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6d Supergravity Blocks

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POSTECH

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Swampland & Finiteness Conjecture

Landscape

EFTs consistent with quantum gravity

(finite!)

↔

Swampland

EFTs that look fine
but have no UV completion

(vast!)

"Finiteness Conjecture" — [Vafa '05], [Douglas '05], [Acharya–Douglas '06], [Hamada-Montero-Vafa-Valenzuela '21]

Why 6d $\mathcal{N}=(1,0)$ supergravity is a perfect testing ground:

Highly constrained
Anomaly cancellation + SUSY + unitarity impose tight restrictions on spectrum and gauge content.

Non-perturbative richness
Tensionless BPS strings, 6d SCFTs, and LSTs appear naturally — testable against their classifications.

Finiteness proven
Kim–Vafa–Xu [2411.19155]: the entire massless spectrum is finite.

F-theory connection
Many solutions come from F-theory on elliptic CY3 — but not all! Non-geometric theories also exist.

Key new ideas HCK–Vafa–Xu [2411.19155]

Assumption 1 : Every **boundary of tensor moduli space** ⇔ a tensionless BPS string.

Assumption 2 : **Classifications of 6d SCFTs and LSTs** are complete.

— BPS string charge cone is dual to the tensor cone, generated by primitive BPS string charges.

— *Existence of universal BPS string called H-string* (related to critical heterotic string).

→ Strong constraints on 6d EFT!

MASSLESS SPECTRUM IS FINITE

$$T \leq 193$$

tensor multiplets
upper bound on the number of tensors

$$rk(V) \leq 480$$

gauge rank
summed over all gauge factors

— Consistent 6d EFT landscape (up to U(1) charges) is a **finite list, in principle classifiable**.

Goal — a systematic construction framework

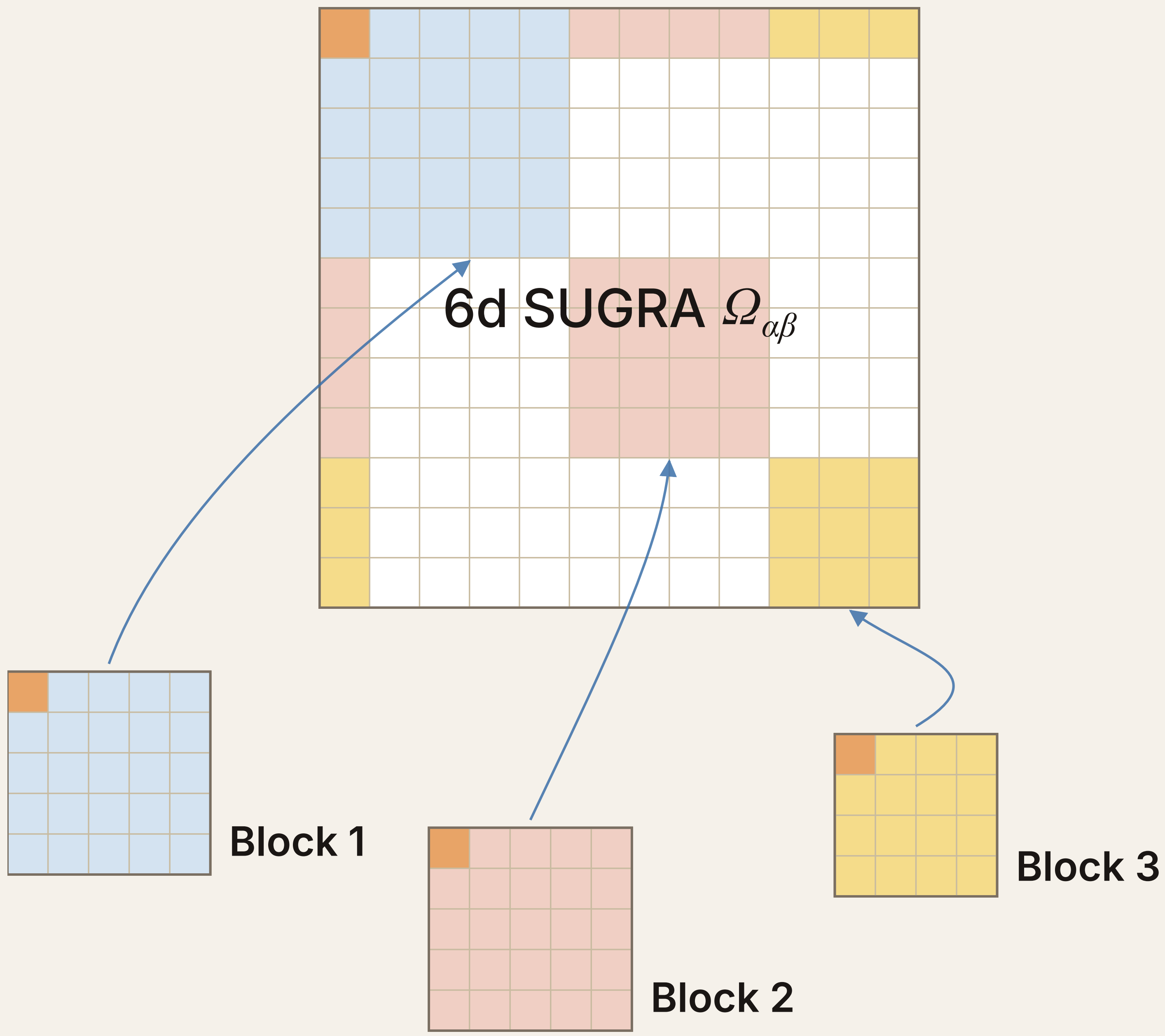
Propose a bottom-up construction framework for 6d (1,0) supergravity.

—Build any 6d (1,0) supergravity from a small set of *basic units*.

—Units : **Supergravity blocks**.

—6d SUGRA = assembly of blocks satisfying gluing rules.

We provide a complete classification of **non-Higgsable supergravity blocks**.



Outline

Field content & anomaly cancellation

MULTIPLET CONTENT

- **Gravity multiplet** (graviton, gravitino, self-dual 2-form).
- T **tensor multiplets** — anti-self-dual 2-forms + scalars.
- V **vector multiplets** — gauge algebra $\mathfrak{g}$.
- H **hypermultiplets** — matter representations.

1-loop anomalies arising from chiral fields can be cancelled by the *Green–Schwarz–Sagnotti* mechanism.

[Green-Schwarz '84] · [Sagnotti '92]

- **Anomaly vectors** $b_0, b_i \in R^{1,T}$ living in the tensor charge lattice together with the **tensor intersection form** $\Omega_{\alpha\beta}$ determine the structure of tensor base.

ANOMALY CANCELLATION CONDITIONS

GRAVITATIONAL · $\mathrm{TR}\, R^4$

$$H - V = 273 - 29\,T$$

$(\mathrm{TR}\, R^2)^2$

$$b_0 \cdot b_0 = 9 - T$$

$\mathrm{TR}\, F_I^4$ · NO IRREDUCIBLE QUARTIC

$$B^i_{\mathrm{adj}} = \sum_r n^i_r\, B^i_r$$

$\mathrm{TR}\, R^2\, \mathrm{TR}\, F_I^2$

$$b_0 \cdot b_i = \frac{\lambda_i}{6}\left(\sum_r n^i_r A^i_r - A^i_{\mathrm{adj}}\right)$$

$(\mathrm{TR}\, F_I^2)^2$

$$b_i \cdot b_i = \frac{\lambda_i^2}{3}\left(\sum_r n^i_r C^i_r - C^i_{\mathrm{adj}}\right)$$

$\mathrm{TR}\, F_I^2\, \mathrm{TR}\, F_J^2$ · $I \neq J$

$$b_i \cdot b_j = 2\lambda_i\lambda_j \sum_{r,s} n^{ij}_{r,s}\, A^i_r A^j_s$$

$\mathrm{tr}_r F^2 = A_r\, \mathrm{tr}\, F^2, \qquad \mathrm{tr}_r F^4 = B_r\, \mathrm{tr}\, F^4 + C_r (\mathrm{tr}\, F^2)^2 .$

Tensor moduli space & BPS strings

Tensor moduli space : T-dimensional coset space $SO(1,T)/SO(T)$ parametrized by scalar J in the tensor multiplets.

