Higgs branch and VOA of 4d N=2 SCFTs from IIB

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Conclusions

- We study the Higgs branch and associated VOA of 4d $\mathcal{N} = 2$ SCFTs $\mathcal{T}_X^{\rm 4d}$ from the geometric engineering of IIB on a non-compact CY3 X with an isolated hypersurface singularity at the origin.

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(2) For $X: x_3^7 + x_4^5 x_3 + x_2^3 + x_1^2 = 0$, $\mathcal{T}_X^{\rm 4d}$ has a one-dimensional Higgs branch, which is a type E_8 Kleinian singularity. (First examples?) VOA is an affine E_8 type W-algebra.

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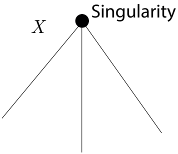
(3) For a large class of singularities, derive the BPS quiver of $\mathcal{T}_X^{\rm 4d}$ from intersection form of the deformed CY3 $\widehat{X}$. Schur index = the vacuum character of VOA.

4d $\mathcal{N} = 2$ SCFTs from IIB on CY3

• Consider IIB superstring on $\mathbb{R}^{3,1} \times X$, where X develops an isolated canonical threefold singularity at the origin



• Geometrically engineer a 4d $\mathcal{N} = 2$ SCFT $\mathcal{T}_X^{\mathrm{4d}}$ (Shapere, Vafa 99')

• Large classes of such singularities, e.g. hypersurface singularities, complete intersection singularities, toric CY3 . . . , leading to a huge class of 4d $\mathcal{N} = 2$ SCFTs (Xie, Yau 15')(Wang, Xie, Yau, Yau 16')(Chen, Xie, Yau, Yau, Zuo 17')(Closset, Schafer-Nameki, YNW 20' 21'). . .

Coulomb branch

- How do we learn about the physical data of $\mathcal{T}_X^{\mathrm{4d}}$ from X?

(1) Deformation $\widehat{X}$ of X, new 3-cycles $\to$ CB of $\mathcal{T}_X^{\mathrm{4d}}$

- CB spectrum: from the generators of the Milnor ring of X
- Take the example of isolated hypersurface singularity (IHS)

$$F(x_1,x_2,x_3,x_4)=0\ ,\ \partial_{x_i}F=0\,.$$

- Deformation of quasi-homogeneous IHS $F(x)=0$: $\widehat{X}$:

$$F(x)\to F(x)+\sum_{l=1}^{\mu}t_l x^{m_l}\,.$$

x^{m_l} are the monomial generators of the Milnor ring

$$\mathcal{M}(F)=\mathbb{C}[x_1,x_2,x_3,x_4]/(\partial_i F)$$

- Quasi-homogeneous: each variable x_i has a weight q_i, such that $F(x)$ has weight 1.

Coulomb branch

• The monomials x^{m_l} correspond to superconformal operators with scaling dimension(Shapere, Vafa 99')

$$\Delta = \frac{1 - q(x^{m_l})}{\sum_i q_i^{-1} - 1} .$$

(1) Operators with $\Delta > 1$: CB parameters of $\mathcal{T}_X^{4d}$
(2) Operators with $\Delta = 1$: flavor current operators of the Cartan subalgebra of G_F

• Use CB spectrum, can compute the central charges for $\mathcal{T}_X^{4d}$(Shapere, Tachikawa 08')(Xie, Yau 15'):

$$a = \frac{R(A)}{4} + \frac{R(B)}{6} + \frac{5\hat{r}}{24} , \quad c = \frac{R(B)}{3} + \frac{\hat{r}}{6} .$$

$$R(A) = \sum_{\Delta_l > 1} (\Delta_l - 1) , \quad R(B) = \frac{\mu}{4(\sum_{i=1}^4 q_i - 1)} = \frac{1}{4}\mu\Delta_{\max} ,$$

Coulomb branch

• BPS quiver defined on 4d CB, i.e. IIB on the deformed $\widehat{X}$.

