Few-Cycle GW X-ray Pulses with Mode-Locked Amplifier FELs

Neil Thompson[1]

David Dunning[1], Brian McNeil[1]

[1]ASTeC, STFC Daresbury Laboratory and Cockcroft Institute, UK

[2]Department of Physics, SUPA, University of Strathclyde, UK

Workshop on Generation of Single-Cycle pulses with FELs, 16-17 May 2016, Minsk

Outline

- Our motivation has been to work out how to produce the shortest possible pulse durations from FELs. This means we need the ***fewest number of cycles*** at the ***shortest wavelengths***

- We hope to circumvent some of the effects that would otherwise place a lower bound on pulse duration - normally in a SASE FEL ***the slippage determines the temporal profile*** of the output pulse through the cooperation length l_c – this controls the length of each SASE spike and the minimum duration of an isolated pulse that can be amplified

- Our work involves the ***artifical manipulation of the slippage*** which leads to the synthesis of axial optical modes which we then lock together to produce pulse durations $<< l_c$

- Using this technique, in a 'standard' FEL lattice pulse durations of a ***few tens of cycles*** are possible in simulation

- To push further, in a more practical implementation, a special afterburner undulator can be added to a normal FEL to produce few cycle pulses, with predicted durations into the ***zeptosecond regime in the hard X-ray***

Pulse Durations *vs* Year

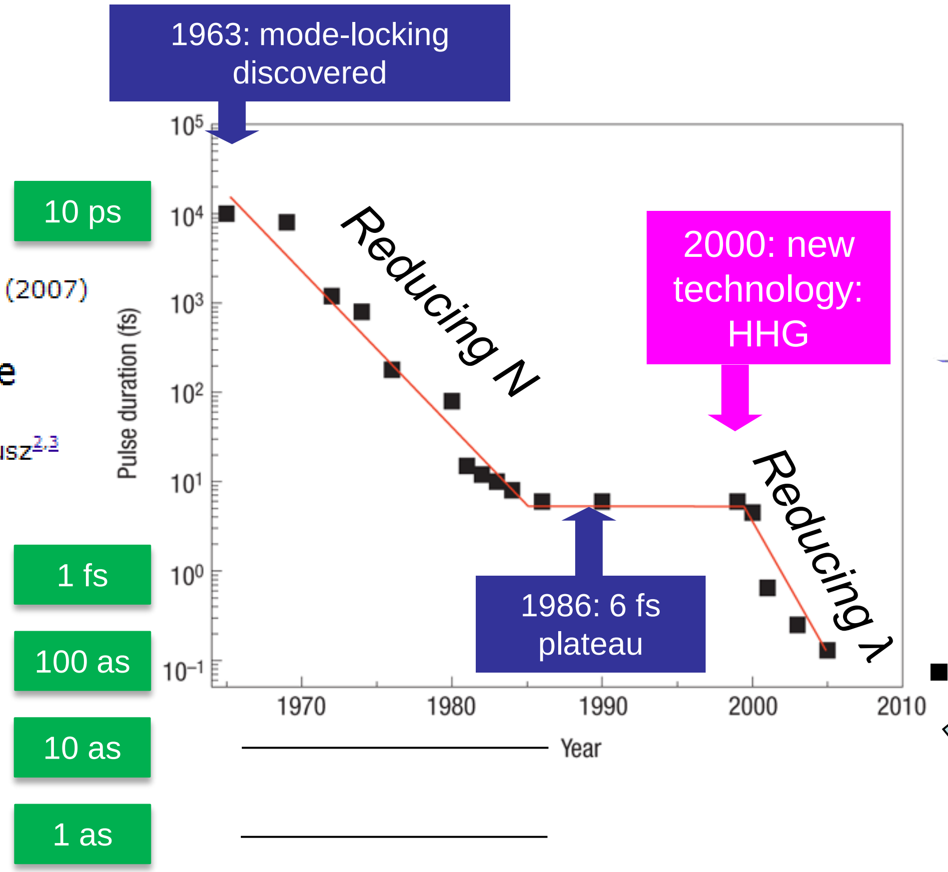
- Progress in the record for shortest pulse of light against year comes through a combination of reducing the number of cycles per pulse, and reducing the wavelength of the light.
- Present HHG sources at ~10 nm have generated ~67 attoseconds.

Nature Physics **3**, 381 - 387 (2007)
doi:10.1038/nphys620

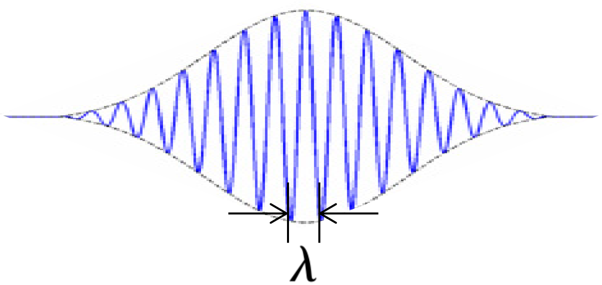
Attosecond science

P. B. Corkum[1] & Ferenc Krausz[2,3]

+ modified to include recent HHG result and possible future development



Pulse duration = N × λ / c



Short-pulse potential of FELs

- Table shows duration of light pulse for a given number of cycles (N), at certain wavelengths.
- Reducing N for FELs, shows potential to reach atto-zeptosecond scales.

	N=1000	N=100	N=10	N=1
Lasers @~800nm	3 ps	300 fs	30 fs	3 fs
HHG @~10nm	30 fs	3 fs	300 as	30 as
FEL @~0.1nm	300 as	30 as	3 as	300 zs

