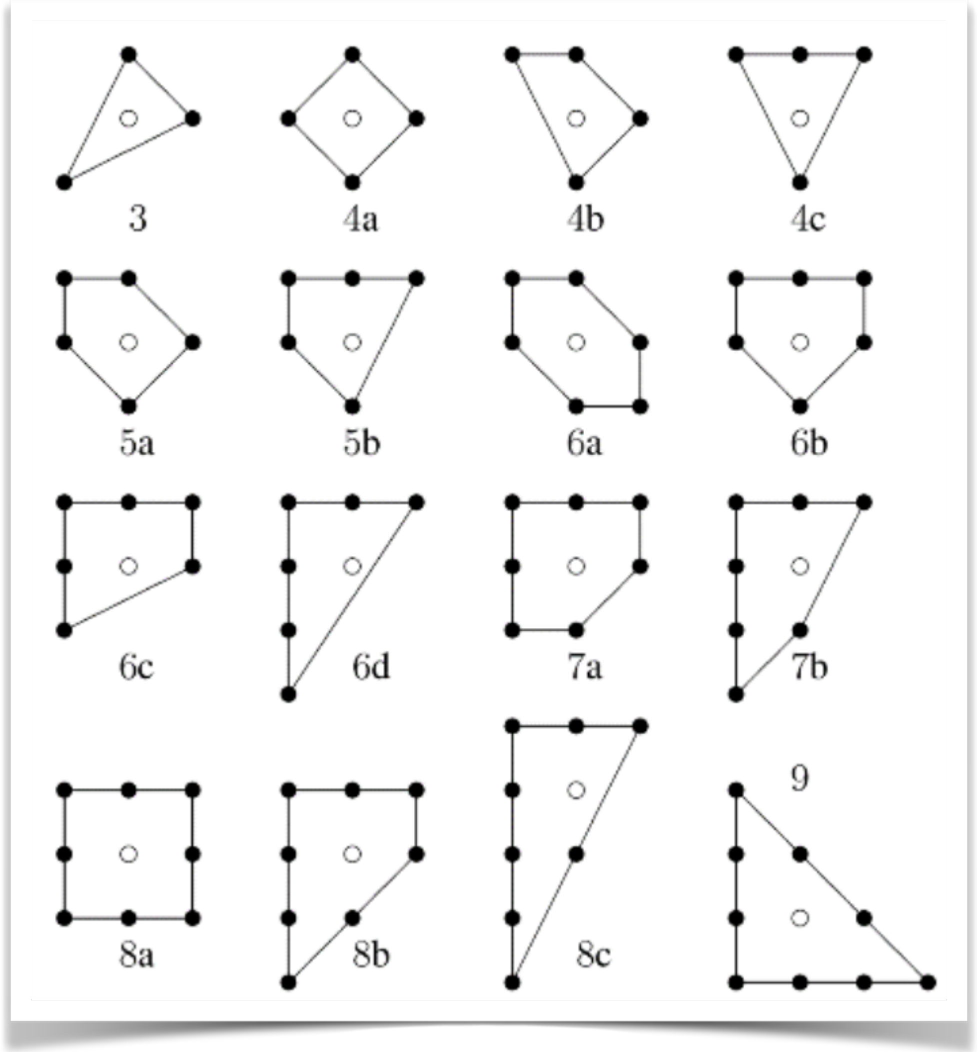
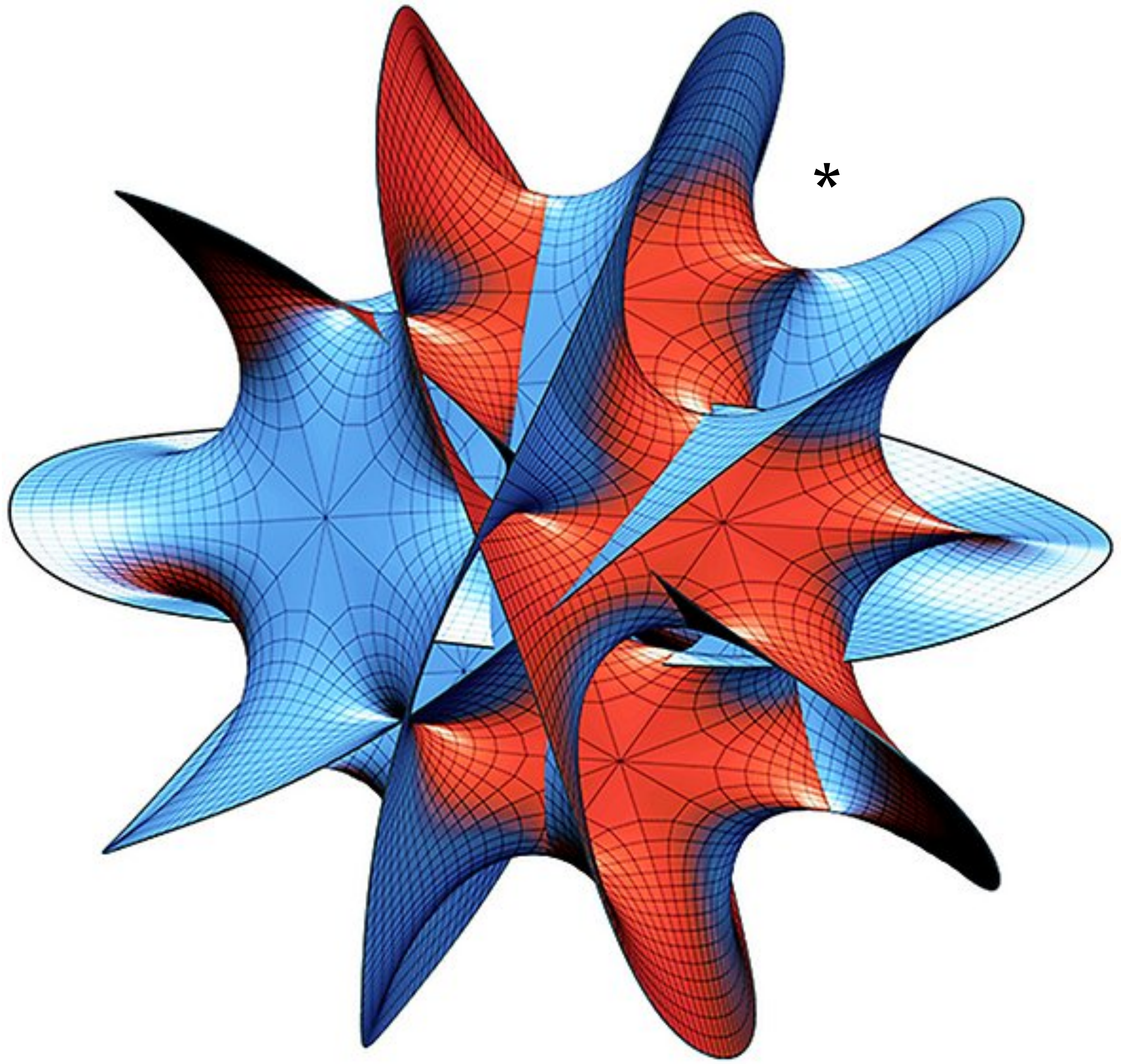
Combinatorial F-Theory Uplifts

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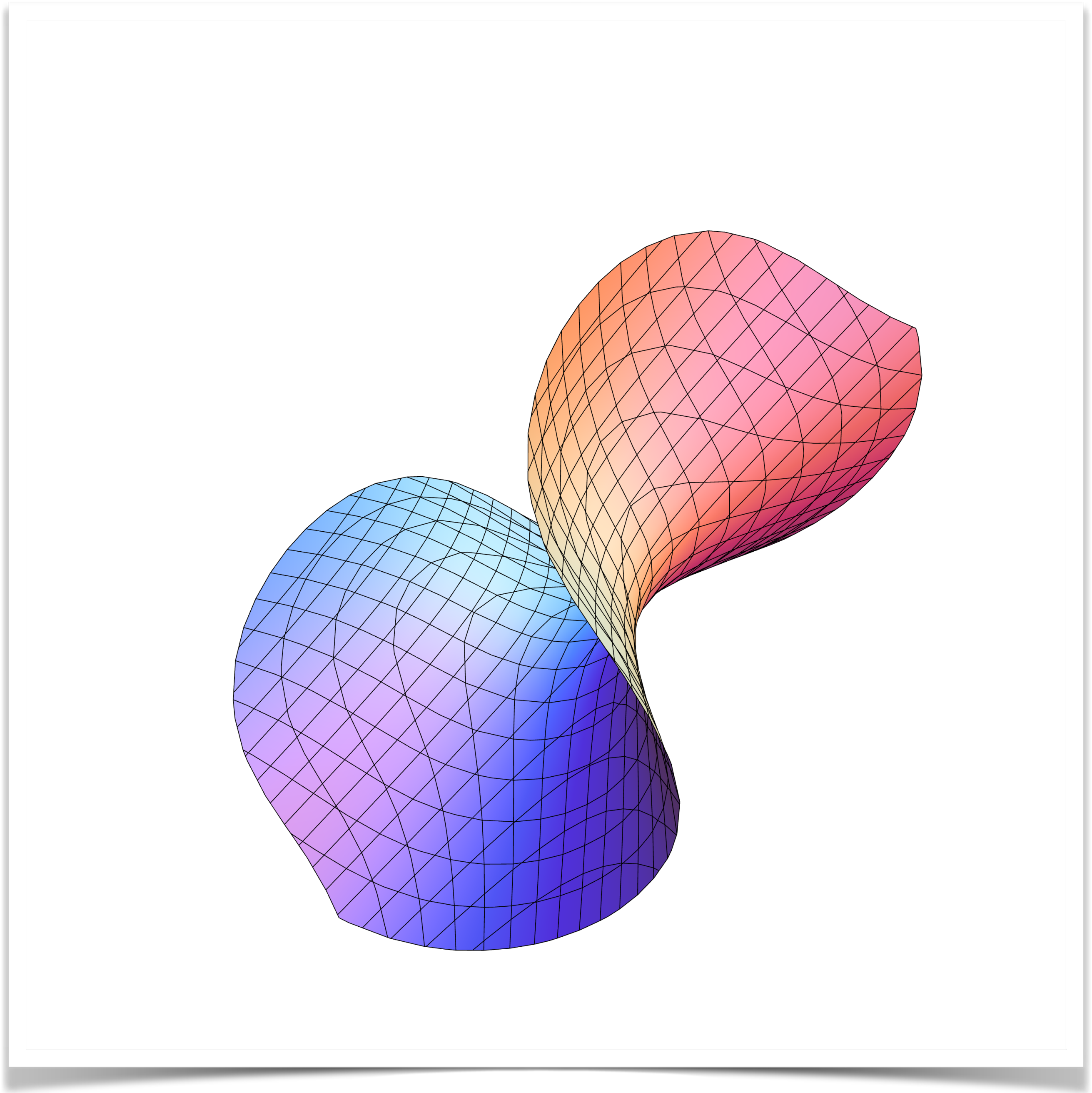

Jakob Moritz

05/21/2026 at Strings and Geometry, Uppsala University

* CY-image credit: Centre for Geometry and Physics

Outline

1. Motivation: String Landscape and Flux superpotentials
2. Systematics of O3/O7 orientifolds of Calabi-Yau hypersurfaces
3. "Combinatorial" F-theory uplifts
4. Conclusions

based on upcoming work with postdoc Björn Hassfeld
(formerly known as Björn Friedrich)

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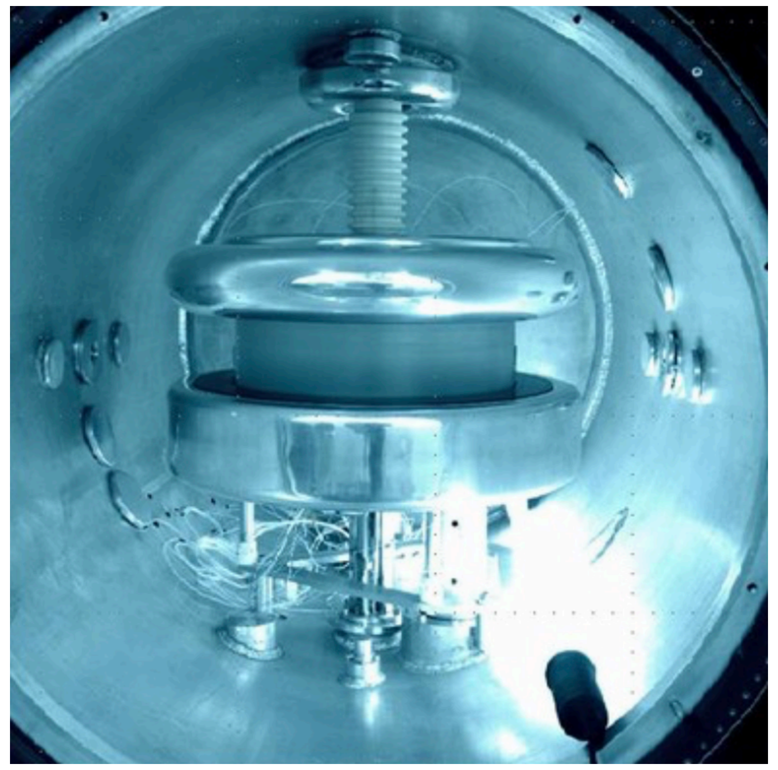
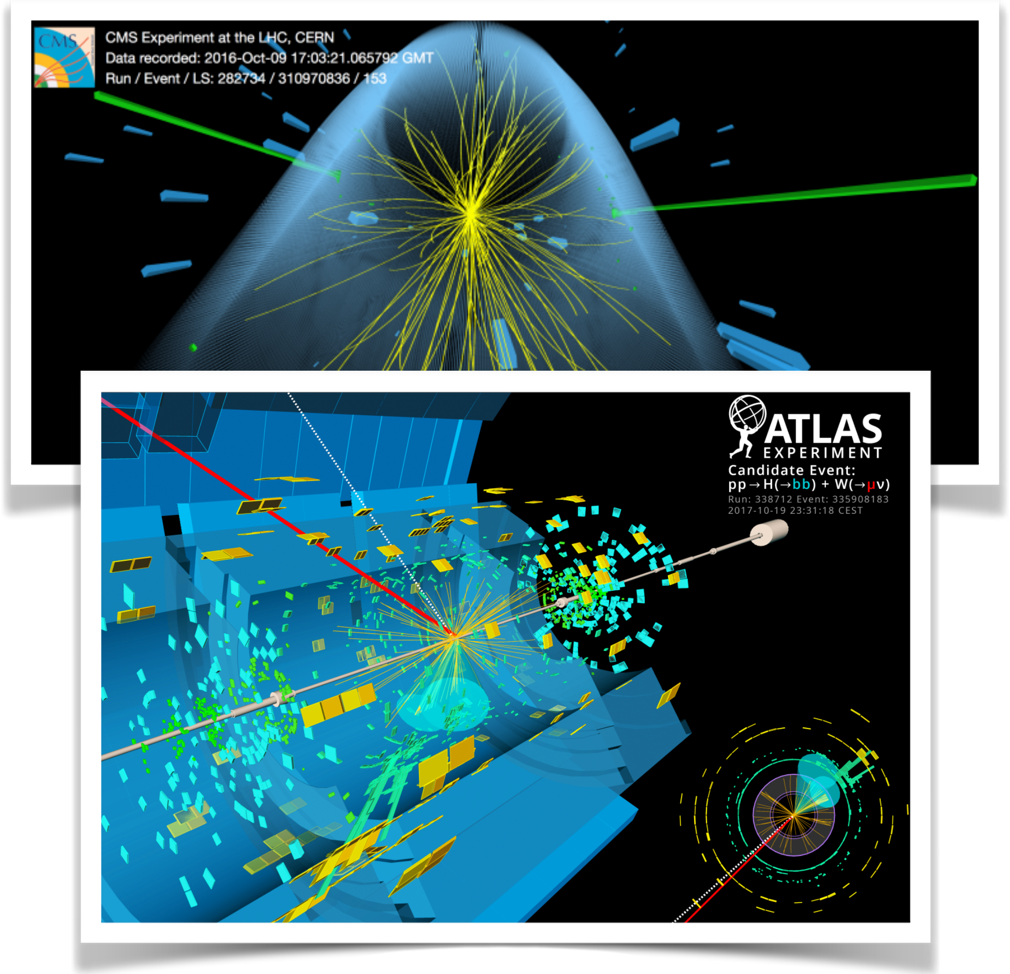
The Big Open Questions

$\theta_{QCD} < 10^{-10}$

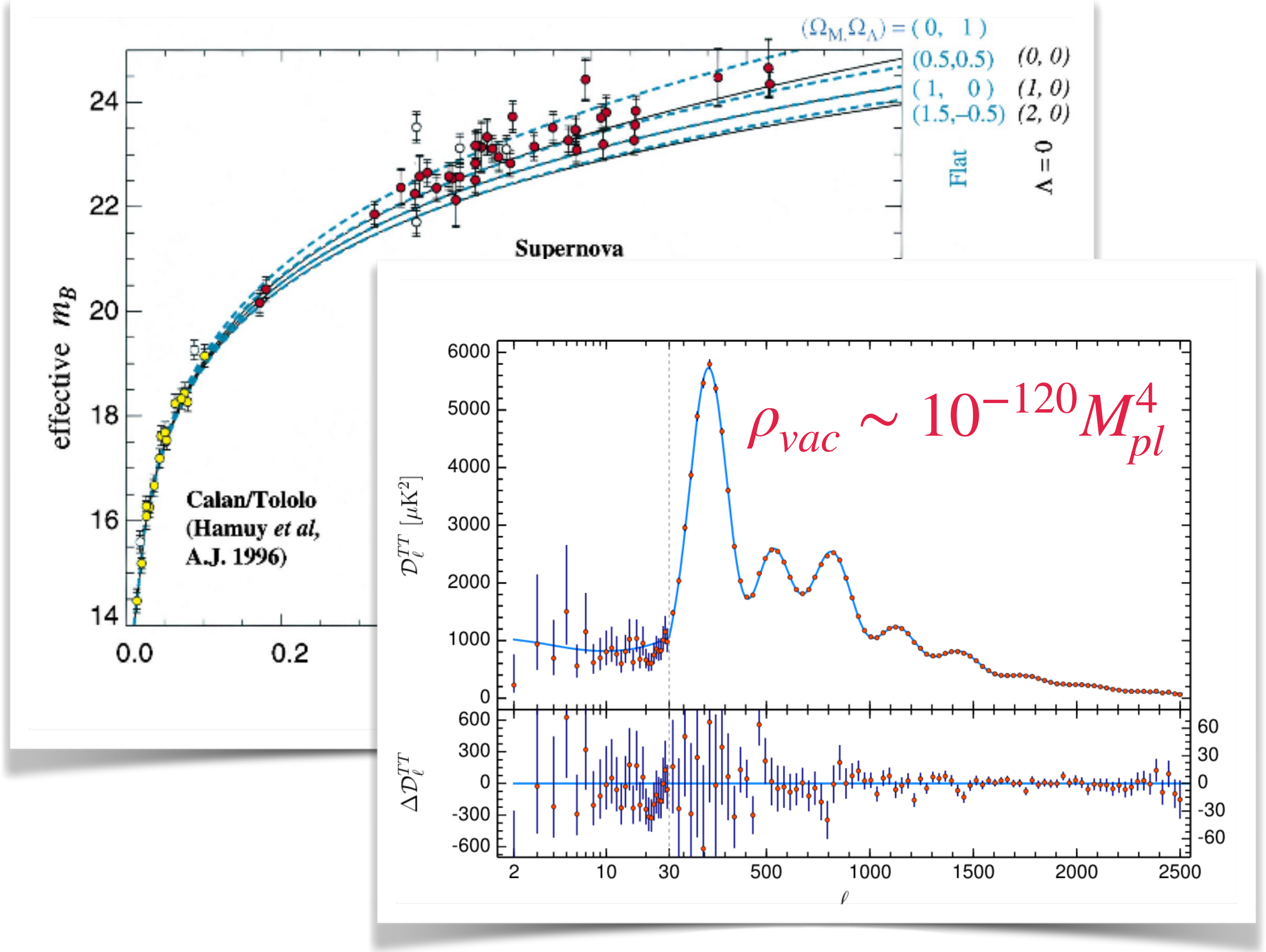


nEDM collaboration

Hierarchy Problems:

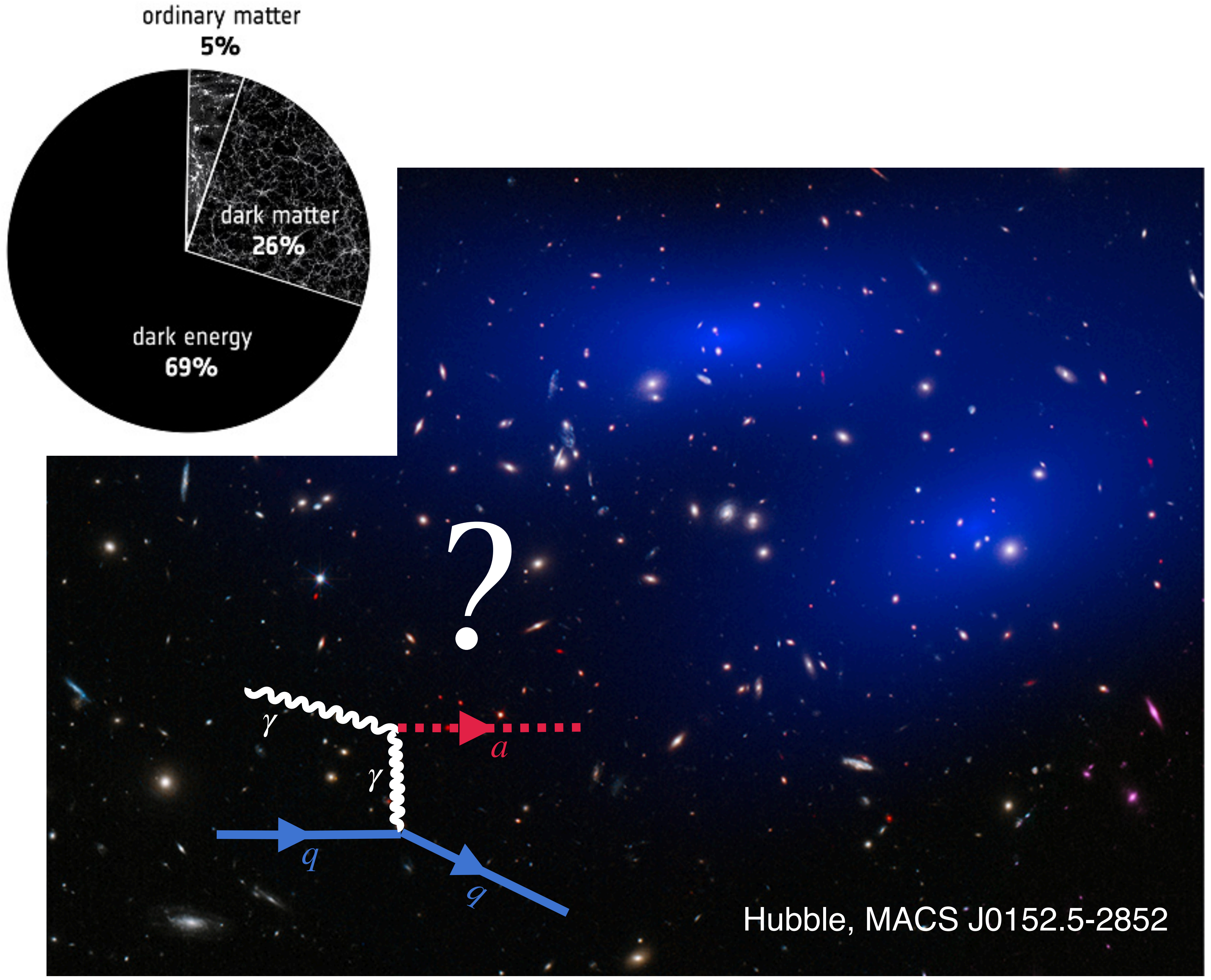


$v_{Higgs} = 250\,GeV \sim 10^{-15} M_p$



+ small Yukawa's,
neutrino masses,
small CMB anisotropies, ...

The nature of dark matter:



String Landscape to the Rescue?

Fine tuning issues in SM and ΛCDM (by definition) UV-sensitive, so it is possible that a detailed understanding of the string landscape will shed new light on these problems.

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Taylor, Wang '15

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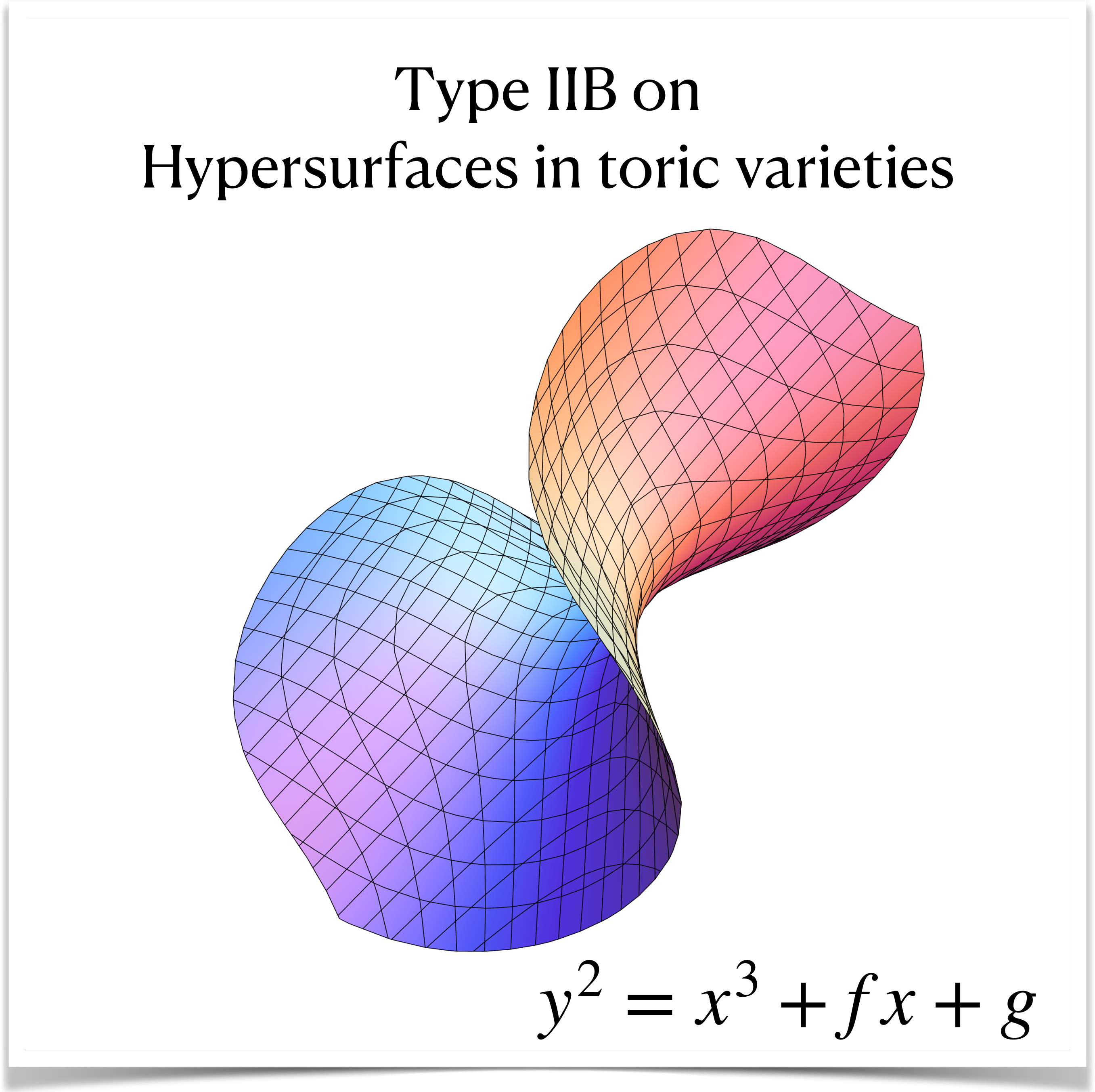
Taylor, Wang '15

Need powerful computational tools/algorithms to study compactifications beyond what's feasible with pencil-and-paper work!