• On the CB, BPS states have electric and magnetic charges, and mutual Dirac pairings

$$n_{i,j} = q_i m_j - q_j m_i$$

• In IIB, BPS states are constructed from D3-branes wrapping special-Lagrangian 3-cycles Σ_i on $\widehat{X}$,

$$n_{i,j} = \Sigma_i \cdot \Sigma_j$$

• No practical algorithm to compute the BPS quiver for a generic X, but many examples are studied (Alim, Cecotti, Cordova, Espahbodi, Rastogi, Vafa 11')(Cecotti, Del Zotto 11' 12' 13' 15')...

(2) Crepant resolution $\widetilde{X}$ of X, new 2/4-cycles $\rightarrow$ HB of $\mathcal{T}_X^{\rm 4d}$

• Quaternionic dimension of HB of $\mathcal{T}_X^{\rm 4d}$, $\widehat{d}_{H,4d}$=# of Kähler parameters of $\widetilde{X}$, i.e. $h_{1,1}(\widetilde{X})$.

• Related to the rank r_{5d} and flavor rank f of the 5d SCFT from M-theory on the X (Closset, Schafer-Nameki, YNW 20')...:

$$\widehat{d}_{H,4d} = r_{5d} + f$$

• How to derive the Higgs branch of the 4d $\mathcal{N}=2$ SCFT from the geometry?

This work: analyze examples.

• How to relate the CB and HB of the same 4d $\mathcal{N}=2$ SCFT?

The bridge: vertex operator algebra (VOA)

VOA

• The vertex operator algebra (VOA) is an important information for 4d SCFT, corresponding to a 2d non-unitary CFT with $c_{2d} = -12c_{4d}$ (Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees 13')...

• HB of 4d SCFT = Associated Variety of the VOA

• Schur index of 4d SCFT $\leftrightarrow$ Vacuum character $X_V(\tau)$ of the VOA, can be computed from the Schur index of the BPS quiver of $\mathcal{T}_X^{\mathrm{4d}}$, which is a CB data!

• a, c and CB spectrum of $\mathcal{T}_X^{\mathrm{4d}}$ also contains the information of VOA

$$a - c = -\frac{1}{48}\mathcal{G}(V)$$

• $\mathcal{G}(V)$ is the asymptotic growth of the character of the VOA

$$X_V(\tau) \sim \exp\left(\frac{\pi i \mathcal{G}(V)}{12\tau}\right) .$$

Examples 1: generalized conifold

(1) Generalized conifold, $\mathcal{T}_X^{\rm 4d} = (A_1, A_{2N-1})$ Argyres-Douglas theory

$$X : x_1^2 + x_2^2 + x_3^2 + x_4^{2N} = 0\,.$$

- After the small resolution, the exceptional curve C has normal bundle $N_{C|\widetilde{X}} = \mathcal{O} \oplus \mathcal{O}(-2)$. There are a multiplicity of N 2-cycles shrinking to zero at the singularity, in the same homology class.
- Expand the IIB fields and Kähler form along the single $\omega_1^{(1,1)}$ as

$$B_2 = b_1 \omega_1^{(1,1)}\,,\ C_2 = c_1 \omega_1^{(1,1)}\,,\ C_4 = d_{2,1} \wedge \omega_1^{(1,1)}\,,\ J = J_1 \omega_1^{(1,1)}$$

- Hypermultiplet scalars: $(b_1, c_1, d_1 = (\text{EM dual of } d_{2,1}), J_1)\,.$

Examples 1: generalized conifold

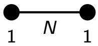
- The metric of scalars in the hypermultiplet (Ooguri, Vafa 96')(Saueressig, Vandoren 07'):

$$\mathrm{d}s^2 = \tau_2^{-2}[V^{-1}(\mathrm{d}d_1 - \vec{A}\cdot \mathrm{d}\vec{y})^2 + V|\mathrm{d}\vec{y}|^2].$$

with $\vec{y} = (-(c_1 - \tau_1 b_1), z\tau_2, \bar{z}\tau_2)$, $z = b_1 + iJ_1$

$$V = N\left(\frac{1}{4\pi}\ln\left(\frac{1}{z\bar{z}}\right) + \frac{1}{2\pi}\sum_{m\neq 0} K_0(2\pi\tau_2|mz|)\mathrm{e}^{2\pi\mathrm{i}m(c_1-\tau_1 b_1)}\right)$$

