48}$ |

III)



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|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{N}=2$ $SU(N)$ | 1 | 1 | 1 | 0 | 0 | $N+2$ | $N+2$ | $\frac{7N^2+3N-4}{24}$ | $\frac{13N^2+3N-10}{48}$ |
| $\mathcal{N}=2$ $SU(N)$ | 1 | 0 | 0 | 1 | 1 | $N-2$ | $N-2$ | $\frac{7N^2-3N-4}{24}$ | $\frac{13N^2-3N-10}{48}$ |
| $\mathcal{N}=1$ $SU(N)$ | 2 | 0 | 0 | 0 | 0 | $N$ | $N$ | $\frac{7N^2-5}{24}$ | $\frac{13N^2-11}{48}$ |
| $\mathcal{N}=1$ $SU(N)$ | 1 | 0 | 1 | 1 | 0 | $N-4$ | $N+4$ | $\frac{7N^2-4}{24}$ | $\frac{13N^2-10}{48}$ |
| $\mathcal{N}=1$ $USp(2N)$ | – | 1 | – | 1 | – | $2N+6$ | – | $\frac{14N^2+15N-1}{24}$ | $\frac{26N^2+21N-1}{48}$ |
| $\mathcal{N}=1$ $USp(2N)$ | – | 2 | – | 0 | – | $2N+10$ | – | $\frac{14N^2+21N-2}{24}$ | $\frac{26N^2+27N-2}{48}$ |
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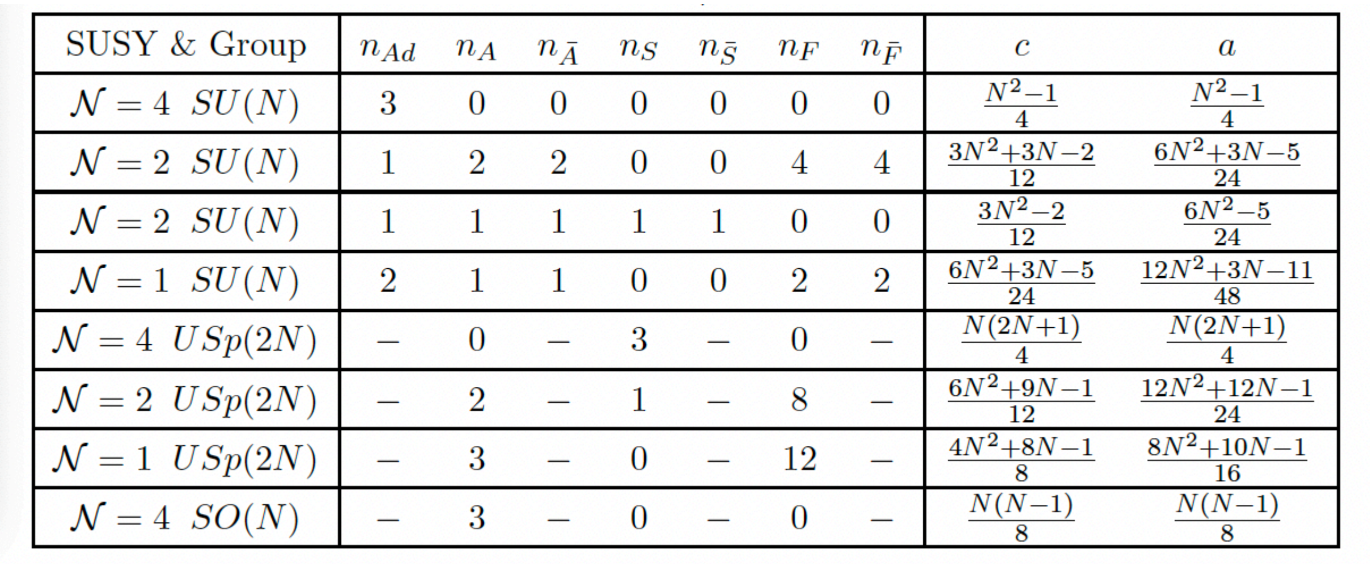

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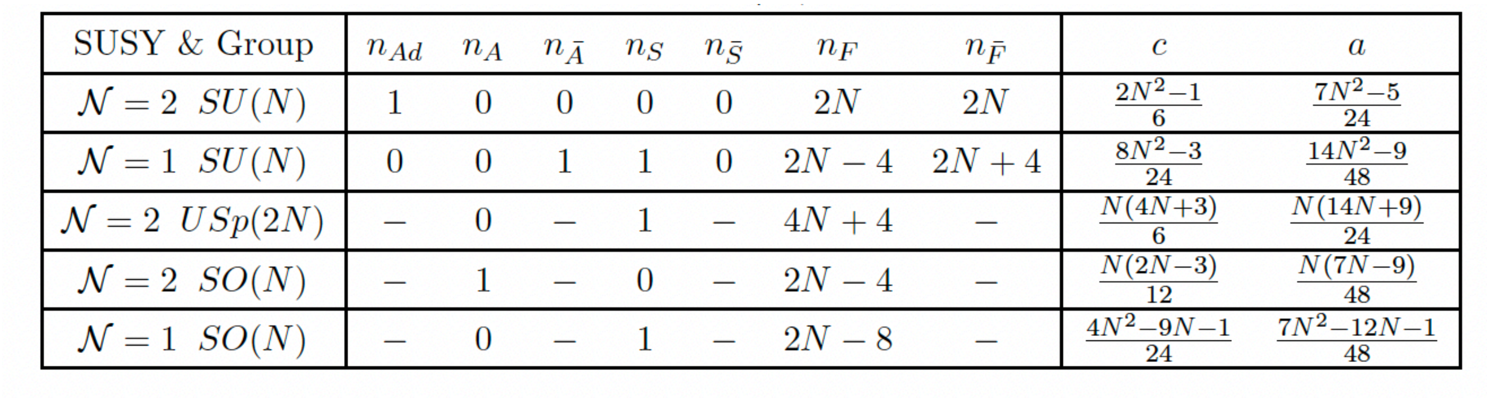
| $\mathcal{N}=4$ $SO(N)$ | – | 3 | – | 0 | – | 0 | – | $\frac{N(N-1)}{8}$ | $\frac{N(N-1)}{8}$ |

$$\alpha = \frac{1}{\sqrt{2}}$$

II)



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|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{N}=2$ $SU(N)$ | 1 | 0 | 0 | 0 | 0 | $2N$ | $2N$ | $\frac{2N^2-1}{6}$ | $\frac{7N^2-5}{24}$ |
| $\mathcal{N}=1$ $SU(N)$ | 0 | 0 | 1 | 1 | 0 | $2N-4$ | $2N+4$ | $\frac{8N^2-3}{24}$ | $\frac{14N^2-9}{48}$ |
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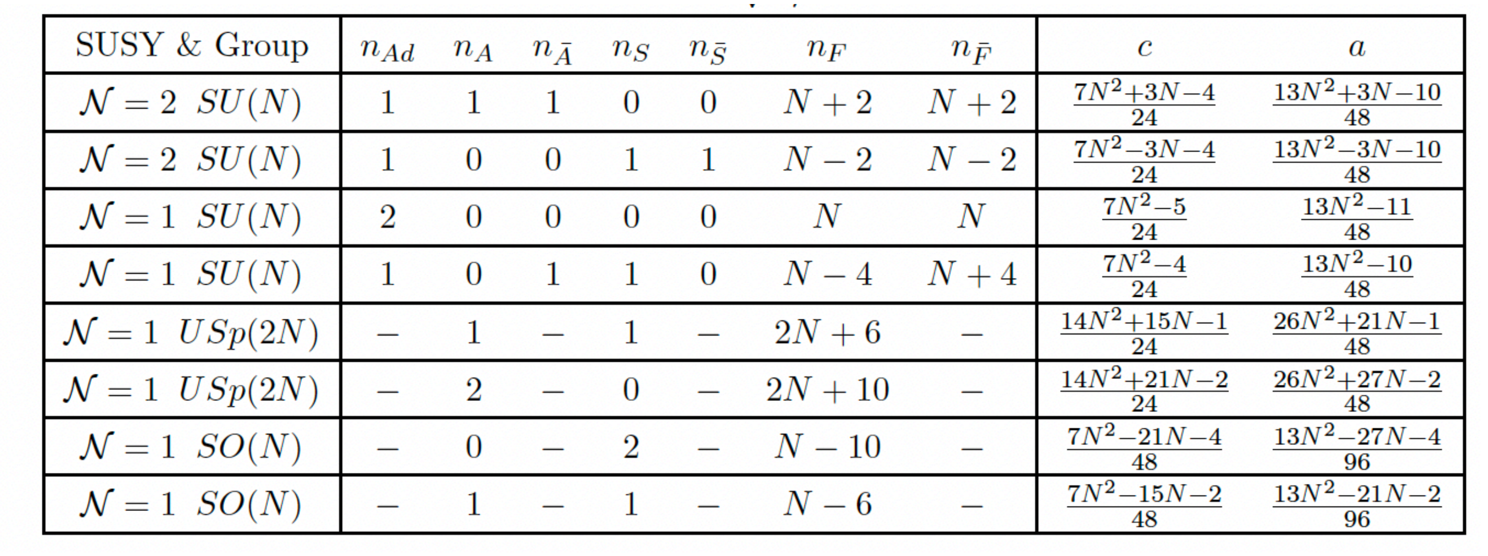
| $\mathcal{N}=1$ $SO(N)$ | – | 0 | – | 1 | – | $2N-8$ | – | $\frac{4N^2-9N-1}{24}$ | $\frac{7N^2-12N-1}{48}$ |

$$\alpha = \sqrt{\frac{2}{3}}$$

III)



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|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{N}=2$ $SU(N)$ | 1 | 1 | 1 | 0 | 0 | $N+2$ | $N+2$ | $\frac{7N^2+3N-4}{24}$ | $\frac{13N^2+3N-10}{48}$ |
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| $\mathcal{N}=1$ $SO(N)$ | – | 0 | – | 2 | – | $N-10$ | – | $\frac{7N^2-21N-4}{48}$ | $\frac{13N^2-27N-4}{96}$ |
| $\mathcal{N}=1$ $SO(N)$ | – | 1 | – | 1 | – | $N-6$ | – | $\frac{7N^2-15N-2}{48}$ | $\frac{13N^2-21N-2}{96}$ |

$$\alpha = \sqrt{\frac{7}{12}}$$

# CLASSIFICATION OF LIMITS IN 4d SCFTs

(Strings&Geometry 2024 talk)

Considering all large N 4d SCFTs with simple gauge factor:

We got only three values for the exponential rate of the HS tower, corresponding to three different types of tensionless strings

I) $\alpha = \frac{1}{\sqrt{2}} \qquad \text{for } \frac{a}{c} = 1$

II) $\alpha = \sqrt{\frac{2}{3}} \qquad \text{for } \frac{a}{c} = \frac{7}{8}$

III) $\alpha = \sqrt{\frac{7}{12}} \qquad \text{for } \frac{a}{c} = \frac{13}{14}$

They have a different Hagedorn-like density of states (different Haggedorn temperature) [Calderon-Infante,IV'24]

(see [Calderon-Infante,Mohseni'26] for generalization to SU(N) quivers for more than one gauge factor)

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I) $\alpha = \frac{1}{\sqrt{2}}$ for $\frac{a}{c} = 1$ → critical string in all Einstein theories

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III) $\alpha = \sqrt{\frac{7}{12}}$ for $\frac{a}{c} = \frac{13}{14}$

→ what about these ones?

non-critical strings in non-Einstein theories?

