Spin and entanglement in the world of elementary particles

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The world of elementary particles

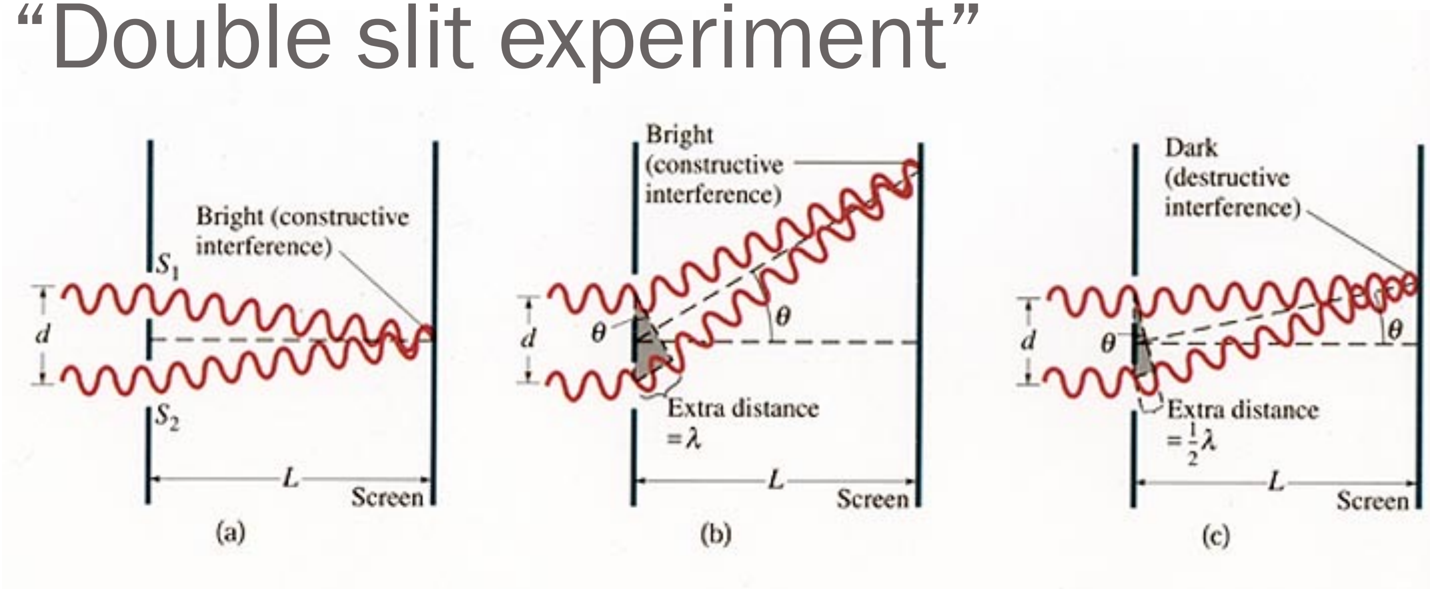
- In APS the name DPF - the division of particles and fields emphasizes the dual (quantum) nature of elementary particles
- Their behavior is described by quantum field theories: QED and QCD
- Quantumness should be engrained in the behavior of elementary particles
- Yet, it is not so easy to observe
- For a typical momentum of 100 GeV/c the precision of measurement is ~1GeV/c, typical position precision is 100 um

$$\Delta x \cdot \Delta p > h/2\pi$$

$$\Delta t \cdot \Delta E > h/2\pi$$

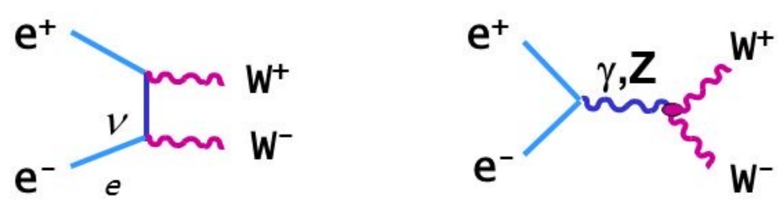
- We are ~10 orders of magnitude away from the uncertainty principle

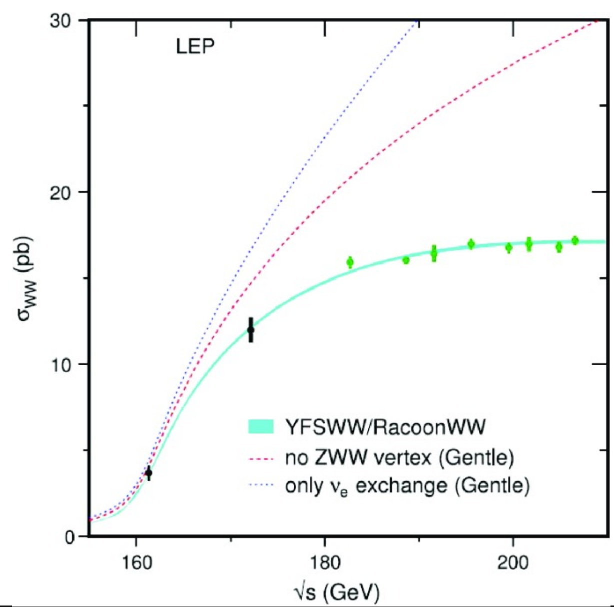
"Double slit experiment"



- Interference is observed in multiple processes

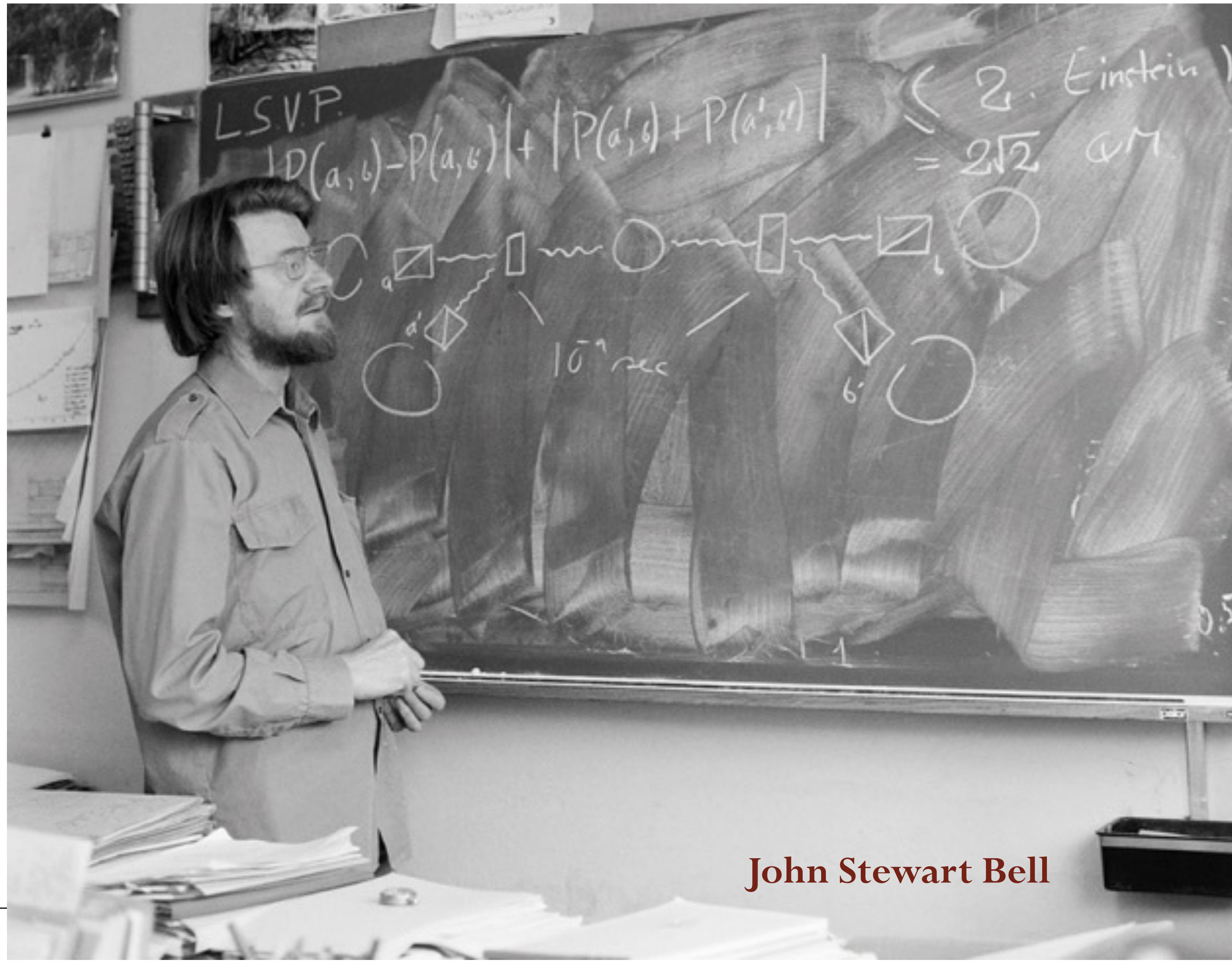
W pair production at LEPII:





- **And then there is spin**

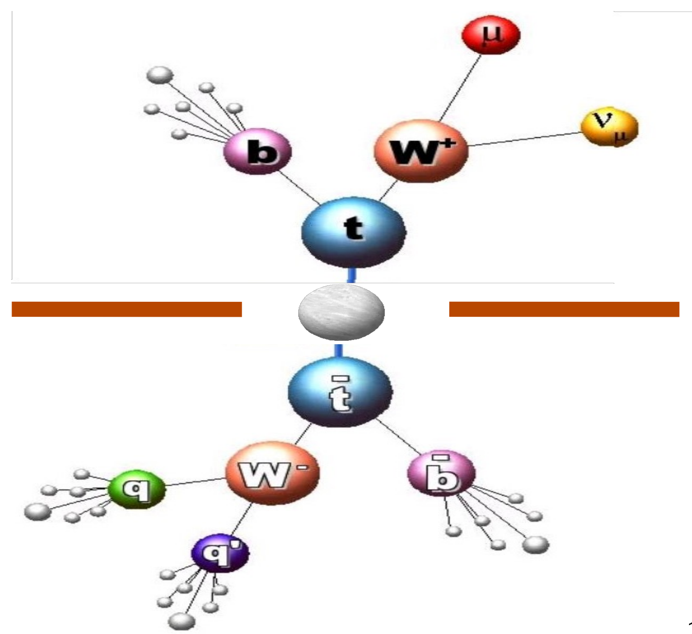
Bell came full circle back to CERN
Will Bell go to BelleII?



John Stewart Bell

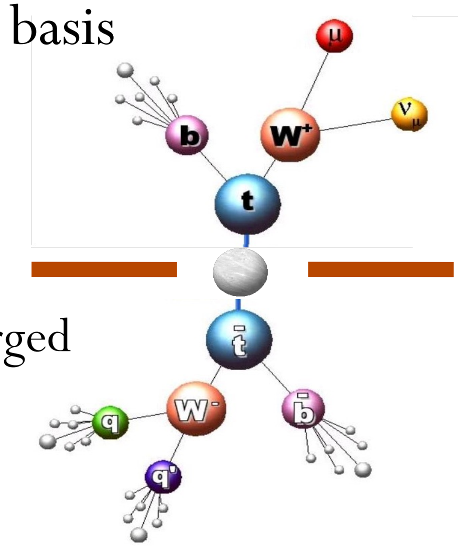
Top-antitop quark pair is a two qubit system

- Top quark has s=1/2, it is a **qubit**
- Top-antitop quark pair is a two-qubit system
- Top quark is very short-lived O(10^{-25}s)< hadronization time O(10^{-24}s), so it decays unlike other quarks before hadronizing (picking up a light quark from "vacuum" to form a bound state – a hadron)
- Top quark spin information is preserved in the angular correlation of its decay products t→Wb, W→lν – or – qq' (=udbar, or csbar)
- When both W bosons decay leptonically – dilepton channel
- When one W boson decays leptonically and the other one hadronically – lepton+jets channel



Spin correlation and entanglement

Polarization, P and spin correlation matrix, C determines the angular distribution of the decay products in the helicity basis as in [1212.4888]



$$\frac{d\sigma}{d\Omega d\bar{\Omega}} = \sigma_{norm}(1+\kappa\vec{P}\cdot\Omega+\bar{\kappa}\vec{\bar{P}}\cdot\bar{\Omega}-\kappa\bar{\kappa}\Omega\cdot C\cdot\bar{\Omega})$$

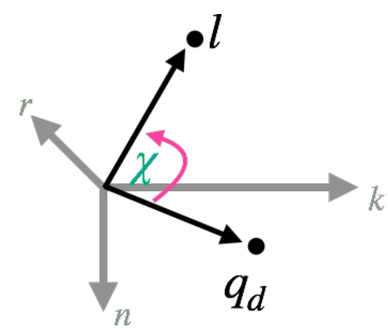
κ - spin analyzing power of top/antitop decay products – max for charged leptons and d-type quarks (d or s)

Ω – unit vector in the direction of the decay product

2x3(P)+3x3(C) =**15 coefficients Q_m**

Or use χ –opening angle between the two decay products in rotated tt frame (a.k.a helicity angle)

$$\frac{d\sigma}{d\cos\chi} = A(1+D\kappa\bar{\kappa}\cos\chi)$$



and $\tilde{\chi}$, where the sign of n-component in one of the decay products is inverted:

$$\frac{d\sigma}{d\cos\tilde{\chi}} = A(1+\tilde{D}\kappa\bar{\kappa}\cos\tilde{\chi})$$

Spin correlation and entanglement in ttbar system

The system is considered **separable** if its density matrix can be factored into individual states

$$\rho = \sum_n p_n \rho_n^t \rho_n^{\bar{t}}$$

Otherwise, it is considered **entangled** → **Peres-Horodecki criterion** [2003.02280]

A. Peres, Phys. Rev. Lett. 77, 1413 (1996).
P. Horodecki, Physics Letters A 232, 333 (1997)

$$\Delta_E = C_{nn} + |C_{rr} + C_{kk}| > 1$$

Entanglement is a result of spin correlation.

There are four pure maximally entangled (Bell) states:

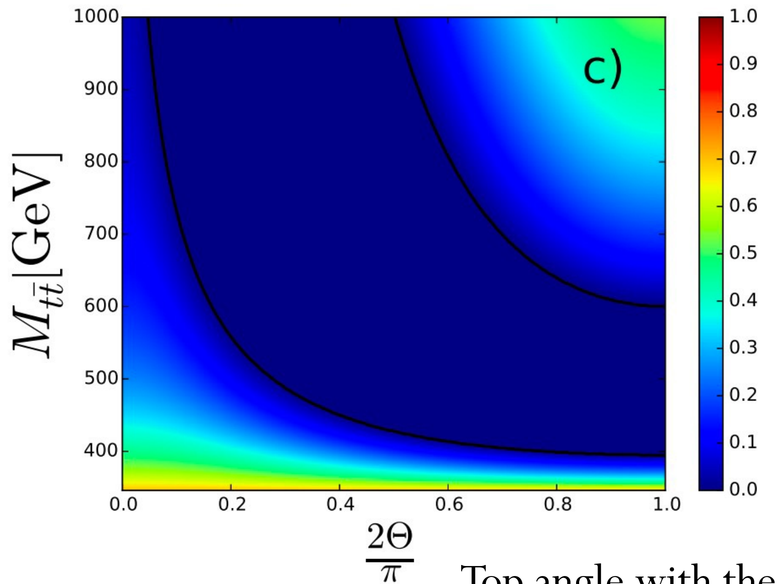
$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle\right),$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right).$$

at low $M_{t\bar{t}}$ singlet pseudoscalar state Ψ^-
Peres-Horodecki criterion

$$\Delta_E = Tr(C) = -3D > 1$$

$$D < -\frac{1}{3}$$



Top angle with the beam axis

at high $M_{t\bar{t}}$ triplet vector state ($\Phi^+ - \Phi^-$, Ψ^+, $\Phi^+ + \Phi^-$)
Peres-Horodecki criterion