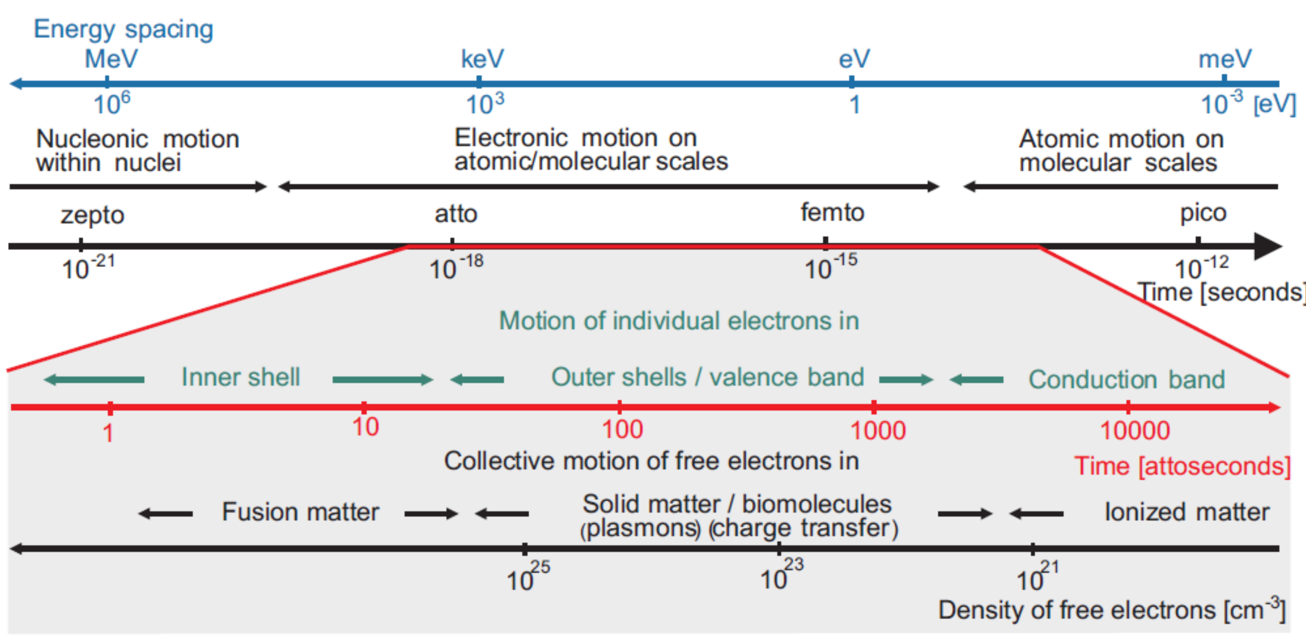


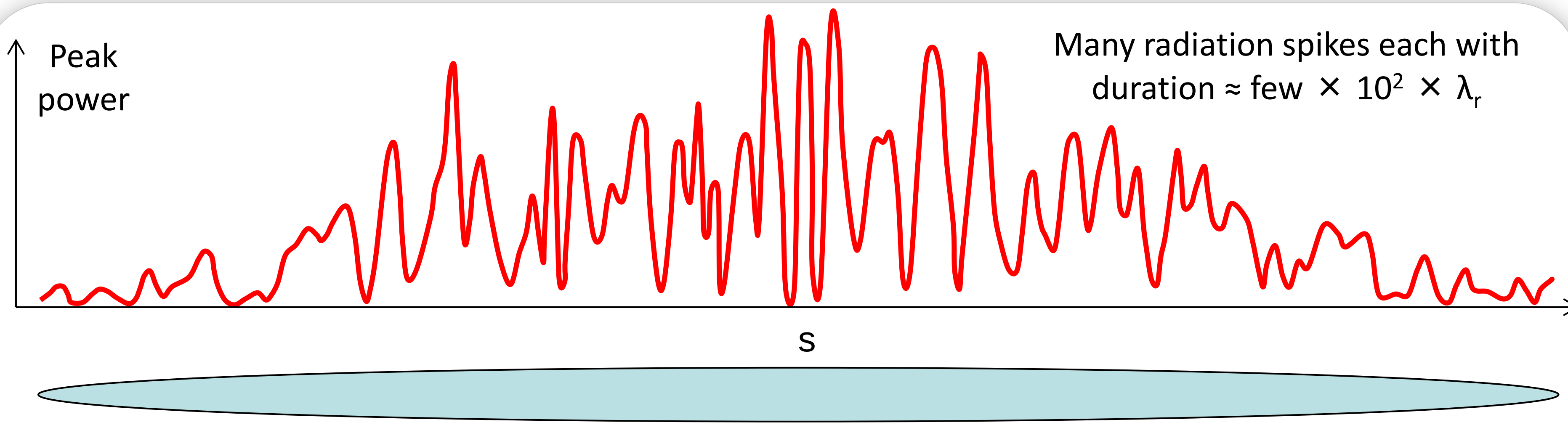
FIG. 2. (Color) Characteristic time scales for microscopic motion and its connection with energy spacing between relevant stationary states (upper panel); characteristic time scales for the motion of one or several electrons and for the collective motion of an electronic ensemble (lower panel).

F.Krausz, M. Ivanov, Rev. Mod. Phys, 81, 163, 2009.

Pulse Durations from SASE

- The total length of the emitted radiation pulse is on the scale of the electron bunch and is relatively long in this context e.g. a few fs corresponds to ~$10^4 \times \lambda_r$ at 0.1nm.
- The slippage between radiation and electrons sets the scale of the sub-structure in the SASE pulse
- The slippage in one gain length is called the **co-operation length** and the length of each SASE spike is about $2\pi l_c$ which is a few hundred $\times\ \lambda_r$ in x-ray FELs.

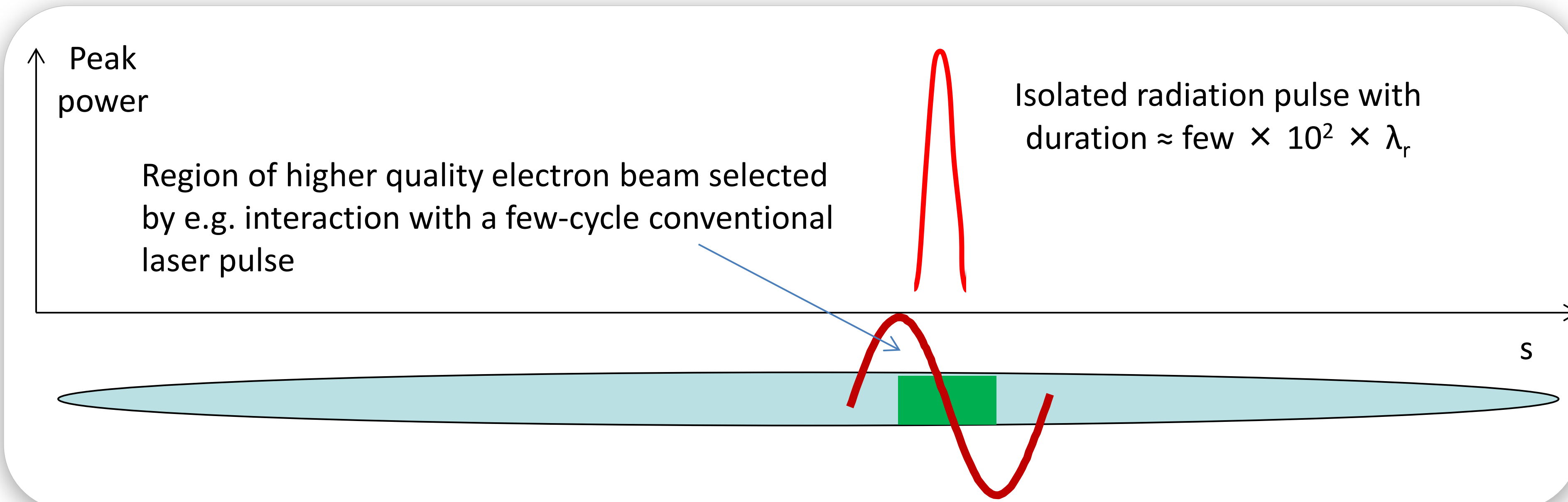
$$\boxed{l_c = \lambda_r / 4\pi\rho}$$



The electron bunch is relatively long, e.g. ~few fs = ~ $10^4 \times \lambda_r$ (not to scale)

