Optical conductivity of microwave superconducting resonators

from non-equilibrium superconductivity in BCS to thin-film iron-based superconductors

Akira Miyazaki
CNRS/IN2P3/IJCLab Université Paris-Saclay

Who am I?

Prof. Koshiba
Nobel Prize 2002
“Neutrino from SN1987”

ICEPP
The University of Tokyo

ICRR
Institute for Cosmic Ray Research
University of Tokyo

Prof Orito

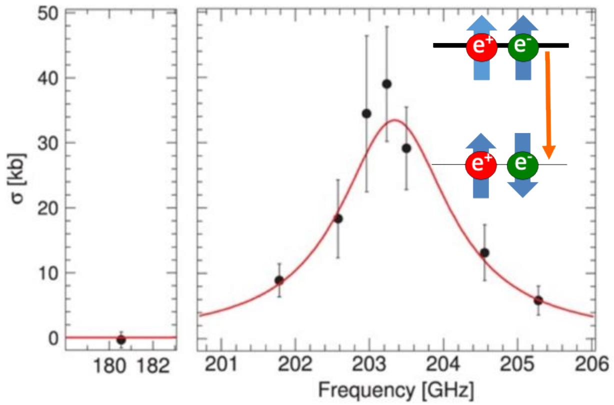
Prof Totsuka

KEK

SUSY
Guru

Prof. Asai
KEK DG 2024-

Prof. Kajita
Nobel Prize 2015
“Neutrino oscillation 1998”



PhD supervisor

Microwaves for fundamental physics

PhD Thesis opponent

IJCLab
Irène Joliot-Curie
Laboratoire de Physique des 2 Infinis

CERN

UPPSALA UNIVERSITET

MANCHESTER 1824
The University of Manchester

Microwaves → resonators → dark matter physics

Nuclear and Particle Physics

Search for axion dark matter with high-frequency microwaves

by Dr Akira Miyazaki (IJCLab, Université Paris-Saclay)

Thursday 9 Apr 2026, 10:30 → 12:30 Europe/Stockholm

https://indico.uu.se/event/2114/

Beurlingrummet (Å10238) (Ångströmlaboratoriet)

Description

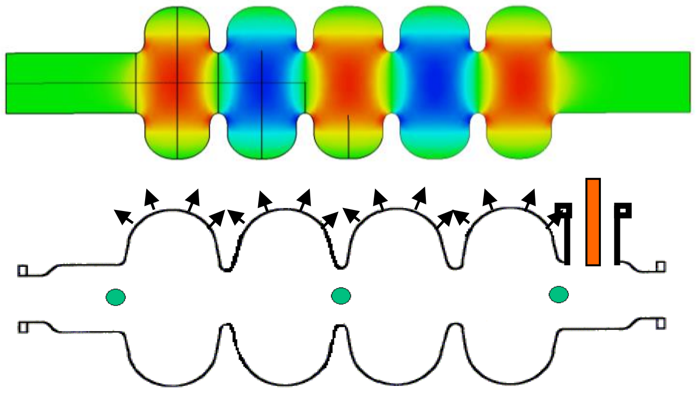
Abstract

One of the pressing problems of the established Standard Model (SM) of particle physics is the absence of a viable dark matter (DM) candidate. A promising candidate, the axion—a pseudo–Nambu–Goldstone boson—emerges in minimal extensions of the SM that introduce an additional global U(1) symmetry to naturally resolve the lack of CP violation in QCD (the strong CP problem). Axions interact very weakly with photons through mixing with pions, and dark matter axions may convert into microwave photons in the presence of a static magnetic field via the inverse Primakoff effect.

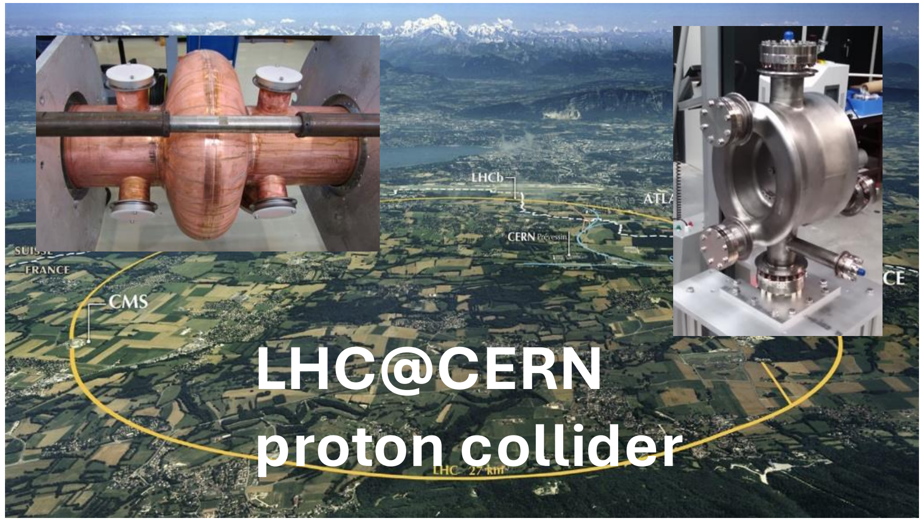
To overcome the energy–momentum mismatch between microwave photons and non-relativistic axions, specially designed resonator structures are employed. These microwave resonators exhibit strong technical synergy with other research fields, including particle accelerators, astrophysics, and telecommunications. In this seminar, we discuss the basic physics of axion detection schemes worldwide, with particular focus on two competing projects in Sweden (ALPHA) and France (MADMAX). We highlight their complementary approaches to probing dark matter physics. If time permits, we will also briefly review the current state of the art in quantum detection schemes for dark matter axions, especially their connection to quantum optics—an area not traditionally emphasized in particle physics.

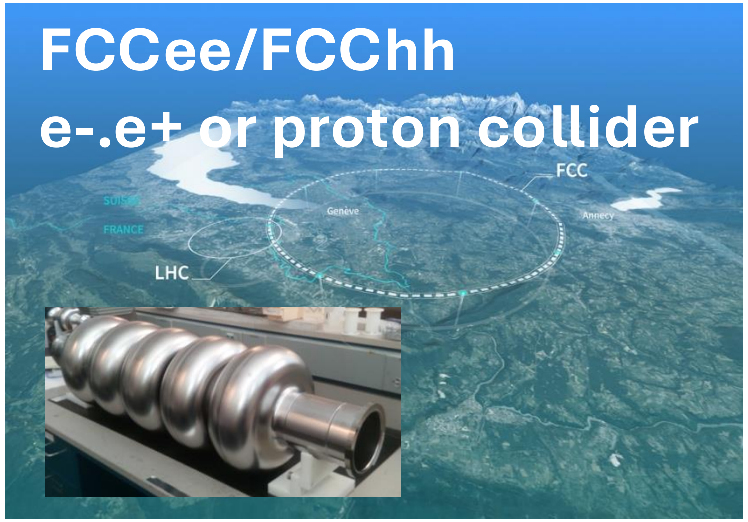
This is my side business ☺ → what is my main role?

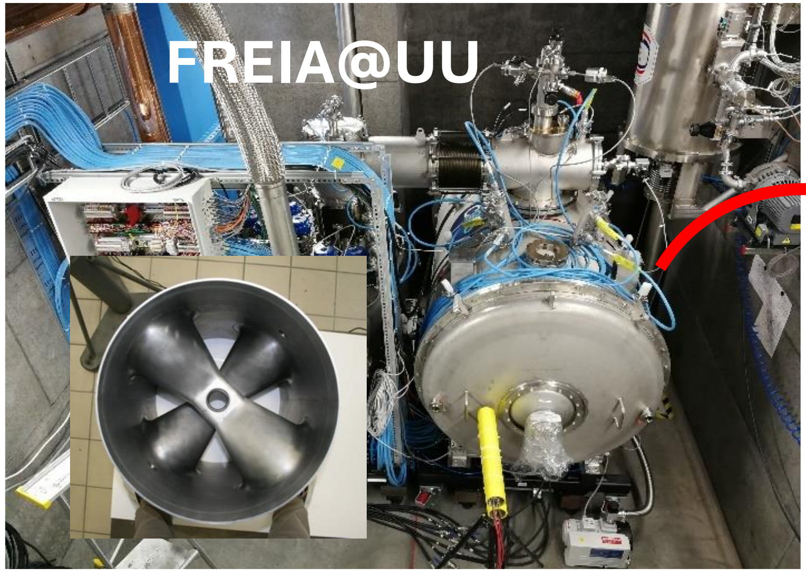
Superconducting Radio Frequency cavities for accelerators

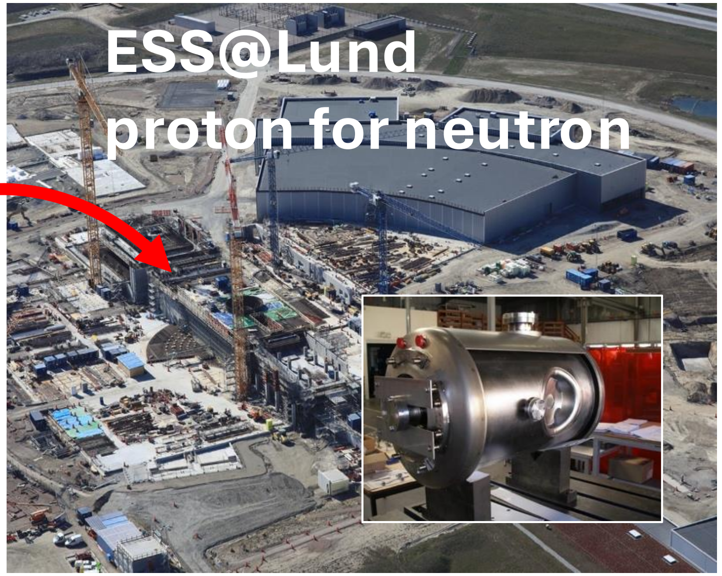


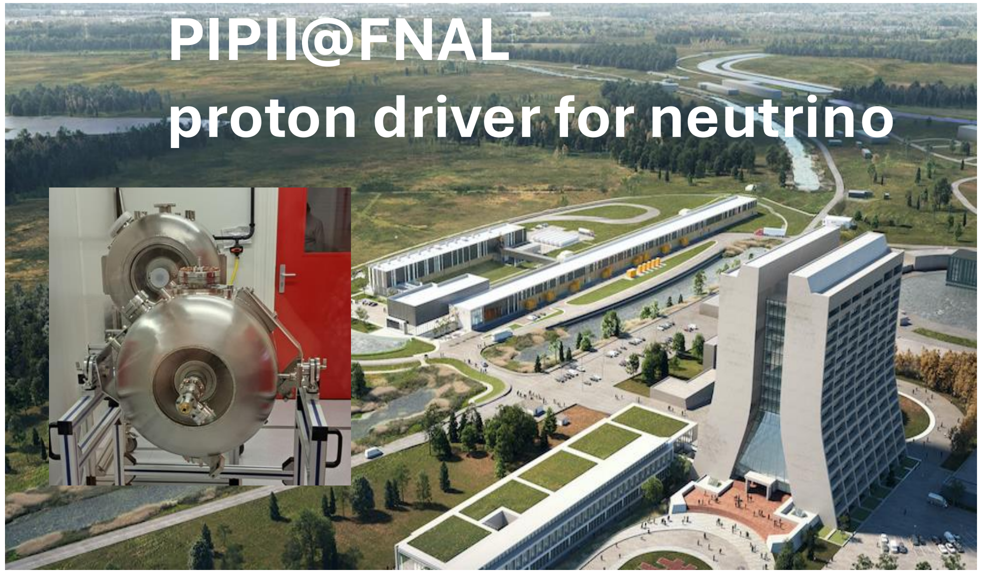
Courtesy: F. Bouly





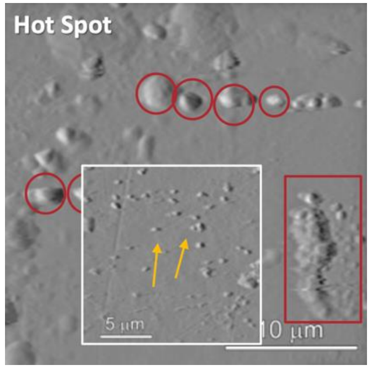


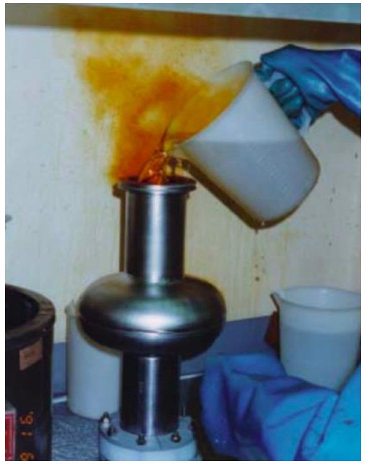


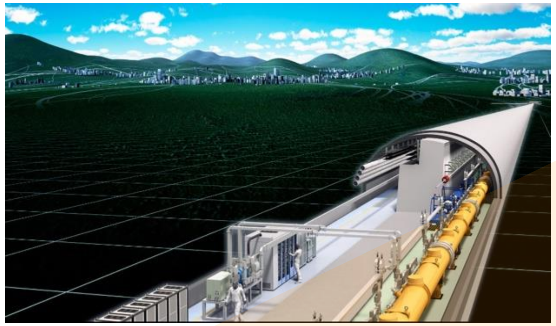


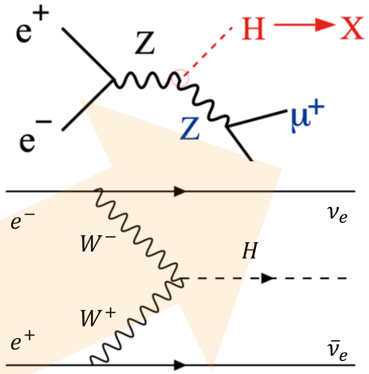
→ Just an engineering challenge of large infrastructure?

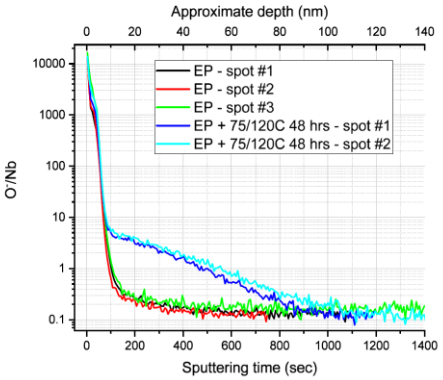
My vision in superconducting RF accelerators

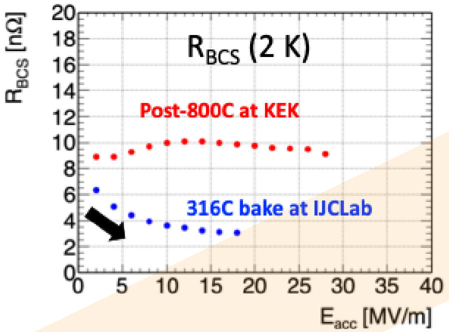


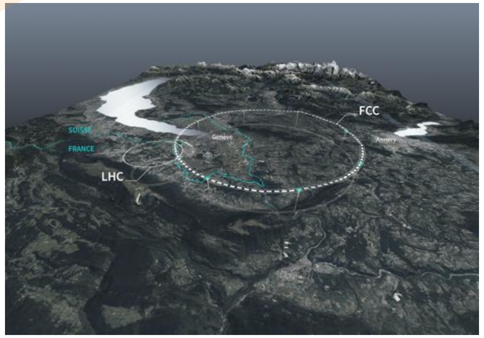


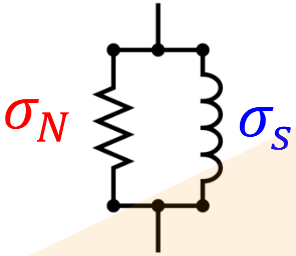


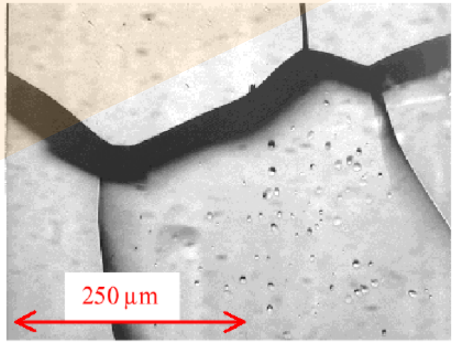


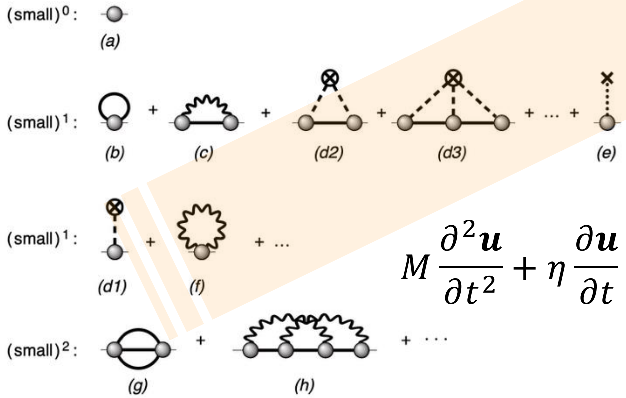








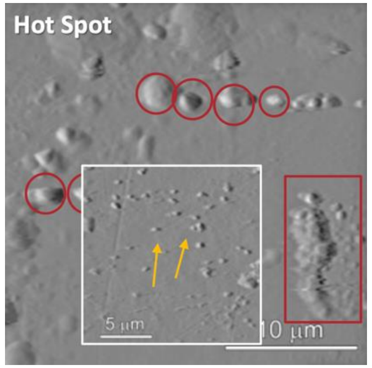


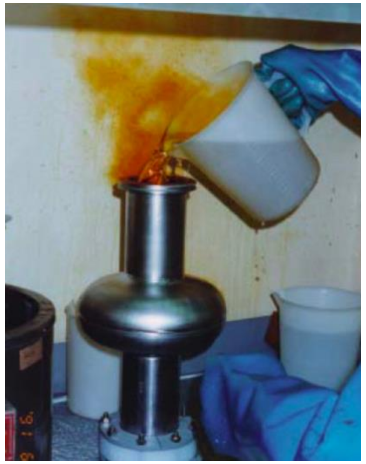


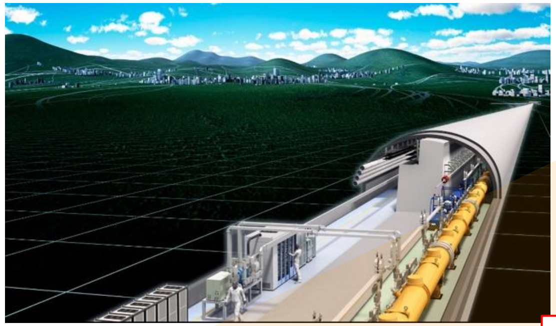
$$M\frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \eta\frac{\partial \boldsymbol{u}}{\partial t} - \epsilon\frac{\partial^2 \boldsymbol{u}}{\partial z^2} + \nabla U(z,u) = \boldsymbol{J}_{RF}(z,u) \times \boldsymbol{B}_{\mathrm{ext}}$$

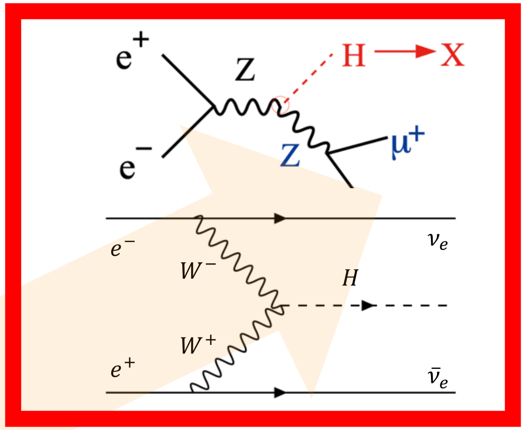
From superconductivity to particle physics through huge MW & cryogenic infrastructure

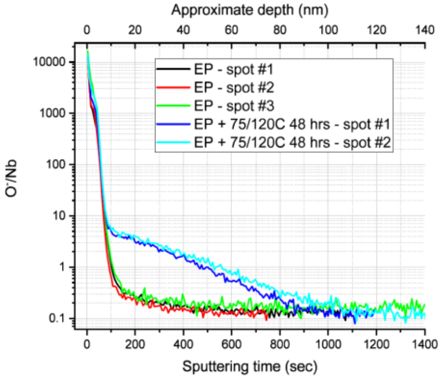
My vision in superconducting RF accelerators

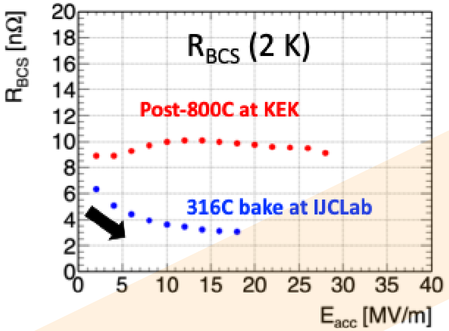


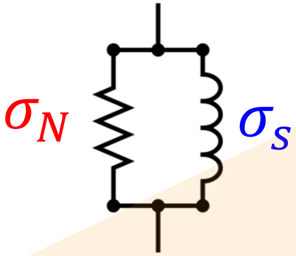


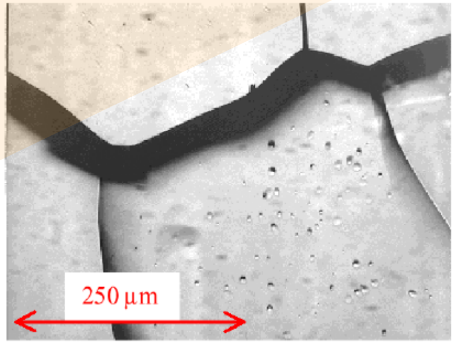




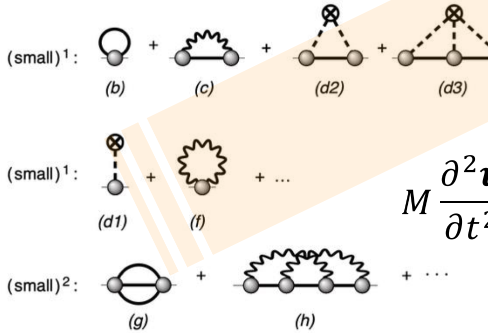








Interesting as a particle physicist (DFSZ-2HDM)



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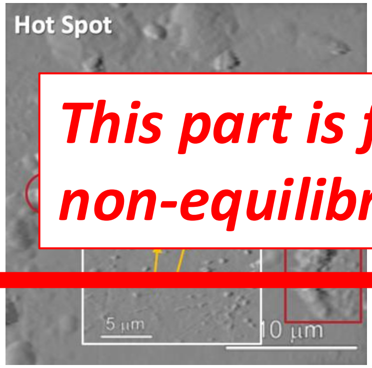
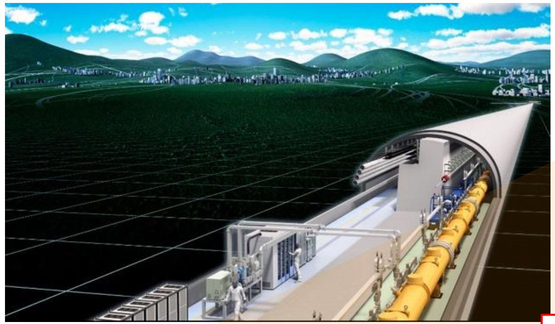
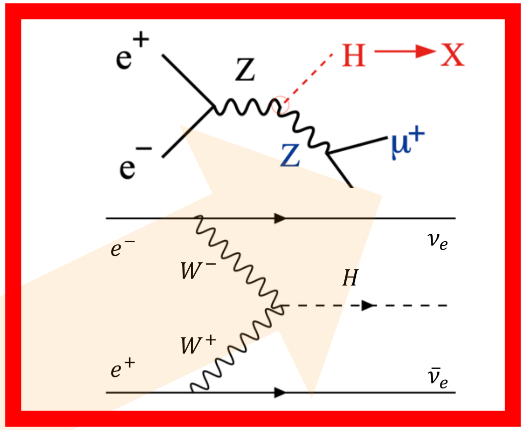
From superconductivity to particle physics through huge MW & cryogenic infrastructure

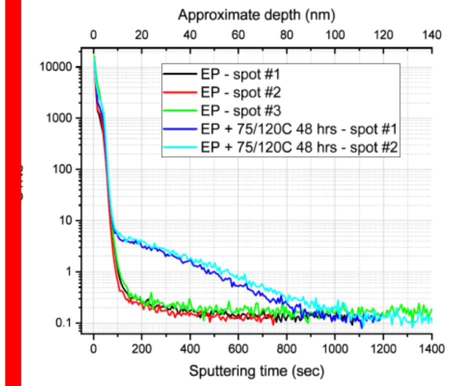
My vision in superconducting RF accelerators

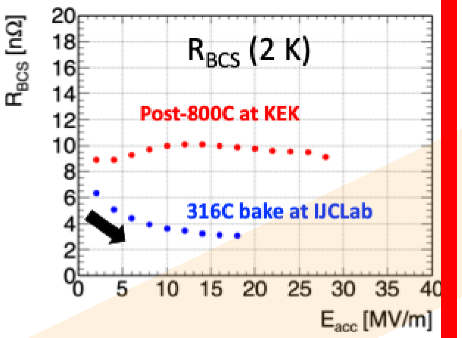


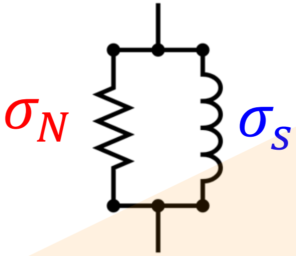


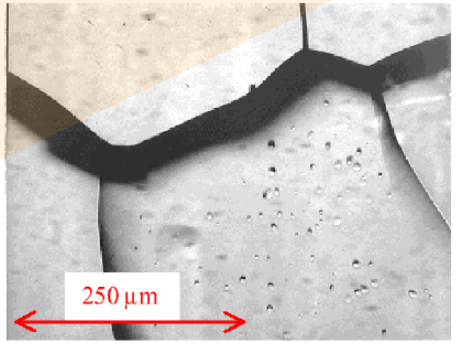


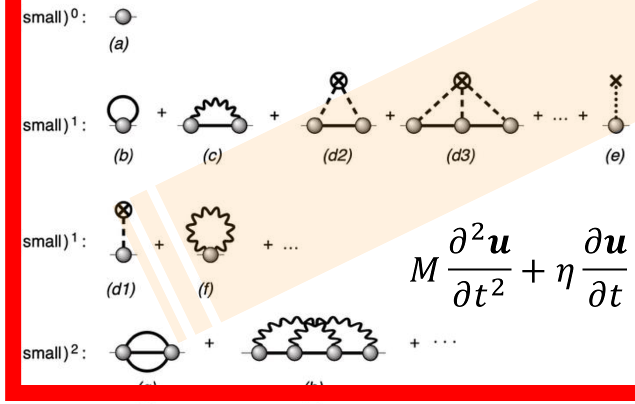
This part is fundamentally interesting non-equilibrium superconductivity











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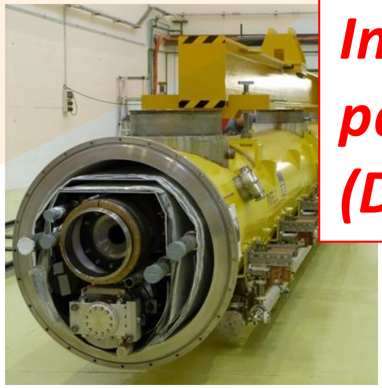

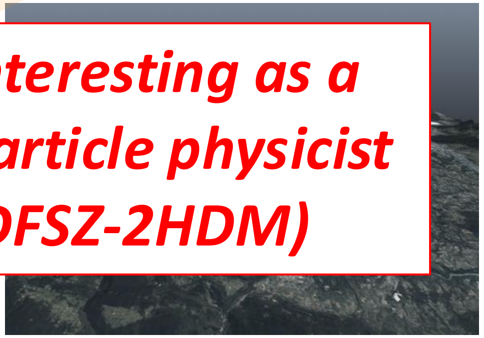

Interesting as a particle physicist (DFSZ-2HDM)

From superconductivity to particle physics through huge MW & cryogenic infrastructure