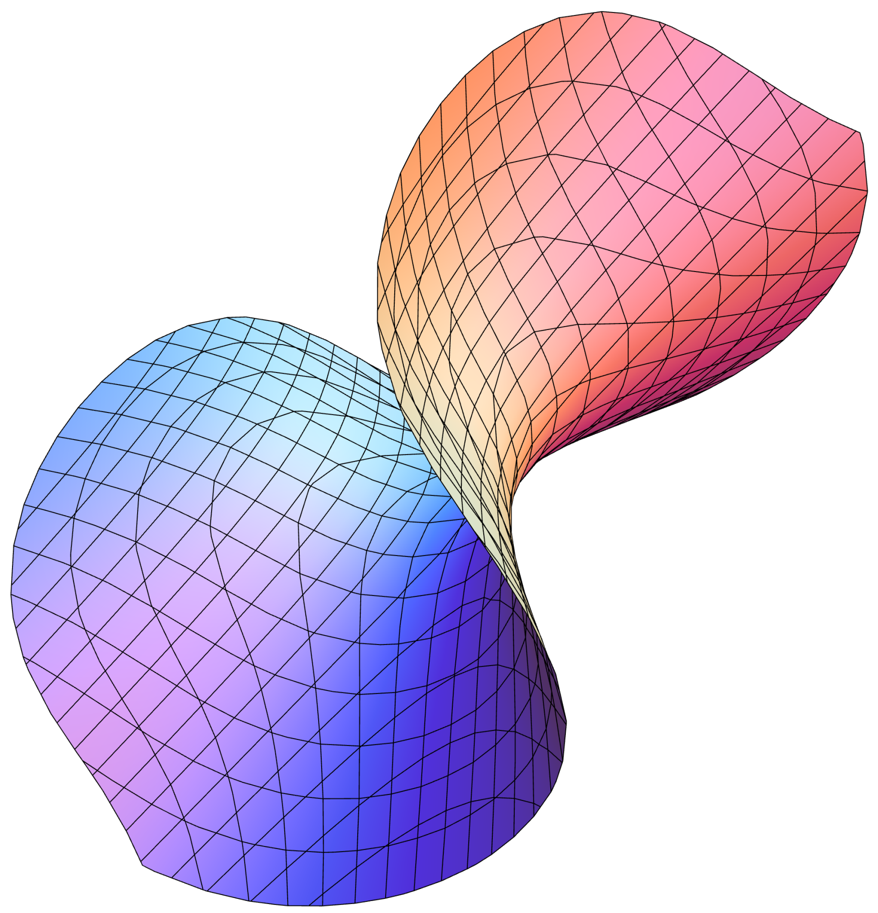
Kreuzer-Skarke



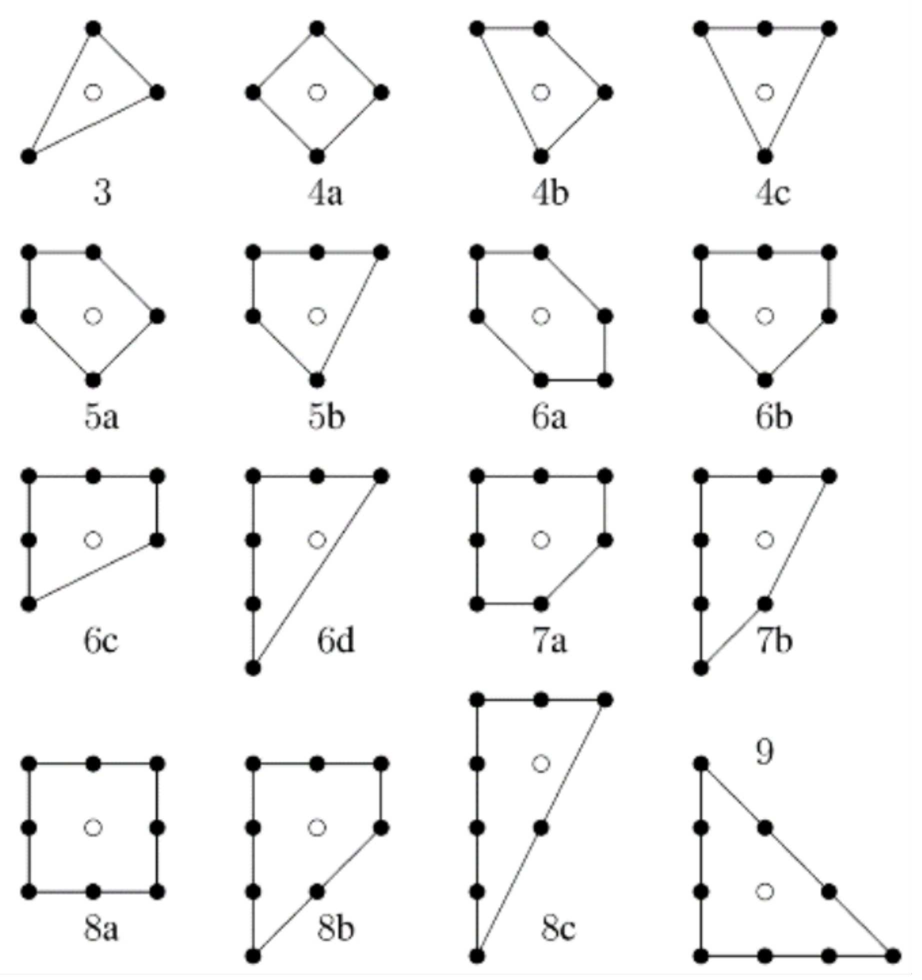
Kreuzer-Skarke

toric varieties can be constructed combinatorially: Batyrev '93

Type IIB on
Hypersurfaces in toric varieties



$$y^2 = x^3 + fx + g$$



473,800,776 (reflexive) polytopes in 4d
Kreuzer, Skarke '00

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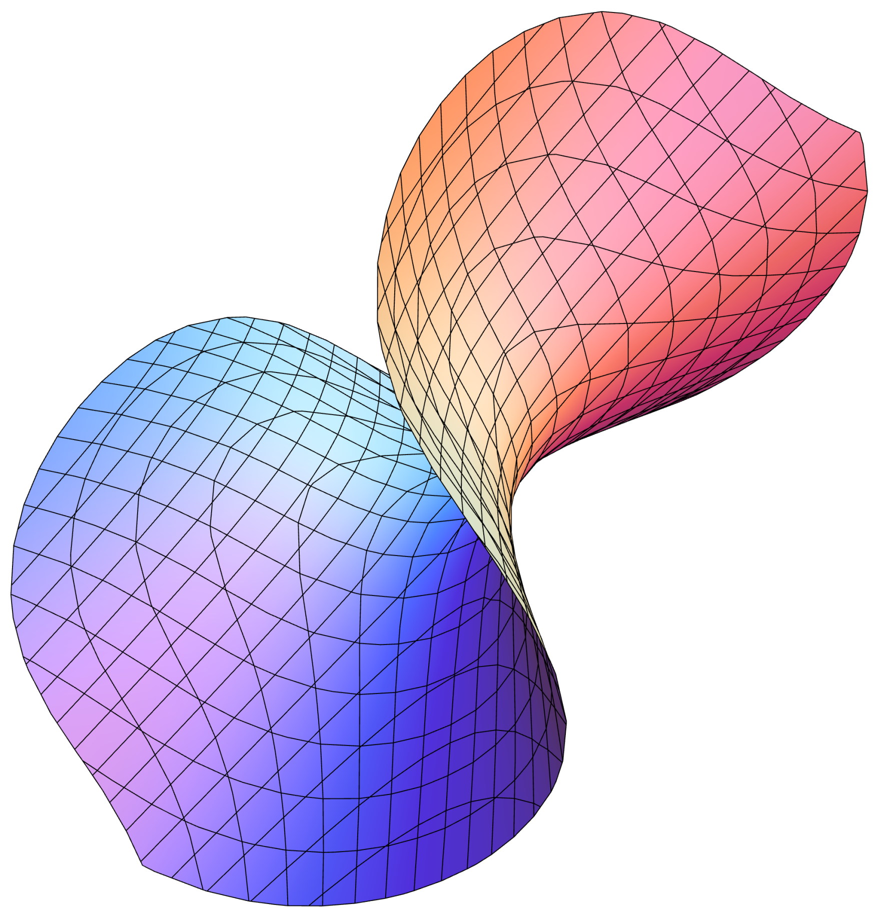

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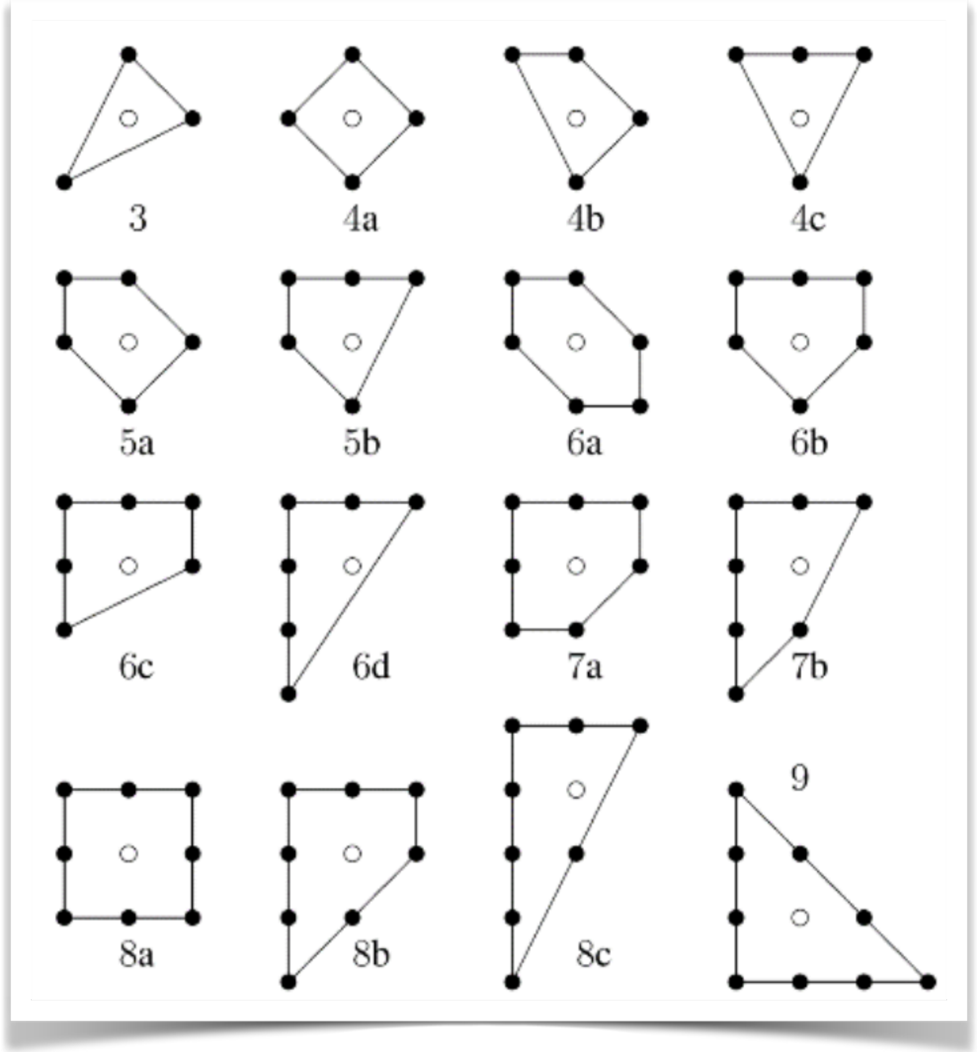

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triangulate faces
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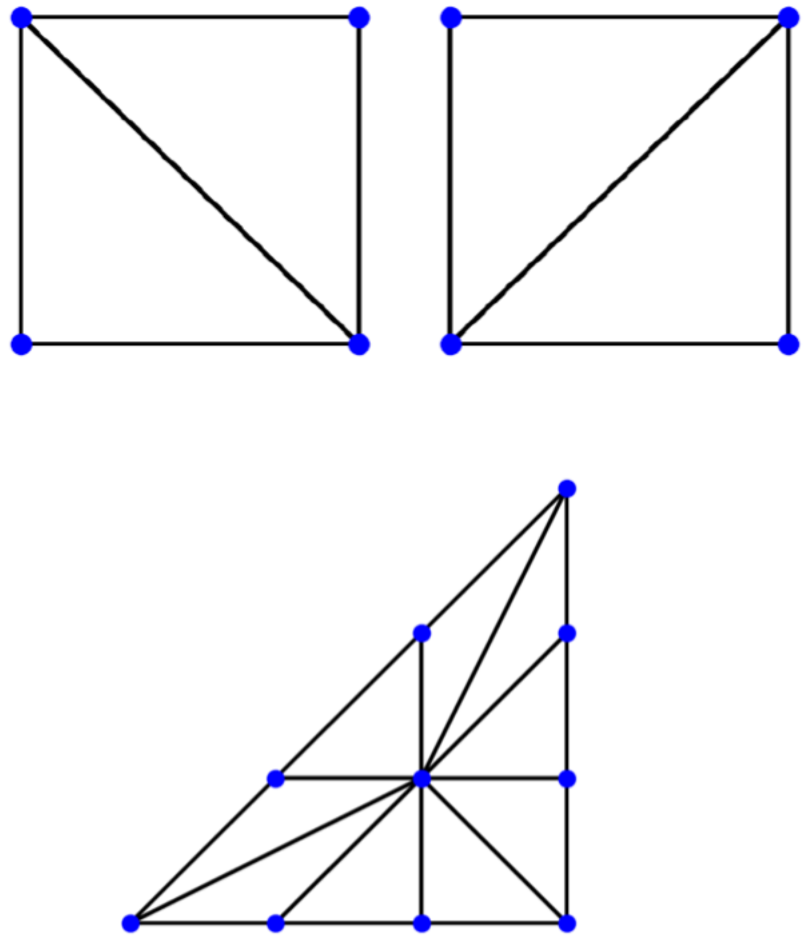

$\lesssim 10^{296}$ Calabi-Yau threefolds
Demirtas, McAllister, Rios-Tascon '20
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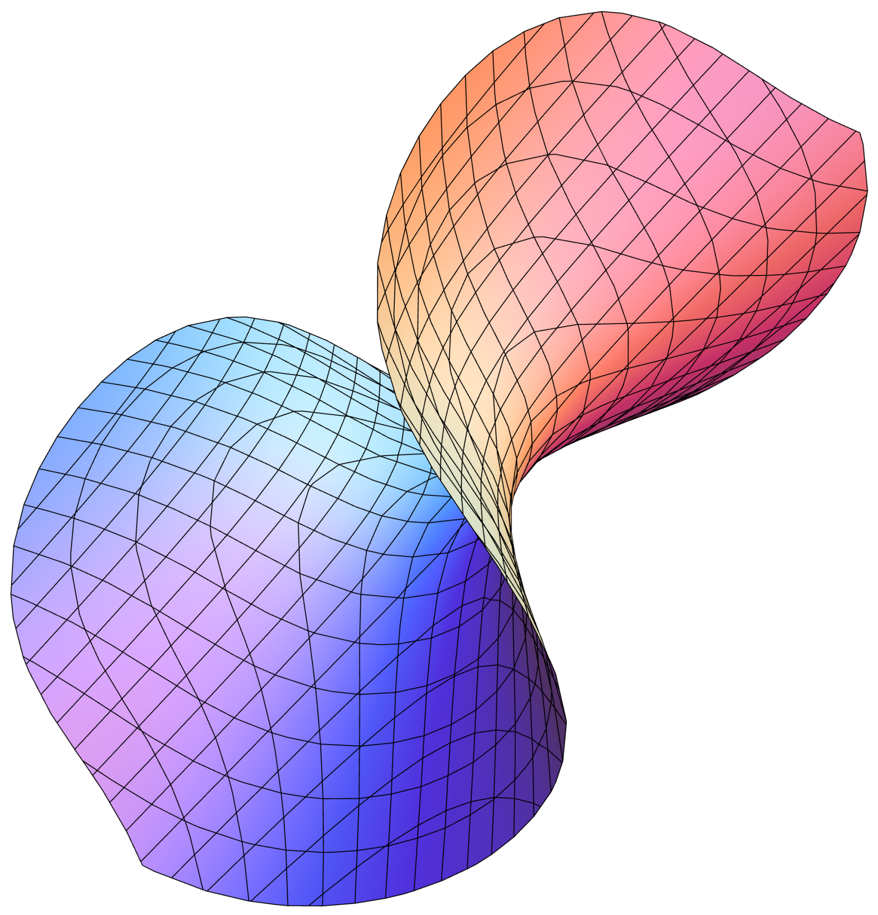

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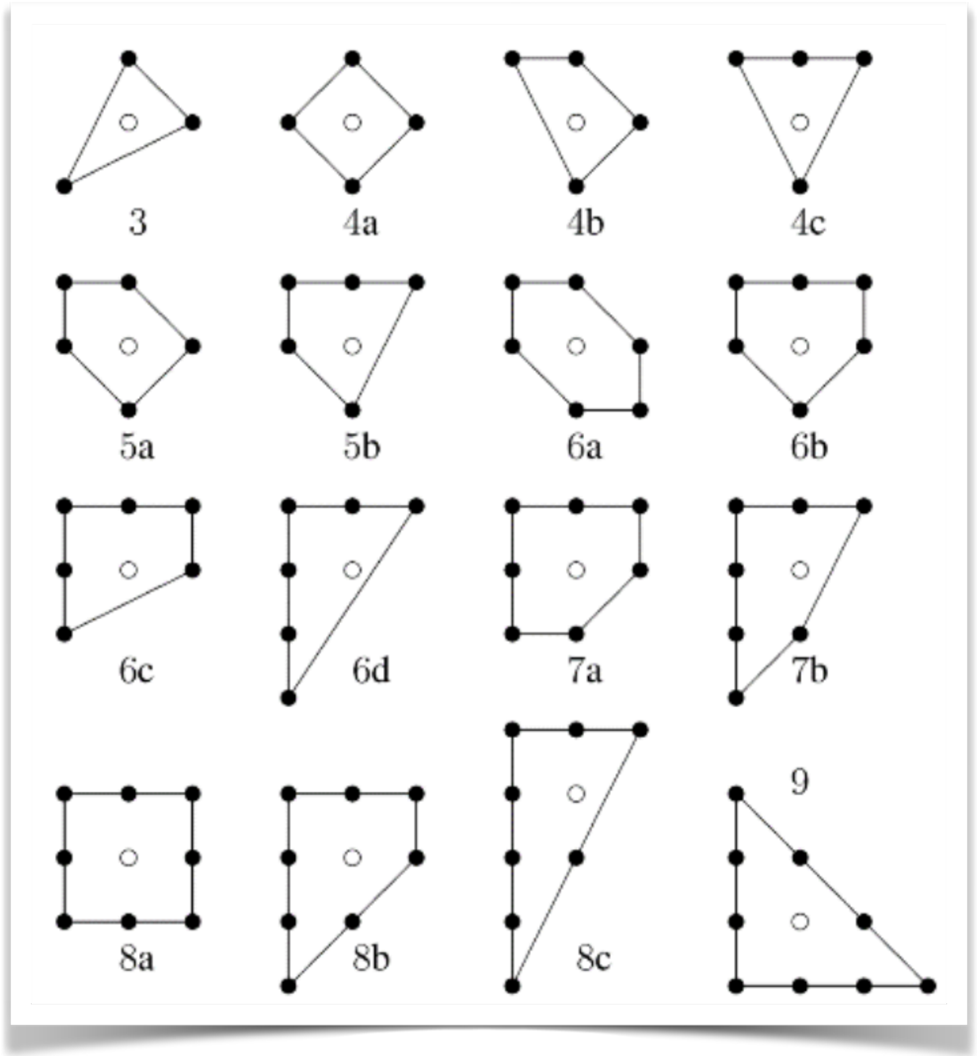

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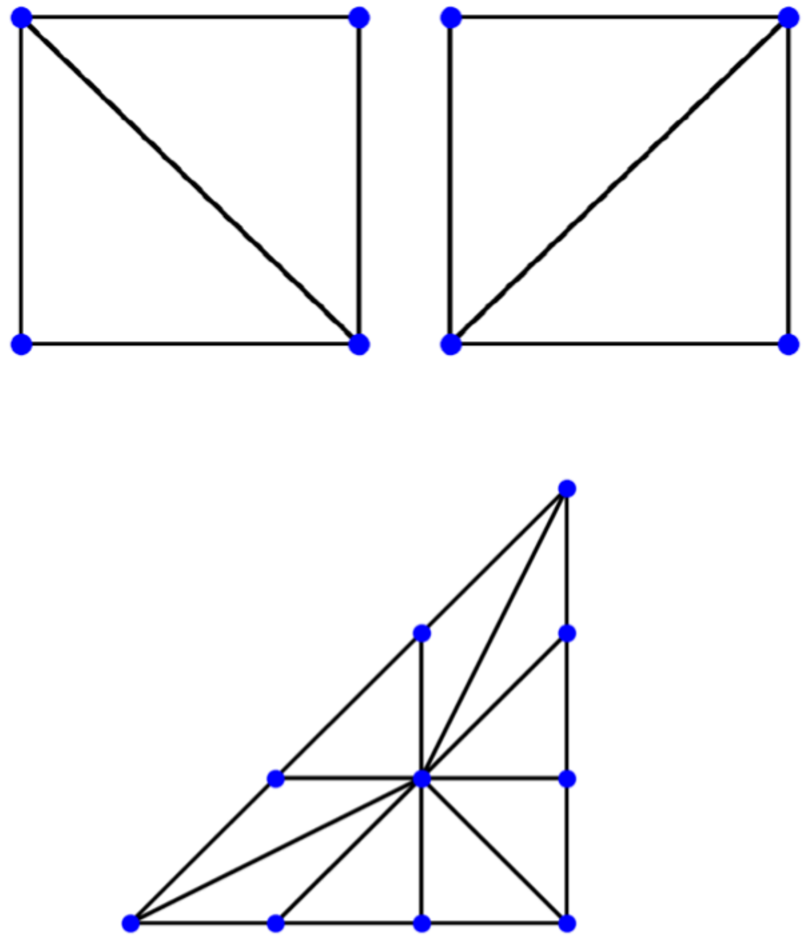

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Open source library of algorithms to treat and analyze Calabi-Yau hypersurfaces

e.g. GV-invariants: Demirtas, Kim, McAllister, JM, Rios-Tascon '23

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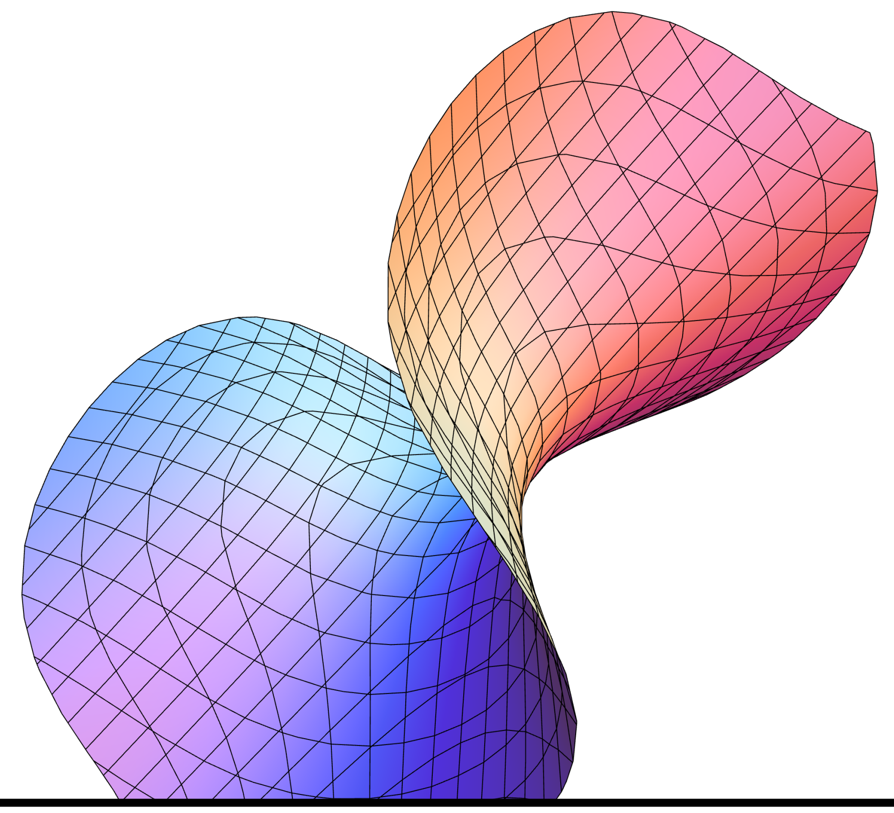

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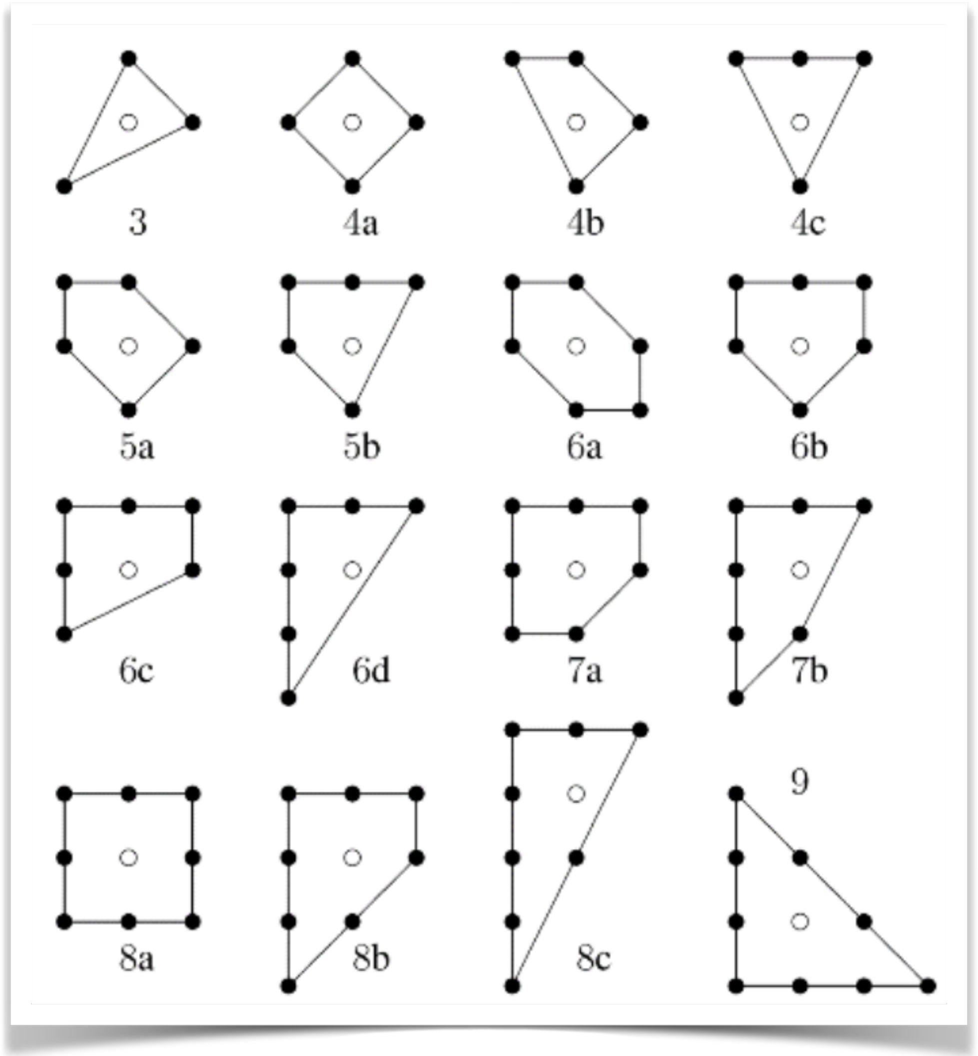

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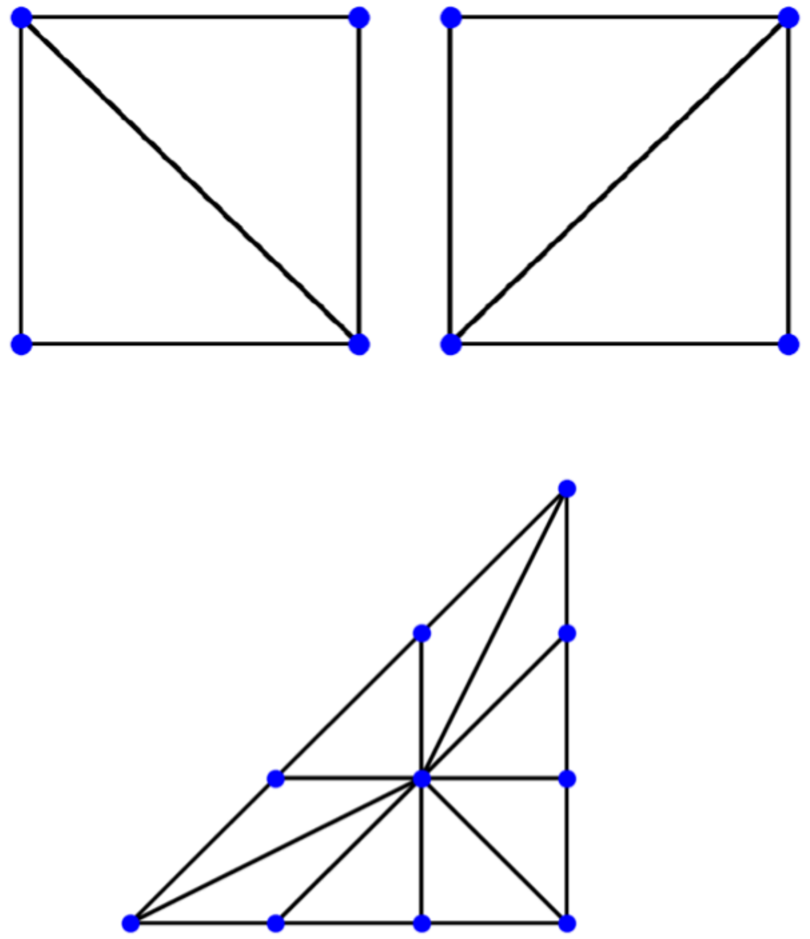

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Calabi-Yau varieties and their properties can be encoded in purely combinatorial data of 4d reflexive polytopes!