—BPS string tension of charge Q :

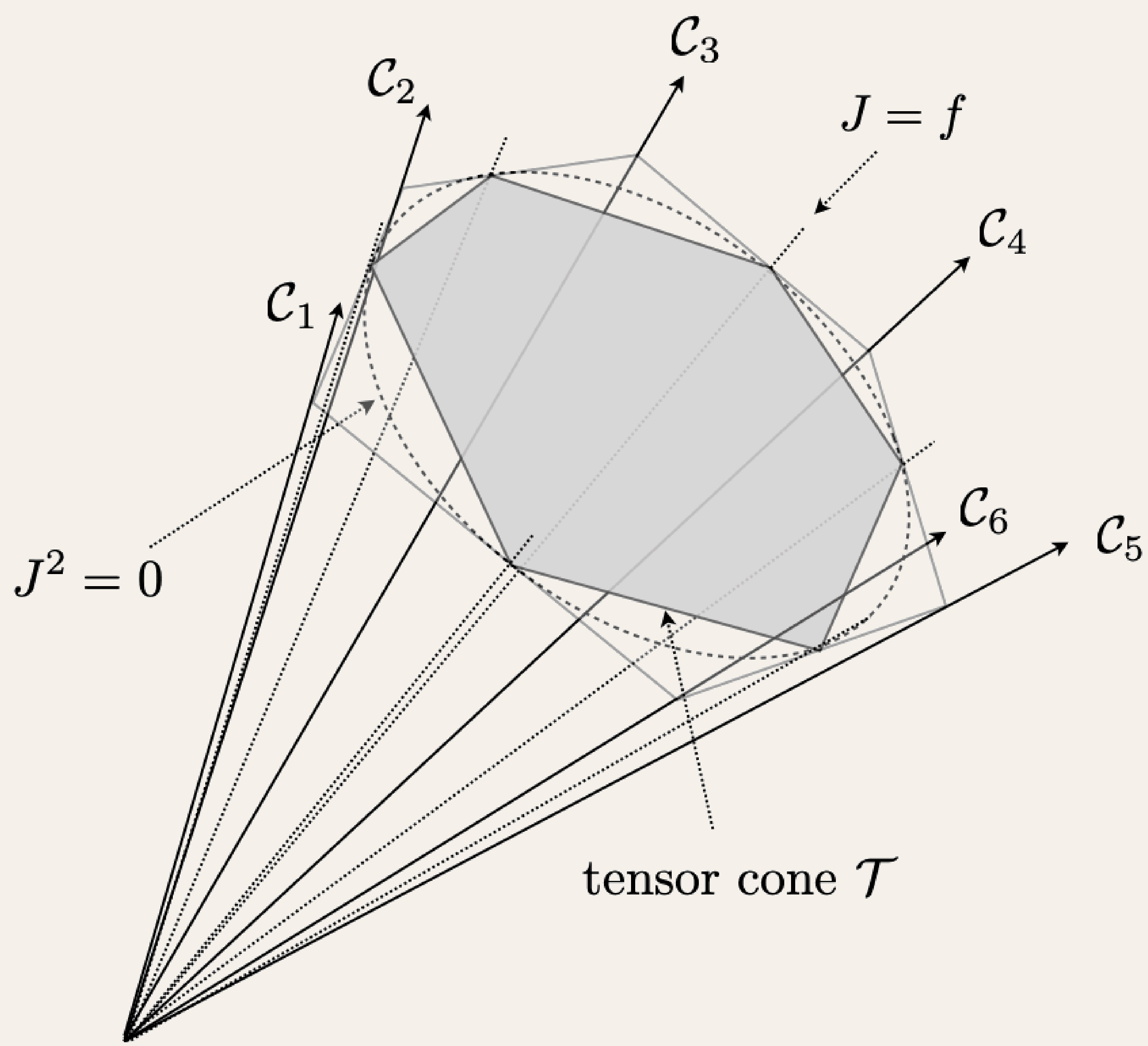
$$T_Q \sim J \cdot Q \geq 0.$$

—Every boundary point of tensor moduli space hosts some *tensionless BPS strings*.

⇒ **BPS cone, cone of BPS string charges, is dual to tensor cone.**

—**BPS generators** C_i span the BPS cone and satisfy $C_i^2 \leq 0,\ C_i \cdot C_j \geq 0.$

⇒ Corresponding BPS strings can become "tensionless".



Classification of BPS generators

[Morrison–Taylor '12], [Heckman–Morrison–Rudelius–Vafa '15], [Bhardwaj–Del Zotto, Heckman–Morrison–Rudelius–Vafa '15], [Bhardwaj '15, '19], [Hamada–Loges '23, '24] ....

INSTANTONIC STRINGS ($b_i^2 \leq 0$)

G_I	H_I (CHARGED HYPERS)	b_i^2	$b_0 \cdot b_i$	NOTES
$\mathfrak{g}$	Adj	0	0	
$\mathfrak{su}_N$	$(2N) \times \mathbf{N}$	-2	0	
$\mathfrak{su}_N$	$(N-8) \times \mathbf{N} \oplus \mathbf{N(N+1)/2}$	-1	-1	$N \geq 8$
$\mathfrak{su}_N$	$(N+8) \times \mathbf{N} \oplus \mathbf{N(N-1)/2}$	-1	1	
$\mathfrak{su}_N$	$16 \times \mathbf{N} \oplus 2 \times \mathbf{N(N-1)/2}$	0	2	
$\mathfrak{su}_N$	$\mathbf{N(N-1)/2} \oplus \mathbf{N(N+1)/2}$	0	0	
$\mathfrak{su}_6$	$15 \times \mathbf{6} \oplus \tfrac{1}{2}\cdot\mathbf{20}$	-1	1	
$\mathfrak{su}_6$	$17 \times \mathbf{6} \oplus \mathbf{15} \oplus \tfrac{1}{2}\cdot\mathbf{20}$	0	2	
$\mathfrak{su}_6$	$18 \times \mathbf{6} \oplus \mathbf{20}$	0	2	
$\mathfrak{su}_6$	$\mathbf{6} \oplus \tfrac{1}{2}\cdot\mathbf{20} \oplus \mathbf{21}$	0	0	
$\mathfrak{so}_N$	$(N-8) \times \mathbf{N}$	-4	-2	$N \geq 8$
$\mathfrak{so}_N$	$(N-7) \times \mathbf{N} \oplus 2^{\lfloor (10-N)/2 \rfloor}\cdot\mathbf{2^{\lfloor (N-1)/2 \rfloor}}$	-3	-1	$12 \geq N \geq 7$
$\mathfrak{so}_N$	$(N-6) \times \mathbf{N} \oplus 2\cdot 2^{\lfloor (10-N)/2 \rfloor}\cdot\mathbf{2^{\lfloor (N-1)/2 \rfloor}}$	-2	0	$13 \geq N \geq 6$
$\mathfrak{so}_N$	$(N-5) \times \mathbf{N} \oplus 3\cdot 2^{\lfloor (10-N)/2 \rfloor}\cdot\mathbf{2^{\lfloor (N-1)/2 \rfloor}}$	-1	1	$12 \geq N \geq 5$
$\mathfrak{so}_N$	$(N-4) \times \mathbf{N} \oplus 4\cdot 2^{\lfloor (10-N)/2 \rfloor}\cdot\mathbf{2^{\lfloor (N-1)/2 \rfloor}}$	0	2	$14 \geq N \geq 4$
$\mathfrak{sp}_N$	$(2N+8) \times \mathbf{2N}$	-1	1	
$\mathfrak{sp}_N$	$16 \times \mathbf{2N} \oplus (N-1)(2N+1)$	0	2	
$\mathfrak{sp}_3$	$\tfrac{35}{2}\cdot\mathbf{6} \oplus \tfrac{1}{2}\cdot\mathbf{14'}$	0	2	
$\mathfrak{e}_8$	—	-12	10	
$\mathfrak{e}_7$	$(k/2) \times \mathbf{56}$	k-8	k-6	$k \leq 8$
$\mathfrak{e}_6$	$k \times \mathbf{27}$	k-6	k-4	$k \leq 6$
$\mathfrak{f}_4$	$k \times \mathbf{26}$	k-5	k-3	$k \leq 5$
$\mathfrak{g}_2$	$(3k+1) \times \mathbf{7}$	k-3	k-1	$k \leq 3$

