



Implication of superscaling cross-section model at FD

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Developed by Lounès Amziane from University of Geneva

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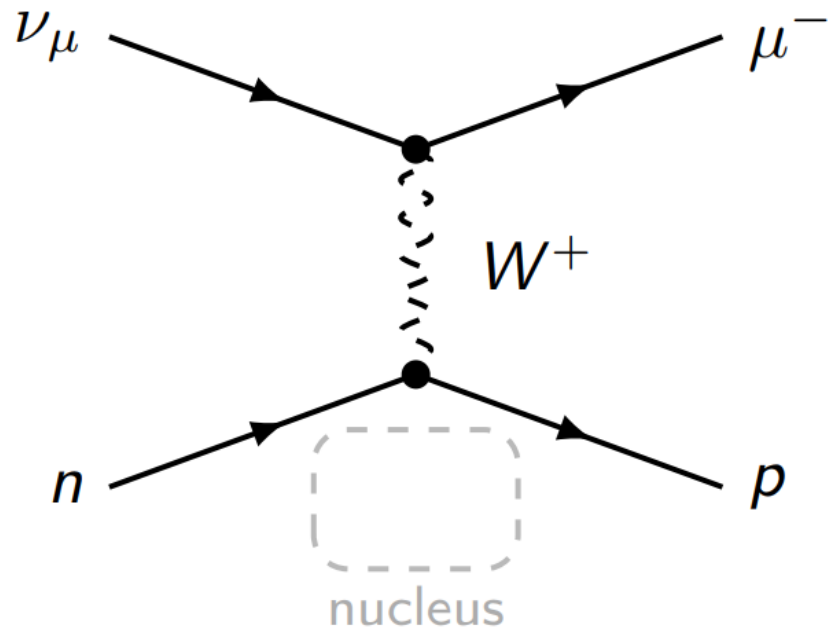
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Energy reconstruction and superscaling framework

CCQE neutrino-nucleus interaction



Energy conservation

$$E_\nu = E_\mu + E_p - M_n + S$$

Outgoing particles information

- Leptonic: $\vec{p}_\mu$ — always measurable
- Hadronic: $\nu \rightarrow E_p$ — not accessible at far detector
 $\bar{\nu} \rightarrow E_n$ — neutron is hard to reconstruct
- Removal energy: S — not directly measurable

Lepton quasielastic reconstruction (LQE)

- TAR:** target-at-rest hypothesis:

$$E_\nu^{\text{reco}} = \frac{m_\ell^2 - 2E_\ell \widetilde{M}_n + \widetilde{M}_n^2 - M_p^2}{2(E_\ell - \widetilde{M}_n - p_\ell \cos \theta_\ell)}$$

$$\widetilde{M}_n \equiv M_n - \bar{S}_{\text{LQE}}$$

Superscaling variable

The neutrino energy is reconstructed from the measured lepton kinematics: $E_\nu^{\text{true}} \rightarrow E_\nu^{\text{reco}}$

How good is the reconstruction?



Superscaling variable ψ'

Dimensionless measure of $\Delta E_\nu \equiv E_\nu^{\text{true}} - E_\nu^{\text{reco}}$

By construction, $\psi' = 0$ for $\Delta E_\nu = 0$. Taylor expansion around this point at leading order gives,

$$\psi' = \chi_F L_1(\vec{p}_\ell, \theta_\ell) \Delta E_\nu + \mathcal{O}(\Delta E_\nu^2)$$

L_1 depends only on the measured lepton kinematics. L_1 can be measured without any hadronic information:

$$L_1(\vec{p}_\ell) = \frac{\sqrt{2}}{M_n} \frac{(\widetilde{M}_n + p_{z\ell} - E_\ell)^2}{\sqrt{[p_{T\ell}^2 + (M_p - \widetilde{M}_n - p_{z\ell} + E_\ell)^2][p_{T\ell}^2 + (M_p + \widetilde{M}_n + p_{z\ell} - E_\ell)^2]}}$$

