D2-brane Probes of Non-Toric cDV Threefolds
via Monopole Superpotentials

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based on:
[arXiv:2604.09428] with A. Collinucci and M. Moleti

D-Branes at Singularities

D-brane at a **singular** point of a CY3 $\longleftrightarrow$ supersymmetric **quiver gauge theory**.
Its vacuum moduli space *reconstructs the local geometry* of the CY3.

This has taught us about QFT and geometry:

- it translates **singular geometry** into **gauge theory**;
- physics perspectinve on **quiver representations** and **resolutions of singularities**;
- **SCFTs** engineered by branes at singularities.

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A beautiful dictionary — but how far can we read it?

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The powerful tools we have to build these probe theories

— brane tilings, dimer models, toric diagrams —

confined to the toric geometry.

For **non-toric** CY3: only **case-by-case** constructions (e.g. matrix factorisations [Aspinwall–Morrison '12]); even worse for **non-resolvable** ones.

Non-toric threefolds

Interesting class of **NON-TORIC** threefolds: **compound Du Val (cDV) singularities**:

- natural threefold analogue of **ADE** surface singularities;
- engineer **Argyres–Douglas theories** (IIB) ;
- rank zero **SCFTs** (M-theory).

Our approach: replace combinatorics with an **algebraic object** that encodes the full geometry:

a **Higgs field** $\Phi(w)$ valued in the ADE Lie algebra [Collinucci's talk].

$\longrightarrow$ $\Phi(w)$ determines the D2-brane **3d** quiver gauge theory with superpotential.

Relation to previous work

- **Cachazo-Katz-Vafa**: D3/D5-branes on ADE fibrations;
- **Karmazyn**: cDV classification via noncommutative algebras.

The 3d perspective adds a new handle (e.g. on the collapsing nodes mechanism).

Compound Du Val Singularities

cDV singularity:

$$P_G(x,y,z) + w\,g(x,y,z,w) = 0 \qquad \subset \mathbb{C}^4$$

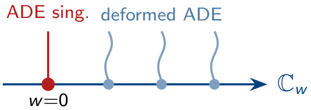
ADE surface singularities. $P_G = 0$**:**

$$A_n : x^2 + y^2 + z^{n+1} = 0$$
$$D_n : x^2 + zy^2 + z^{n-1} = 0$$
$$E_6 : x^2 + y^3 + z^4 = 0$$
$$E_7 : x^2 + y^3 + yz^3 = 0$$
$$E_8 : x^2 + y^3 + z^5 = 0$$

cDV three-fold:

$w\,g(x,y,z,w)$ produces fibration over $\mathbb{C}_w$:

- fiber at $w = 0$: ADE singularity
- fiber at $w \neq 0$: *deformed* ADE surface



Singular point of the threefold at $w = 0$.

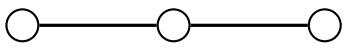
Partial Resolution: the Colored Dynkin Diagram

ADE surface singularity ($w=0$):

Resolved by a collection of holomorphic 2-spheres intersecting as the **Dynkin diagram**.

One sphere $\leftrightarrow$ one simple root α_i.

(A_3)



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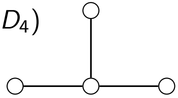


Partial Resolution: the Colored Dynkin Diagram

ADE surface singularity (w=0):

Resolved by a collection of holomorphic 2-spheres intersecting as the **Dynkin diagram**.

cDV threefold:

Exceptional locus of the threefold: only a subset of these 2-spheres:

⇒ **partial simultaneous resolution.**

Examples of simple flops.

Colored Dynkin diagram

- **white node**: resolvable root (globally defined sphere)
- **colored node**: non-resolvable (sphere undergoes monodromy)

The Higgs Field $\Phi(w)$

$\Phi(w)$: matrix-valued field taking values in the **ADE Lie algebra** $\mathfrak{g}$.

What $\Phi(w)$ encodes:

- **hypersurface equation**: $\det(z\,\mathbf{1} - \Phi(w)) = 0$; [Collinucci's talk]
- **(partial) resolution pattern**: white vs. coloured nodes in the Dynkin diagram.