GENERATORS with NO GAUGE ALGEBRA

STRING	C^2	$b_0 \cdot C$	TENSIONLESS LIMIT
E-string	-1	+1	finite distance
M-string	-2	0	finite distance
H-string	**0**	**+2**	**infinite distance**
Type II	0	0	infinite distance

Instantonic strings (carrying gauge algebras) + four generators without gauge algebra = Complete set of primitive BPS generators C_i.

Refined structure of the tensor moduli space

KEY STATEMENT: HCK–Vafa–Xu [2411.19155]

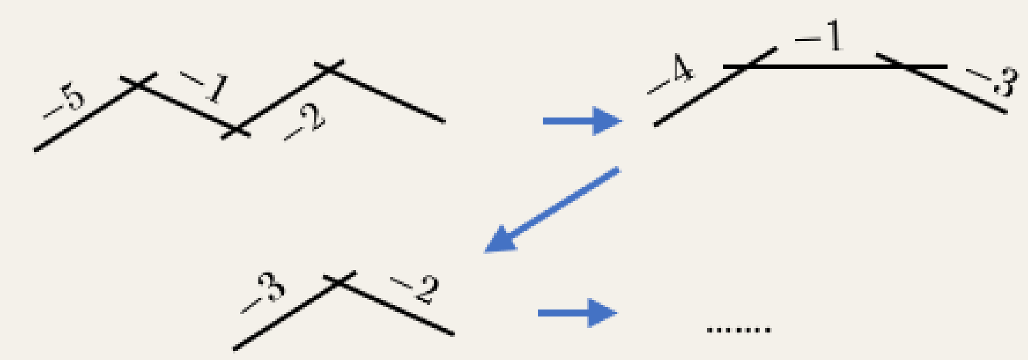
1. The intersection structure of tensors matches a Kähler surface — $\mathbb{P}^2$, $\mathbb{F}_n$, or blowups thereof *(except $T=9$ with $b_0^2=0$ cases)*

2. Every consistent 6d (1,0) theory contains the *"H-string"* of charge f:

$$f^2 = 0 \;\&\; f\cdot b_0 = 2$$

—An EFT statement with no geometric assumption.

Sequential Blowdowns



$$\text{Blowdown} \;:\; \mathcal{C}_j'^2 = \mathcal{C}_j^2 + \mathcal{C}_j\cdot e_i(\mathcal{C}_j\cdot e_i - 1)$$

- Sequential blowdowns of any tensor cone terminate at a 2d cone of $\mathbb{F}_n$.

Any supergravity base (for tensor multiplets) is a blowup of $\mathbb{P}^2$ or $\mathbb{F}_n$

–**String Lamppost Principle** holds for tensor structure of 6d supergravity

–Systematically enumerate tensor bases by successively blowing up points — finite problem!

Constraints from the H-string

Every consistent 6d (1,0) theory (except $T = 9,\ b_0^2 = 0$) contains the H-string of charge f with $f^2 = 0$ and $f \cdot b_0 = 2$.

01 · EXTERNAL GENERATORS & RANK BOUND — Shiu–HCK–Vafa [1905.08261], Baykara–Dierigl–HCK–Vafa–Xu [2511.09613]

- External generators C_i^{ext} with $f \cdot C_i^{\text{ext}} > 0$ induce a worldsheet current algebra at level $k_i = f \cdot C_i^{\text{ext}}$.
- Unitarity of the worldsheet CFT on the H-string requires

$$\sum_i c_{\mathfrak{g}_i} \le c_L \quad \longrightarrow \quad \sum_i \mathrm{rank}(\mathfrak{g}_i^{\text{ext}}) \le 20 \quad (\text{or} \le 16 \text{ for } T > 1)$$

02 · CONTAINMENT RELATION — HCK–Vafa–Xu [2411.19155]

- Generators with $f \cdot C_i = 0$ must be contained in an LST of H-string charge f: $C_i \subset \mathrm{LST}(f)$ for all $f \cdot C_i = 0$
- LST classification ⇒ strong constraints on every generator orthogonal to f.

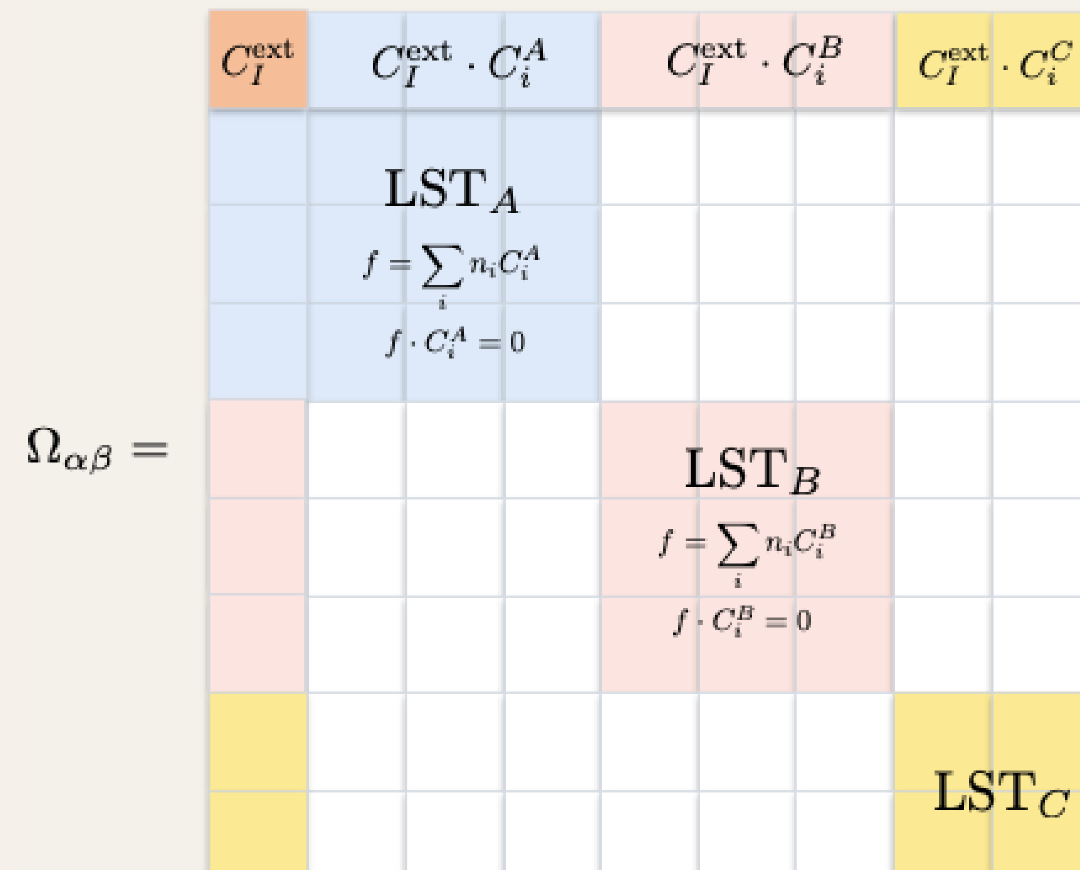
03 · P-TYPE ENDPOINT FOR LSTS — Bhardwaj–Del Zotto–Heckman–Morrison–Rudelius–Vafa ['15], Bhardwaj ['19]