L1 meaning

The superscaling hypothesis implies that the quasielastic region is defined by

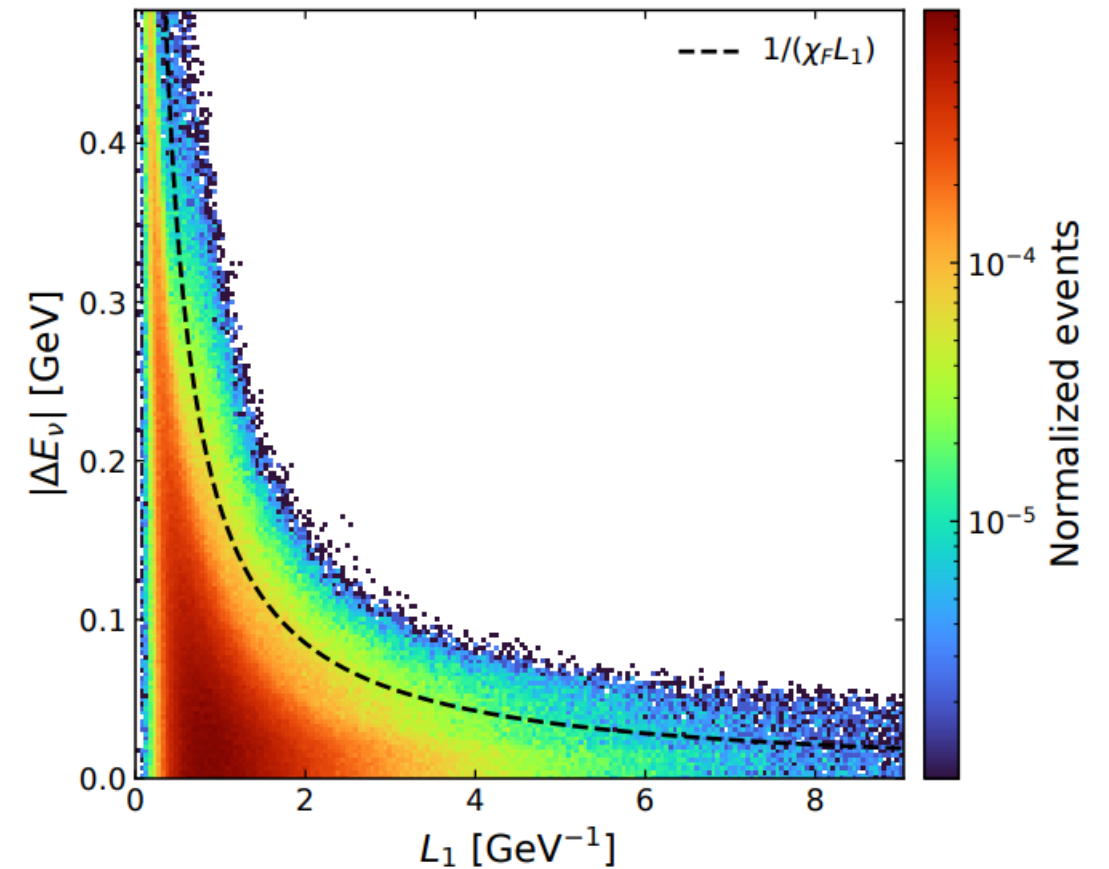
$$-1 < \psi' < 1$$

Using the linear-order relation,

$$\psi' \simeq \chi_F L_1 \Delta E_\nu$$

L_1 provides an event-by-event upper bound on the neutrino energy bias:

$$|\Delta E_\nu| < \frac{1}{\chi_F L_1}$$



<https://arxiv.org/abs/2609.15431>

Energy reconstruction effect in neutrino oscillation analysis

Neutrino energy reconstruction effect in oscillation analysis

- A neutrino of flavour α is produced at a time $t = 0$ as a mixture of the mass eigenstates:

$$|\nu_\alpha\rangle = \sum_{i=1}^3 U_{\alpha i}^* |\nu_i\rangle$$

2-flavour oscillation probability:

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \underbrace{\sin^2 2\theta}_{\substack{\text{Amplitude} \\ \text{How much mixing?}}} \sin^2 \underbrace{\left(\frac{\Delta m^2}{4E} L \right)}_{\substack{\text{Phase} \\ \text{How fast?}}}$$

- **Oscillation length depends on the energy of the neutrino**

Experiment

Data in bins of reconstructed energy

$$N_e^{\text{exp}}(E_\nu^{\text{reco}})$$

Simulation

Predicted events are computed as the product of flux, oscillation probability, and cross-section at true energy, and then migrated to reco-energy bins,