Outline

- Accelerators offer a playground for nonequilibrium superconductivity
  - Complementary to quantum sensing / qubit applications
- Fundamental of BCS and Mattis Bardeen theories
  - Linear response theory
- Nonequilibrium physics and surface treatment
- Thin-film: alternative approach
- Conclusion

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Particle acceleration with RF resonant cavities

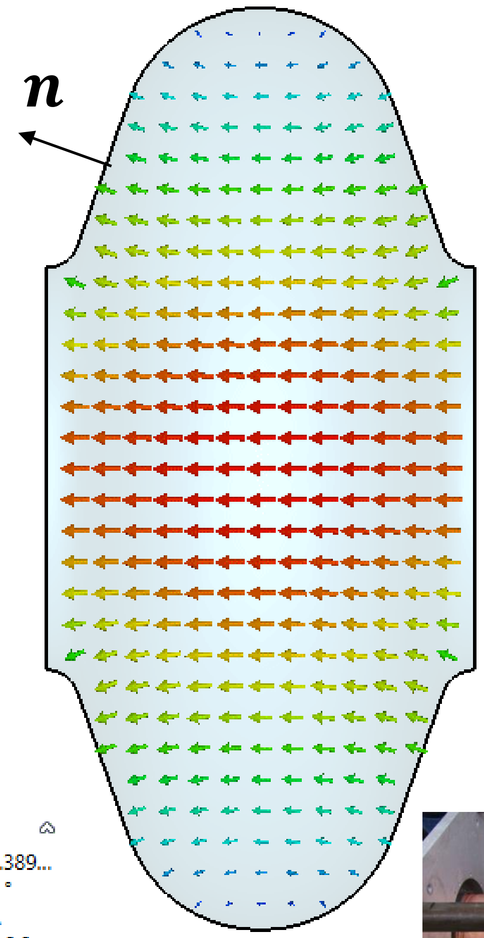
Maxwell equation

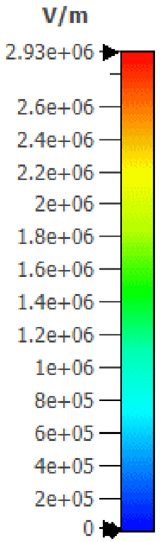
$$
\begin{cases}
\nabla \cdot \boldsymbol{E} = 0 \\
\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\
\nabla \cdot \boldsymbol{B} = 0 \\
\nabla \times \boldsymbol{B} = \dfrac{1}{c^2}\dfrac{\partial \boldsymbol{E}}{\partial t}
\end{cases}
$$

Boundary condition

$$
\begin{cases}
\boldsymbol{n} \times \boldsymbol{E} = 0 \\
\boldsymbol{n} \cdot \boldsymbol{B} = 0
\end{cases}
$$

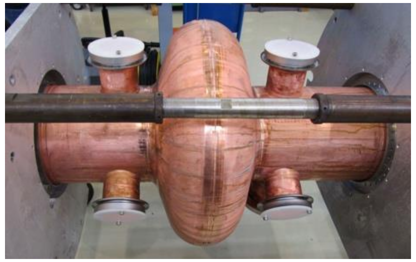
LHC

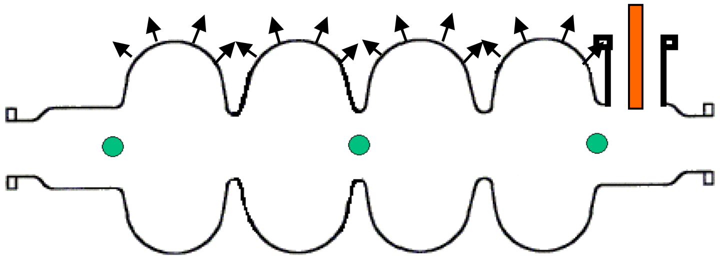


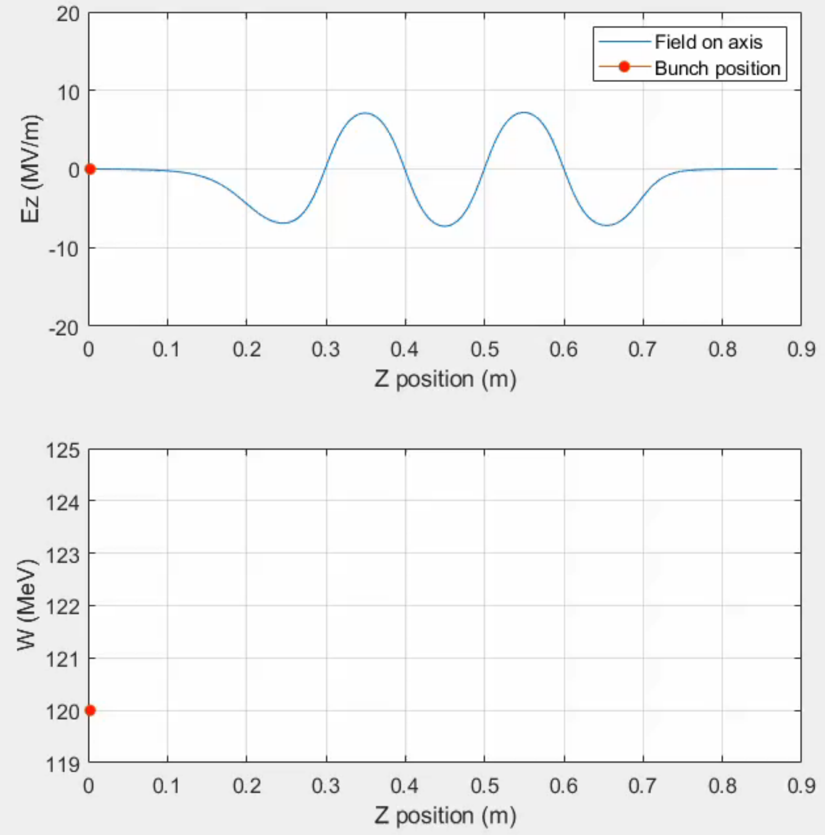


Mode 1 E-Field

Frequency	0.389...
Phase	1 °
Cross section	A
Cutplane at Y	0.0...
Maximum on Plane (Pl...	2.932...
Maximum (Plot)	2.930...





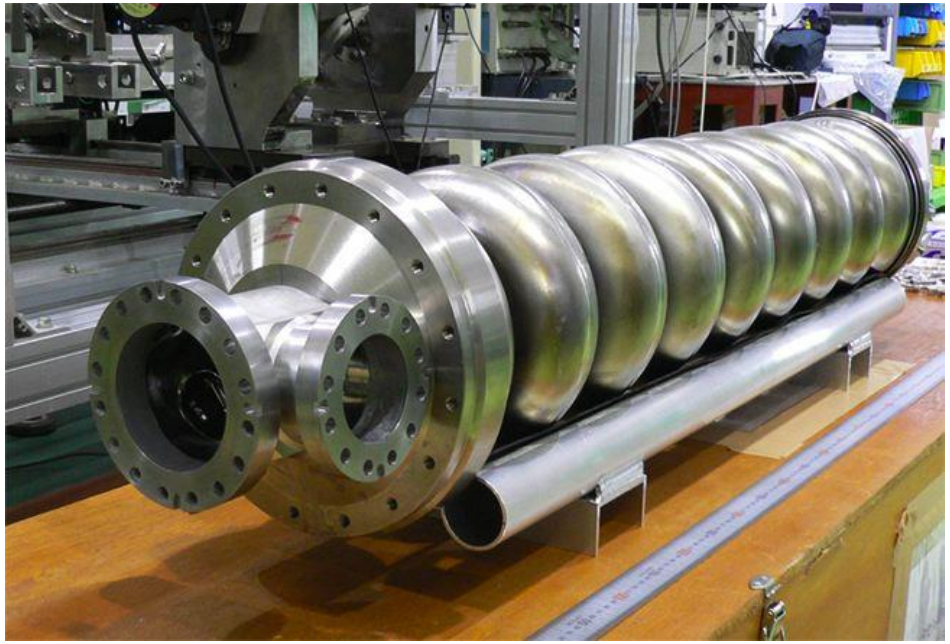


Courtesy: F. Bouly

Superconducting

cavity

cryogenics



Cryolab
@CERN

https://www.123rf.com/stock-photo/old_radio.html?sti=mbqiuc5egnu4ynov6q|&mediapopup=41753240

Unloaded quality factor

Higher Q → higher field E_{acc} with smaller power dissipation P_c

Geome trical

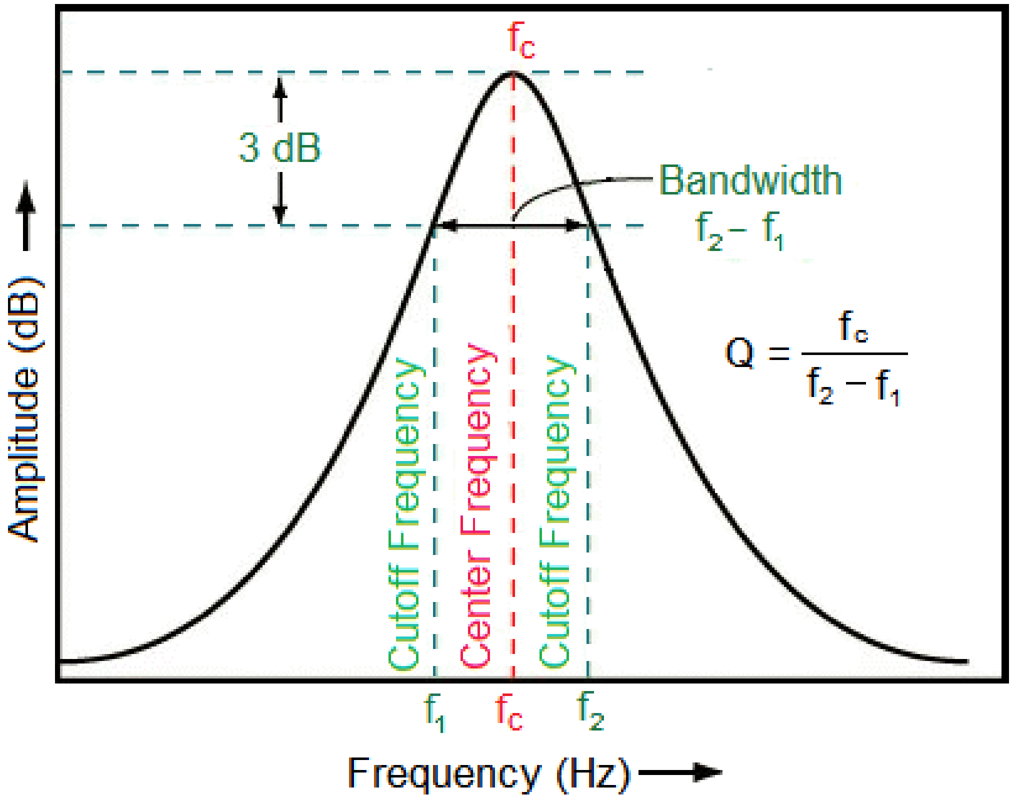
$$Q_0 = \frac{\omega U}{P_c} = \frac{\kappa E_{acc}^2}{P_c}$$

Smaller surface resistance R_s → high Q & low P_c

Experimental observable $$Q_0 = \frac{G}{R_s}$$ Geometrical; From material

Experimental observable $$P_c = \frac{\kappa R_s}{G} E_{acc}^2$$

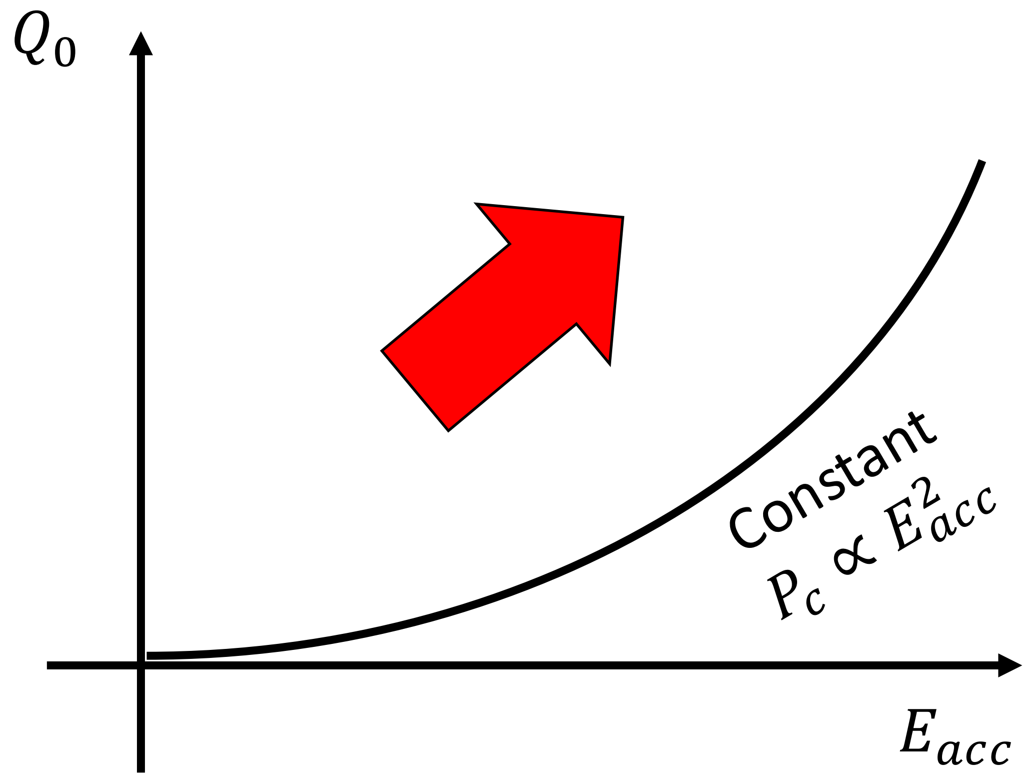
http://lossenderosstudio.com/glossary.php?index=q



<u>G is a geometrical factor</u>

- Elliptical cavity $G \sim 250\ \Omega$
- Spoke cavity $G \sim 130\ \Omega$
- Quarter-wave resonator $G \sim 30\ \Omega$

High-Q (Q_0) and high-gradient (E_{acc}) is the keyword



One of our goals in SRF is to go

High-gradient (G): E_{acc}

with lower power consumption P_c

High-Q: $Q_0 = G/R_s$

Accelerator projects offer an extreme environement for superconductors: R_s at high E_{acc} near **phase transition**

← Quench field of Nb $\sim 200\ \mathrm{mT} = 50\ \mathrm{MV/m} \gtrsim 30\ \mathrm{MV/m}$

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Superconductors for DC fields

Ohm's law

Applied DC electric field *E*

DC Current *J*

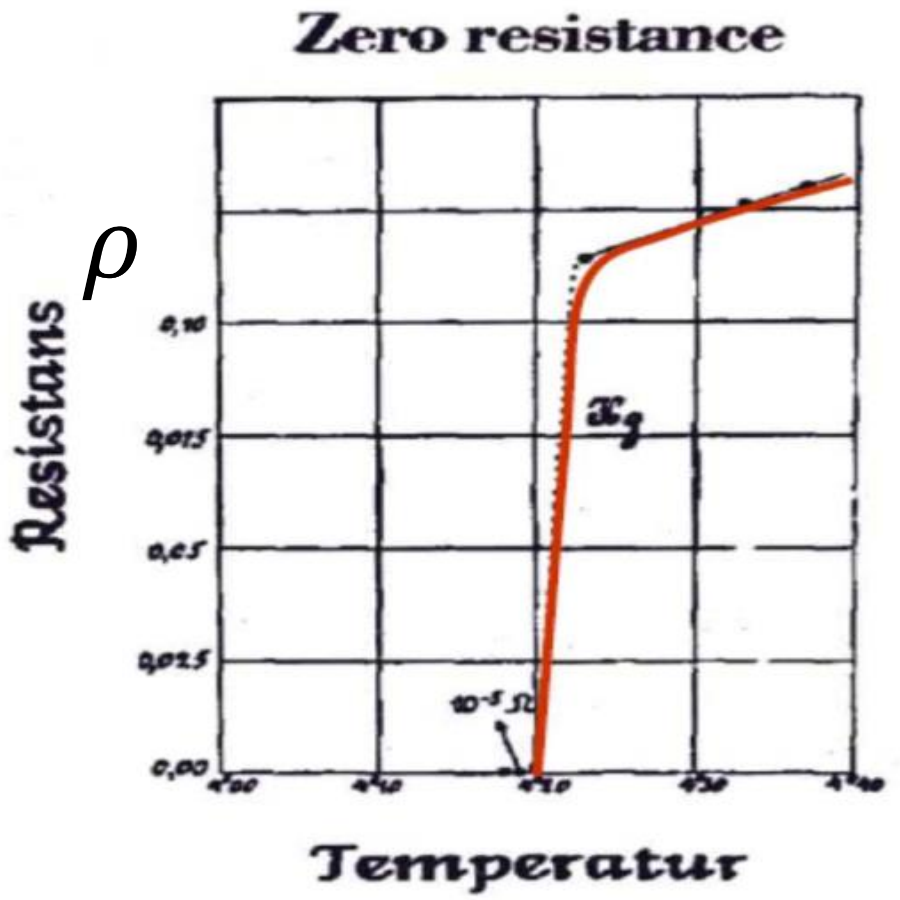
DC resistivity ρ

$$\rho \equiv \frac{E}{J}$$

DC conductivity σ

$$\sigma = \frac{1}{\rho} \equiv \frac{J}{E}$$

Cool down the resistor…



Heike Kamerlingh Onnes

Nobel prize in 1913

$\boldsymbol{\rho = 0}$ below transition temperature T_c

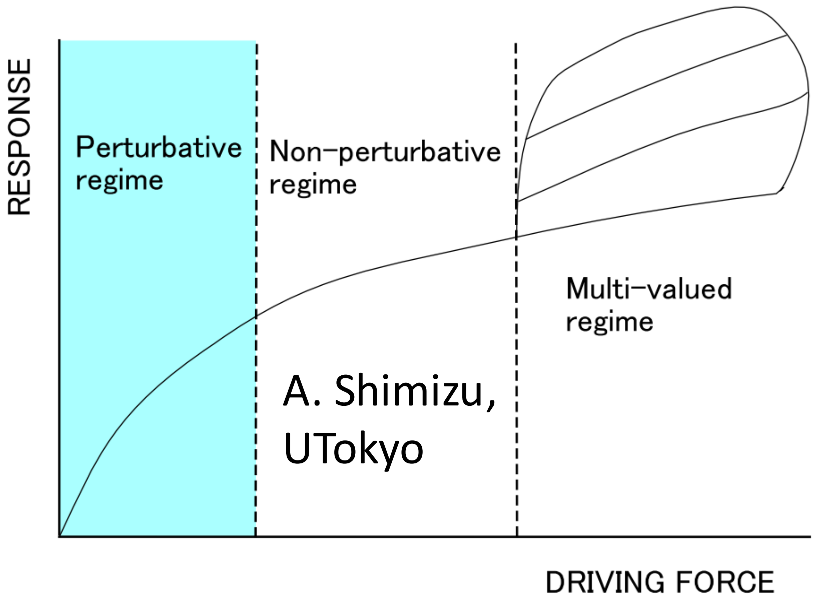
Linear response to RF

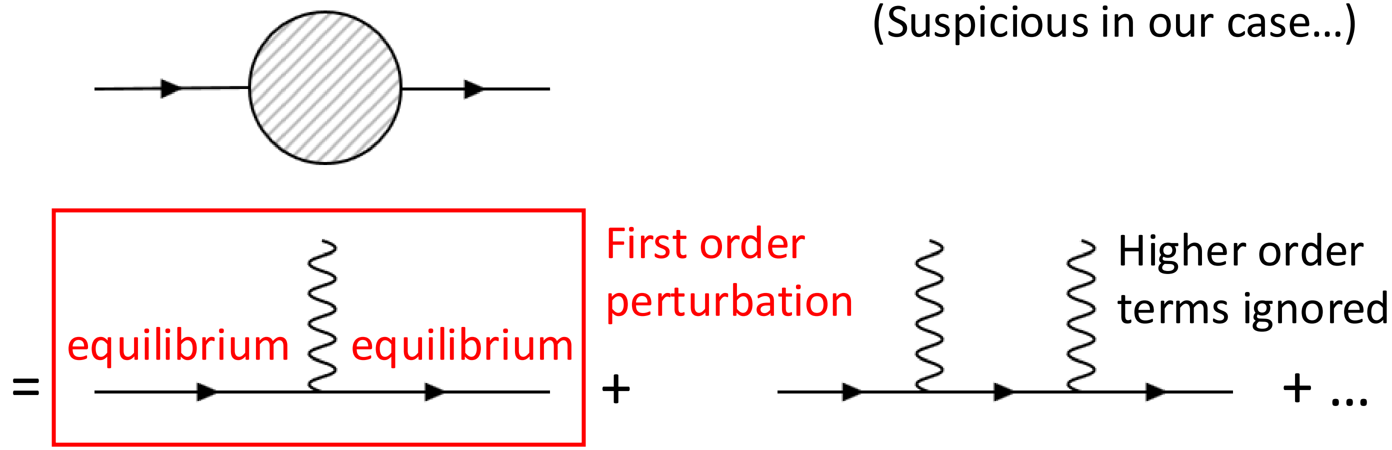
Quamtum mechanical ***derivation*** of R_s requires quantum many body theory

$$\mathcal{H} = \mathcal{H}_0 + \boxed{\mathcal{H}_{RF}(t)}$$

If the RF field is "small"

(Suspicious in our case…)





The responding current is given by the equilibrium state (fluctuation-dissipation theorem)

$\mathcal{H}_0$: Fermi Liquid → Ohm's law

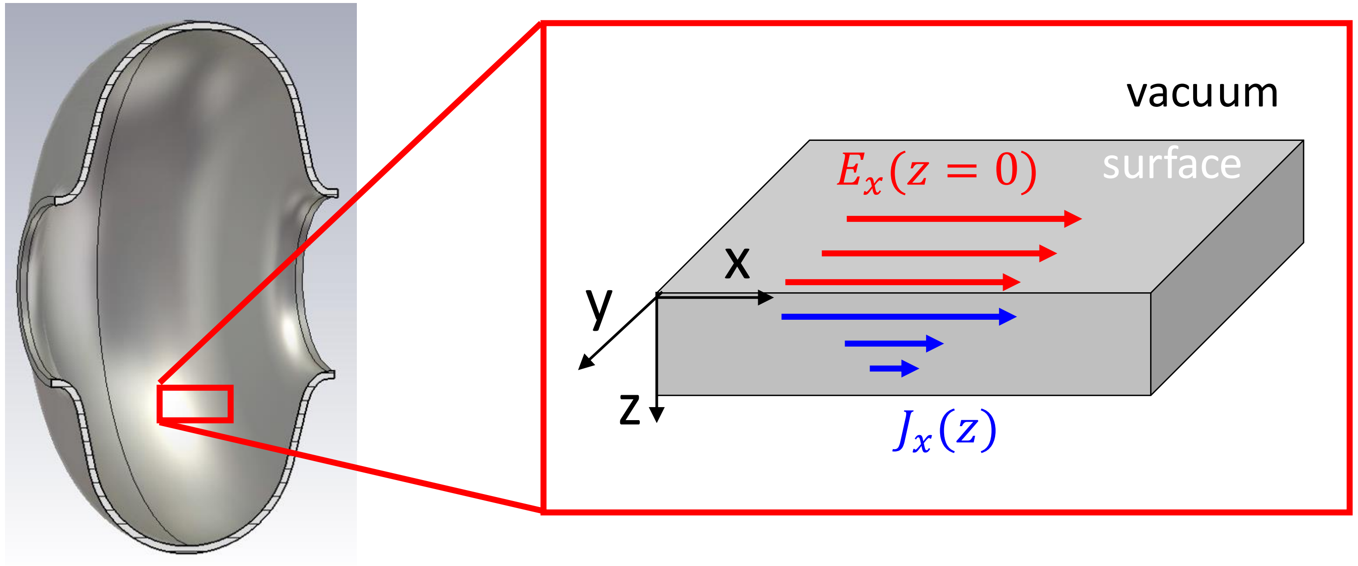
$\mathcal{H}_0$: BCS → Mattis Bardeen

Quantum ***derivation*** *of* Ohm's law via Kubo theory

$$\left.\begin{aligned} \sigma &= -\frac{1}{i\omega}[\Phi^R(\omega) - \Phi^R(0)] \\ \Phi^R &= \frac{i}{\hbar V}\theta(t)\langle \hat{J}(t)\hat{J}(0) - \hat{J}(0)\hat{J}(t)\rangle \end{aligned}\right\} \rightarrow \sigma = \frac{ne^2\tau_{\boldsymbol{k}}}{m}\frac{\widetilde{\rho_0}}{\rho_0}$$

RF resistance R_s is non zero

Materials provide boundary conditions with finite power dissipation

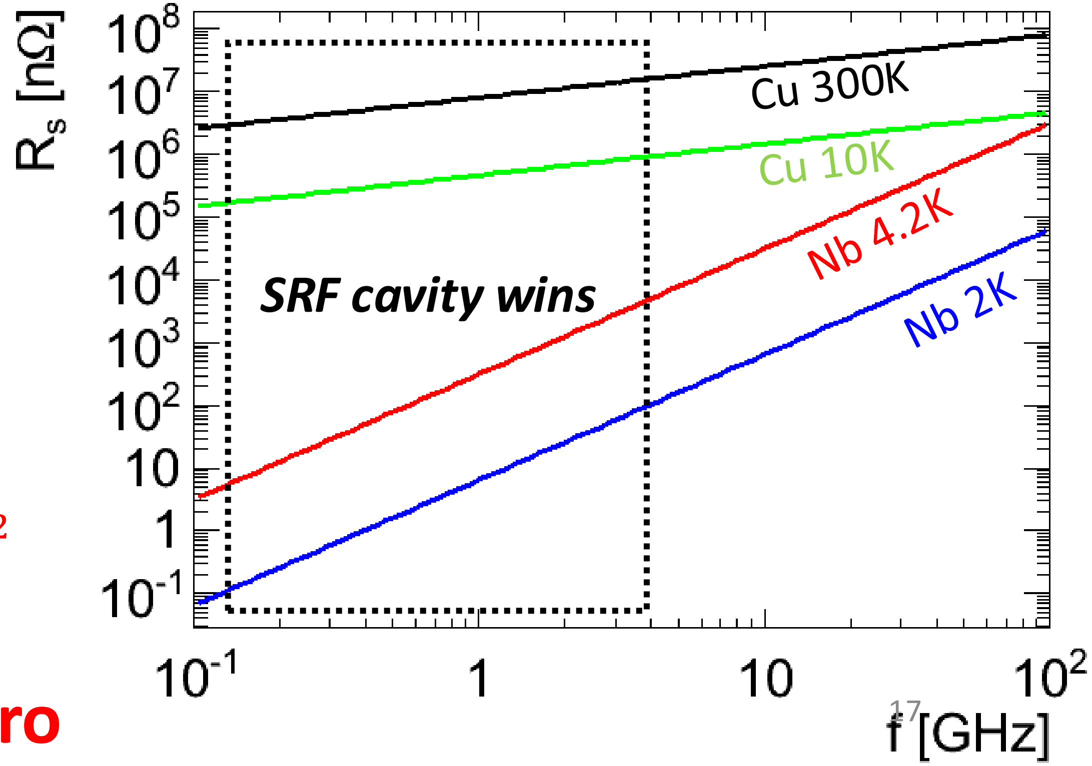


Local surface resistance

$$R_s \equiv \mathrm{Re}\left(\frac{E_x(z=0)}{\int_0^\infty J_x(z)dz}\right)$$

Normal conducting (Cu) $\quad R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} \propto f^{1/2}$

Super- conducting (Nb) $\quad R_s = \frac{Af^2}{T}\exp\left(-\frac{\Delta}{k_B T}\right) \propto f^2$



Superconducting R_s is small but non zero

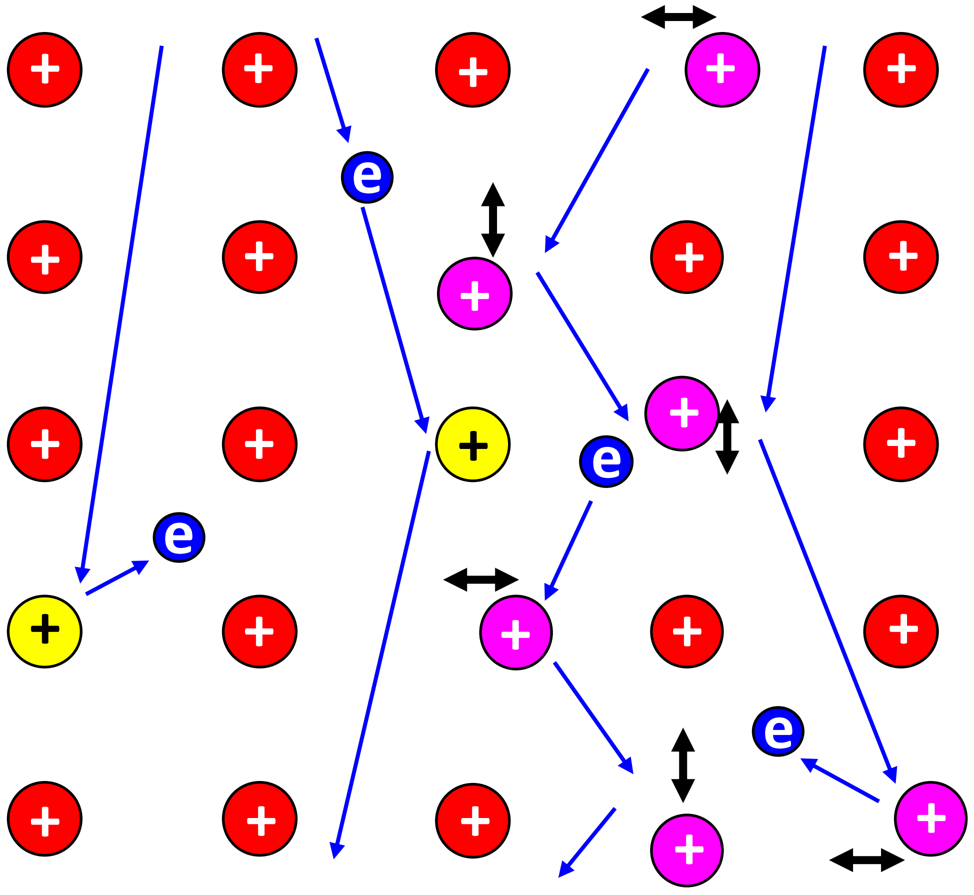
Electrons in normal conducting metals show Ohmic loss

<u>Imperfections causes ***local*** scattering</u>

1. Impurity, defects (scattering time τ_{def})
2. Lattice vibration, phonon (τ_{ph})

Total scattering time

$$\frac{1}{\tau} = \frac{1}{\tau_{\mathrm{def}}} + \frac{1}{\tau_{ph}}$$



<u>Phenomenological explanation ≠ derivation</u>

An electron accelerated by an electric field

$$m^* \frac{dv}{dt} = -eE$$

is scattered by imperfections per τ, and its velocity relaxes to a mean velocity

$$\langle v \rangle = -\frac{e}{m^*} E\tau$$

Electric current is a collective flow of n electrons

$$j = -en\langle v \rangle = \boxed{\frac{e^2 n\tau}{m^*}} E$$

Electrical conductivity σ

Ohm's law

$$\boxed{\boldsymbol{j} = \sigma \boldsymbol{E}}$$

Is this unavoidable? → No!