- LSTs sharing the H-string charge f have **P-type endpoints** after blowdowns.
  No circular intersections; frozen singularity clusters excluded from P-type LSTs.
- With T & V bounds, allowed LSTs with P-type endpoints form a **finite set** — classifiable!

Structure of 6d (1,0) SUGRA

Tensor intersection form $\Omega_{\alpha\beta}$ of 6d (1,0) SUGRA exhibits a **block structure** organized around its LSTs.

- Each diagonal block = one LST sector (A,B,C,..) sharing the H-string charge f.
- Generators inside each block satisfy $f \cdot C_i = 0$ — contained in the LST.
- **External generators** C_I^{ext} glue the LST sectors together.



Next, given intersection form Ω, we place gauge algebras and associated charged matters on tensors.

✓ **Classified**

Gauge algebras on the BPS generators C_I^{ext}, $C_i^{A,B,C}$. These are tensors with $Q^2 \leq 0$.

○ **Yet to classify**

Gauge algebras on positive self-intersection tensors ($Q^2 > 0$) — we will not talk about

Supergravity blocks

DEFINITION

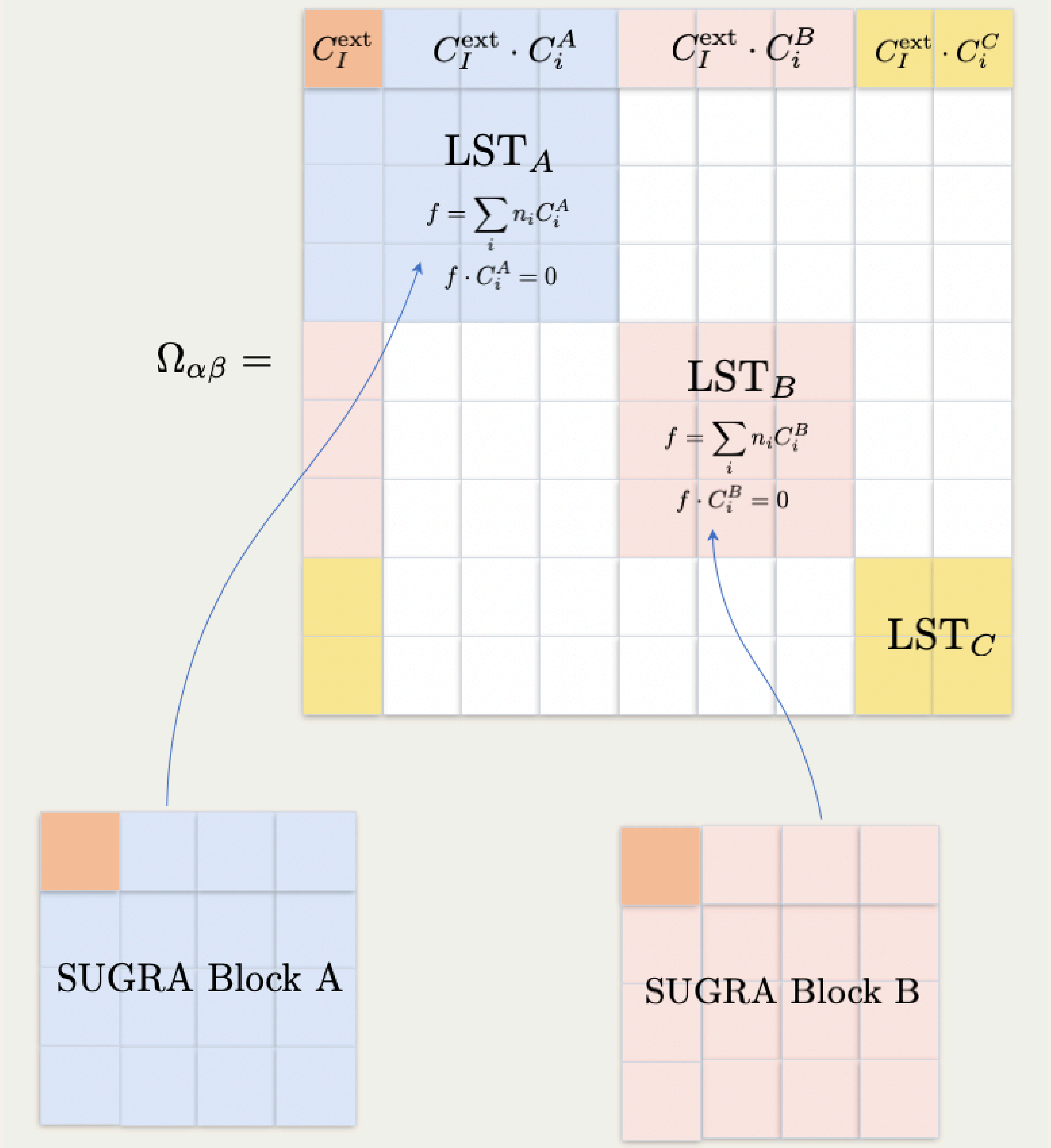
A **supergravity block** is one LST of H-string charge f, together with all external BPS generators attached to it.

Defining feature. The block's Gram matrix has *exactly one positive eigenvalue*.

$$\mathrm{sig}(\mathcal{G}_{IJ}) = (+,-,-,\cdots,-)$$

Consequence: BPS strings inside a block cannot all become tensionless simultaneously.

$\Rightarrow$ Every supergravity block contains intrinsic **gravitational tensors**.



Building a supergravity from blocks

Assemble the full tensor intersection form $\Omega_{\alpha\beta}$ as **block-diagonal**, glued together by the common external generators C_I^{ext}.