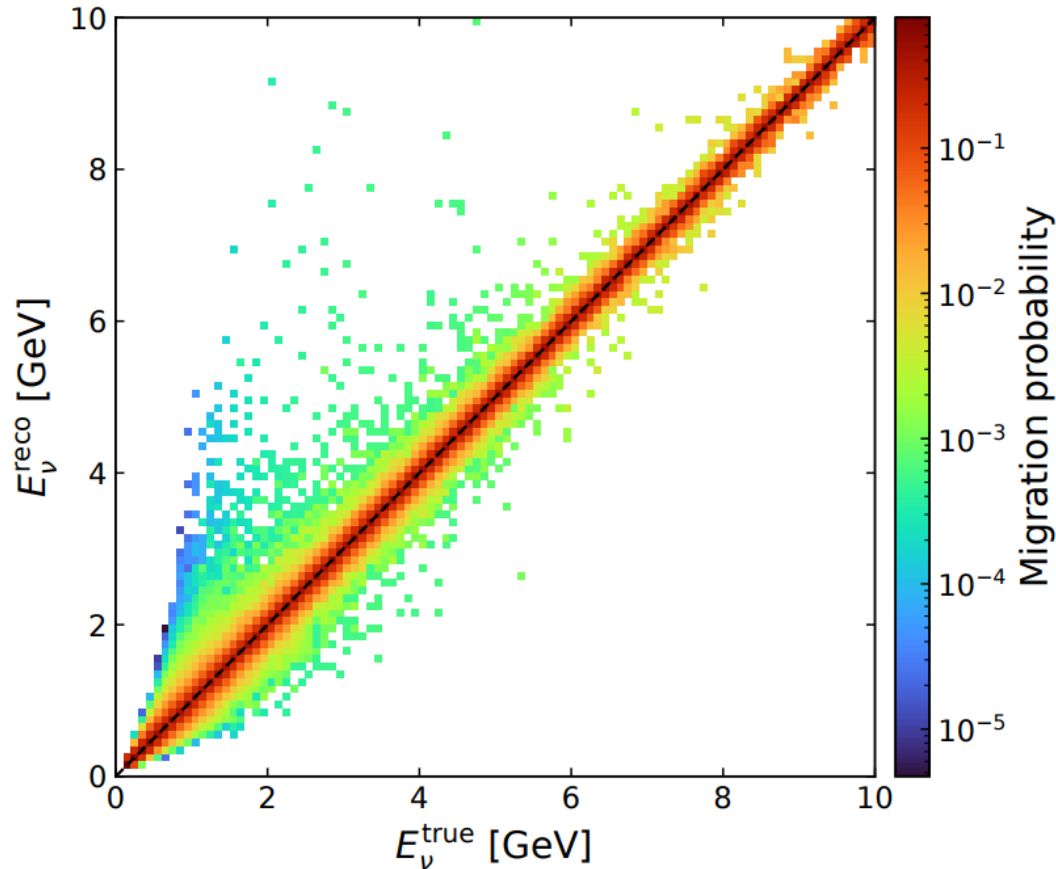
$$N_e^{\text{th}}(E_\nu^{\text{reco}}) = \int \phi_\mu(E_\nu^{\text{true}}) P_{\mu e}(x, E_\nu^{\text{true}}) \sigma_e(E_\nu^{\text{true}}) \epsilon R(E_\nu^{\text{true}}, E_\nu^{\text{reco}}) dE_\nu^{\text{true}}$$

Migration matrix

Smears the effect of varying an oscillation parameter over the reconstructed-energy bins.

1D: E_ν^{reco} analysis

<https://arxiv.org/abs/2609.15431>



1D migration matrices: E_ν^{reco} vs E_ν^{true}

HK flux, MC sample restricted to CCQE interactions

- Probability of a neutrino of true-energy bin j is reconstructed as reco-energy bin i