- Singular limit $\rightarrow \mathcal{M}_H(\mathcal{T}_X^{4\mathrm{d}}) = \mathbb{C}^2/\mathbb{Z}_N$.
- Consistent with the result from magnetic quiver (Xie 12')

$\mathrm{MQ}^{(4)} =$ 

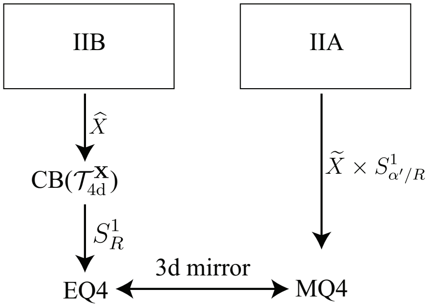
- Associated VOA is known(Song, Xie, Yan 17')(Beem, Rastelli 17')

$$W_{-\frac{N^2}{N+1}}(\mathfrak{sl}_N, [N-1, 1])\,.$$

Examples 2: terminal singularities

(2) General terminal singularities

- $\widetilde{X}$ has f 2-cycles and no compact 4-cycle
- Read off the magnetic quiver of $\mathcal{T}_X^{4d}$, $\text{MQ}^{(4)}$ from Gopakumar-Vafa (GV) invariants! Following T-duality arguments in (Seiberg, Shenker 96')(Hori, Ooguri, Vafa 97')



Examples 2: terminal singularities

- IIA on $\widetilde{X}$: f $U(1)$ gauge fields in 4d:

$$C_3 = \sum_{\alpha=1}^{f} A_\alpha^{(4d)} \wedge \omega_\alpha^{(1,1)},$$

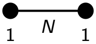
- Further reduce on $S^1_{\alpha'/R}$:

$$-\int_{\mathbb{R}^{2,1}\times\widetilde{X}\times S^1} \frac{1}{2} dC_3 \wedge \star dC_3 = -\sum_{\alpha=1}^{f}\int_{\mathbb{R}^{2,1}} \frac{1}{2}\left(dA_\alpha \wedge \star dA_\alpha\right) \int_{\widetilde{X}} \frac{\alpha'}{R} \omega_\alpha^{(1,1)} \wedge \star\omega_\alpha^{(1,1)}$$
$$= -\sum_{\alpha=1}^{f}\int_{\mathbb{R}^{2,1}} \frac{1}{2}\left(dA_\alpha \wedge \star dA_\alpha\right) \frac{\alpha'}{R} \mathrm{Vol}(-K_{D_\alpha}).$$

- D_α are non-compact divisors, in the limit of $\frac{\alpha'}{R} \to 0$, $\mathrm{Vol}(-K_{D_\alpha}) \to \infty$, and $\frac{\alpha'}{R}\mathrm{Vol}(-K_{D_\alpha})$ finite, get 3d $\mathcal{N}=4$ $U(1)^f$ quiver gauge theory with finite gauge coupling!

Examples 2: terminal singularities

- In $\mathrm{MQ}^{(4)}$, there are also charged hypermultiplets from D2-brane wrapping 2-cycles C, with charge $q_\alpha = C \cdot D_\alpha$ under the α-th $U(1)$ gauge group
- These modes are exactly characterized by the $g = 0$ GV invariants of the degree q_α!
- E.g. in the example (A_1, A_{2N-1}) Argyres-Douglas theory
- non-zero GV invariant: $n^{g=0}_{\mathbf{d}=1} = N$, giving rise to

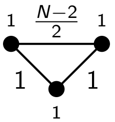
$\mathrm{MQ}^{(4)} =$ 

- Can be generalized to compute the $\mathrm{MQ}^{(4)}$ for many X, following (Collinucci, Sangiovanni, Valandro 21')(Collinucci, De Marco, Sangiovanni, Valandro 21', 22')(De Marco, Sangiovanni, Valandro 22')..., including AD (A_m, A_n) and (A_m, D_n) theories