Example: Reid's Pagoda: $x^2 + y^2 = z^{2k} - w^2$ with $k = 2$

$$\Phi(w) = \begin{pmatrix} 0 & \mathbf{1} & & \\ w & 0 & & \\ & & 0 & \mathbf{1} \\ & & -w & 0 \end{pmatrix}$$

$$\det(z\,\mathbf{1} - \Phi) = 0$$
$$\downarrow$$
$$x^2 + y^2 = z^4 - w^2.$$

deformed A_3 family

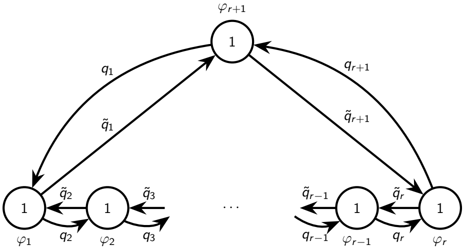
[Collinucci-De Marco-Sangiovanni-RV]

From $\Phi(w)$ to a Deformed Probe Theory

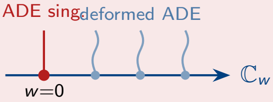
Starting point: $S_{\mathrm{ADE}} \times \mathbb{C}_w$

- D-brane probing the **undeformed ADE surface singularity** S_{ADE}

  $\Rightarrow$ quiver gauge theory with the shape of the **affine Dynkin diagram**.
- φ_i: parametrize positions of fractional D-branes on $\mathbb{C}_w$



Key idea [Klebanov–Witten; Cachazo-Intriligator-Katz-Vafa]:
D3 probing ADE **fibration** over $\mathbb{C}_w$ $\rightsquigarrow$ deform superpotential by $\delta W(\varphi_i)$.



Our claim: Since fibration encoded in $\Phi(w)$, expect that δW is dictated by $\Phi(\varphi_i)$.

D2-brane probes

Instead of D3, we take D2 probes, with a 3d worldvolume.

Specific to 3d:

- the 3d photon dualizes to a scalar, whose shift symmetry is the **topological** $U(1)$;
- **monopole operators** (the 3d analogue of 't Hooft operators) are charged under it, and parametrize the 3d $\mathcal{N} = 4$ **Coulomb branch**.

When *colored nodes* are present, the superpotential deformation involves monopole operators, tractable in 3d via **mirror symmetry**.

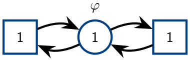
Final goal: obtain **quiver** and its **superpotential** (the same for D2 and D3); the **3d perspective** is what makes the monopole deformations computable.

Superpotential deformation generated by Φ

In 3d $\mathcal{N} = 4$ probe theory, topological symmetry given by **ADE Lie algebra** $\mathfrak{g}$ (the same as the sing). The associated **moment map** μ transforms in the **adjoint of** $\mathfrak{g}$ — just like the Higgs field Φ.

Each quiver node (fractional brane) has its own μ in $A_1 \subset \mathfrak{g}$:

$$\mu_{A_1} = \begin{pmatrix} \varphi & \mathsf{m}_{+\alpha} \\ \mathsf{m}_{-\alpha} & -\varphi \end{pmatrix}$$



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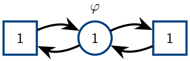


Φ encodes the geometric deformation, μ packages the probe-theory fields; both live in adj($\mathfrak{g}$). Natural coupling through *Killing form* — as in the constant case [Moleti–Valandro]:

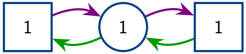
$$\boxed{\delta W = \kappa\big(\Phi(\varphi),\, \mu\big)}$$

$\hookrightarrow$ **Reduces to Cachazo-Katz-Vafa** on case with no colored nodes, and in particular to Klebanov-Witten on the conifold.

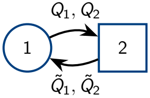
Trading Monopoles for Polynomials: $U(1)$ with 2 flavors

Setup. Single A_1 block: $U(1)$ gauge theory with 2 flavors $(q_i, \tilde{q}_i)$. Superpotential deformation:

$$W = \phi(q_1\tilde{q}_1 - q_2\tilde{q}_2) + \mathsf{m}_{-\alpha} + \varphi\, \mathsf{m}_{+\alpha}.$$