Paired electrons can avoid Ohmic loss in DC

If electrons ***in a distance*** (>39 nm) are bounded, ***local*** (< 0.5 nm) scattering can be avoided

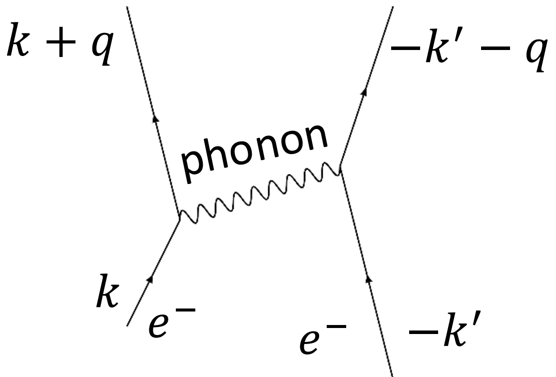
Any small attractive interaction V between electrons can lead to a ***Cooper pair*** coupled with an energy 2Δ, below critical temperature T_c

<u>BCS gap equation (1957)</u>

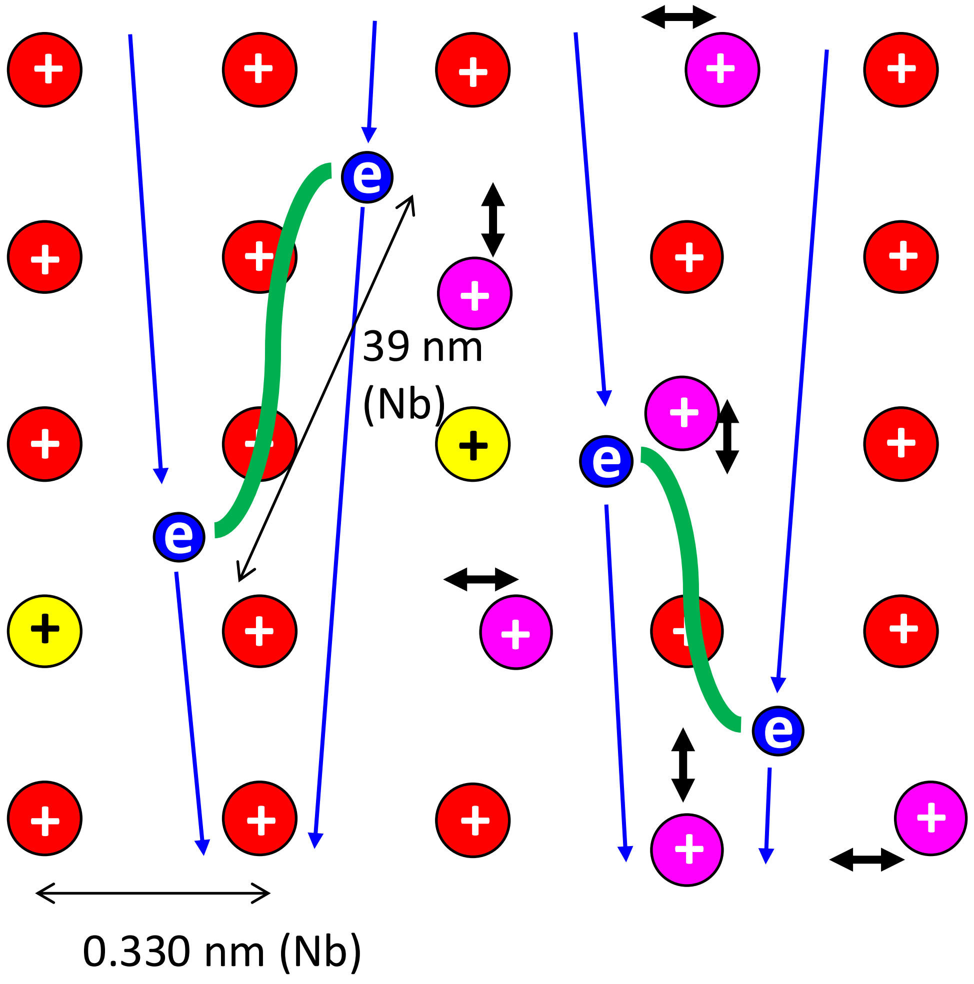
Non-perturbative!

$$\Delta = n(E_F)V\int_{\Delta}^{\hbar\omega_D}\frac{\Delta}{\sqrt{\xi^2+\Delta^2}}\tanh\left(\frac{1}{2}\frac{\sqrt{\xi^2+\Delta^2}}{k_BT}\right)d\xi$$

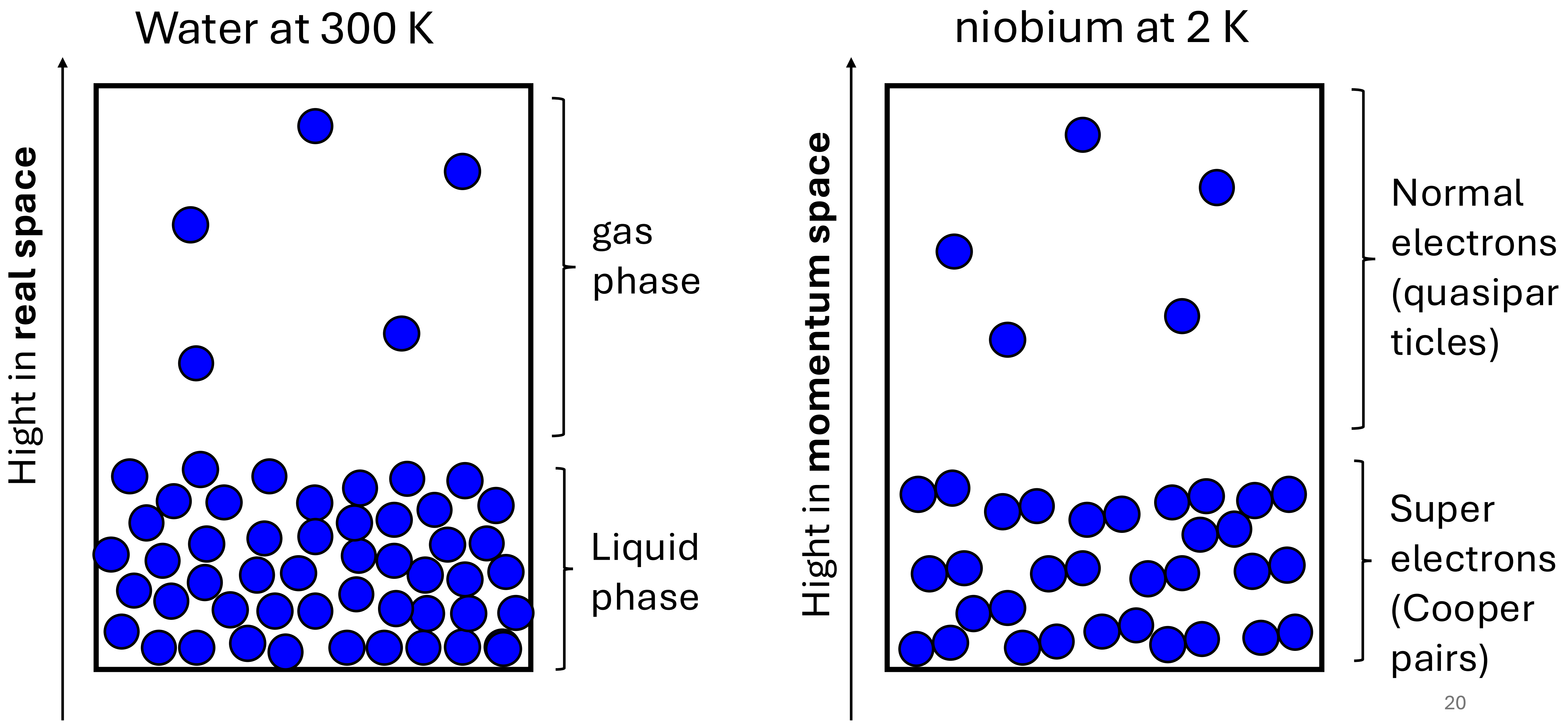
Classical superconductors' attractive potential is from ***longitudinal mode of lattice vibration***



If energy transfer $|\epsilon_{k+q}-\epsilon_k|$ is smaller than phonon energy the interaction is attractive (Flöhlich)
→ Eliashberg's strong coupling superconductor (1960)



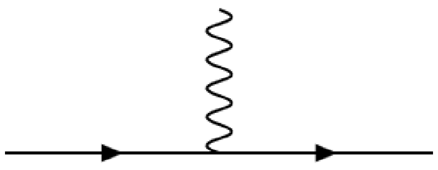
Only part of electrons form Cooper pairs at > 0K



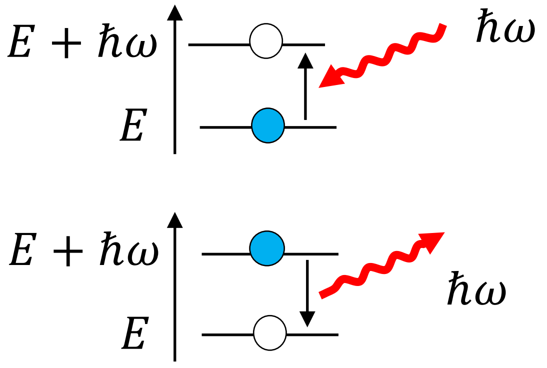
Semi-classical **quantum** mechanical **derivation**

Fermi's Golden rule [Z. Physik 266 p.209 (1974)]

$$R_s \propto P \cong \Sigma_{p,p',h\vec{k}} \left| \quad \right|^2 \propto \text{(photon energy) x (net \# of absorbed photons)}$$



$$= \hbar\omega(n_+ - n_-) = \hbar\omega \int_{\Delta}^{\infty} dE\, [f(E) - f(E+\hbar\omega)] \times N(E)N(E+\hbar\omega)$$



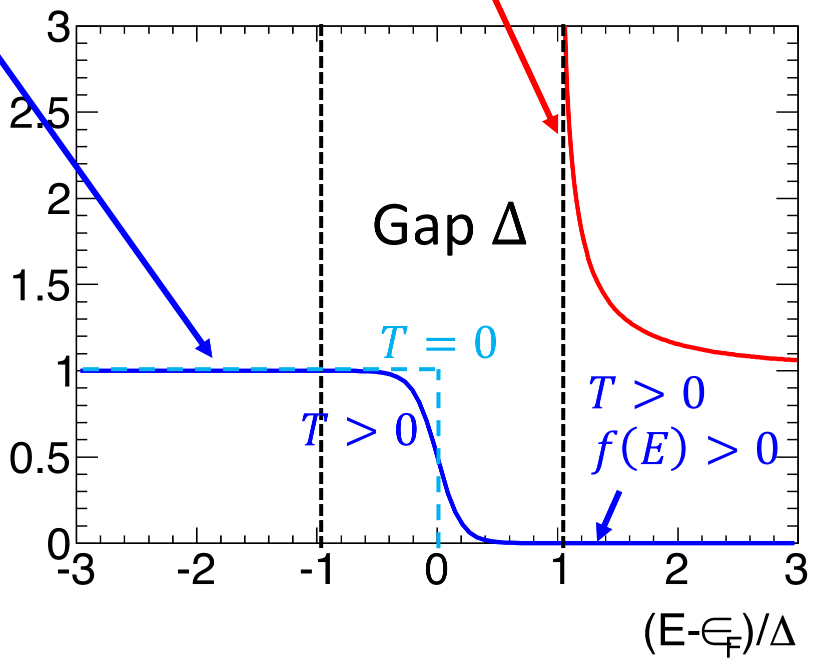
All quasiparticles
$\Delta < E < \infty$

$N(E)$: density of states (how many quantum states at energy E: a kind of degeneracy)

$f(E)$: distribution function (how many electrons are in one state at energy E)

Fermi-Dirac function in equilibrium state

$$f(E) = \frac{1}{\exp(-\mathrm{E}/k_B T) + 1}$$



$$R_s \propto \frac{\omega^{1.5}}{T} \exp\left(-\frac{\Delta}{k_B T}\right)$$

$$R_s(T = 0) = 0$$

Remark: why **semi**-classical?

- The right picture is just schematic for simplicity
- $\hbar\omega$ looks like **a photon** but it is classical microwaves $E_0\cos(\omega t)$
- We treat quantized electrons interacting with classical microwave $E_0\cos(\omega t)$ → **semi**-classical!
- This approach is valid if the intensity of the microwave is high:

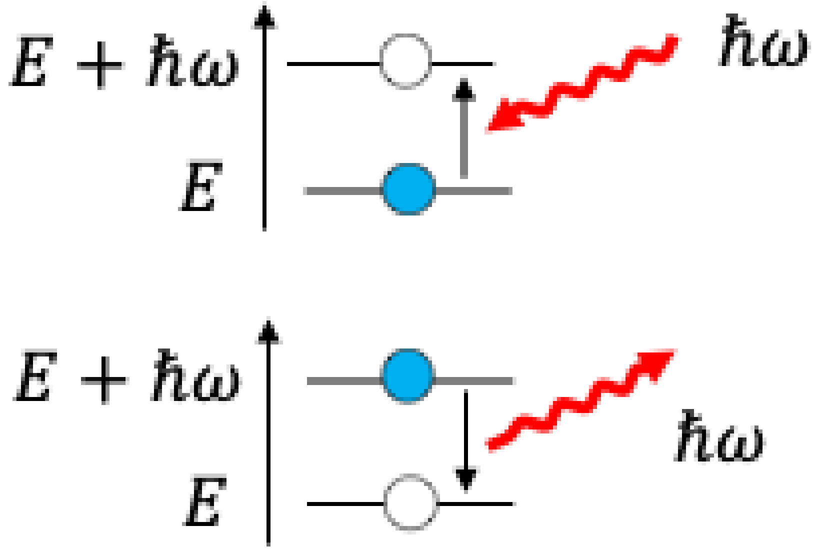
$$50\ \mathrm{dBm}(100\ \mathrm{W}) \gg -210\ \ \mathrm{dBm/Hz}\ (1\ \mathrm{GHz\ single\ photon})$$

- A true photon is described by quantum field operator

$$\hat{\mathcal{E}}(\boldsymbol{r},t) = i\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}\left[\hat{a}e^{-i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})} - \hat{a}^\dagger e^{i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})}\right]$$

- Only important if you study **quantum sensing and qubits** (eg. Vacuum Rabi oscillation, etc)

Maybe a misleading schematic ☹

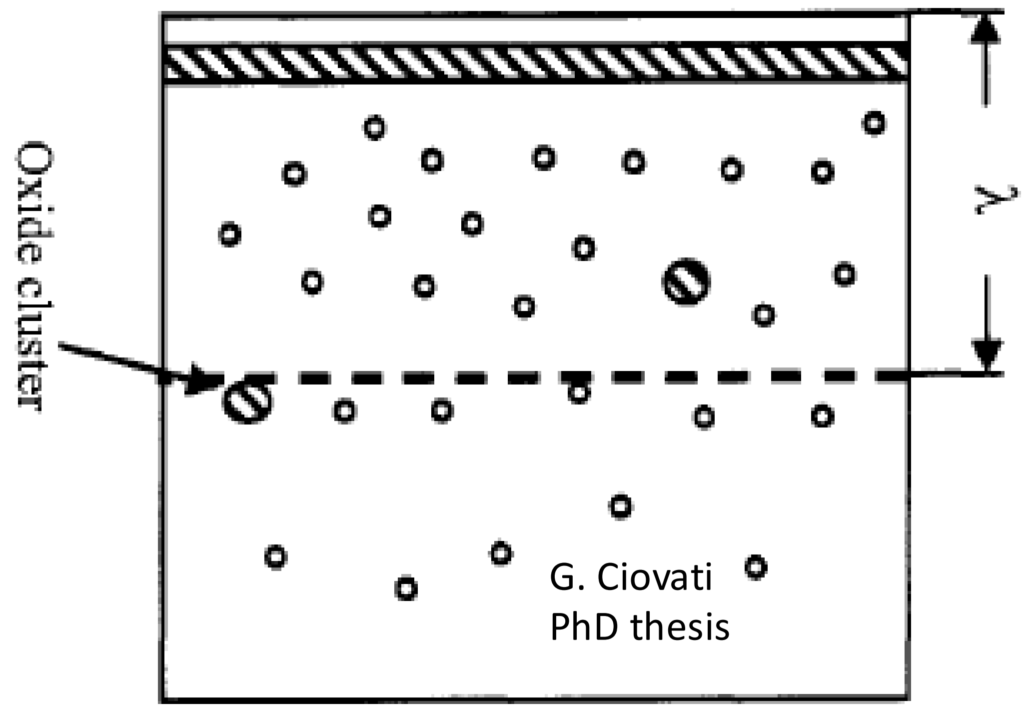


Outline

- Accelerators offer a playground for nonequilibrium superconductivity
  - Complementary to quantum sensing / qubit applications
- Fundamental of BCS and Mattis Bardeen theories
  - Linear response theory
- Nonequilibrium physics and surface treatment
- Thin-film: alternative approach
- Conclusion

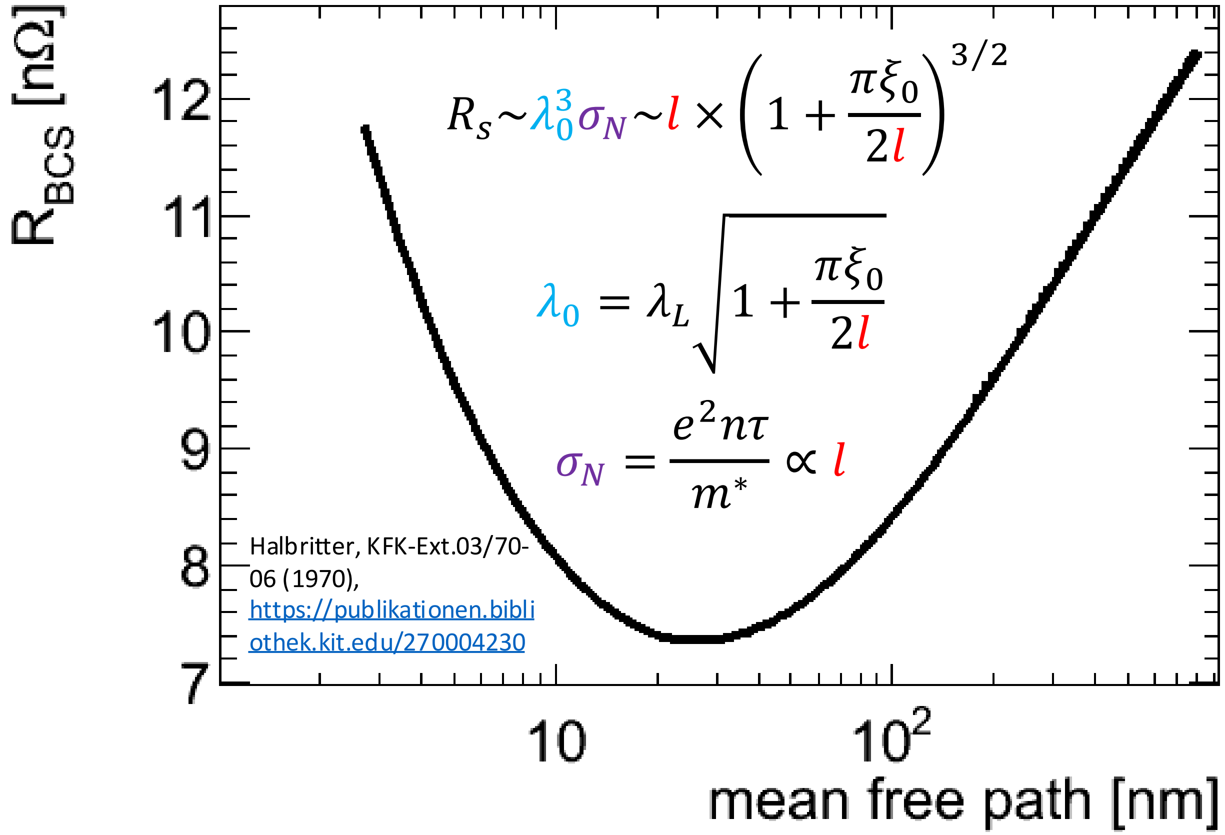
Surface engineering for lower R_s

Bulk niobium
+ surface engineering



G. Ciovati PhD thesis

BCS-MB for niobium



$$R_s \sim \lambda_0^3 \sigma_N \sim l \times \left(1 + \frac{\pi \xi_0}{2l}\right)^{3/2}$$

$$\lambda_0 = \lambda_L \sqrt{1 + \frac{\pi \xi_0}{2l}}$$

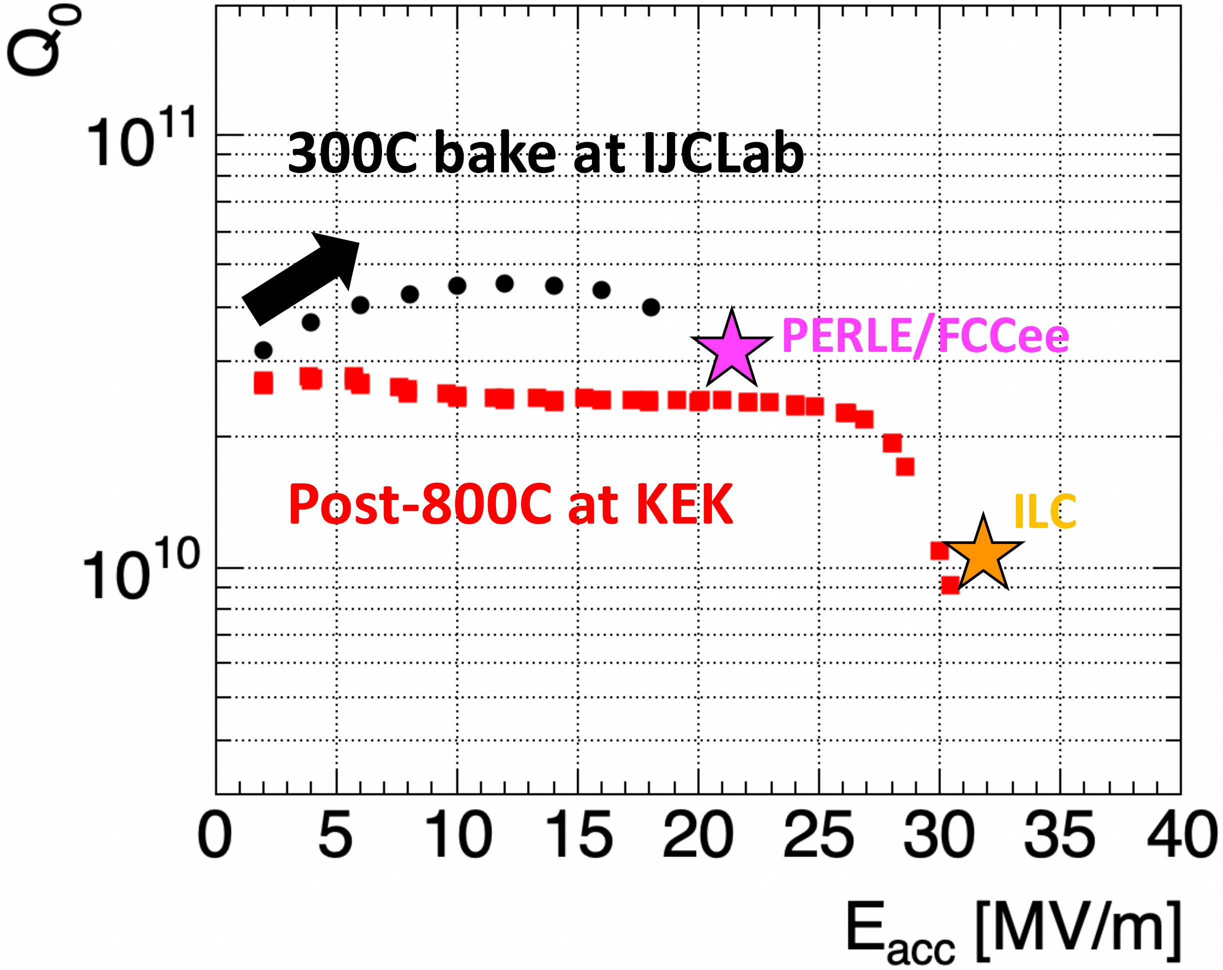
$$\sigma_N = \frac{e^2 n \tau}{m^*} \propto l$$