CYTools

Demirtas, Rios-Tascon, McAllister '22

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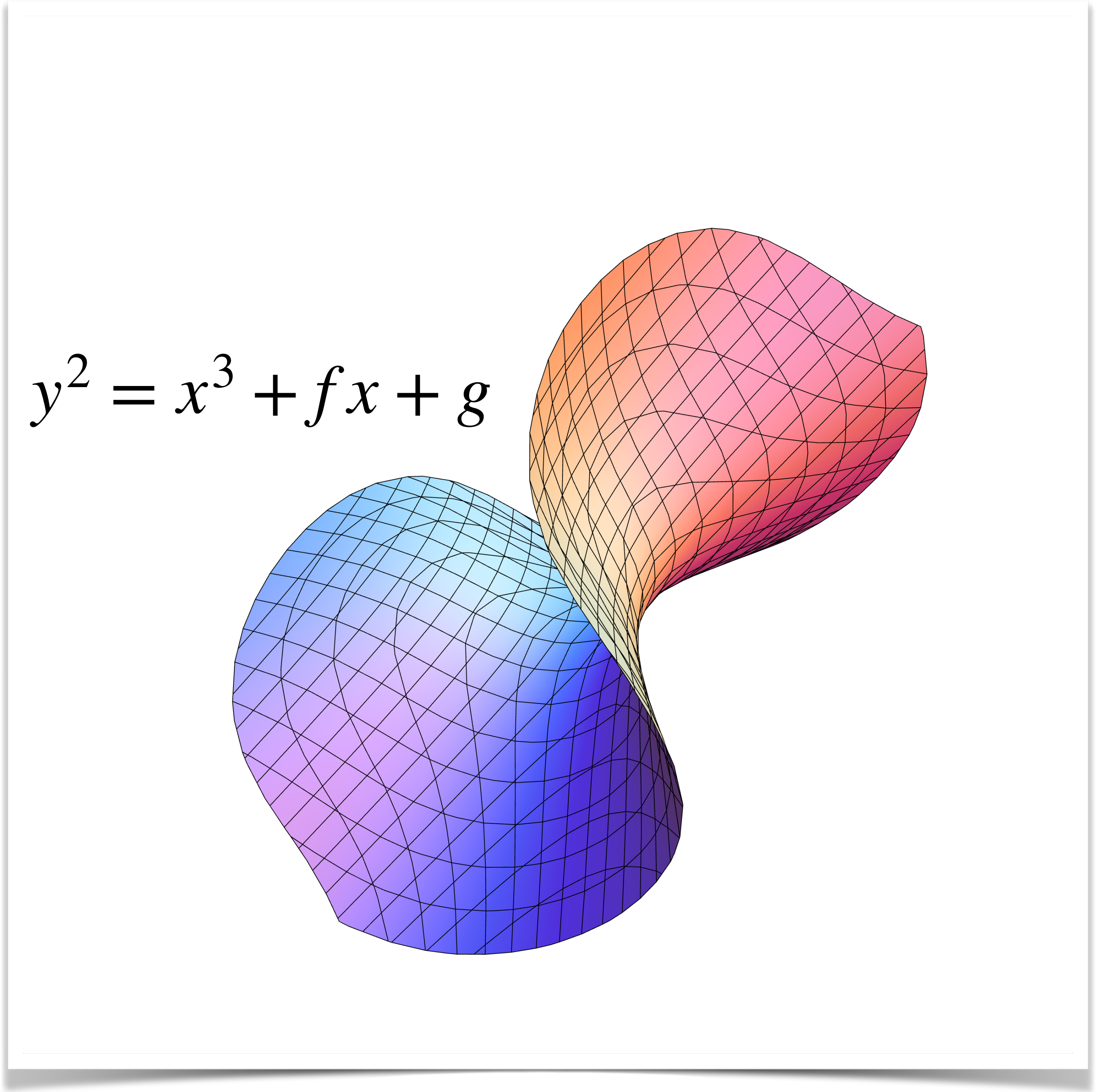
Calabi-Yau Orientifolds

Type IIB/F-theory corner of landscape: need to specify holomorphic orientifold



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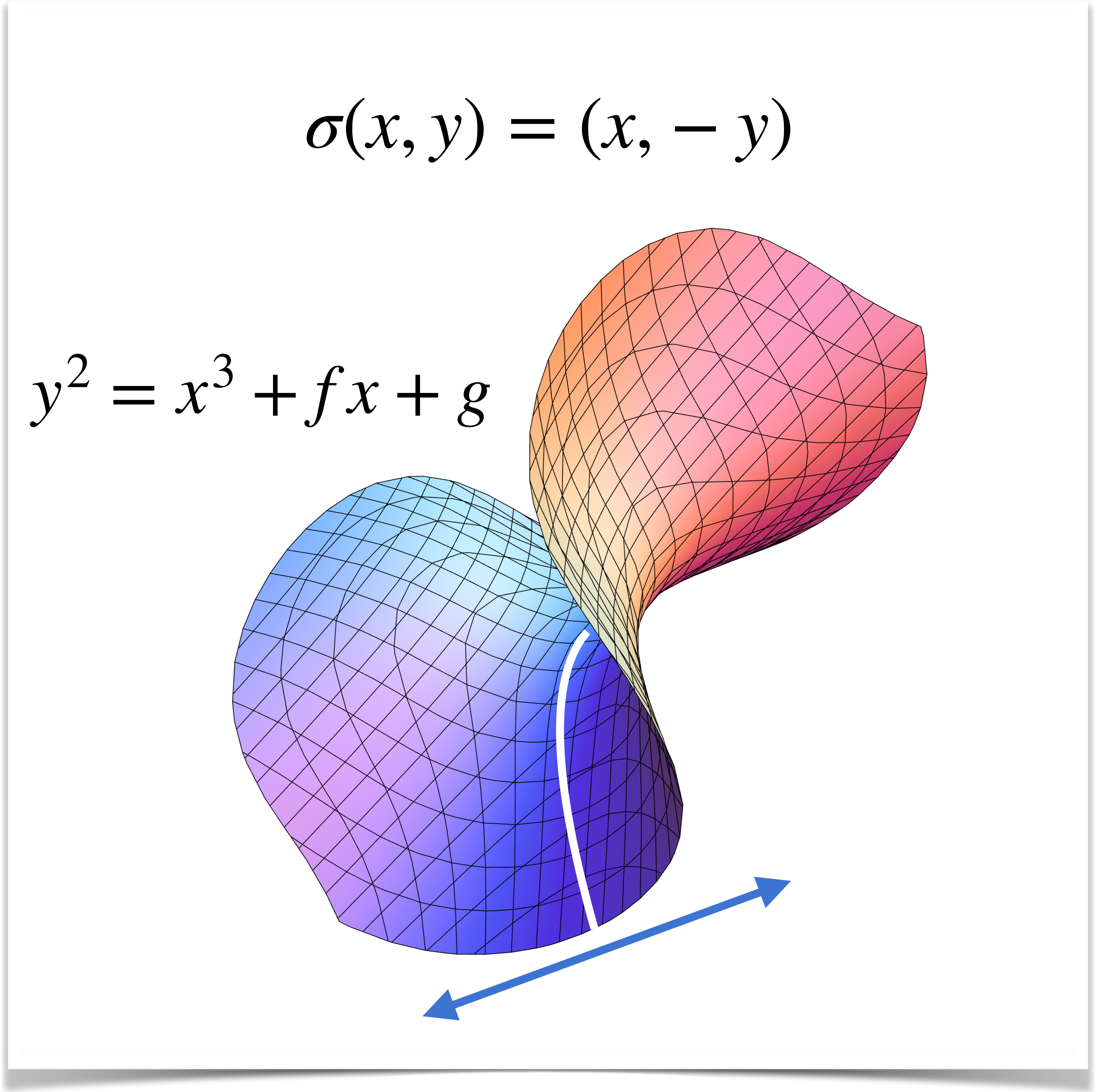

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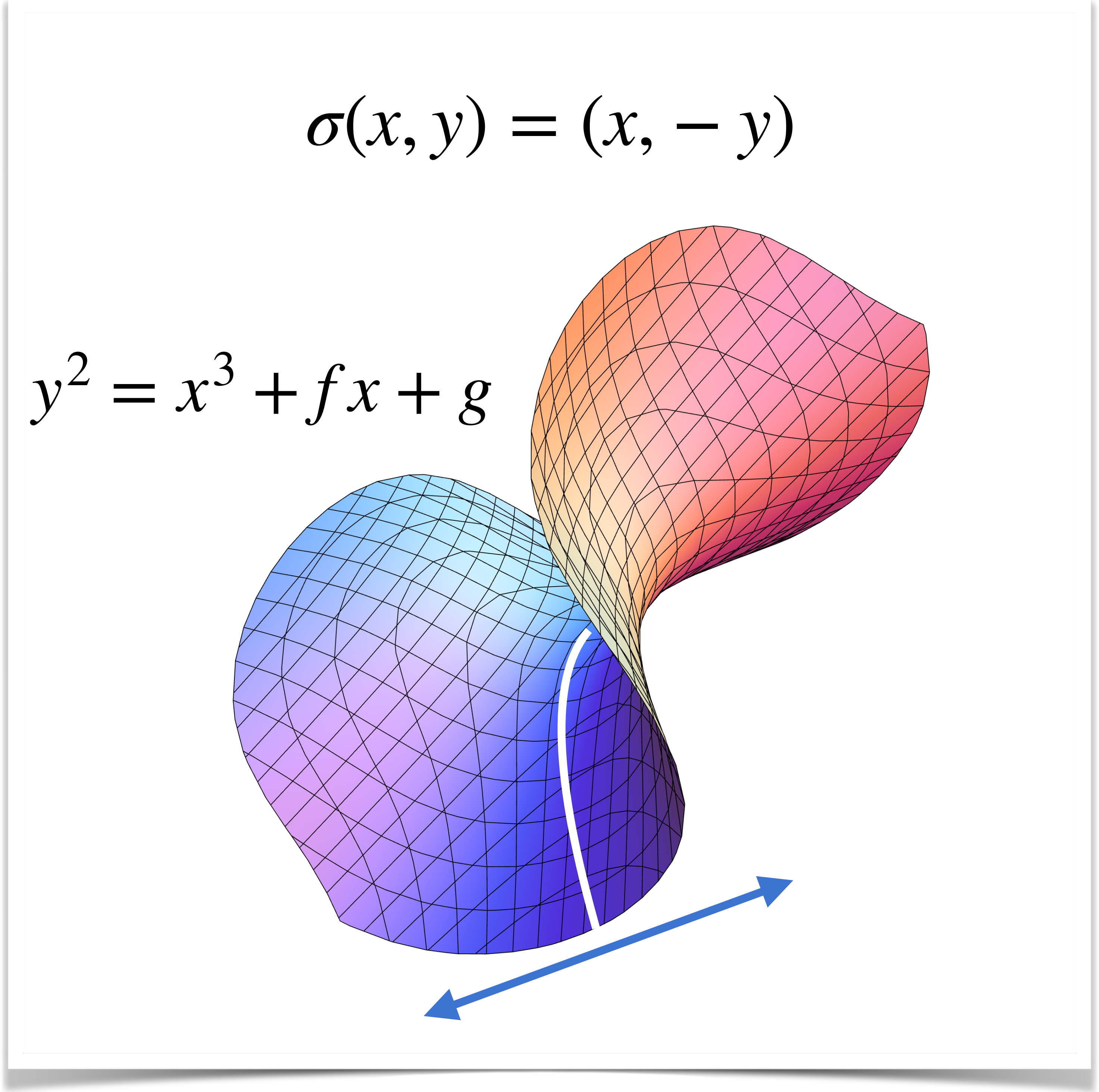

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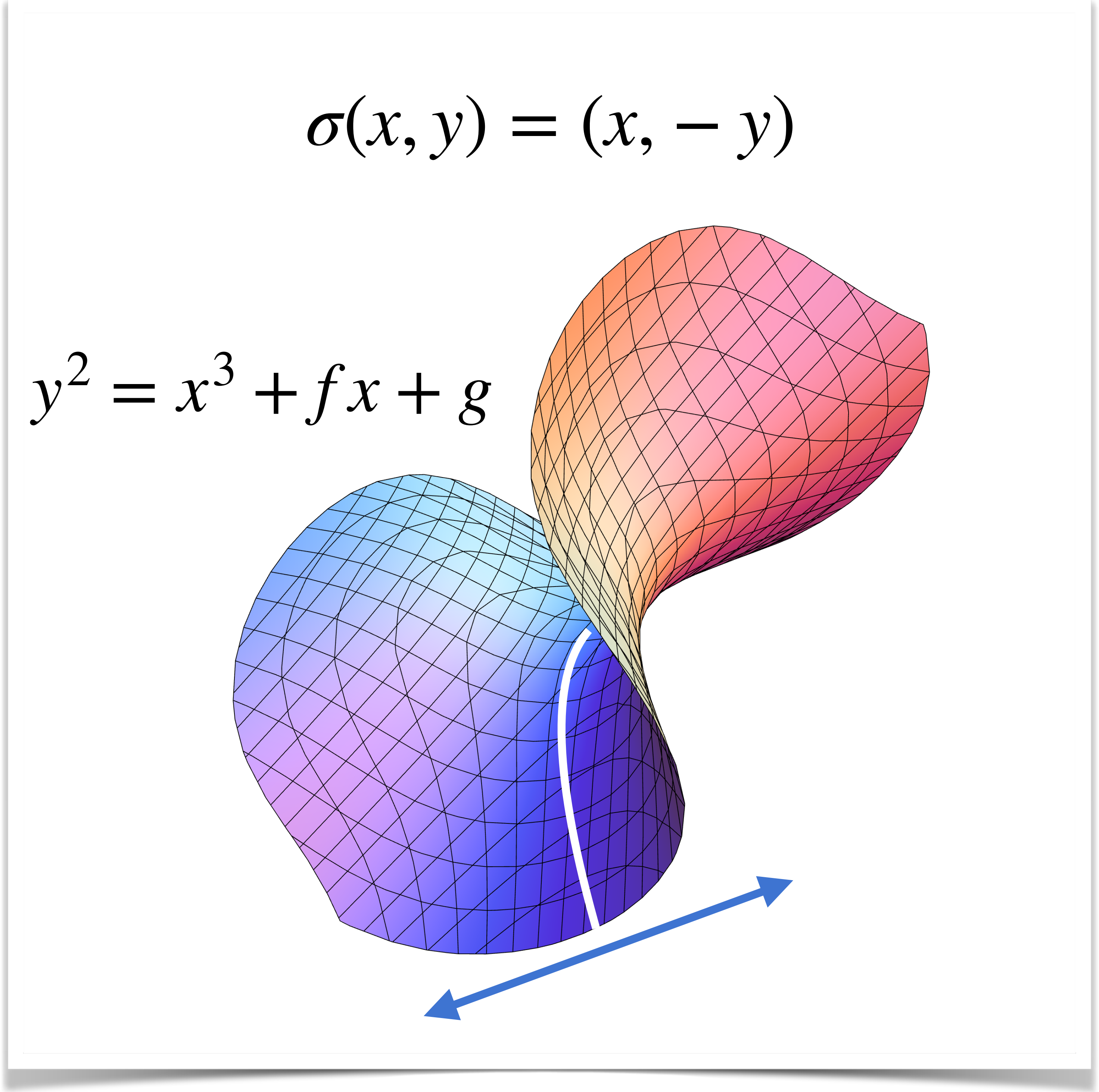

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Orientifolds of Calabi-Yau hypersurfaces in toric fourfolds are classified

JM '23

EFT Data: 4D supergravity

- Real Kähler potential $K(\Phi, \overline{\Phi})$
- Holomorphic superpotential $W(\Phi)$

Φ = set of chiral multiplets

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$$V(\Phi, \overline{\Phi}) = e^K \left((\partial_\Phi \partial_{\overline{\Phi}} K)^{-1} D_\Phi W \, \overline{D_\Phi W} - 3\,|W|^2 \right) \qquad D_\Phi W := \partial_\Phi W + W \partial_\Phi K$$

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Need computational handle on superpotential and Kähler potential!

The Superpotential

classical superpotential, exact up to non-perturbative corrections:

$$W(z,\tau,\phi) = \int_{Y_4} G_4 \wedge \Omega_4(z,\tau,\phi) = \overrightarrow{N} \cdot \overrightarrow{\Pi}(\tau,z,\phi)$$

Gukov, Vafa, Witten '99

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For all D7's on top of O7s, "strict Sen-limit":

$$W(z, \tau, \phi = 0) = \int_{X_3} (F_3 - \tau H_3) \wedge \Omega_3(z)$$

Giddings, Kachru, Polchinski '01

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To compute classical superpotential beyond strict Sen-limit, need to construct F-theory uplifts!

F-THEORY UPLIFTS

Thus, to compute moduli potential, need algorithm to move between

"F-theory uplift" →

Calabi-Yau threefold orientifolds ⇄ Torus fibered Calabi-Yau fourfolds

← "Sen limit"

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4d reflexive polytopes + Orientifold symmetry ⟷ 6d reflexive polytopes + degrees of two polynomials

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At least for subset, mirror symmetry is well-understood on both sides.
We can start investigating D7-brane moduli potentials in vast ensemble of models!

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In general, complex structure moduli of CY hypersurface $X_3 := \{f = 0\} \subset V_4$ need to be tuned to make it (projectively) invariant under orientifold involution

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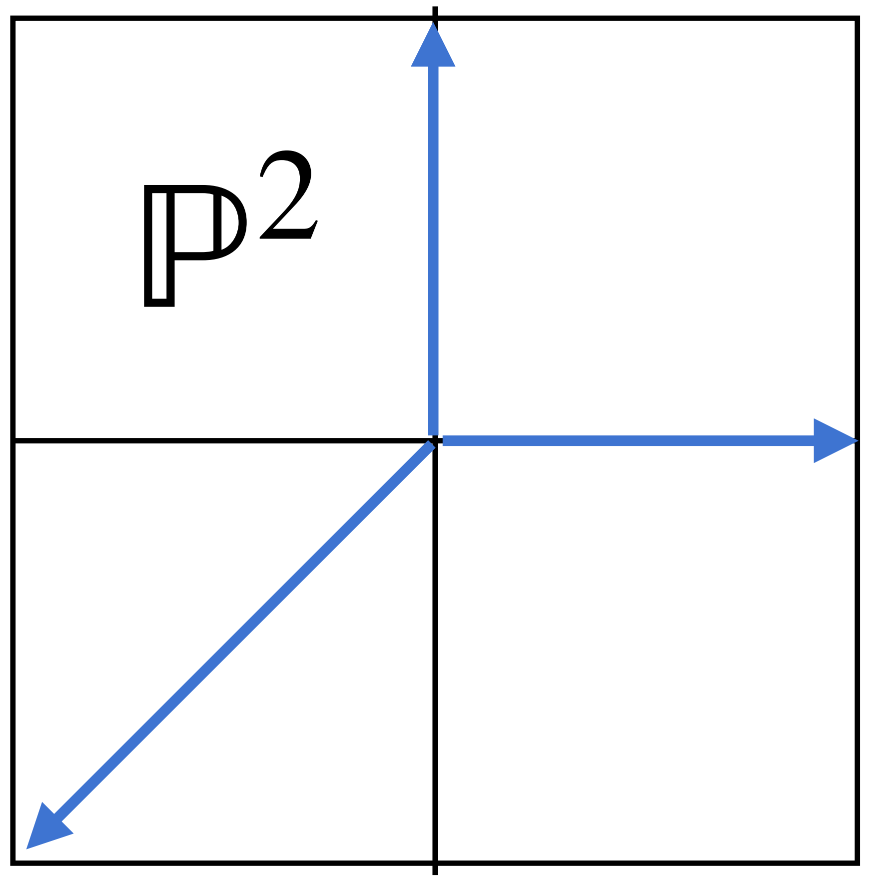

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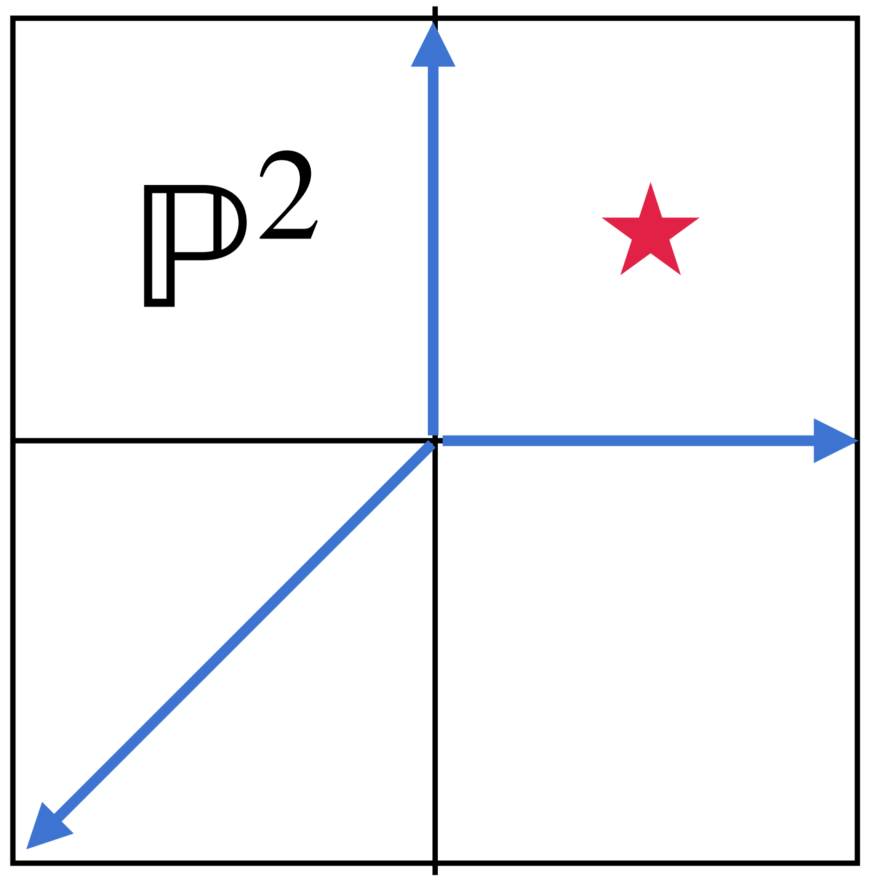

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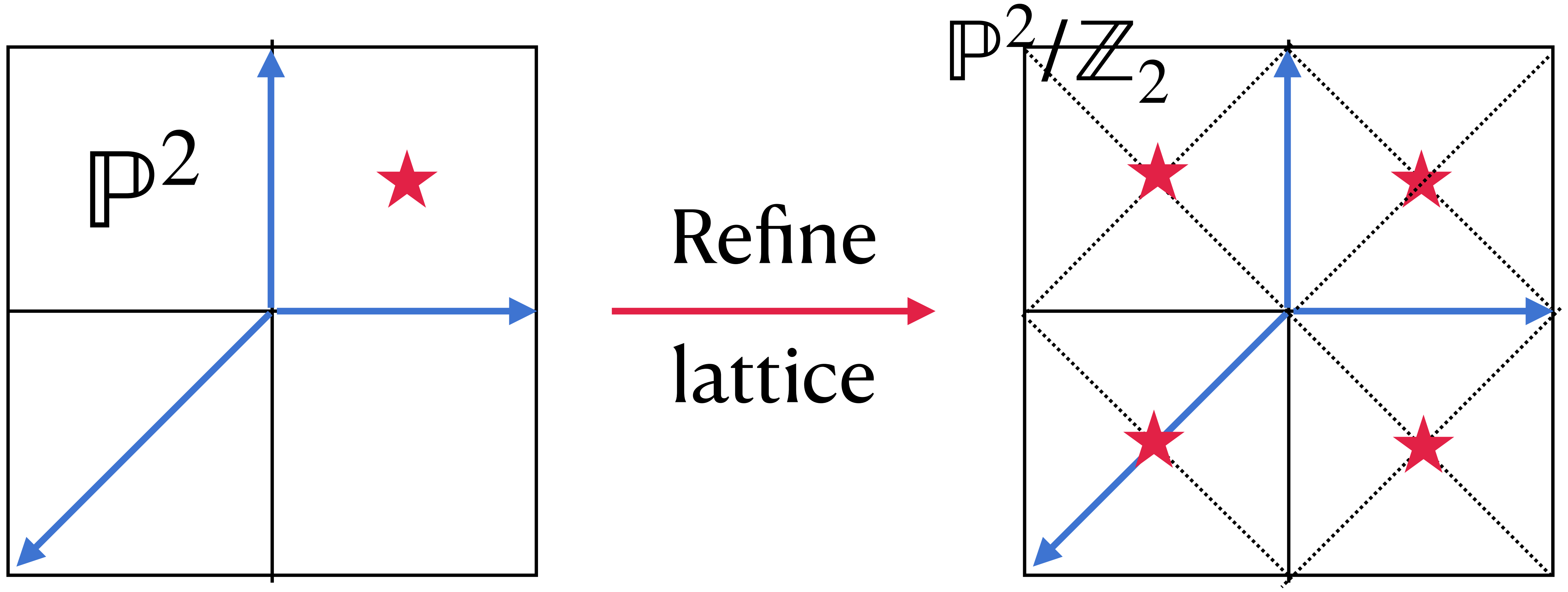

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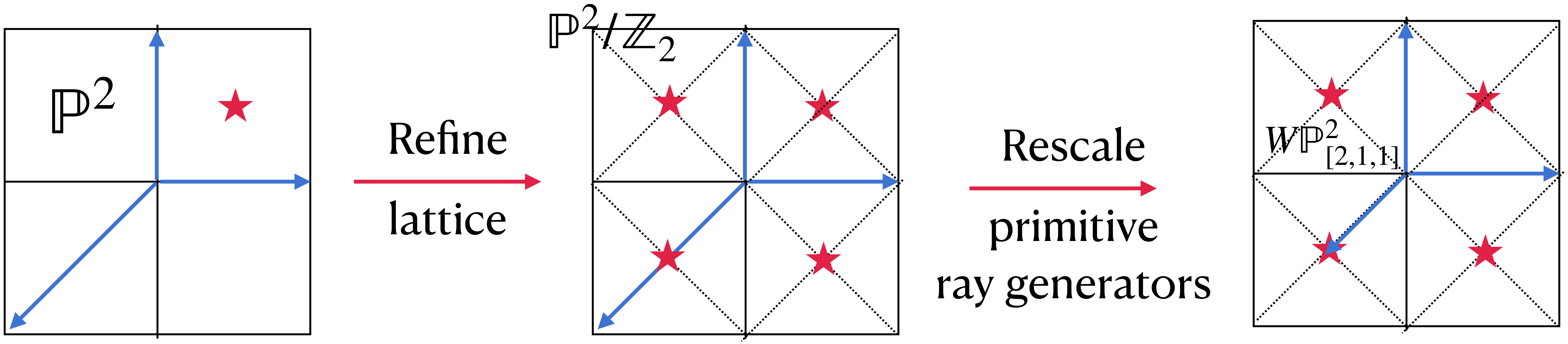
as a general hypersurface $f_B = 0$ in orbifold fourfold $\widetilde{V}_4 := V_4/\mathbb{Z}_2$

$\widetilde{V}_4$ is also a toric fourfold, so we can continue to harness the power of toric geometry!

Toric fan of $\widetilde{V}_4$ obtained by refining underlying lattice by half-integral points:



Outline

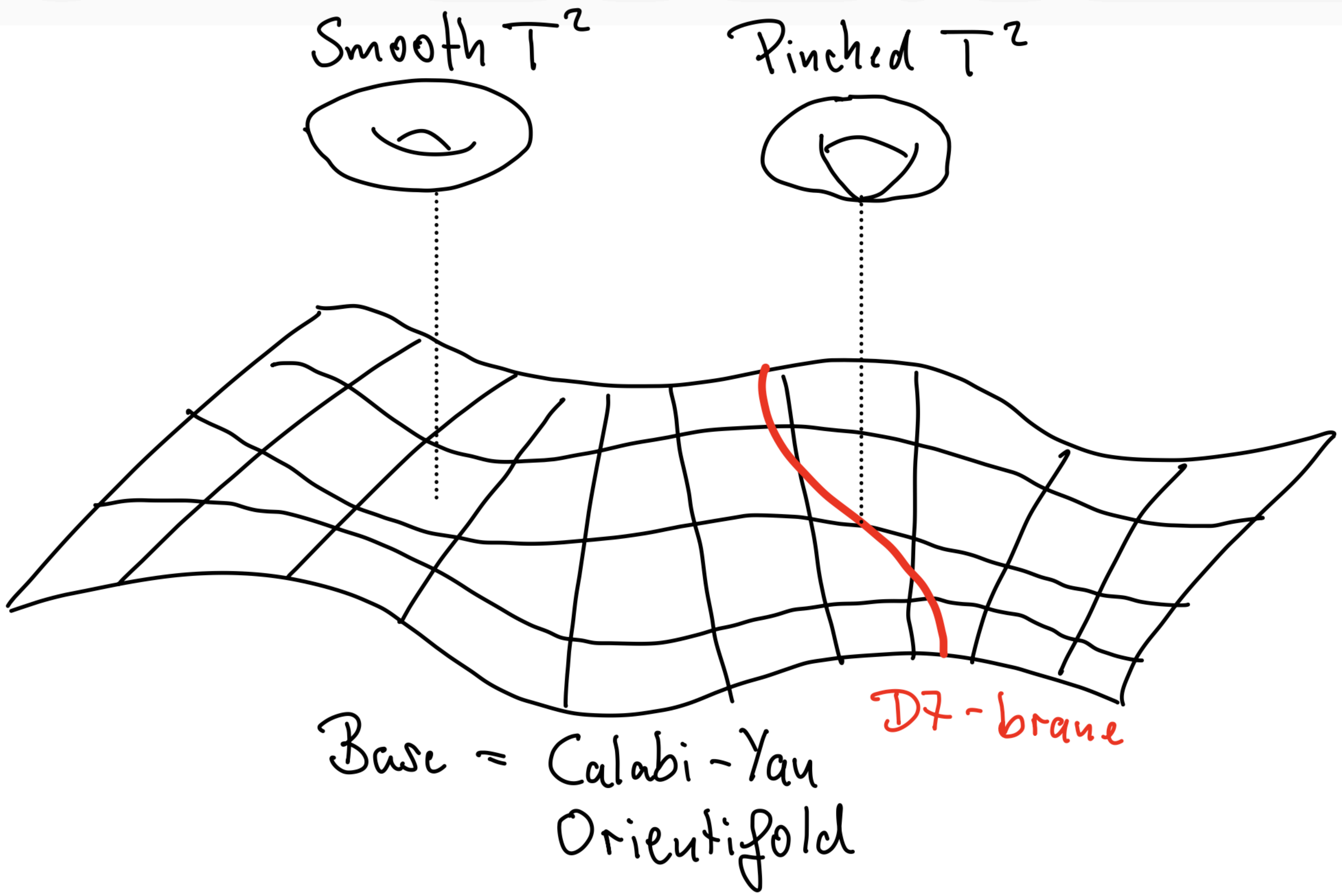
1. Motivation: String Landscape and Flux superpotentials
2. Systematics of O3/O7 orientifolds of Calabi-Yau hypersurfaces
3. "Combinatorial" F-theory uplifts
4. Conclusions

Step II: F-Theory Uplifts

Having constructed the toric fan for $\widetilde{V}_4 := V_4/\mathbb{Z}_2$, and the divisor $B := \{f_B = 0\}$ next task is to construct the F-theory uplift.

Fourfold Y_4 is a of torus-fibration over our six-dimensional Calabi-Yau orientifold:

Vafa '96



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$$W\mathbb{P}^2_{[2,3,1]} \to V_6 \to \widetilde{V}_4$$

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Toric fan of V_6 is uniquely determined by toric fan of $\widetilde{V}_4$ and Calabi-Yau property of Weierstrass model

$$y^2 = x^3 + fxz^4 + gz^6$$

$f \in \Gamma(4\overline{K}_B)$

$g \in \Gamma(6\overline{K}_B)$

Smooth F-theory Uplifts

Key technical obstacle: Defining a Calabi-Yau orientifold requires tuning complex structure moduli: $h^{2,1}_{-} \leq h^{2,1}$...

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Example: "frozen" conifold singularities:

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Solution: after discarding models with the most severe singularities (e.g., reducible varieties; easy to detect)
remaining models can be smoothed with appropriate birational morphism.

(small resolution in conifold case)

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Computational Algorithm: (in the spirit of Batyrev '93)

1. Construct "Minkowski sum" Δ_M of Newton polytopes associated to $D_B, D_{6\overline{K}_B}$
2. Define singular toric fan for $\widetilde{V}_4$ as normal fan of Δ_M.
3. "Maximally" refine toric fan: add blow-ups subject to $(\text{ord}(f), \text{ord}(g)) \leq (2,3)$ and s.t. no sections of $6\overline{K}_B$ are removed

This systematically resolves singularities like conifolds that we saw before.