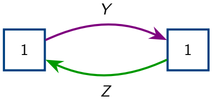
$\longrightarrow$ 3d mirror



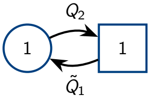
$$\delta W = \mathsf{m}_{-\alpha}.$$

$$\delta W = Q_1 \tilde{Q}_2.$$

$\downarrow$ integrate out



$\longleftarrow$ 3d mirror



$$W_{\mathrm{XYZmodel}} = X\,(Y\,Z + T^2)$$

[Collinucci-Giacomelli-Savelli-RV]

Reid's Pagoda ($k = 2$): Geometry and Higgs Field

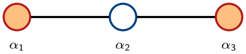
Threefold (deformed A_3):

$$uv \;=\; z^4 - w^2.$$

Higgs field, block-diagonal:

$$\Phi(w) = \left(\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ w & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & -w & 0 \end{array}\right) \in A_1^{(L)} \oplus A_1^{(R)} \subset A_3$$

A_3 **quiver** — external nodes coloured, only α_2 resolvable



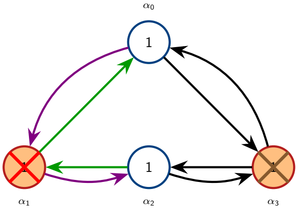
Superpotential deformation

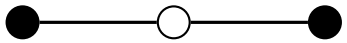
$$\delta W \;=\; \underbrace{\mathfrak{m}_{-\alpha_1} + \varphi_1\, \mathfrak{m}_{+\alpha_1}}_{A_1^{(L)}} + \underbrace{\mathfrak{m}_{-\alpha_3} - \varphi_3\, \mathfrak{m}_{+\alpha_3}}_{A_1^{(R)}}$$

Reid's Pagoda ($k = 2$): Handling Monopoles via 3d Mirror Symmetry

$$\delta W = \mathsf{m}_{-\alpha_1} + \varphi_1\,\mathsf{m}_{+\alpha_1} + \mathsf{m}_{-\alpha_3} - \varphi_3\,\mathsf{m}_{+\alpha_3}.$$

Affine A_3 quiver

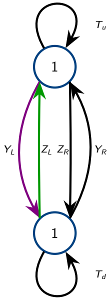




$\Longrightarrow$

3d mirror
+
integrate
out

Effective 2-node quiver



$$W_{\text{eff}} = (T_d - T_u)\,(Y_L Z_L + Y_R Z_R) - \tfrac{2}{3}\left(T_d^3 - T_u^3\right)$$

Construct gauge invariants and impose relations from $W_{\text{eff}} \longrightarrow$ Reid's pagoda hypersurface equations.

Reproduced quiver and superpotential of [Cachazo-Katz-Vafa,Aspinwall-Katz].

Simple Flop of Length 2: Geometry and Higgs Field

Threefold (deformed D_4):

$$x^2+zy^2-(z+4w)\left[\,16zw^2+(z+4w-4w^2)^2\,\right]=0$$

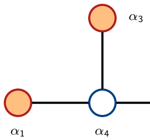
a single $\mathbb{CP}^1$, length $\ell=2$.

$$\Phi \in A_1^{(\alpha_1)} \oplus A_1^{(\alpha_2)} \oplus A_1^{(\alpha_3)} \oplus \langle \alpha_4^* \rangle \subset D_4$$

Higgs field $\Phi(w)$:

$$\begin{pmatrix} w/2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4w & w/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & w & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -w/2 & -4w & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -w/2 & 0 & 0 \\ 0 & 0 & 0 & -w & 0 & 0 & 0 & -w \\ 0 & 0 & w & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

D_4 **quiver** — three external nodes coloured, central α_4 resolvable

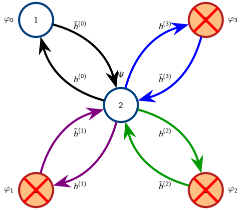


Superpotential deformation involves monopole operators relative to the coloured nodes.