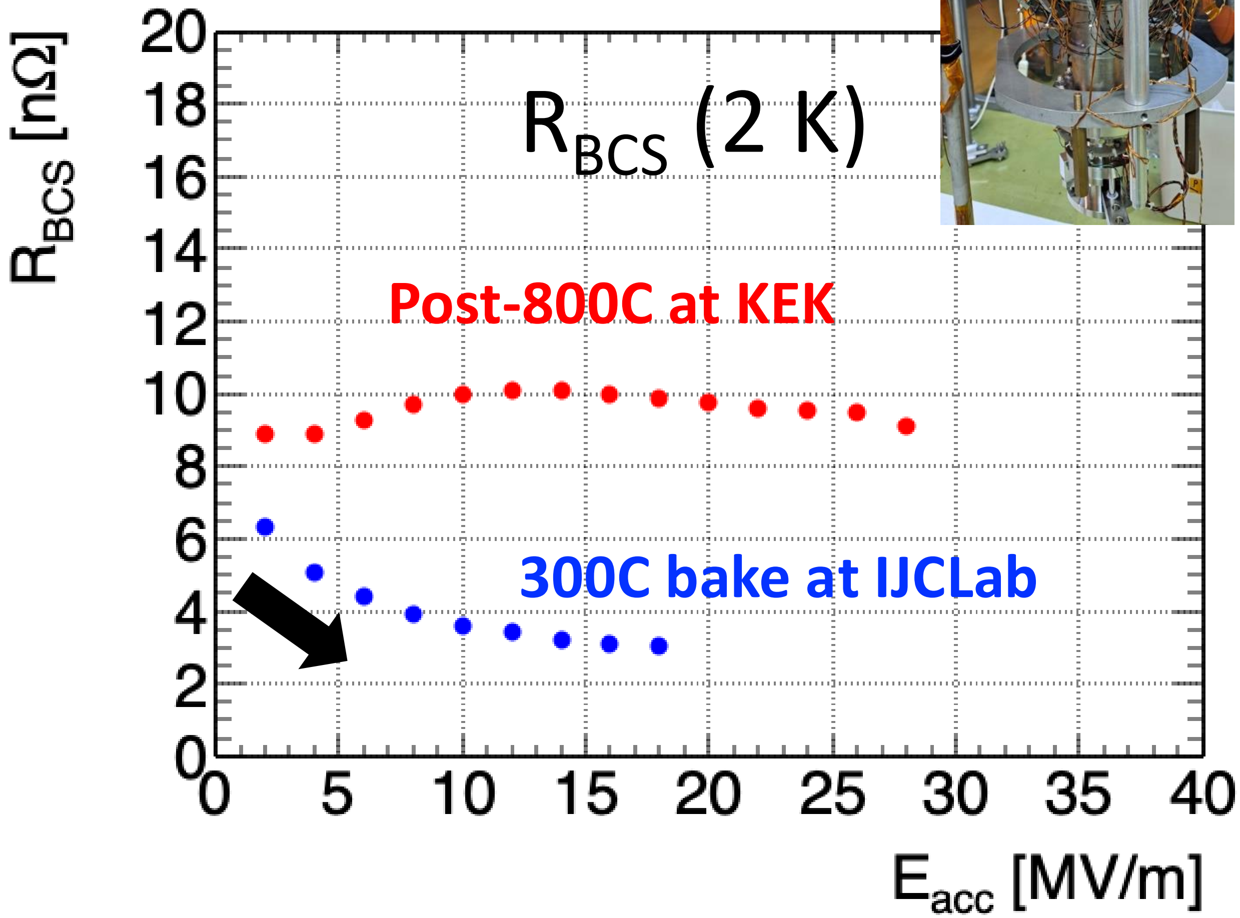
Halbritter, KFK-Ext.03/70-06 (1970), https://publikationen.bibliothek.kit.edu/270004230

Recipes have been found through try & error sometime via mistakes (serendipity)

high-Q or high-G in the **same** 1.3 GHz cavity

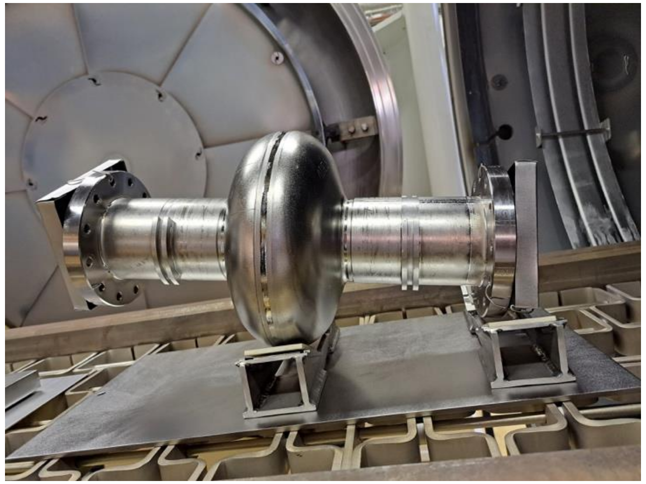
1DE08 cavity @ IJCLab-CEA-KEK-DESY collaboration

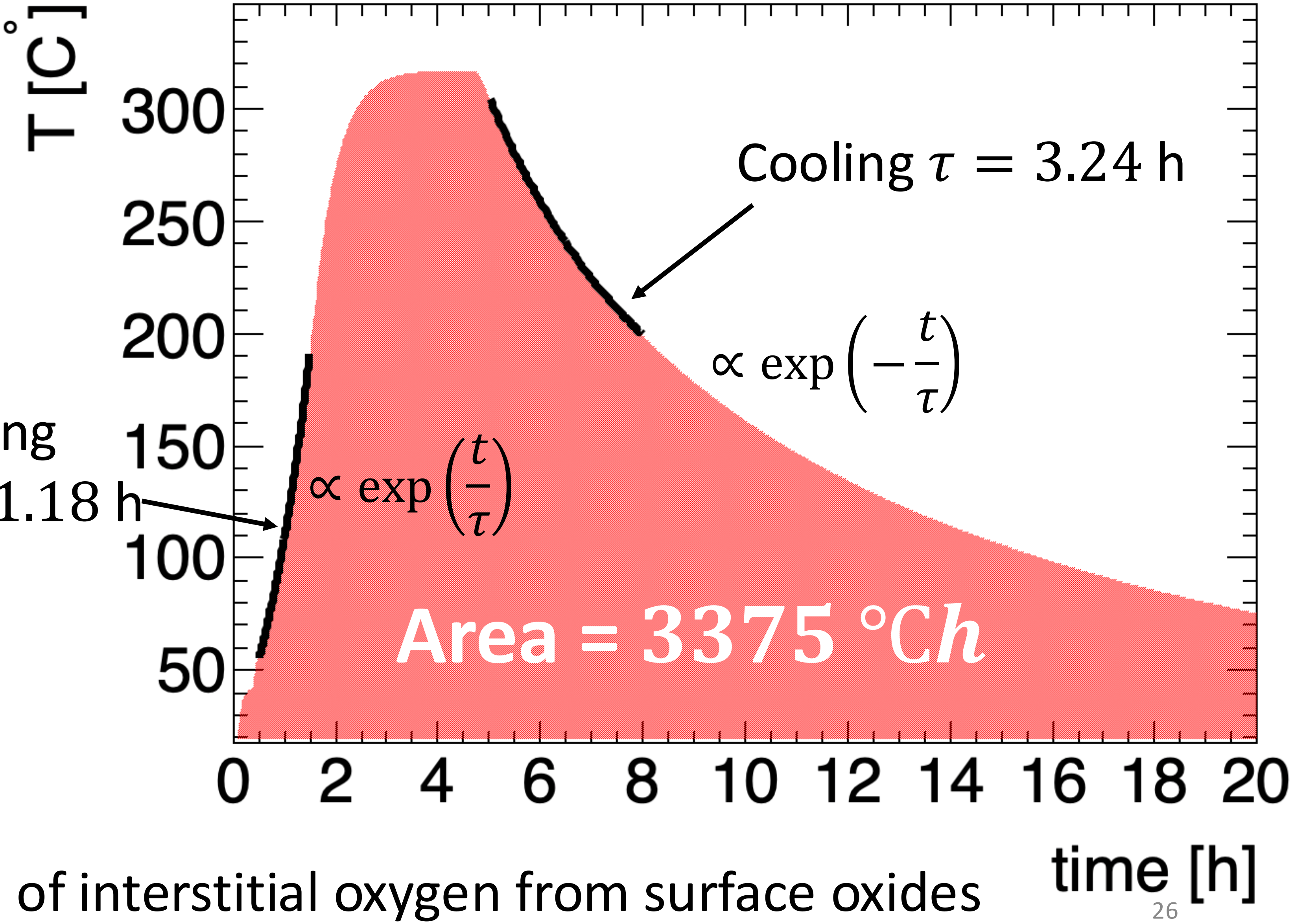




→ ***300C baking*** for high-Q applications like FCC

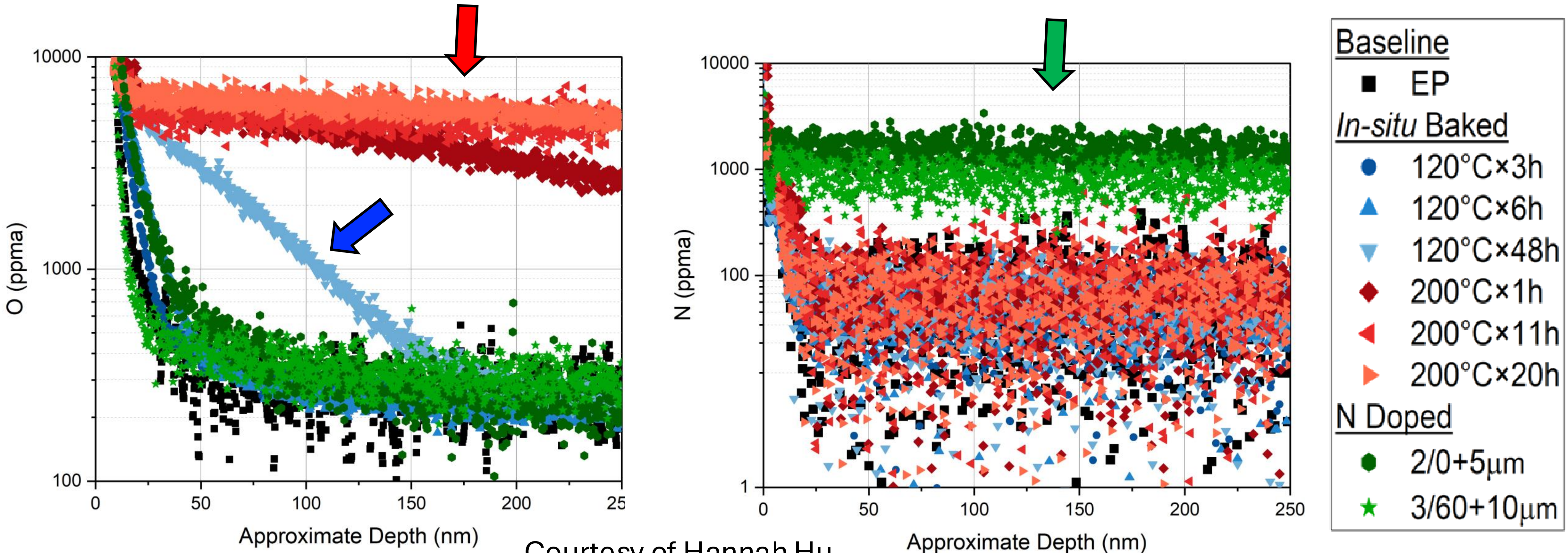
300C baking in a clean vacuum furnace





Heating causes diffusion of interstitial oxygen from surface oxides

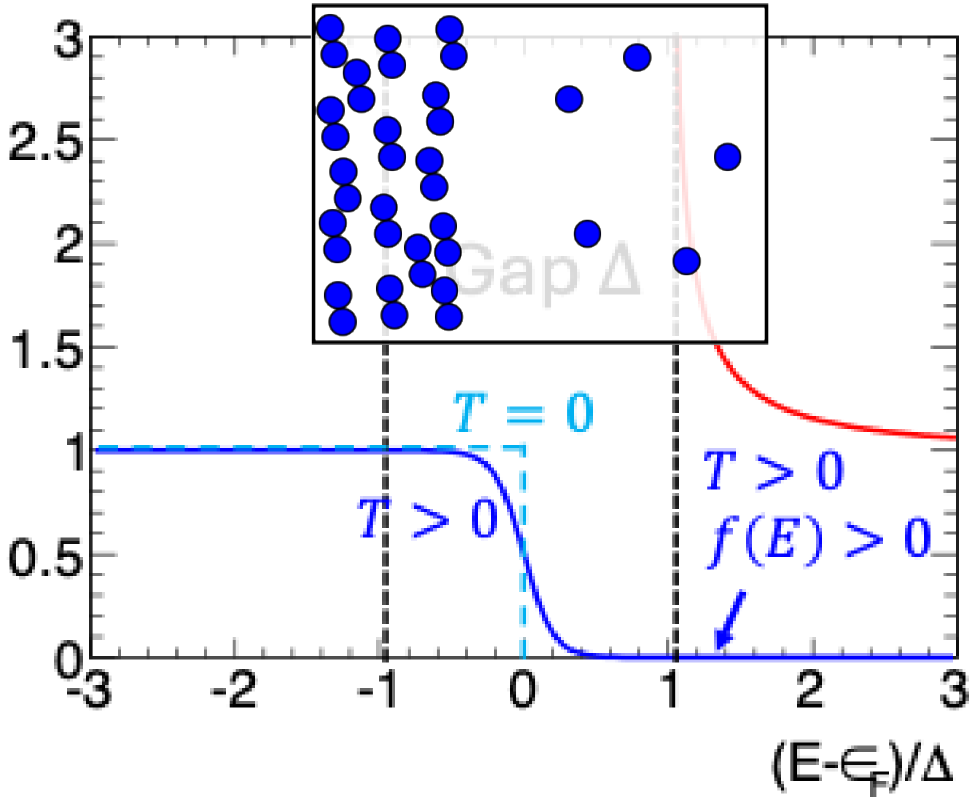
Materials studies (SIMS) by Fermilab

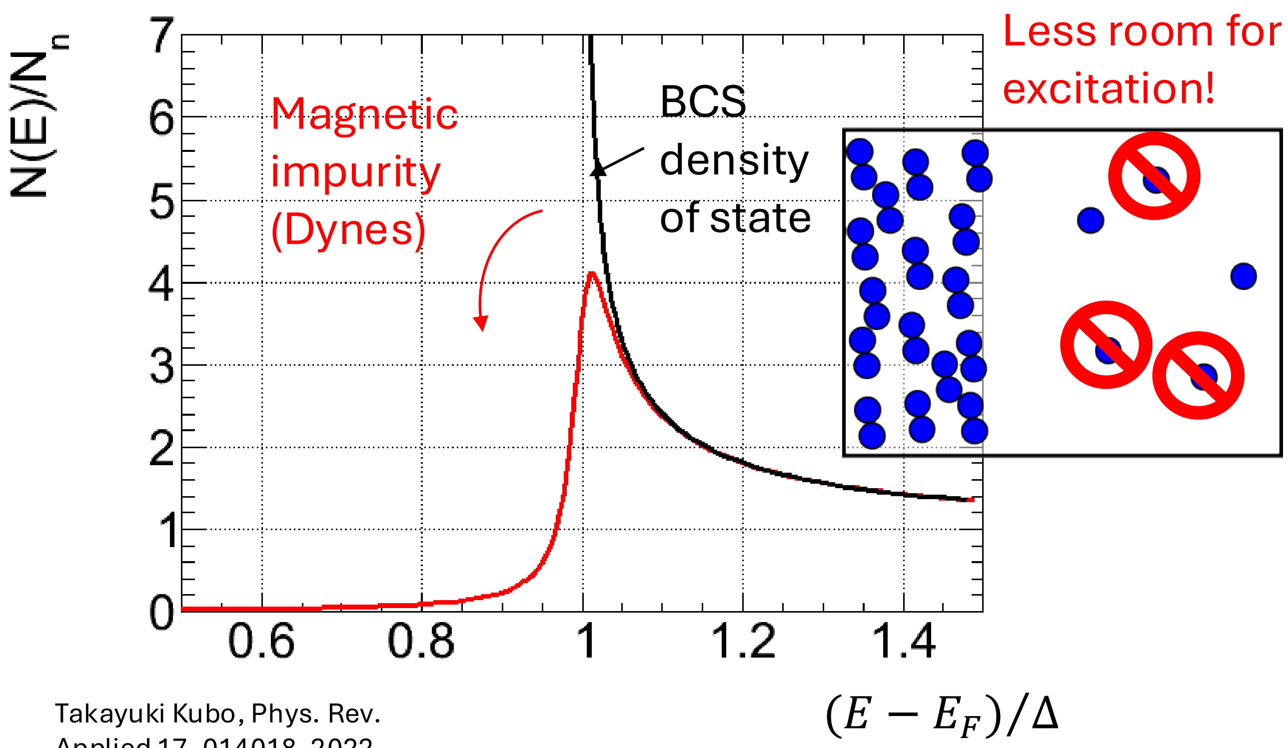


Courtesy of Hannah Hu

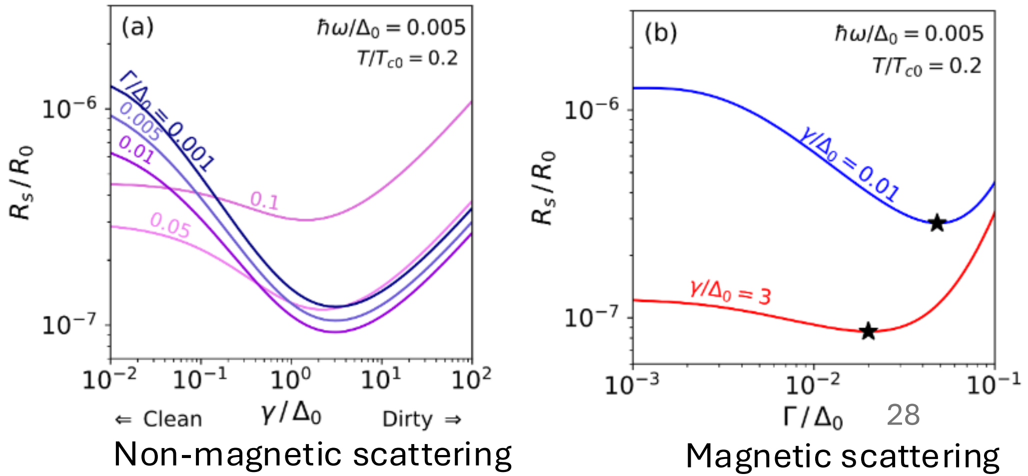
- 120C baking → inhomogeneous impurity distribution in the RF layer (100 nm)
- >200C baking → homogeneous impurity (O or N) distribution in the RF layer

Why mid-T baking / N-doping helps high-Q? *Hypothesis*





Takayuki Kubo, Phys. Rev. Applied 17, 014018, 2022



<u>The overlap between $f(E)$ and $N(E)$ causes loss</u>

The pole at Δ cause R_{BCS}; the states below Δ causes R_{res}

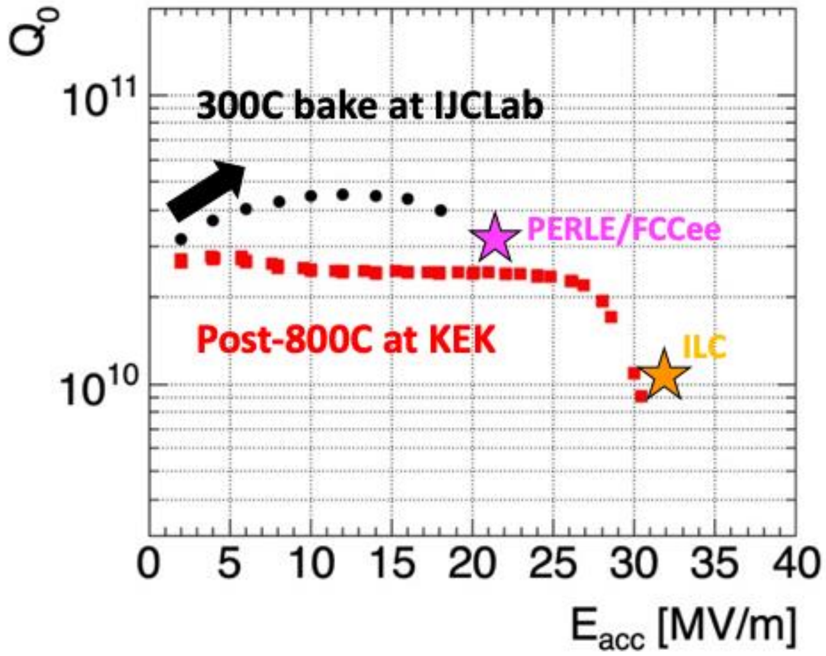
→ Smearing of $N(E)$ decreases R_{BCS} and increases R_{res}

→ Combination of magnetic (O?) and nonmagnetic (N?) **impurity scattering** can control a minimum of $R_{BCS} + R_{res}$

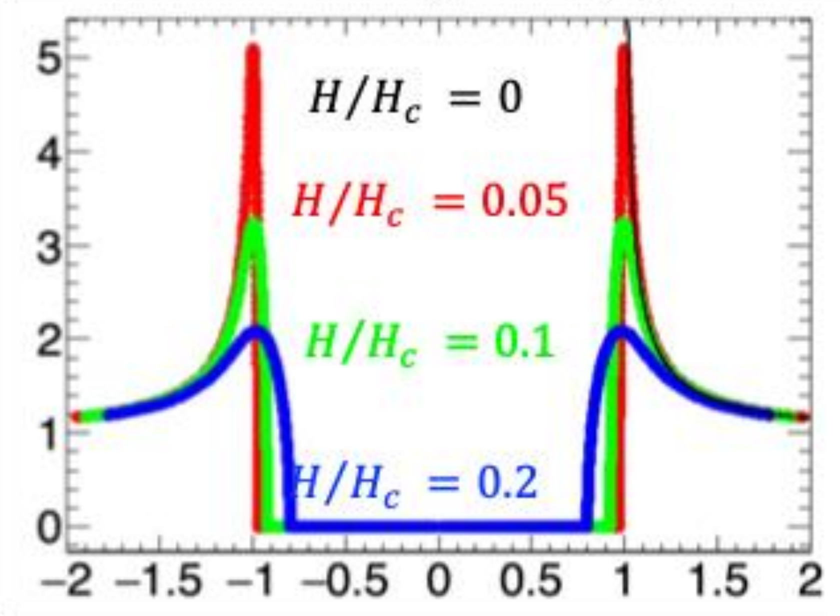
The theory is valid only at low field (1st order perturbation)

Anti-Q-slope in 300C bake? → two research directions

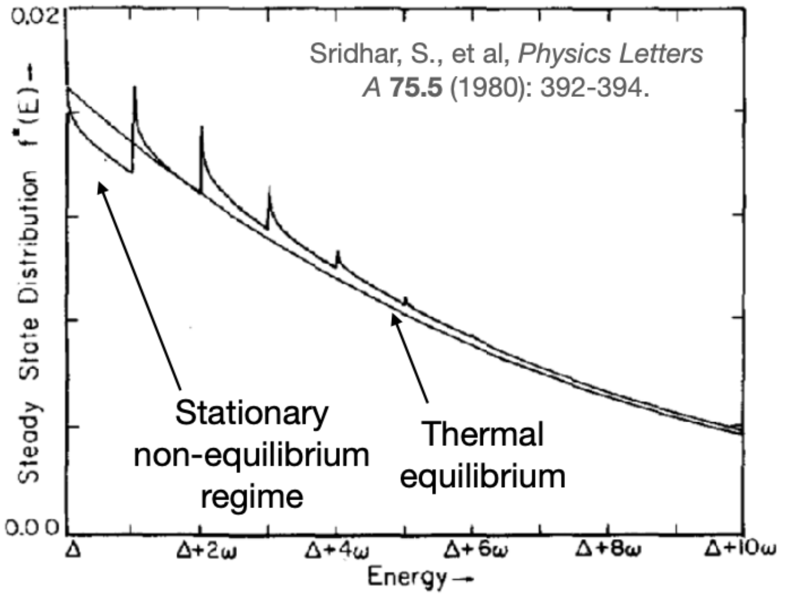
Anti-Q-slope



Smearing of $N(E)$



Non-equilibrium $f(E)$



$$R_s \sim \hbar\omega \int_{\Delta}^{\infty} dE\, [f(E) - f(E+\hbar\omega)] \times N(E)N(E+\hbar\omega)$$

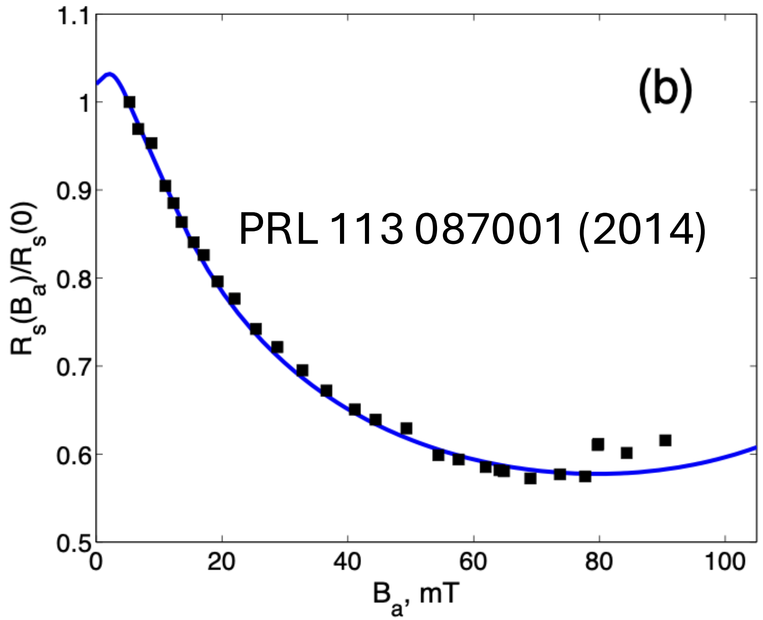
Manipulation of quantum states or distribution function under strong fields

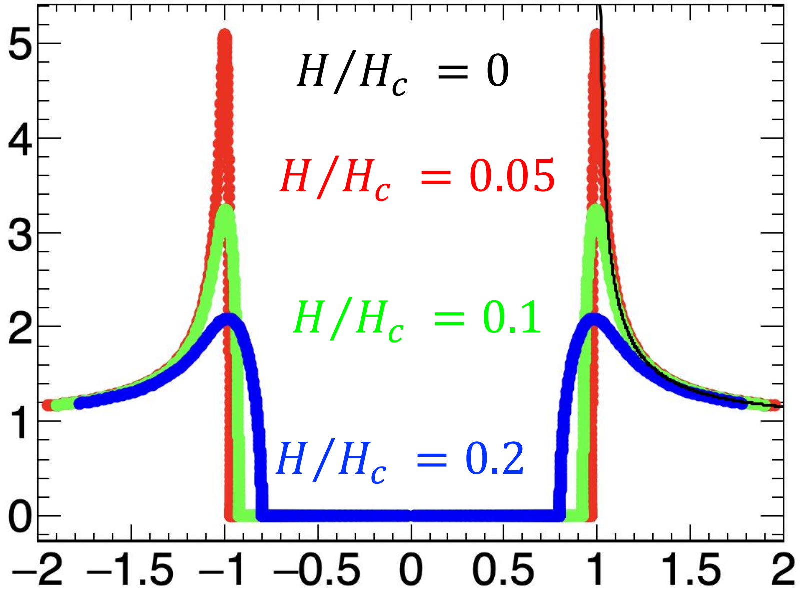
In my view, no experimental direct evidence and thus no consensus!

→Centre of my recent research interest!