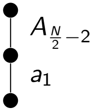
Associated VOA from geometry

(a) (A_1, D_N) theory with even N: $x^2 + y^2 + z^{N-1} + w^2 z = 0$

- $\text{MQ}^{(4)}$ from GV invariants and crepant resolution:



- Hasse diagram of the 4d HB:



- From the known Class S description, the associated VOA is $(N = 2k-2)$ $W_{-k+\frac{k}{k-1}}(\mathfrak{sl}_k, [k-2, 1^2])$ (Song, Xie, Yan 17'), whose associated variety is consistent with the MQ4 computation

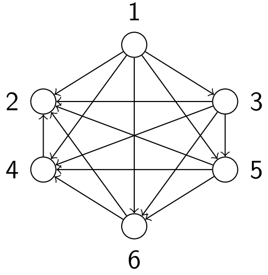
Associated VOA from geometry

(b) $D_N^N[k]$ singularity given by $x^2 + zy^2 + z^{N-1} + w^k y = 0$

- Derive the BPS quiver, from the intersection of 3-cycles on $\widehat{X}$
- Following Orlik and Randell's conjecture (1977), applicable for

$$f(z_0, \ldots, z_n) = z_0^{a_0} + z_0 z_1^{a_1} + \ldots + z_{n-1} z_n^{a_n}, \quad n \geq 1.$$

- For the example of $N = 8$, $k = 1$, the BPS quiver is given by



Associated VOA from geometry

- Such 4d $\mathcal{N}=2$ SCFT has a trivial, 0-dimensional HB, corresponds to the associated variety of a lisse VOA.
- We can compute its vacuum character $\mathcal{I}(q)$, i.e. the Schur index of the BPS quiver following (Cordova, Shao 15')(Cordova, Gaiotto, Shao 16'):

$$\mathcal{I}(q,z_1,\cdots,z_n)=(q)_\infty^{2r}\mathrm{Tr}\left[\mathcal{O}(q)\right]\left(\mathrm{Tr}[X_{\gamma_{f_1}}],\cdots,\mathrm{Tr}[X_{\gamma_{f_i}}]\right)\,,$$

where $\mathcal{O}(q)$ is the Kontsevich-Soibelman operator determined by the quiver mutation

- For general N and $k=1$,

$$\mathcal{I}(q)=(q;q)_\infty^{N-2}\sum_{n_i=0}^{\infty}\prod_{i=1}^{N-2}\frac{(-q)^{n_i}}{(q;q)_{n_i}^2}q^{n_i\cdot A\cdot n_i^T}$$

- For $N=8$, $k=1$:

$$1-9q^2+7q^3+18q^4-27q^5+q^6+57q^7-45q^8+\mathcal{O}(q^9)\,.$$

Example 3: Cases with compact 4-cycles in $\widetilde{X}$

- In the IIA derivation the gauge coupling of $\mathrm{MQ}^{(4)}$ $1/g^2 \sim \frac{\alpha'}{R}\mathrm{Vol}(S_i)$, $g \to \infty$ when $\alpha'/R \to 0$. Strongly coupled dynamics arise, not clear how to derive $\mathrm{MQ}^{(4)}$
- Discuss special cases

(a) One-dimensional 4d HB (Closset, Schafer-Nameki, YNW 21'):

sing.	$F(x)$	f	r_{5d}	$d_{H,5d}$	$\widehat{r}_{4d}$	$\widehat{d}_{H,4d}$
sing(E_8)	$x_3^7 + x_4^5 x_3 + x_2^3 + x_1^2$	0	1	29	29	1
sing(E_7)	$x_2^5 + x_3^3 x_2 + x_3 x_4^3 + x_1^2$	0	1	17	17	1
sing(E_6)	$x_2^4 + x_3^2 x_2 + x_1^3 + x_3 x_4^2$	0	1	11	11	1

- The resolved $\widetilde{X}$ is smooth, but with a singular divisor S, that is a del Pezzo surface dP_n with type E_n surface singularity (flavor mass parameters are frozen).
- M-theory on X gives a rank-1 5d SCFT $\mathcal{T}_X^{\mathrm{5d}}$ with HB dimension $d_{H,5d} = h^\vee(E_n) - 1$