Each diagonal block $=$ one supergravity block

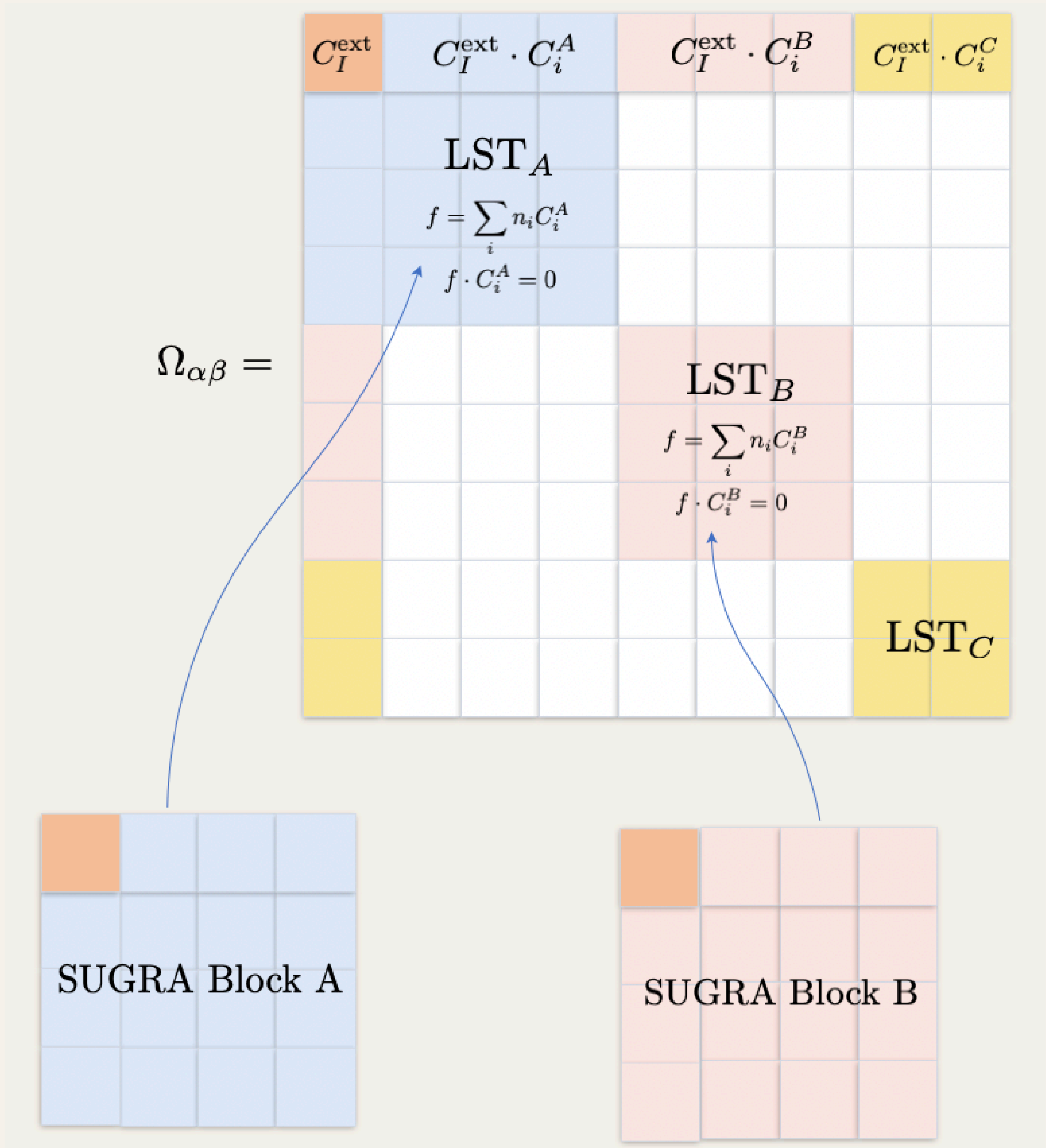
CONSISTENCY CHECK

After assembly and blowdowns, the final intersection form Ω must reduce to that of a Hirzebruch surface $\mathbb{F}_n$.

- After this, we can also place gauge algebras and matter on tensors with positive self-intersection.

Note. The resulting SUGRA is not yet guaranteed to have a consistent UV completion.

THREE-BLOCK ASSEMBLY — INTERSECTION FORM Ω_{AB}

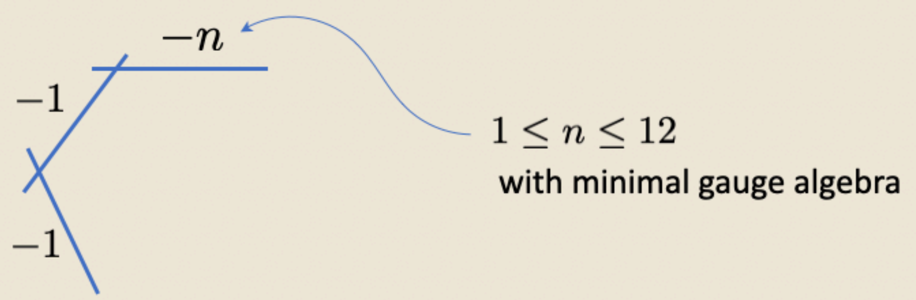


Result: 6d (1,0) SUGRA satisfying all known consistency conditions.

F-theory examples of supergravity blocks

EXAMPLE A — SUGRA WITH $T = 2$

Base $\mathbb{F}_n$ with one blowup & minimal gauge algebra, $1 \leq n \leq 12$.



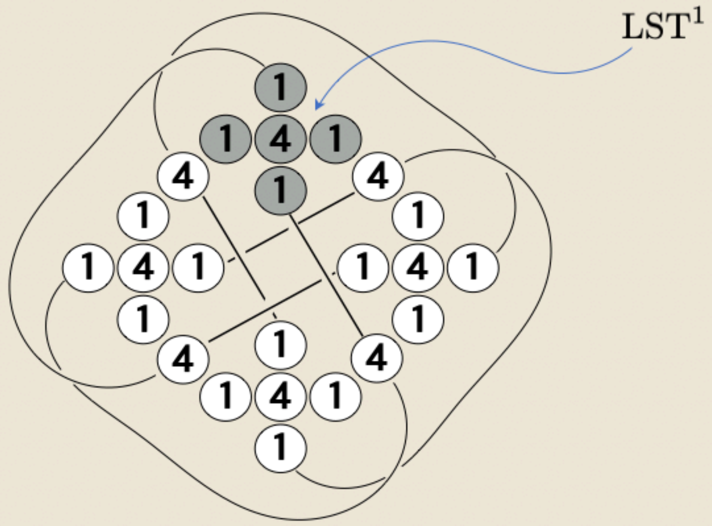
$$\Omega_{\alpha\beta} = \mathcal{G}_{\alpha\beta} = \begin{pmatrix} \overset{C^{\text{ext}}}{-n} & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\underbrace{1 \quad -1}_{\text{LST}}$$

One external generator C^{ext} + one LST for the H-string.

EXAMPLE B — SUGRA WITH $T = 17$

Base $T^4/\mathbb{Z}_2 \times \mathbb{Z}_2$ & **eight $\mathfrak{so}_8$** gauge algebras.



$$\mathcal{G}^{a=1,2,3,4}_{\alpha\beta} = \begin{pmatrix} -4 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -4 \end{pmatrix}$$

$$\underbrace{\qquad\qquad\qquad}_{\text{LST}^a} \quad \underbrace{\qquad\qquad\quad}_{C^{\text{ext}}_{1,2,3,4}}$$

Four external generators $C^{\text{ext}}_{1,2,3,4}$ glue four LSTs

[Hayashi–HCK–Ohmori–Vafa '19]

These are examples; the framework itself does not require an F-theory origin.

Non-Higgsable supergravity blocks

Supergravity blocks built *entirely* from **non-Higgsable clusters (NHCs)**.

NHC : tensor multiplet configuration whose gauge algebra cannot be Higgsed. [Morrison-Taylor '12]. *NHCs are classified!*

WHY THIS SUBCLASS

It cleanly separates the *topology of the tensor base* from the choice of gauge enhancements & matter charges.

- More general SUGRA blocks are obtained by **enhancing gauge algebras** and **adding charged hypers** on top.

- We give a **complete classification of the non-Higgsable supergravity blocks** .