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(see [Calderon-Infante,Mohseni'26] for generalization to SU(N) quivers for more than one gauge factor)

# CLASSIFICATION OF LIMITS IN 4d SCFTs

**UPDATE:**

We have explicitely confirmed the previous expectations and identify each type of string

by constructing the dual brane picture to each of these SCFTs

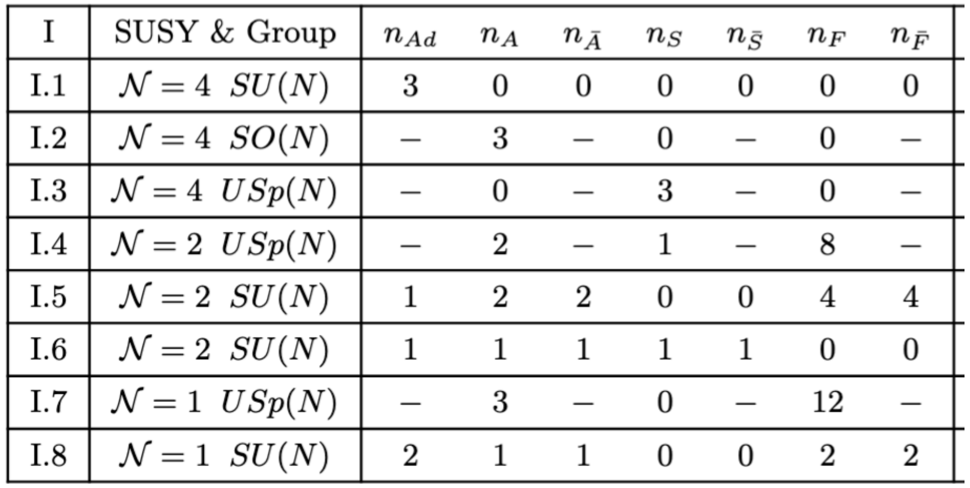
(including all the non-Einstein SCFTs)

[Calderon-Infante,UrangaIV'ongoing]

# CLASSIFICATION OF LIMITS IN 4d SCFTs

Full classification of 4d (Lagrangian) SCFTs with large N and simple gauge factor:

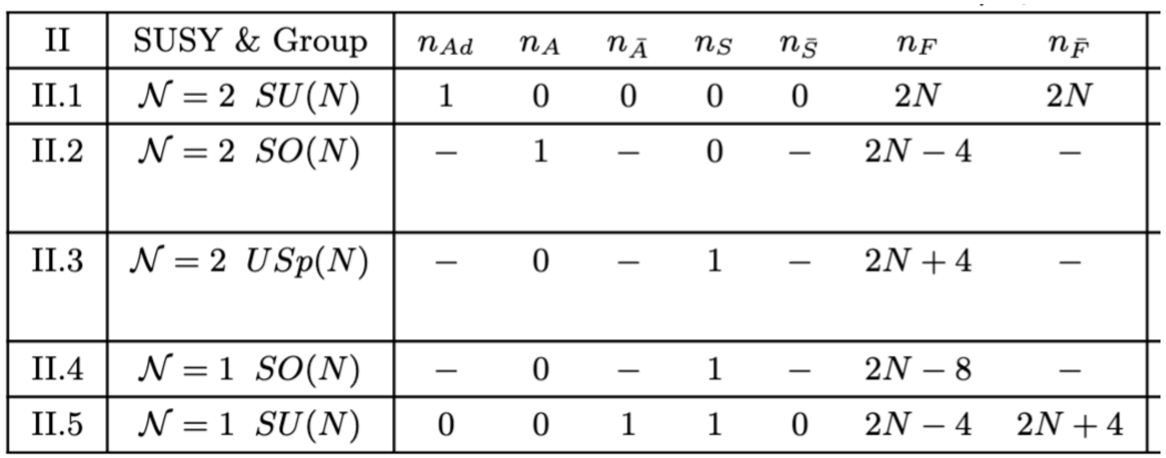
I)



| I | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ |
|---|---|---|---|---|---|---|---|---|
| I.1 | $\mathcal{N}=4\ SU(N)$ | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| I.2 | $\mathcal{N}=4\ SO(N)$ | – | 3 | – | 0 | – | 0 | – |
| I.3 | $\mathcal{N}=4\ USp(N)$ | – | 0 | – | 3 | – | 0 | – |
| I.4 | $\mathcal{N}=2\ USp(N)$ | – | 2 | – | 1 | – | 8 | – |
| I.5 | $\mathcal{N}=2\ SU(N)$ | 1 | 2 | 2 | 0 | 0 | 4 | 4 |
| I.6 | $\mathcal{N}=2\ SU(N)$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| I.7 | $\mathcal{N}=1\ USp(N)$ | – | 3 | – | 0 | – | 12 | – |
| I.8 | $\mathcal{N}=1\ SU(N)$ | 2 | 1 | 1 | 0 | 0 | 2 | 2 |

$$\alpha = \frac{1}{\sqrt{2}}$$

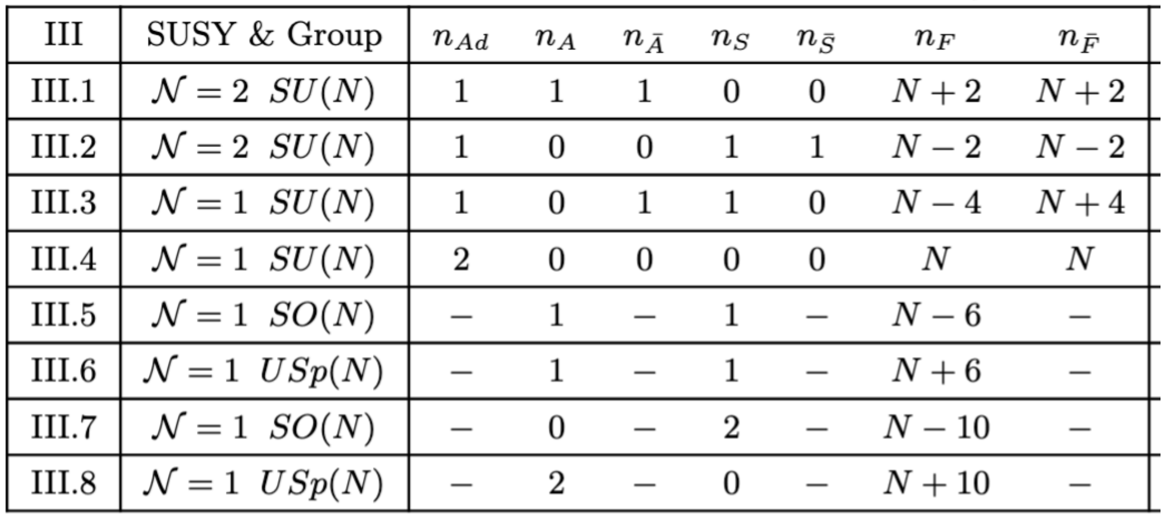
II)



| II | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ |
|---|---|---|---|---|---|---|---|---|
| II.1 | $\mathcal{N}=2\ SU(N)$ | 1 | 0 | 0 | 0 | 0 | $2N$ | $2N$ |
| II.2 | $\mathcal{N}=2\ SO(N)$ | – | 1 | – | 0 | – | $2N-4$ | – |
| II.3 | $\mathcal{N}=2\ USp(N)$ | – | 0 | – | 1 | – | $2N+4$ | – |
| II.4 | $\mathcal{N}=1\ SO(N)$ | – | 0 | – | 1 | – | $2N-8$ | – |
| II.5 | $\mathcal{N}=1\ SU(N)$ | 0 | 0 | 1 | 1 | 0 | $2N-4$ | $2N+4$ |

$$\alpha = \sqrt{\frac{2}{3}}$$

III)



| III | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ |
|---|---|---|---|---|---|---|---|---|
| III.1 | $\mathcal{N}=2\ SU(N)$ | 1 | 1 | 1 | 0 | 0 | $N+2$ | $N+2$ |
| III.2 | $\mathcal{N}=2\ SU(N)$ | 1 | 0 | 0 | 1 | 1 | $N-2$ | $N-2$ |
| III.3 | $\mathcal{N}=1\ SU(N)$ | 1 | 0 | 1 | 1 | 0 | $N-4$ | $N+4$ |
| III.4 | $\mathcal{N}=1\ SU(N)$ | 2 | 0 | 0 | 0 | 0 | $N$ | $N$ |
| III.5 | $\mathcal{N}=1\ SO(N)$ | – | 1 | – | 1 | – | $N-6$ | – |
| III.6 | $\mathcal{N}=1\ USp(N)$ | – | 1 | – | 1 | – | $N+6$ | – |
| III.7 | $\mathcal{N}=1\ SO(N)$ | – | 0 | – | 2 | – | $N-10$ | – |
| III.8 | $\mathcal{N}=1\ USp(N)$ | – | 2 | – | 0 | – | $N+10$ | – |

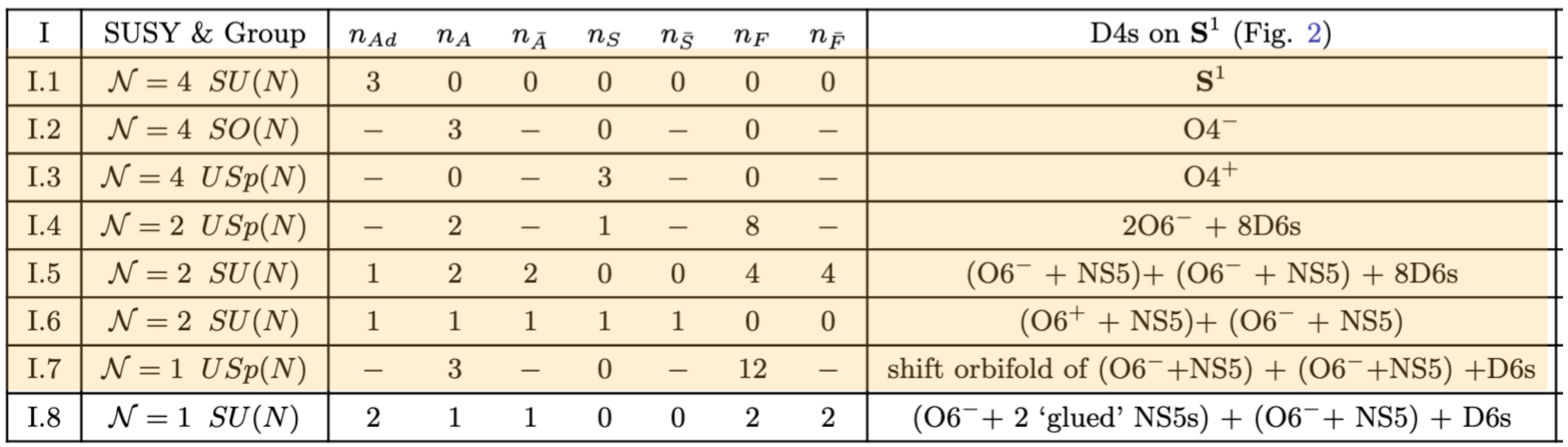
$$\alpha = \sqrt{\frac{7}{12}}$$

# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

Full classification of 4d (Lagrangian) SCFTs with large N and simple gauge factor:

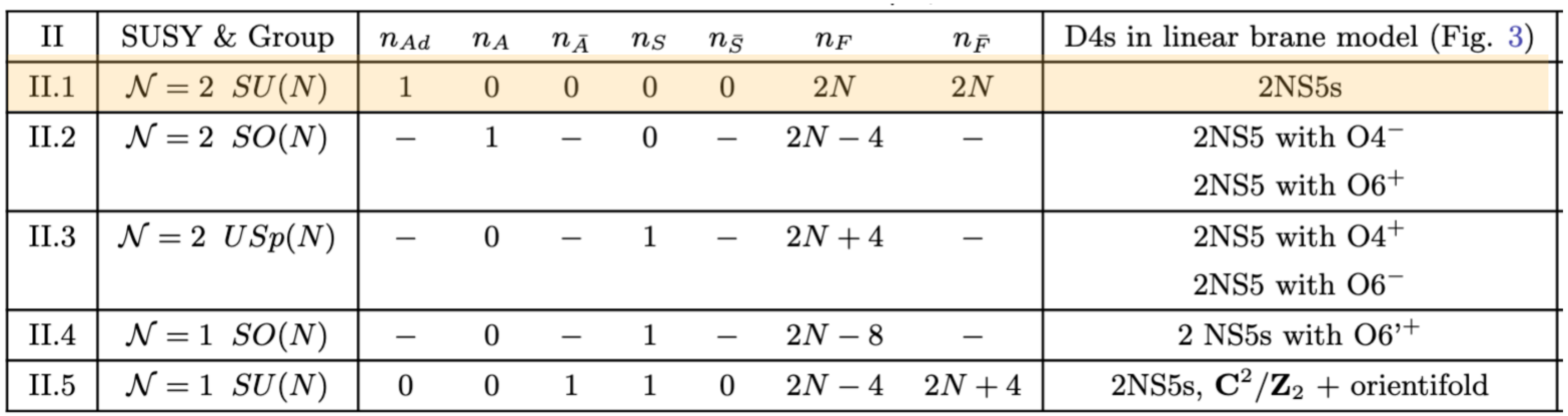
I)



| I | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s on $\mathbf{S}^1$ (Fig. 2) |
|---|---|---|---|---|---|---|---|---|---|
| I.1 | $\mathcal{N}=4$ $SU(N)$ | 3 | 0 | 0 | 0 | 0 | 0 | 0 | $\mathbf{S}^1$ |
| I.2 | $\mathcal{N}=4$ $SO(N)$ | – | 3 | – | 0 | – | 0 | – | $\mathrm{O4}^-$ |
| I.3 | $\mathcal{N}=4$ $USp(N)$ | – | 0 | – | 3 | – | 0 | – | $\mathrm{O4}^+$ |
| I.4 | $\mathcal{N}=2$ $USp(N)$ | – | 2 | – | 1 | – | 8 | – | $2\mathrm{O6}^- + 8\mathrm{D6s}$ |
| I.5 | $\mathcal{N}=2$ $SU(N)$ | 1 | 2 | 2 | 0 | 0 | 4 | 4 | $(\mathrm{O6}^- + \mathrm{NS5}) + (\mathrm{O6}^- + \mathrm{NS5}) + 8\mathrm{D6s}$ |
| I.6 | $\mathcal{N}=2$ $SU(N)$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | $(\mathrm{O6}^+ + \mathrm{NS5}) + (\mathrm{O6}^- + \mathrm{NS5})$ |
| I.7 | $\mathcal{N}=1$ $USp(N)$ | – | 3 | – | 0 | – | 12 | – | shift orbifold of $(\mathrm{O6}^- + \mathrm{NS5}) + (\mathrm{O6}^- + \mathrm{NS5}) + \mathrm{D6s}$ |
| I.8 | $\mathcal{N}=1$ $SU(N)$ | 2 | 1 | 1 | 0 | 0 | 2 | 2 | $(\mathrm{O6}^- +$ 2 'glued' NS5s$) + (\mathrm{O6}^- + \mathrm{NS5}) + \mathrm{D6s}$ |