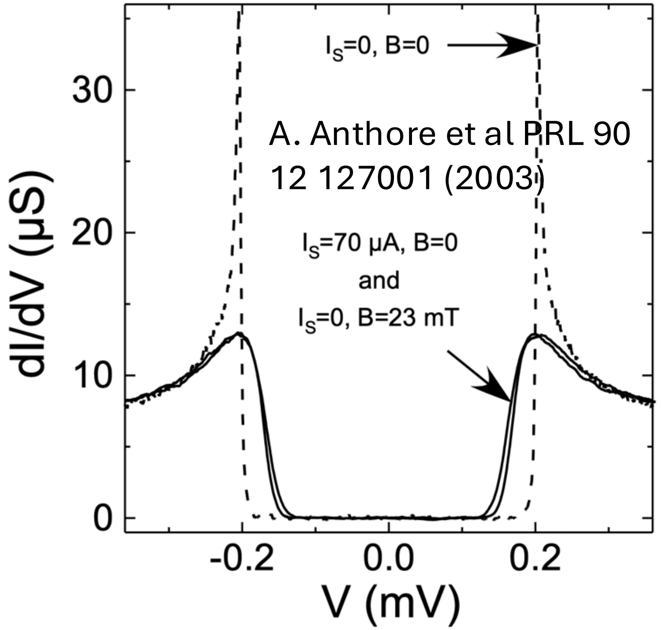
Gurevich-Kubo theory

Smearing of $N(E)$ → R_{BCS} ↓ caused by field not impurity





Smearing of $N(E)$ is real in **DC** tunneling



- **DC** field is known to smear $N(E)$ [Usadel / Eilenberger equation under DC current]
  - → reduction of net number of quasi-particles
- Extrapolation of DC result to RF may be justified if relaxation time of SC is faster than RF (quasi-DC approximation)
  - Non-stationary DOS oscillates with RF
- Eliashberg excitation of $f(E)$

$$f_0 > 1.73/2\pi\tau_E \sim \mathbf{15\ GHz} \gg 1\ \text{GHz (SRF)}$$

→ Extrapolation of DC may be valid for SRF

Energy relaxation time

Metal	T_c (K)	$\tau_E{}^a$ (s)	$k_B T_c$ (μeV)	γ_r (μeV)
Zn	0.88	$9.3\ 10^{-8}$	76	$7.1\ 10^{-3}$
Al	1.18	$1.3\ 10^{-8}$	102	$5.2\ 10^{-2}$
In	3.41	$9.5\ 10^{-11}$	294	6.9
Sn	3.72	$2.7\ 10^{-10}$	321	2.4
Nb	9.25	$1.8\ 10^{-11}$	798	37

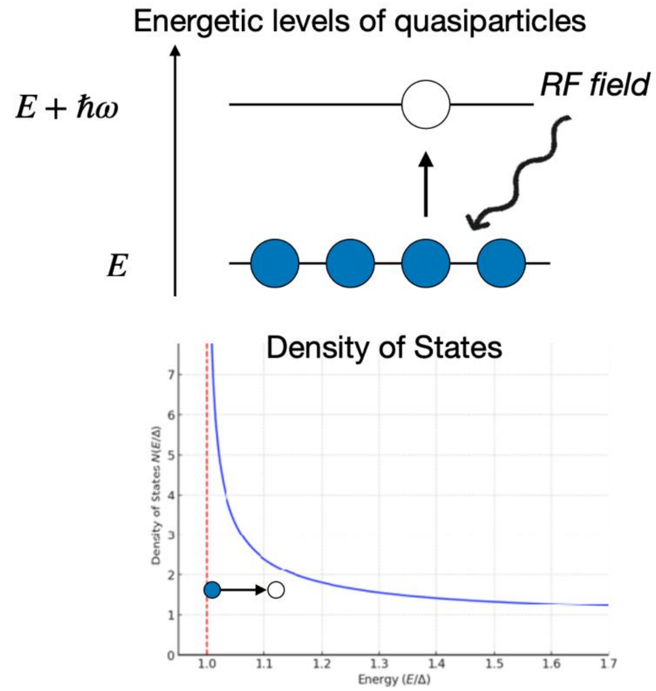
1 GHz corresponds to $\hbar\omega$ = 4.1 μeV.

Mooij, J. E. Enhancement of Superconductivity, chap. 9, 191–287 (Plenum Press, New York, 1981).

Romanenko-Martinello theory

The hypothesis that the anti-Q slope of N-doped cavities may originate from a deviation of the quasiparticle energy distribution from thermal equilibrium was first proposed by Romanenko [32] and further corroborated in this Letter.

In agreement with the Eliashberg theory [33], superconductors start showing nonequilibrium effects above a certain frequency threshold at which quasiparticles populate high-energy states far from the gap edge. This redistribution of quasiparticles decreases the probability of photon absorption, lowering the dissipation and hence the surface resistance. As previously reported for aluminum cavities [34], such a regime of stimulated superconductivity



Martinello, M., et al, *Physical Review Letters* **121.22** (2018): 224801.

However, this was inconsistent with the consensus in the Eliashberg theory

→ The threshold of nonequilibrium excitation of the distribution function is **15 GHz** for Nb

→ We need yet another idea for **1 GHz** SRF

Metal	T_c (K)	$\tau_E{}^a$ (s)	$k_B T_c$ (μeV)	γ_r (μeV)
Zn	0.88	$9.3\ 10^{-8}$	76	$7.1\ 10^{-3}$
Al	1.18	$1.3\ 10^{-8}$	102	$5.2\ 10^{-2}$
In	3.41	$9.5\ 10^{-11}$	294	6.9
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Nb	9.25	$1.8\ 10^{-11}$	798	37

1 GHz corresponds to $\hbar\omega$ = 4.1 μeV.

Our idea: quantify the idea by Romanenko & Martinello

Chang & Scalapino just after Eliashberg

$$\frac{df(E)}{dt} = I_{qp}(E) - \frac{2\pi}{\hbar}\int_0^\infty d\Omega\, \alpha^2(\Omega)F(\Omega)\rho(E+\Omega)\left(1-\frac{\Delta^2}{E(E+\Omega)}\right)$$
$$\times\{f(E)[1-f(E+\Omega)]n(\Omega) - f(E+\Omega)[1-f(E)][n(\Omega)+1]\}$$
$$-\frac{2\pi}{\hbar}\int_0^{E-\Delta} d\Omega\, \alpha^2(\Omega)F(\Omega)\rho(E-\Omega)\left(1-\frac{\Delta^2}{E(E-\Omega)}\right)$$
$$\times\{f(E)[1-f(E-\Omega)][n(\Omega)+1] - [1-f(E)]f(E-\Omega)n(\Omega)\}$$
$$-\frac{2\pi}{\hbar}\int_{E+\Delta}^\infty d\Omega\, \alpha^2(\Omega)F(\Omega)\rho(\Omega-E)\left(1+\frac{\Delta^2}{E(\Omega-E)}\right)$$
$$\times\{f(E)f(\Omega-E)[n(\Omega)+1] - [1-f(E)][1-f(\Omega-E)]n(\Omega)\}$$

and

$$\frac{dn(\Omega)}{dt} = I_{ph}(\Omega) - \frac{8\pi}{\hbar}\frac{N(0)}{N}\int_\Delta^\infty dE \int_\Delta^\infty dE'\, \alpha^2(\Omega)\rho(E)\rho(E')$$
$$\times\Bigg(\left(1-\frac{\Delta^2}{EE'}\right)\{f(E)[1-f(E')]n(\Omega)$$
$$-f(E')[1-f(E)][n(\Omega)+1]\}$$
$$\times\delta(E+\Omega-E') + \frac{1}{2}\left(1+\frac{\Delta^2}{EE'}\right)\{[1-f(E)][1-f(E')]n(\Omega)$$
$$-f(E)f(E')[n(\Omega)+1]\}\delta(E+E'-\Omega)\Bigg) - \frac{n(\Omega)-n(\Omega,T)}{\tau_{es}}$$

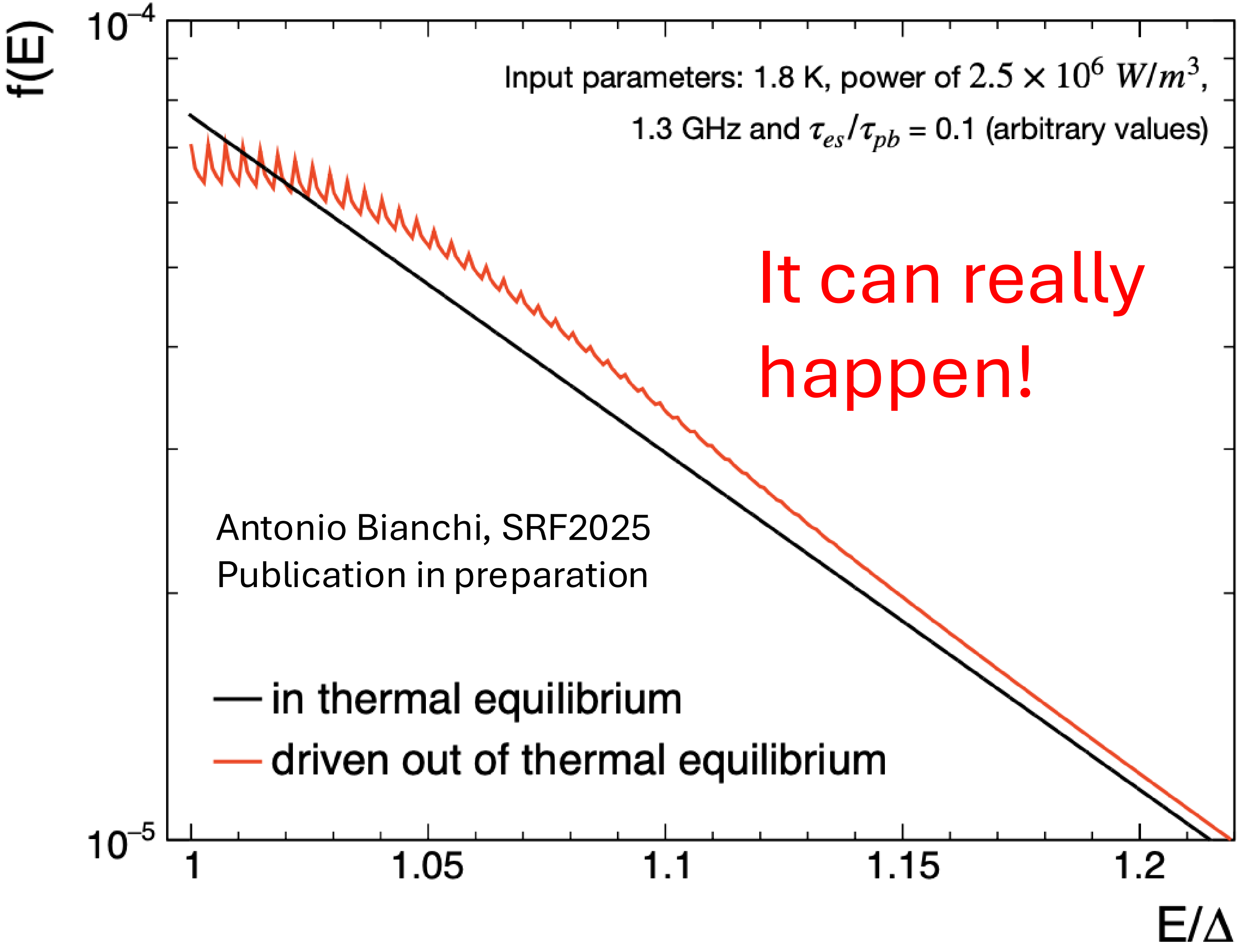
Metal	T_c (K)	$\tau_E{}^a$ (s)	k_BT_c (μeV)	γ_r (μeV)
Zn	0.88	$9.3\ 10^{-8}$	76	$7.1\ 10^{-3}$
Al	1.18	$1.3\ 10^{-8}$	102	$5.2\ 10^{-2}$
In	3.41	$9.5\ 10^{-11}$	294	6.9
Sn	3.72	$2.7\ 10^{-10}$	321	2.4
Nb	9.25	$1.8\ 10^{-11}$	798	37

1 GHz corresponds to $\hbar\omega$ = 4.1 μeV.

We ague...

- Gurevich-Kubo (Eliashberg) calculated the distribution function of only quasi-particles
- Simultaneous calculation of **phonon distribution coupled with quasi-particles** could reduce the threshold frequency down from **15 GHz**

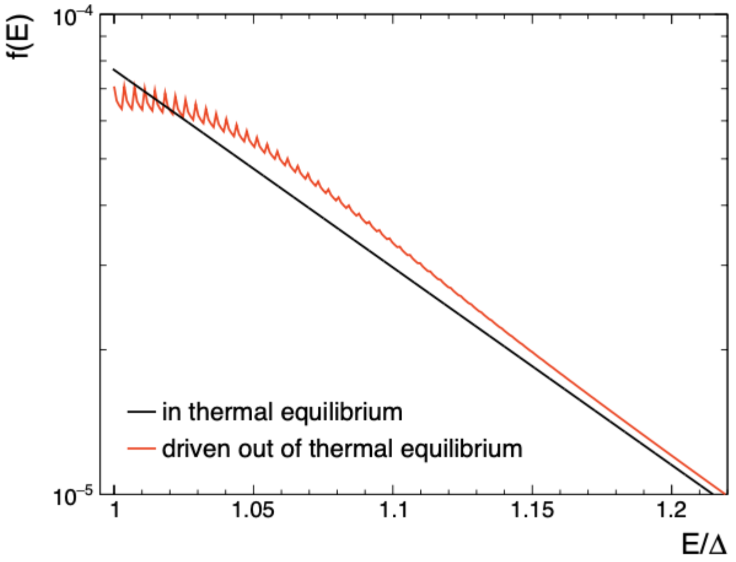
1st demonstration of distortion of FD dist. at 1.3 GHz



Anti-Q-slope can be explained!

Calculations:

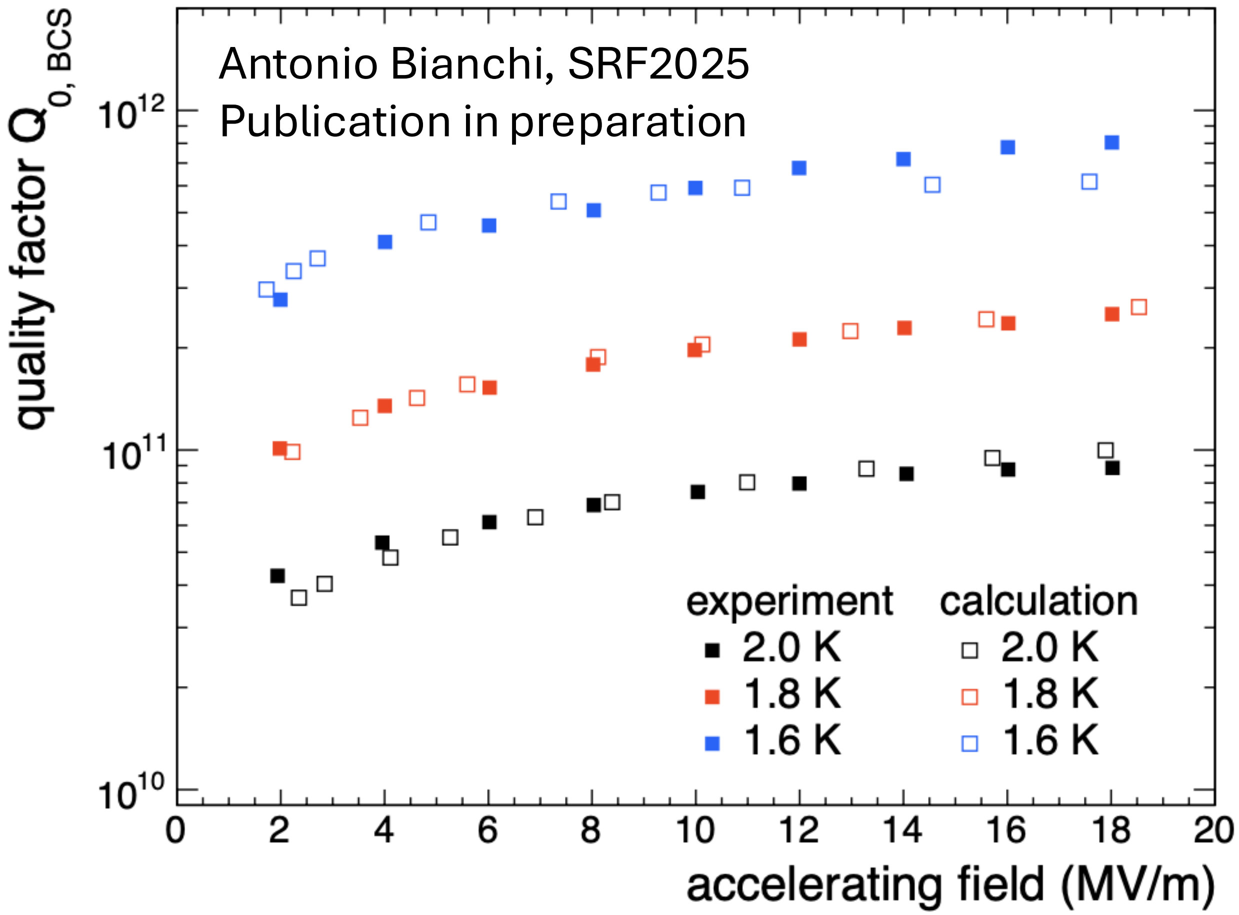
1. Calculation of quasiparticle distribution function under 1.3 GHz field at 1.6 K, 1.8 K, and 2 K



2. Calculation of surface resistance:

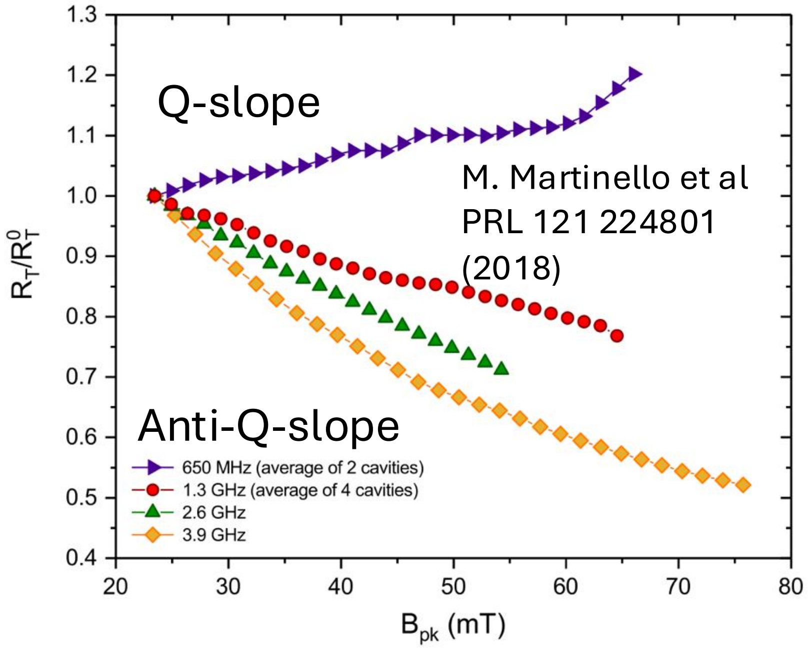
$$R_s \propto \hbar\omega \int_{\Delta}^{\infty} dE\, [f(E) - f(E+\hbar\omega)]\, \rho(E)\, \rho(E+\hbar\omega)$$

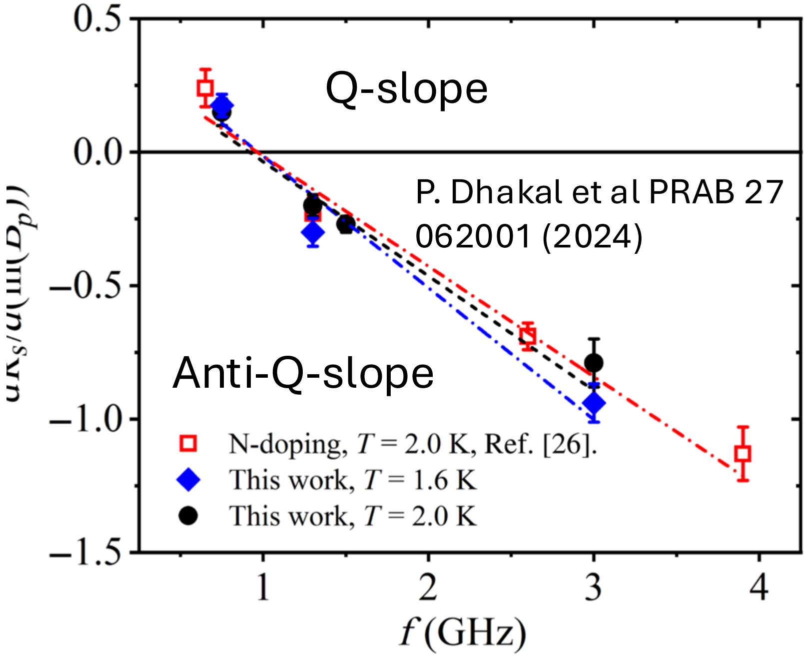
Important: valid only at low fields

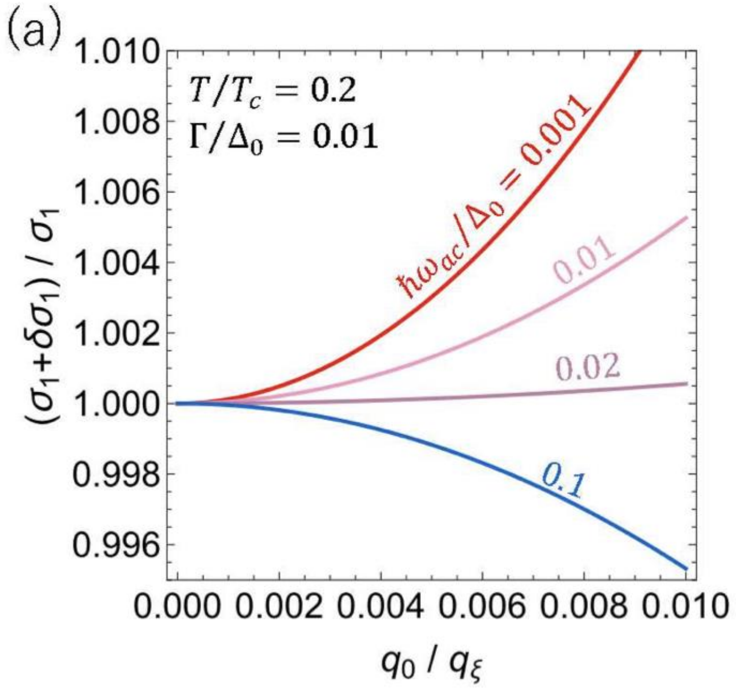


Note: DoS is kept at BCS for simplicity

Frequency dependence of anti-Q-slope theory







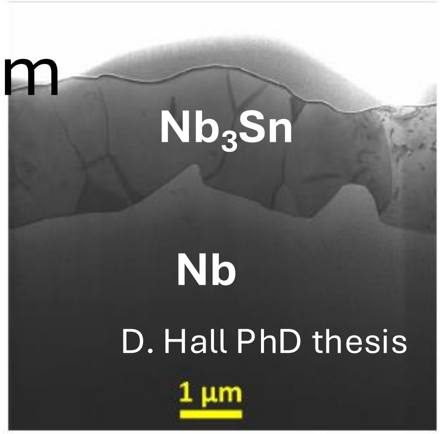
- Frequency dependence (higher/lower 1 GHz) was NOT explained by Gurevich theory
- If the relaxation process is dominated by $\tau_{e,ph}$, corresponding cross-over is **1.2 GHz**
- Extension of Gurevich theory (+ impurity self-energy and Higgs mode) enhances anti-Q-slope at higher frequency WITHIOUT $\tau_{e,ph}$ discussions (T. Kubo arXiv:2509.09766)
- DoS smearing + e-ph coupling (?)
- Any theory is valid only at low field (limitation of linear response theory)

Outline

- Accelerators offer a playground for nonequilibrium superconductivity
  - Complementary to quantum sensing / qubit applications
- Fundamental of BCS and Mattis Bardeen theories
  - Linear response theory
- Nonequilibrium physics and surface treatment
- Thin-film: alternative approach
- Conclusion

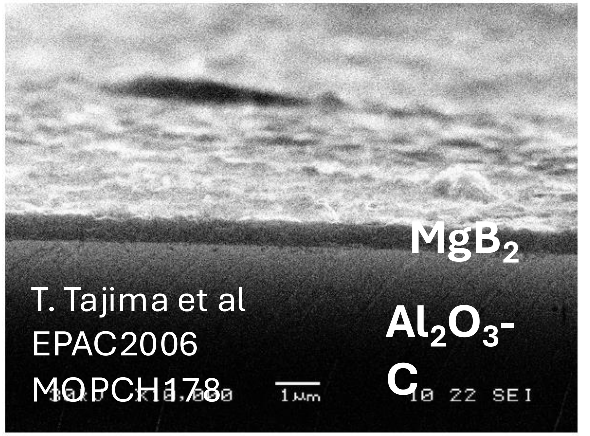
Beyond Nb for sustainability and higher performance

- Niobium material (RRR=300) is getting more and more expensive
  - Over 20 times more expensive than copper (?)
- Nb cavities are typically operated in 2K liquid helium
  - Crisis in He supply (Russia & mid-east)
  - Very expensive cryogenic infrastructure
- On-going researches
  - Nb-coating on copper substrates
  - Nb_3Sn on Nb to be operated at 4K
    - Cryocooler
    - Nb_3Sn on Cu
  - NbTiN, MgB_2, etc...
- Another point: HTS market is growing
  - Magnet, cavity, detector communities
  - Does HTS have any potential for the particle accelerator application?