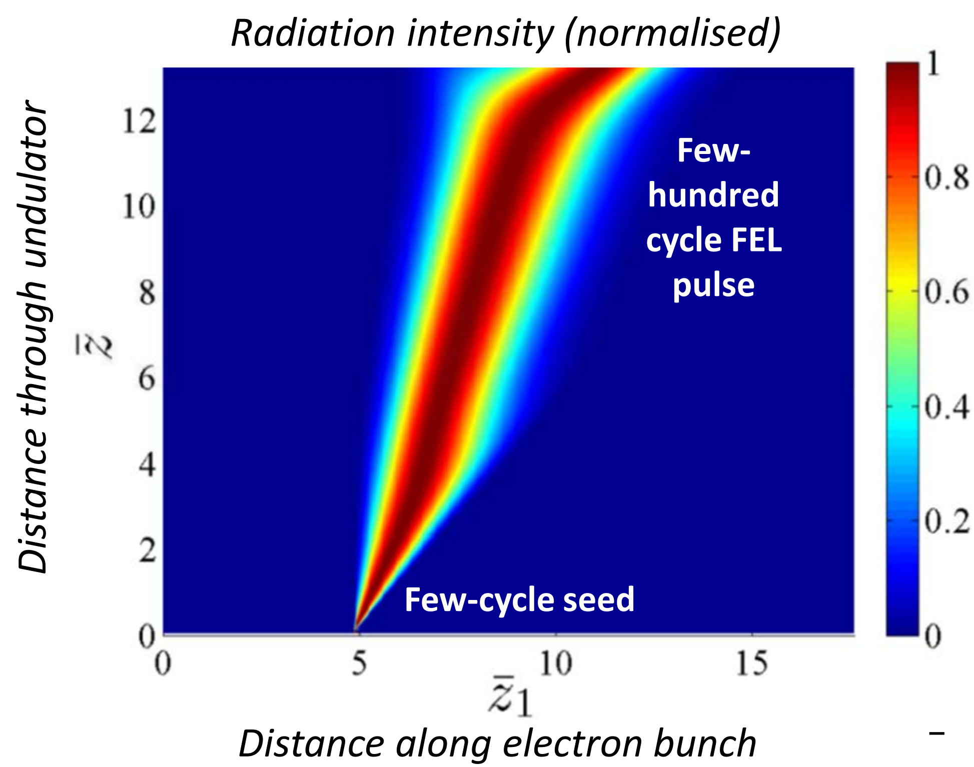
Producing a single SASE spike

- Can reduce the bunch length or 'slice' the electron beam quality so only one spike occurs
- There are several proposals and experiments:
  - Reducing bunch length: e.g. Y. Ding et al. PRL, 102, 254801 (2009).
  - Emittance spoiling: e.g. P. Emma et al. Proc. 26th FEL Conf. 333 (2004), Y. Ding et al. PRL, 109, 254802 (2012).
  - Energy modulation: e.g. E.L. Saldin et al. PRST-AB 9, 050702, (2006), L. Giannessi et al. PRL 106, 144801, (2011).
- The minimum pulse duration is usually one SASE spike. For hard x-ray FEL parameters this is around 100 as – close to record from HHG – but at shorter wavelength and higher power.
- But there is still potential for a further two orders of magnitude reduction with fewer cycles per pulse.



Shorter than a SASE spike?

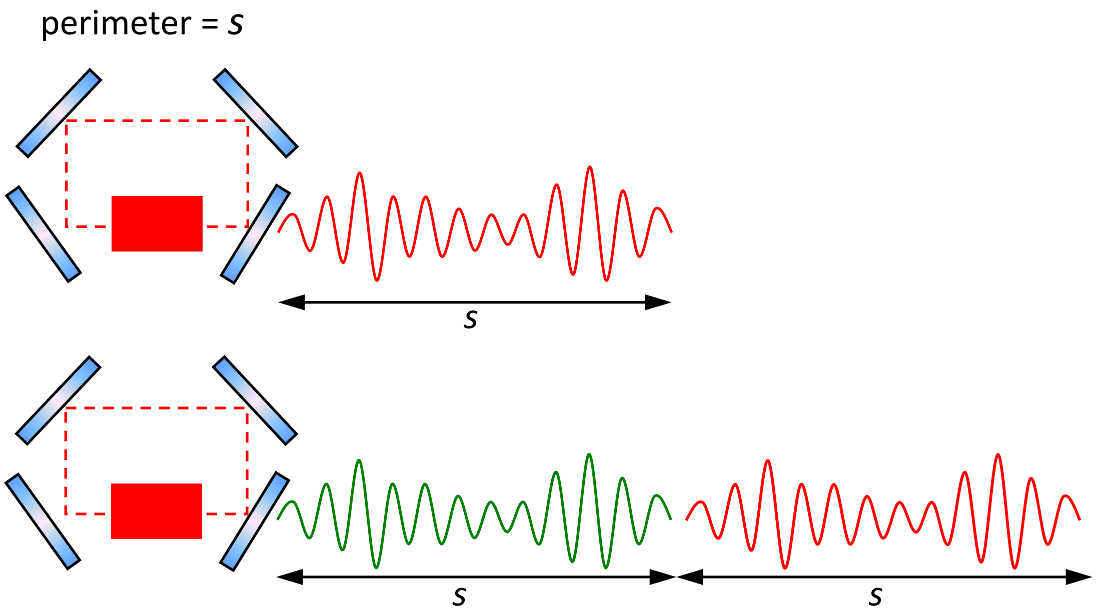
- ***So why can't you just slice a region of electron bunch which is shorter than a SASE spike?***
- For a bunch ***shorter than l_c*** the radiation has slipped out of the front of the bunch before it is amplified.
- Even if you start with a long bunch and a single cycle seed it is immediately broadened by the slippage as it is amplified

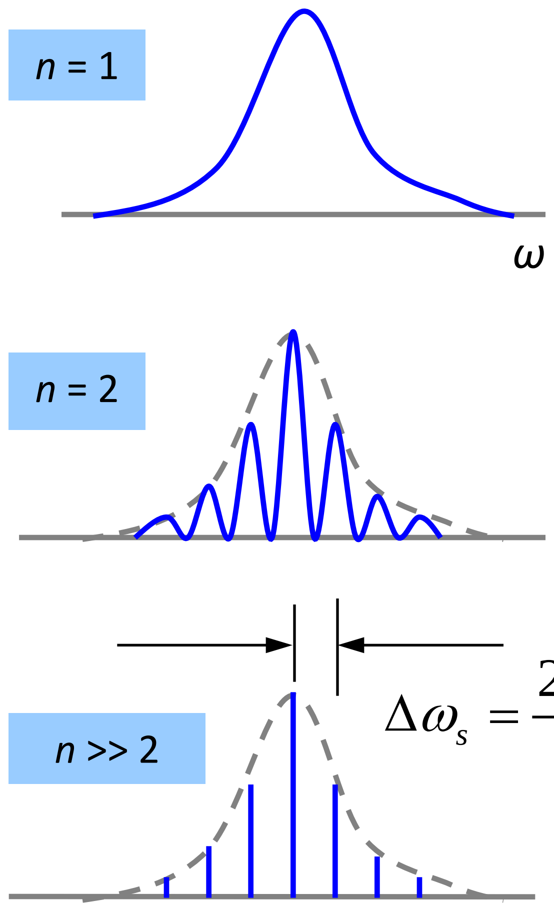


Minimum radiation pulse length from a standard FEL is ~few-hundred cycles
"FEL co-operation length"

Mode-locking in lasers allowed access to a new regime of shorter pulses – can mode-locking do the same for FELs?