$$\alpha = \frac{1}{\sqrt{2}}$$

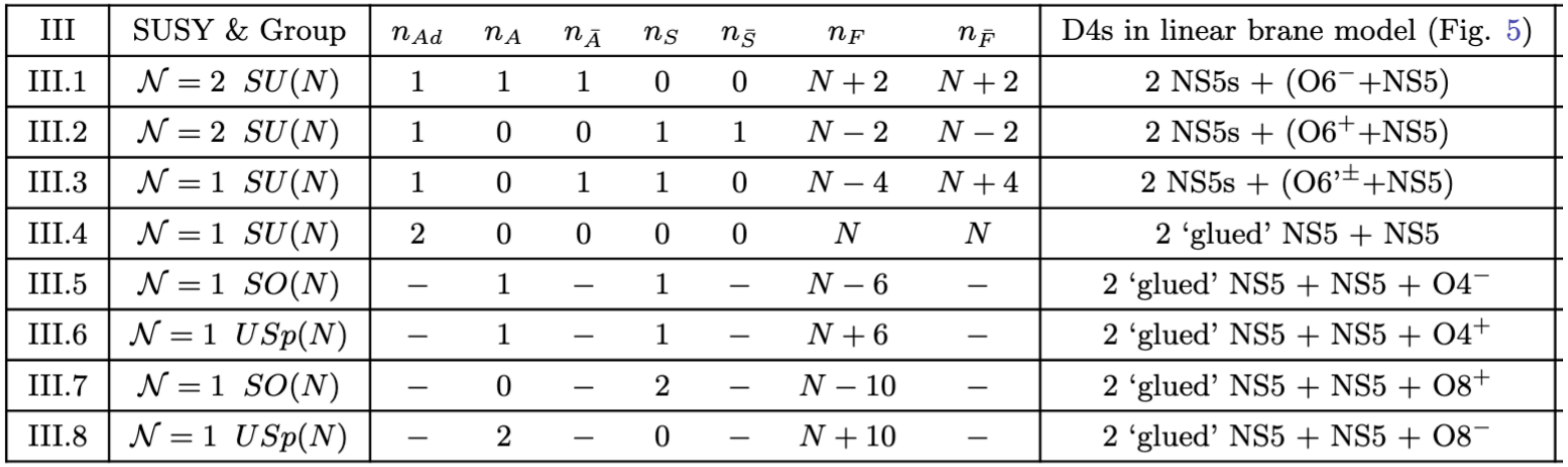
II)



| II | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s in linear brane model (Fig. 3) |
|---|---|---|---|---|---|---|---|---|---|
| II.1 | $\mathcal{N}=2$ $SU(N)$ | 1 | 0 | 0 | 0 | 0 | $2N$ | $2N$ | 2NS5s |
| II.2 | $\mathcal{N}=2$ $SO(N)$ | – | 1 | – | 0 | – | $2N-4$ | – | 2NS5 with $\mathrm{O4}^-$<br>2NS5 with $\mathrm{O6}^+$ |
| II.3 | $\mathcal{N}=2$ $USp(N)$ | – | 0 | – | 1 | – | $2N+4$ | – | 2NS5 with $\mathrm{O4}^+$<br>2NS5 with $\mathrm{O6}^-$ |
| II.4 | $\mathcal{N}=1$ $SO(N)$ | – | 0 | – | 1 | – | $2N-8$ | – | 2 NS5s with $\mathrm{O6'}^+$ |
| II.5 | $\mathcal{N}=1$ $SU(N)$ | 0 | 0 | 1 | 1 | 0 | $2N-4$ | $2N+4$ | 2NS5s, $\mathbf{C}^2/\mathbf{Z}_2$ + orientifold |

$$\alpha = \sqrt{\frac{2}{3}}$$

III)



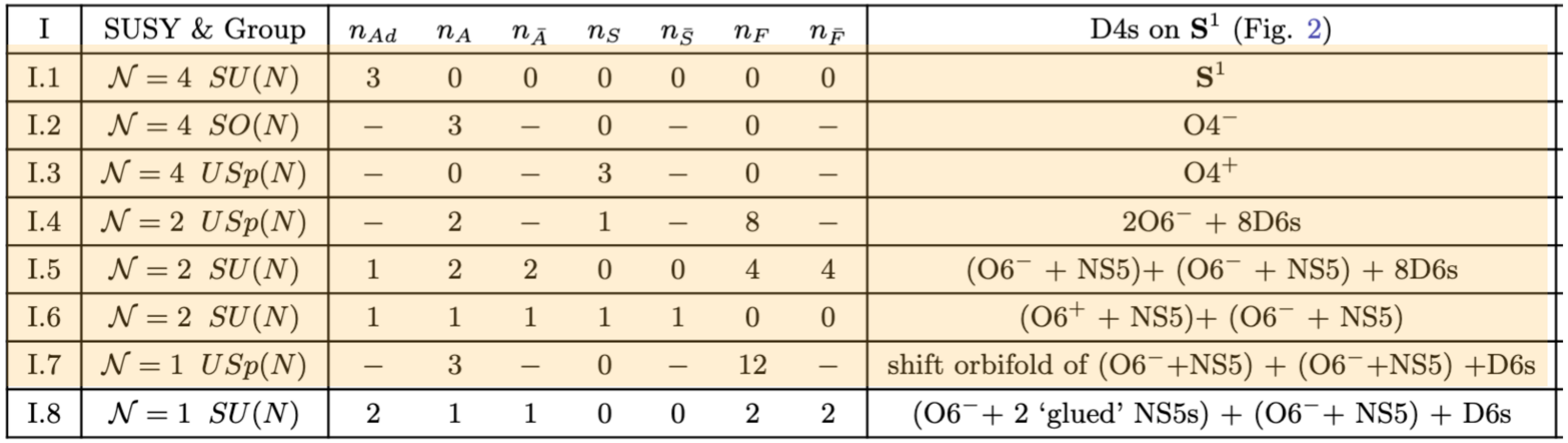
| III | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s in linear brane model (Fig. 5) |
|---|---|---|---|---|---|---|---|---|---|
| III.1 | $\mathcal{N}=2$ $SU(N)$ | 1 | 1 | 1 | 0 | 0 | $N+2$ | $N+2$ | 2 NS5s + $(\mathrm{O6}^- + \mathrm{NS5})$ |
| III.2 | $\mathcal{N}=2$ $SU(N)$ | 1 | 0 | 0 | 1 | 1 | $N-2$ | $N-2$ | 2 NS5s + $(\mathrm{O6}^+ + \mathrm{NS5})$ |
| III.3 | $\mathcal{N}=1$ $SU(N)$ | 1 | 0 | 1 | 1 | 0 | $N-4$ | $N+4$ | 2 NS5s + $(\mathrm{O6'}^{\pm} + \mathrm{NS5})$ |
| III.4 | $\mathcal{N}=1$ $SU(N)$ | 2 | 0 | 0 | 0 | 0 | $N$ | $N$ | 2 'glued' NS5 + NS5 |
| III.5 | $\mathcal{N}=1$ $SO(N)$ | – | 1 | – | 1 | – | $N-6$ | – | 2 'glued' NS5 + NS5 + $\mathrm{O4}^-$ |
| III.6 | $\mathcal{N}=1$ $USp(N)$ | – | 1 | – | 1 | – | $N+6$ | – | 2 'glued' NS5 + NS5 + $\mathrm{O4}^+$ |
| III.7 | $\mathcal{N}=1$ $SO(N)$ | – | 0 | – | 2 | – | $N-10$ | – | 2 'glued' NS5 + NS5 + $\mathrm{O8}^+$ |
| III.8 | $\mathcal{N}=1$ $USp(N)$ | – | 2 | – | 0 | – | $N+10$ | – | 2 'glued' NS5 + NS5 + $\mathrm{O8}^-$ |

$$\alpha = \sqrt{\frac{7}{12}}$$

# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

I)



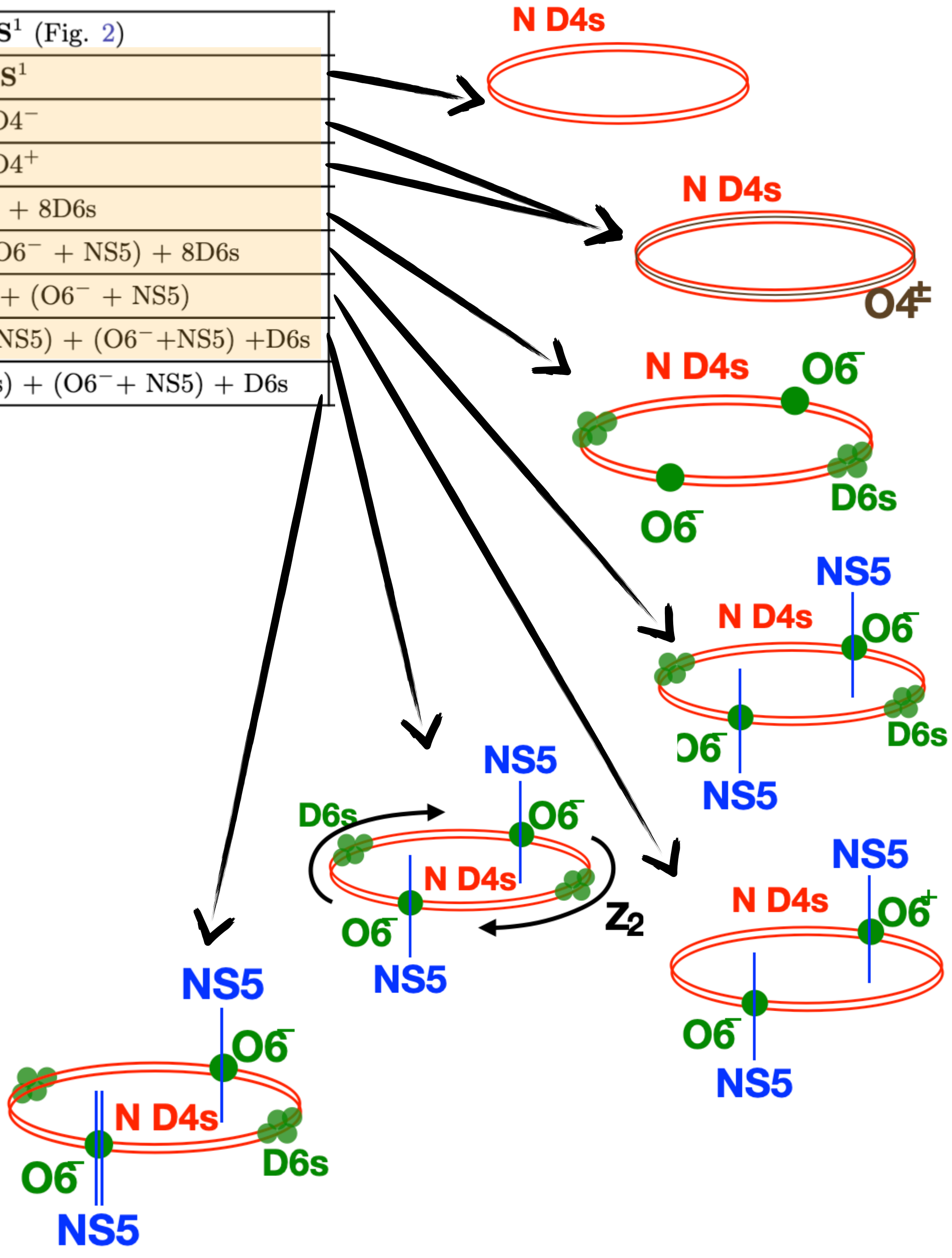
| I | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s on $\mathbf{S}^1$ (Fig. 2) |
|---|---|---|---|---|---|---|---|---|---|
| I.1 | $\mathcal{N}=4\ SU(N)$ | 3 | 0 | 0 | 0 | 0 | 0 | 0 | $\mathbf{S}^1$ |
| I.2 | $\mathcal{N}=4\ SO(N)$ | – | 3 | – | 0 | – | 0 | – | $O4^-$ |
| I.3 | $\mathcal{N}=4\ USp(N)$ | – | 0 | – | 3 | – | 0 | – | $O4^+$ |
| I.4 | $\mathcal{N}=2\ USp(N)$ | – | 2 | – | 1 | – | 8 | – | $2O6^-$ + 8D6s |
| I.5 | $\mathcal{N}=2\ SU(N)$ | 1 | 2 | 2 | 0 | 0 | 4 | 4 | ($O6^-$ + NS5)+ ($O6^-$ + NS5) + 8D6s |
| I.6 | $\mathcal{N}=2\ SU(N)$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | ($O6^+$ + NS5)+ ($O6^-$ + NS5) |
| I.7 | $\mathcal{N}=1\ USp(N)$ | – | 3 | – | 0 | – | 12 | – | shift orbifold of ($O6^-$+NS5) + ($O6^-$+NS5) +D6s |
| I.8 | $\mathcal{N}=1\ SU(N)$ | 2 | 1 | 1 | 0 | 0 | 2 | 2 | ($O6^-$+ 2 'glued' NS5s) + ($O6^-$+ NS5) + D6s |

# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

I)

| I | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s on $\mathbf{S}^1$ (Fig. 2) |
|---|---|---|---|---|---|---|---|---|---|
| I.1 | $\mathcal{N}=4\ SU(N)$ | 3 | 0 | 0 | 0 | 0 | 0 | 0 | $\mathbf{S}^1$ |
| I.2 | $\mathcal{N}=4\ SO(N)$ | – | 3 | – | 0 | – | 0 | – | $O4^-$ |
| I.3 | $\mathcal{N}=4\ USp(N)$ | – | 0 | – | 3 | – | 0 | – | $O4^+$ |
| I.4 | $\mathcal{N}=2\ USp(N)$ | – | 2 | – | 1 | – | 8 | – | $2O6^- + 8D6s$ |
| I.5 | $\mathcal{N}=2\ SU(N)$ | 1 | 2 | 2 | 0 | 0 | 4 | 4 | $(O6^- + NS5)+ (O6^- + NS5) + 8D6s$ |
| I.6 | $\mathcal{N}=2\ SU(N)$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | $(O6^+ + NS5)+ (O6^- + NS5)$ |
| I.7 | $\mathcal{N}=1\ USp(N)$ | – | 3 | – | 0 | – | 12 | – | shift orbifold of $(O6^-+NS5)$ + $(O6^-+NS5)$ +D6s |
| I.8 | $\mathcal{N}=1\ SU(N)$ | 2 | 1 | 1 | 0 | 0 | 2 | 2 | $(O6^-+$ 2 'glued' NS5s) + $(O6^-+ NS5)$ + D6s |

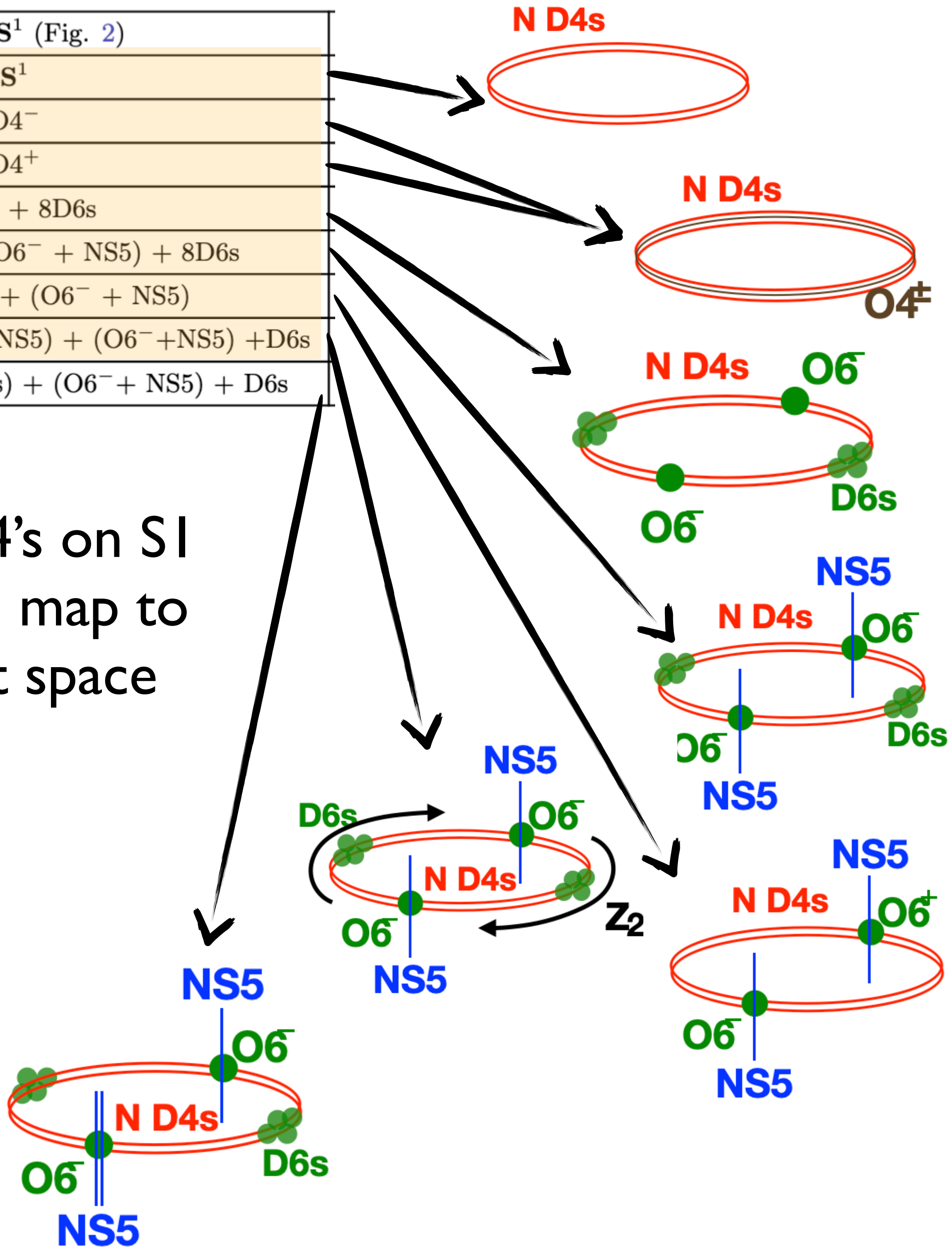


# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

I)

| I | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s on $\mathbf{S}^1$ (Fig. 2) |
|---|---|---|---|---|---|---|---|---|---|
| I.1 | $\mathcal{N}=4\ SU(N)$ | 3 | 0 | 0 | 0 | 0 | 0 | 0 | $\mathbf{S}^1$ |
| I.2 | $\mathcal{N}=4\ SO(N)$ | – | 3 | – | 0 | – | 0 | – | $O4^-$ |
| I.3 | $\mathcal{N}=4\ USp(N)$ | – | 0 | – | 3 | – | 0 | – | $O4^+$ |
| I.4 | $\mathcal{N}=2\ USp(N)$ | – | 2 | – | 1 | – | 8 | – | $2O6^- + 8D6s$ |
| I.5 | $\mathcal{N}=2\ SU(N)$ | 1 | 2 | 2 | 0 | 0 | 4 | 4 | $(O6^- + NS5)+ (O6^- + NS5) + 8D6s$ |
| I.6 | $\mathcal{N}=2\ SU(N)$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | $(O6^+ + NS5)+ (O6^- + NS5)$ |
| I.7 | $\mathcal{N}=1\ USp(N)$ | – | 3 | – | 0 | – | 12 | – | shift orbifold of $(O6^-$+NS5) + $(O6^-$+NS5) +D6s |
| I.8 | $\mathcal{N}=1\ SU(N)$ | 2 | 1 | 1 | 0 | 0 | 2 | 2 | $(O6^-$+ 2 'glued' NS5s) + $(O6^-$+ NS5) + D6s |



Bulk dual is always Einstein, obtained from N D4's on S1 (+ different branes/orientifolds/orbifolds) which map to N D3's on orientifold/orbifold quotients of flat space

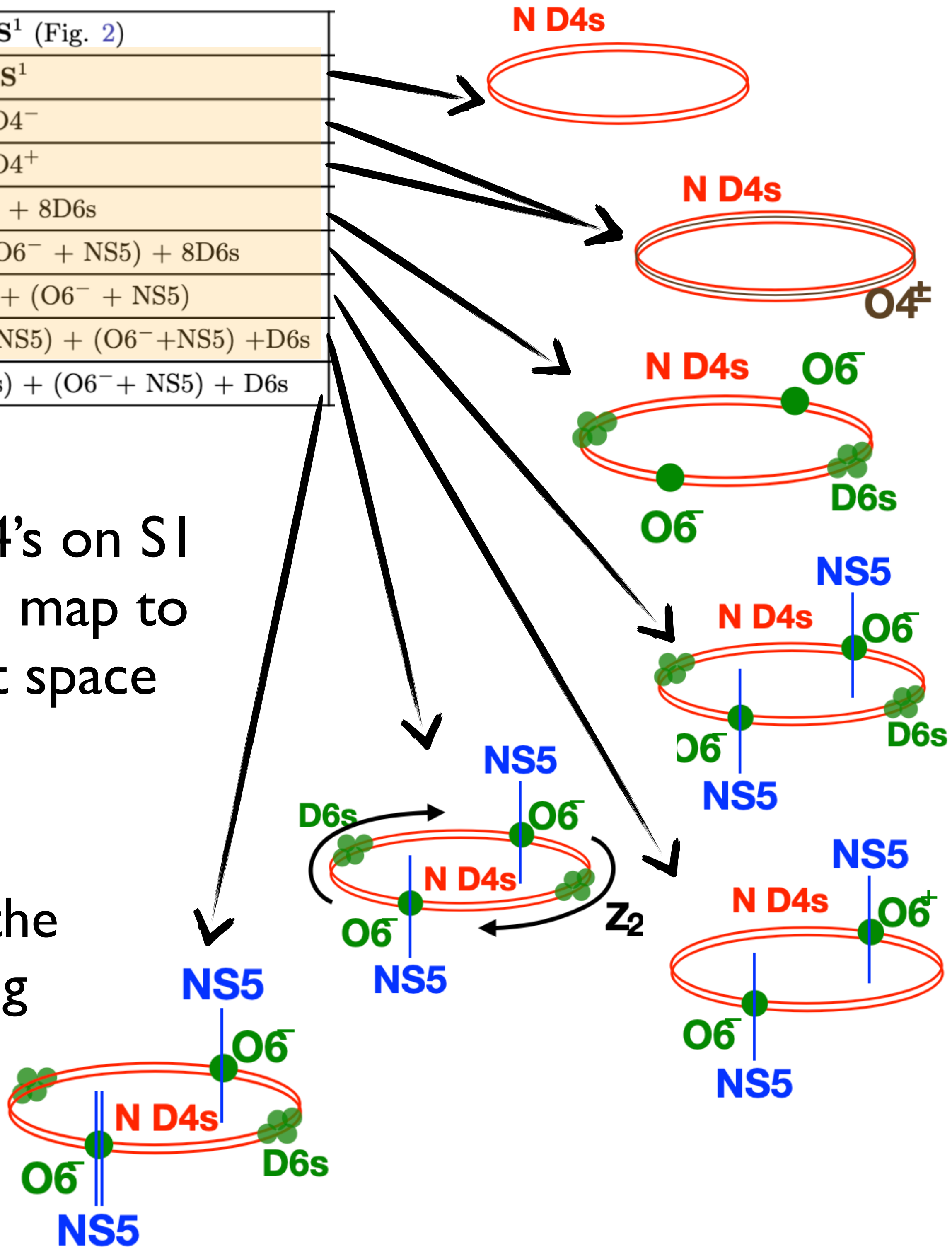
↪ subleading on 1/N

# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

I)

| I | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s on $\mathbf{S}^1$ (Fig. 2) |
|---|---|---|---|---|---|---|---|---|---|
| I.1 | $\mathcal{N}=4$ $SU(N)$ | 3 | 0 | 0 | 0 | 0 | 0 | 0 | $\mathbf{S}^1$ |
| I.2 | $\mathcal{N}=4$ $SO(N)$ | – | 3 | – | 0 | – | 0 | – | O4$^-$ |
| I.3 | $\mathcal{N}=4$ $USp(N)$ | – | 0 | – | 3 | – | 0 | – | O4$^+$ |
| I.4 | $\mathcal{N}=2$ $USp(N)$ | – | 2 | – | 1 | – | 8 | – | 2O6$^-$ + 8D6s |
| I.5 | $\mathcal{N}=2$ $SU(N)$ | 1 | 2 | 2 | 0 | 0 | 4 | 4 | (O6$^-$ + NS5)+ (O6$^-$ + NS5) + 8D6s |
| I.6 | $\mathcal{N}=2$ $SU(N)$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | (O6$^+$ + NS5)+ (O6$^-$ + NS5) |
| I.7 | $\mathcal{N}=1$ $USp(N)$ | – | 3 | – | 0 | – | 12 | – | shift orbifold of (O6$^-$+NS5) + (O6$^-$+NS5) +D6s |
| I.8 | $\mathcal{N}=1$ $SU(N)$ | 2 | 1 | 1 | 0 | 0 | 2 | 2 | (O6$^-$+ 2 'glued' NS5s) + (O6$^-$+ NS5) + D6s |