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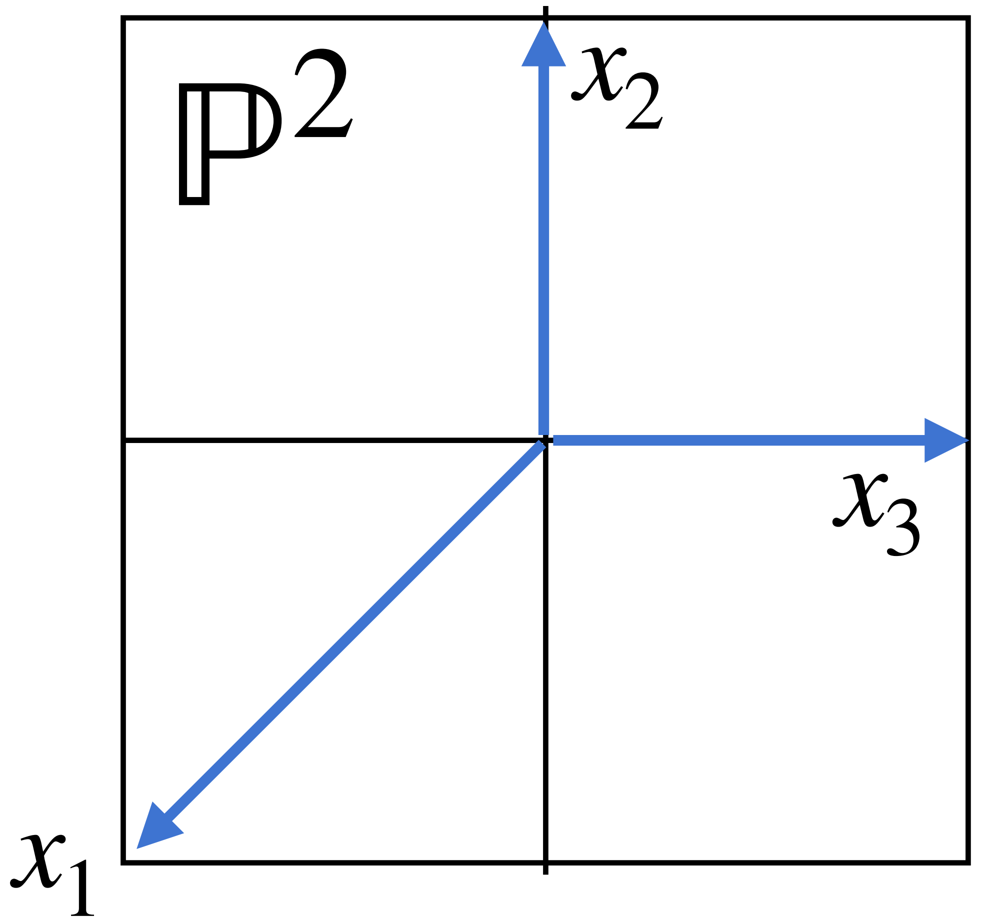

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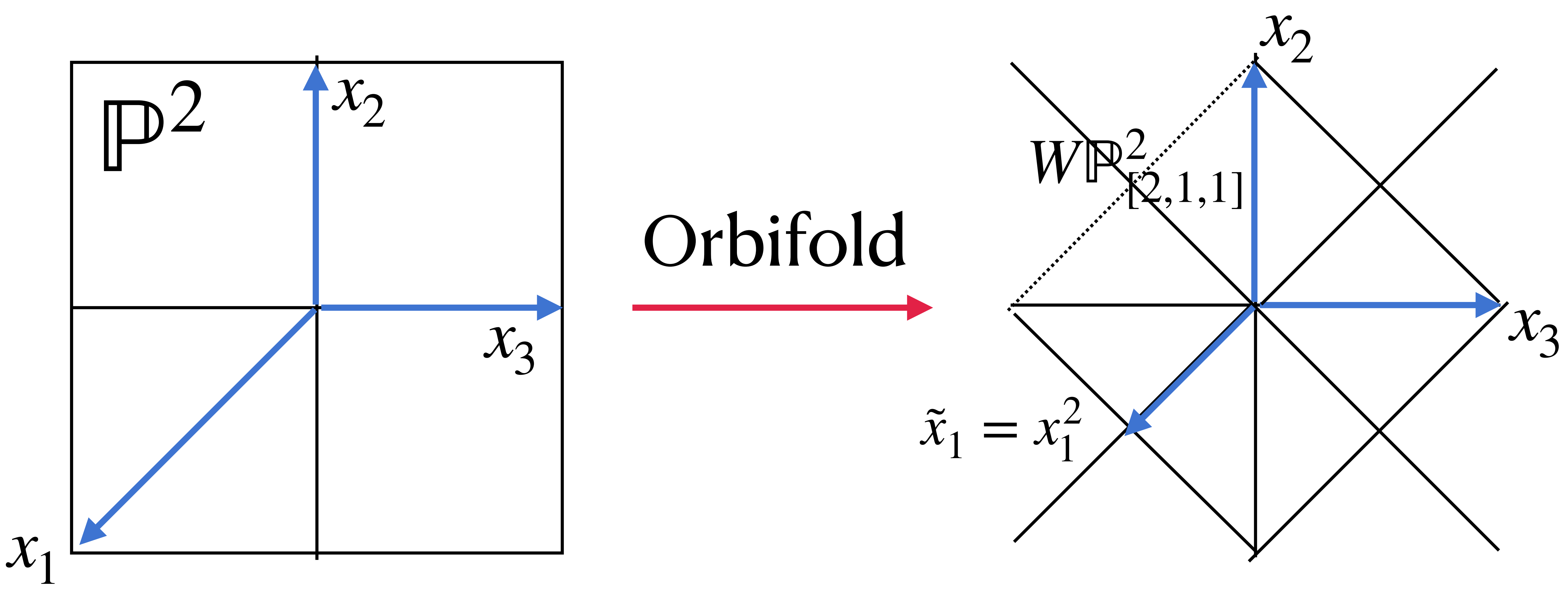

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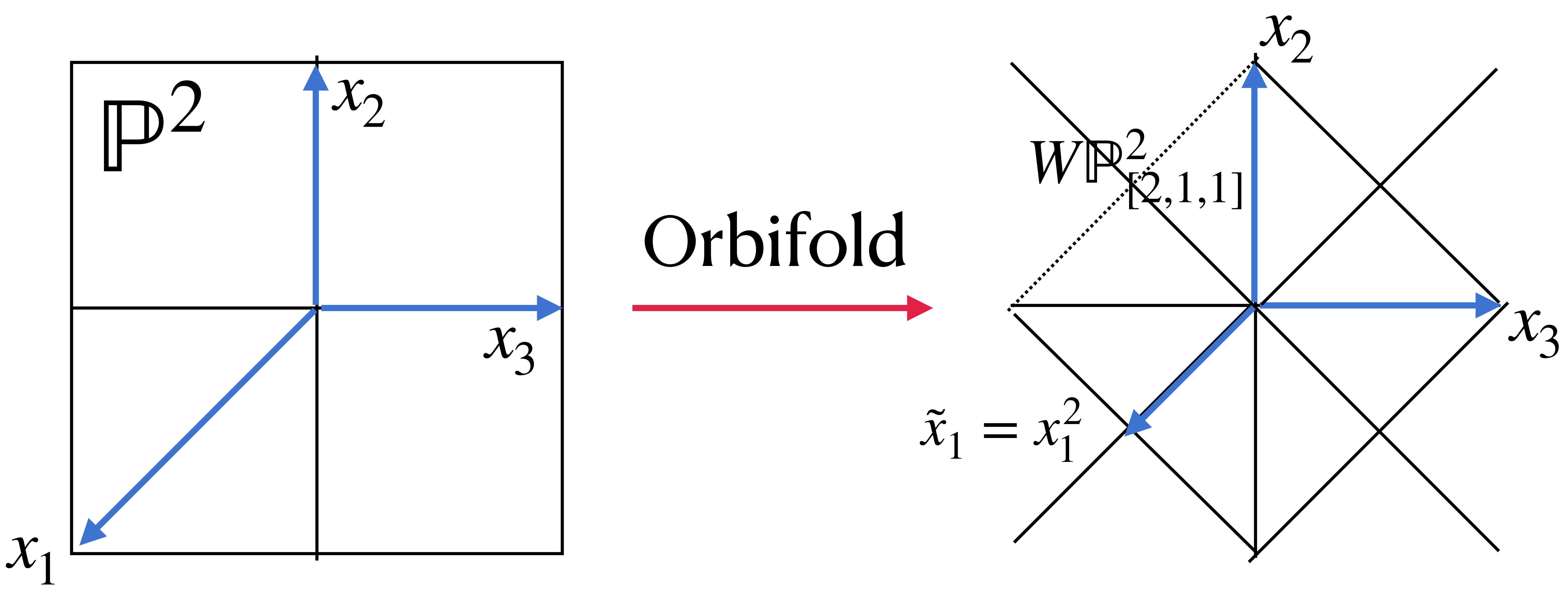

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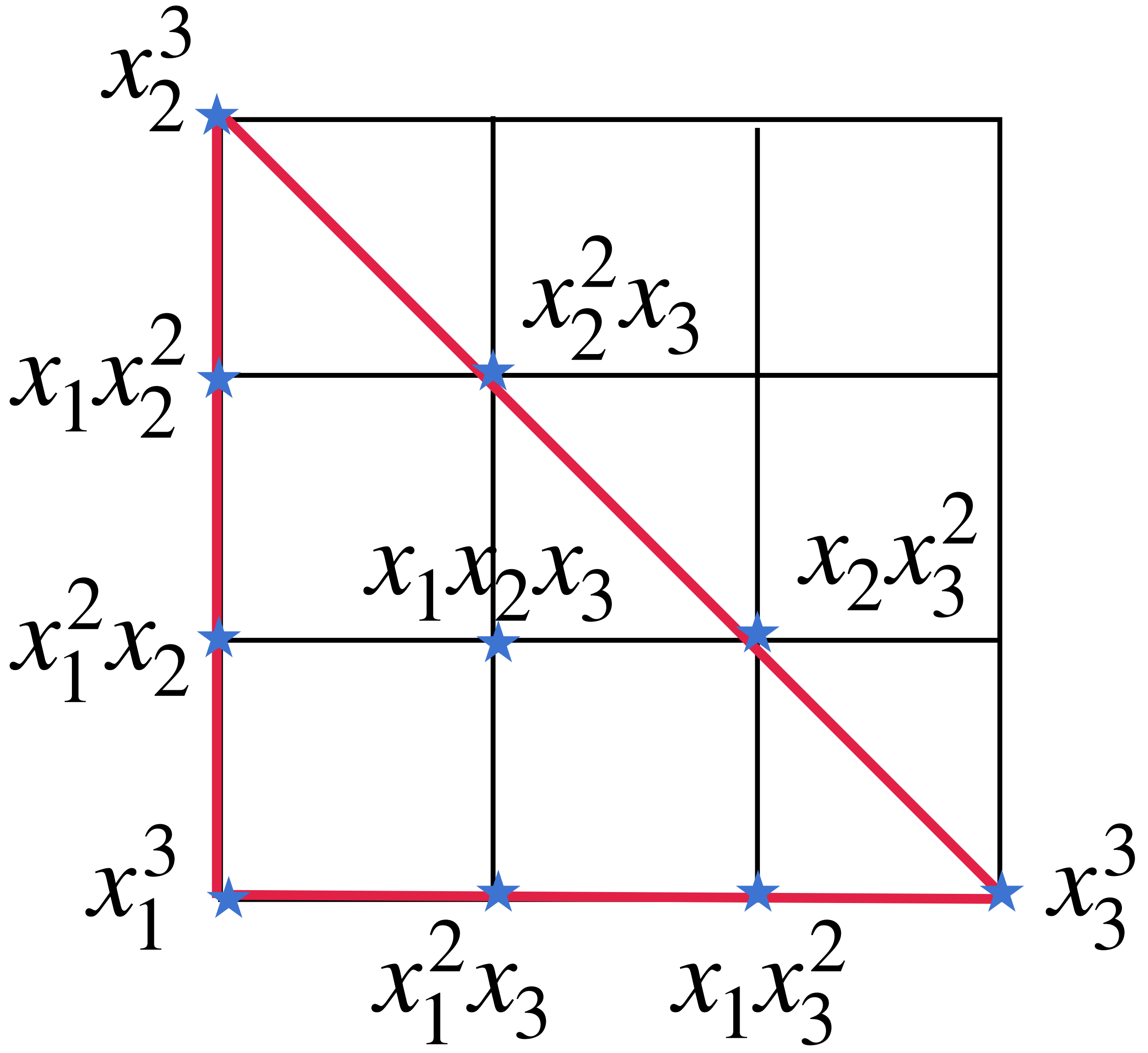

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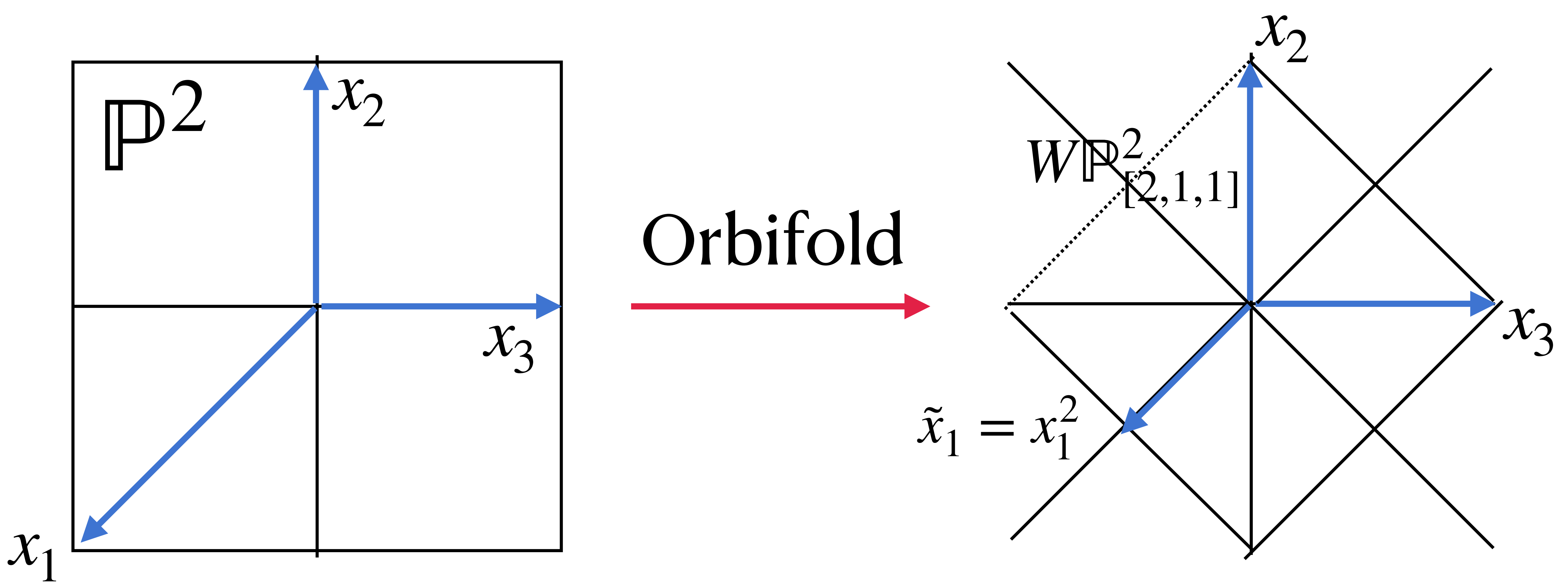

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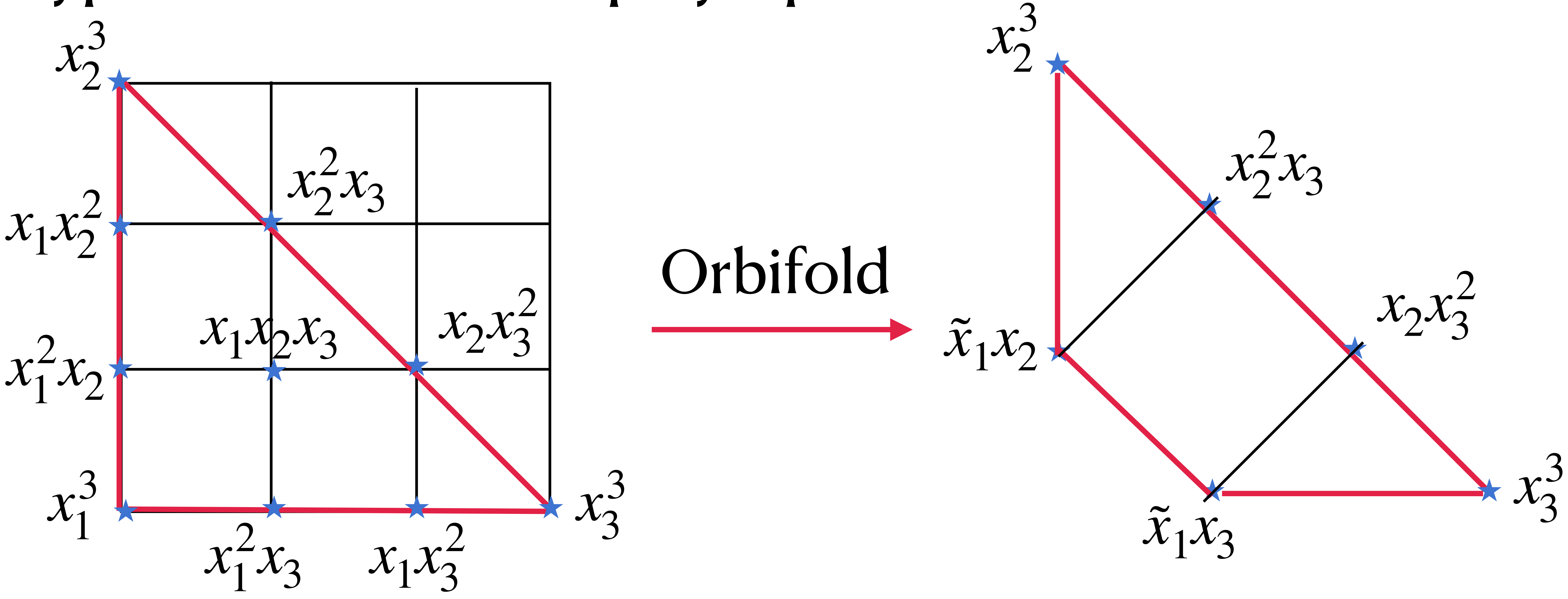

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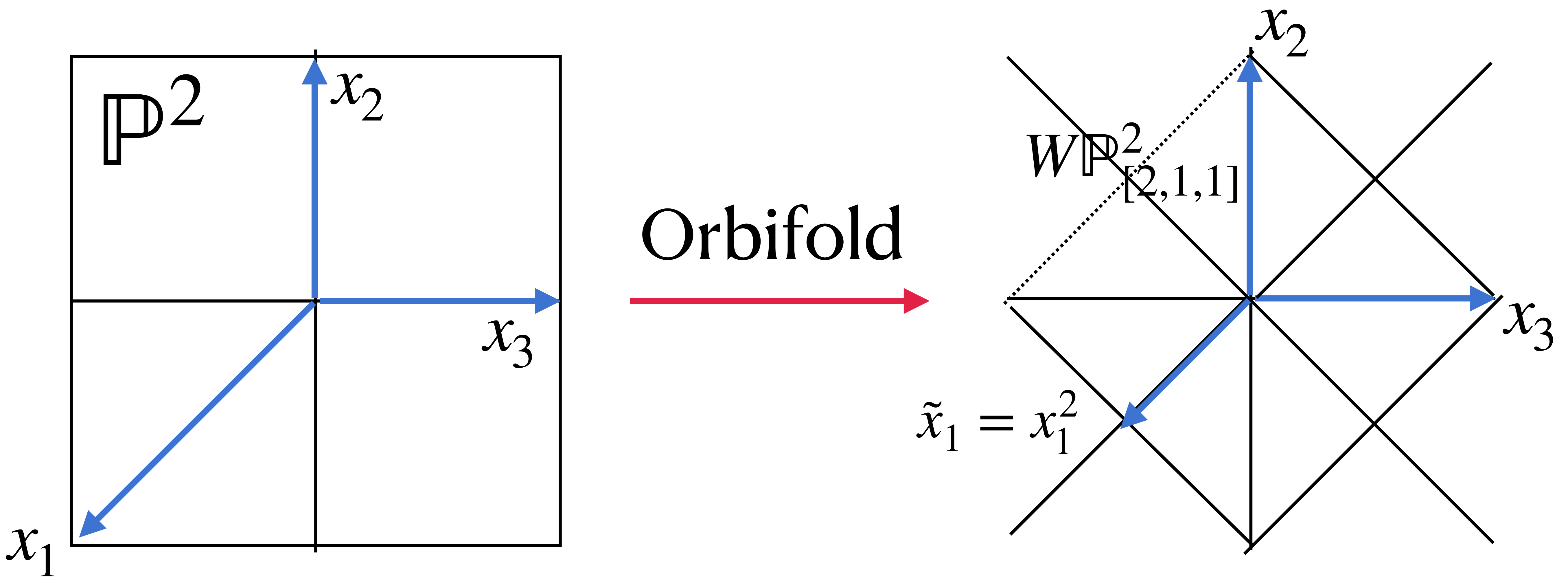

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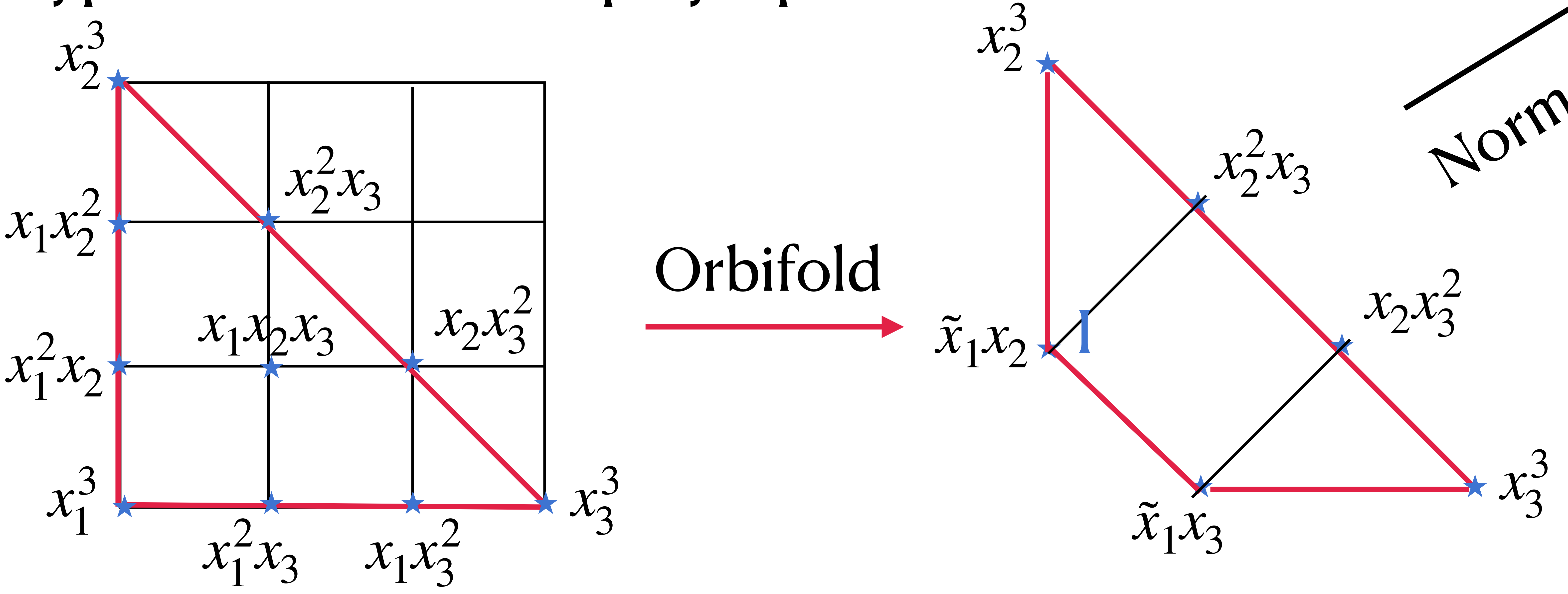

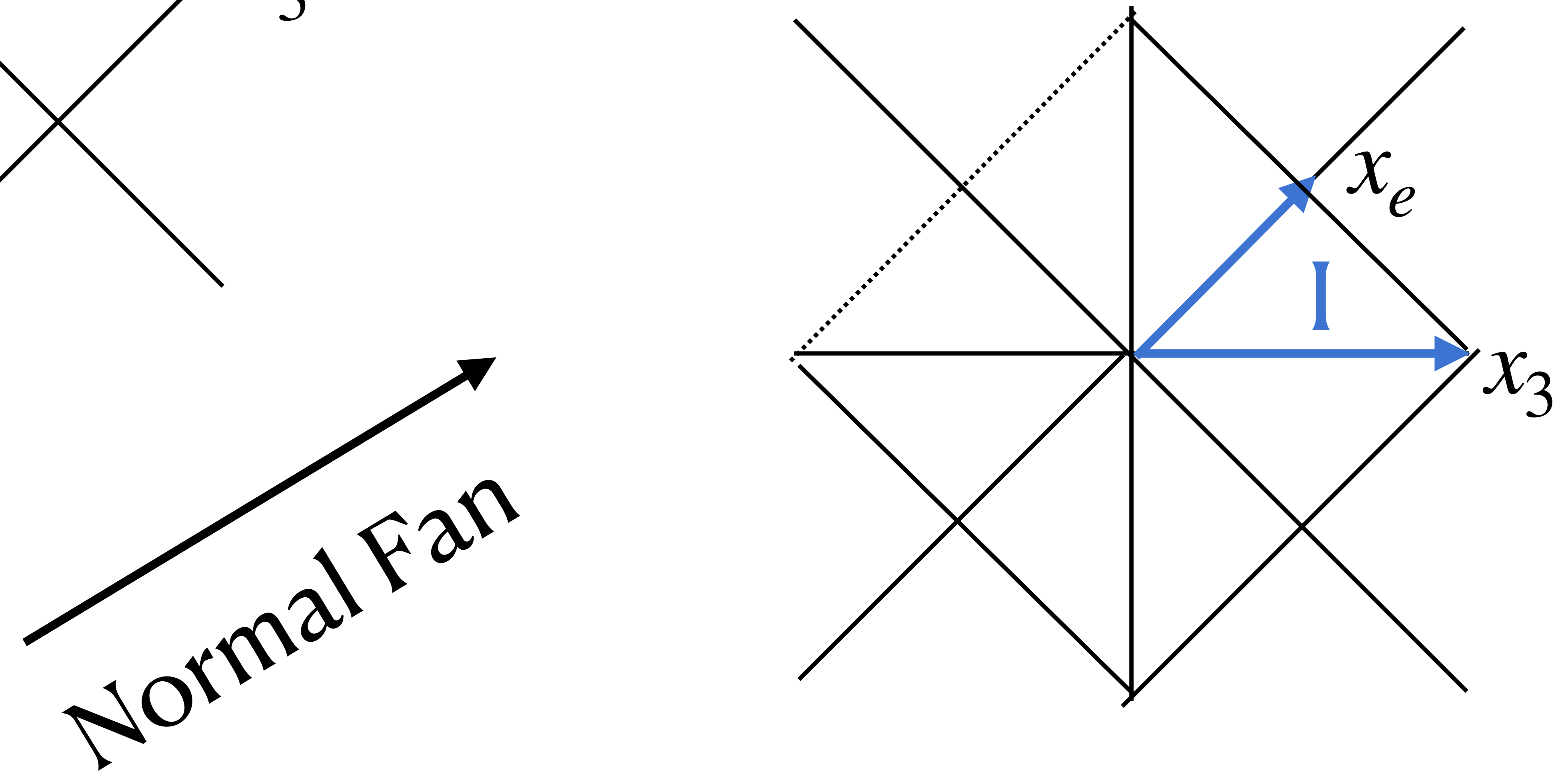

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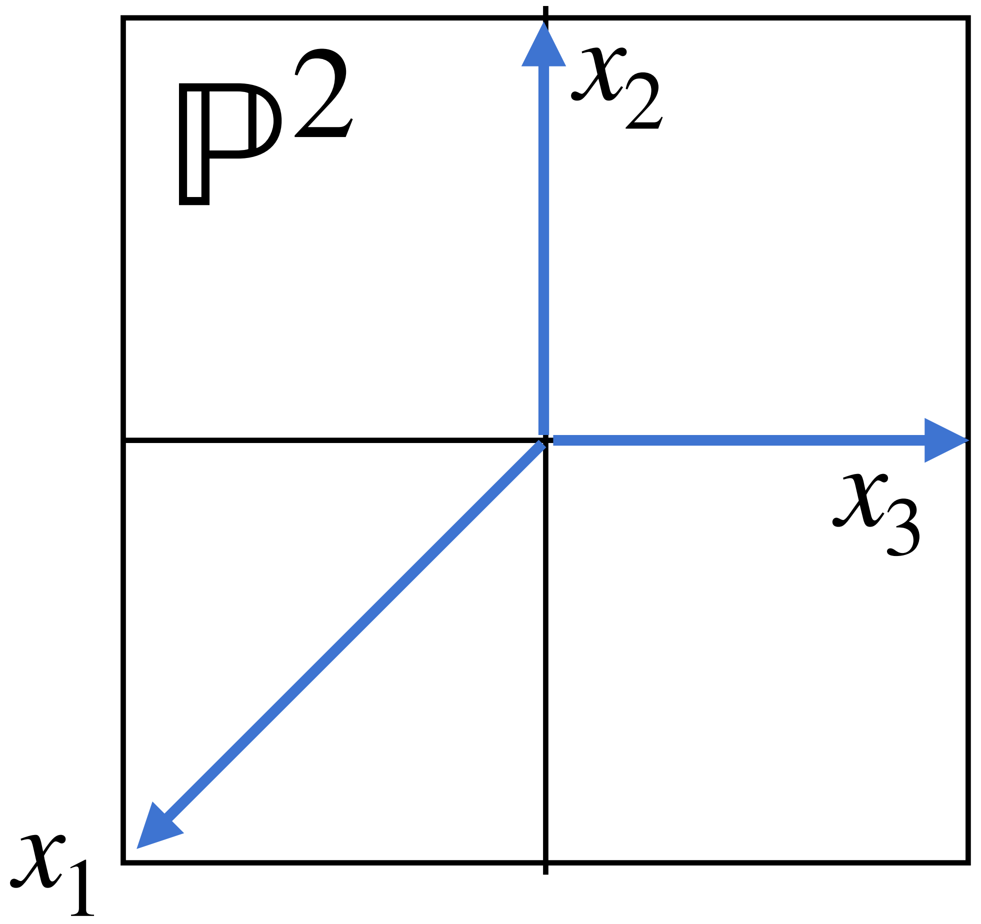