Simple Flops of Length 2

D_4 quiver with δW

three coloured A_1 blocks

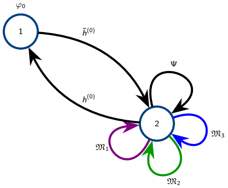


$\Longrightarrow$

3d mirror + integr. out

Effective quiver

three coloured nodes collapsed



$$W_{\text{eff}} = \operatorname{tr}\left[\Psi\left(\sum_{i=1}^{3}\mathfrak{M}_i + h^{(0)}\tilde{h}^{(0)}\right)\right] - \varphi_0\,\tilde{h}^{(0)}h^{(0)} - \sum_{i=1}^{3}\frac{1}{3c_i}\operatorname{tr}\left(\mathfrak{M}_i^3\right) + \operatorname{tr}\Psi^2 - 2\varphi_0^2 \qquad (c_1 = 4,\, c_2 = c_3 = 1)\,.$$

Non-Resolvable (A_2, D_4): Geometry and Higgs Field

Threefold (deformed D_4):

$$x^2 + zy^2 + z^3 + w^3 = 0$$

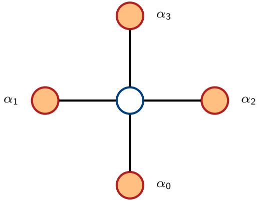
isolated singularity, no crepant resolution.

$$\Phi \in A_1^{(\alpha_1)} \oplus A_1^{(\alpha_2)} \oplus A_1^{(\alpha_3)} \oplus A_1^{(\alpha_0)} \subset D_4$$

Higgs field $\Phi(w)$:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & -\frac{3}{4}w & 0 & 0 \\ \frac{1}{4}w & 0 & 0 & 0 & \frac{3}{4}w & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{4}w & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -\frac{1}{4}w & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{4}w & 0 & 0 & 0 & -\frac{1}{4}w \\ 0 & 0 & \frac{1}{4}w & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

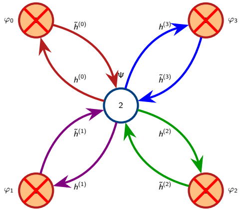
D_4 **quiver** — all four external nodes coloured, no resolvable sphere



Superpotential deformation involves monopole operators relative to all four coloured nodes.

D_4 quiver with δW

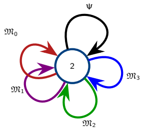
all four external nodes coloured: no resolvable sphere



$\Longrightarrow$ 3d mirror + integr. out

Effective quiver

all external nodes collapsed



one $U(2)$ node

$$W_{\text{eff}} = \text{tr}\left[\Psi \sum_{i=0}^{3} \mathfrak{M}_i\right] - \sum_{i=0}^{3} \frac{1}{3c_i}\,\text{tr}\left(\mathfrak{M}_i^3\right) \qquad \left(c_1 = c_2 = c_3 = \tfrac{1}{4},\ c_0 = -\tfrac{3}{4}\right)$$

Conclusions

- D2-brane probe of a class of non-toric singularities.
- A **systematic framework** to obtain quiver and superpotential for cDV threefolds, even for non-resolvable ones, via Higgs field $\Phi(w)$.
- Quiver-collapsing mechanism match with Cachazo-Katz-Vafa and subsequently used by Karmazyn for his classification of cDV threefolds via noncommutative algebras.
- What the 3d approach adds: A genuine superpotential directly, with **no restriction on the type of cDV**.

Open problems:

- E-type quivers: collapsing non-abelian nodes make both the monopole deformations and the mirror-symmetry procedure harder.
- Probing with N D2-branes, again need non-abelian generalization.

Thank you!

Example: The Conifold

Setup: non-monodromic A_1 fibration.

Higgs field and threefold equation:

$$\Phi(w) = \begin{pmatrix} w & 0 \\ 0 & -w \end{pmatrix}, \quad uv = z^2 - w^2$$

Deformation of the $\widetilde{A}_1$ quiver:

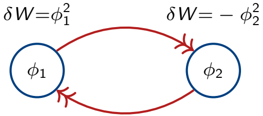
$$\delta W_1 = \phi_1^2, \qquad \delta W_2 = -\phi_2^2$$

Full superpotential (Klebanov–Witten):

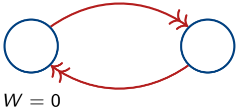
$$W = \phi_-(q_1\tilde{q}_1 - q_2\tilde{q}_2 + \phi_+)$$

After integrating out $\phi_\pm$: $W = 0$, $\widetilde{A}_1$ quiver $\Rightarrow$ $\mathcal{M}_{\mathrm{HB}} \simeq \{uv = z^2 - w^2\}$ ✓

$\widetilde{A}_1$ quiver:



After integrating out:



Standard D-brane probe of the conifold.