$$\Delta_E = C_{nn} - C_{rr} - C_{kk} = 3\tilde{D} > 1$$

$$\tilde{D} > \frac{1}{3}$$

Plot from Afik, De Nova EPJP136(2021)9,907 hep-ph:2003.02280

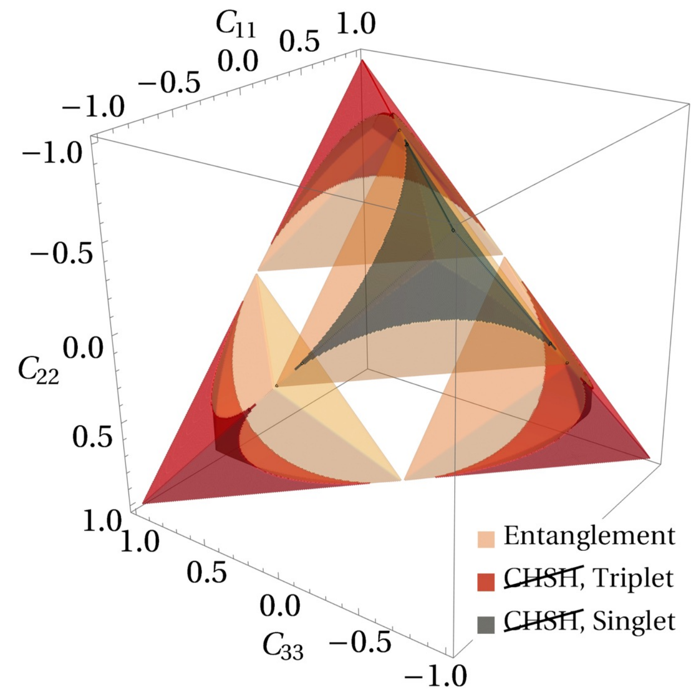
Bell inequality

- **Bell inequality** is formulated based on conventional logic, which is violated in QM
- **Entanglement is a necessary but not sufficient condition for Bell inequality violation**
- It can be phrased in terms of Clauser, Horne, Shimony, and Holt (CHSH) inequality [PRL, 23(15), 1969] which states that measurements a, a' and b, b' on subsystems A and B, respectively (with absolute values <=1) classically must satisfy:

$$\left|\langle ab\rangle - \langle ab'\rangle + \langle a'b\rangle + \langle a'b'\rangle\right| \leq 2$$

- For ttbar system the CHSH can formatted as

2 leading conditions for CHSH violation

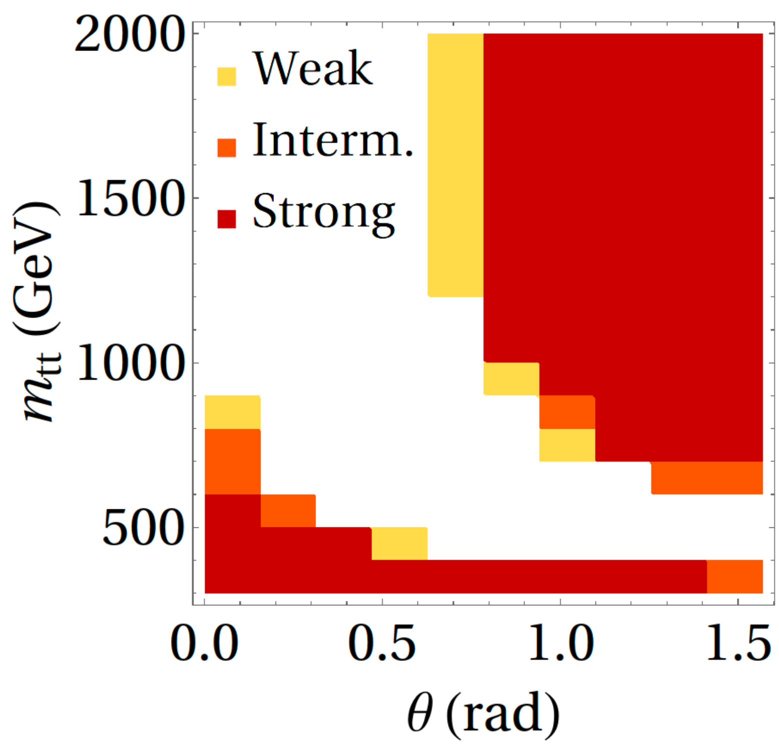


$$B_1 = |C_{rr} - C_{nn}| > \sqrt{2}$$

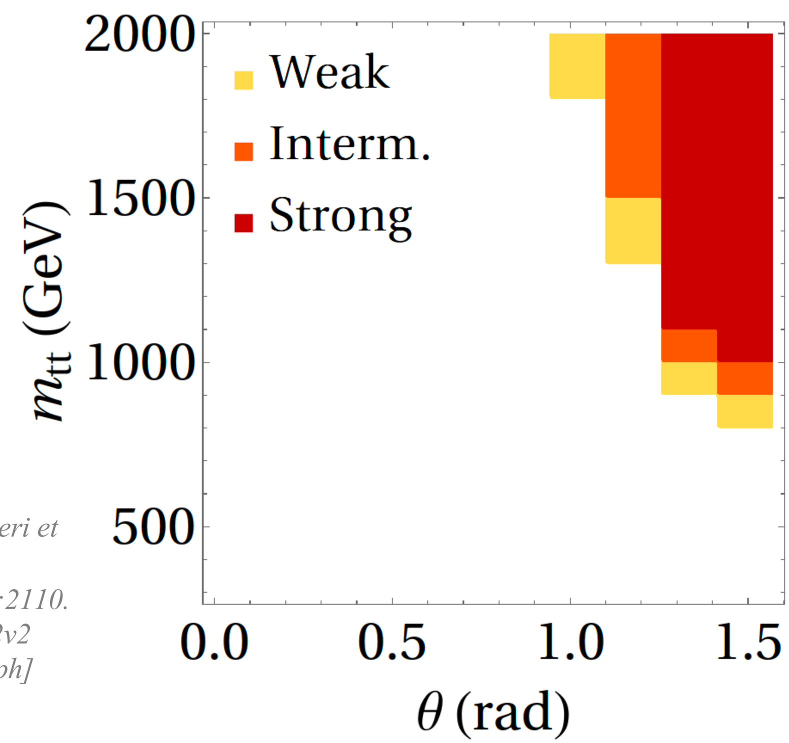
$$B_2 = |C_{rr} + C_{kk}| > \sqrt{2}$$

C Severi et al. arXiv:2110.10112v2 [hep-ph]

Entanglement



Bell inequality violation



C Severi et al. arXiv:2110.10112v2 [hep-ph]

While entanglement can be observed at the threshold (singlet) and in high Mtt (triplet) regions, for the observation of Bell inequality violation we must probe even higher Mtt (>1TeV), central production

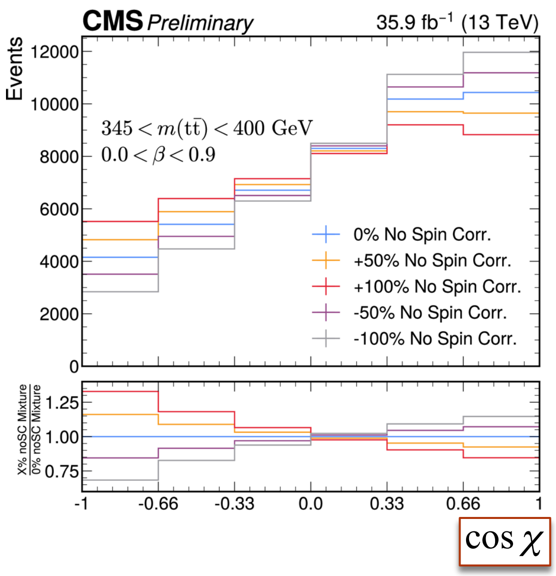
Dilepton channel [ROPP 87 (2024) 117801]

$$\frac{d\sigma}{d\cos\chi} = A(1 + D\kappa\bar{\kappa}\cos\chi)$$

Advantages of dilepton channel:

clean, have 2 charged leptons(max spin analyzing power)