Bulk dual is always Einstein, obtained from N D4's on S1 (+ different branes/orientifolds/orbifolds) which map to N D3's on orientifold/orbifold quotients of flat space

⟶ subleading on 1/N

Weak coupling limit with

$$\alpha = \frac{1}{\sqrt{2}}$$

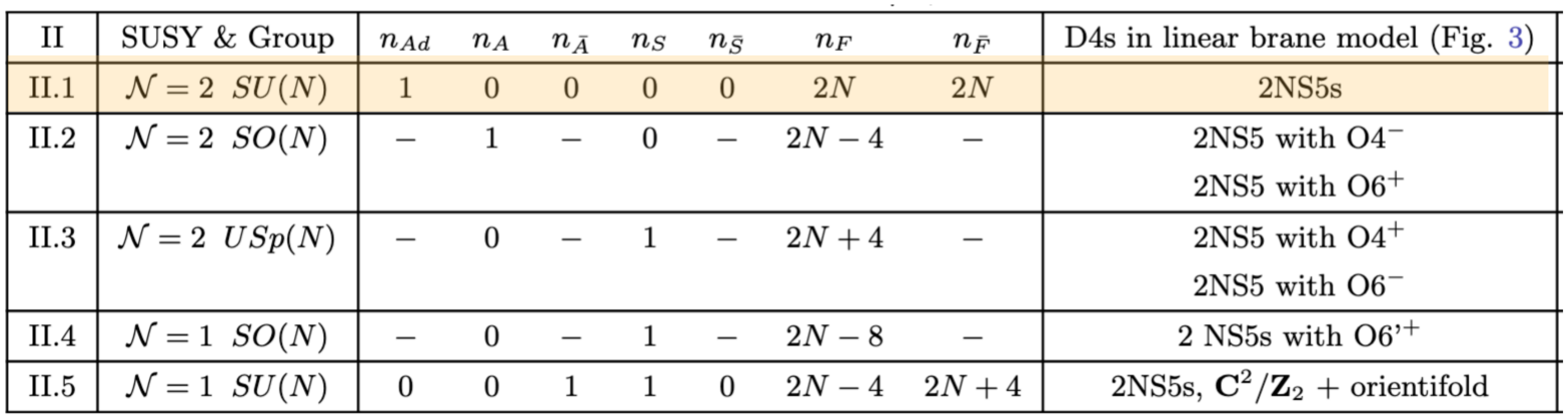
= tensionless limit of the critical Type II string

$(g_s \to 0)$

# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

II)

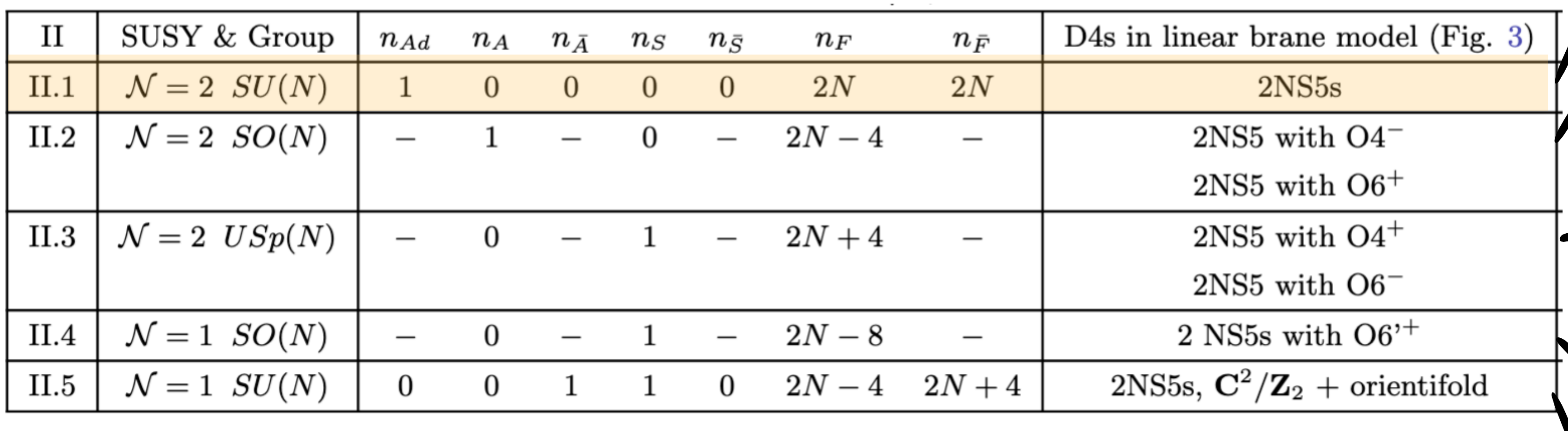


| II | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s in linear brane model (Fig. 3) |
|---|---|---|---|---|---|---|---|---|---|
| II.1 | $\mathcal{N}=2\ SU(N)$ | 1 | 0 | 0 | 0 | 0 | $2N$ | $2N$ | 2NS5s |
| II.2 | $\mathcal{N}=2\ SO(N)$ | – | 1 | – | 0 | – | $2N-4$ | – | 2NS5 with O4$^-$<br>2NS5 with O6$^+$ |
| II.3 | $\mathcal{N}=2\ USp(N)$ | – | 0 | – | 1 | – | $2N+4$ | – | 2NS5 with O4$^+$<br>2NS5 with O6$^-$ |
| II.4 | $\mathcal{N}=1\ SO(N)$ | – | 0 | – | 1 | – | $2N-8$ | – | 2 NS5s with O6'$^+$ |
| II.5 | $\mathcal{N}=1\ SU(N)$ | 0 | 0 | 1 | 1 | 0 | $2N-4$ | $2N+4$ | 2NS5s, $\mathbf{C}^2/\mathbf{Z}_2$ + orientifold |

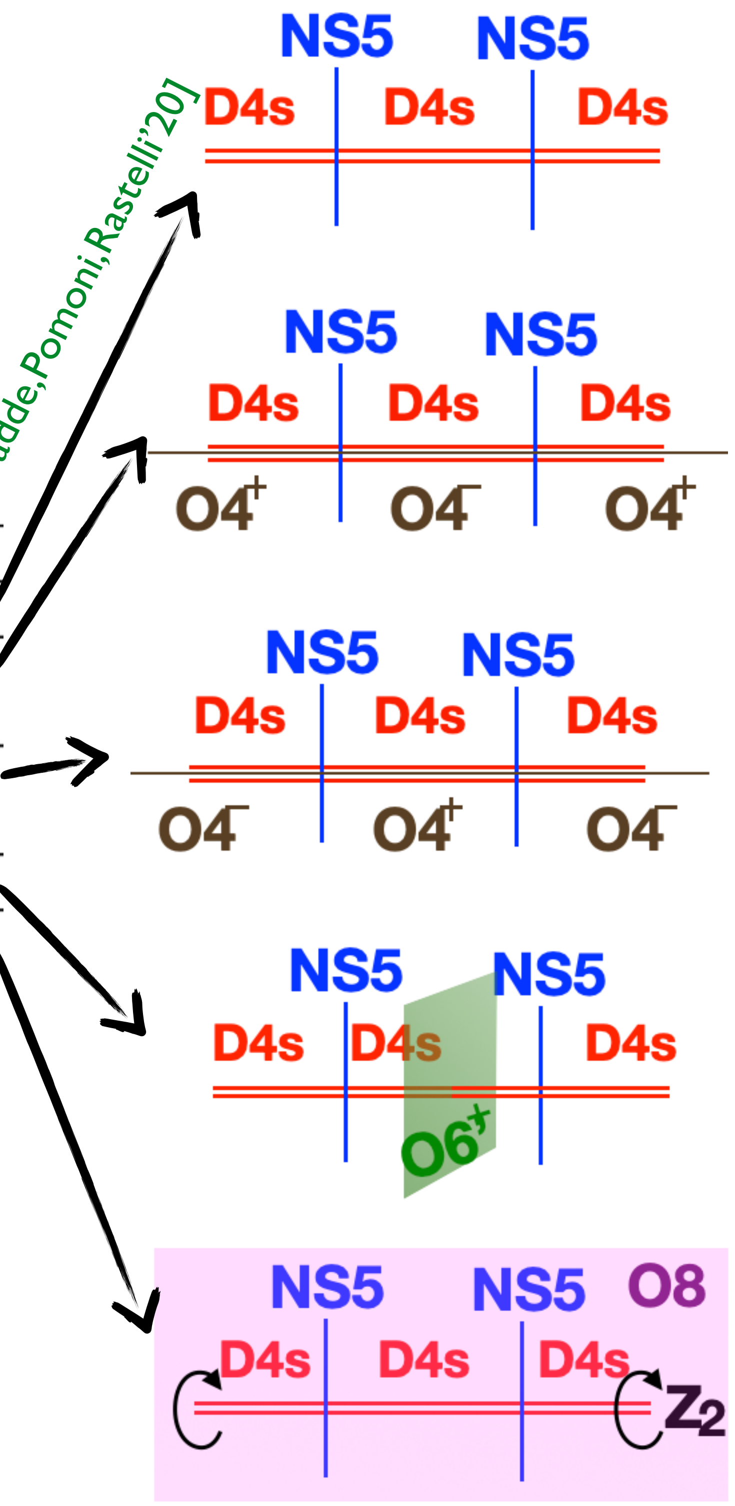
# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

II)



| II | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s in linear brane model (Fig. 3) |
|---|---|---|---|---|---|---|---|---|---|
| II.1 | $\mathcal{N}=2$ $SU(N)$ | 1 | 0 | 0 | 0 | 0 | $2N$ | $2N$ | 2NS5s |
| II.2 | $\mathcal{N}=2$ $SO(N)$ | – | 1 | – | 0 | – | $2N-4$ | – | 2NS5 with $O4^-$<br>2NS5 with $O6^+$ |
| II.3 | $\mathcal{N}=2$ $USp(N)$ | – | 0 | – | 1 | – | $2N+4$ | – | 2NS5 with $O4^+$<br>2NS5 with $O6^-$ |
| II.4 | $\mathcal{N}=1$ $SO(N)$ | – | 0 | – | 1 | – | $2N-8$ | – | 2 NS5s with $O6'^+$ |
| II.5 | $\mathcal{N}=1$ $SU(N)$ | 0 | 0 | 1 | 1 | 0 | $2N-4$ | $2N+4$ | 2NS5s, $\mathbf{C}^2/\mathbf{Z}_2$ + orientifold |



[Gadde,Pomoni,Rastelli'20]

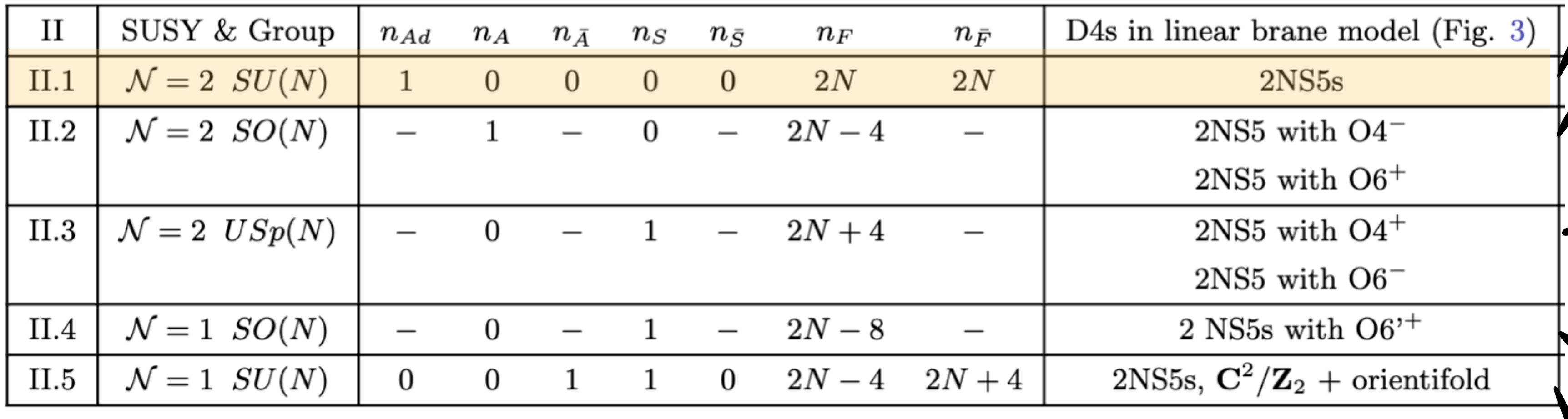
# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