NON-HIGGSABLE CLUSTERS

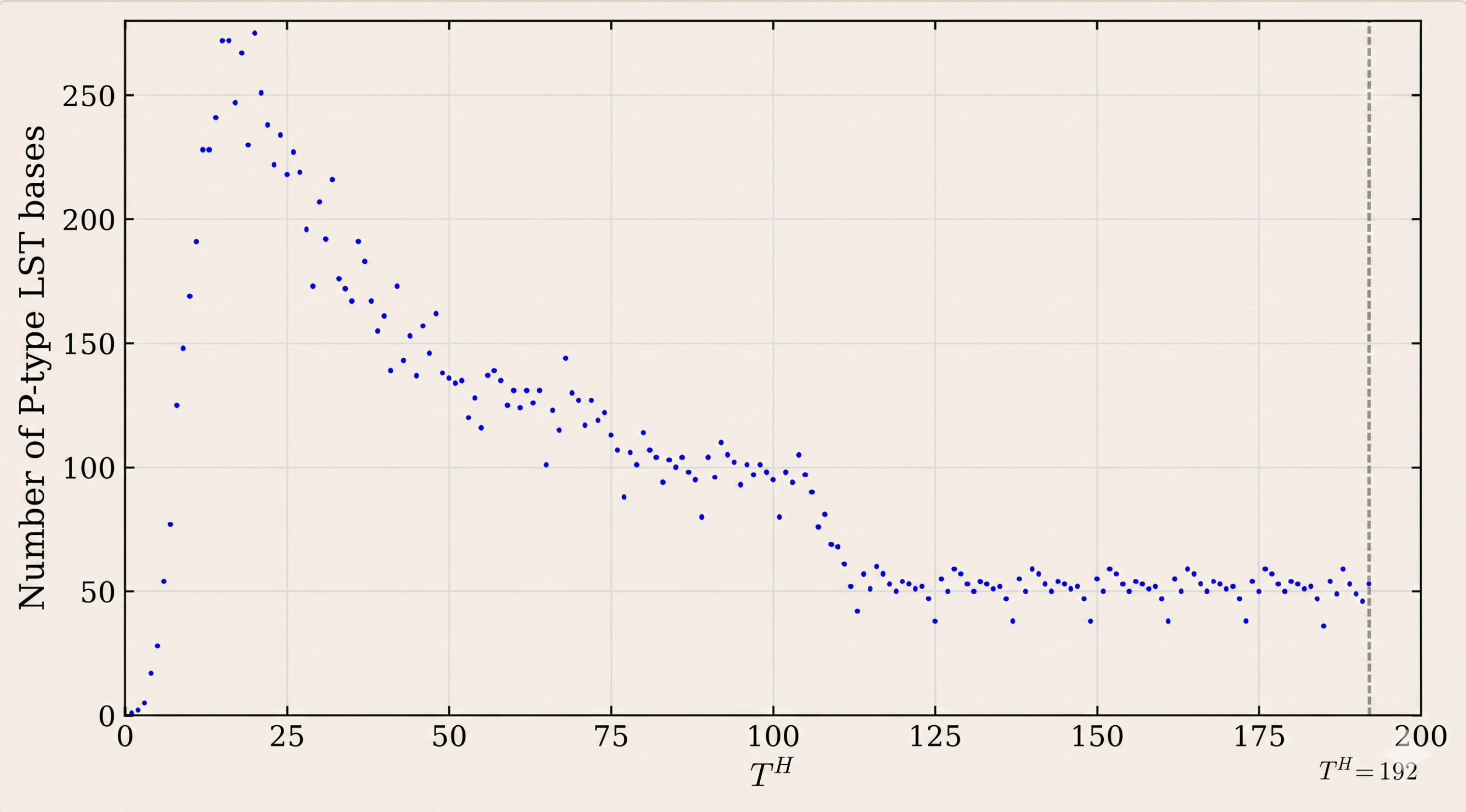
	GAUGE ALGEBRA
-1	—
-2	—
-3	$\mathfrak{su}_3$
-4	$\mathfrak{so}_8$
-5	$\mathfrak{f}_4$
-6	$\mathfrak{e}_6$
-7	$\mathfrak{e}_7'$
-8	$\mathfrak{e}_7$
-12	$\mathfrak{e}_8$
$-2,\ -3$	$\mathfrak{su}_2 \oplus \mathfrak{g}_2$
$-2,\ -3,\ -2$	$\mathfrak{su}_2 \oplus \mathfrak{so}_7 \oplus \mathfrak{su}_2$
$-3,\ -2,\ -2$	$\mathfrak{g}_2 \oplus \mathfrak{sp}_1$
$\hat{1}$	$\mathfrak{su}_8$ non-geometric
$-2,\ \hat{1}$	$\mathfrak{su}_8 \oplus \mathfrak{su}_{16}$ non-geometric
$-2 = -4$	$\mathfrak{su}_8 \oplus \mathfrak{so}_{16}$ non-geometric

Geometric NHCs + **frozen-singularity NHCs**.

[Morrison–Taylor '12], [Heckman–Morrison–Vafa '13], [Bhardwaj-del Zotto- Heckman-Morrison-Rudelius-Vafa '15], [Bhardwaj–Morrison–Tachikawa–Tomasiello '18], [Bhardwaj '15, '18]

Classification of P-type LSTs

We first classify all allowed P-type LSTs for the H-string (involving only NHCs).



19,270 distinct **P-type LSTs** (with NHCs)

The full set is on github.

LST CONFIGURATIONS FOR $T^H < 7$

$$T^H=1 \ : 11$$

$$T^H=2 \ : 121,\ 212,$$

$$T^H=3 \ : 1221,\ \overset{2}{2}21,\ \overset{1}{1}31,\ 3\overset{2}{1}\mathfrak{e}_6,\ 2131,\ 2213,$$

$$T^H=4 \ : 12221,\ 2\overset{2}{2}21,\ 2\overset{1}{\underset{1}{3}}2, 12\overset{1}{3}1,\ 22\overset{1}{2}3,\ 2\overset{1}{2}31,\ 2\overset{2}{3}12,\ 31\overset{1}{3}2,\ 12312,\ 13131,\ 23123$$

$$31231,\ 31313,\ 1\overset{1}{\underset{1}{\mathfrak{so}}}1,\ \mathbf{I}^{\oplus 2}\overset{1}{\mathfrak{so}}1,\ \mathbf{I}^{\oplus 2}\mathfrak{so}\mathbf{I}^{\oplus 2},\ \mathbf{I}^{\oplus 3}\mathfrak{so}1,\ \mathbf{I}^{\oplus 4}\mathfrak{so},\ 3\overset{2}{2}1\mathfrak{e}_7'$$

$$T^H=5 \ : 122221,\ 2\overset{2}{2}221,\ \mathbf{I}^{\otimes 5}5,\ 122\overset{1}{3}1,\ 12\overset{1}{3}21,\ 2\overset{2}{3}131,\ 31\overset{2}{3}21,\ \overset{4,2}{\otimes}2,\ 131321,\ 312321$$

$$1\overset{1}{\underset{1}{5}}12,\ 21\overset{1}{5}12, 221\overset{1}{5}1,\ 221512,\ 222151,\ 222215,\ \overset{2,2}{\otimes}\overset{1}{\mathfrak{so}}1,\ 22\overset{1}{3}1\mathfrak{so},\ \overset{1}{2}31\mathfrak{so}1,\ 2\overset{1}{3}21\mathfrak{so}$$

$$31\overset{1}{\mathfrak{so}}13,\ \overset{2,2}{\otimes}\mathfrak{so}\mathbf{I}^{\oplus 2},\ \overset{2,3}{\otimes}\mathfrak{so}1,\ \overset{2,4}{\otimes}\mathfrak{so},\ 23131\mathfrak{so},\ 3\overset{2,2}{\otimes}\mathfrak{so}1,\ 31321\mathfrak{so},\ 321\mathfrak{so}13,\ 3\overset{2}{2}1\mathfrak{e}_7 1,$$