To extract D we measure the distribution in the sensitive variable – cos χ

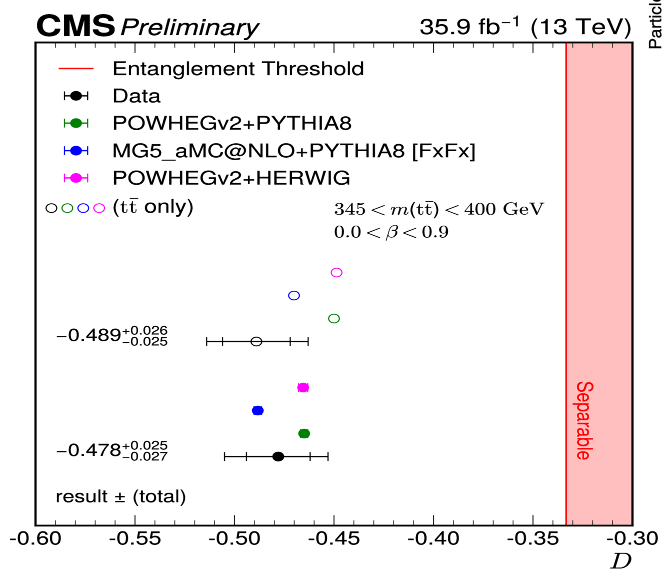


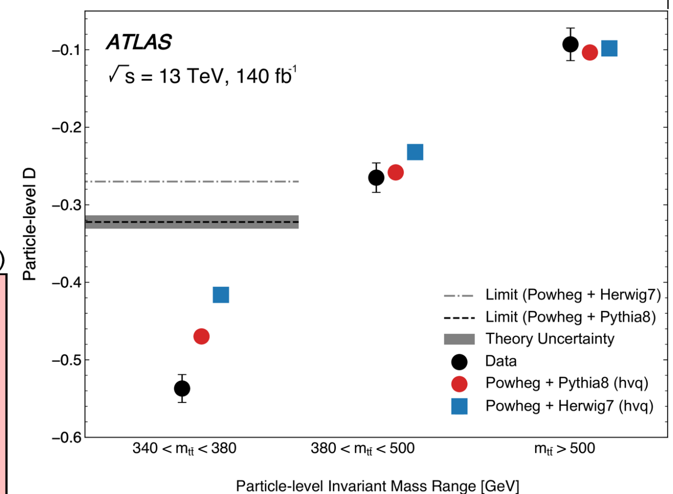
w/out **Toponium**

w/ **Toponium**

~1.5σ tension with the expectation if toponium is not included

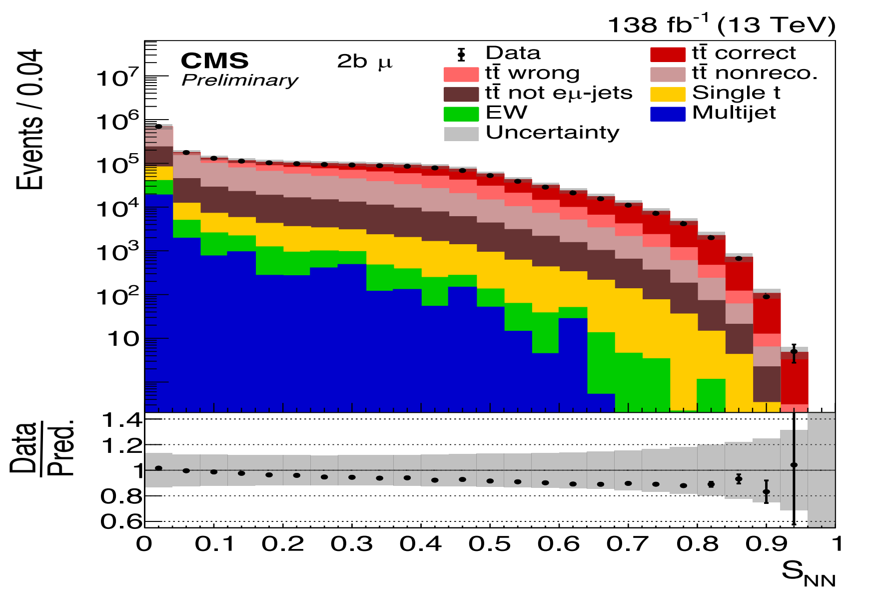
The ttbar entanglement is observed (expected) at the 5.0(4.7)σ level for 345<Mtt<400 GeV, β<0.9



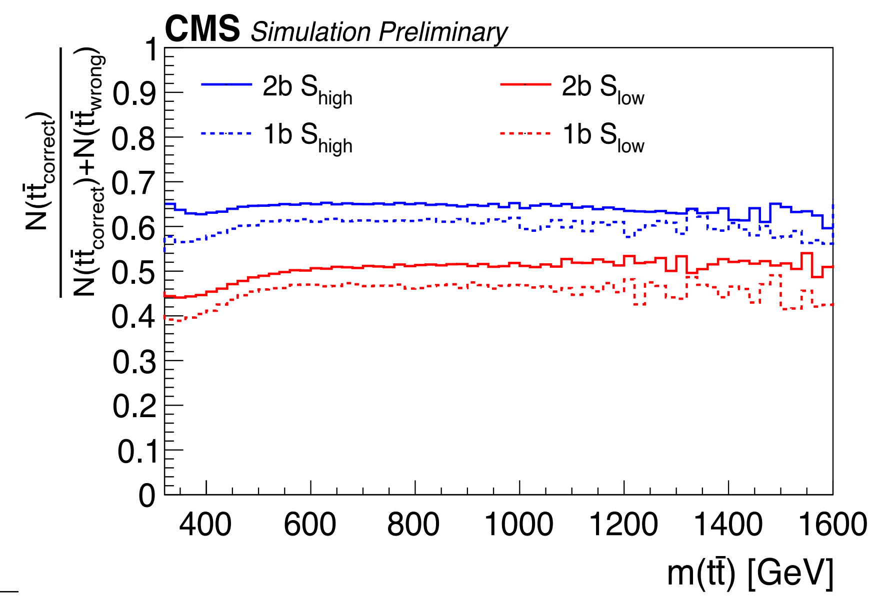


L+jets channel PRD 110 (2024) 112016

- We pursue both strategies – evaluation of the full correlation matrix C and polarization vectors P as well as D and $D\sim$ measurements
- The measurements are done inclusively and differentially in bins of M_{tt}, $cos\theta$ and top quark p_T
- Event reconstruction (jet-parton assignment) is performed using NN
- Lepton+jets have higher statistics, but
- **Major challenge – identify *d*-type jets from W boson decay**
- Divide events into categories based on lepton flavor, number of b-tags, and NN score



Fraction of tt events with correctly assigned jets to partons including d-type quark



Extraction of spin correlation

- We observe the distribution over top quark decay products:

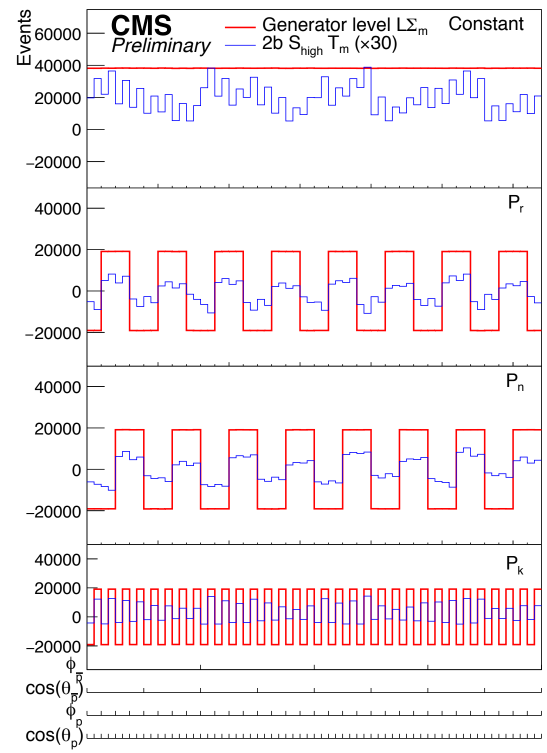
$$\frac{d\sigma}{d\Omega d\bar{\Omega}} = \sigma_{norm}(1 + \kappa\vec{P}\cdot\Omega + \bar{\kappa}\vec{\bar{P}}\cdot\bar{\Omega} - \kappa\bar{\kappa}\Omega\cdot C\cdot\bar{\Omega})$$

- Full measurement of the vectors P and matrix C is performed using templates defined based on the functions of angles of top and antitop decay products

$$\Sigma_m = \{\kappa\sin\theta_p\cos\phi_p, \ldots - \kappa\bar{\kappa}\cos\theta_p\cos\theta_{\bar{p}}\}$$

- The total cross section is a linear combination of these templates with coefficients Q_m that are the components of P and C

$$\Sigma_{tot} = \Sigma_0 + \sum_{m=1}^{15} Q_m \Sigma_m$$



Σ_m Theoretically predicted distributions in angles of decay products

T_m Reconstruction level distributions, which take into the account selection criteria, efficiency and resolution of the detector.