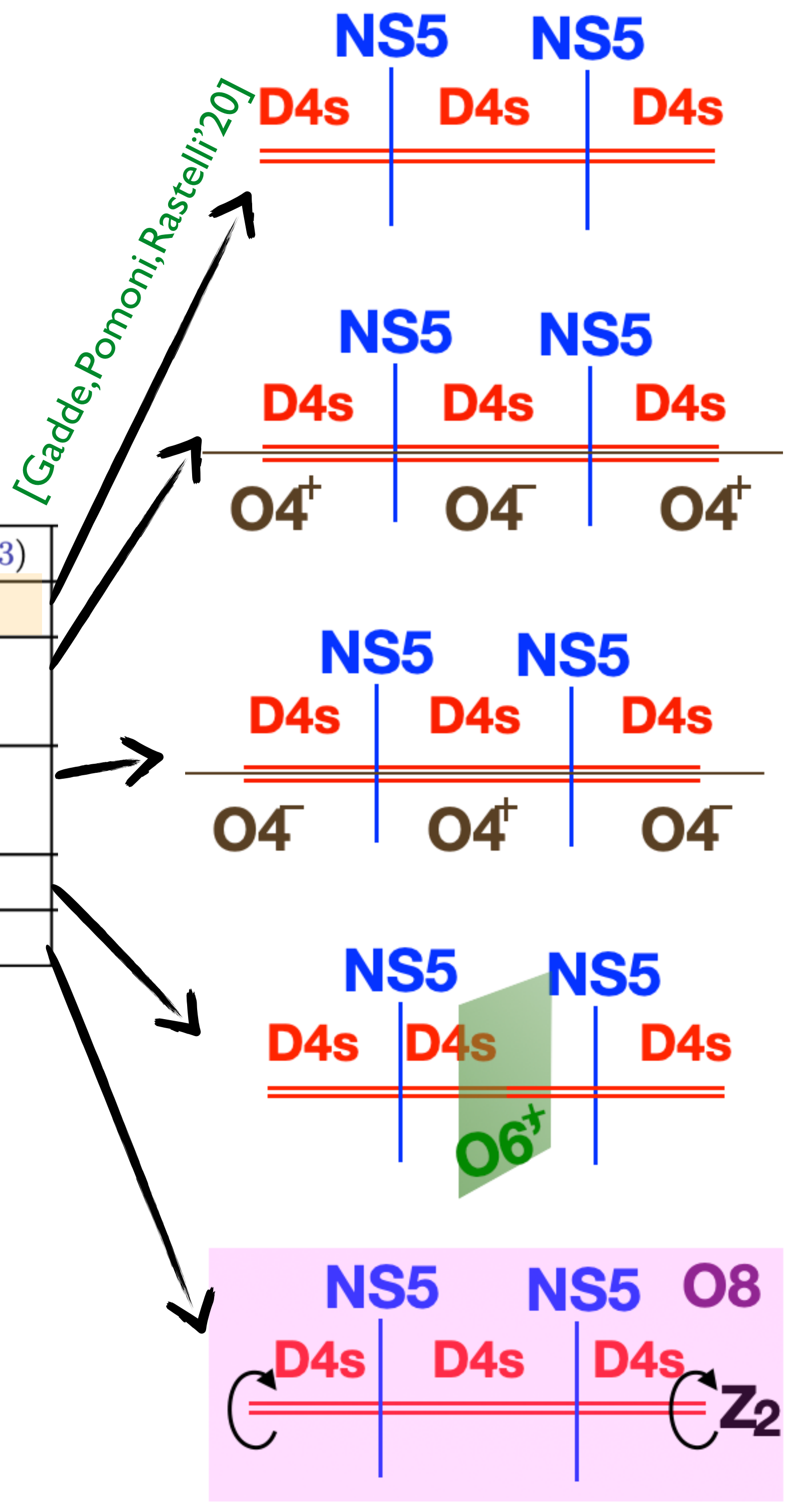
Bulk dual is non-Einstein, obtained from N D4's probing a background obtained from approaching 2 NS5's (in a double-scaling limit) + different content of orientifolds, flavour branes

subleading on 1/N

II)



| II | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s in linear brane model (Fig. 3) |
|---|---|---|---|---|---|---|---|---|---|
| II.1 | $\mathcal{N}=2$ $SU(N)$ | 1 | 0 | 0 | 0 | 0 | $2N$ | $2N$ | 2NS5s |
| II.2 | $\mathcal{N}=2$ $SO(N)$ | – | 1 | – | 0 | – | $2N-4$ | – | 2NS5 with $O4^-$<br>2NS5 with $O6^+$ |
| II.3 | $\mathcal{N}=2$ $USp(N)$ | – | 0 | – | 1 | – | $2N+4$ | – | 2NS5 with $O4^+$<br>2NS5 with $O6^-$ |
| II.4 | $\mathcal{N}=1$ $SO(N)$ | – | 0 | – | 1 | – | $2N-8$ | – | 2 NS5s with $O6'^+$ |
| II.5 | $\mathcal{N}=1$ $SU(N)$ | 0 | 0 | 1 | 1 | 0 | $2N-4$ | $2N+4$ | 2NS5s, $\mathbf{C}^2/\mathbf{Z}_2$ + orientifold |



[Gadde,Pomoni,Rastelli'20]

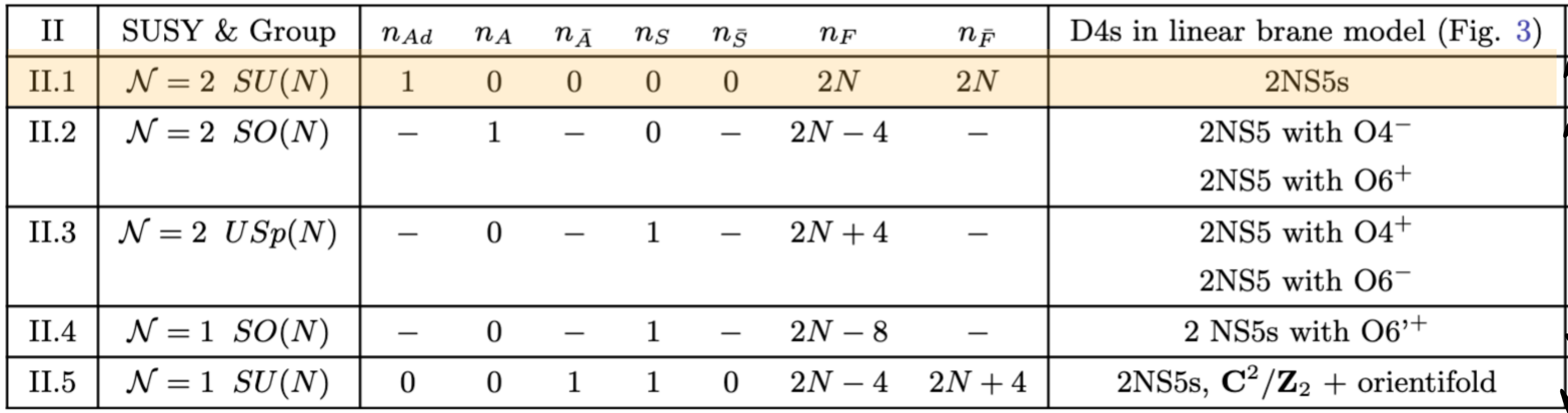
# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

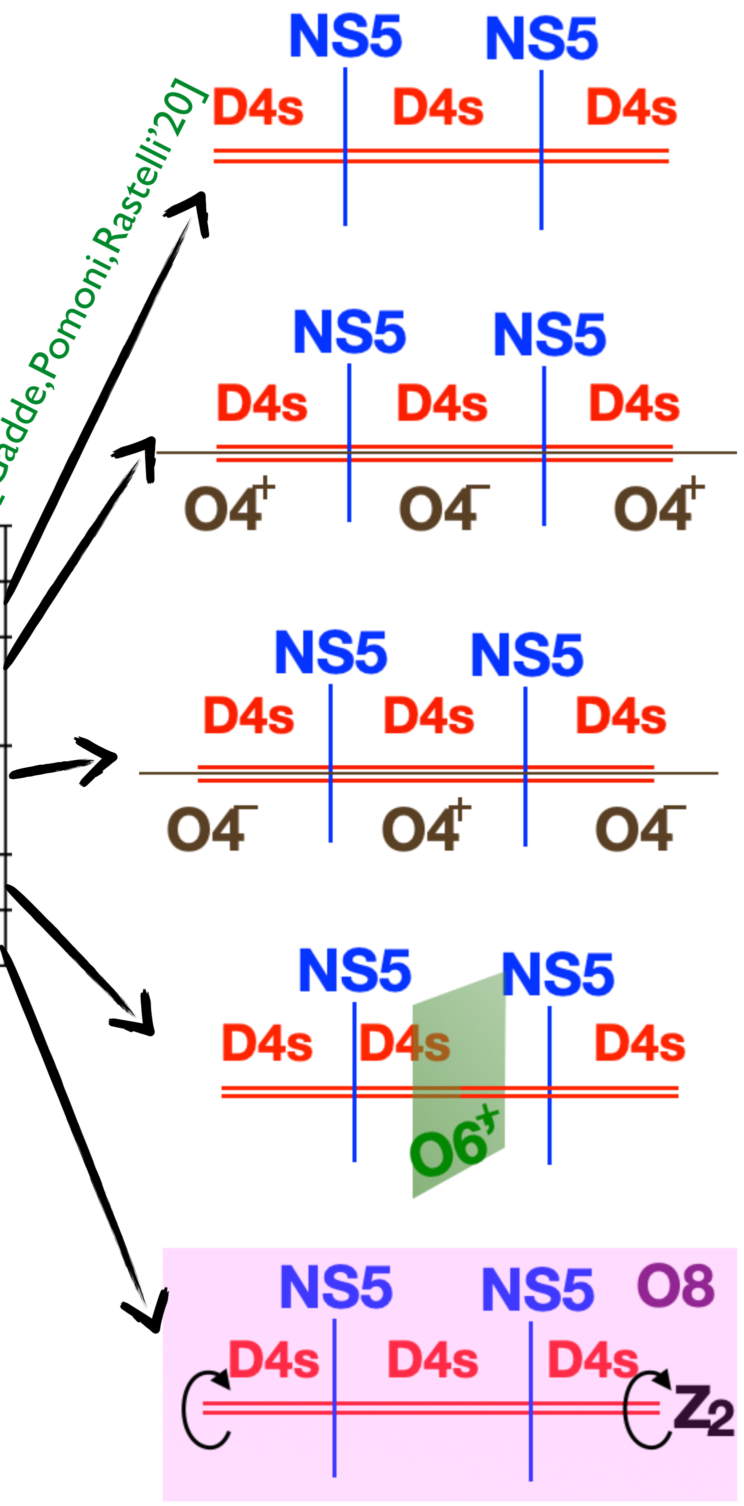
Bulk dual is non-Einstein, obtained from N D4's probing a background obtained from approaching 2 NS5's (in a double-scaling limit)
+ different content of orientifolds, flavour branes

subleading on 1/N

II)



| II | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s in linear brane model (Fig. 3) |
|---|---|---|---|---|---|---|---|---|---|
| II.1 | $\mathcal{N}=2\ SU(N)$ | 1 | 0 | 0 | 0 | 0 | $2N$ | $2N$ | 2NS5s |
| II.2 | $\mathcal{N}=2\ SO(N)$ | – | 1 | – | 0 | – | $2N-4$ | – | 2NS5 with $O4^-$<br>2NS5 with $O6^+$ |
| II.3 | $\mathcal{N}=2\ USp(N)$ | – | 0 | – | 1 | – | $2N+4$ | – | 2NS5 with $O4^+$<br>2NS5 with $O6^-$ |
| II.4 | $\mathcal{N}=1\ SO(N)$ | – | 0 | – | 1 | – | $2N-8$ | – | 2 NS5s with $O6'^+$ |
| II.5 | $\mathcal{N}=1\ SU(N)$ | 0 | 0 | 1 | 1 | 0 | $2N-4$ | $2N+4$ | 2NS5s, $\mathbf{C}^2/\mathbf{Z}_2$ + orientifold |



Weak coupling limit with $\alpha = \sqrt{2/3}$ = tensionless limit of the non-critical string