Mode Locking in Lasers: Cavity Modes*





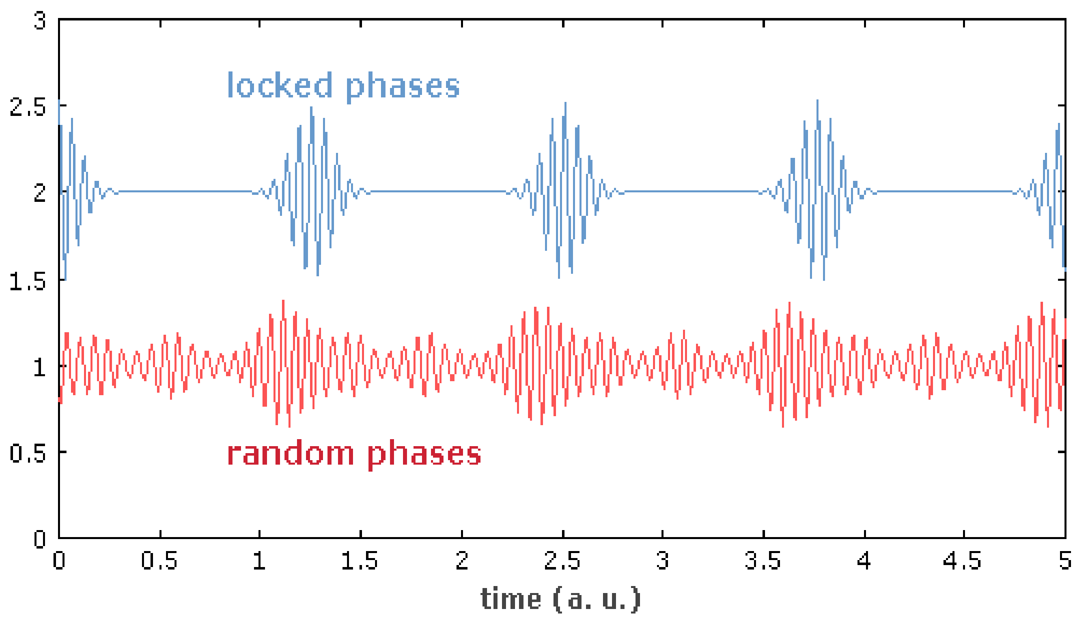
$$\Delta\omega_s = \frac{2\pi c}{s}$$

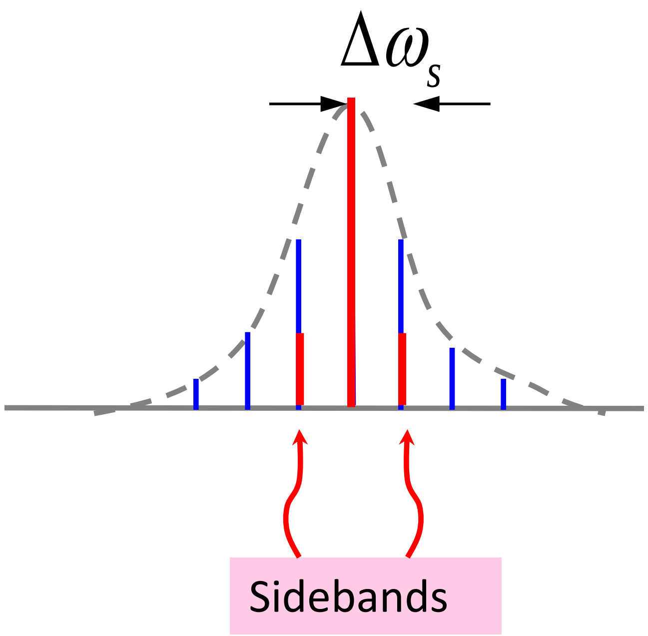
"It is the fixed time delay or time shift between successive round trips that gives the axial mode character to a laser output signal" - *Siegman*

*A. E. Siegman, *Lasers* (University Science Books, Sausalito, USA, 1986). See Chap. 27.

Mode-Locking in Lasers: Locking Modes

- The modes are locked by establishing a ***fixed phase relationship*** between the axial modes.
  - Application of modulation (*e.g. cavity length modulation, gain modulation, frequency modulation*) causes axial modes to develop sidebands.
  - If modulation frequency is at mode spacing $\Delta\omega_s$ sidebands overlap neighbouring modes which then **couple and phase lock.**
  - The output consists of ***one dominant repeated short pulse***. $\tau_p \approx 0.5/\sqrt{N_0} f_m$