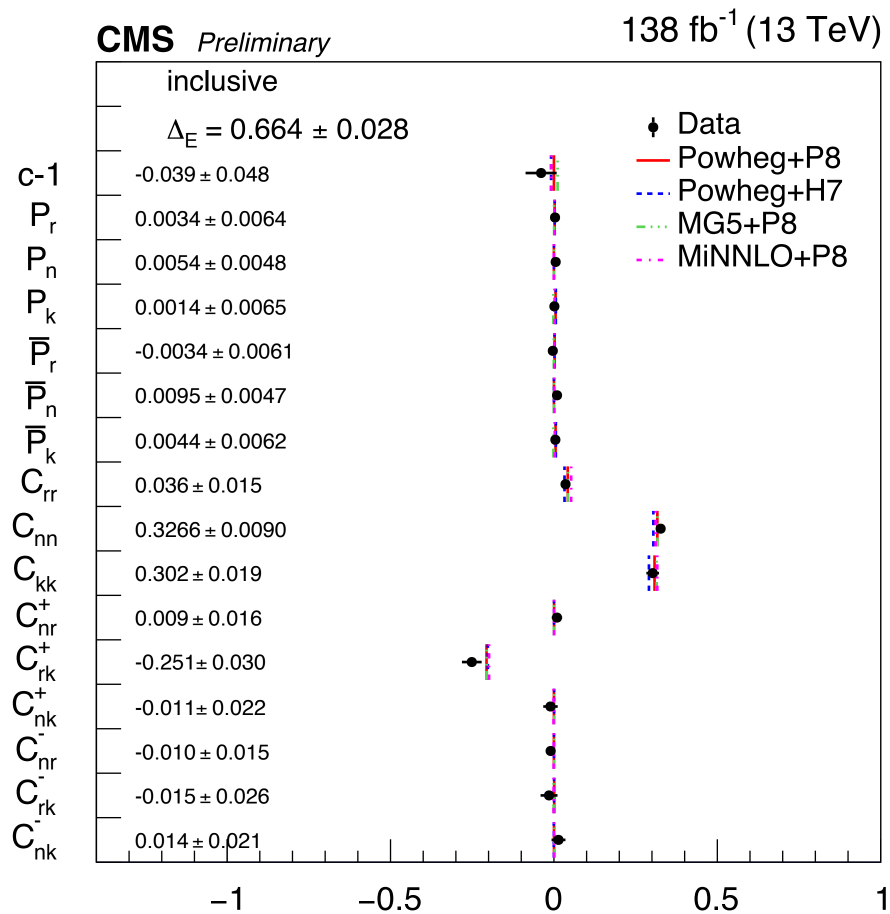
In other words, encode the effects of the environment

Data distribution is fit the sum of T_m with free coefficients – P_i and C_{ij}

PRD 110 (2024) 112016

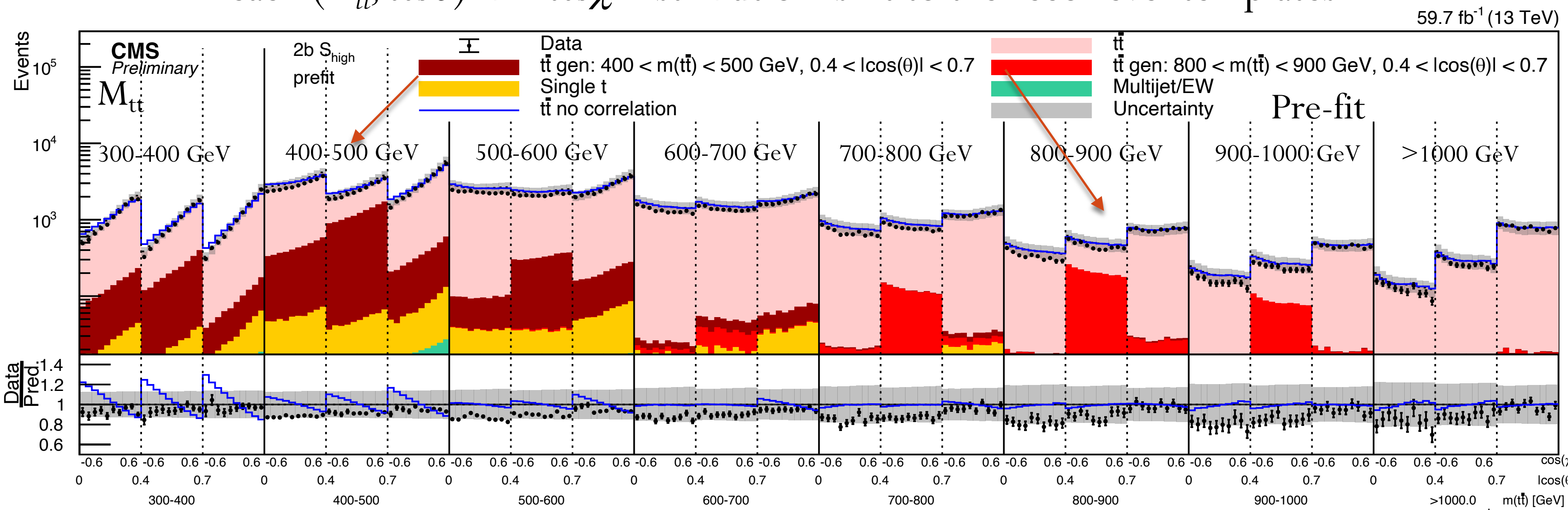
Spin correlation matrix measurement

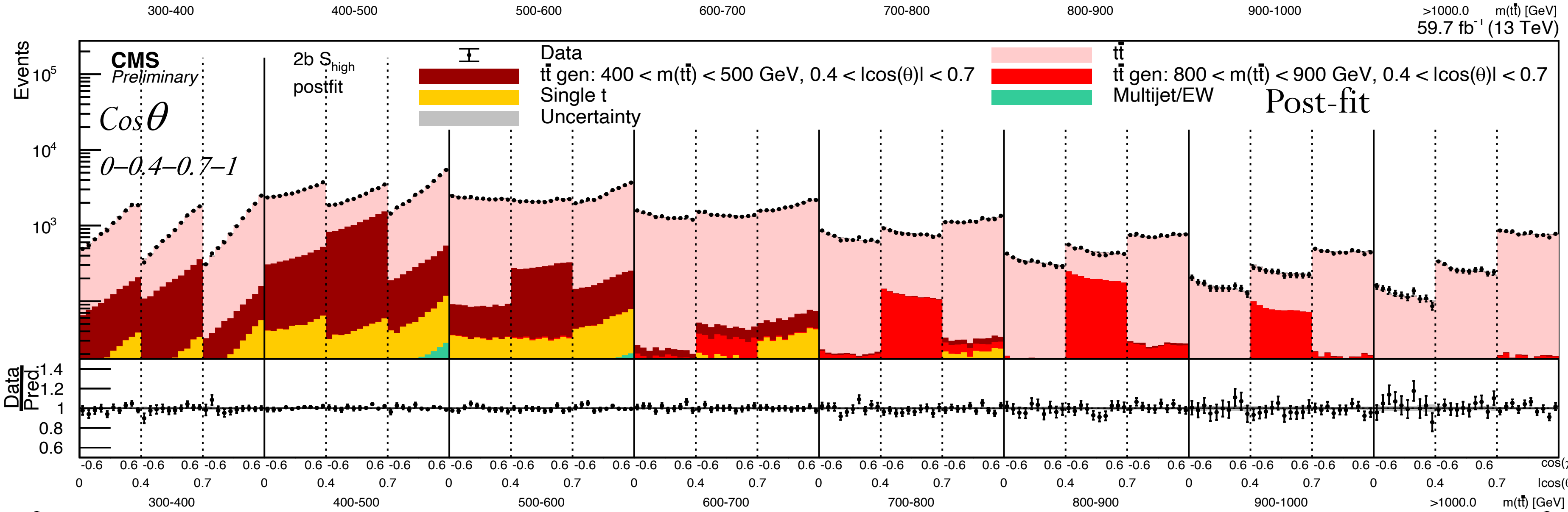
- Full measurement of the P and C is performed inclusively and differentially in bins of M_{tt}, $cos\theta$ and top p_T
- Full covariance matrix will be provided with the published result
- A good agreement with the SM prediction is observed