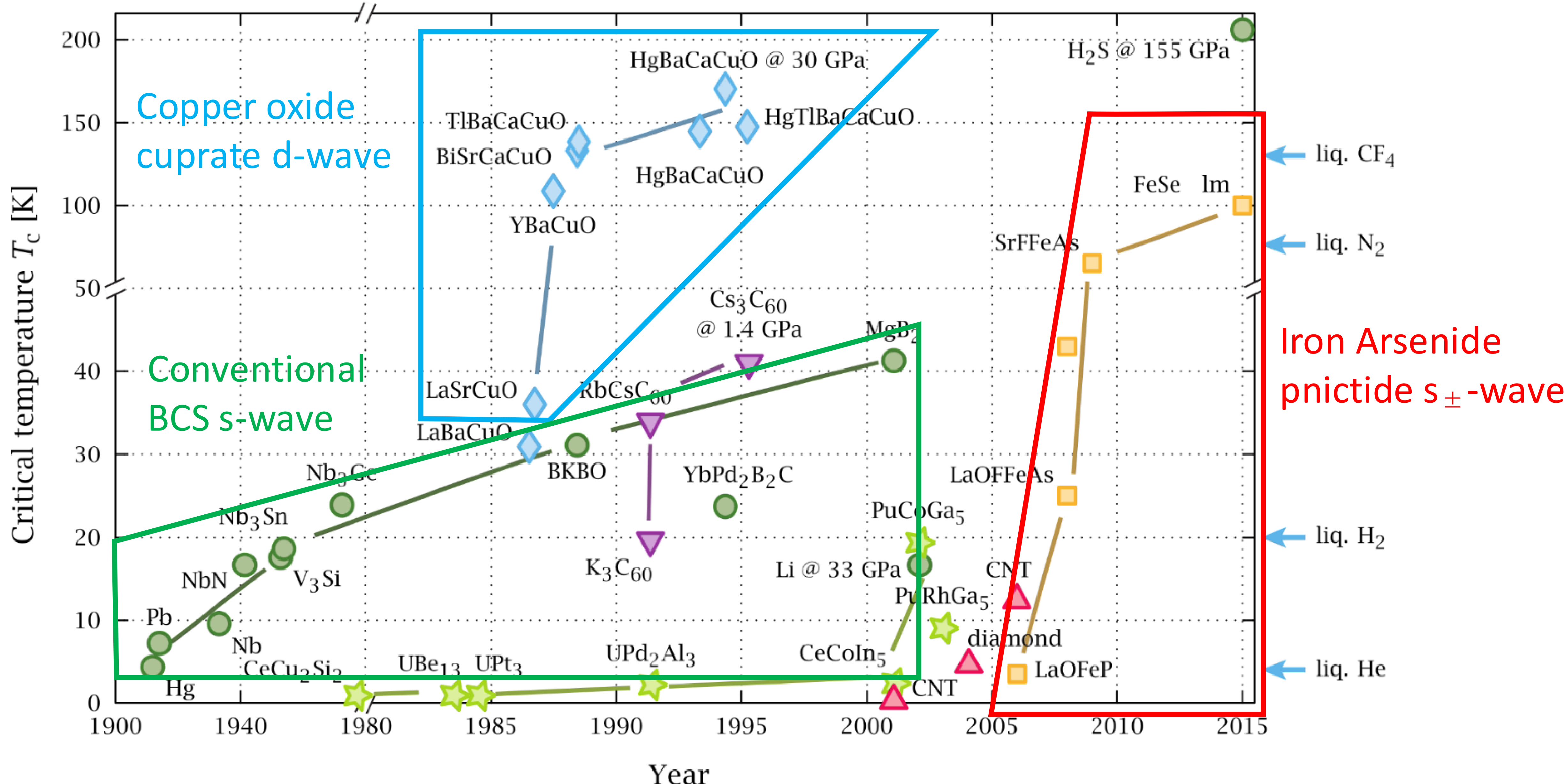


Material	$\lambda(T=0)$ [nm]	$\xi(T=0)$ [nm]	$\mu_0 H_{sh}$ [mT]	T_c [K]	$\Delta/k_B T_c$
Nb	50	22	219	9.2	1.8
Nb_3Sn	111	4.2	425	18	2.2
MgB_2	185	4.9	170	37	0.6-2.1
NbN	375	2.9	214	16	2.2

S. Posen PhD thesis

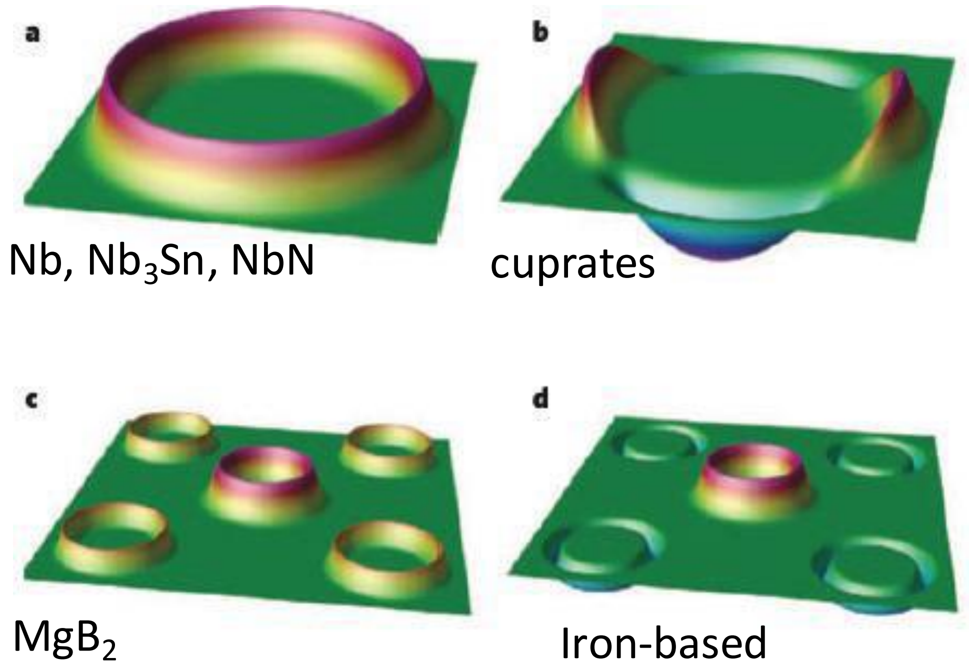


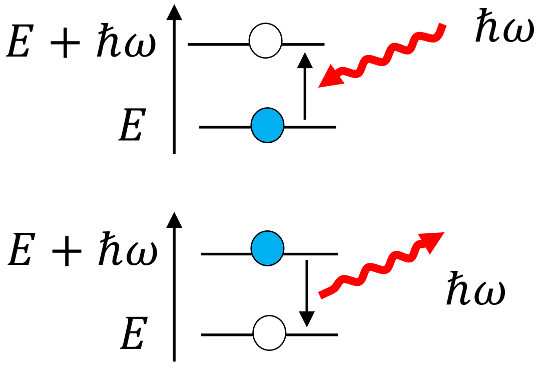
Three different families of superconductors

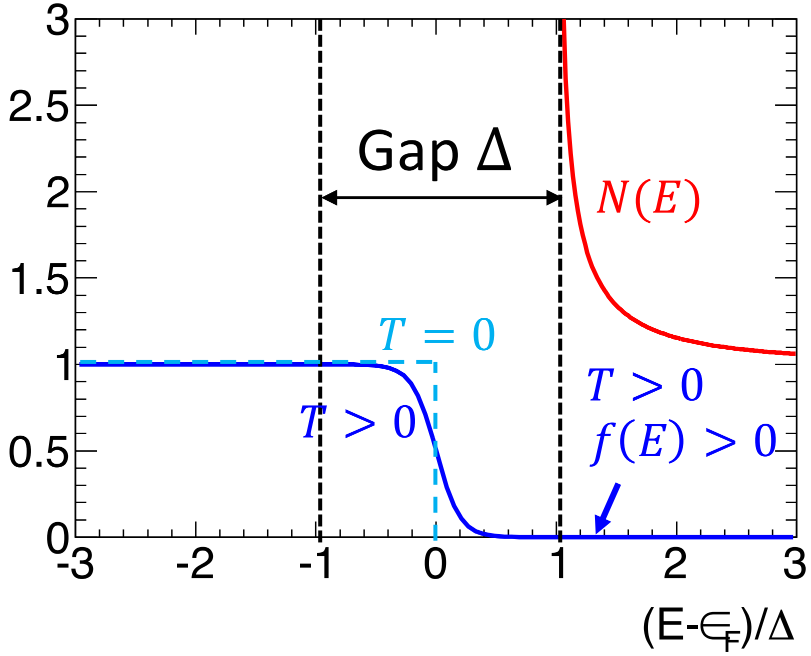


Surface resistance: naivest possible argument

$$R_s \propto \hbar\omega(n_+ - n_-) = \hbar\omega \int_{\Delta}^{\infty} dE\,[f(E) - f(E+\hbar\omega)] \times N(E)N(E+\hbar\omega)$$







- Gapless SC may have too much thermal excitation of quasiparticles → low R_s
  - Cuprate may not be OK but interesting as “slightly better Cu"
- Gap-full is the minimum requirement
  - Iron-based SC and MgB_2 would be OK as improvement from Nb

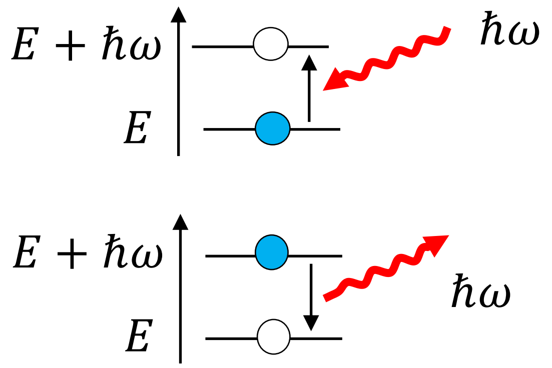
Optical conductivity in the Meissner state

$$\sigma_1 = \frac{2\sigma_n}{\hbar\omega}\int_0^{\infty}[f(\epsilon) - f(\epsilon+\hbar\omega)][\mathrm{Re}G^R(\epsilon)\mathrm{Re}G^R(\epsilon+\omega) + \mathrm{Re}F^R(\epsilon)\mathrm{Re}F^R(\epsilon+\omega)]d\epsilon$$

S. N. Nam, Phys Rev 156 470 (1967)

$$\sim \frac{2\sigma_n}{\hbar\omega}(1-e^{-\omega/T})\int_0^{\infty} e^{-\epsilon/kT}N(\epsilon)N(\epsilon+\hbar\omega)d\epsilon$$

J. Halbritter Z. Physik 266 p.209 (1974)



Quasi-classical Green functions

Conventional s-wave (Dynes)

$$\frac{N(\epsilon)}{N_0} = \mathrm{Re}\left(\frac{\epsilon+i\delta}{\sqrt{(\epsilon+i\delta)^2-\Delta_0^2}}\right)$$

$$\Delta_0(T) = \Delta_0[\cos(\pi T^2/2T_c^2)]^{1/2}$$

Cuprate d-wave

$$\frac{N(\epsilon)}{N_0} = \mathrm{Re}\left(\left\langle\frac{\epsilon+i\delta}{\sqrt{(\epsilon+i\delta)^2-\Delta^2(\theta)}}\right\rangle\right)$$

$$\Delta(\theta) = \Delta_0\cos 2\theta$$

P. Coleman "Introduction to Many-Body Physics"

Pnictide $s_{\pm}$-wave

$$\frac{N(\epsilon)}{N_0} = \mathrm{Re}\left(\left\langle\frac{\epsilon+i\delta}{\sqrt{(\epsilon+i\delta)^2-\Delta^2_{\alpha_{1,2},\beta_{1,2}}(\phi_{1,2})}}\right\rangle\right)$$

$$\Delta_{\alpha_{1,2},\beta_{1,2}}(\phi_{1,2}) = \Delta_0\Phi_{\alpha_{1,2},\beta_{1,2}}$$

$$\Phi_{\alpha_{1,2}} = -\Phi_a$$

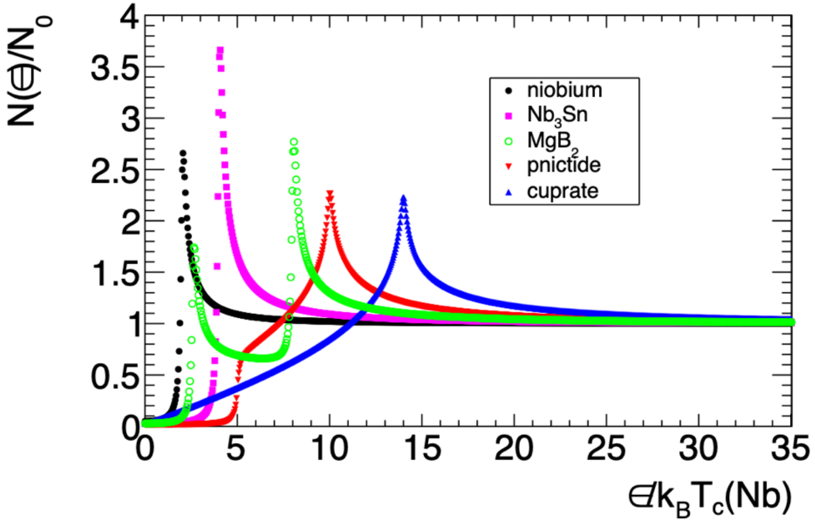
$$\Phi_{\beta_{1,2}} = \frac{1+\Phi_{\beta_{min}}}{2} \pm \frac{(1-\Phi_{\beta_{min}})}{2}\cos(2\phi_{1,2})$$

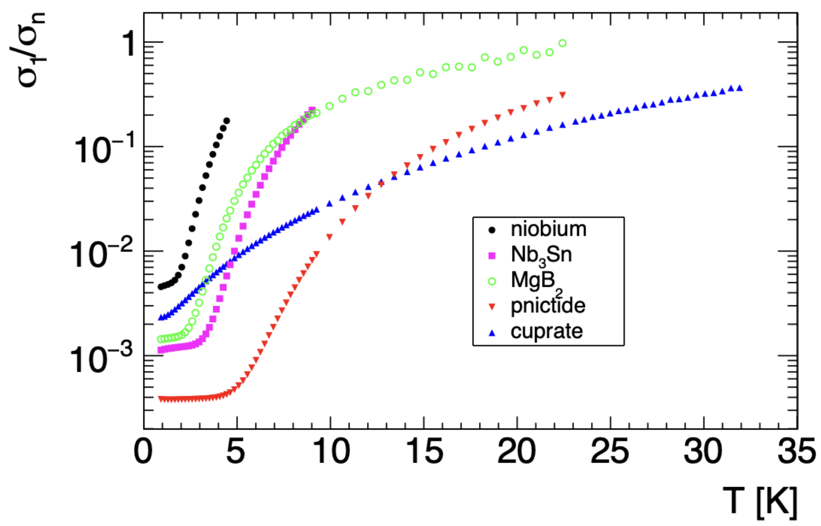
Y. Nagai et al New J. Phys. 10 103026 (2008)

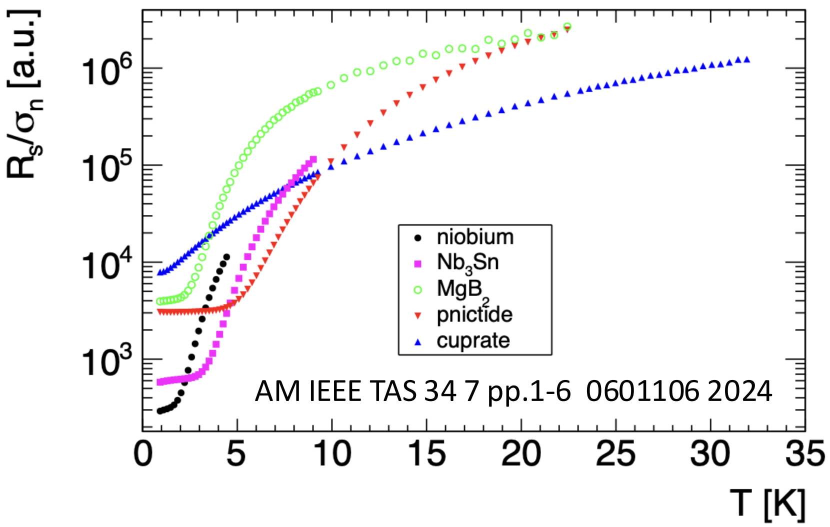
Assumption

- Meissner state = thermodynamical state
- Optical conductivity formulae for BCS SC may be still valid in 1st order approximation

Surface resistance of *bulk* non-conventional SCs







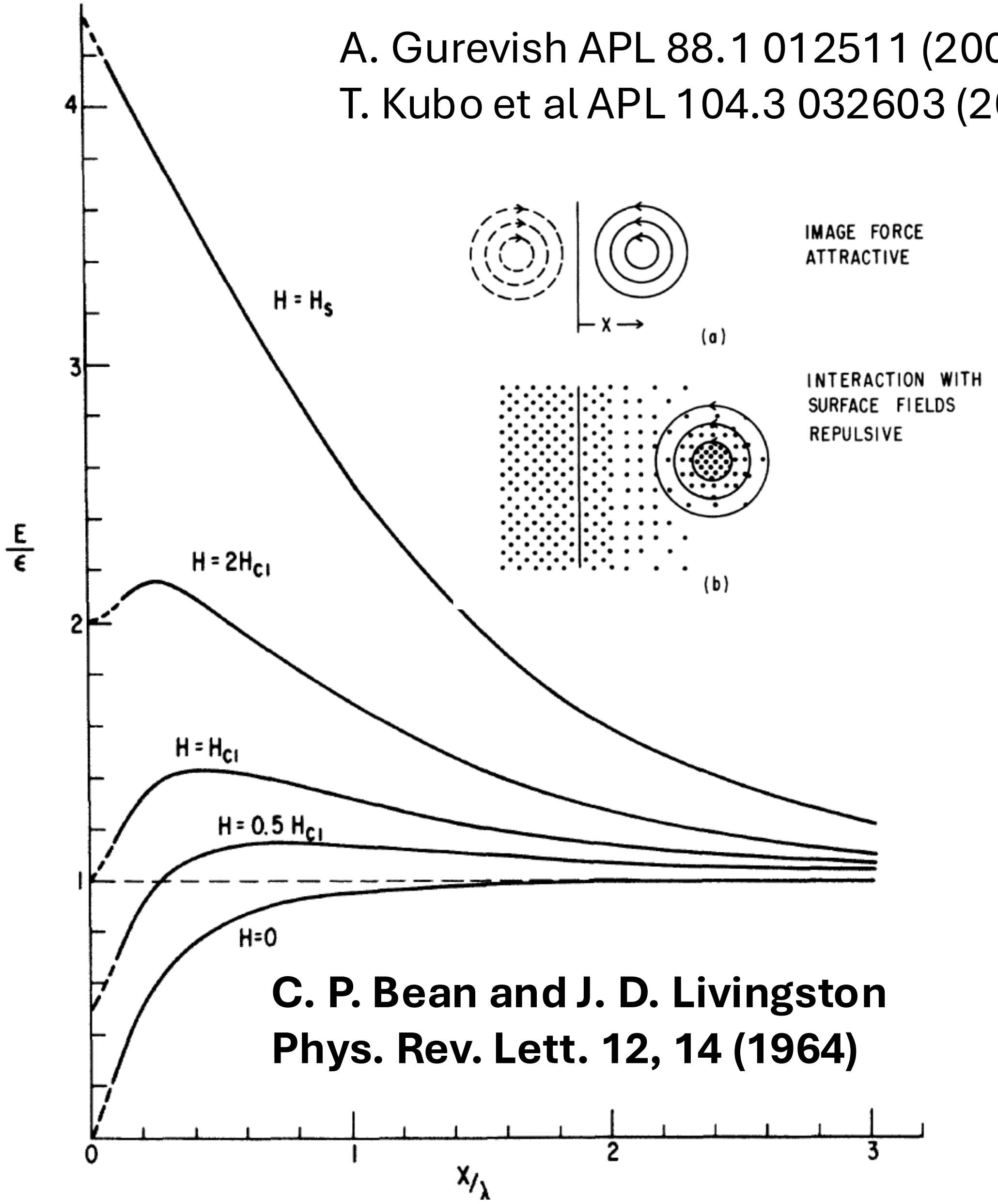
- This simplest (naive) approach gives is universal comparison of different SC materials
- The results reflect the symmetry of the gap as expected
  - d-wave: $\sigma_1(T) \propto T^a$
  - s-wave(s): $\sigma_1(T) \propto \exp(-\Delta/k_B T)$
- Non-conventional SC shows good $\sigma_1(T)$ but what does matter is surface resistance:

$$R_s \equiv \mathrm{Re}\left(\frac{E_x(z=0)}{\int_0^\infty J_x(z)dz}\right) = \sqrt{\frac{i\omega\mu_0}{\sigma_1 - i\sigma_2}} \xrightarrow{T \ll T_c,\, \sigma_1 \ll \sigma_2} \sqrt{\frac{\mu_0}{\omega\sigma_2^3}}\left(\frac{1}{2}\sigma_1 + i\sigma_2\right) \to R_s = \mathrm{Re}(Z_s) = \frac{\mu_0\omega^2\lambda^3}{2}\sigma_1(T)$$

→ Good SCs of long λ increase the amount of materials exposed to RF and cause joule heating ☹

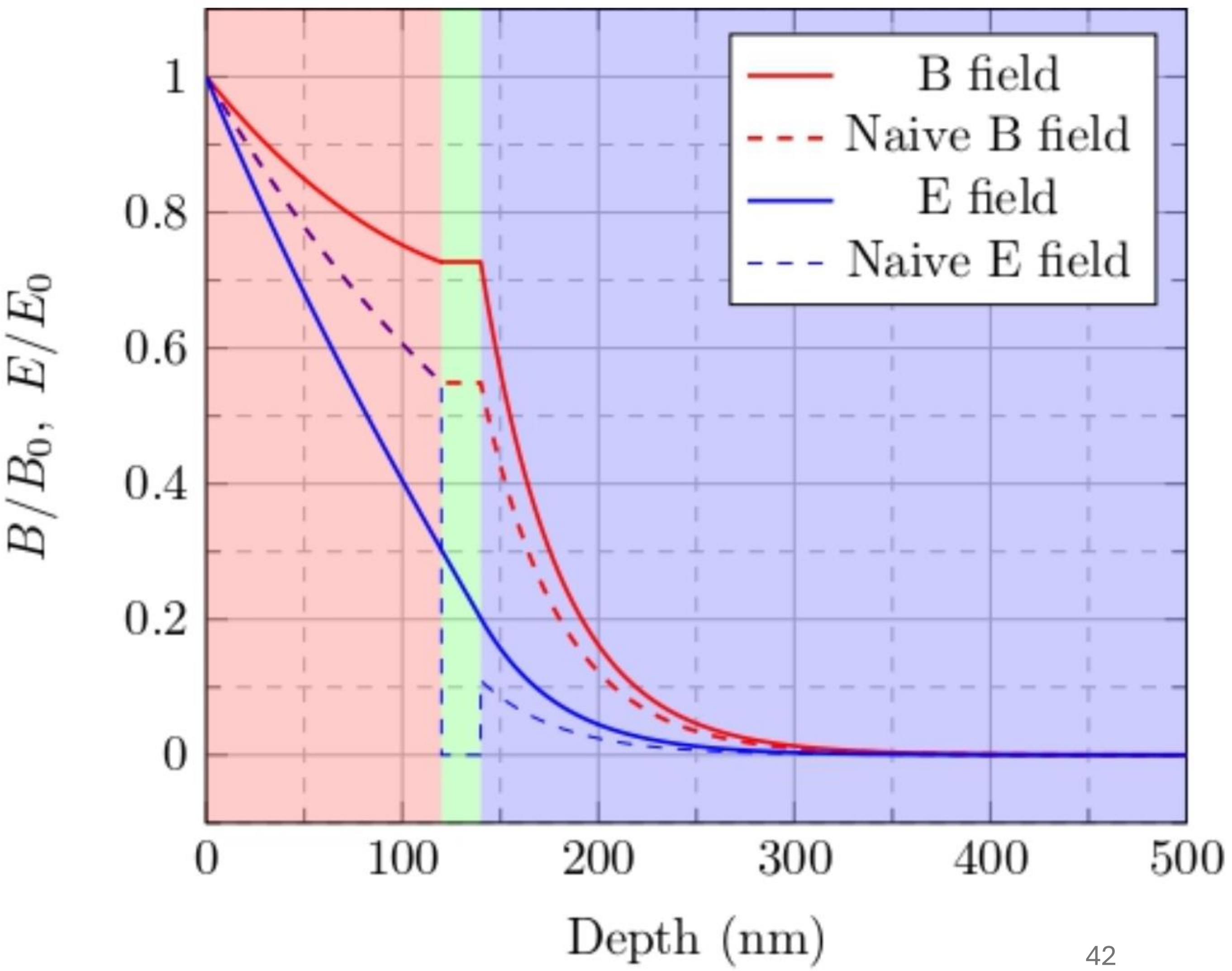
Multilayer: surface barrier from layer thick < λ

A. Gurevish APL 88.1 012511 (2006)
T. Kubo et al APL 104.3 032603 (2014)



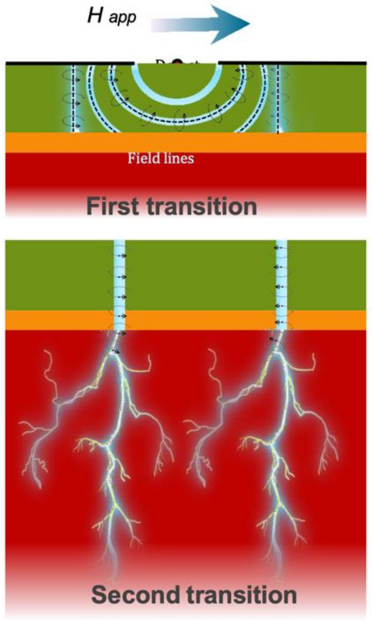
C. P. Bean and J. D. Livingston
Phys. Rev. Lett. 12, 14 (1964)

Carlos Redondo-Herrero, AM, arXiv:2604.03702



Multilayer of conventional SCs

A. M. Valente SRF2023 tutorial lecture

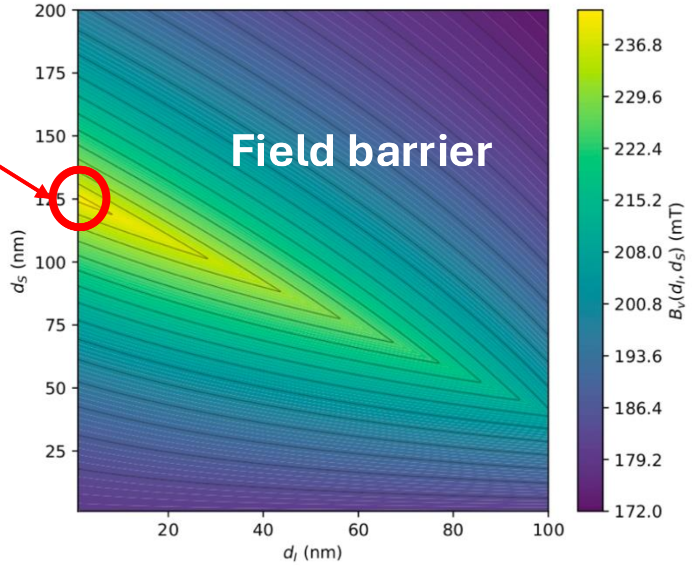


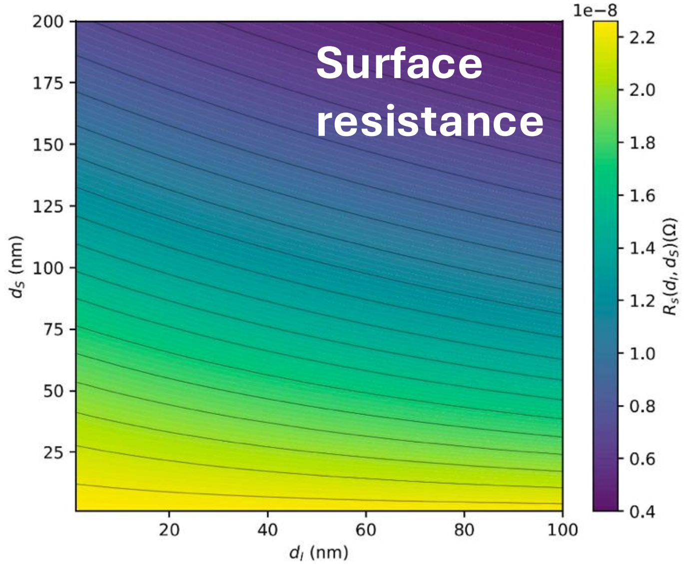
Why do we need insulator?