$$T^H=6 \ : 1222221,\ 2\overset{2}{2}2221,\ 2\overset{2}{3}1321,\ 1231321,\ 1313221,\ 1\overset{1}{5}1\overset{1}{3}2,\ 1\underset{1}{5}\overset{12,2}{\otimes},\ 31\overset{1}{\underset{1}{5}}13,\ 31\overset{\mathbf{I}^{\oplus 2}}{5}13,$$

$$31\overset{3}{\overset{1}{5}}12,\ 1512\overset{1}{3}2,\ 151\overset{1}{3}22,\ 15\overset{13,2}{\otimes},\ 1\overset{1}{5}1313,\ 2151\overset{1}{3}2,\ 215\overset{12,2}{\otimes},\ 31\overset{1}{5}123,\ 1512231,$$

$$1512313,\ 2151231,\ 2151313,\ 2215131,\ 2231315,\ 2313151,\ 2313215,\ 3215123,$$

$$3221513,\ 5122313,\ 2\overset{2}{3}1\overset{1}{\mathfrak{so}}1,\ \overset{3,2}{\otimes}\overset{1}{\mathfrak{so}}1,\ 12\overset{2}{3}1\mathfrak{so}1,\ 2\overset{2}{3}1\mathfrak{so}\mathbf{I}^{\oplus 2},\ \overset{3,2}{\otimes}\mathfrak{so}\mathbf{I}^{\oplus 2},\ \overset{3,3}{\otimes}\mathfrak{so}1,\ \overset{2,2}{\otimes}\mathfrak{so}\overset{2,2}{\otimes},$$

$$\mathbf{I}^{\oplus 2}\overset{1}{\underset{1}{\mathfrak{e}_6}}\mathbf{I}^{\oplus 2},\ \mathbf{I}^{\oplus 2}\overset{\mathbf{I}^{\oplus 2}}{\mathfrak{e}_6}\mathbf{I}^{\oplus 2},\ \mathbf{I}^{\oplus 3}\overset{1}{\underset{1}{\mathfrak{e}_6}}1,\ \mathbf{I}^{\oplus 3}\overset{1}{\mathfrak{e}_6}\mathbf{I}^{\oplus 2},\ \mathbf{I}^{\oplus 4}\overset{1}{\mathfrak{e}_6}1,\ \mathbf{I}^{\oplus 3}\mathfrak{e}_6\mathbf{I}^{\oplus 3},\ \mathbf{I}^{\oplus 4}\mathfrak{e}_6\mathbf{I}^{\oplus 2},\ \mathbf{I}^{\oplus 5}\mathfrak{e}_6 1,$$

$$\mathbf{I}^{\oplus 6}\mathfrak{e}_6,\ 1\overset{1}{\mathfrak{so}}1\overset{1}{\mathfrak{so}}1,\ 1\overset{1}{\mathfrak{so}}1\mathfrak{so}\mathbf{I}^{\oplus 2},\ 231\overset{1}{\mathfrak{so}}1\mathfrak{so},\ 31\overset{1}{\mathfrak{so}}1\mathfrak{so}1,\ 1\mathfrak{so}\overset{2,2}{\otimes}\mathfrak{so}1,\ 1\mathfrak{so}1\mathfrak{so}123,$$

$$\mathbf{I}^{\oplus 2}\mathfrak{so}1\mathfrak{so}\mathbf{I}^{\oplus 2},\ 2321\mathfrak{so}1\mathfrak{so},\ \mathfrak{so}\overset{3,2}{\otimes}\mathfrak{so}1,\ \mathfrak{so}\overset{3,3}{\otimes}\mathfrak{so},\ \mathfrak{so}\overset{2,2}{\otimes}\mathfrak{so}13\ .$$

T^H: number of tensors in the LST

Classification of non-Higgsable gravity blocks

276
patterns of external generators

163,183
non-Higgsable blocks
total across all patterns

PATTERNS WITH E_8 AND E_7 EXTERNAL TENSORS

Ext.	NH	Ext.	NH	Ext.	NH	Ext.	NH
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{e}_8}{12}$	78	$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}$	210	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{e}_6}{6}+\overset{\mathfrak{su}_3}{3}$	88	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}$	65
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{e}_7}{8}$	170	$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	155	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{e}_6}{6}+\overset{\mathfrak{su}_2}{2}$	34	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	4
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{e}_7'}{7}$	158	$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	397	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{e}_6}{6}$	412	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	90
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{e}_6}{6}+\overset{\mathfrak{su}_3}{3}$	55	$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}$	542	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{f}_4}{5}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	10	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}$	323
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{e}_6}{6}+\overset{\mathfrak{su}_2}{2}$	10	$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	41	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{f}_4}{5}+\overset{\mathfrak{su}_3}{3}$	99	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	8
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{e}_6}{6}$	234	$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	469	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{f}_4}{5}+\overset{\mathfrak{su}_2}{2}$	136	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	214
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{f}_4}{5}+\overset{\mathfrak{su}_3}{3}$	54	$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	721	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{f}_4}{5}$	543	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	944
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{f}_4}{5}+\overset{\mathfrak{su}_2}{2}$	45	$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_3}{3}$	1208	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{so}_8}{4}$	68	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}$	1059
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{f}_4}{5}$	287	$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	14	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	1	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	2
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{so}_8}{4}$	51	$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_2}{2}$	388	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}$	70	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	77
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}$	57	$\overset{\mathfrak{e}_8}{12}$	900	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	4	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	1124
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	76	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}$	4	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	121	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	2216
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_3}{3}$	223	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_2}{2}$	14	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_3}{3}$	330	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}$	2451
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_2}{2}$	115	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{e}_7}{8}$	255	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	12	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_2}{2}+\overset{\mathfrak{su}_2}{2}$	95
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{so}_8}{4}$	442	$\overset{\mathfrak{e}_7'}{7}+\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}$	5	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{so}_8}{4}+\overset{\mathfrak{su}_2}{2}$	330	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_2}{2}$	1010
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}$	54	$\overset{\mathfrak{e}_7'}{7}+\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_2}{2}$	15	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{so}_8}{4}$	759	$\overset{\mathfrak{e}_7}{8}$	2000
$\overset{\mathfrak{e}_8}{12}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	64	$\overset{\mathfrak{e}_7'}{7}+\overset{\mathfrak{e}_7}{8}$	234	$\overset{\mathfrak{e}_7}{8}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_3}{3}+\overset{\mathfrak{su}_2}{2}$	1		

Ext. = external generator configuration · **NH** = number of distinct NH blocks for that pattern

Note. Patterns involving a *"2"* intersecting *"1"* configuration are excluded here; the full set is on github.

Classification of SUGRA bases with small T

For small tensor number, our framework gives a direct enumeration of **allowed tensor bases**.