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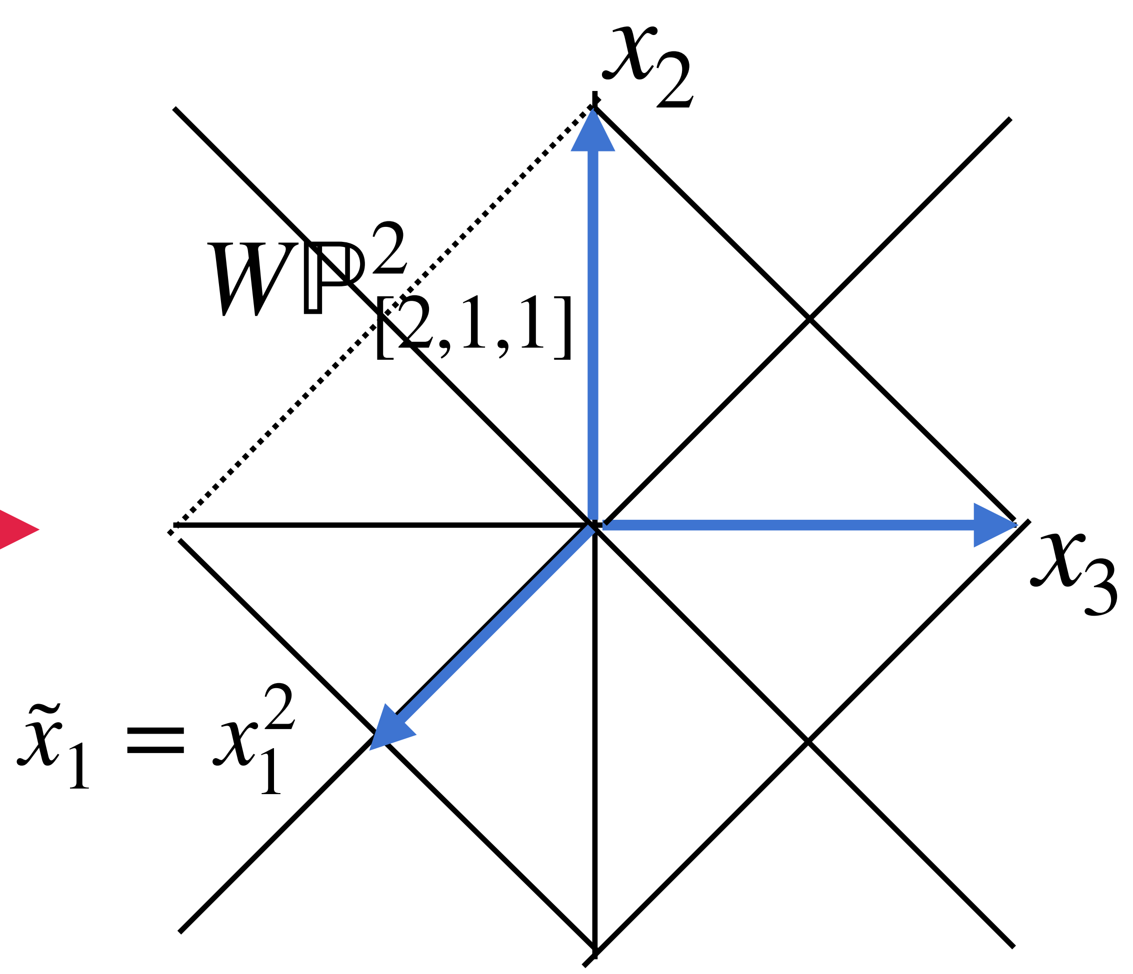

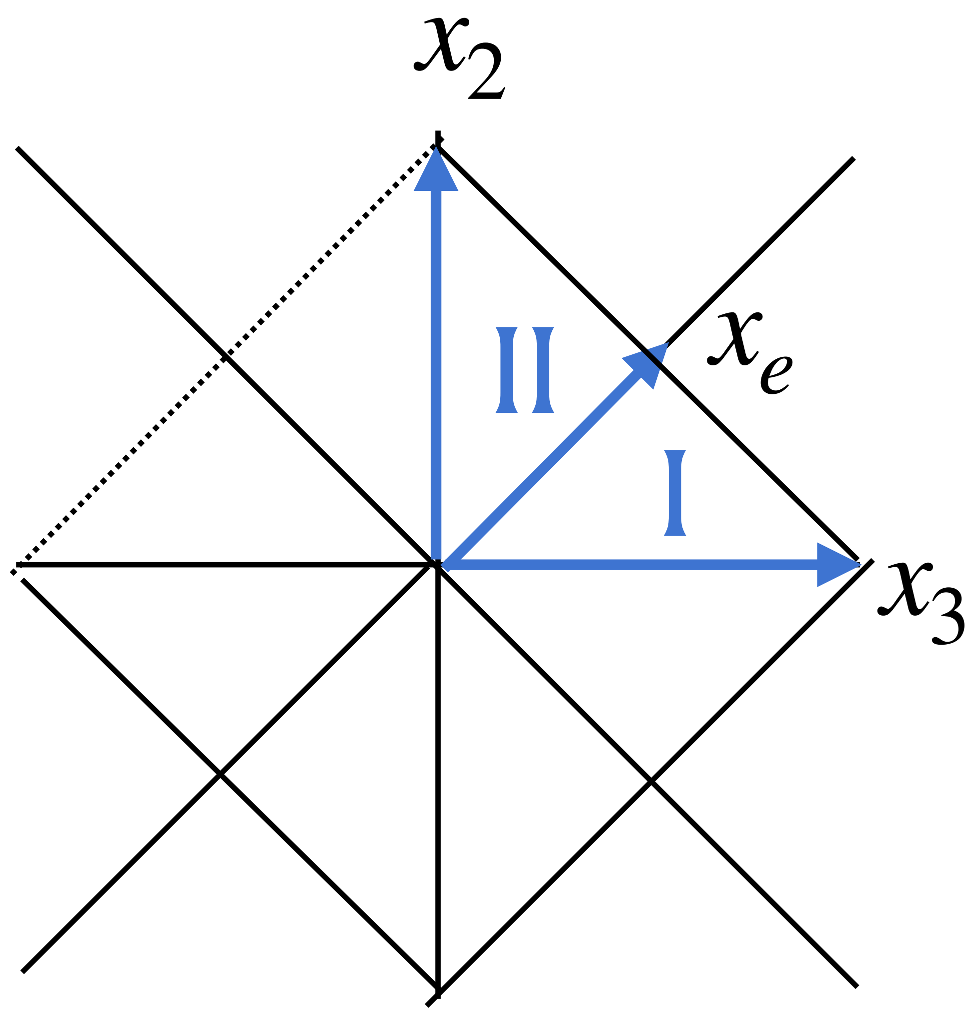

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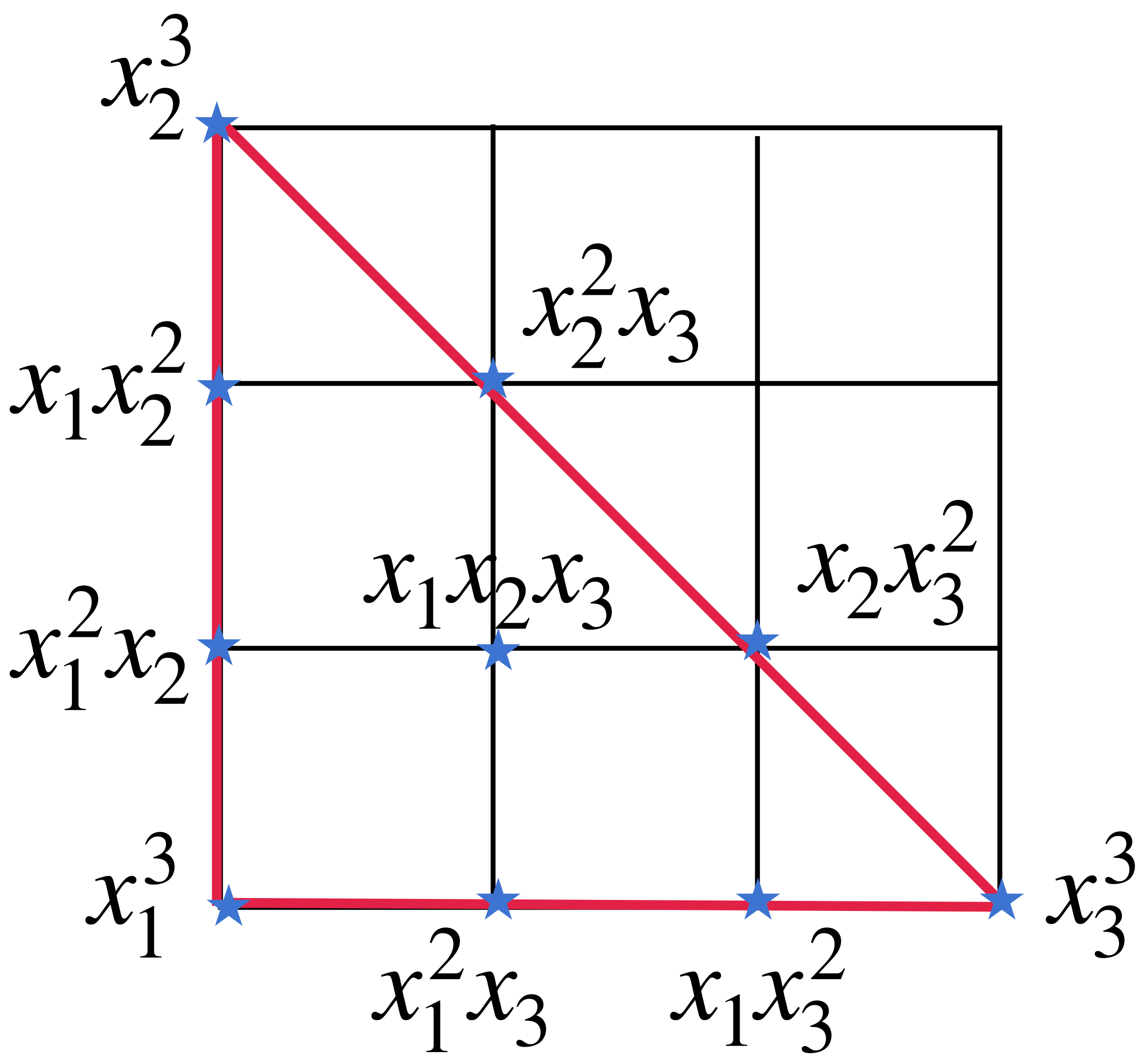

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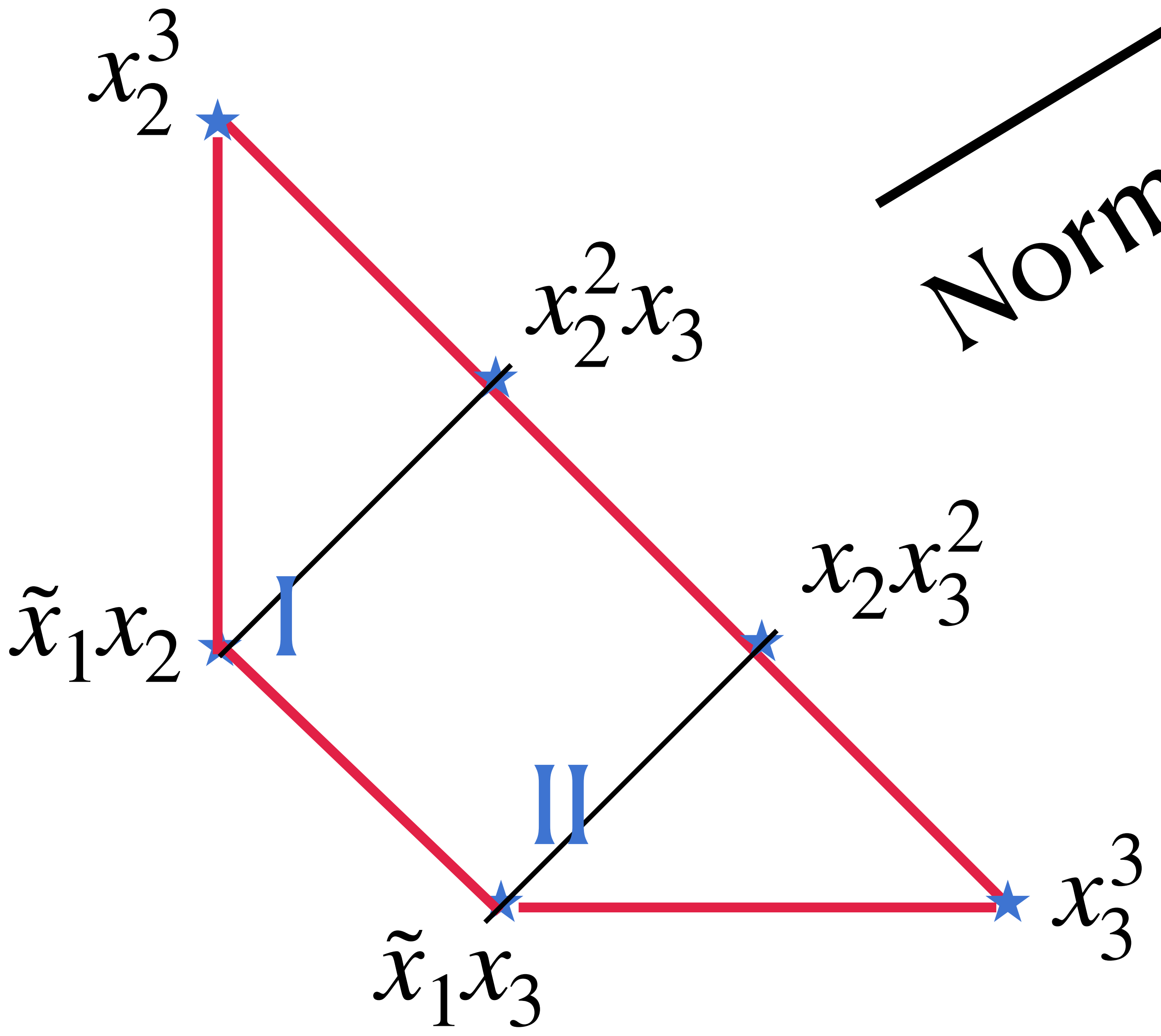

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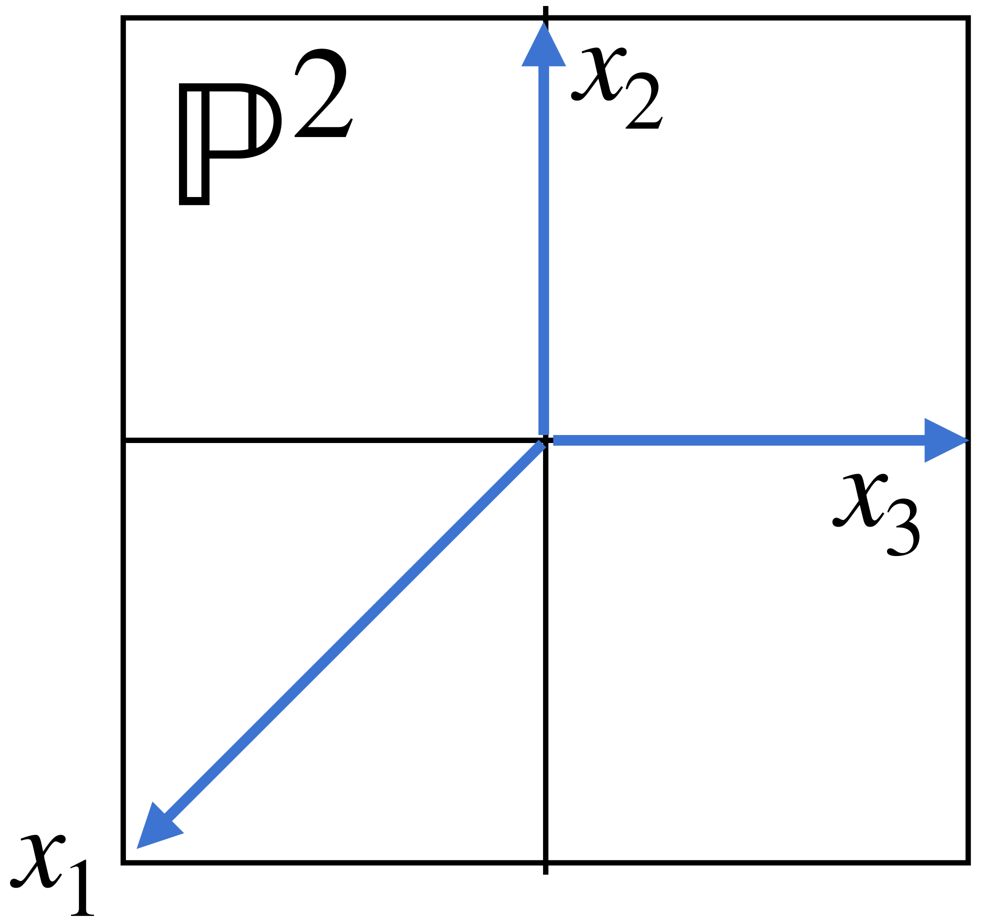

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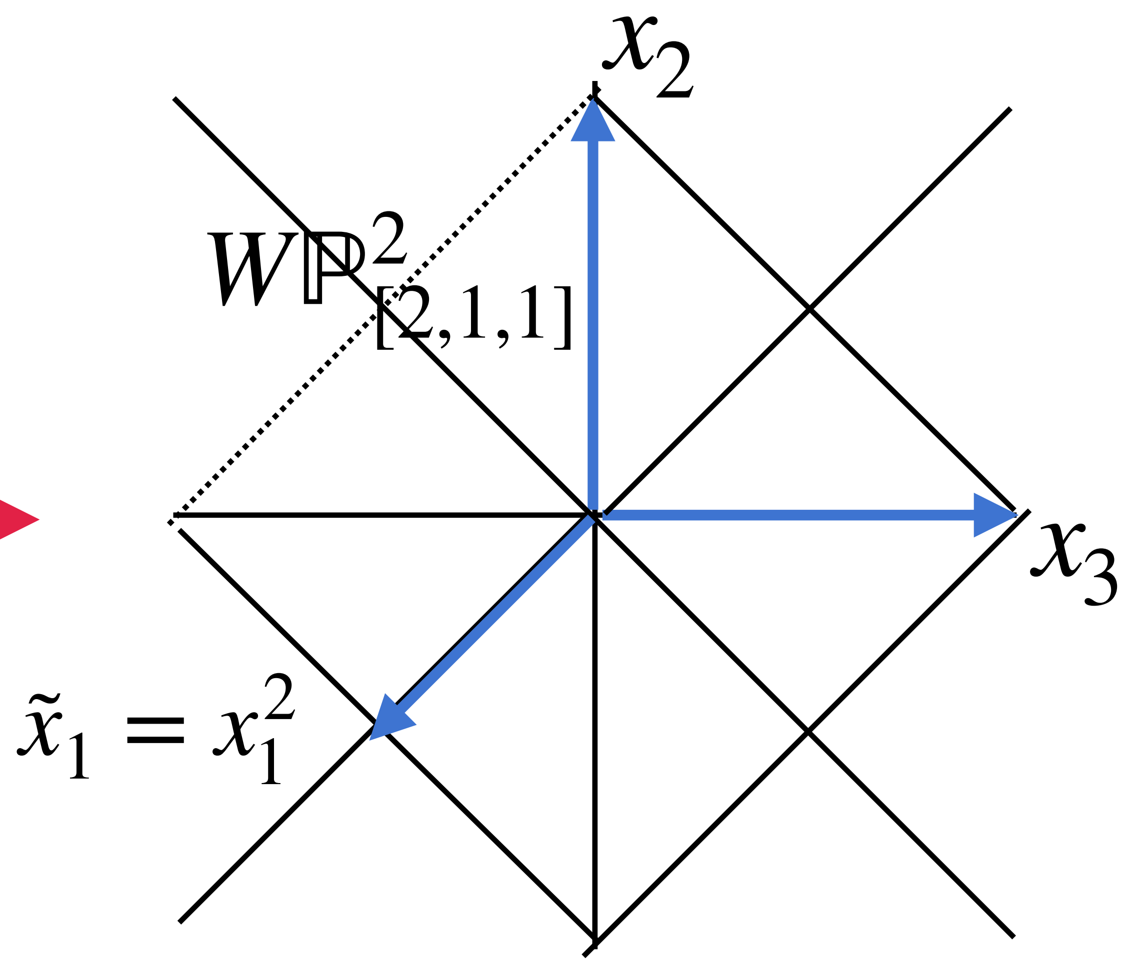

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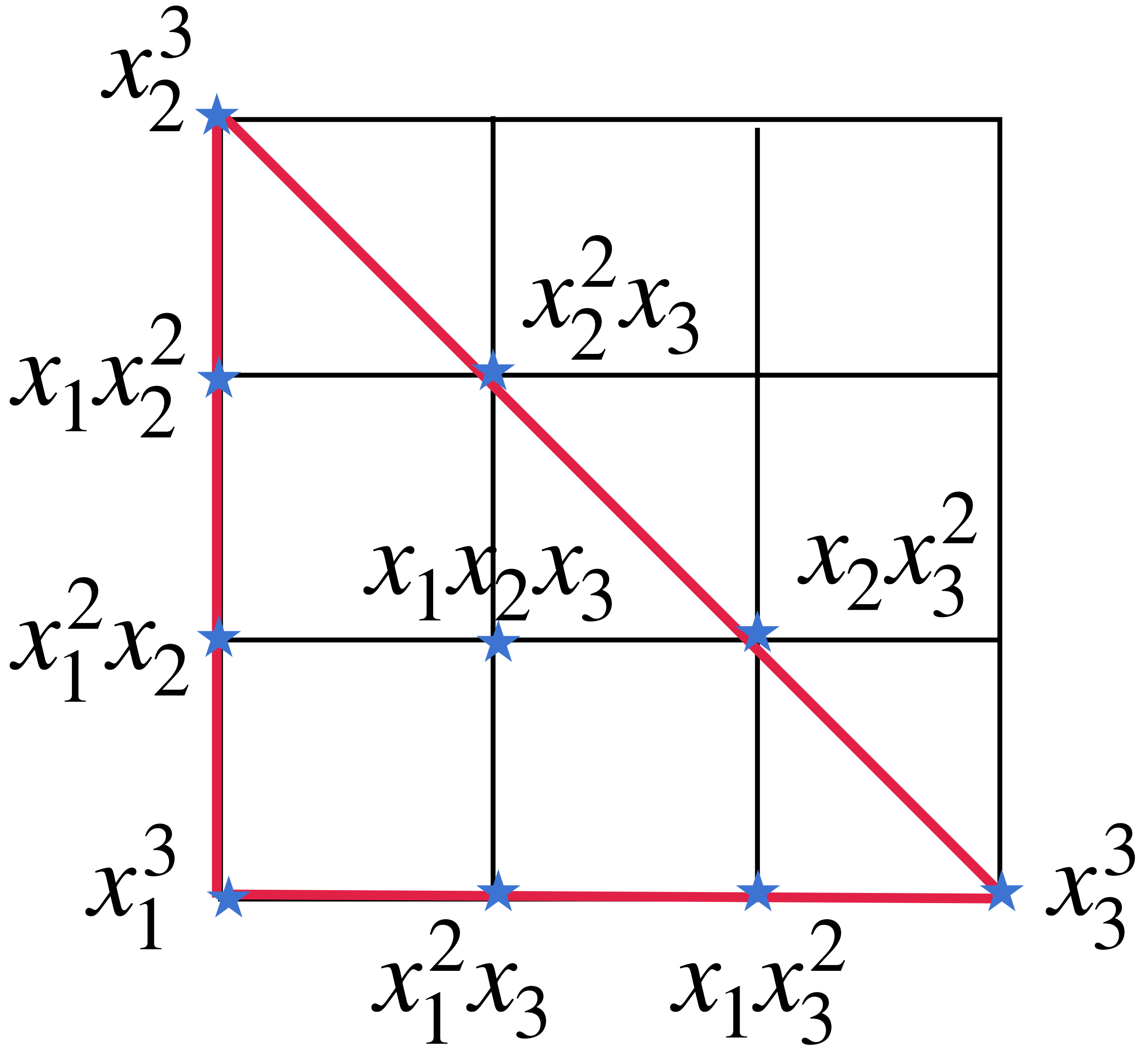

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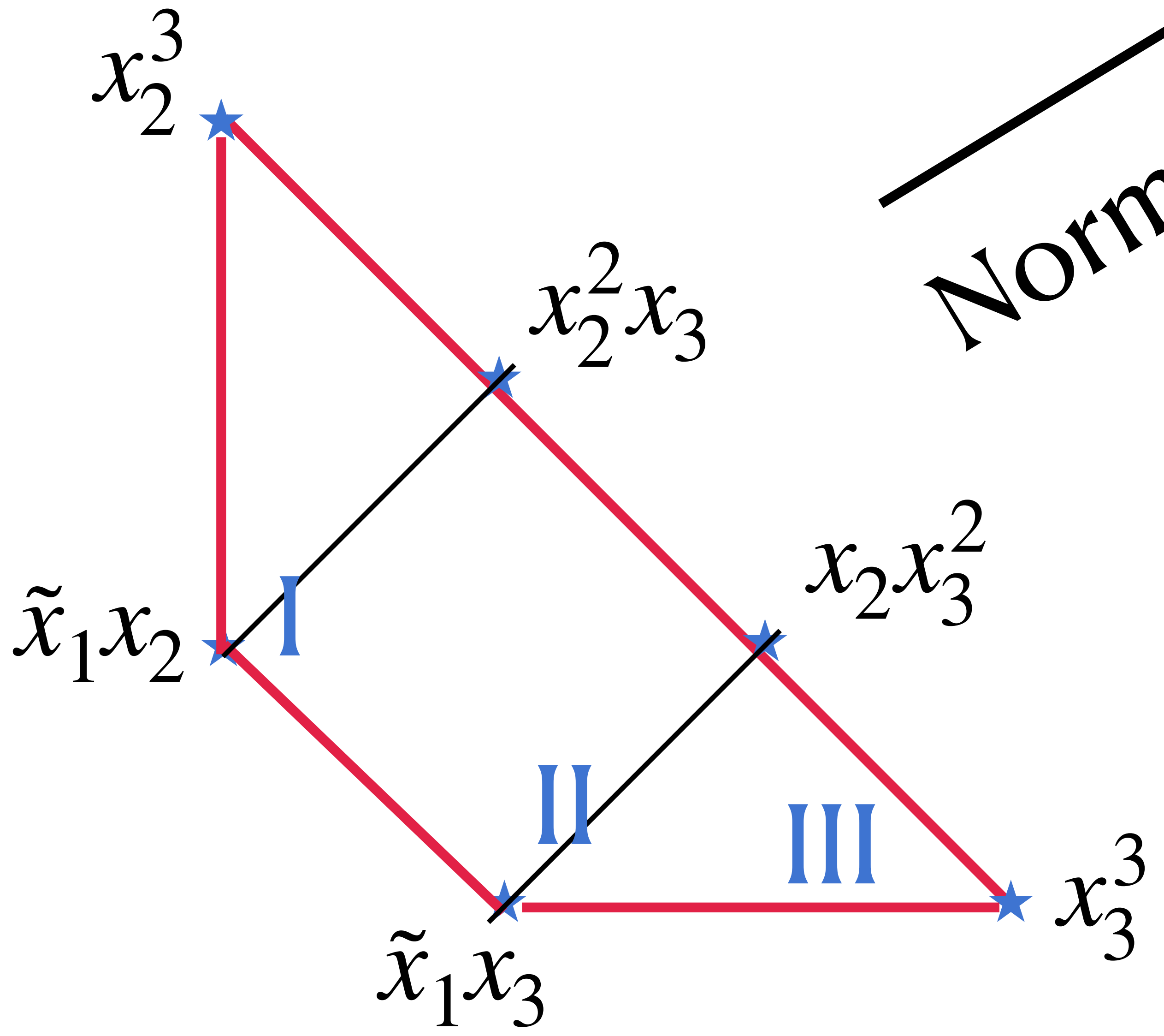

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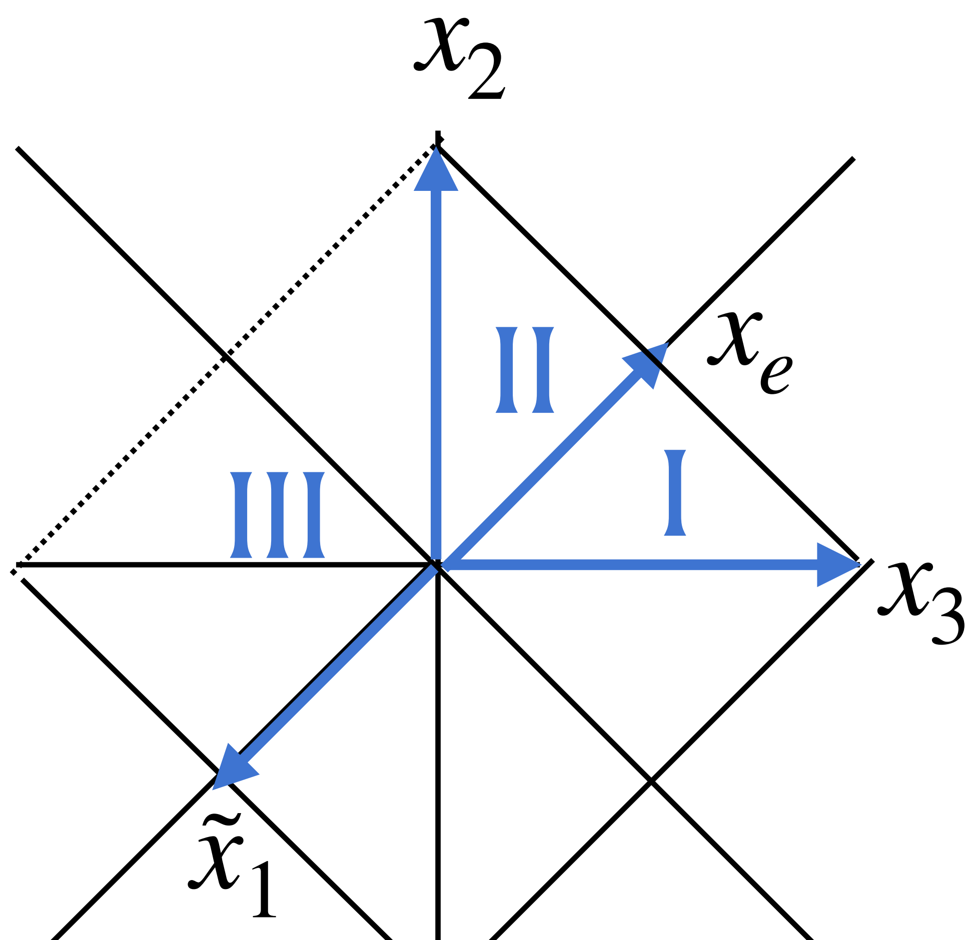