$$R_{ij} = P(E_\nu^{\text{reco}} \in i \mid E_\nu^{\text{true}} \in j)$$

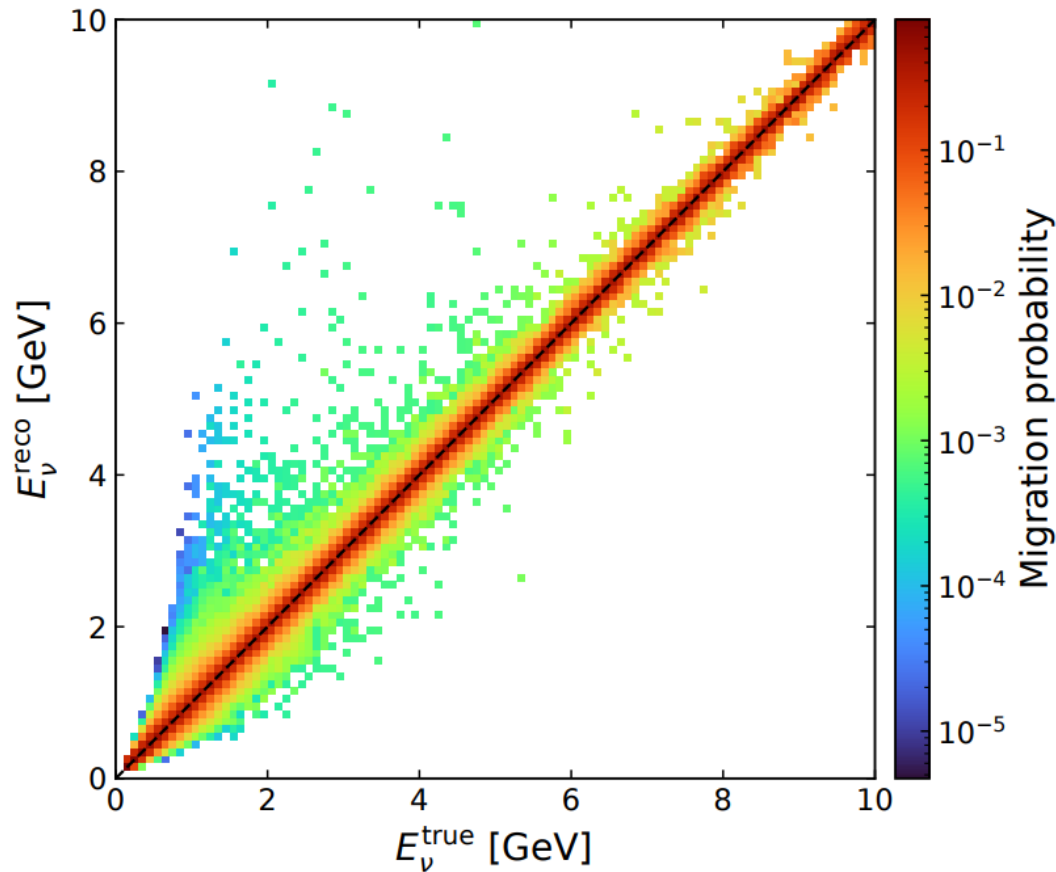
- Migration matrices from GENIE. Each event: $E_\nu^{\text{true}}, (p_\mu, \theta_\mu) \rightarrow E_\nu^{\text{reco}}$.

- Statistical analysis performed in **1 reconstructed variable E_ν^{reco}**

$$\chi_{\text{stat}}^2 = \sum_{i \in \{E_\nu^{\text{reco}} \text{ bins}\}} \frac{((N_e)_i^{\text{th}} - (N_e)_i^{\text{exp}})^2}{(N_e)_i^{\text{exp}}}$$

- ESSnuSB: 1D fit in reconstructed energy $\rightarrow$ loses information on the reconstruction bias.

1D: E_ν^{reco} analysis



1D migration matrices: E_ν^{reco} vs E_ν^{true}

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- Migration matrices from GENIE. Each event: $E_\nu^{\text{true}}, (p_\mu, \theta_\mu) \rightarrow E_\nu^{\text{reco}}$.