→ Protection of flux avalanche from surface defect

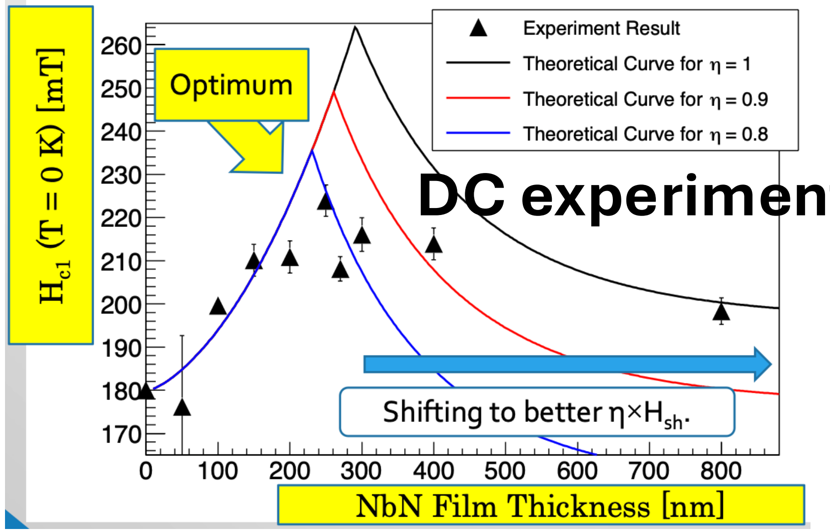
→ $d_I \geq \xi$ prepared to avoid Josephson vortex

NbN/I/Nb

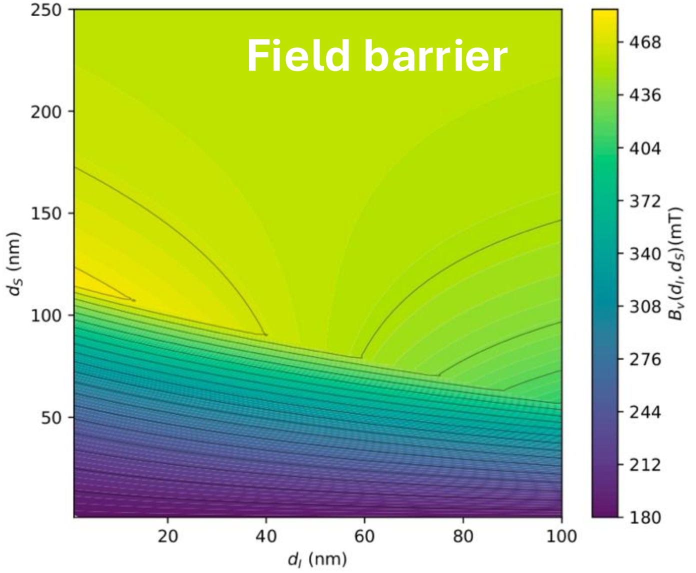


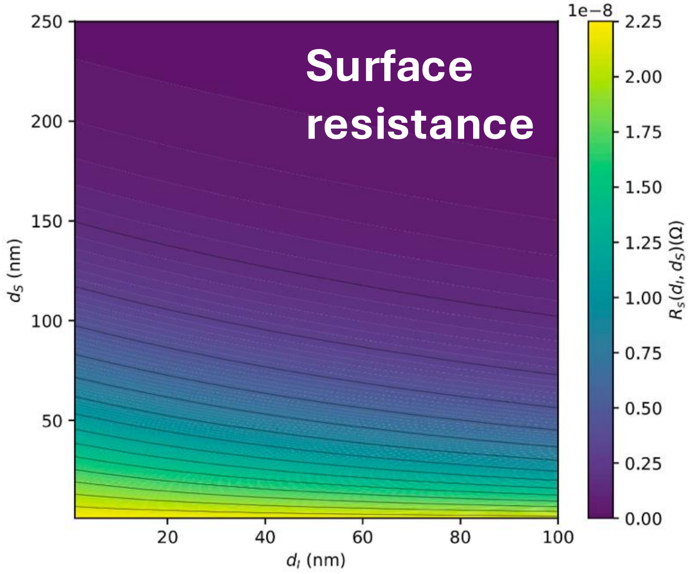


R. Katayama SRF2019



Nb3Sn/I/Nb

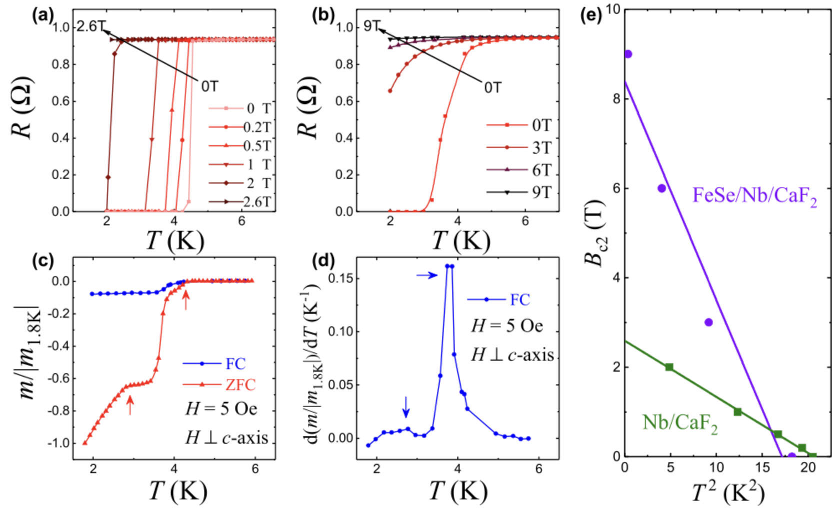


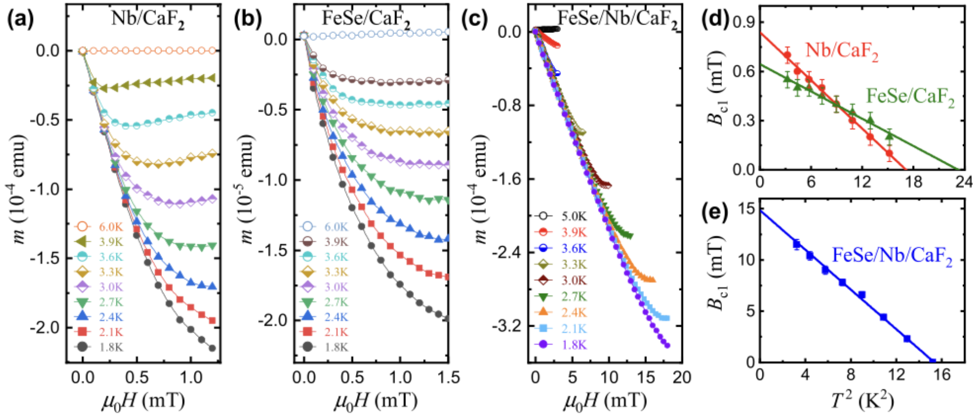


Multilayer of iron-based superconductors

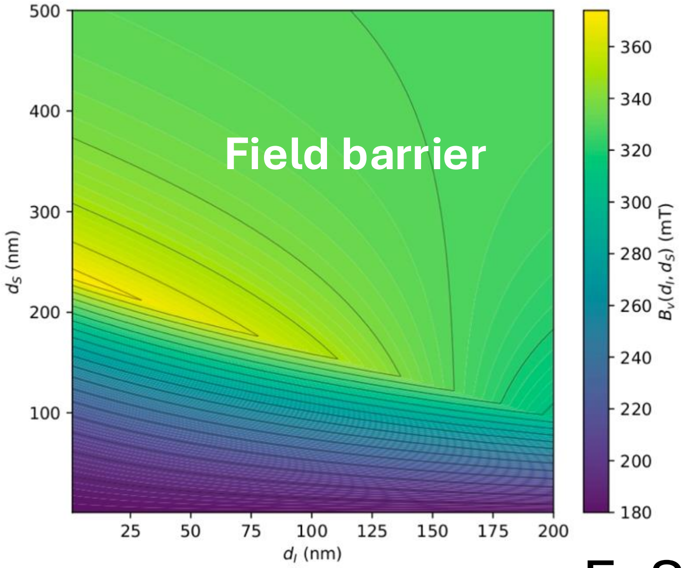
DC experiment

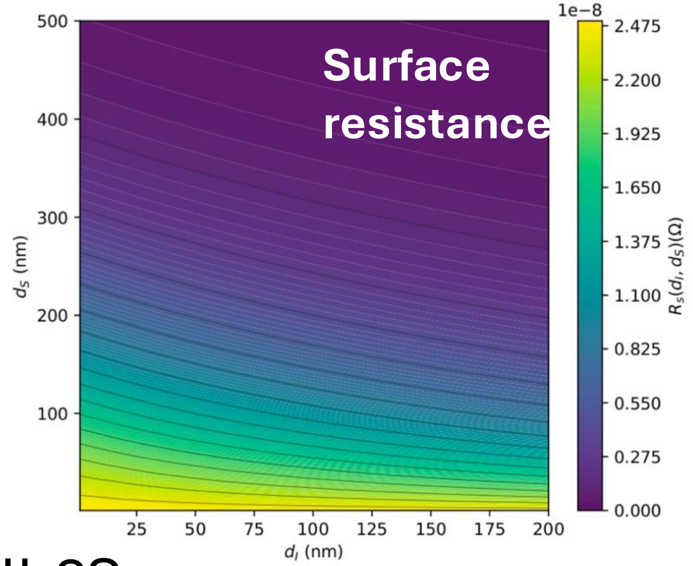
Z. Lin et al SUST 34 015001 (2021)



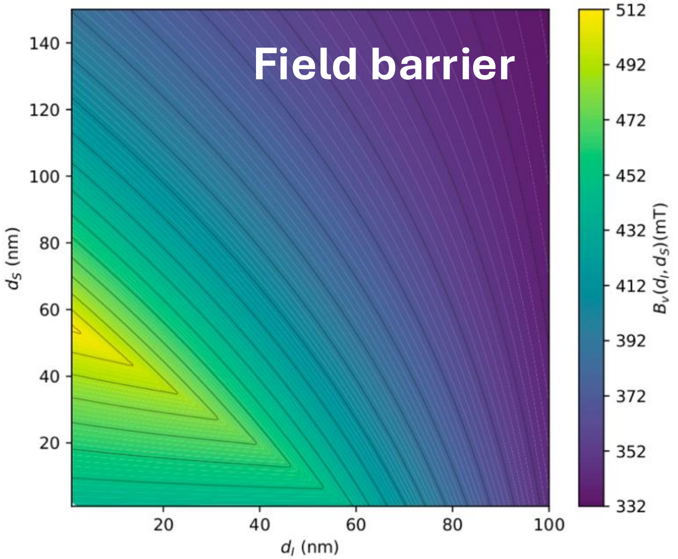


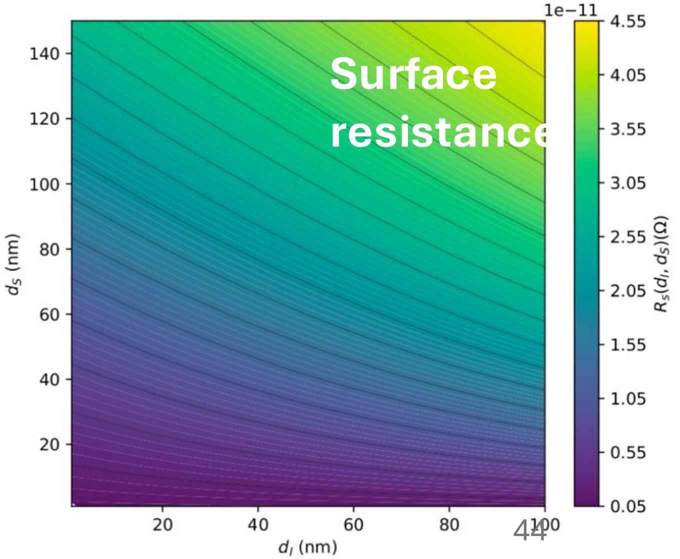
FeSe/I/Nb





FeSe/I/Nb3Sn



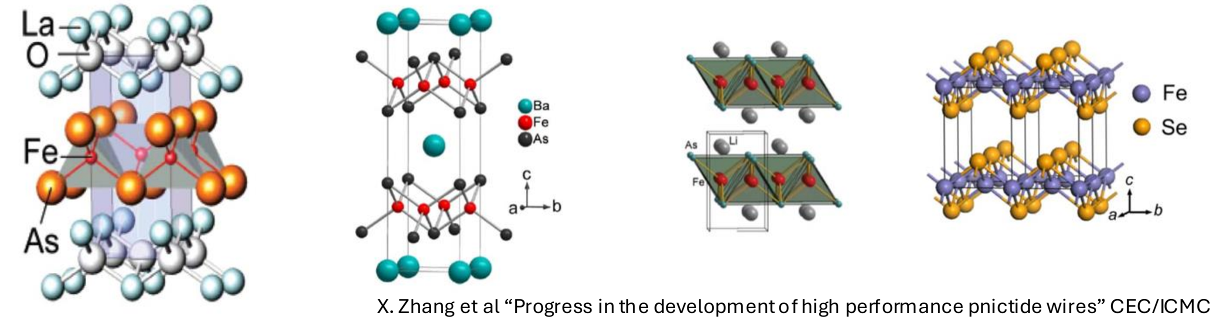


Discussion

	FeSe/I/Nb	NbN/I/Nb	Nb_3Sn/I/Nb	FeSe/I/Nb_3Sn	Nb
$B_v(\mathrm{mT})$	370	243.3	480.8	508.3	180
$R_s(n\Omega)$	5.354	12.40	3.094	$2.770 \cdot 10^{-3}$	22.26
$P(B_v)(\mathrm{Wm}^2)$	232.1	232.4	226.3	0.75	232.1
$d_S(\mathrm{nm})$	215.15	125	110	51	0
$d_I(\mathrm{nm})$	25	5	10	5	0

- Prediction of Nb3Sn/I/Sn is better than FeSe/I/Nb
  - But Nb3Sn is brittle
  - FeSe is elastic metal
- FeSe is selected because its flat sample for RF was already realized
  - Roma 3 / Genova for DM axions
  - Chinese Academy of Science
- FeSe is not the best IBS!
  - As 💀 is the issue for other candidates
- IBS wires for magnets are fabricated
  - IBS is a core not the surface
- GaAs facility for SRF application (?)

1111 Phase LnOFeAs | **122 phase AFe_2As_2 (A=Ba, Sr, Ca)** | **111 phase LiFeAs** | **11 phase FeSe**



X. Zhang et al "Progress in the development of high performance pnictide wires" CEC/ICMC 2017

T_c ~55 K | T_c ~38 K | T_c ~18 K | T_c ~8 K

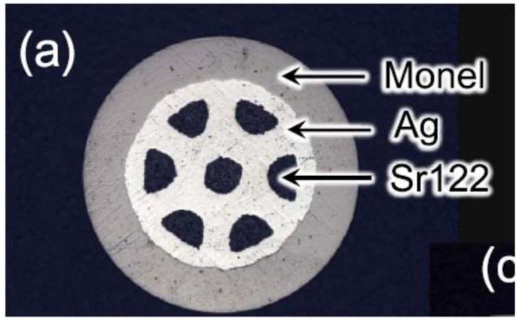
Z. A. Ren et al., *Chin. Phys. Lett.* 25, 2215 (2008)

M. Rotter, et al., *Phys. Rev. Lett.* **101**, 107006 (2008)

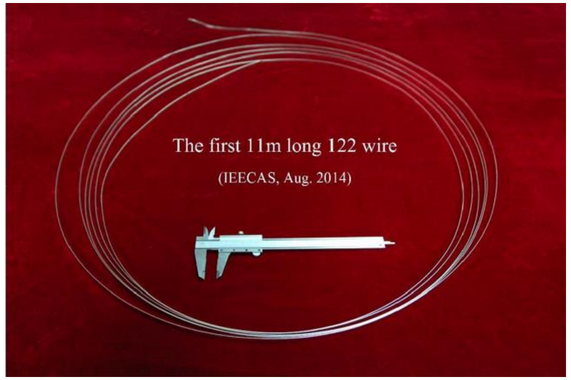
X. C. Wang, et al., *Solid State Commun.* **148**, 538 (2008).

F. C. Hsu, et al., *Proc. Natl. Acad. Sci. U.S.A.* **105**, 14262 (2008).

Y. Ma "Recent progress in Fe-based superconducting wires and tapes"



Monel/Ag, IEECAS



Outline

- Accelerators offer a playground for nonequilibrium superconductivity
  - Complementary to quantum sensing / qubit applications
- Fundamental of BCS and Mattis Bardeen theories
  - Linear response theory
- Nonequilibrium physics and surface treatment
- Thin-film: alternative approach
- Conclusion

Conclusion

- Accelerators offer a playground for nonequilibrium superconductivity
  - Extremely low surface resistance near the quench field
  - Complementary to quantum sensing / qubit applications
- Fundamental of BCS and Mattis Bardeen theories
  - Linear response theory
  - Density of states vs distribution function
- Nonequilibrium physics and surface treatment: different hypotheses
  - high Q: 300C baking → anti-Q-slope → DoS vs f(E)
- Thin-film: alternative approach
  - Multilayer & IBS may open a new research opportunity!

backup

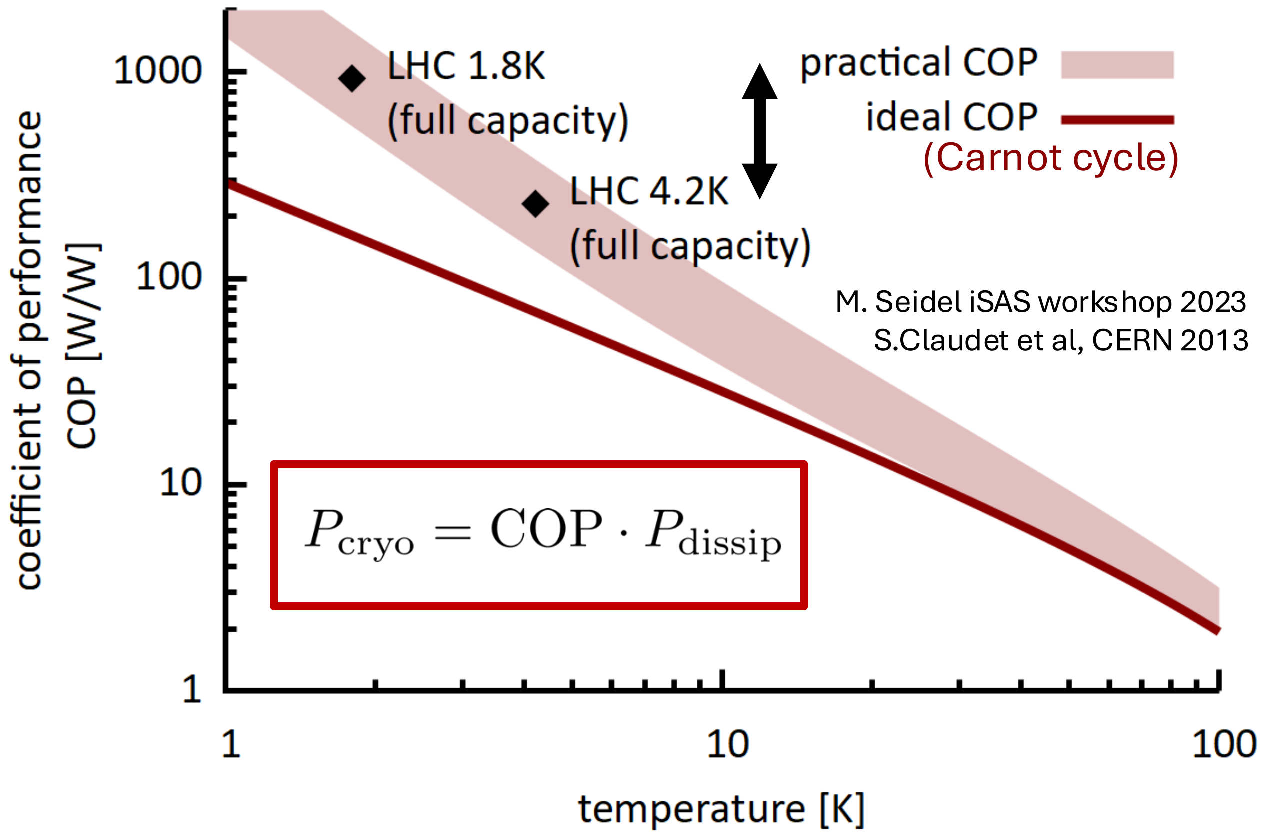
R&D toward high-Q/high-G…in the modern context

Example

~~Higher-Q & higher-G are always better~~

→ Higher T has safety margin (x3-5)
→ Simpler cryogenics is a big plus

Good-Q at higher T can be a competitive option



Depending on the purpose

1. Identify the bottleneck (cavity itself? beam dynamics? cryogenics? amplifier? coupler?, X-ray?…)
2. Find the best compromise → global optimum? Beware of ideology!

However, fundamental R&D of high-Q/high-G is of great **academic interest**

→ We focus on this point in this seminar

Where do we need high-Q/high-G SRF cavities?

Energy frontier storage ring (CW, high current)
→ Higgs boson, new physics search

Electron linac (long pulse / CW, high current)
→ FEL

Proton linac (long pulse / CW, high current)
→ Neutron source, ADS/transmutation, neutrino source, MuCol injector

Heavy ion linac (CW, low-high current)
→ Nuclear physics, material science, medical application, etc

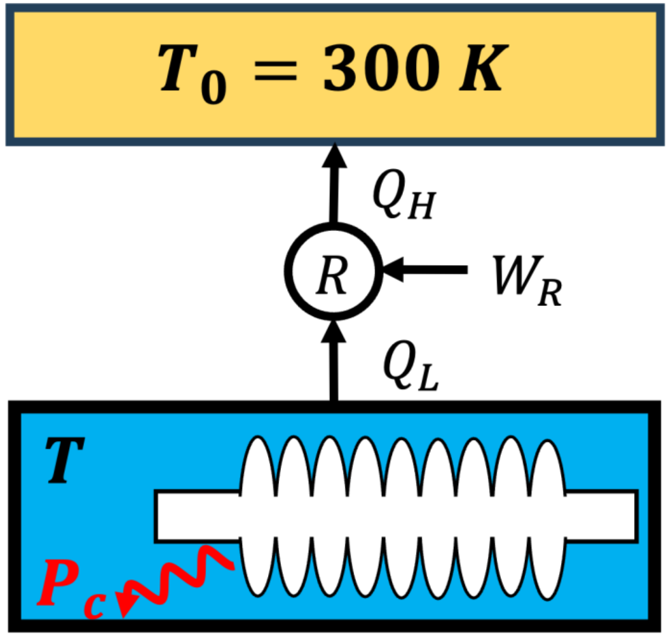
- High-Q SRF cavities may be the only solution
- Gradient may also depend on other aspects

Energy frontier e+/e– linear collider (long pulse / short pulse)

→ High-G is absolute must → CLIC/C^3 is better in this particular aspect

Decision making depends on various things: aperture, cryogenic infrastructure, high-power RF, overall technical readiness, ...

Cooling power 1 W depends on temperature



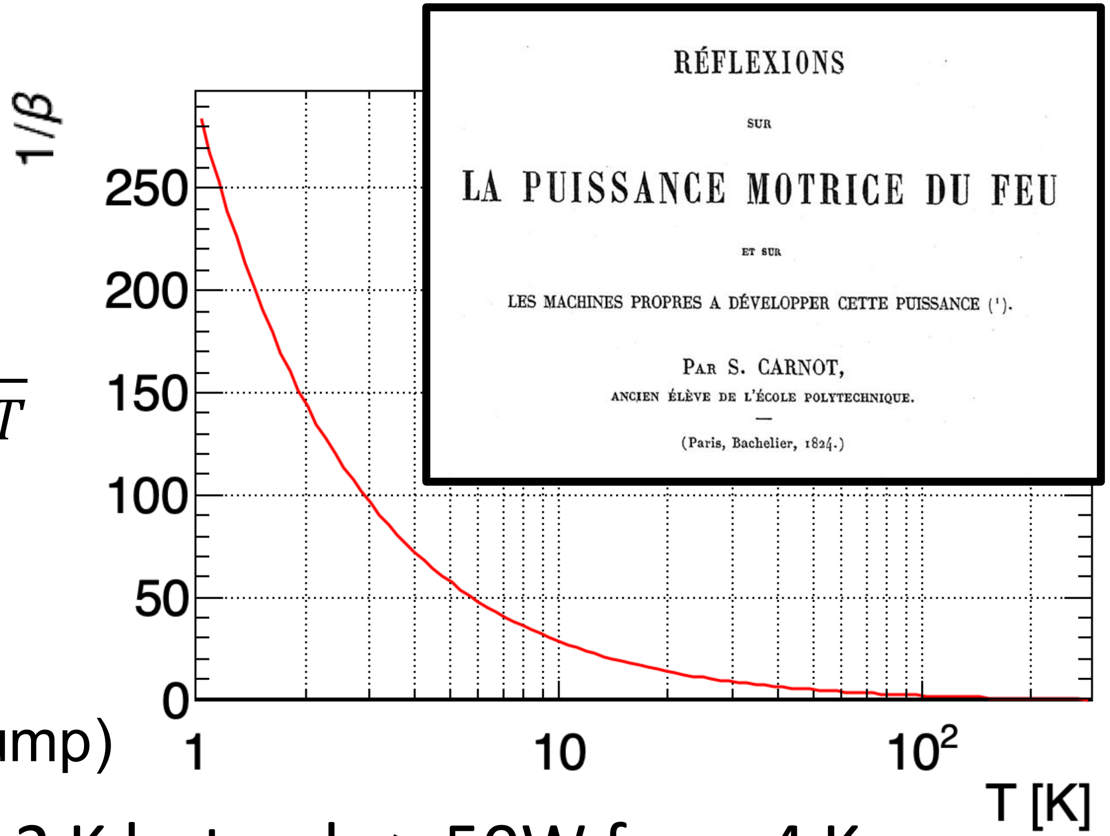
Carnot's theorem

$$\beta = \frac{Q_L}{W_R} = \frac{Q_L}{Q_H - Q_L} = \frac{T}{T_0 - T}$$

Required power

$$P_{cryo} > \frac{P_c}{\beta}$$

(be careful about logical jump)



We may need >150 W to evacuate 1 W from 2 K but only > 50W from 4 K

→ Higher T has safety margin (x3) + simpler cryogenics (no superfluid)

→ Good-Q at higher T can be a competitive option (research trend)

Remark: Carnot cycle is unrealistic at all

Thermodynamics does not include characteristic time constant

→ Carnot cycle gives maximum efficiency in quasi-static process ($\Delta t \to \infty$)

→ Power (work per time) is in trade off with the efficiency

PRL117 190601 (2016)

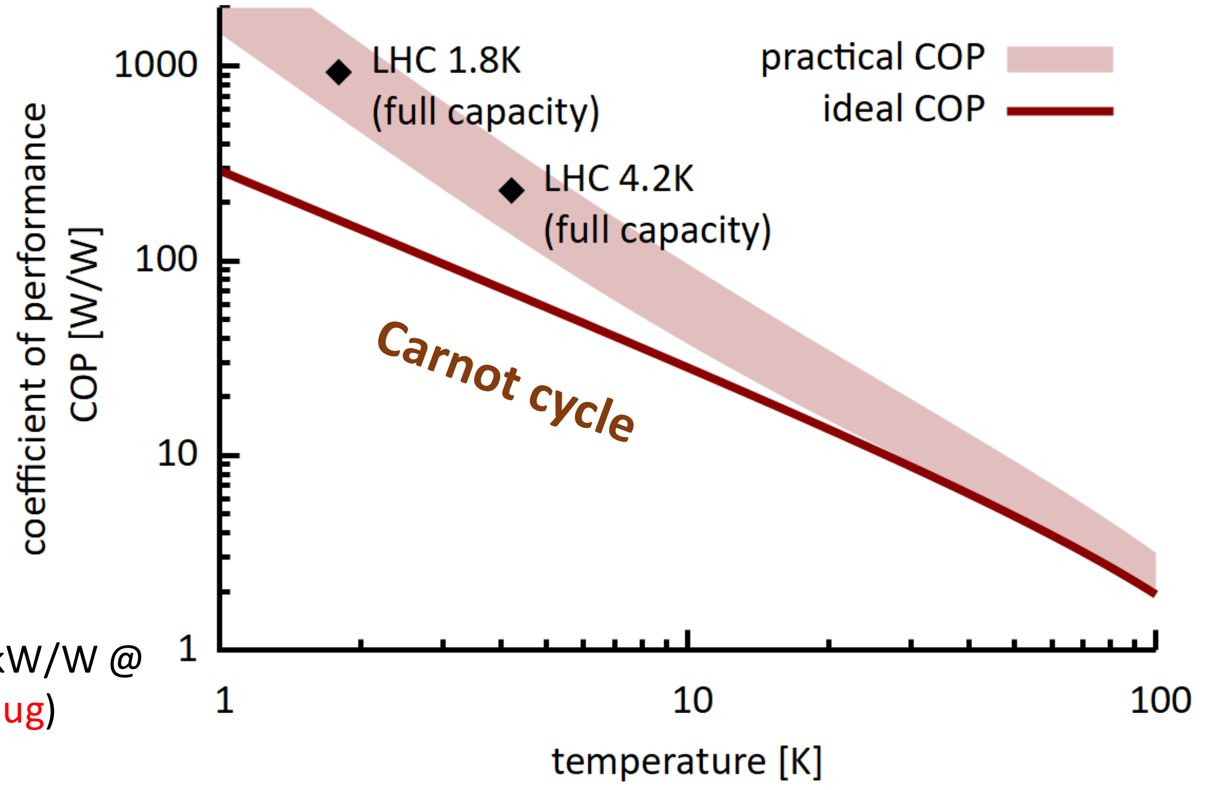
$$P_c \leq \bar{\Theta}\beta_L \eta(\eta_C - \eta)$$

We lose useful power if efficiency η is too good approaching to Carnot η_C

In addition, more practical limitations further degrade the efficiency

→ Around 1 kW is necessary to evacuate 1 W from 2 K

(typically 5 kW/W @ 2 K for AC plug)

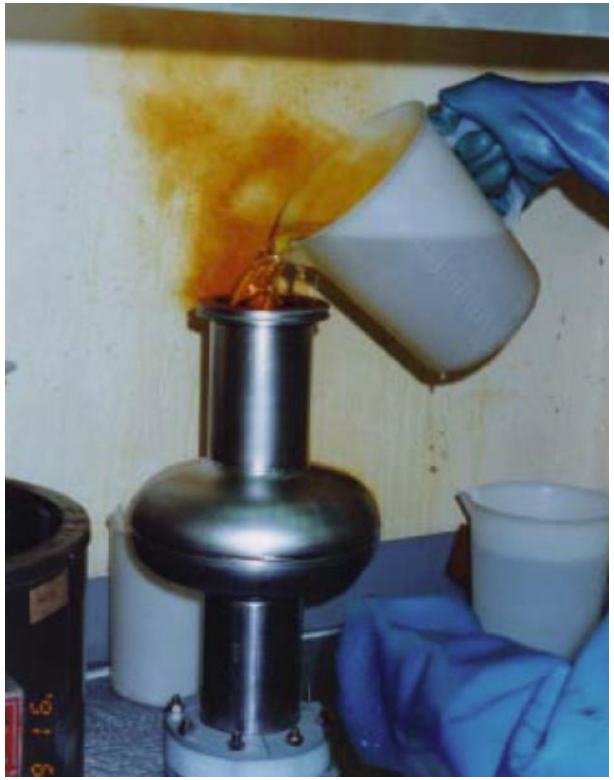


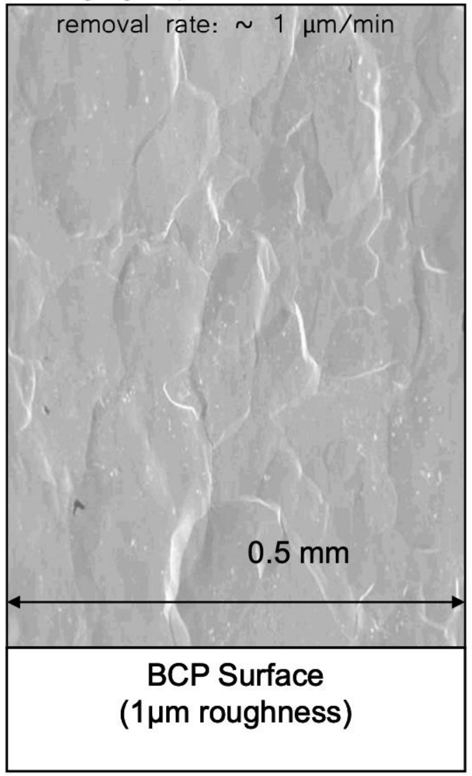
The practical advantage of 4 K over 2 K is even more pronounced!