Example 3: Cases with compact 4-cycles in $\widetilde{X}$

- Conjecture: $\mathcal{M}_H(\mathcal{T}_X^{\mathrm{4d}})$ for X =sing(E_n) is the Kleinian singularity $\mathbb{C}^2/\Gamma_{E_n}$!
- Comparing with rank-1 E_n theory, the 5d theories $\mathcal{T}_X^{\mathrm{5d}}$ has the same HB, which is the minimal nilpotent orbit of E_n, coming from the magnetic quiver $\mathrm{MQ}^{(5)}$ which is an affine E_n shaped $U(N)$ quiver.
- Because the flavor rank $f=0$, $\mathcal{T}_X^{\mathrm{5d}}$ and $\mathcal{T}_X^{\mathrm{4d}}$ are 3d mirror to each other after compactified to 3d $\mathcal{N}=4$!

sing.	$F(x)$	f	r_{5d}	$d_{H,5d}$	$\widehat{r}_{4d}$	$\widehat{d}_{H,4d}$
sing(E_8)	$x_3^7+x_4^5x_3+x_2^3+x_1^2$	0	1	29	29	1
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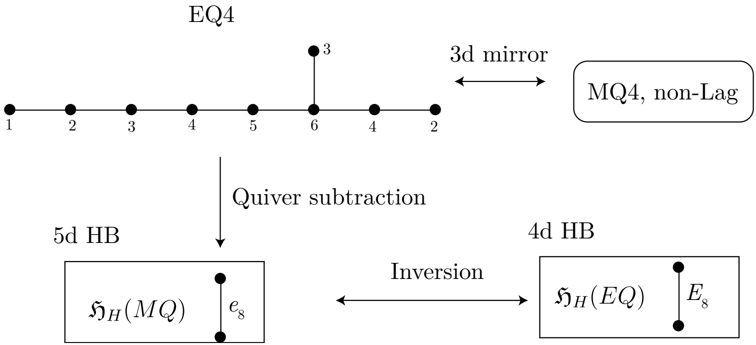
sing(E_6)	$x_2^4+x_3^2x_2+x_1^3+x_3x_4^2$	0	1	11	11	1

- Hence after the reduction of $\mathcal{T}_X^{\mathrm{4d}}$ to S^1, we get an affine E_n shaped $U(N)$ quiver!

Inversion

- The affine E_n shaped $U(N)$ quiver has the Higgs branch $\mathbb{C}^2/\Gamma_{E_n}$, related to the minimal nilpotent orbit of E_n by Inversion (Hanany, Grimminger 20'). Related to the symplectic dualities in math.



Hint from VOA

- The associated VOA is conjectured to be

$$\mathcal{W}_{k_{2d}}(E_n, \mathcal{O}_{\text{subregular}}), \quad k_{2d} = -h^\vee + \frac{h^\vee}{h^\vee + 1} \, .$$

- Central charge matches that of the 4d SCFT, from deformed geometry

$$c_{2d} = -\frac{1342}{13}, -\frac{3706}{19}, -\frac{13978}{31} \text{ for } E_6, E_7, E_8 \, .$$

- Associated varieties are $\mathbb{C}^2/\Gamma_{E_n}$. ✓

Coulomb branch spectrum

- CB spectrums are completely non-integral!
  - sing(E_8) : $x_3^7 + x_4^5 x_3 + x_2^3 + x_1^2$:

$$\left\{ \frac{32}{31}, \frac{38}{31}, \frac{42}{31}, \frac{44}{31}, \frac{48}{31}, \frac{50}{31}, \frac{54}{31}, \frac{60}{31}, \frac{66}{31}, \frac{68}{31}, \frac{72}{31}, \frac{74}{31}, \frac{78}{31}, \frac{80}{31}, \frac{84}{31}, \frac{90}{31}, \frac{102}{31}, \frac{104}{31}, \frac{108}{31}, \frac{110}{31}, \frac{114}{31}, \frac{120}{31}, \frac{138}{31}, \frac{140}{31}, \frac{144}{31}, \frac{150}{31}, \frac{174}{31}, \frac{180}{31}, \frac{210}{31} \right\}$$