- S_{ν}^{reco} is performed in **1 reconstructed**

→ L_1 : provides additional information on the quality of the reconstruction

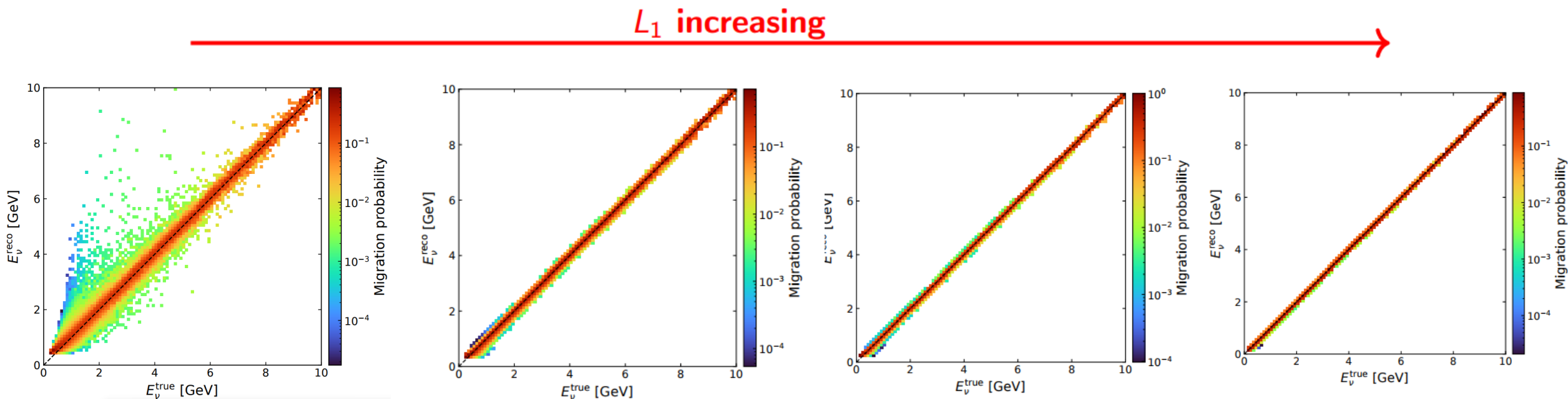
$$\chi_{\text{stat}}^2 = \sum_{i \in \{E_\nu^{\text{reco}} \text{ bins}\}} \frac{(S_i^{\text{reco}} - \mu_i)^2}{\sigma_i^2}$$

- ESSnuSB: 1D fit in reconstructed energy → less information on the reconstruction bias.

2D: $(E_\nu^{\text{reco}}, L_1)$ analysis

- Probability that an event with true energy in bin j is reconstructed in energy bin i and classified in L_1 bin k :

$$R_{ij}^k = P(E_\nu^{\text{reco}} \in i, L_1 \in k \mid E_\nu^{\text{true}} \in j)$$



<https://arxiv.org/abs/2609.15431>

- As L_1 increases, the matrices become progressively more diagonal, events are better reconstructed.

Oscillation analysis in HK: 1D vs 2D

Does including L_1 give better sensitivity to oscillation parameter ?

Oscillation analysis based on a Hyper-K-like simulation with reconstructed variables in

1D

→ (E_ν^{reco})

$$\chi_{\text{stat}}^2 = \sum_{i \in \{E_\nu^{reco} \text{ bins}\}} \frac{((N_e)_i^{\text{th}} - (N_e)_i^{\text{exp}})^2}{(N_e)_i^{\text{exp}}}$$

Currently done in ESSnuSB

2D

→ (E_ν^{reco}, L_1)

$$\chi_{\text{stat}}^2 = \sum_{\substack{i \in E_\nu^{reco} \text{ bins,} \\ k \in L_1 \text{ bins}}} \frac{((N_e)_{ik}^{\text{th}} - (N_e)_{ik}^{\text{exp}})^2}{(N_e)_{ik}^{\text{exp}}}$$