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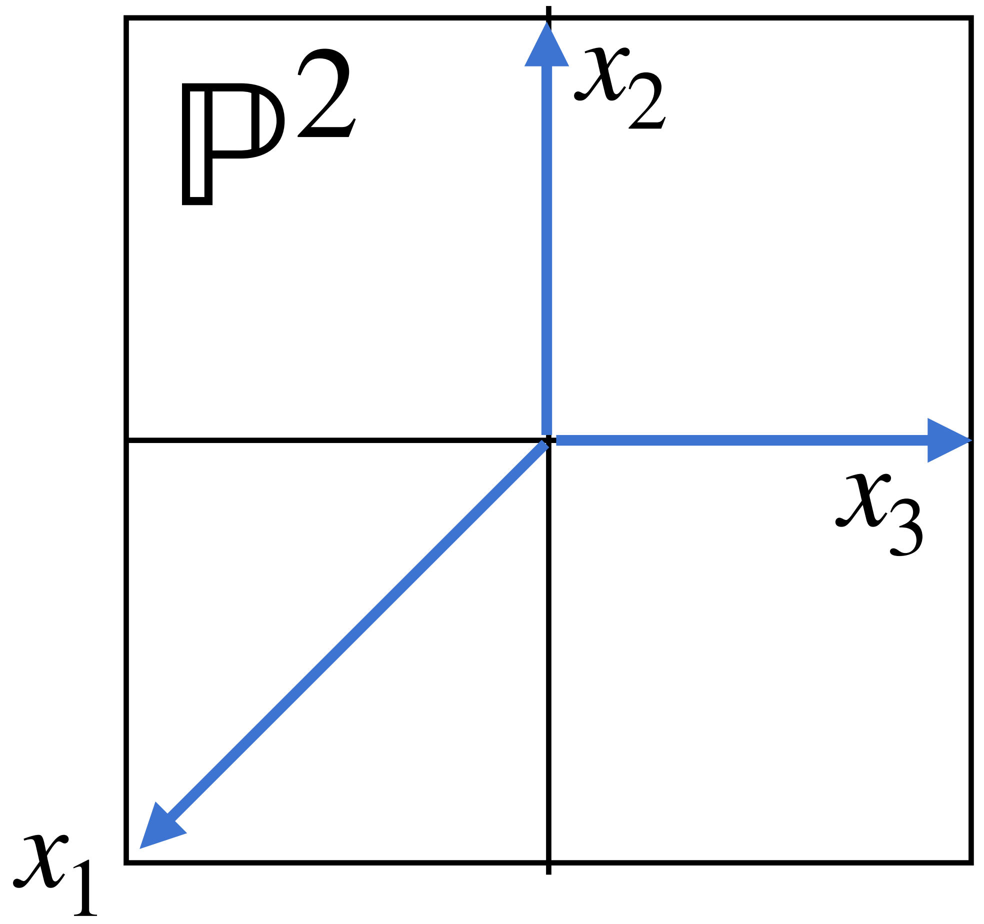

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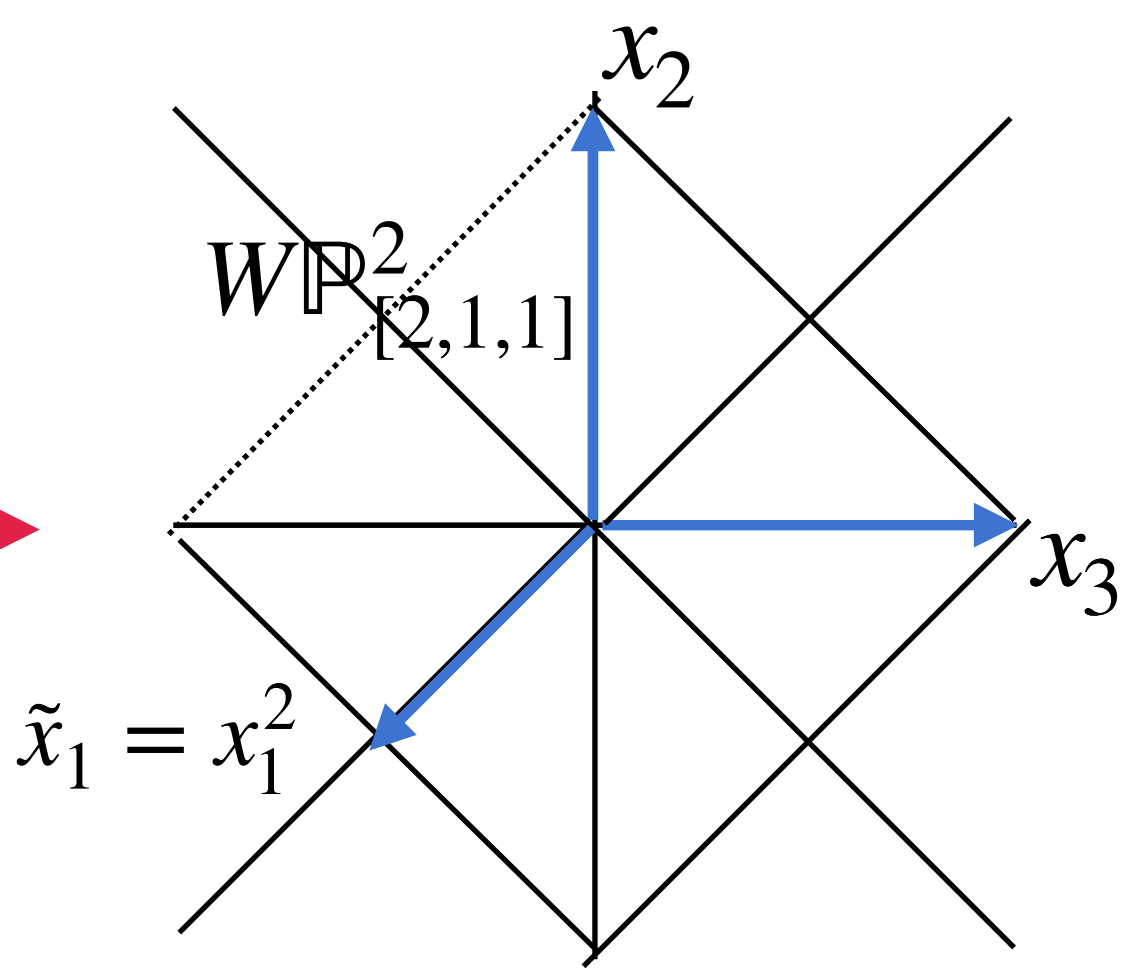

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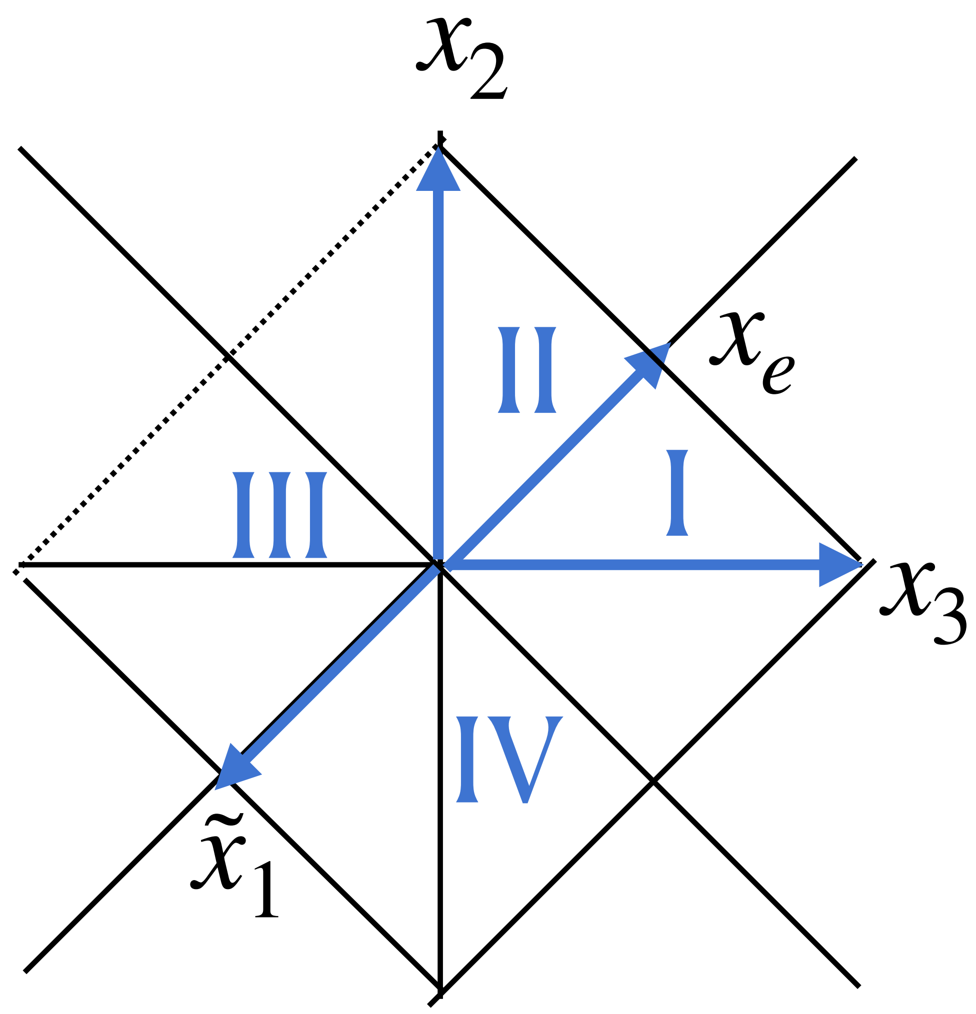

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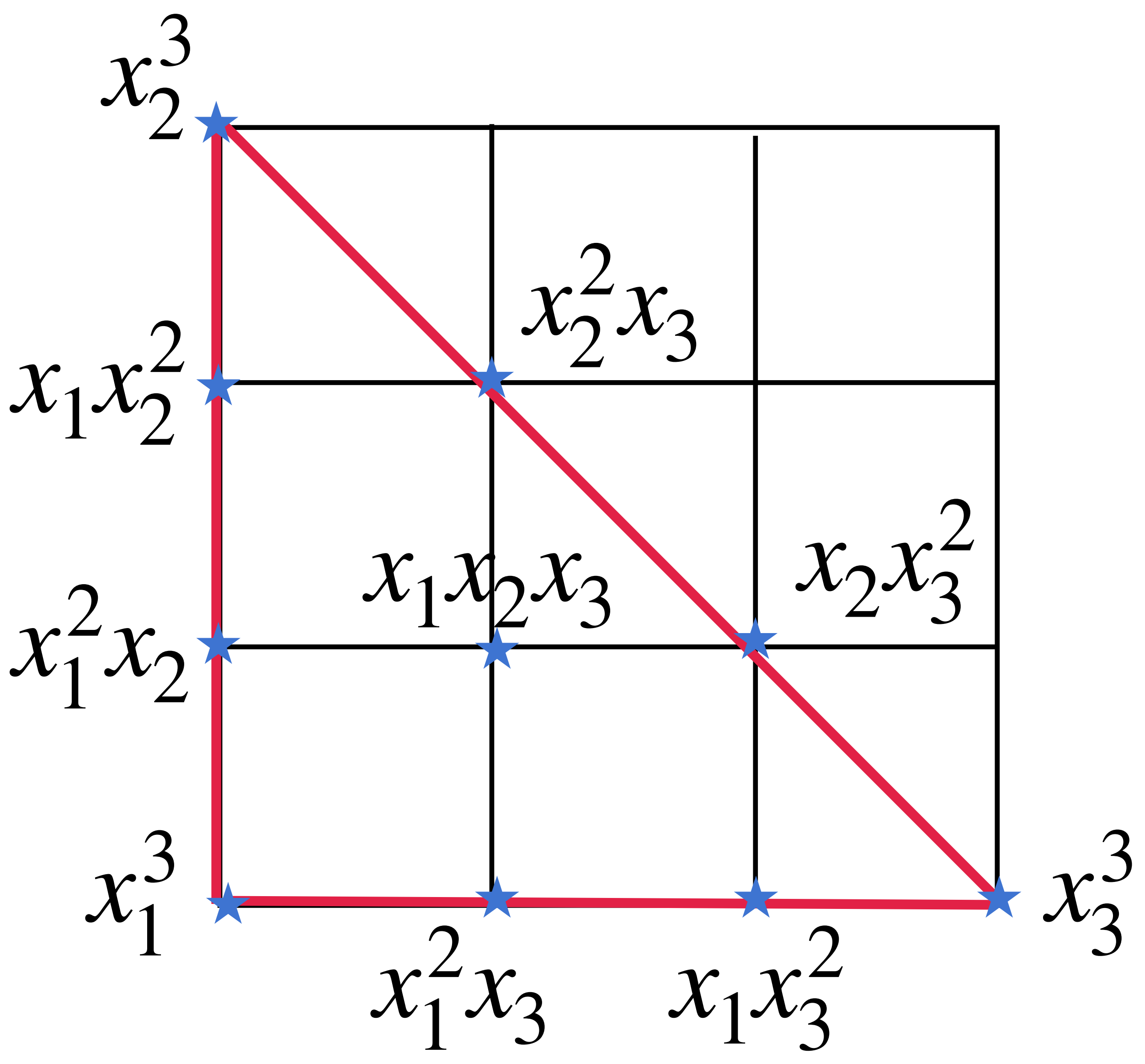

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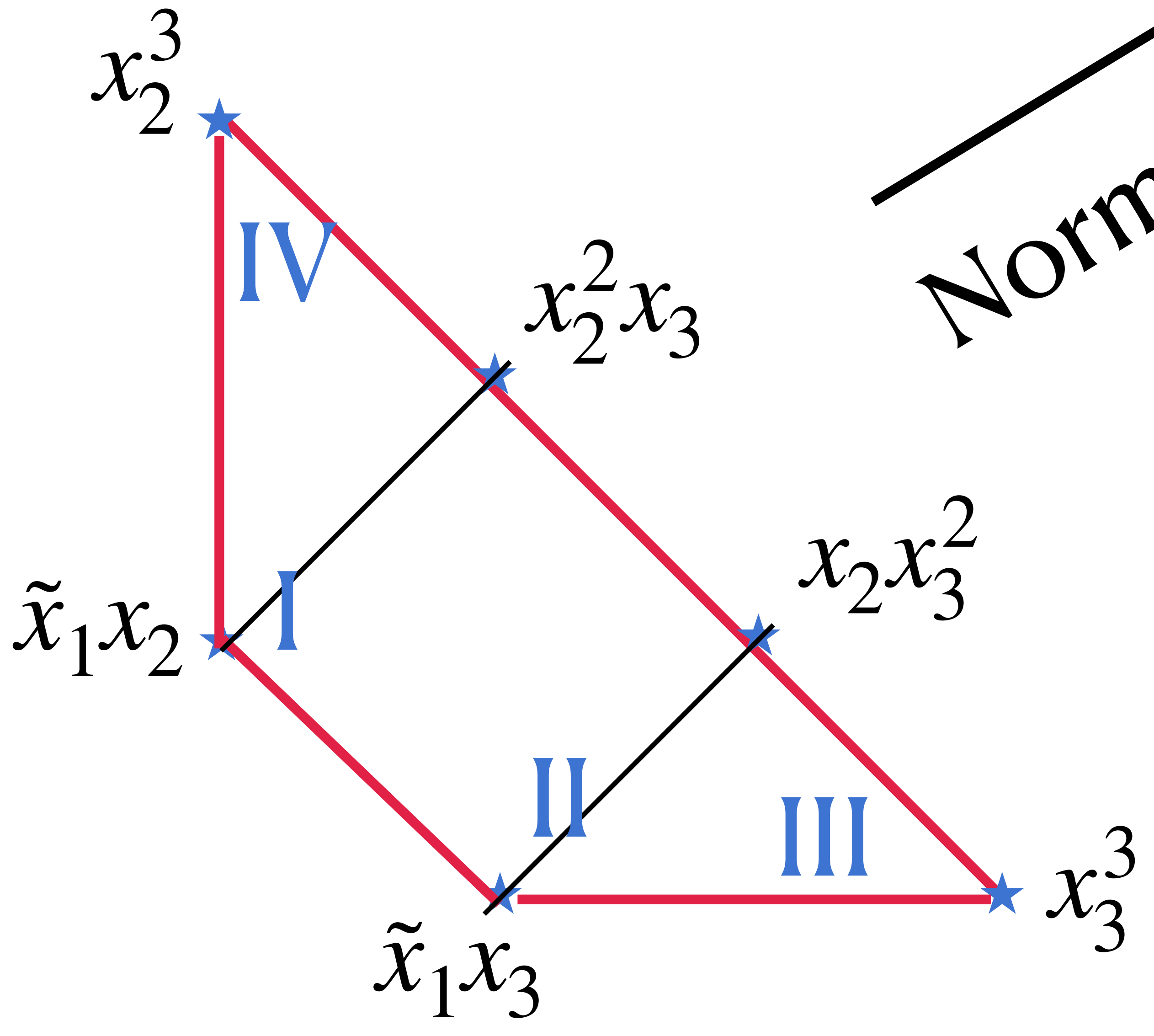

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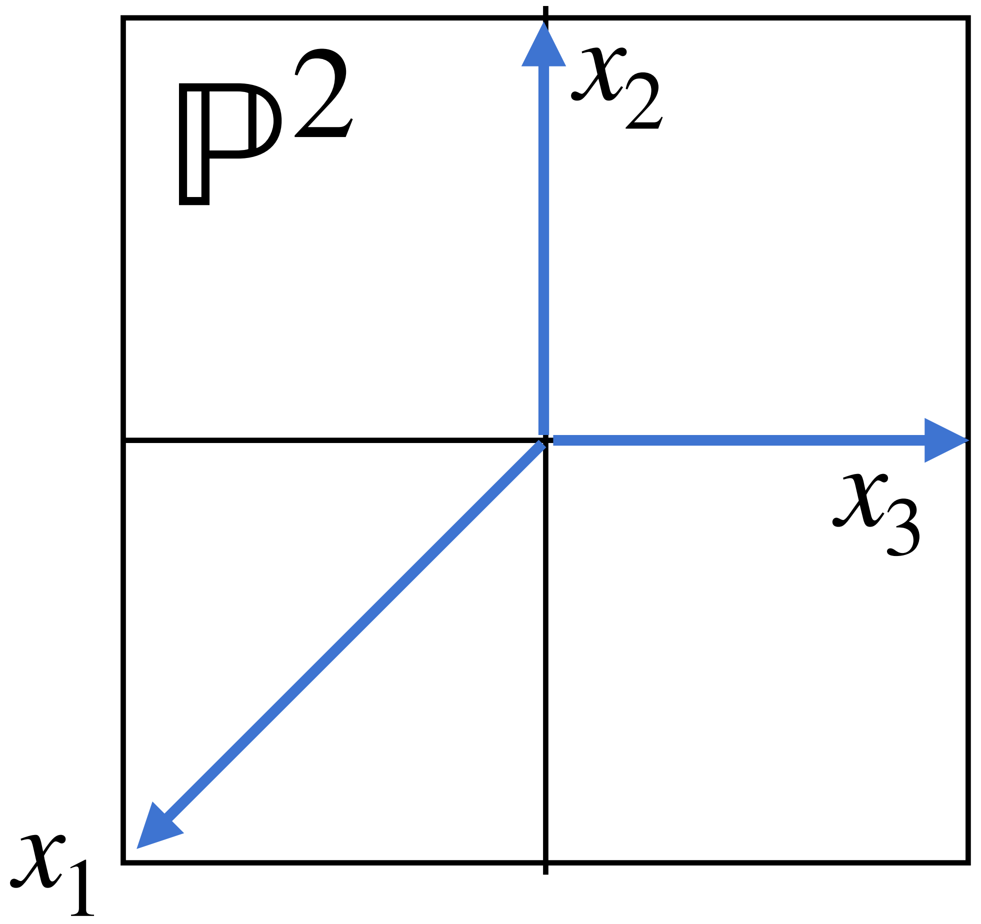

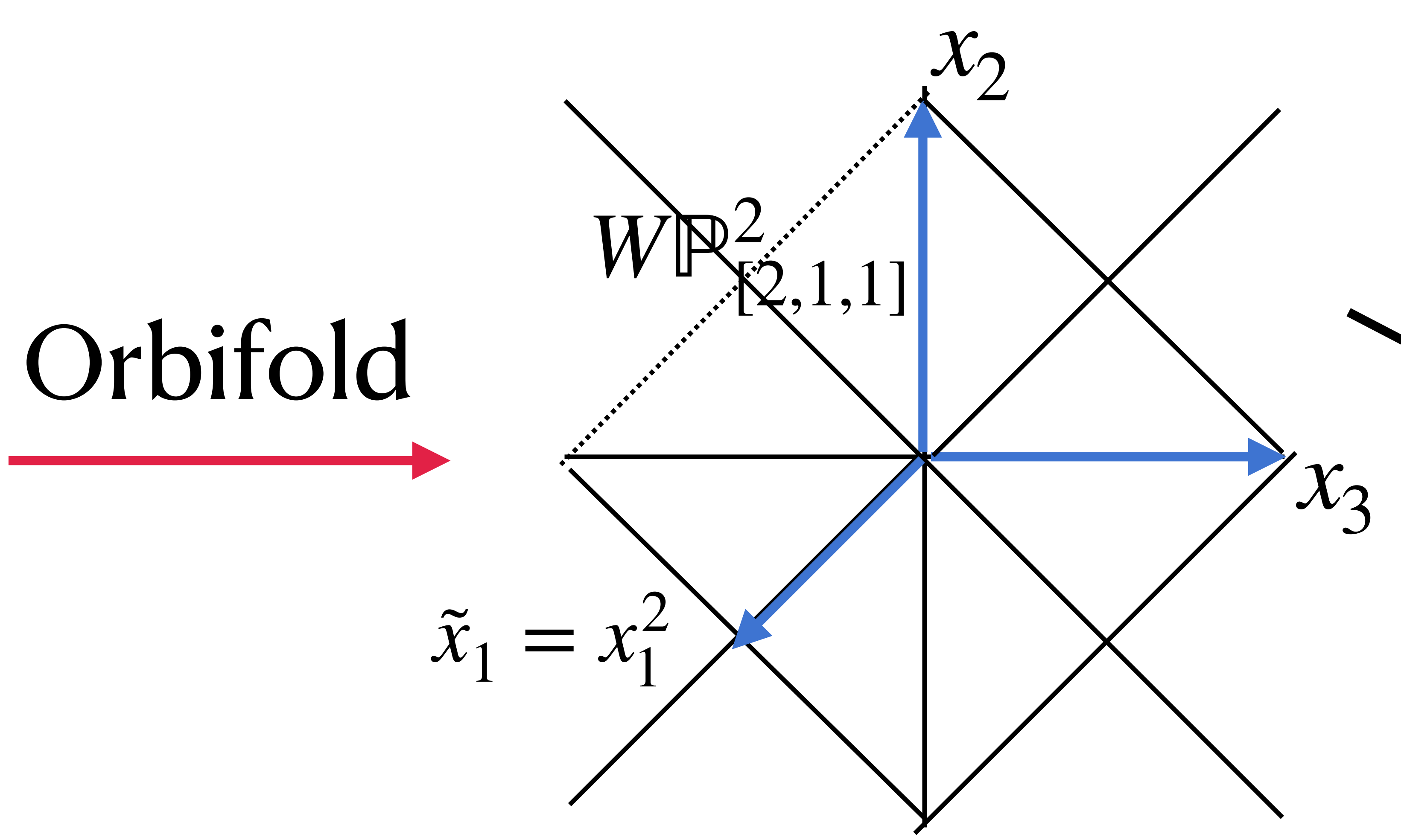

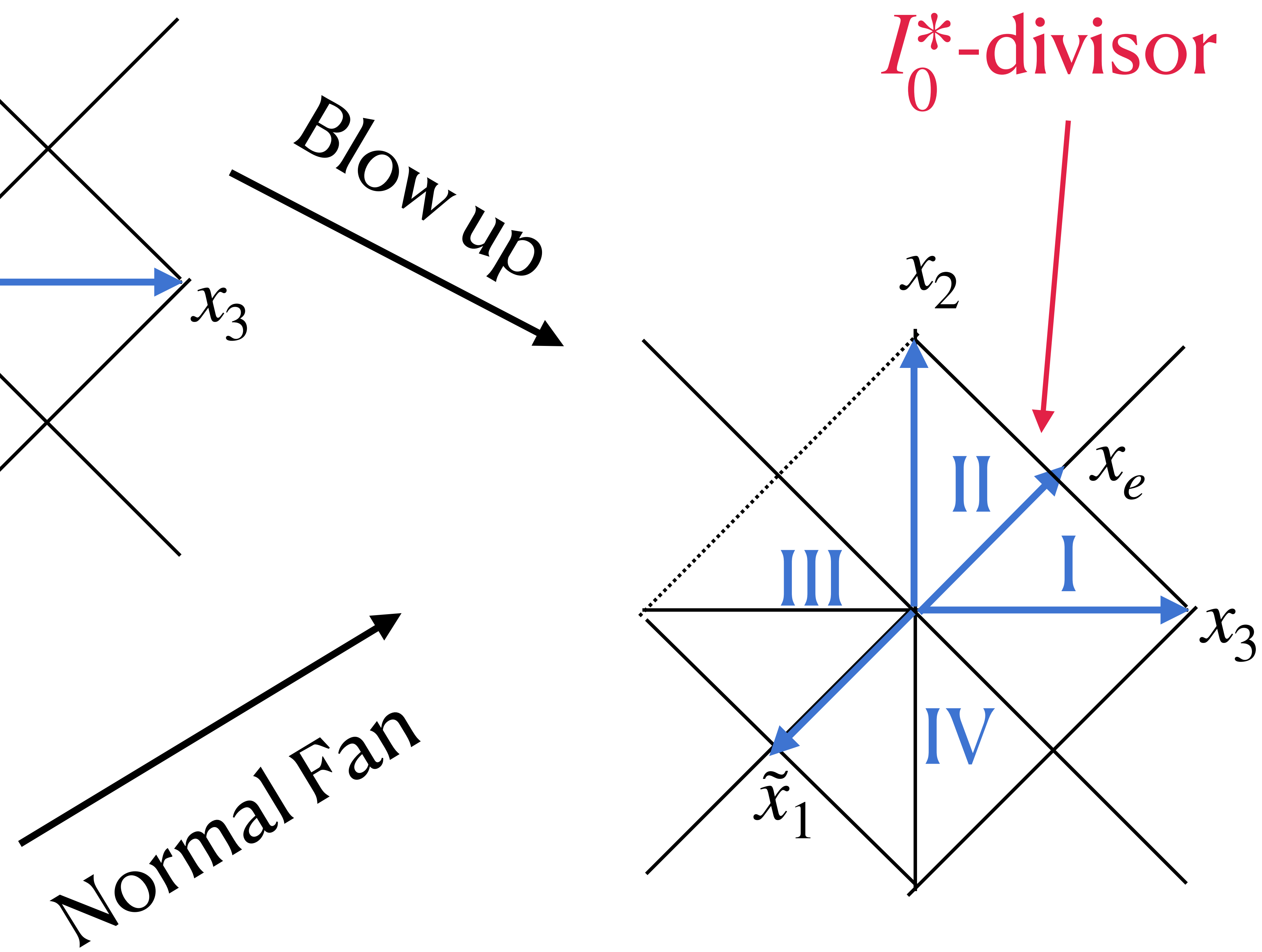

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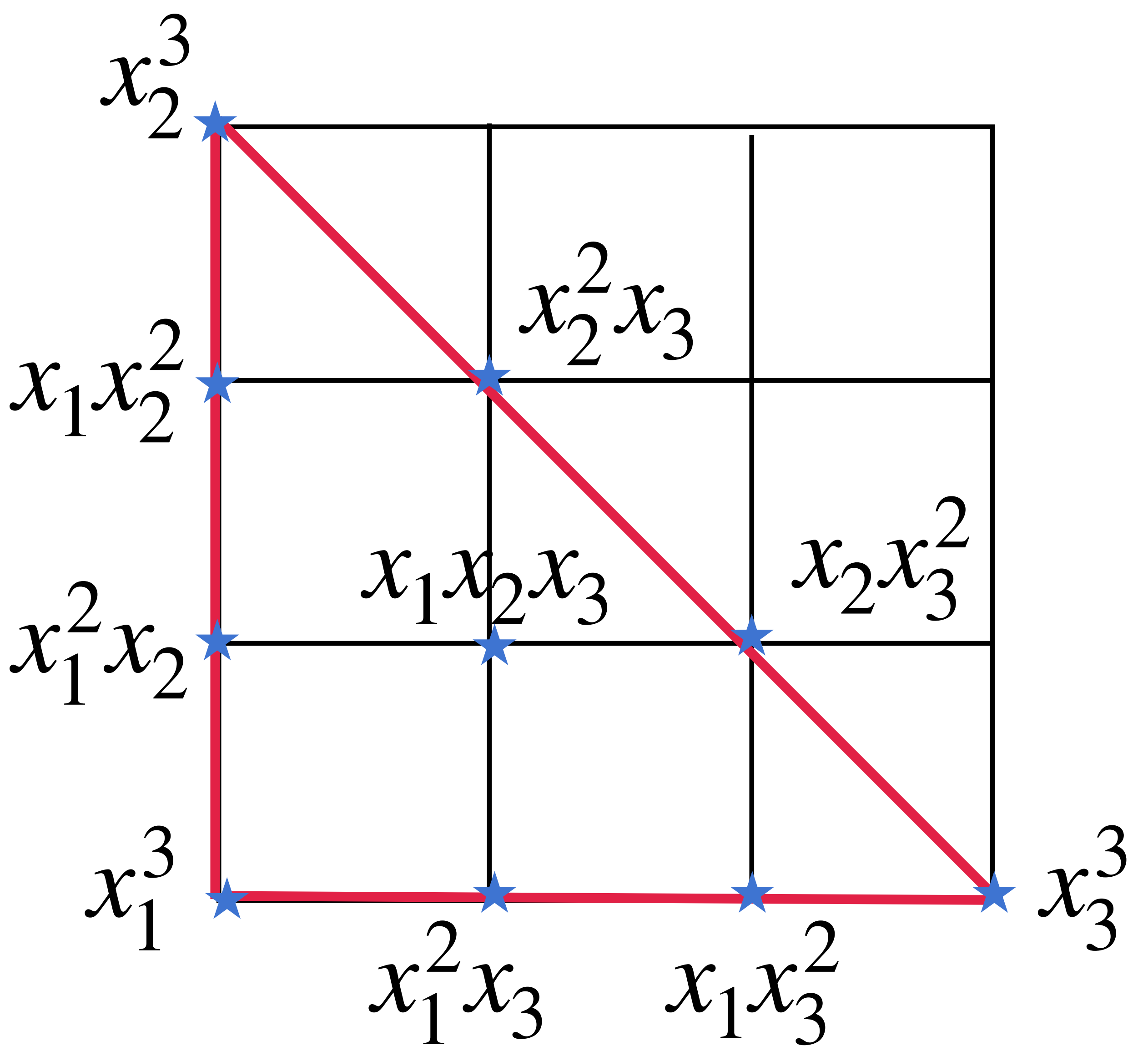