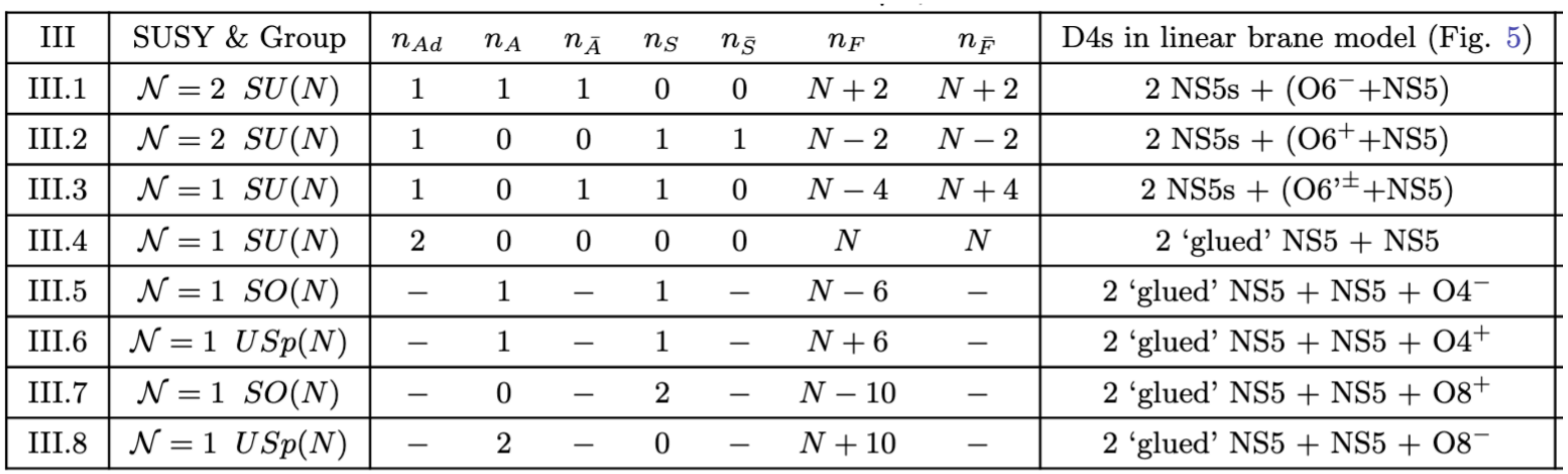
$$\mathbb{R}^{5,1} \times \left( \frac{SL(2,\mathbb{R})_2}{U(1)} \right) / \mathbb{Z}_2$$

(string living in less than 10d, but with vanishing central charge)

# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

III)



| III | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s in linear brane model (Fig. 5) |
|---|---|---|---|---|---|---|---|---|---|
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| III.7 | $\mathcal{N}=1\ SO(N)$ | – | 0 | – | 2 | – | $N-10$ | – | 2 'glued' NS5 + NS5 + O8$^+$ |
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# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

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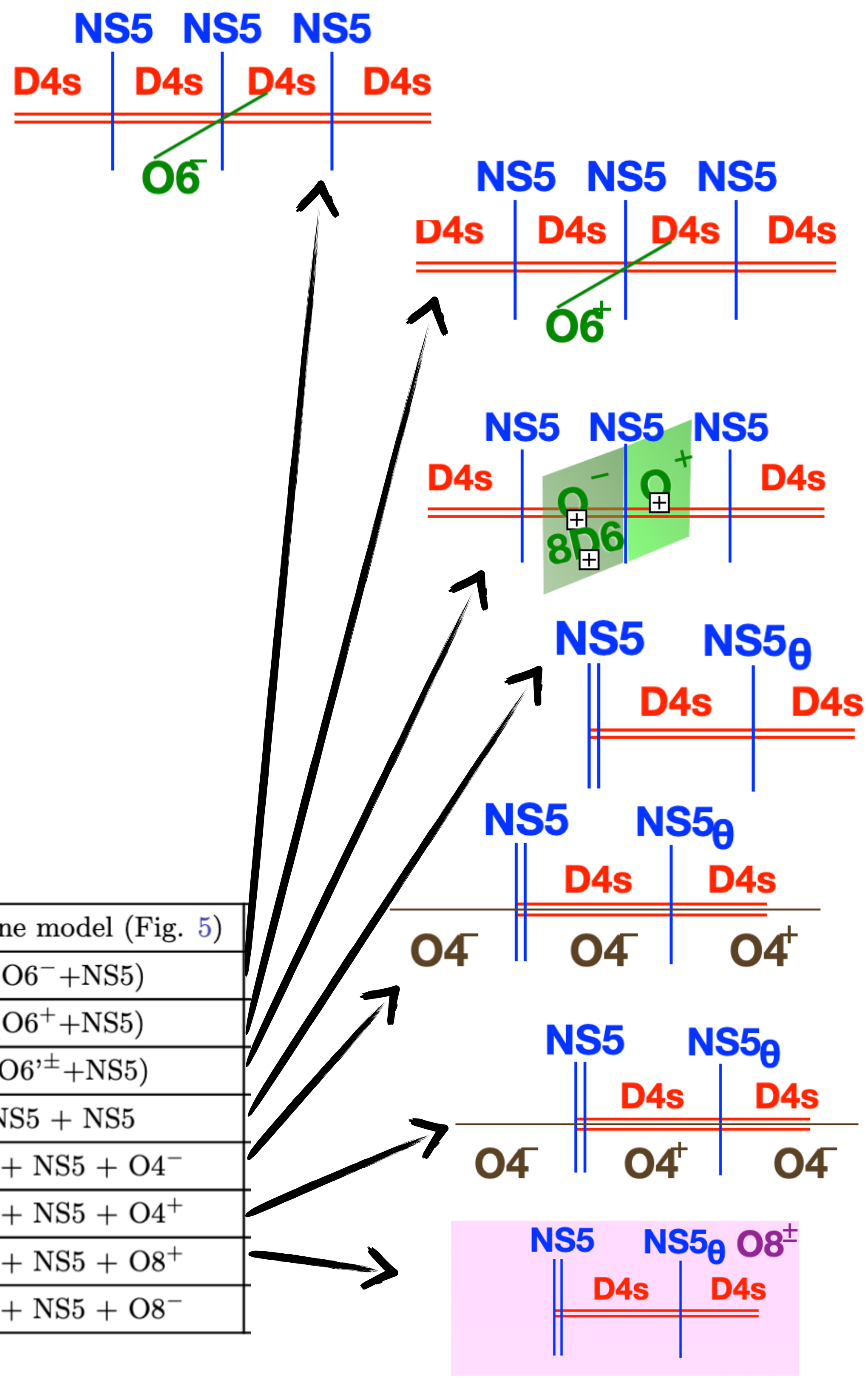

# CLASSIFICATION OF LIMITS IN 4d SCFTs

[Calderon-Infante,UrangaIV'ongoing]

Bulk dual is non-Einstein, obtained from N D4's probing a background obtained from approaching 3 NS5's (in a double-scaling limit) + different content of orientifolds, flavour branes

⟶ subleading on 1/N

III)

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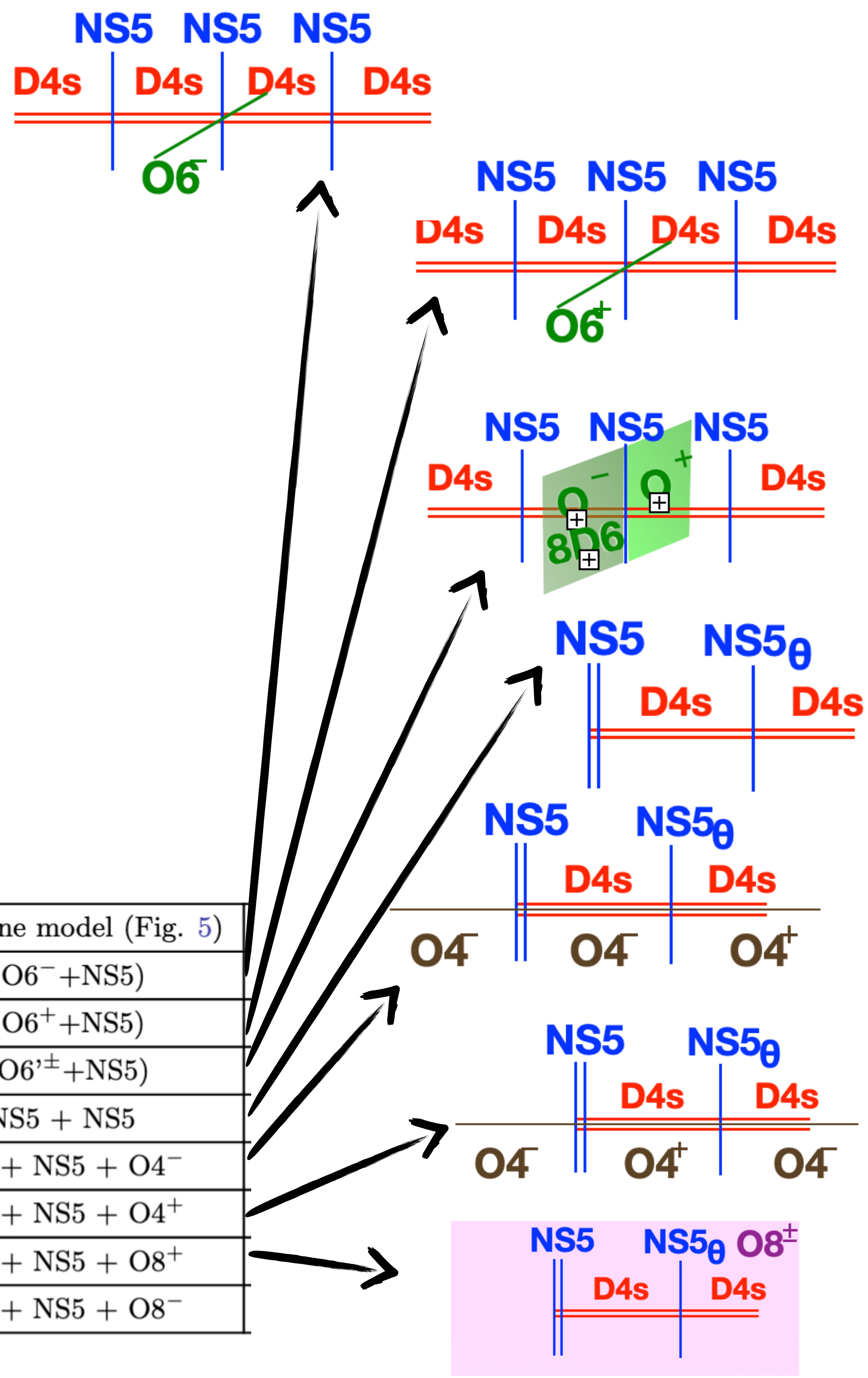

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Bulk dual is non-Einstein, obtained from N D4's probing a background obtained from approaching 3 NS5's (in a double-scaling limit) + different content of orientifolds, flavour branes

⟶ subleading on 1/N

Weak coupling limit with

$$\alpha = \sqrt{7/12}$$

= tensionless limit of (a deformation of) the string worldsheet

$$\mathbb{R}^{5,1} \times \left( \frac{SL(2,\mathbb{R})_3}{U(1)} \times \frac{SU(2)_3}{U(1)} \right) / \mathbb{Z}_3$$

III)

| III | SUSY & Group | $n_{Ad}$ | $n_A$ | $n_{\bar{A}}$ | $n_S$ | $n_{\bar{S}}$ | $n_F$ | $n_{\bar{F}}$ | D4s in linear brane model (Fig. 5) |
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| III.6 | $\mathcal{N}=1$ $USp(N)$ | – | 1 | – | 1 | – | $N+6$ | – | 2 'glued' NS5 + NS5 + O4$^+$ |
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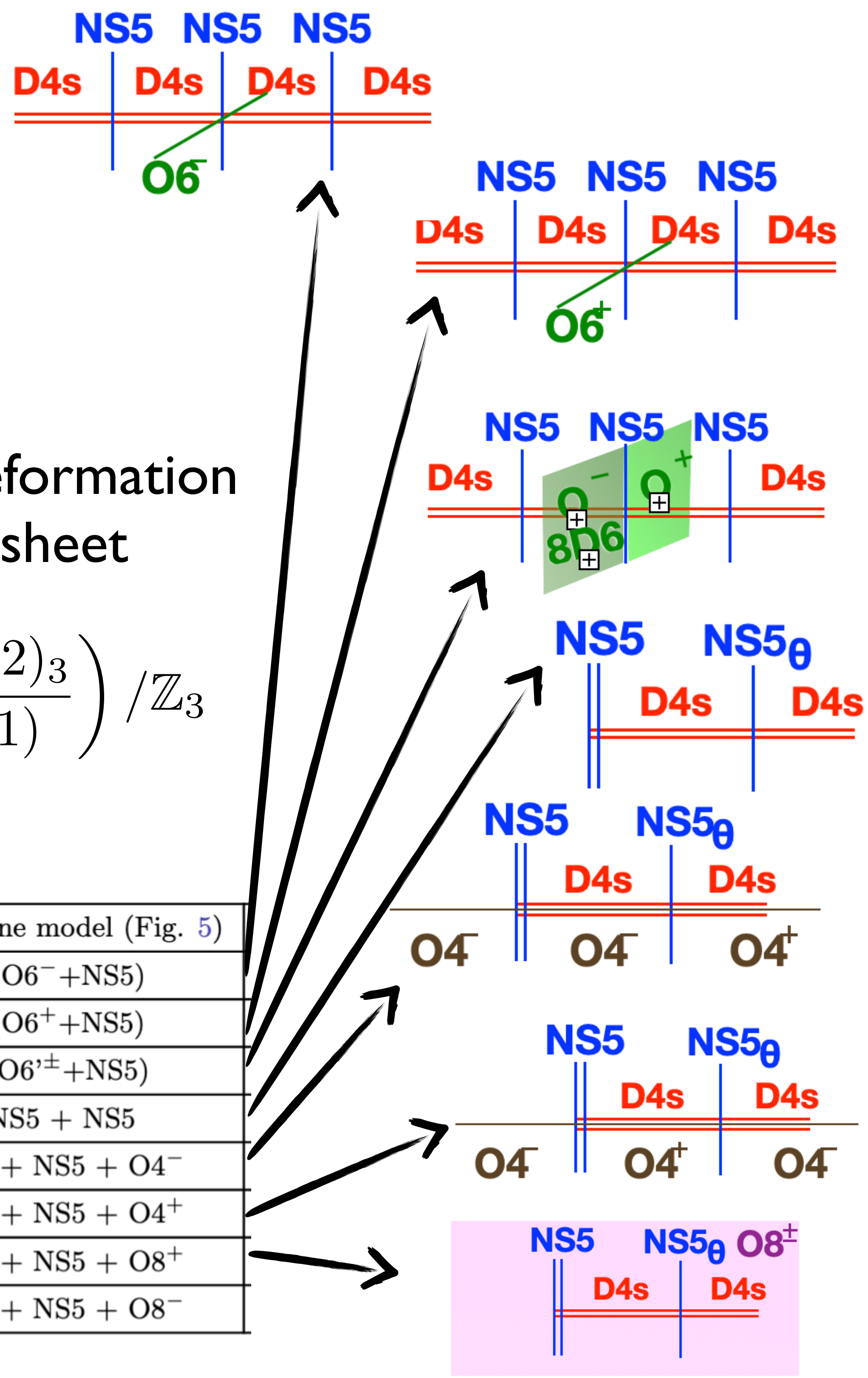