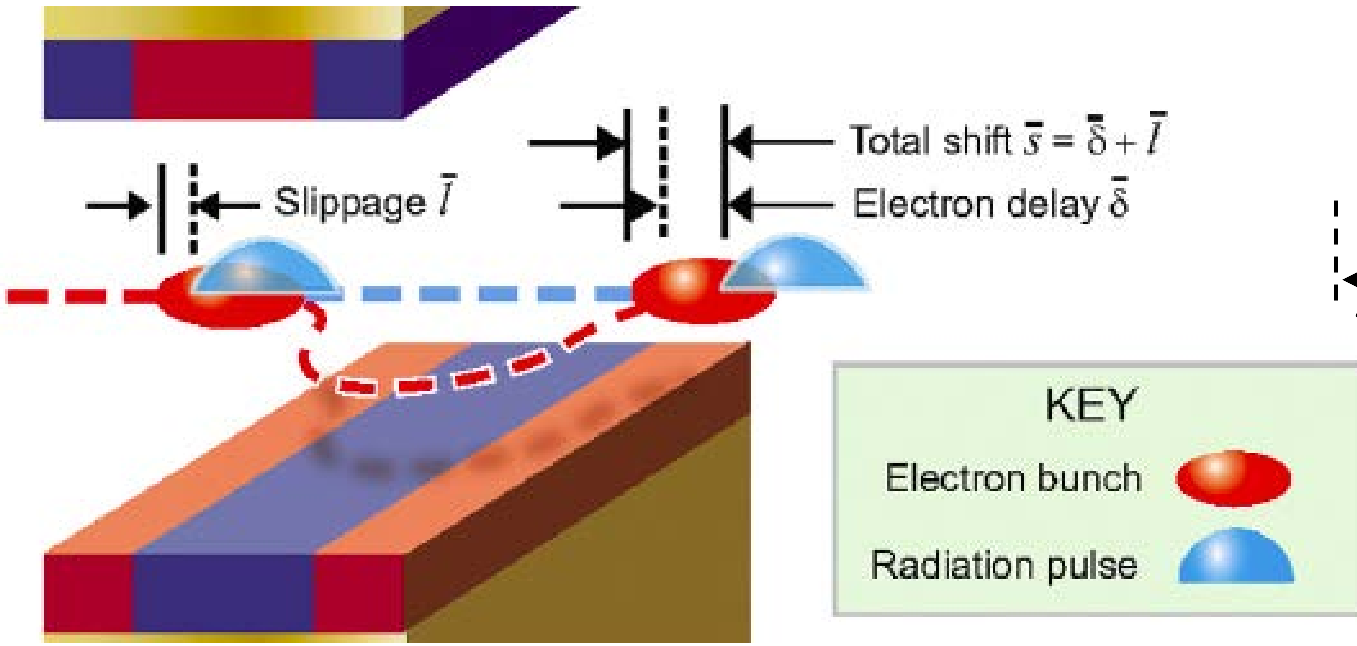


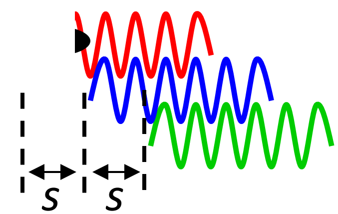


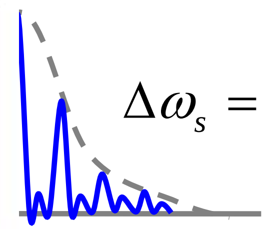
Generating modes in an amplifier FEL

- In the amplifier FEL the axial modes are synthesised by repeatedly delaying the electron bunch in magnetic chicanes between undulator modules
- This produces a sequence of time-shifted copies of radiation from one module, and hence axial modes
- The modes are locked by modulating the input electron beam energy at the mode spacing

n =







$$\Delta\omega_s = \frac{2\pi c}{s}$$

Modal structure of Spontaneous Emission

Starting from universally scaled 1D wave equation

$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial \bar{z}_1} = b_0(\bar{z}_1)$$

spontaneous emission spectrum for N modules and delay s_1 is

$$|\tilde{A}(\bar{\omega})|^2 = |\tilde{b}|^2 \bar{l}^2 \mathrm{sinc}^2\left(\frac{\bar{\omega}\bar{l}}{2}\right) \frac{1-\cos(N\bar{\omega}\bar{s}_1)}{1-\cos(\bar{\omega}\bar{s}_1)}.$$

Comparing this with expression for modes of a cavity laser with round trip period T:

$$I^{(N)}(\omega) \equiv |\tilde{E}^{(N)}(\omega)|^2 = I(\omega)\frac{1-\cos(NT\omega)}{1-\cos(T\omega)}$$

So the delays synthesise the effect of an optical cavity of length equal to the total slippage in undulator + chicane

Universal FEL Scaling

$$\bar{z} = 2k_w \rho z$$

$$\bar{z}_1 = \frac{2k_r\rho}{\bar{\beta}_z}(z - c\bar{\beta}_z t)$$

$$\rho = \frac{1}{\gamma_r}\left(\frac{\bar{a}_w \omega_p}{4ck_w}\right)^{\frac{2}{3}}$$

$$\omega_p = \left(\frac{e^2 n_p}{\epsilon_0 m}\right)^{\frac{1}{2}}$$

$$A = \frac{e\varepsilon}{mc\omega_p\sqrt{\rho\gamma_r}}.$$

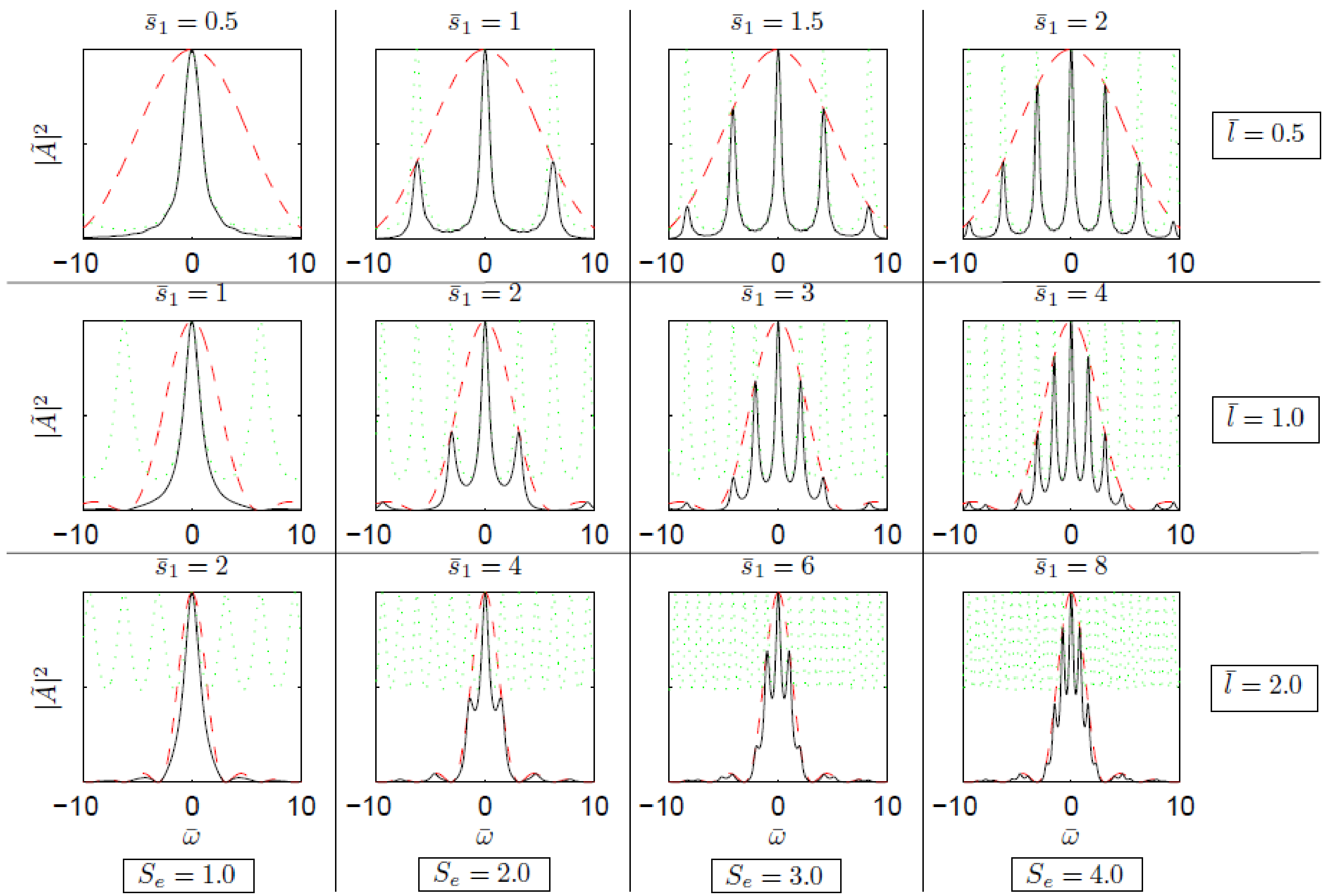
$$\bar{\omega} = \frac{1}{2\rho}\frac{\omega - \omega_r}{\omega_r}$$