L+jets: Example of the fit

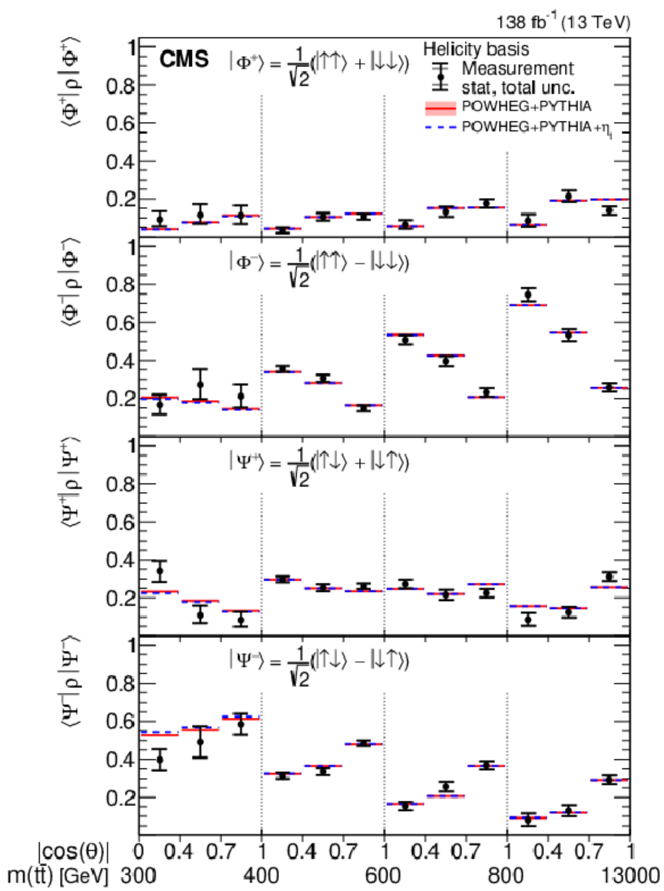
- In each (M_{tt}, $cos\theta$) bin $cos\chi$ distribution is fit to the reco-level templates

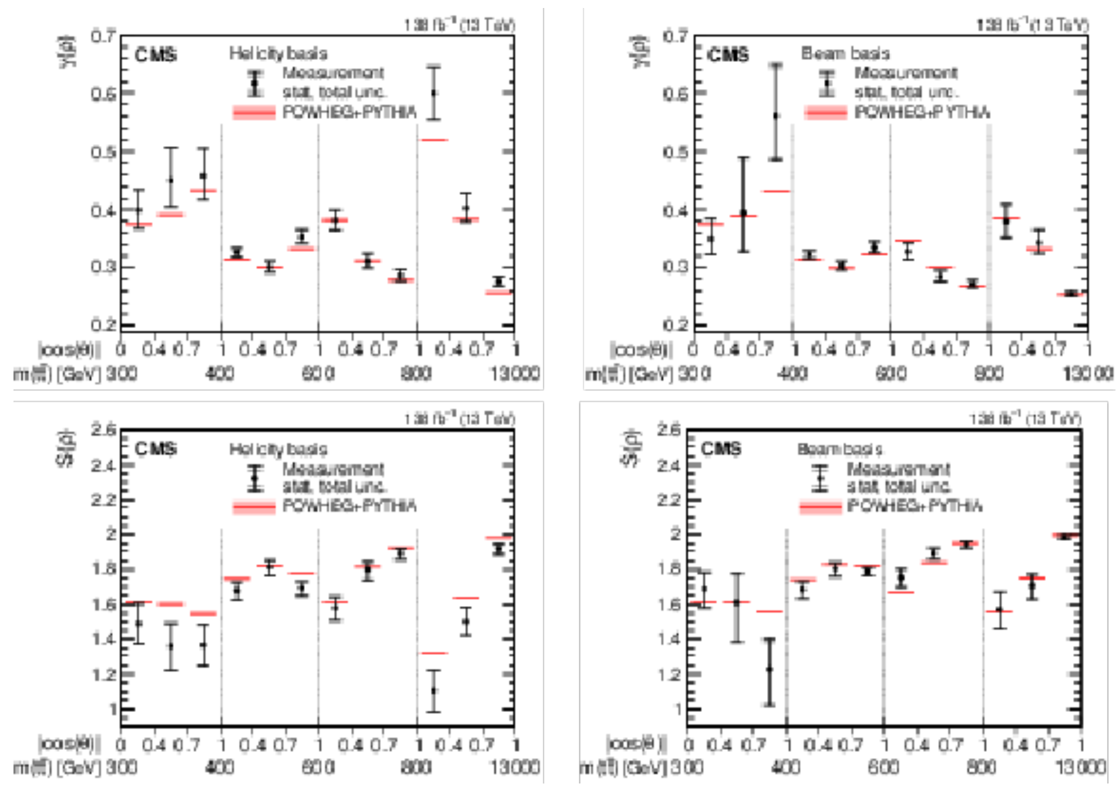




Decomposition into eigenstates

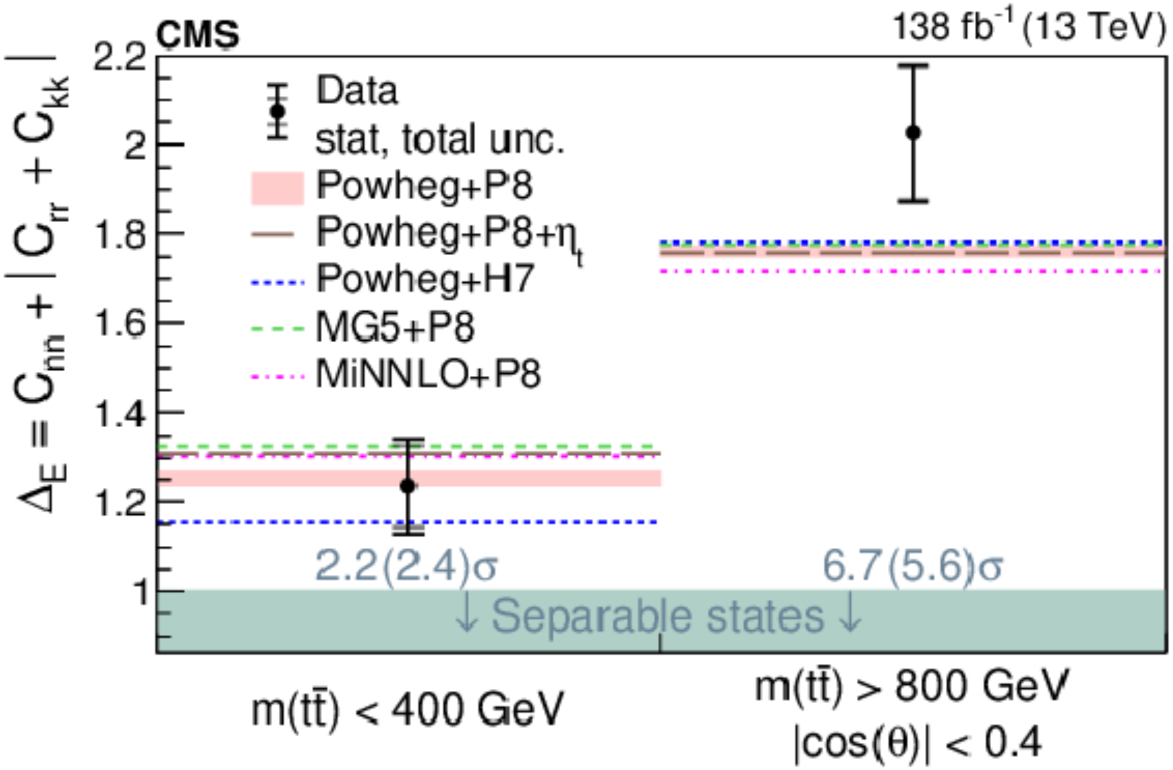
- Based on the measured polarizations and full spin correlation matrix we can construct the probability density matrix and decompose it into its eigenstates, evaluate properties like purity and von Neumann entropy