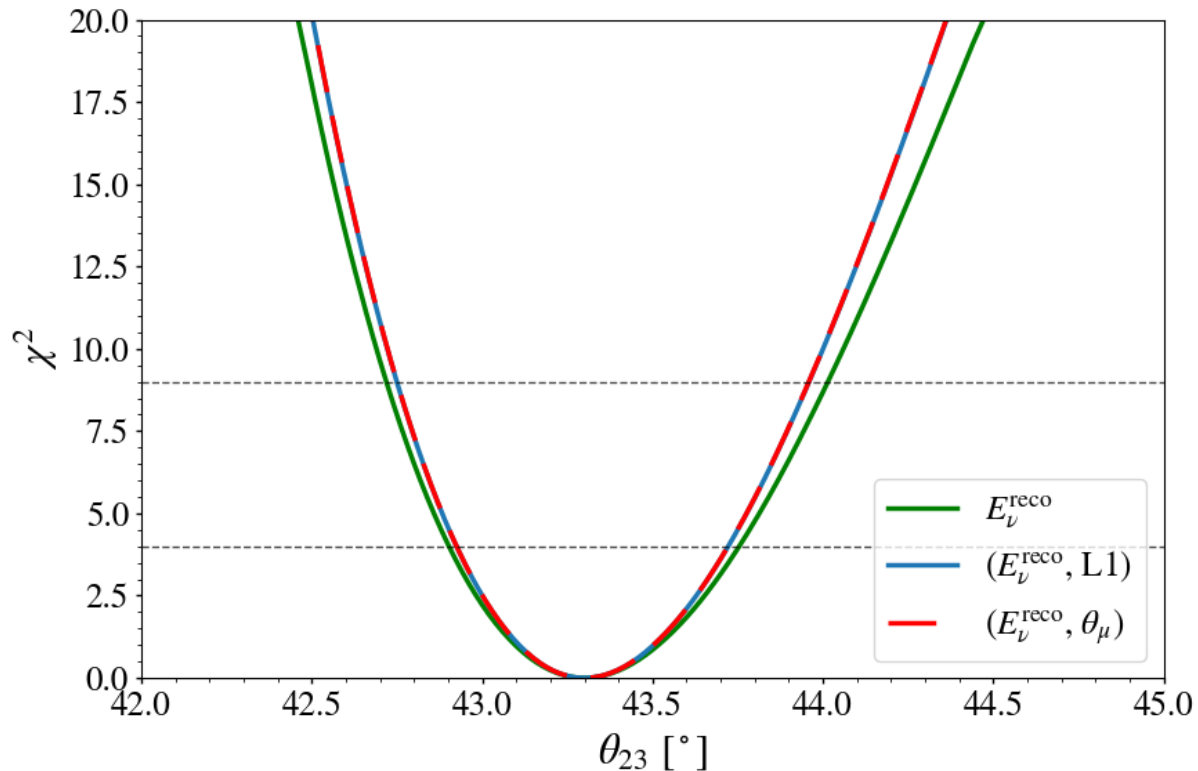
→ $(E_\nu^{reco}, \theta_\mu)$

Currently done in T2K/HK

Results of HK oscillation analysis

χ^2 Profile of θ_{23}

Results: 1D vs 2D sensitivity to θ_{23}



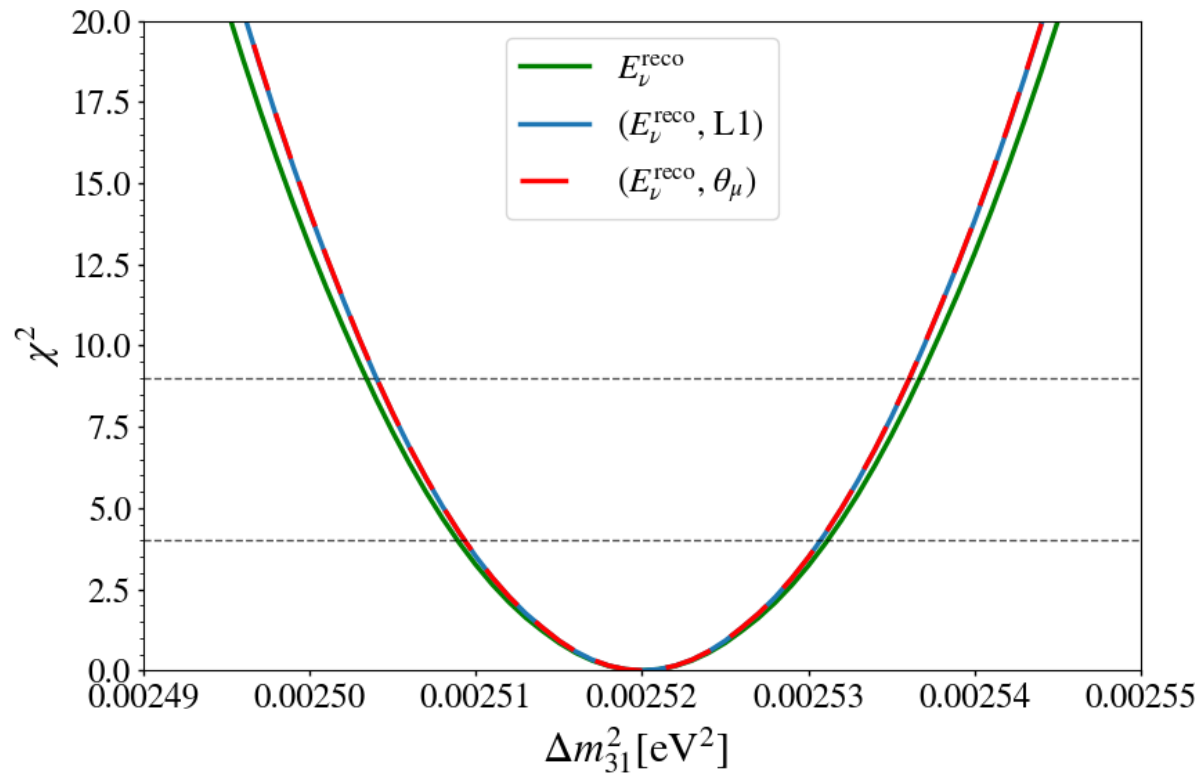
With fixed E_ν^{reco} , $L_1 = L_1(\theta_\mu)$ and θ_μ are equivalent. L_1 does not provide additional information but an interpretation in terms of the reconstruction bias.