Two methods of surface etching

Buffered Chemical Polishing (BCP)

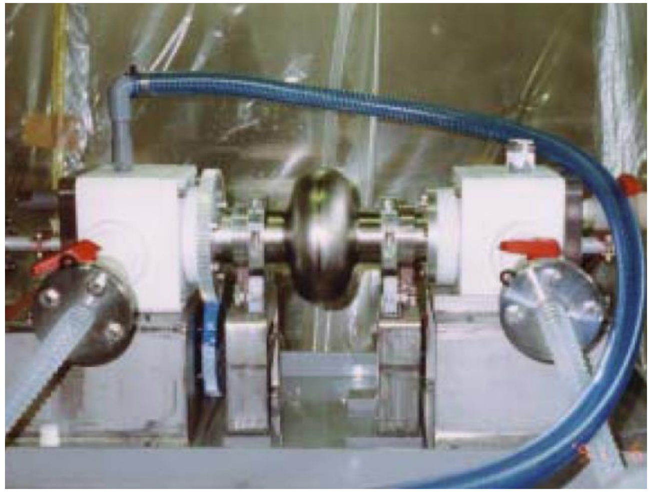
$HF + HNO_3 + H_3PO_4$

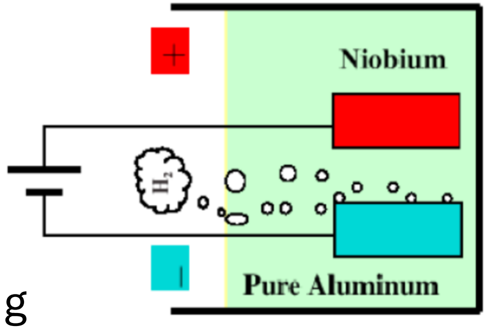


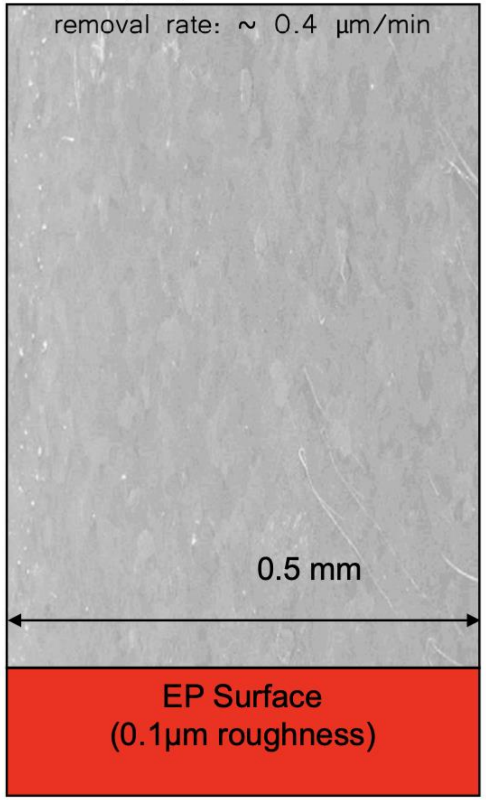


Electropolishing (EP)

$HF + H_2SO_4$ + voltage







Courtesy: Rong-Li Geng

EP is better than BCP for high-Q/high-G (but more complex and expensive…again compromise!)

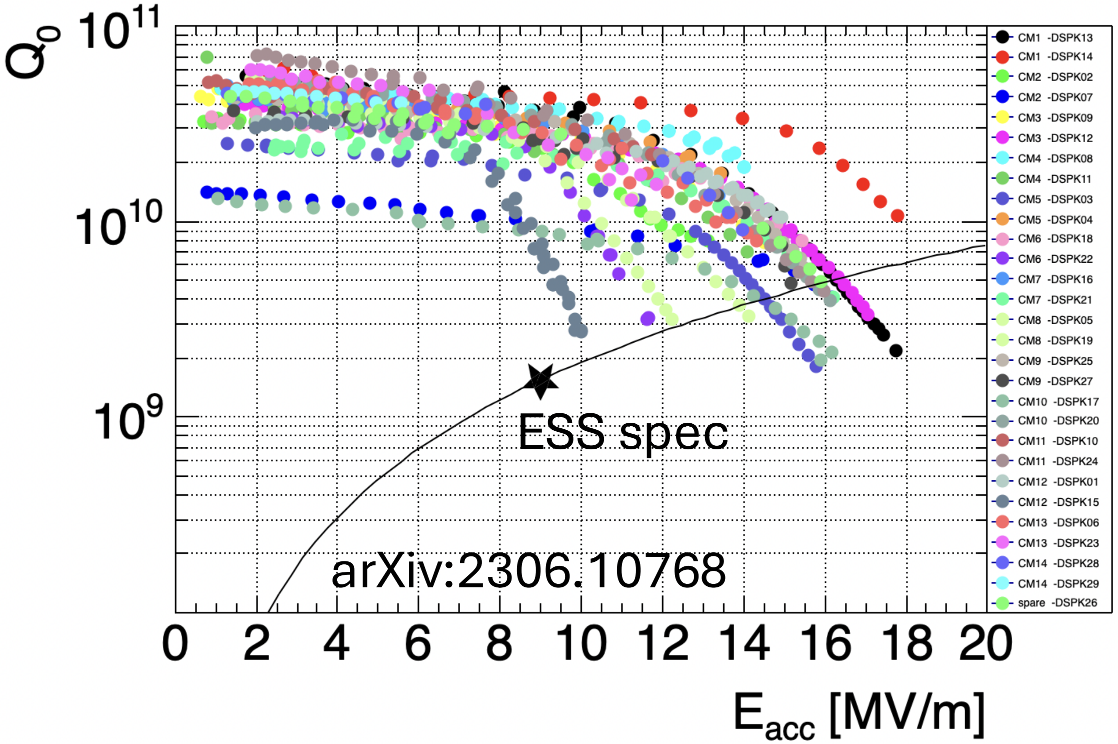
→ Vertical EP (simpler than horizontal bench but challenging in asymmetry removal and H_2 gas traces)

→ Cold EP for even better surface roughness (<0.1 µm)

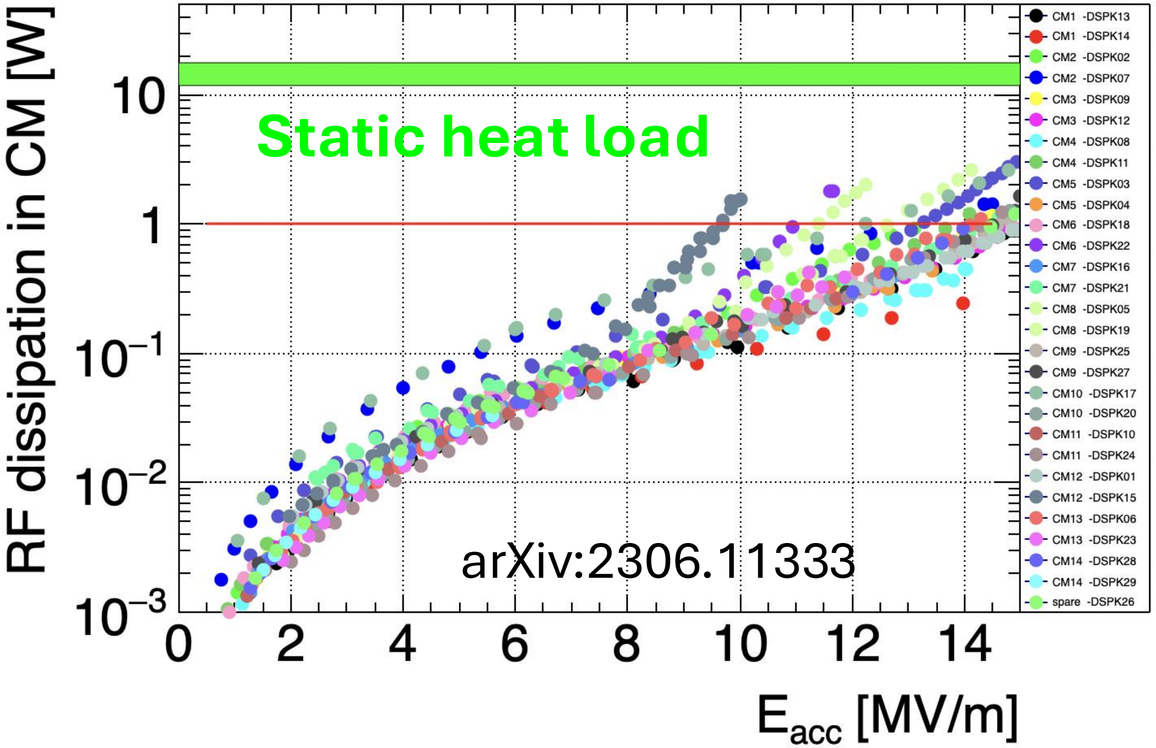
Low-β cavities: high-G / high-Q?

ESS double spoke as an example (352 MHz, $\beta_{\mathrm{opt}} = \mathbf{0.5}$, **pulse**, 2 K)

high-G?



high-Q?

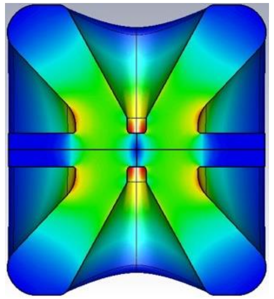


- Most of the cavities reached > 12 MV/m >> 9 MV/m
- $\beta = 0.5$ → shorter linac & less cavities are impossible
- **beam dynamics is the bottle-neck**

- Most of the cavities reached < 5 nΩ @ 9 MV/m
- Duty cycle 4.5 % = 3.2 ms x 14 Hz
- **Static heat load is the bottle-neck**

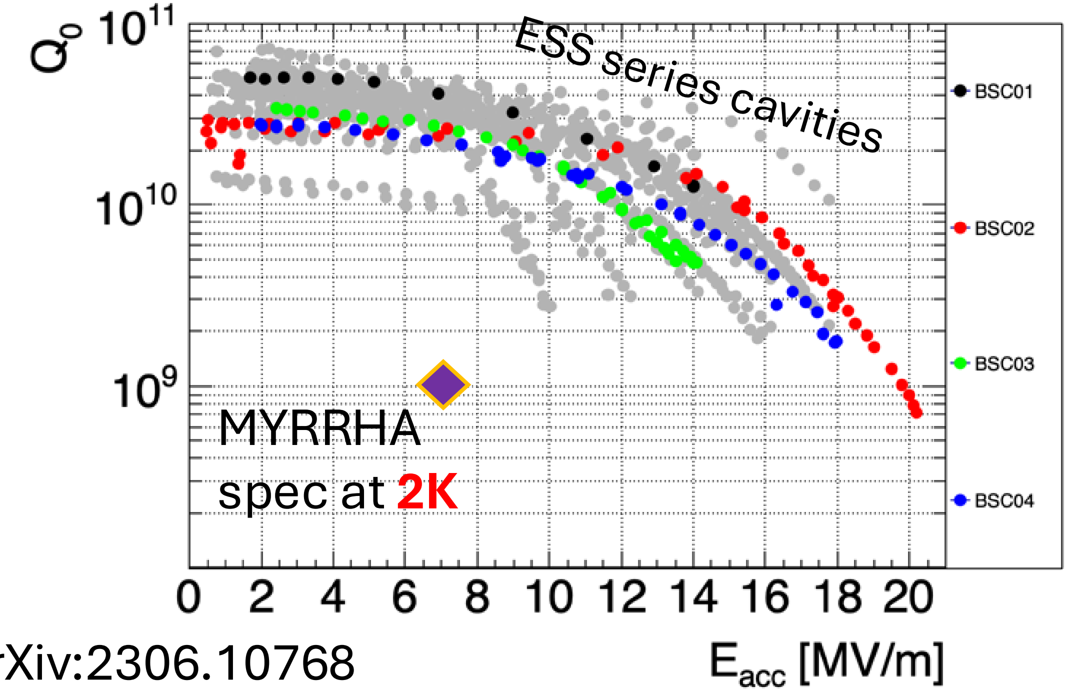
→ For ESS double spoke cavities, R&D for higher-G/higher-Q makes little sense

Low-β cavities: high-Q & 4 K



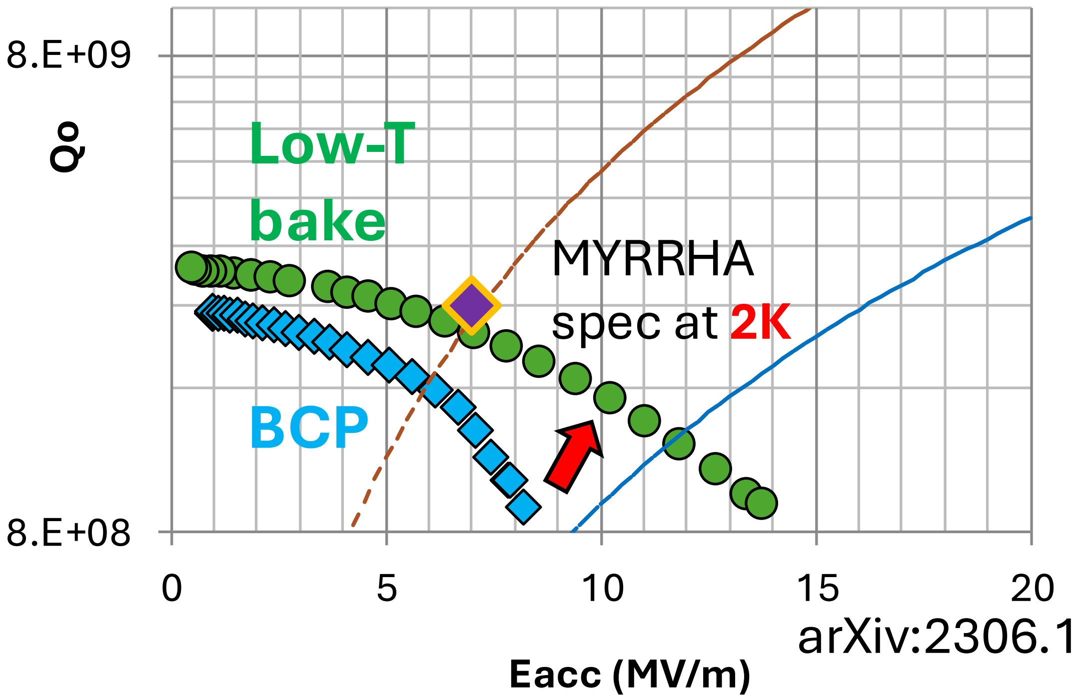
MYRRHA single spoke as an example (352 MHz, $\beta_{opt} = 0.37$, **CW**, **2 K**)

<u>High-G?</u>



arXiv:2306.10768

<u>high-Q at **4K**</u>



arXiv:2306.10768

- All prototype cavities reached > 12 MV/m
- Lower gradient for redundancy in ADS operation
- **beam dynamics & operation are the bottle-neck**

- **Dynamic RF heat load may dominate in CW**
- Appropriate surface treatment → high-Q in 4 K
- Cryogenics COP 2K→4 K: factor 5

→ For MYRRHA single spoke cavities, R&D for higher-Q/higher-T **does** make sense

Good performance → geometry and material

Figures of merit can be decomposed into two factors

Unloaded quality factor

$$Q_0 = \frac{G}{R_s}$$

$R_s = R_s(T;\omega;E_{acc};\text{matetial parameters})$: surface resistance of SC material

(Fundamental interest)

Acceleration efficiency

$$\frac{V_{acc}^2}{P_c} = R_{sh} = \left(\frac{R_{sh}}{Q_0}\right) Q_0 = \left(\frac{R_{sh}}{Q_0}\right) G \left(\frac{1}{R_s}\right)$$

Quench limit in gradient

$$E_{acc} = \left(\frac{B_{pk}}{E_{acc}}\right)^{-1} B_{pk}$$

$B_{pk} < B_{sh}(T;\omega;;\text{matetial parameters})$: quench field of SC material

(Fundamental interest)

Field emission limit in gradient

$$E_{acc} = \left(\frac{E_{pk}}{E_{acc}}\right)^{-1} E_{pk}$$

E_{pk} <field emission onset

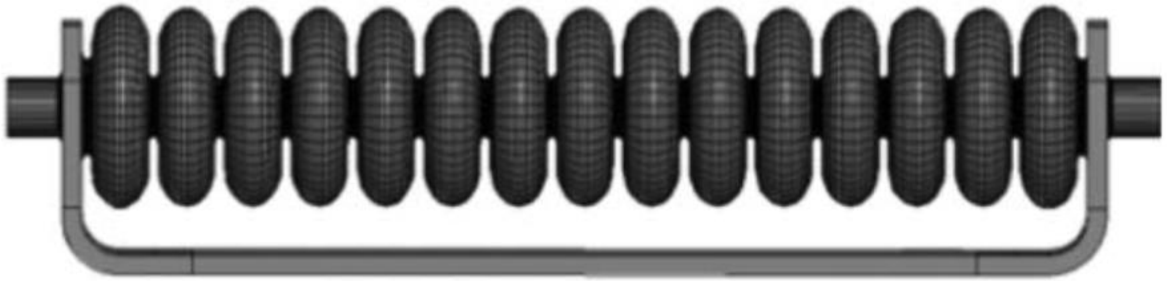
(Practical challenges)

Geometrical solution: Ex) Traveling Wave

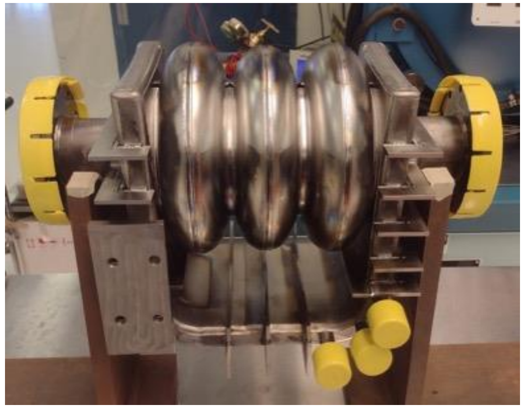
R. Kostin LCWS2024

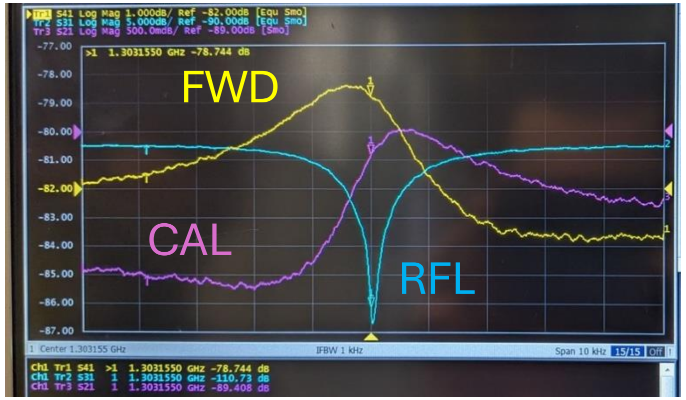
ILC cavity

HELEN cavity



3-cell prototype demonstrated in 2023





TW originally proposed in 1968

FoM	ILC	TW 1 m
E_{pk}/E_{acc}	2.0	1.73
B_{pk}/E_{acc} [mT(MV)$^{-1}$m]	4.26	2.88 ☺
R_{sh}/Q [Ω]	518	2127 ☺
G [Ω]	270	186 ☹
E^*_{acc} [MV/m]	47	69 😎

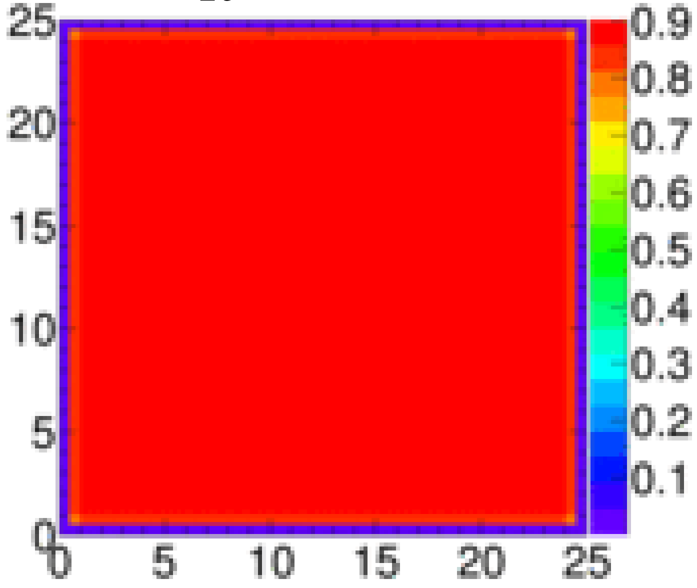
(* Assuming $B_{pk} = B_{sh} = 200$ mT)

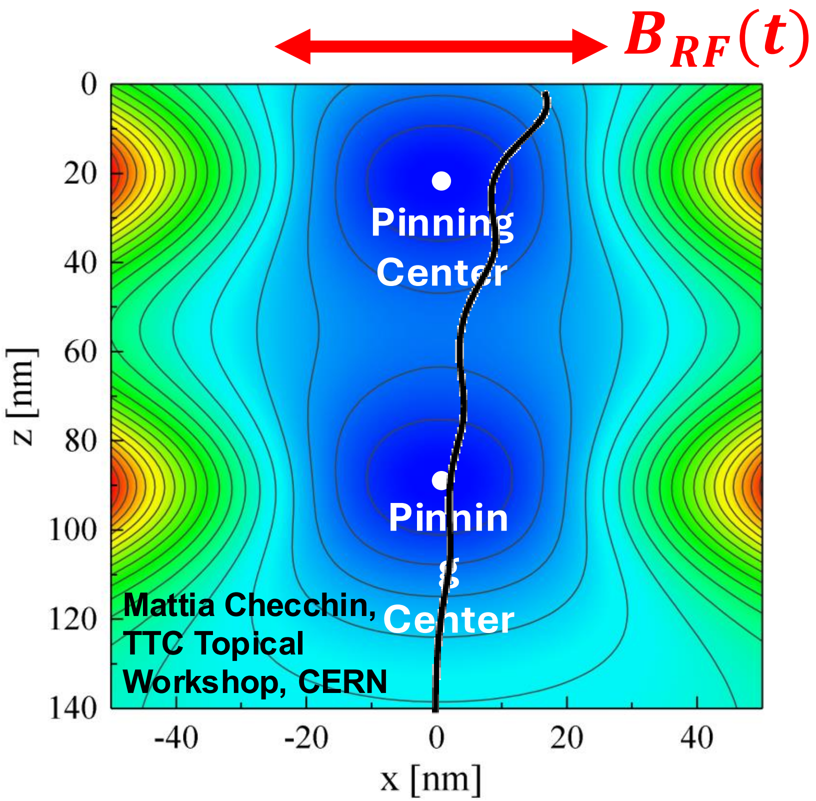
- Still before proof-of-concept of high-G operation of the TW mode
- Possible update of ILC without extending the tunnel
- Practical concerns from SRF experts
  - Phase(?) Series production (?)

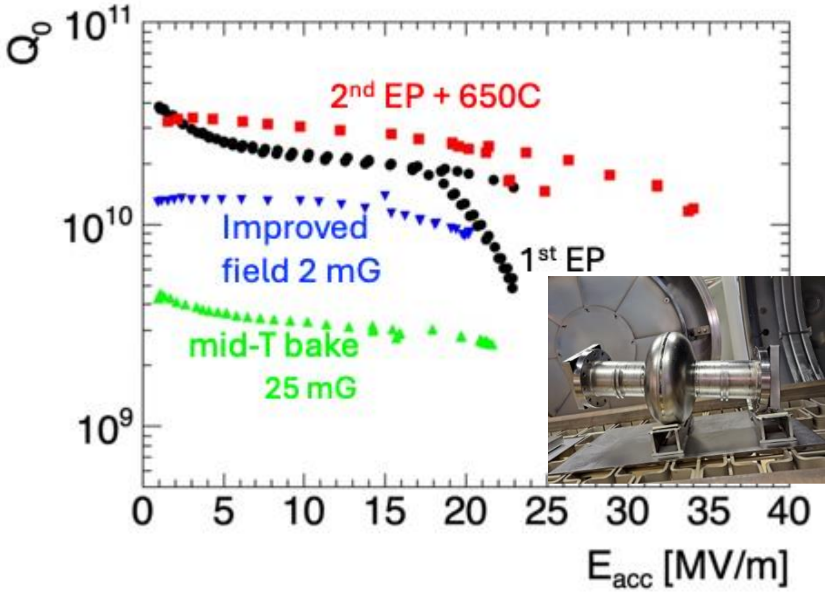
Trapped flux → another key for both bulk and film

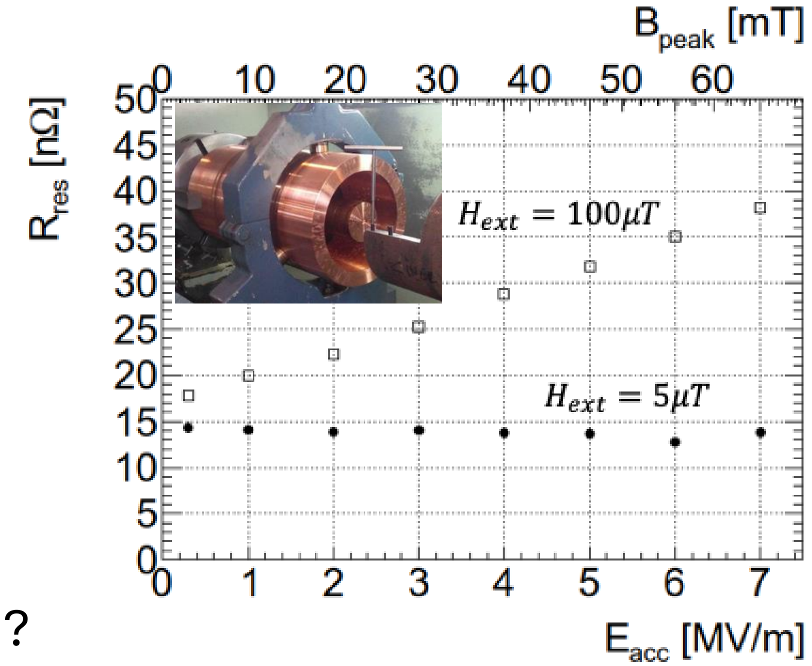
Quantized flux

$$\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}\ \text{Wb}$$









$$R_s = R'_{BCS} + R_{fl} \equiv R'_{BCS} + S \times H_{trap}$$

- The magnetic field is **quantized** inside superconductors and trapped by pinning centres (defects) → additional loss under RF
- bulk niobium: ambient magnetic field reduction is crucial
- Thin film: ambient magnetic field is not important (textbook) but…?