Establishing entanglement in ttbar system

- **The value of Δ_E in the threshold region exceeds the maximal value for separable states by 2.2 σ and in the high Mtt region by 6.7 σ.**



Observed Δ_E exceeds Δ_E =1 (maximal value for separable states) by >5σ

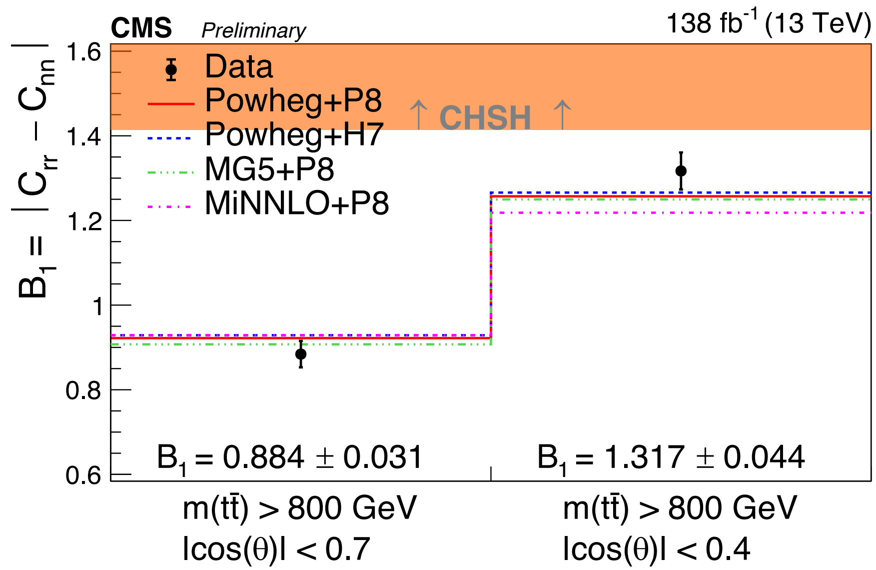
Results in l+jets – reaching for Bell inequality

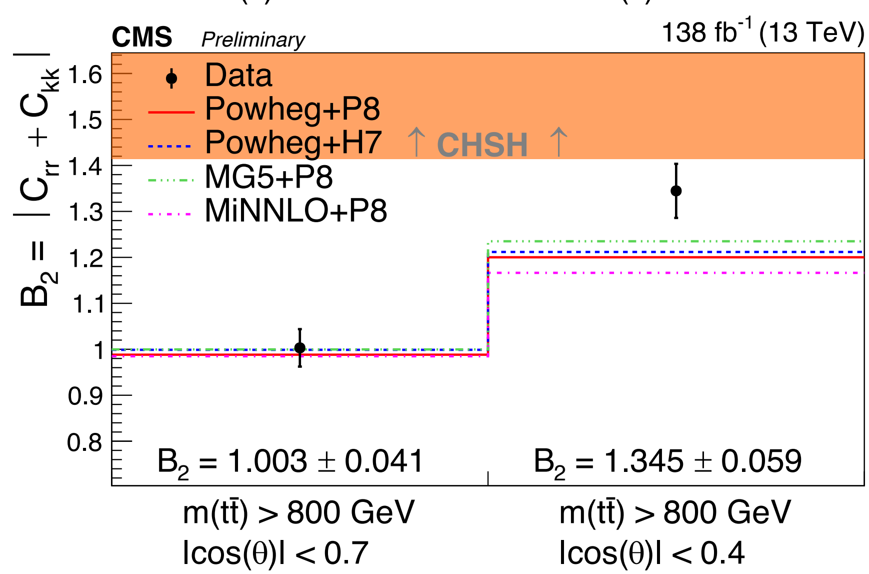
- Criteria for Bell inequality violation (in CHSH definition) are shown in orange

$$B_1 = |C_{rr} - C_{nn}| > \sqrt{2}$$

$$B_2 = |C_{rr} + C_{kk}| > \sqrt{2}$$

- We are not there yet
- Need more data to go to higher Mtt





Focus on the methodology

- Rely on the angular correlation of the decay products to infer spin orientation of the parent particle
- Use angular templates adjusted for detector effects to fit to data to extract polarization and spin correlation matrix
- Do this in bins of phase space (in this case cosθ and Mtt)
- Account for migration between bins – 1 generator level bin contributes to many bins at reconstruction level
- This is important when resolution in a particular variable of interest (Mtt in our case) is not negligible

What can we do at Belle II?

- The system of ττ is very identical to tt from the point of view of QIS – 2 qubit system

The dependence of the τ leptons production amplitude squared on the scattering angle θ is given by

$$P(\theta) = 1 + c_\theta^2 + 4\alpha_\tau^2 s_\theta^2, \quad (1)$$

where $\alpha_\tau = \frac{m_\tau}{\sqrt{s}}$, m_τ is τ lepton mass, s - is the center of mass energy squared of SuperKEKB, and s_θ and c_θ are the sine and cosine of the scattering angle.

The 4x4 probability density matrix is given by

$$\rho = \frac{\mathbf{1}\otimes\mathbf{1} + B_{1i}\,\sigma_i\otimes\mathbf{1} + B_{2j}\,\mathbf{1}\otimes\sigma_j + C_{ij}\,\sigma_i\otimes\sigma_j}{4}, \quad (2)$$

- Spin correlation matrix

$$C = \frac{1}{P(\theta)}\begin{pmatrix} (4\alpha_\tau^2-1)s_\theta^2 & 0 & 0 \\ 0 & (4\alpha_\tau^2+1)s_\theta^2 & 4\alpha_\tau s_\theta c_\theta \\ 0 & 4\alpha_\tau s_\theta c_\theta & 1+c_\theta^2-4\alpha_\tau^2 s_\theta^2 \end{pmatrix}. \quad (3)$$

- Concurrence, if >1/3 – entangled state

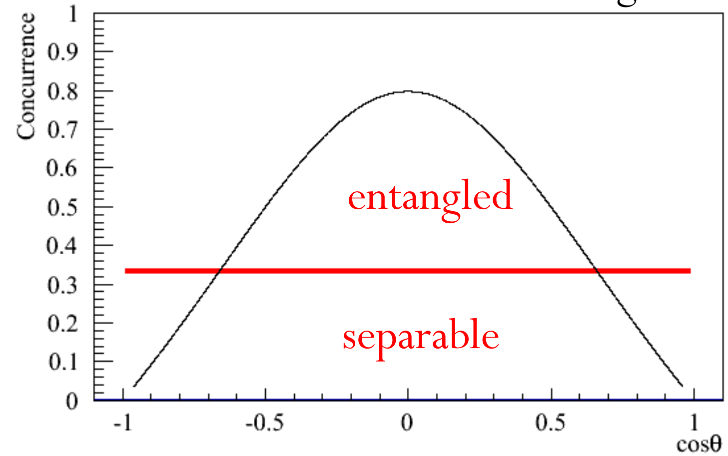
$$C(\rho) = \frac{(1-4\alpha_\tau^2)s_\theta^2}{P(\theta)}.$$