Analysis	3σ interval
1D (E_ν^{reco})	$[42.72^\circ, 44.01^\circ]$
2D (E_ν^{reco}, L_1)	$[42.75^\circ, 43.96^\circ]$
2D ($E_\nu^{\text{reco}}, \theta_\mu$)	$[42.75^\circ, 43.96^\circ]$

- *Gain = 1.15 $\rightarrow$ 1D needs ~15% more exposure to match the 2D 3σ sensitivity*

χ^2 Profile of Δm_{31}^2

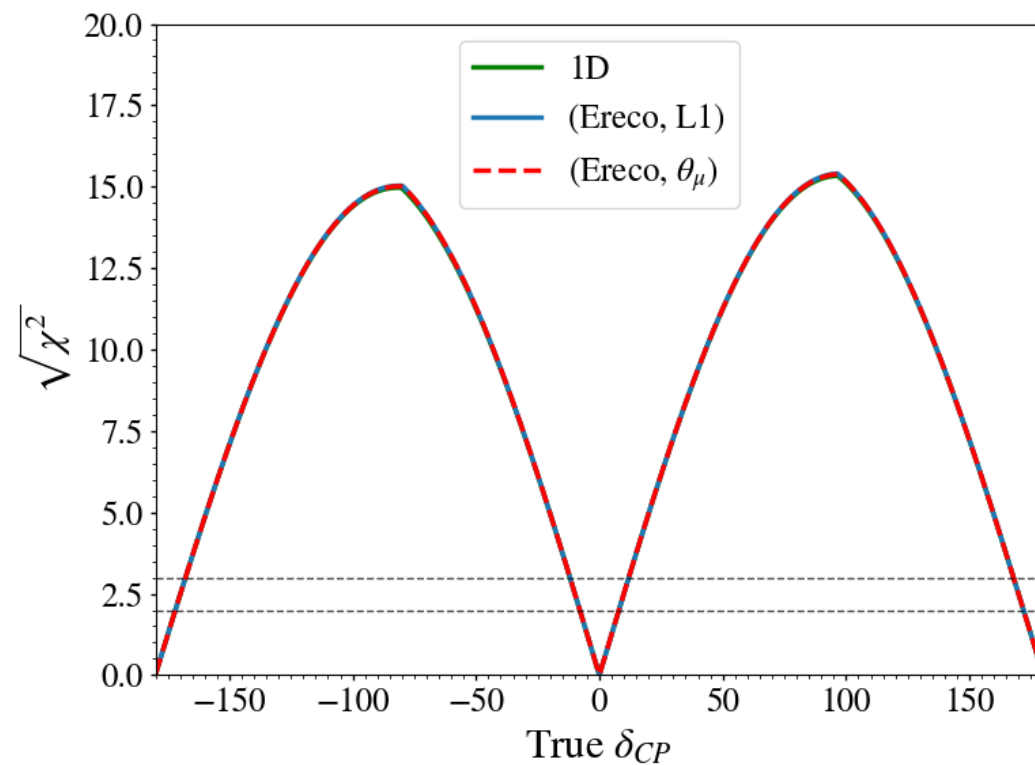
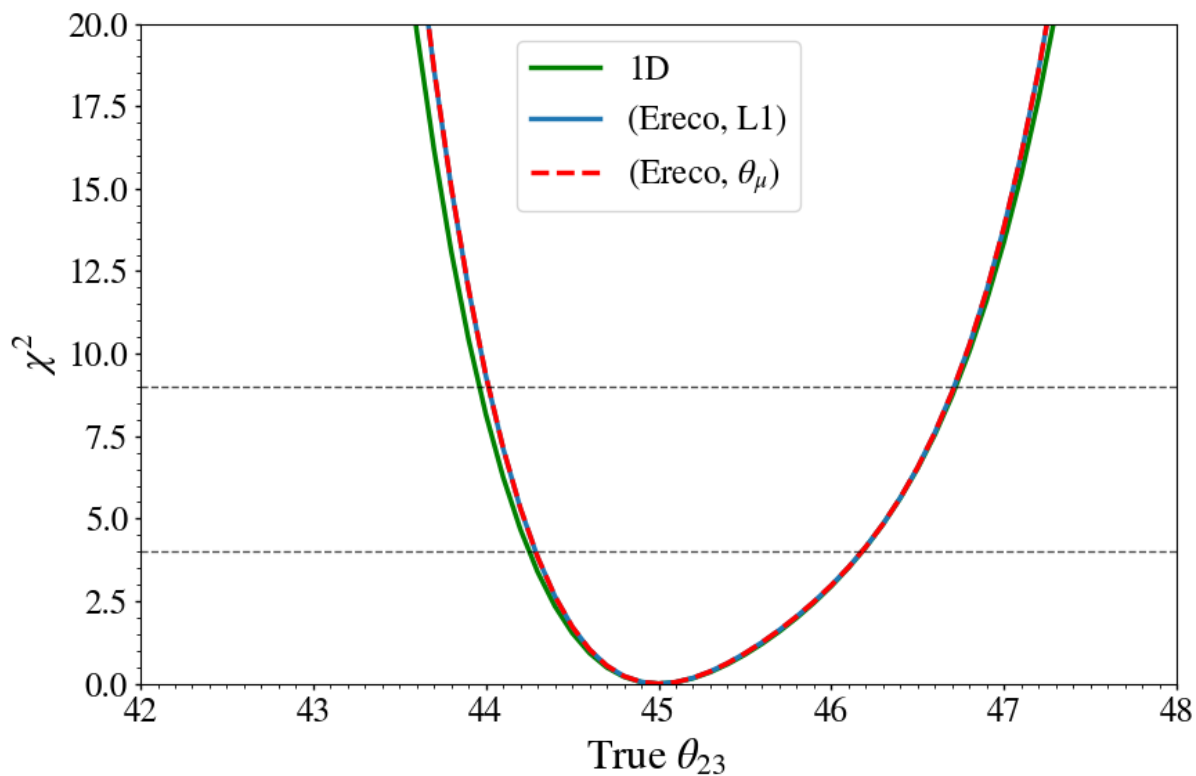
Results: 1D vs 2D sensitivity to Δm_{31}^2



Analysis	3σ interval [10^{-3} eV^2]
1D (E_ν^{reco})	[2.503, 2.537]
2D ($E_\nu^{\text{reco}}, L1$)	[2.504, 2.536]
2D ($E_\nu^{\text{reco}}, \theta_\mu$)	[2.504, 2.536]

- *Gain = 1.08 $\rightarrow$ 1D needs ~8% more exposure to match the 2D 3σ sensitivity*

Sensitivity studies



Conclusions

- L_1 provides an event-by-event measurement on the quality of energy reconstruction.
- Migration matrices smear the effect of varying an oscillation parameter.
- 1D fit in reconstructed energy loses information on the reconstruction bias.
- HK results: Including the superscaling variable L_1 in the análisis (2D: E_ν^{reco}, L_1) improves sensitivity to θ_{23} and Δm_{31}^2 compared to the 1D análisis.
- ESSnuSB: currently uses 1D análisis. Adopting the 2D approach could improve its sensitivity.

Backup

Antineutrinos