We can also add a simple gain term so that each module amplifies by a factor e^{α}

$$|\tilde{A}(\bar{\omega},\alpha)|^2 = |\tilde{b}|^2 \bar{l}^2 \mathrm{sinc}^2\left(\frac{\bar{\omega}\bar{l}}{2}\right) \frac{1+e^{2N\alpha} - 2e^{N\alpha}\cos(N\bar{\omega}\bar{s}_1)}{1+e^{2\alpha} - 2e^{\alpha}\cos(\bar{\omega}\bar{s}_1)}$$

Emission Spectra: N=8

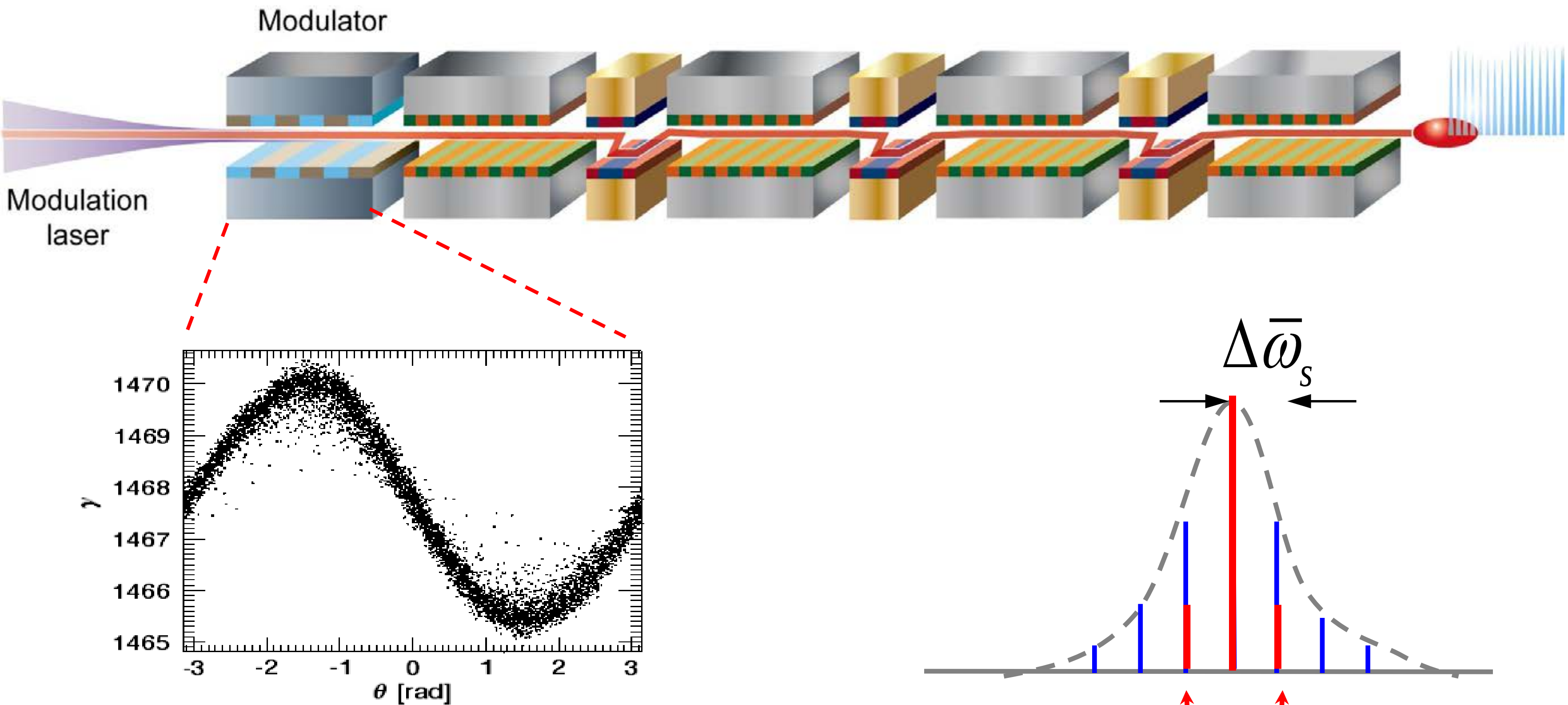
Gain included: $\alpha = \sqrt{3}/2 \times l$