  - sing(E_7) : $x_2^5 + x_3^3 x_2 + x_3 x_4^3 + x_1^2$:

$$\left\{ \frac{20}{19}, \frac{24}{19}, \frac{26}{19}, \frac{28}{19}, \frac{30}{19}, \frac{32}{19}, \frac{36}{19}, \frac{42}{19}, \frac{44}{19}, \frac{46}{19}, \frac{48}{19}, \frac{50}{19}, \frac{54}{19}, \frac{66}{19}, \frac{68}{19}, \frac{72}{19}, \frac{90}{19} \right\}$$

  - sing(E_6) : $x_2^4 + x_3^2 x_2 + x_1^3 + x_3 x_4^2$:

$$\left\{ \frac{14}{13}, \frac{17}{13}, \frac{18}{13}, \frac{20}{13}, \frac{21}{13}, \frac{24}{13}, \frac{30}{13}, \frac{32}{13}, \frac{33}{13}, \frac{36}{13}, \frac{48}{13} \right\}$$

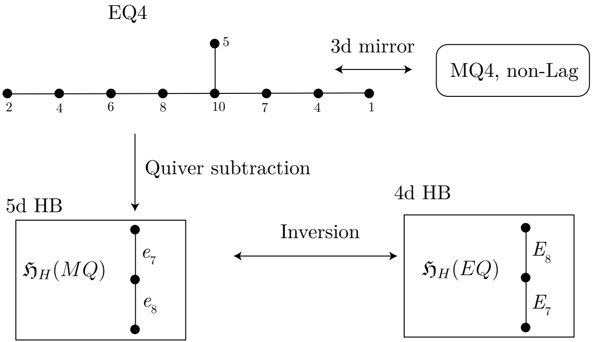
Example 3: Cases with compact 4-cycles in $\widetilde{X}$

(b) Cases with two compact (but singular) divisors, $f = 0$, $\widehat{d}_{H,4d} = 2$.

$$X : x_1^2 + x_2^5 + x_3^{11} + x_3 x_4^3 = 0$$

- Its resolution and $d_{H,5d}$ are the same as the other singularity

$$X' : x_1^2 + x_2^5 + x_3^{10} + x_3 x_4^3 = 0$$

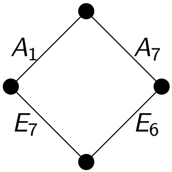


Associated VOA

- From the CB spectrum and central charges, we conjecture the associated VOA to be

$$W_{-30+\frac{30}{31}}(\mathfrak{e}_8, E_8(a_2))\,.$$

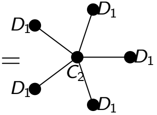
- We also found a number of new VOAs with no W-algebra descriptions, but with associated varieties e.g.



(c) Example: (D_4, D_4) AD theory

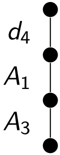
$$x_1^3 + x_2^3 + x_3^3 + x_4^3 = 0\,.$$

- The $\mathrm{MQ}^{(4)}$ for $\mathcal{T}_X^{\mathrm{4d}}$ was proposed (Carta, Giacomelli, Mekareeya, Mininno 21')

$MQ^{(4)} =$  $/\ \mathbb{Z}_2$

- Can compute the HB Hasse diagram from "Decay and Fission"

(Bourget, Sperling, Zhong 23' 24')(Lawrie, Mansi, Sperling, Zhong 24')



- Generalized to (D_m, D_n) AD theory

Future directions

- How to derive the magnetic quiver and HB for $\mathcal{T}_X^{\mathrm{4d}}$ from IIB on more general singularities X, i.e. capture the quantum corrections and derive the metric of the Higgs branch
- Modular properties of the new VOAs
- Local mirror symmetries, i.e. what are the CB, HB and VOA of the 4d $\mathcal{N}=2$ SCFT from IIA on X?
- Thank you very much!