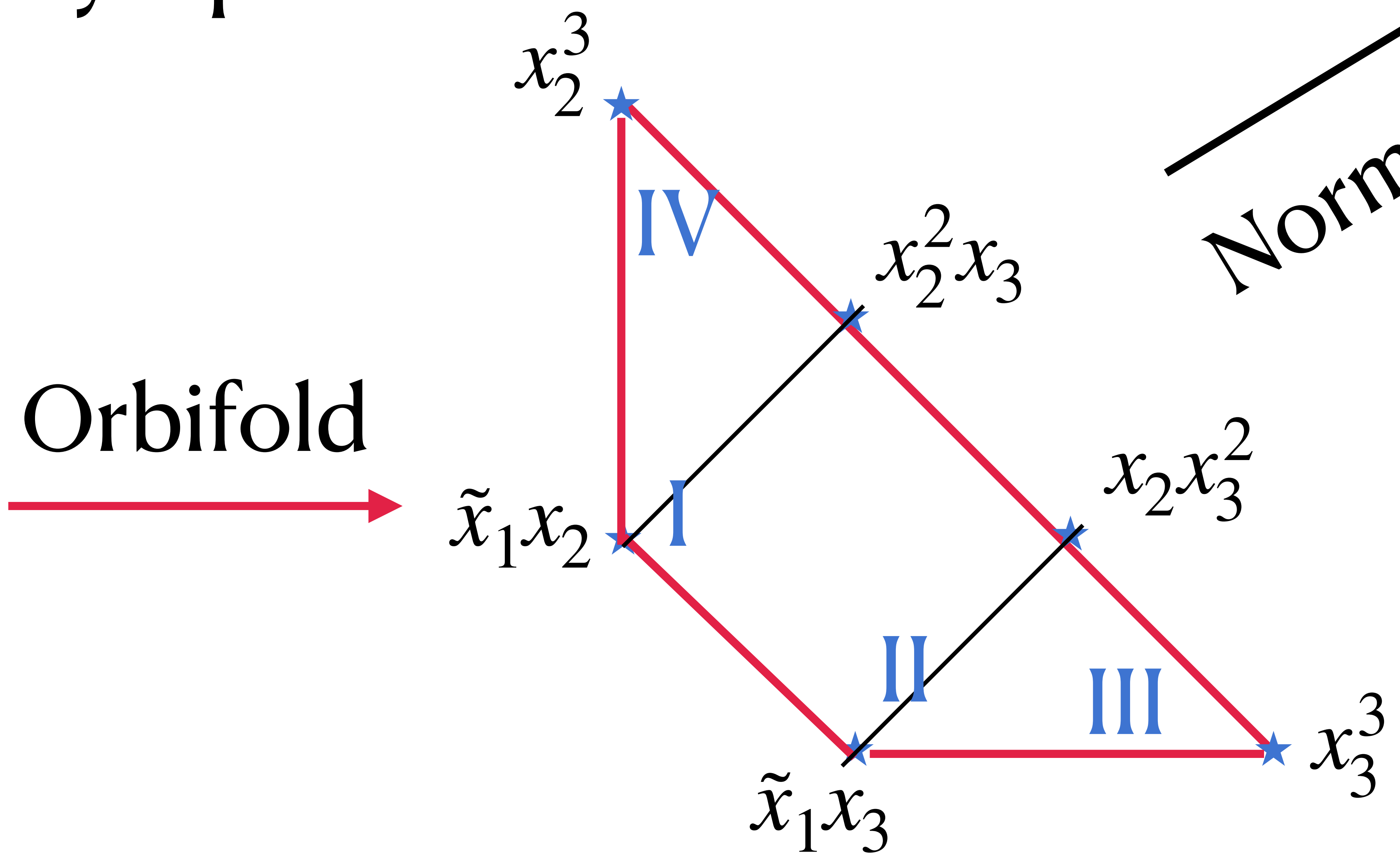

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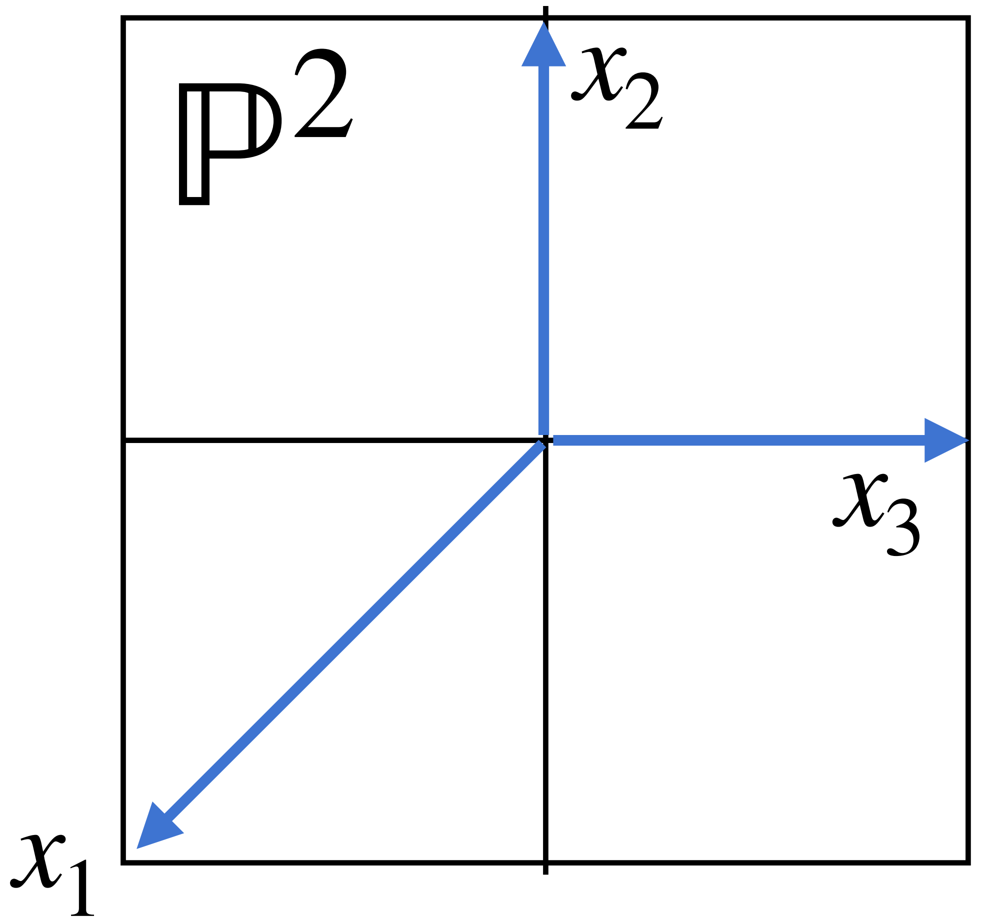

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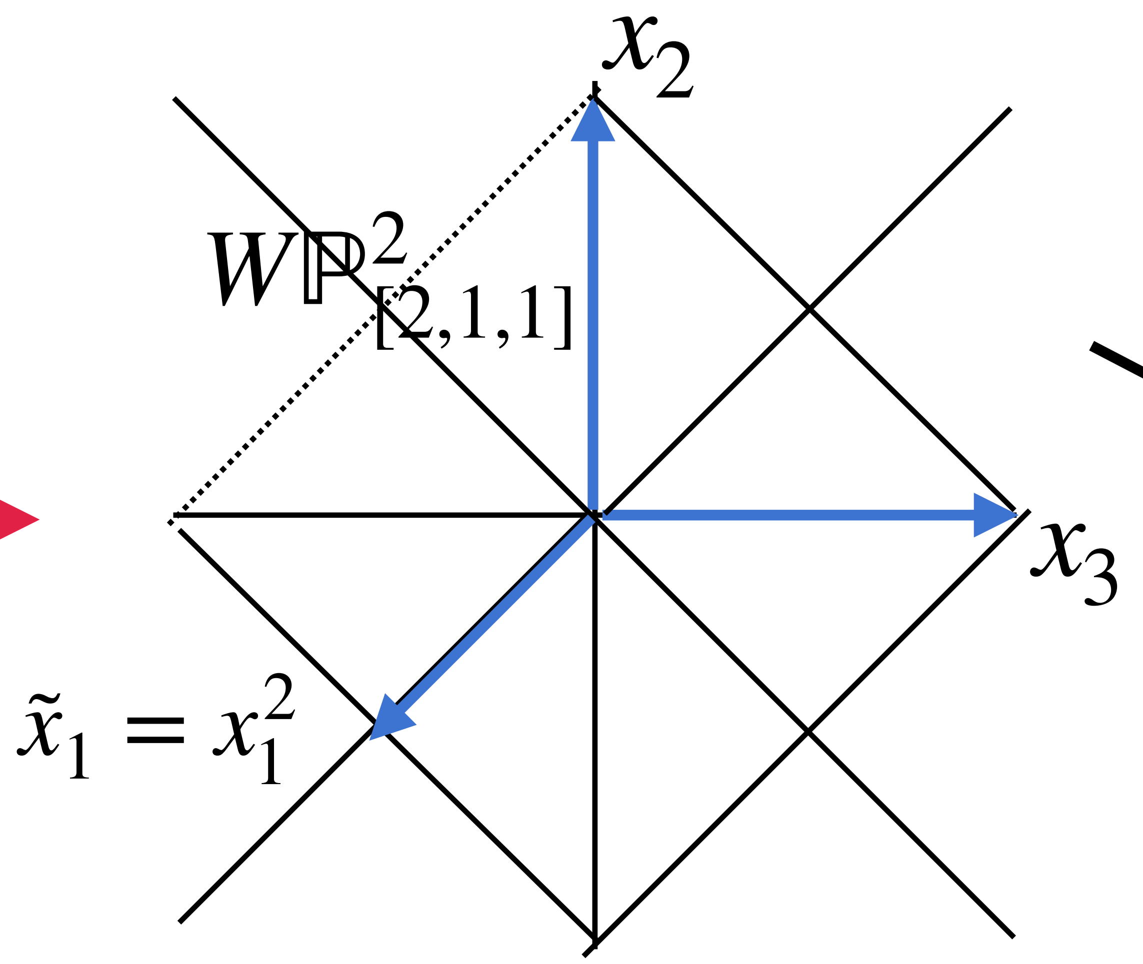
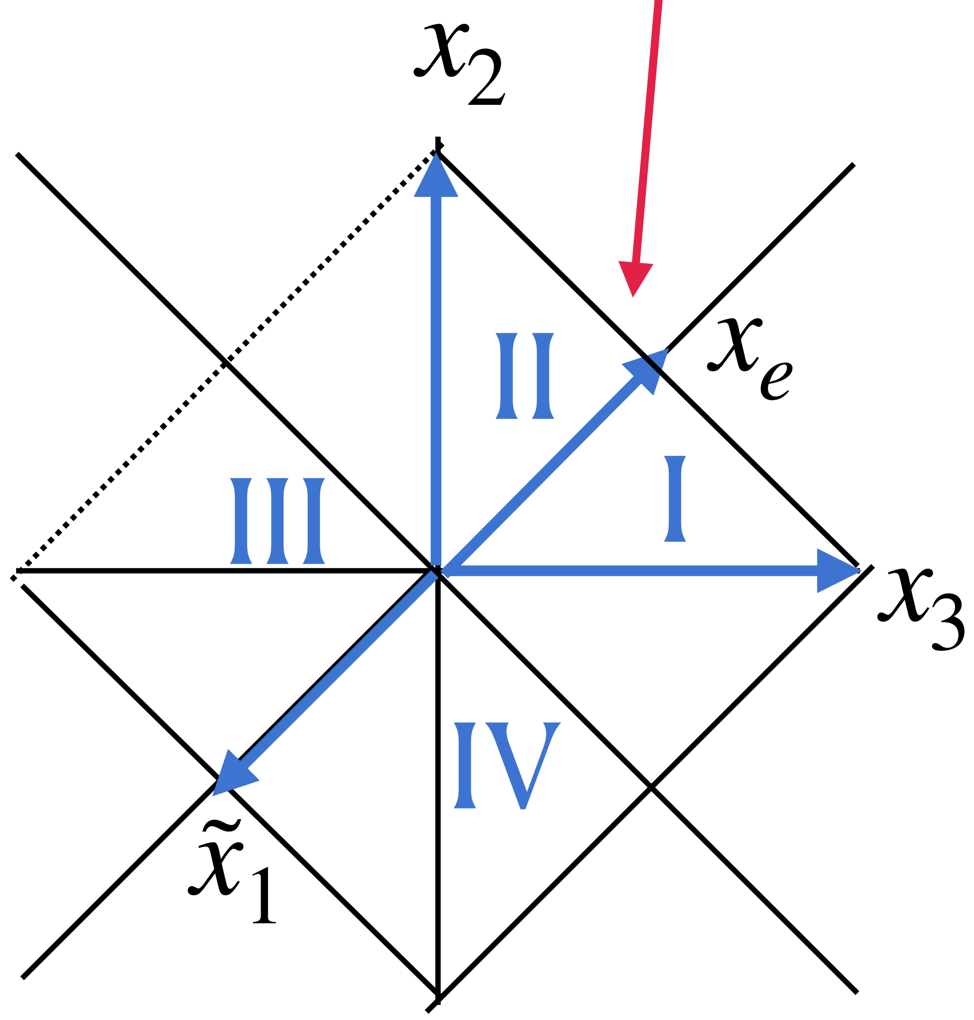


Blow up

I_0^*-divisor



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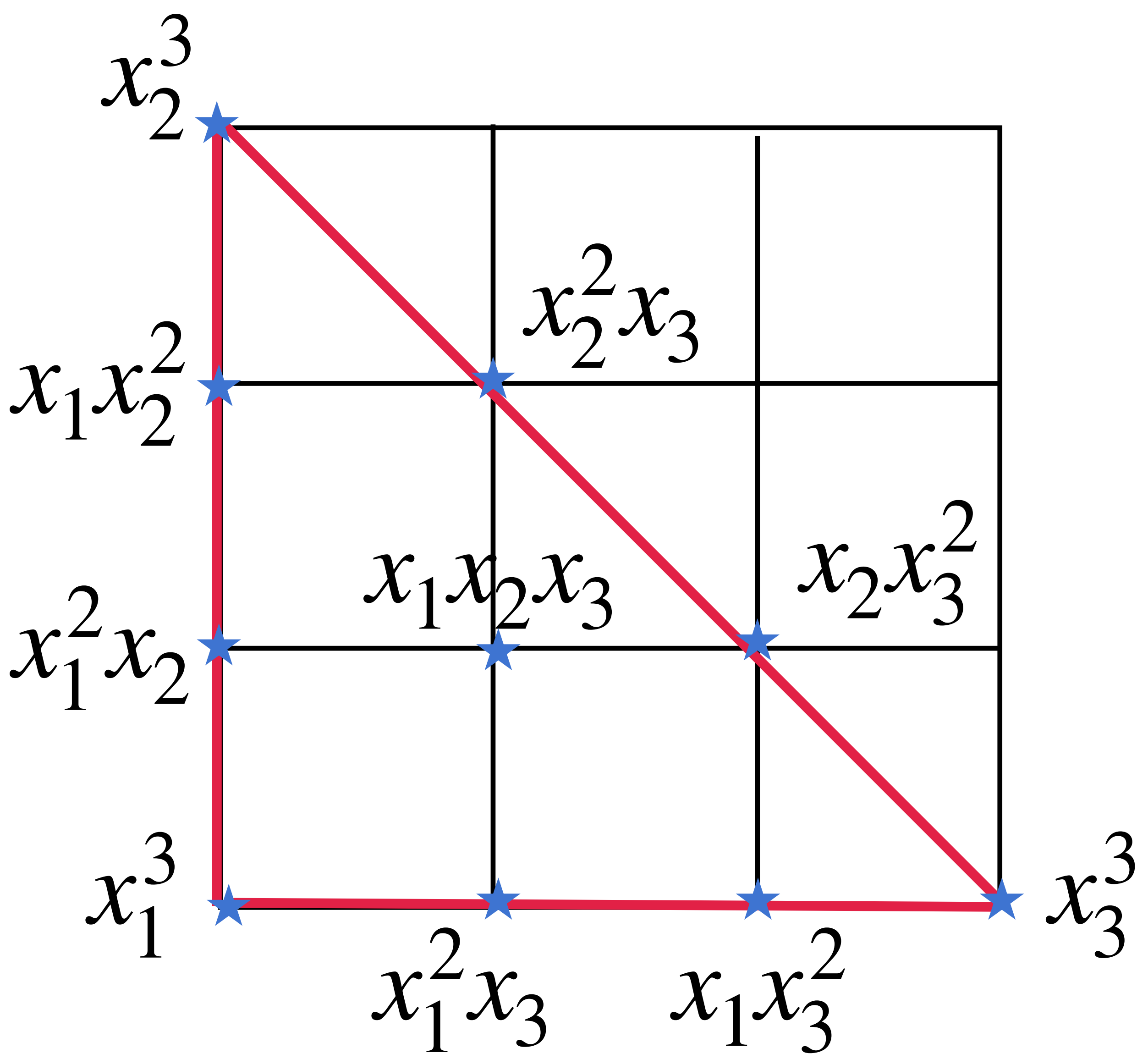

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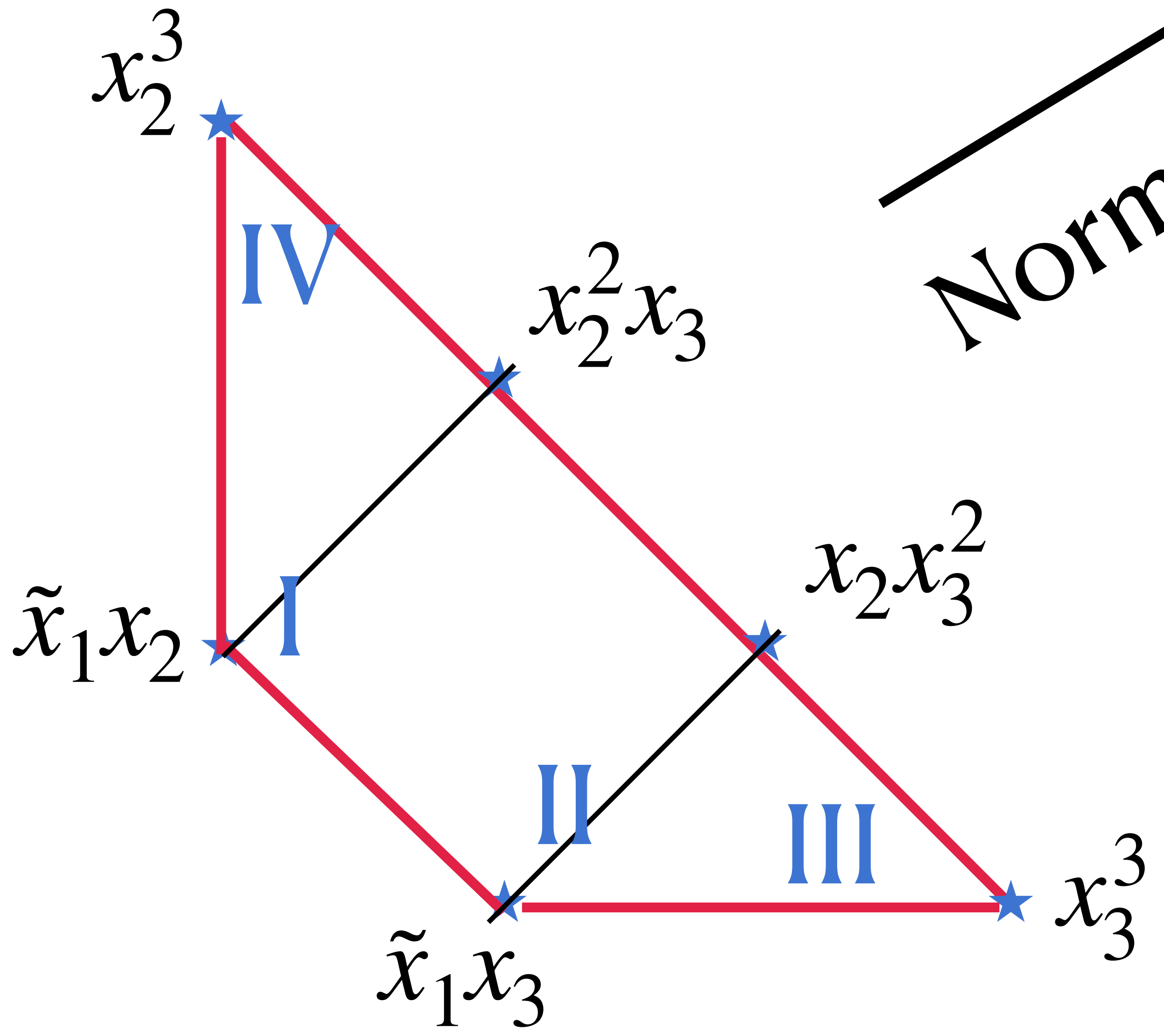
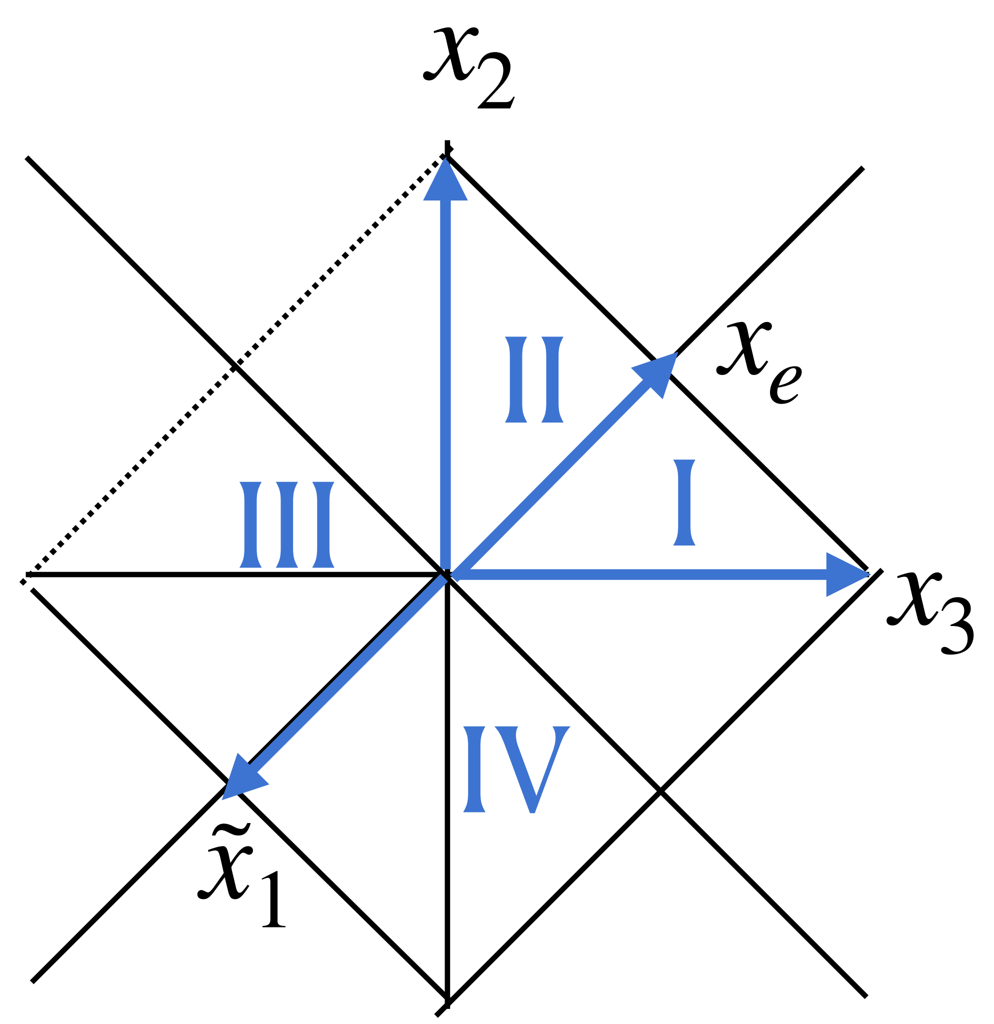


Normal Fan

Normal fan has eliminated A_1 singularity!

Normal Fan Construction

What about further blow-ups?



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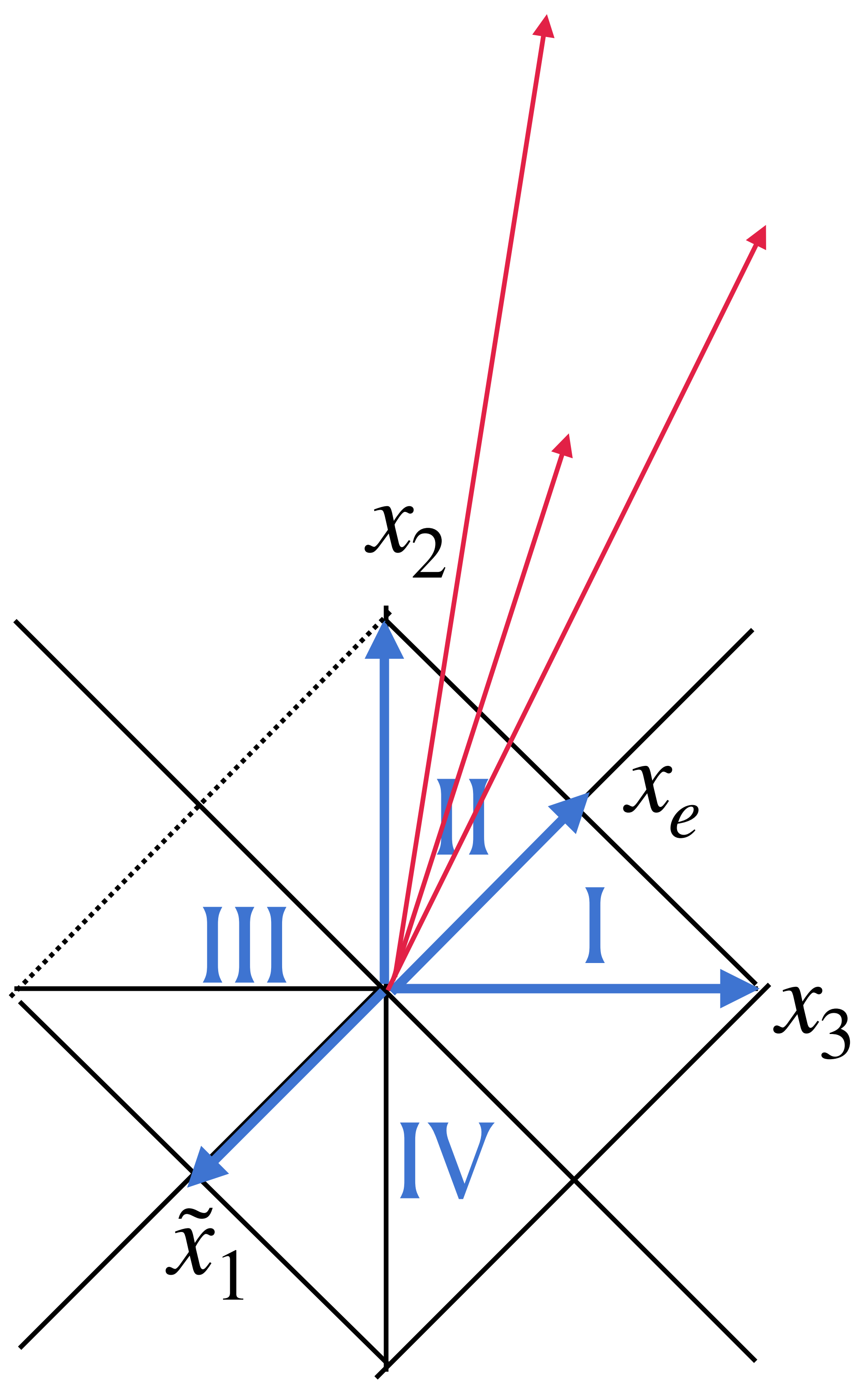