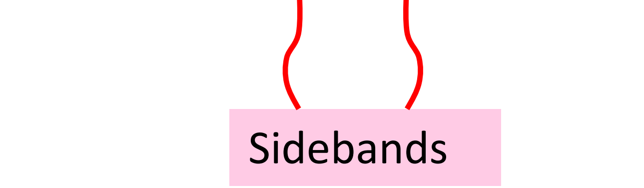


Increasing module length
Width of sinc function is $4\pi/l$

Increasing chicane delay
Mode spacing = $2\pi/s$

Locking the generated modes



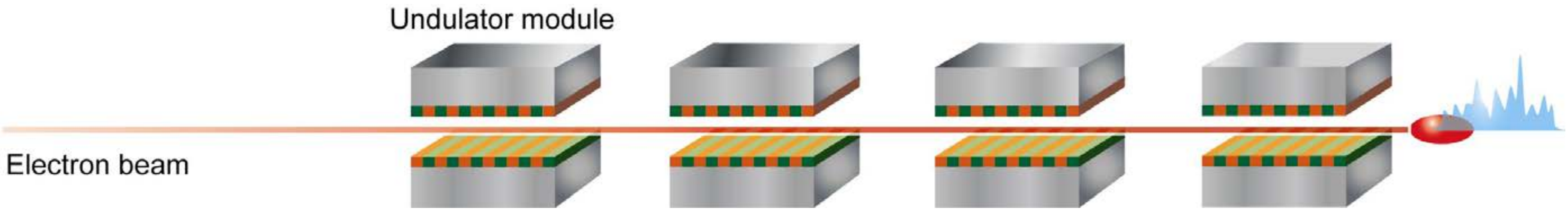


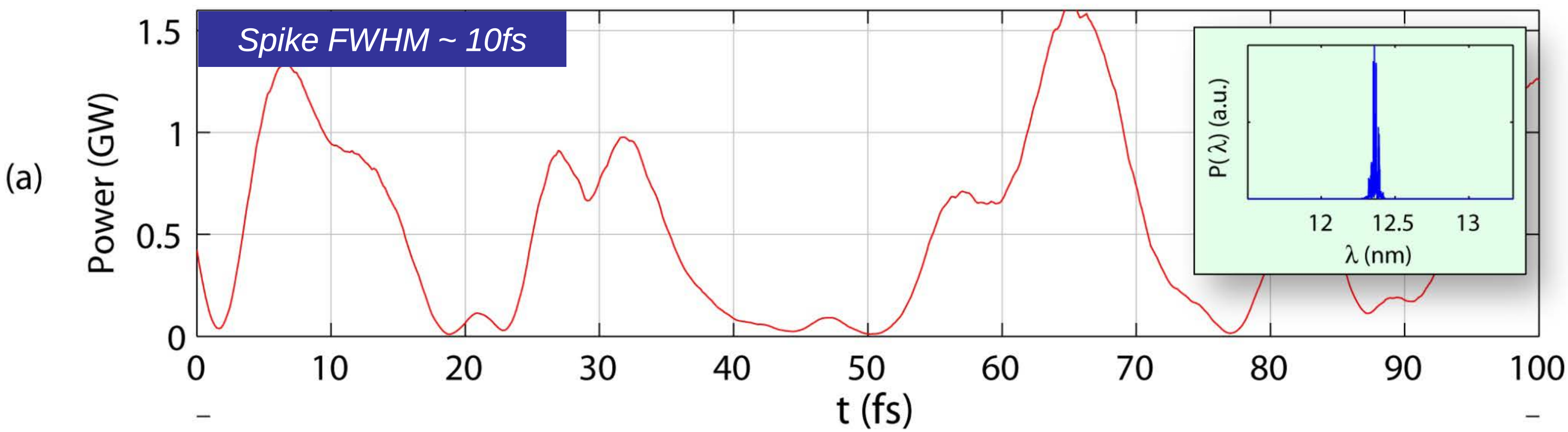
3D Simulation Parameters: SASE FEL @ 12.4nm

TABLE I. XUV and x-ray simulation parameters.

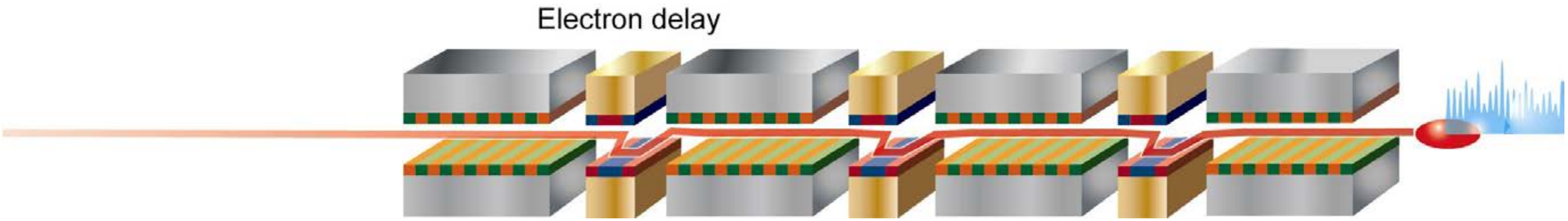
	XUV	X-ray
Bunch energy E (GeV)	0.75	14.3
Bunch peak current I (kA)	3	3.4
Normalized emittance ϵ_n (mm-mrad)	2	1.2
RMS fractional energy spread σ_γ/γ_0	10^{-4}	8×10^{-5}
Undulator period λ_u (cm)	3.1	3
Resonant wavelength λ_r (Å)	124	1.5
Undulator module length (units l/λ_u)	12	72
FEL parameter ρ	2.5×10^{-3}	5×10^{-4}
Chicane delay $N_c = \delta/\lambda_r$	48	228
Modulation period (units of λ_r)	61	303
Modulation amplitude (MeV)	5.8	14.3
Slippage enhancement S_e	5	5

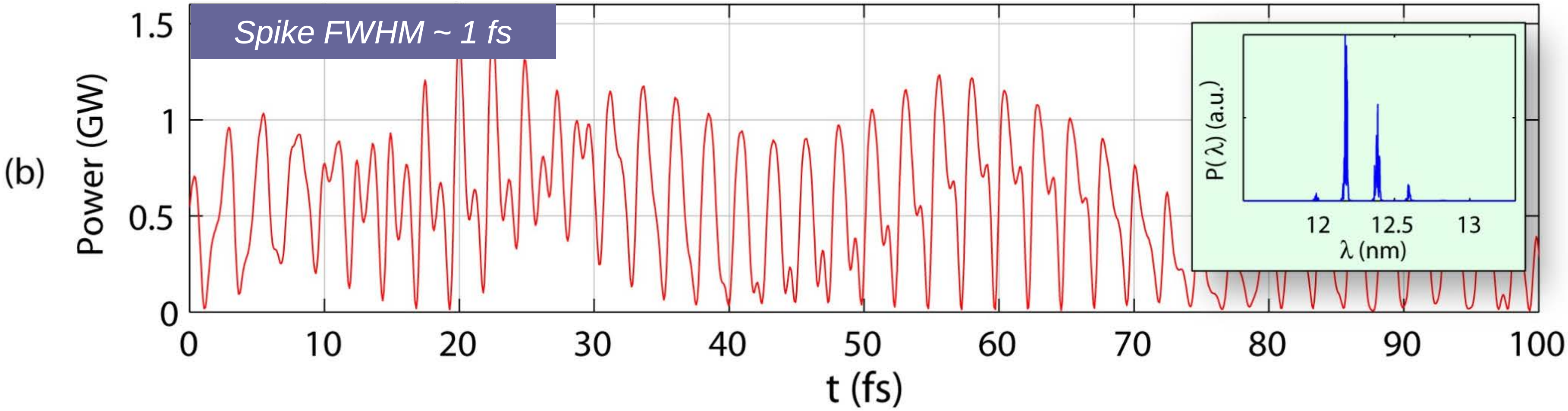
3D Simulation Results: SASE XUV-FEL @ 12.4nm