K. Ehat¨aht, M. Fabbrichesi, L. Marzola, and C. Veelken, Phys. Rev. D 109, 032005 (2024), arXiv:2311.17555 [hep-ph]

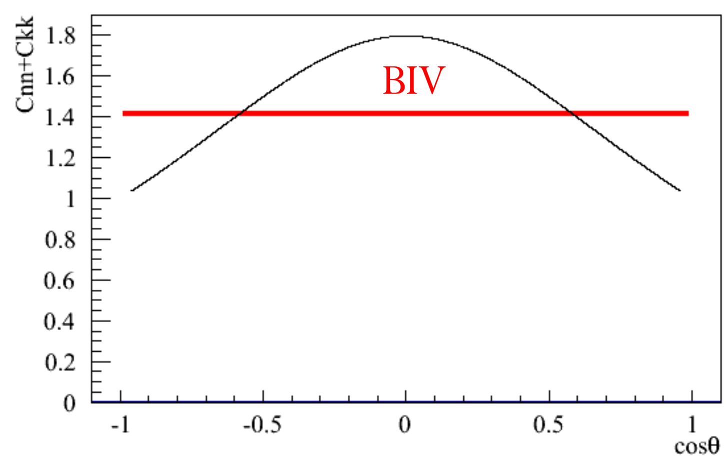
Analysis strategy for time-independent entanglement and Bell inequality test

- $\tau \rightarrow \pi\nu$ (~10.8%)
- $\tau \rightarrow \rho\nu \rightarrow \pi\pi^0\nu$ (~25.5%)
- $\tau \rightarrow e\nu\bar{\nu}$ (~17.8%)
- $\tau \rightarrow \mu\nu\bar{\nu}$ (~17.4%)
- Use π, ρ, e, μ as polarimeters → >70% of τ decays, → 50% of ττ are analyzable
- Define basis based on the τ direction, boost decay products in the rest frame of parent τ
- Design templates based on the angles of polarimeters at reconstruction level
- Divide the sample into bins of cosθ (how good is this resolution?)
- Account for migration across bins in the templates
- Fit for the full spin correlation matrix
- Evaluate the concurrence and check for entanglement, Bell inequality violation (BIV)
- **Expect to observe both**

Peres-Horodecki test for entanglement



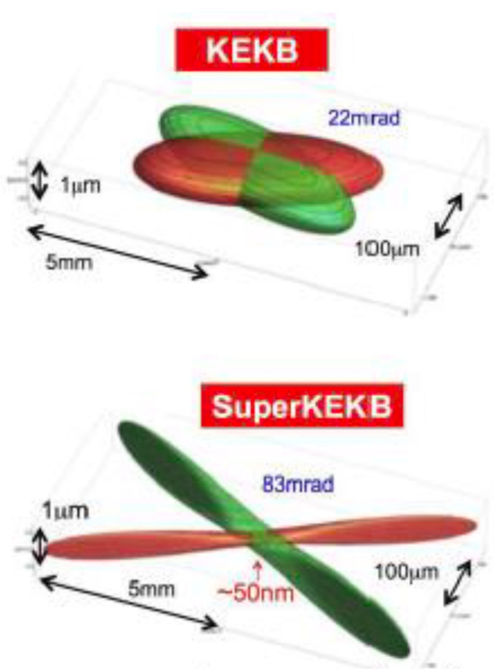
CHSH test for Bell inequality violation



Time dependence

- Account for decoherence (quantum-to-classical transition due to the interaction with the environment) using Lindblad master equation $t=min(t_1,t_2)$, decoherence length L_{decoh} is a free parameter

$$C(\rho, ct) = C(\rho, 0)e^{-ct/L_{decoh}},$$



τ leptons have an advantage over top quarks since their lifetime ($c\tau$=87 µm) is long enough to be measured, but not so long that the decay happens outside of the detector volume

SuperKEKB has a very small beam spot and BelleII has excellent Pixel Vertex detector (PXD)

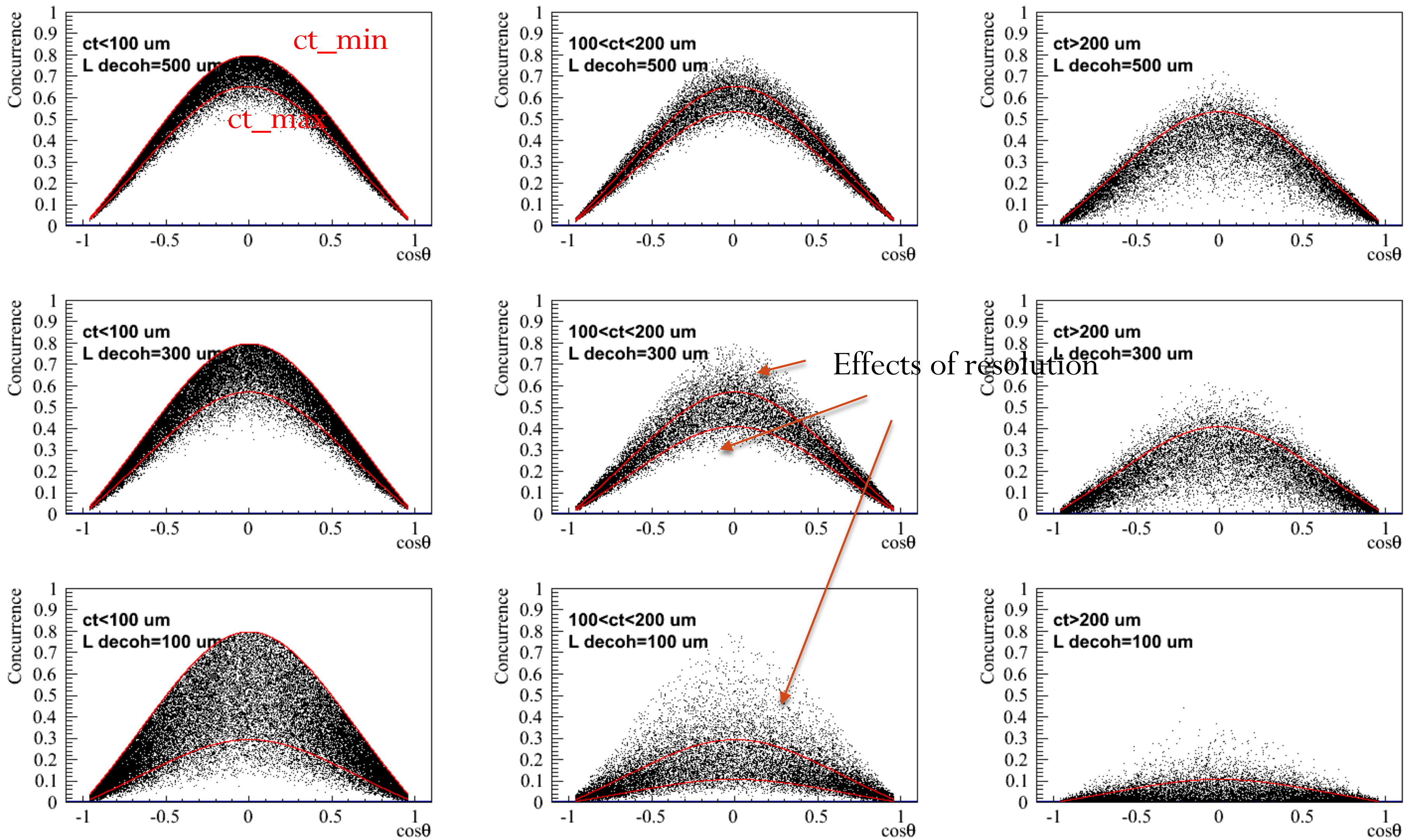
Nevertheless, time resolution cannot be neglected

The approach that we used in CMS to account for migration across bins (in Mtt) can be applied here for migration across time bins

Concurrence vs time

Decoherence →

Assumed resolution on decay length of 50 μm



Each point is not one event – it is concurrence evaluated on an ensemble of events

Analysis strategy for time dependence

- We would like to have at least 2 tracks to have a reliable decay vertex
- $\tau+ \rightarrow h+h-h+$ (~15%) → 2.25% of $\tau\tau$ are analyzable
- Construct polarimeter from 3 hadrons
- Design templates based on the angles of polarimeters at reconstruction level
- Divide the sample into bins of $\cos\theta$ and ct
- Account for migration across bins in the templates
- Fit for the full spin correlation matrix
- Evaluate concurrence, and its dependence on time
- Fit for decoherence parameter L_{decoh}

Conclusion

- Quantum tests at colliders are in their infancy
- It is an exciting new field at the intersection of particle physics and quantum informatics
- A new community is forming – a workshop at CERN in April, following a series of workshops held in Florence, Oxford, Shanghai
- Quantum tomography opens new possibilities to study the foundations of quantum mechanics with unstable particles
- Belle II is ideally suited for several of such studies