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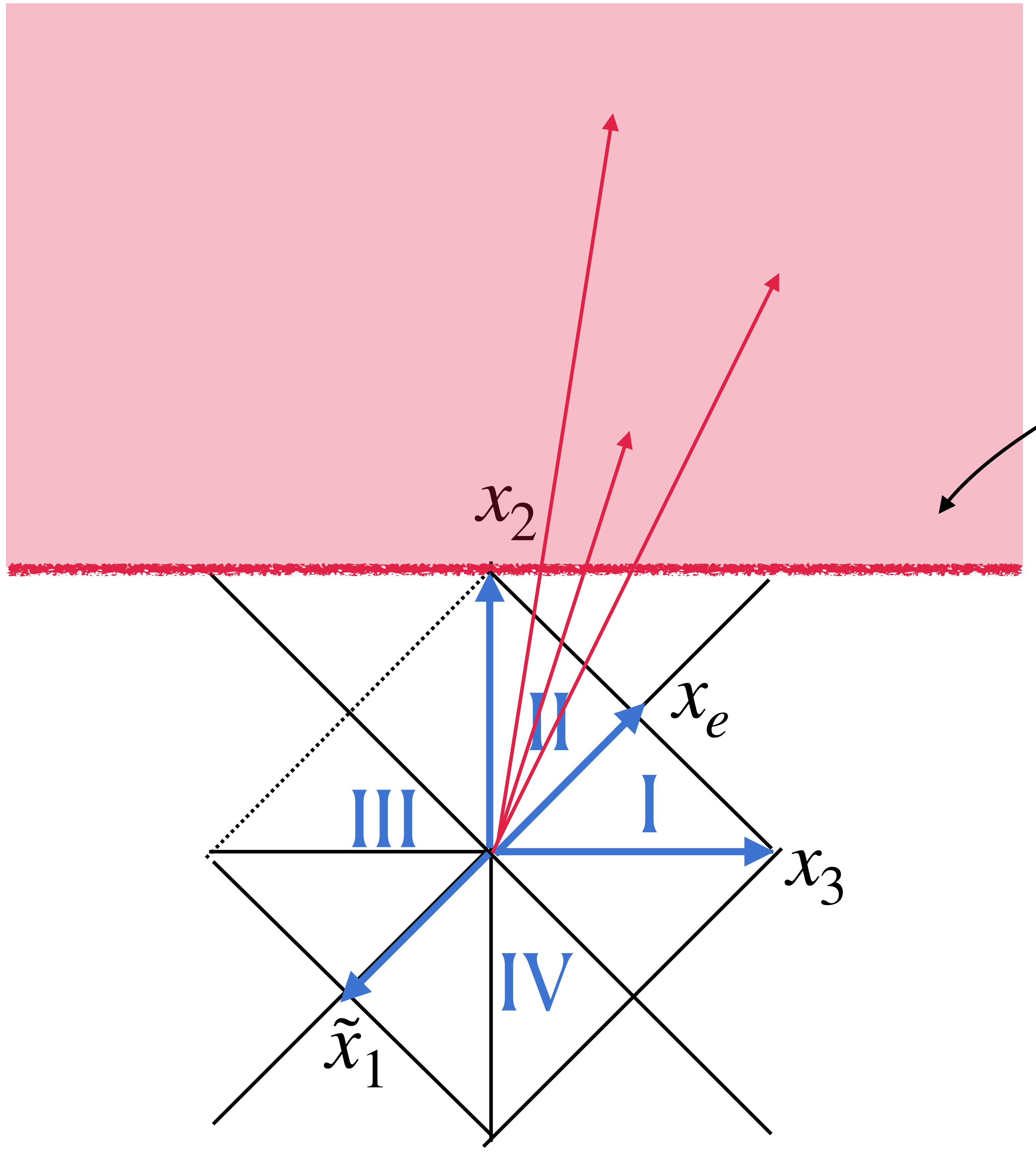

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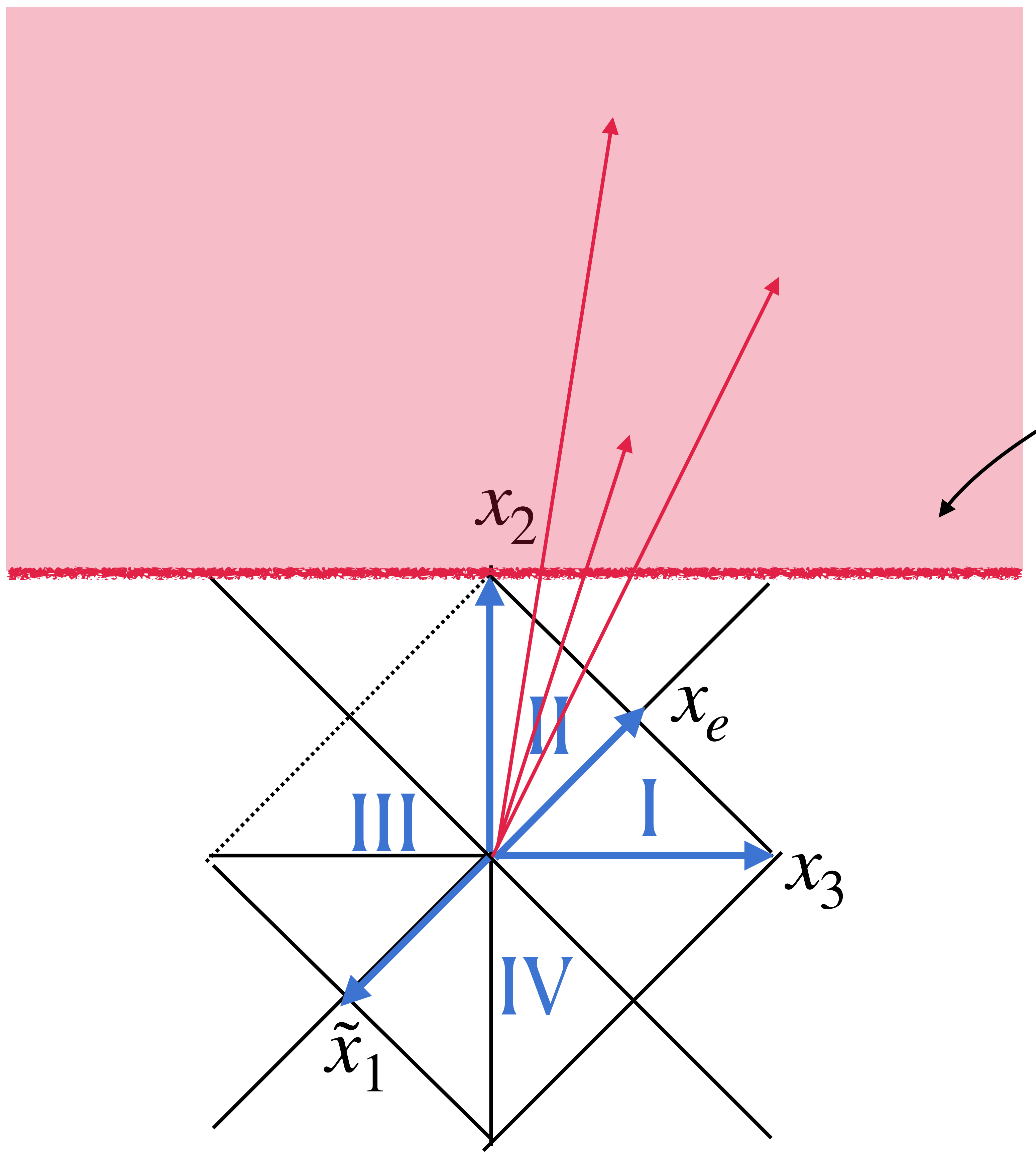

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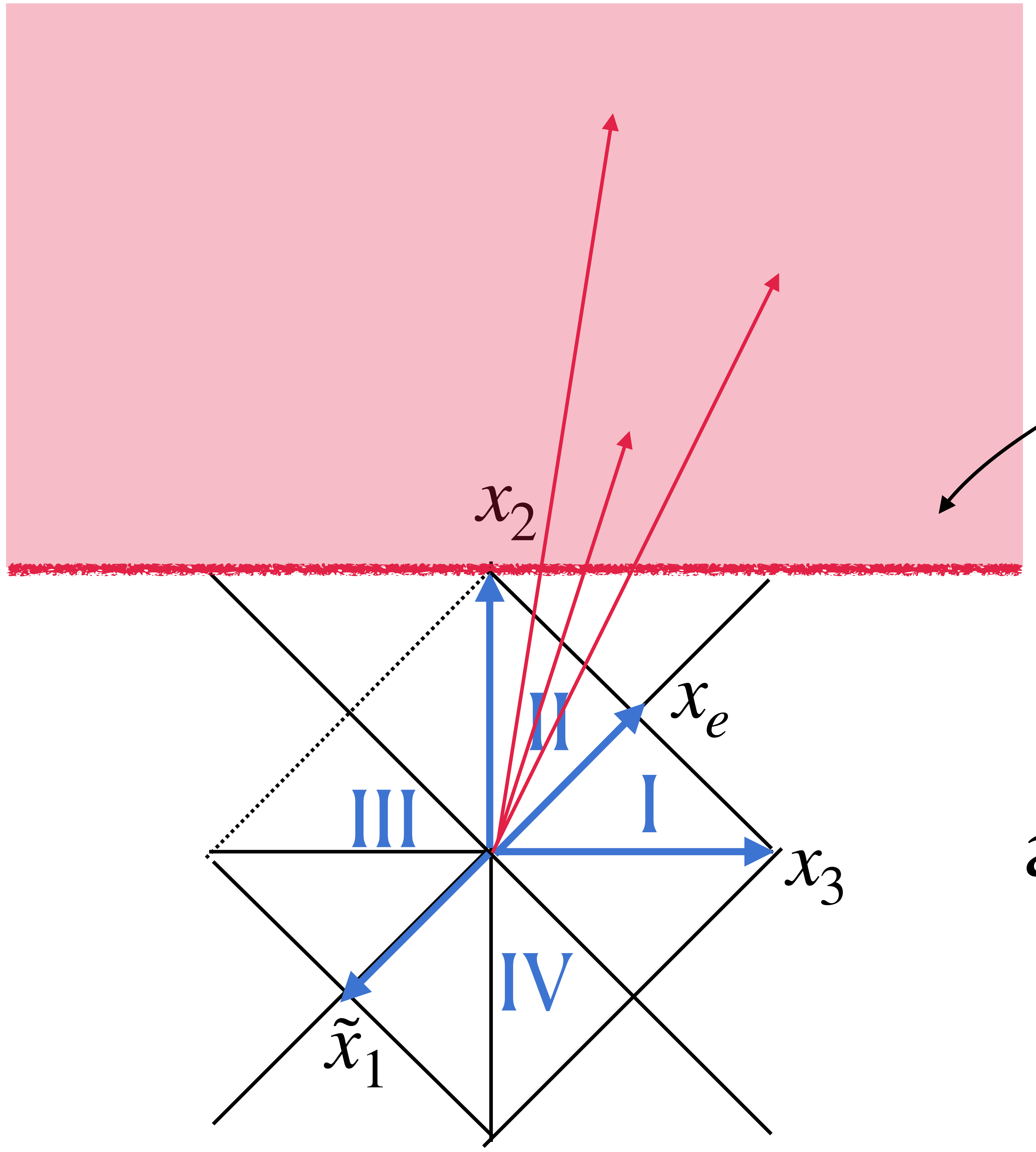

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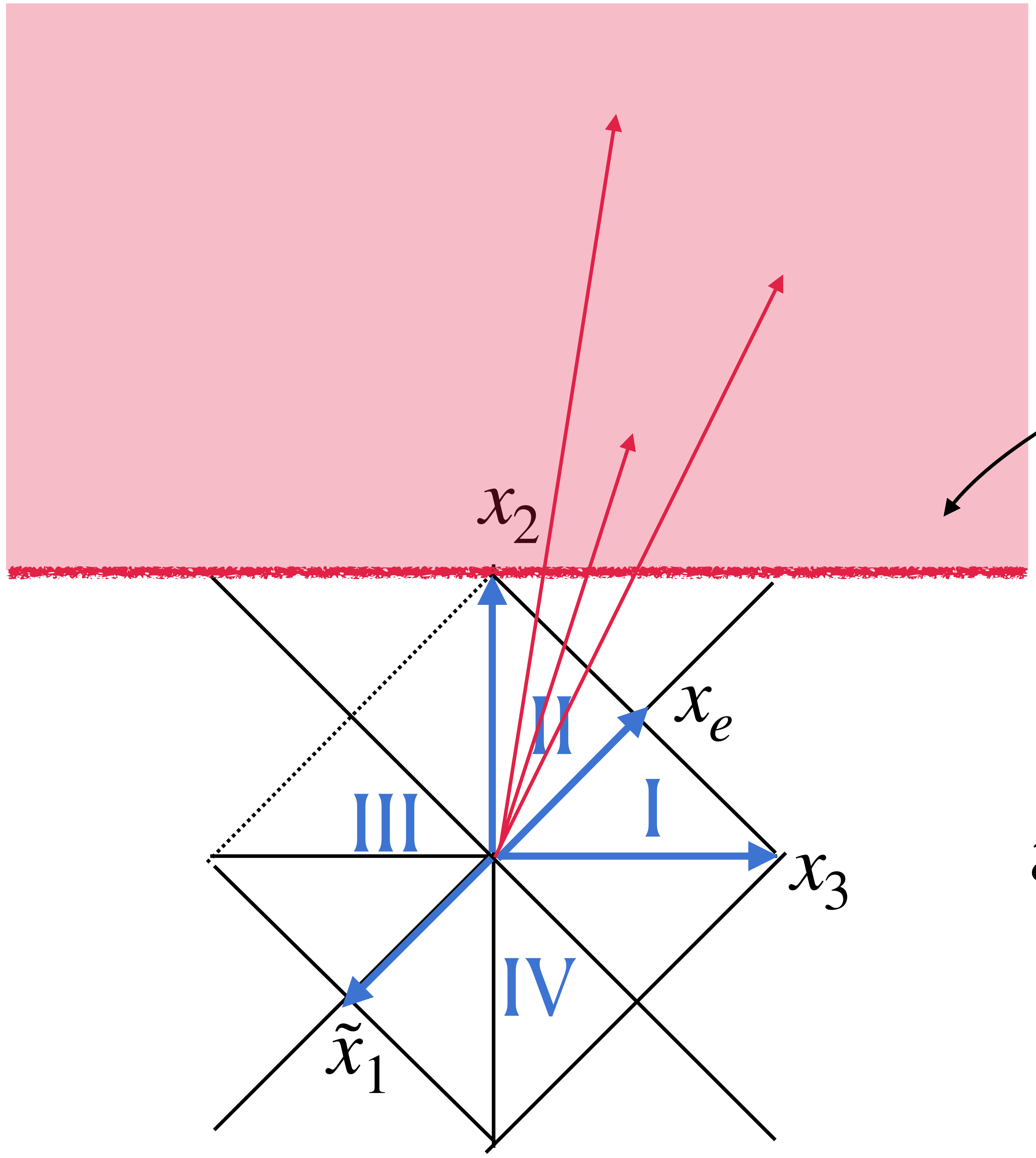

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related construction of toric F-theory bases: Halverson, Long, Sung '17; Taylor, Wang '17

"Failure" Modes

Not all naive orientifolds can be uplifted:

- Generic orientifold invariant hypersurface might factorize.
- Normal fan generators can be incompatible with either $(\mathrm{ord}(f), \mathrm{ord}(g), \mathrm{ord}(Disc)) < (4,6,12)$ or generic Weierstrass.
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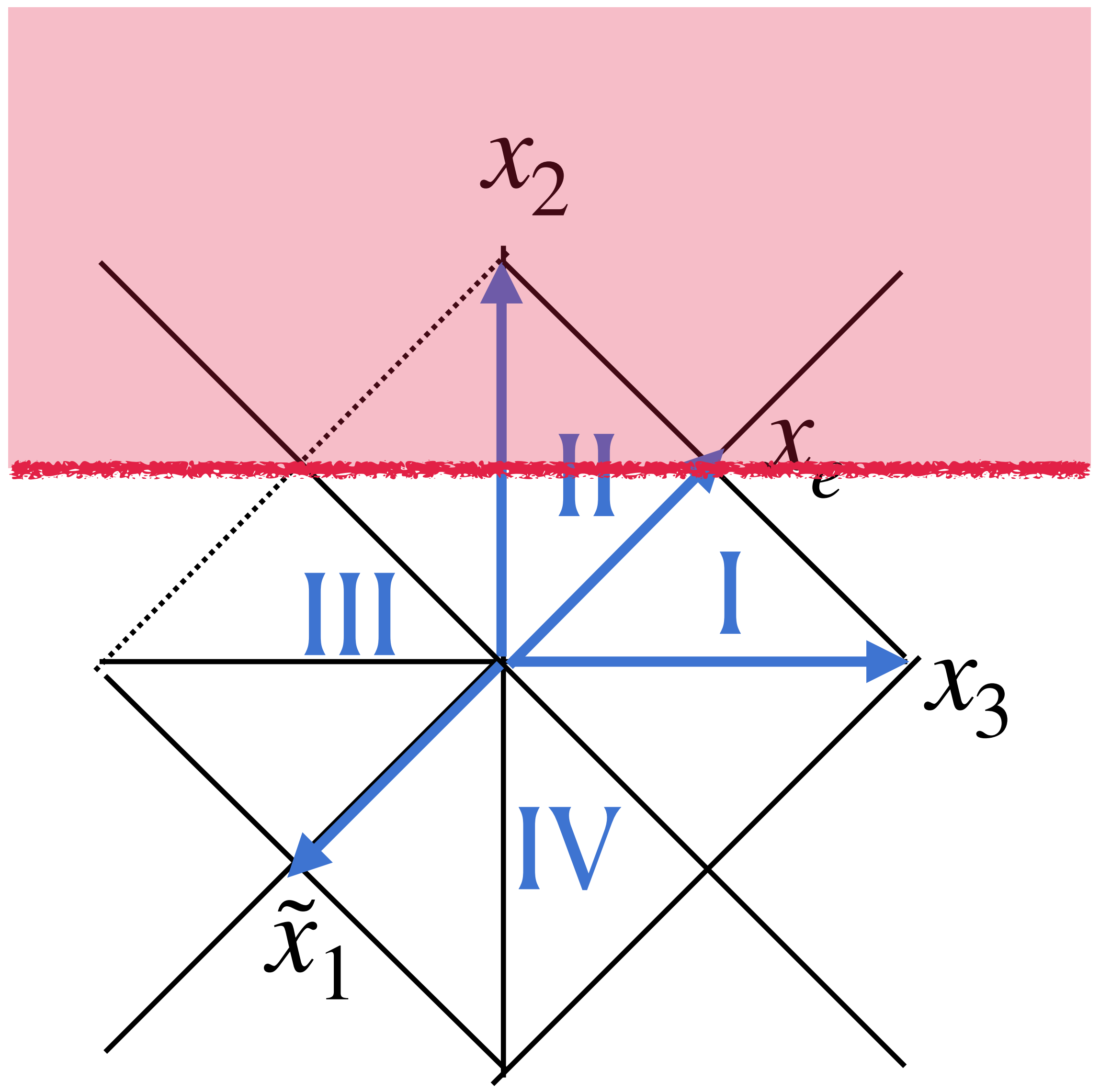

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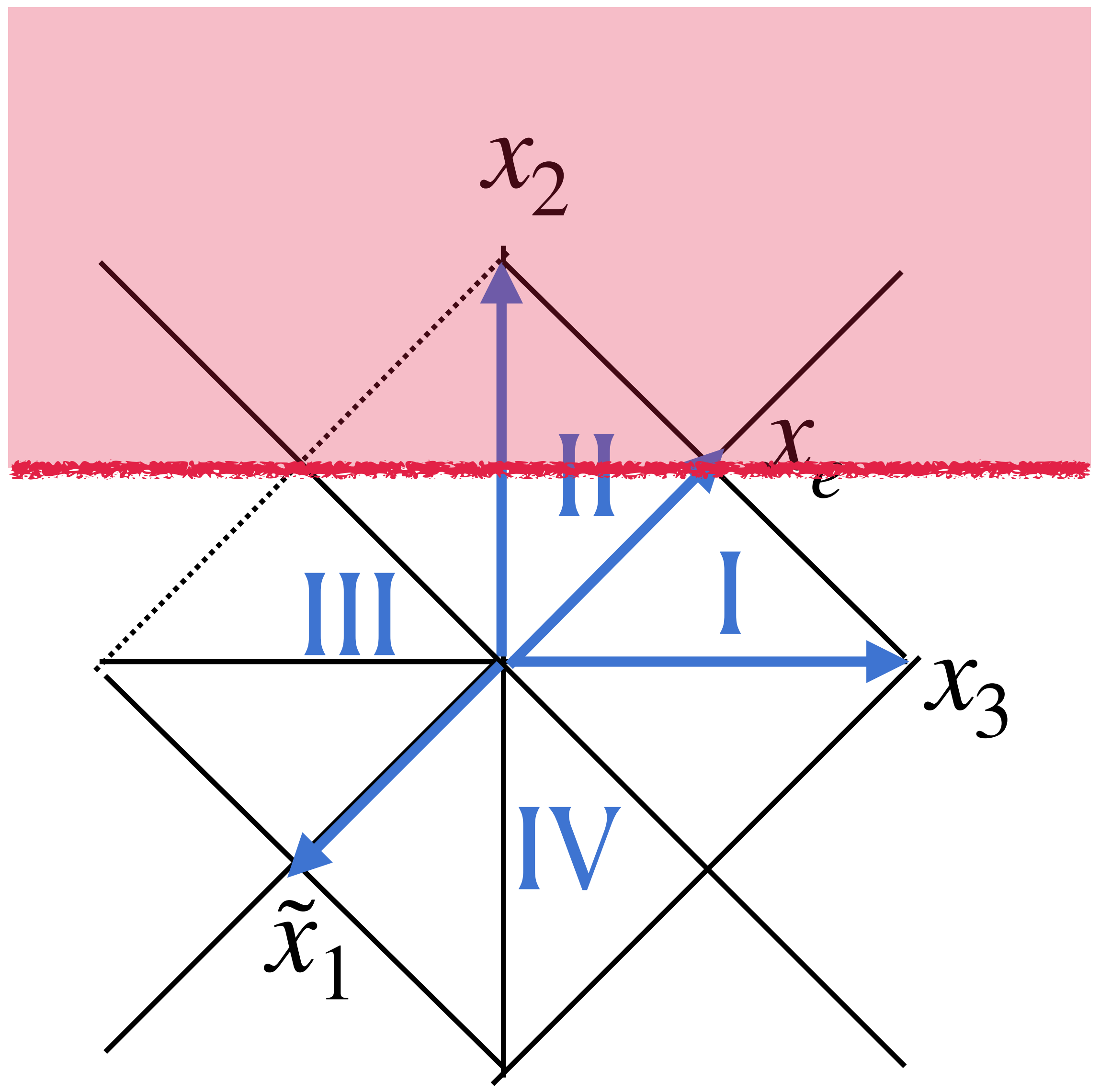

But, these pathologies are easy to detect ($\implies$ discard orientifold model).
Remaining models are guaranteed to be smooth (up to terminal $\mathbb{Z}_2$ singularities).

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Higgs and 3d Coulomb branches

Nef Partitions

In all our (surviving) models, by construction, D_B and D_W are nef Cartier divisors.

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This is the setup of Batyrev-Borisov '94, where mirror symmetry is perfectly understood:

$$\Delta^\circ := \mathrm{Conv}(\nabla_B \cup \nabla_W) \quad \xleftrightarrow{\text{reflexive pairs}} \quad \Delta = \Delta_B + \Delta_W$$

(toric fan of V_6) (monomial sections of anti-canonical divisor of V_6)

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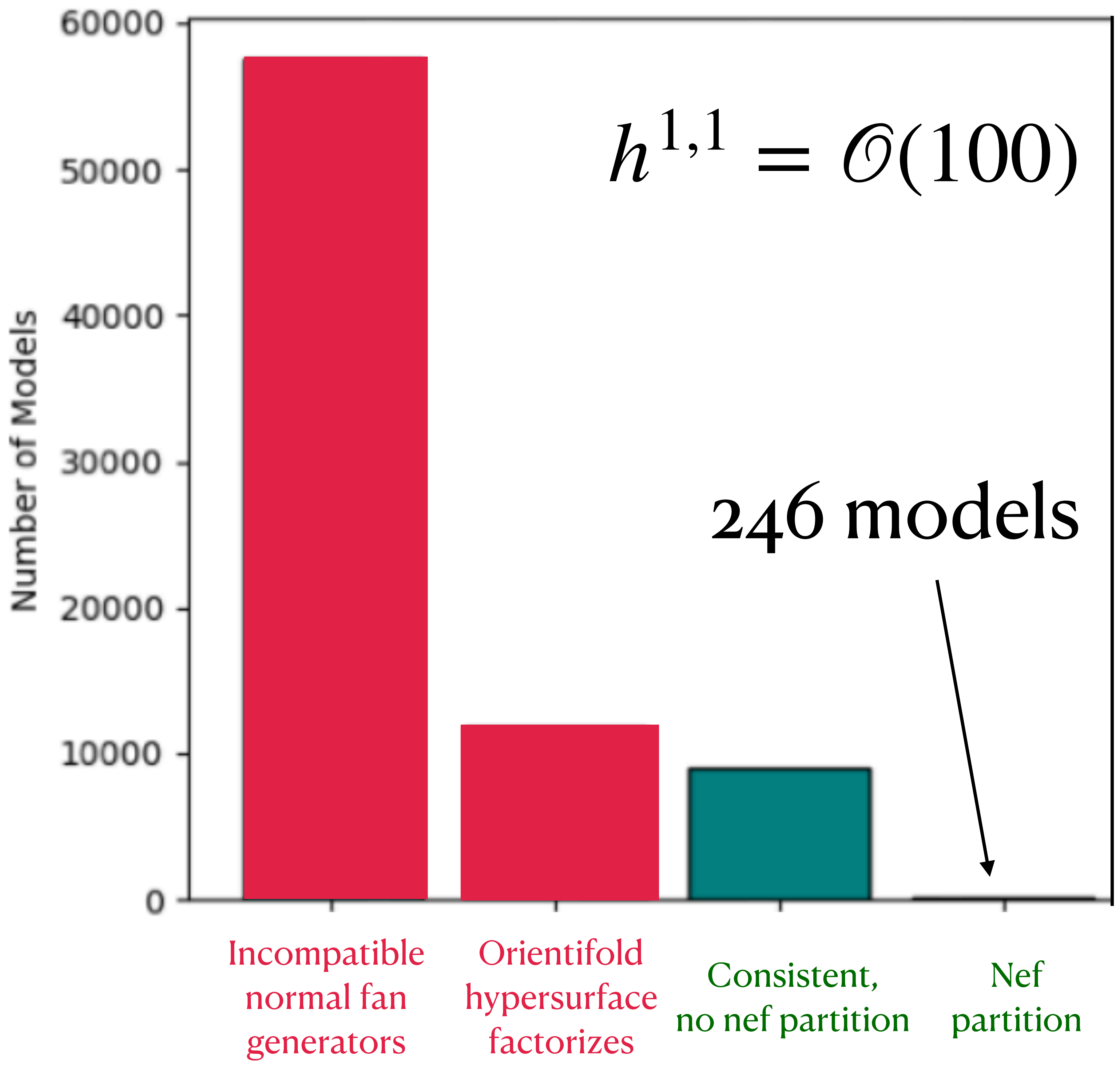
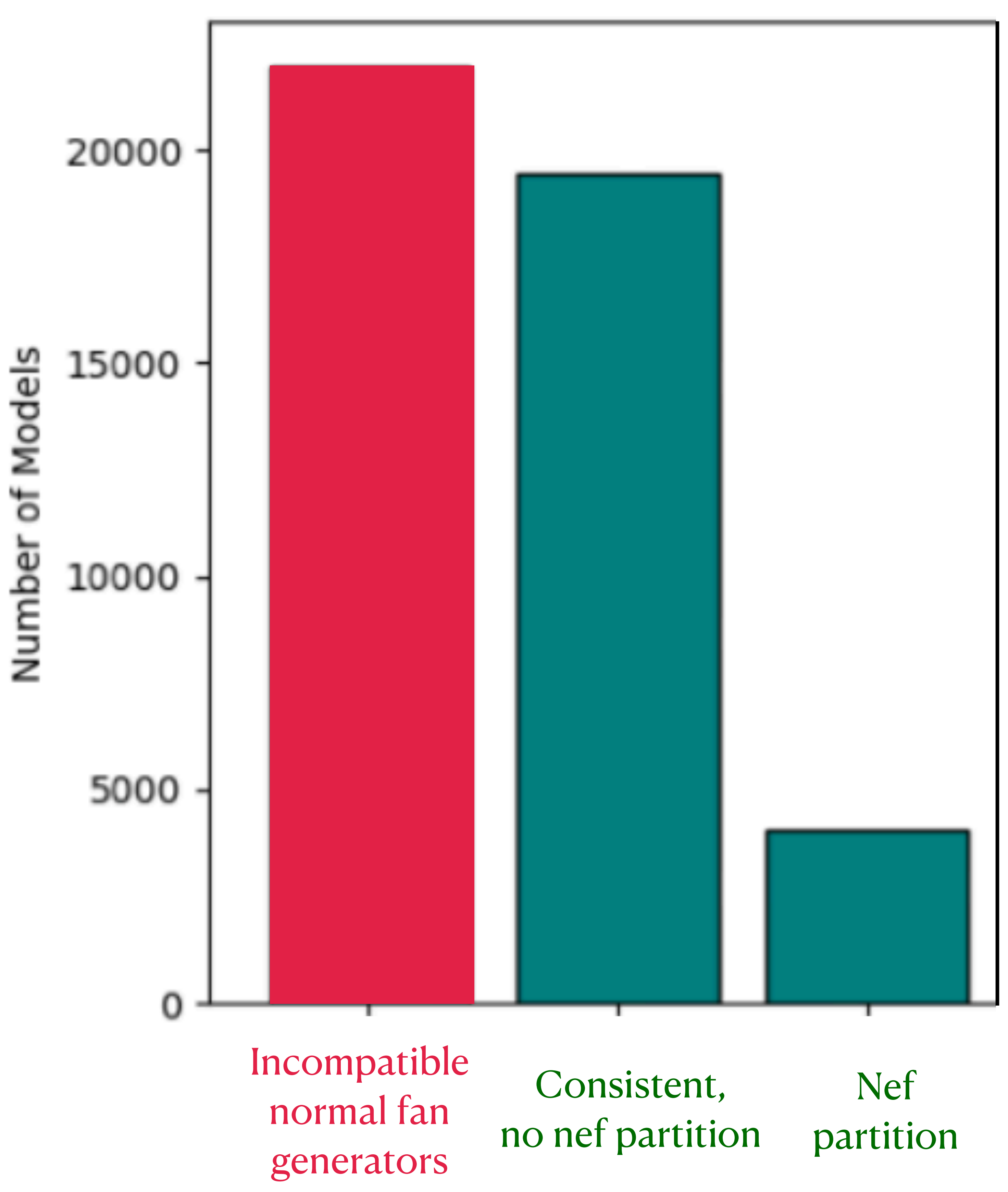
Preliminary Statistics

All uplifts for $h^{2,1} \leq 9$



All uplifts for $h^{1,1} \leq 9$



Outline

1. Motivation: String Landscape and Flux superpotentials
2. Systematics of O3/O7 orientifolds of Calabi-Yau hypersurfaces
3. "Combinatorial" F-theory uplifts
4. Conclusions

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- New computational machinery for uplifting CY3 orientifolds to F-theory
- Combinatorial in the sense of Batyrev '93
- Allows investigating D7-superpotential away from $so(8)$ limit +matching with threefold results.
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Future directions:

- Masses of D7-moduli in explicit type IIB flux vacua?
- Higgs potentials in intersecting D7-brane standard models?
- Mirror symmetry for co-dimension two complete intersections without nef-partitions?

Thank You!