- We classify **all tensor bases with $T \leq 10$** generated by the construction.
- The output grows rapidly, but remains **finite** and **algorithmic**.

$$
\begin{aligned}
T=1\ &:\ \overset{\text{any}}{\mathcal{C}}\,0\\
T=2\ &:\ \overset{\text{any}}{\mathcal{C}}\,11\\
T=3\ &:\ 11\overset{\text{any}}{\mathcal{C}}11,\ \overset{\text{any}}{\mathcal{C}}121,\ \mathfrak{g}_2212,\ 1\overset{\mathfrak{g}_2}{2}1\\
T=4\ &:\ (T=3)\mathcal{C}11,\ \overset{\text{any}}{\mathcal{C}}1221,\ \overset{\mathfrak{e}_6,\mathfrak{f}_4,\mathfrak{so}_8,\mathfrak{g}_2}{\mathcal{C}}1\overset{1}{3}1,\ 1\underset{\mathfrak{su}_2}{\overset{1}{3}}1,\ \mathfrak{g}_2\overset{1}{2}21,\ 2131\overset{\mathfrak{e}_6,\mathfrak{f}_4,\mathfrak{so}_8,\mathfrak{g}_2}{\mathcal{C}},\ \mathfrak{g}_22131,\ 21\overset{\mathfrak{su}_2}{3}1,\ 2213\mathfrak{su}_2,\ \mathfrak{g}_22213\\
T=5\ &:\ (T=4)\mathcal{C}11,\ (T=3)\mathcal{C}(T=3),\ \overset{\text{any}}{\mathcal{C}}12221,\ \overset{\leq\mathfrak{so}_8}{\mathcal{C}}1\underset{1}{\overset{1}{4}}1,\ \overset{\leq\mathfrak{e}_7}{\mathcal{C}}12\overset{1}{3}1,\ 12\overset{1}{3}1\overset{\leq\mathfrak{f}_4}{\mathcal{C}},\ 12\underset{\mathfrak{su}_2}{\overset{1}{3}}1\ 21\overset{1}{4}1\overset{\leq\mathfrak{so}_8}{\mathcal{C}},\\
&\quad \mathfrak{g}_221\overset{1}{4}1,\ \mathfrak{su}_231\overset{1}{3}2,\ \overset{\leq\mathfrak{e}_7}{\mathcal{C}}12312,\ 12\overset{\mathfrak{su}_2}{3}12,\ 12312\mathfrak{g}_2,\ \overset{\leq\mathfrak{e}_6}{\mathcal{C}}13131,\ 1\overset{\mathfrak{su}_2}{3}131,\ \mathfrak{g}_221412,\ 22141\overset{\leq\mathfrak{so}_8}{\mathcal{C}},\\
&\quad \mathfrak{g}_222141,\ 23123\mathfrak{su}_2,\ 31231\overset{\leq\mathfrak{f}_4}{\mathcal{C}},\ 312\overset{\mathfrak{su}_2}{3}1,\ \mathfrak{su}_231231,\ \mathfrak{su}_231313,\ 2\overset{1}{2}31\overset{\leq\mathfrak{f}_4}{\mathcal{C}}.\\
T=6\ &:\ (T=5)\mathcal{C}11,\ (T=4)\mathcal{C}(T=3),\ \overset{\text{any}}{\mathcal{C}}122221,\ \mathfrak{su}_31^{\otimes5}5\ \mathfrak{su}_31\underset{1}{\overset{1}{5}}12,\ 1\underset{1}{\overset{1}{5}}12\mathfrak{g}_2,\ \overset{\text{any}}{\mathcal{C}}122\overset{1}{3}1,\ 122\overset{1}{3}1\overset{\leq\mathfrak{f}_4}{\mathcal{C}},\\
&\quad \overset{\leq\mathfrak{e}_7}{\mathcal{C}}12\overset{1}{3}21,\ 12\overset{\leq\mathfrak{so}_8}{\overset{1}{3}}21,\ \overset{\leq\mathfrak{e}_6}{\mathcal{C}}131\overset{1}{4}1,\ 131\overset{1}{4}1\overset{\leq\mathfrak{so}_8}{\mathcal{C}},\ 131\overset{\leq\mathfrak{so}_8}{\overset{1}{4}}1,\ 1\overset{\mathfrak{su}_2}{3}1\overset{1}{4}1,\ \mathfrak{g}_221\overset{1}{5}12,\ 21\overset{\mathfrak{su}_3}{\overset{1}{5}}12,\ 2\overset{1}{3}141\overset{\leq\mathfrak{so}_8}{\mathcal{C}},\\
&\quad \mathfrak{su}_231\overset{1}{4}13,\ \mathfrak{su}_231\overset{2}{3}21,\ \overset{\text{any}}{\mathcal{C}}122312,\ 122312\mathfrak{g}_2,\ \overset{\leq\mathfrak{e}_6}{\mathcal{C}}131321,\ 131321\overset{\leq\mathfrak{e}_7}{\mathcal{C}},\ 1\overset{\mathfrak{su}_2}{3}1321,\ 131\overset{\mathfrak{su}_2}{3}21,\\
&\quad \overset{\leq\mathfrak{e}_6}{\mathcal{C}}131412,\ 131412\mathfrak{g}_2,\ 1\overset{\mathfrak{su}_2}{3}1412,\ \overset{\leq\mathfrak{f}_4}{\mathcal{C}}132141,\ 132141\overset{\leq\mathfrak{so}_8}{\mathcal{C}},\ 1\overset{\mathfrak{su}_2}{3}2141,\ \overset{\leq\mathfrak{f}_4}{\mathcal{C}}132214,\ \mathfrak{g}_2221512,\\
&\quad 221512\mathfrak{g}_2,\ 222151\mathfrak{su}_3,\ \mathfrak{su}_2312321,\ 312321\overset{\leq\mathfrak{e}_7}{\mathcal{C}},\ \mathfrak{su}_2313141,\ 313141\overset{\leq\mathfrak{so}_8}{\mathcal{C}},\ \mathfrak{su}_2313214,\\
&\quad \mathfrak{su}_2321413,\ 321413\mathfrak{su}_2.
\end{aligned}
$$

- Red labels mark the allowed external NHC generator C.
- The results agree with the geometric base classification in **Taylor-Wang** ['15].

Novel discovery : supergravity with frozen singularity

Containing NHCs that cannot be realised in F-theory geometry, but with *frozen singularity*.

$$\overset{\mathfrak{su}_8}{\hat{1}} \quad , \quad \overset{\mathfrak{su}_8}{2}\ \overset{\mathfrak{su}_{16}}{\hat{1}} \quad , \quad \overset{\mathfrak{su}_8}{2} = \overset{\mathfrak{so}_{16}}{4}$$

T=10

Supergravity with external $\mathcal{C} = \overset{\mathfrak{su}_8}{\hat{1}}$

- Tensor base = 10 blowup points of P2
- Gauge algebra on canonical class b_0 with $b_0^2 = -1$
- Configuration:

$$\overset{\mathfrak{su}_8}{\hat{1}}\ 122222222\overset{2}{2}$$

T=11

Supergravity with external $\overset{\mathfrak{su}_8}{2} = \overset{\mathfrak{so}_{16}}{4}$

- Tensor base configuration:

$$21\ \overset{\mathfrak{su}_8}{2} = \overset{\mathfrak{so}_{16}}{4}\ 12222\overset{2}{2}2$$

- Cannot be realized in F-theory as this base involves a tangential intersection

- All SCFT, LST subsets can be realised in Type II string theory with O^+-plane.
- Instanton strings for every gauge algebra have a known UV completion.

  $\Rightarrow$ Likely to be consistent supergravity theories. Can we construct them in string compactification?

Conclusion & outlook

CONCLUSION

- We give a bottom-up, systematic framework for constructing 6d (1,0) supergravities.
- The basic building blocks are supergravity blocks — units that carry an intrinsic gravitational tensor.
- We obtain a complete classification of the non-Higgsable supergravity blocks.
- The framework also uncovers candidate supergravities with frozen singularities — theories that are not realised in ordinary F-theory geometry.

OUTLOOK

- A complete classification of 6d (1,0) supergravities (including gauge algebras on tensors with positive self-intersection).
- Further consistency checks: discrete anomalies, unitarity of non-perturbative objects, and additional UV-completion constraints.