# CLASSIFICATION OF LIMITS IN 4d SCFTs

All SCFTs of each universality class share the same worldsheet at large N (the differences are subleading in 1/N)

Each class is charaterized by the number of NS5 colliding in a double scaling limit of the dual brane picture

# CLASSIFICATION OF LIMITS IN 4d SCFTs

All SCFTs of each universality class share the same worldsheet at large N (the differences are subleading in 1/N)

Each class is charaterized by the number of NS5 colliding in a double scaling limit of the dual brane picture

This confirms that the three types of weak coupling limits correspond to three types of tensionless strings backgrounds (described by different worldsheets)

different story than in flat space!

The story can be easily generalized to SCFTs with more gauge factors, which will be obtained from >3 NS5's

(more details in [Calderon-Infante,UrangaIV'ongoing])

# CONCLUSIONS

Thank you!

1) We have studied the KK spectrum (at large KK momentum) in Minkowski warped compactifications

For codim-1 backgrounds, the exponential KK mass decay rate is:

$$\alpha_{\rm KK} = \sqrt{\frac{d-1}{d-2}} \left(1 + \frac{4(d-2)}{(d-1)\gamma^2}\right)^{-1/2} \quad \text{for} \quad V(\hat{\varphi}) = \kappa_D^{-2} V_0 e^{\gamma \hat{\varphi}}$$

Hence: $\alpha_{\rm KK} \geq \frac{1}{\sqrt{d-2}}$ only if $\gamma \geq \frac{2}{\sqrt{d-1}} = \frac{2}{\sqrt{D-2}}$

(*sharpened bound* for Distance Conjecture)

(Strong deSitter conjecture/TCC) in the higher dim. theory

2) In AdS moduli spaces, we can have other types of towers associated to different strings becoming tensionless (which can even be non-cirtical)

We have started a classification of possible limits/ tensionless strings in 4d SCFTs, and constructed their brane dual configuration.

back-up slides

# INTEGER SCALING WEIGHT

[Calderón-Infante,Uranga,IV'20]
[Etheredge,Heidenreich,Kaya,Qiu,Rudelius'22]
[Calderón-Infante,Castellano,Herraez,Ibañez'23]
[Etheredge,Heidenreich,(McNamara),Rudelius,Ruiz,IV'23-24]
[Castellano,Ruiz,IV'23]

Define the "*scaling vectors*" (or *scalar charge-to-mass ratio vectors*):

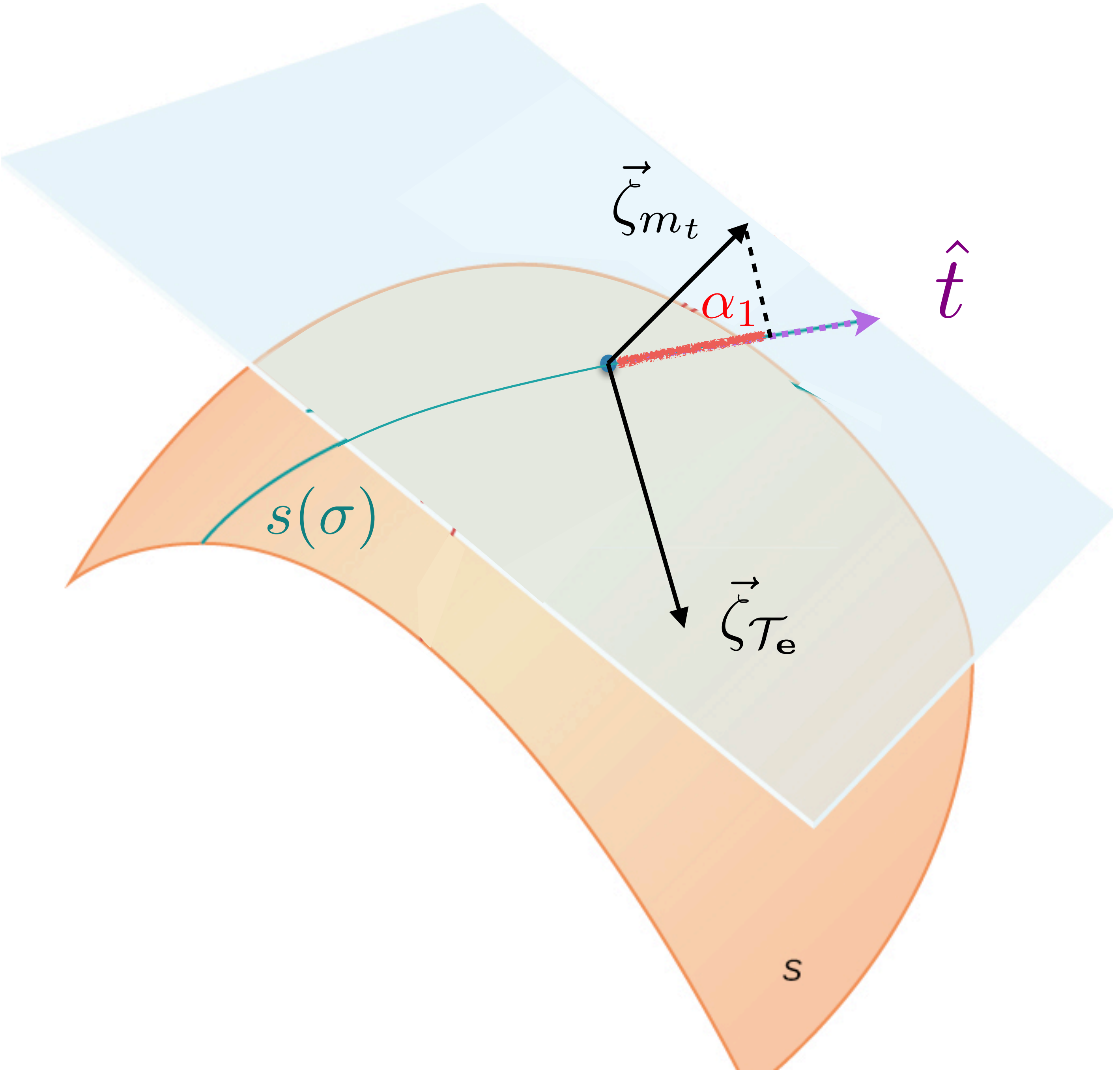
$$\vec{\zeta}_{m_t} = -\vec{\nabla}_\phi \log \frac{m(\vec{\varphi})}{M_{\text{Pl},d}} \qquad \vec{\zeta}_{\mathcal{T}_{\mathbf{e}}} = -\vec{\nabla}_\phi \log \frac{\mathcal{T}_{\mathbf{e}}}{M_{\text{Pl},d}}$$

(locally in the moduli space)

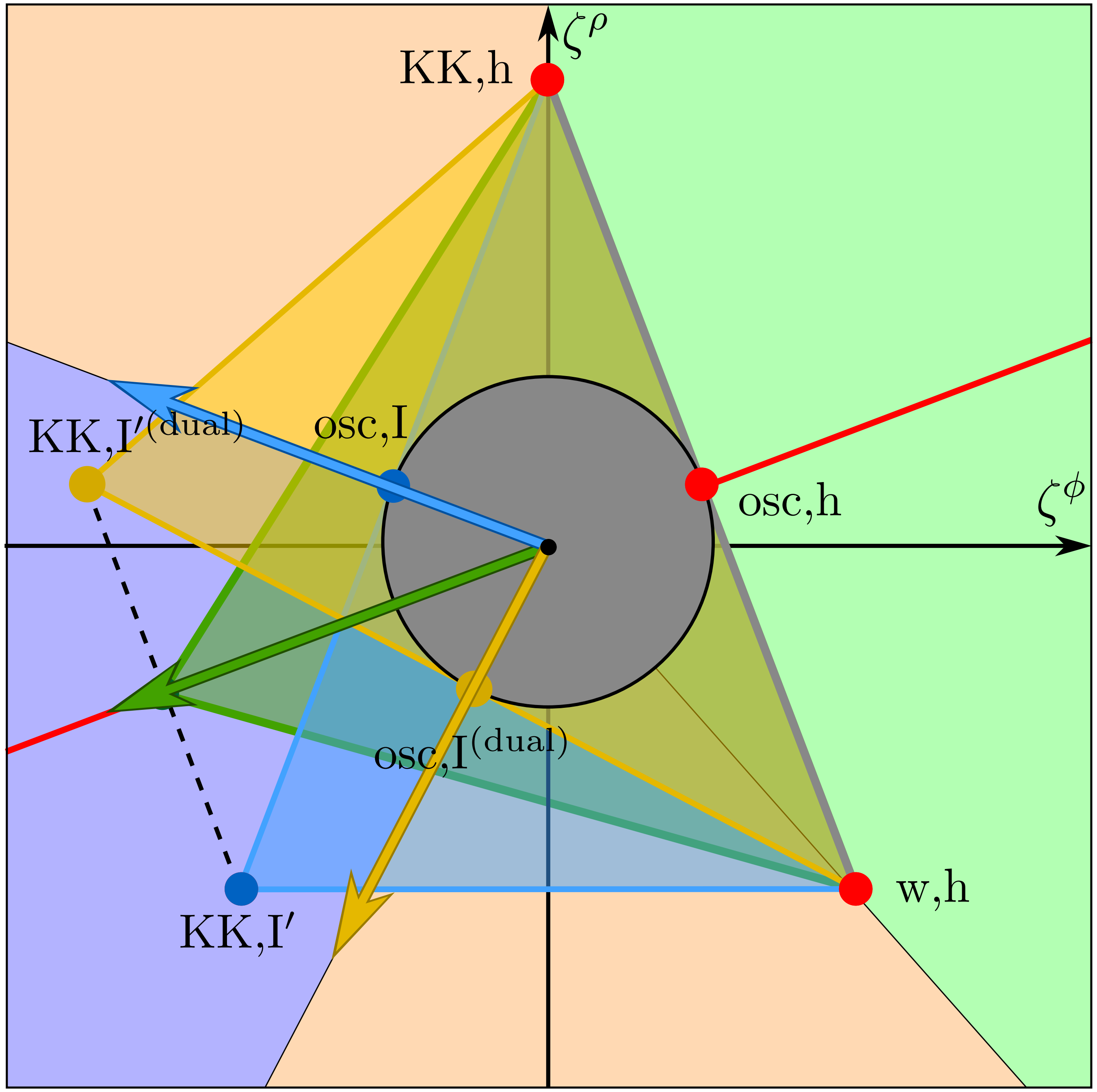
They capture information about the exponential rate of the characteristic mass of the towers

$$m_{\text{tower}} \sim m_0 e^{-\alpha \Delta\phi}$$

$$\alpha = \vec{\zeta}_{m_t} \cdot \hat{t}$$



# Type I’: Example of running decompactification



[Etheredge,Heidenreich,McNamara,Rudelius,Ruiz,IV’23]

# STRING THEORY MENTORING PROGRAM

Targeted to underrepresented groups working on string theory across the world

https://sites.google.com/view/mentoring-program-string-th

Applications will open soon and close in September!