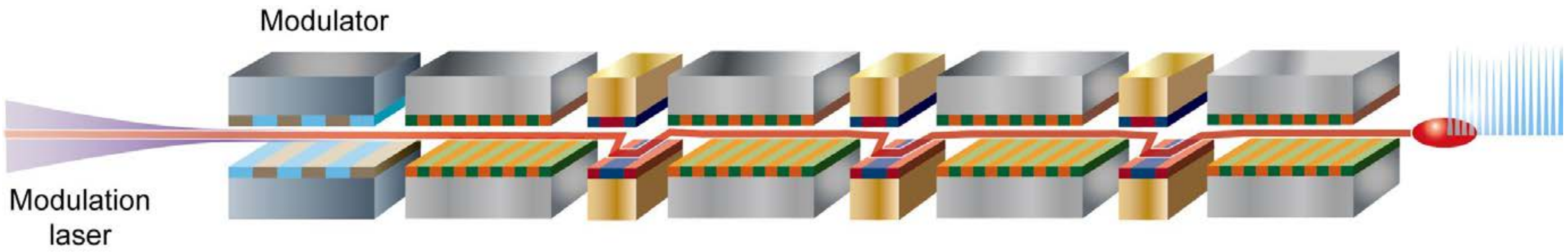


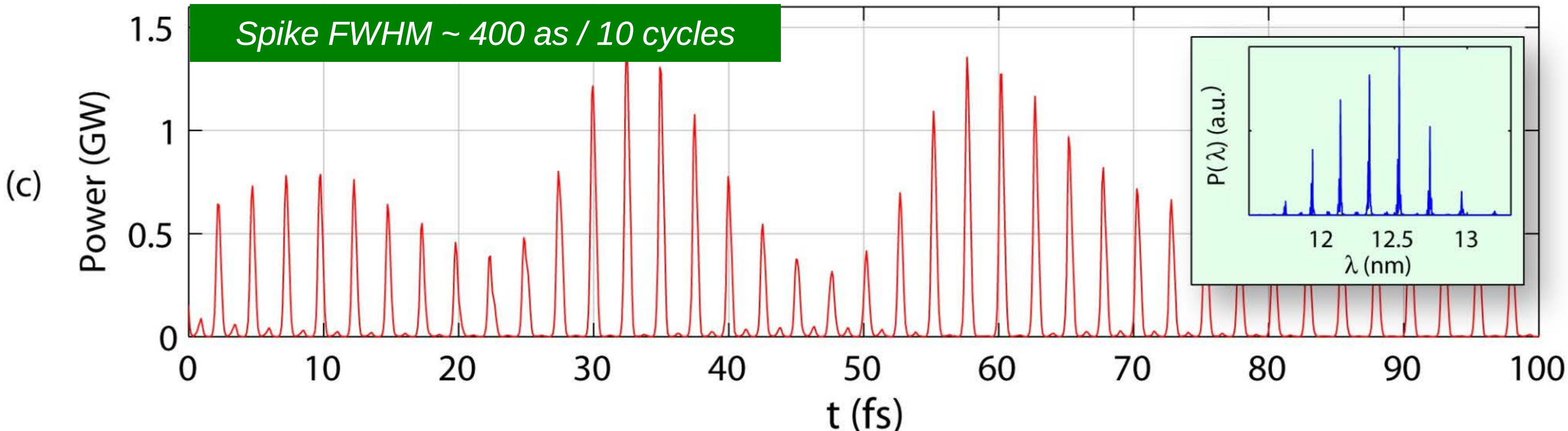
Mode-Coupled SASE XUV-FEL @ 12.4nm





Mode-Locked SASE XUV-FEL @ 12.4nm



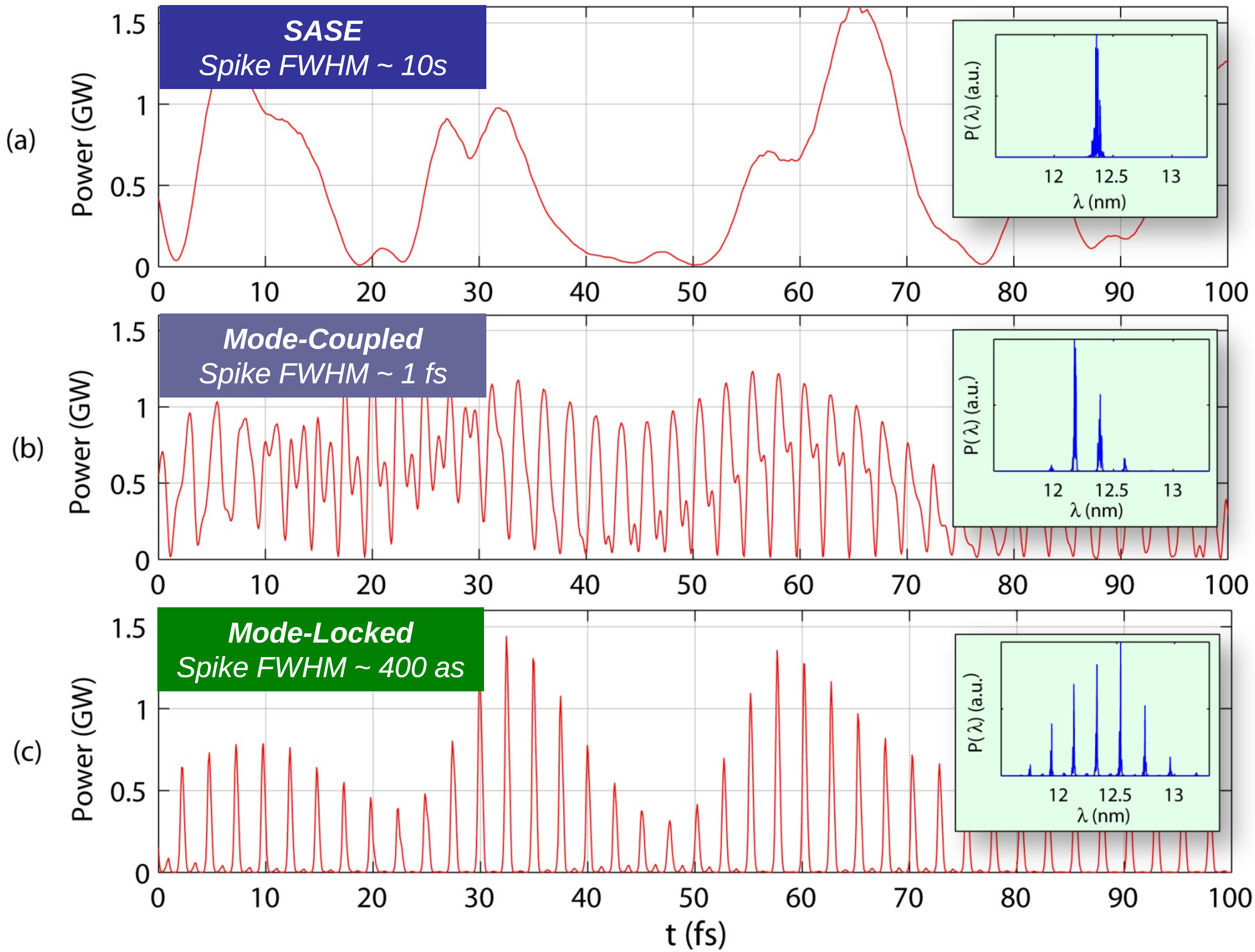


From conventional cavity analysis: $\tau_p \approx 0.5/\sqrt{N_0} f_m$

f_m the modulation frequency N_0 is the number of oscillating modes

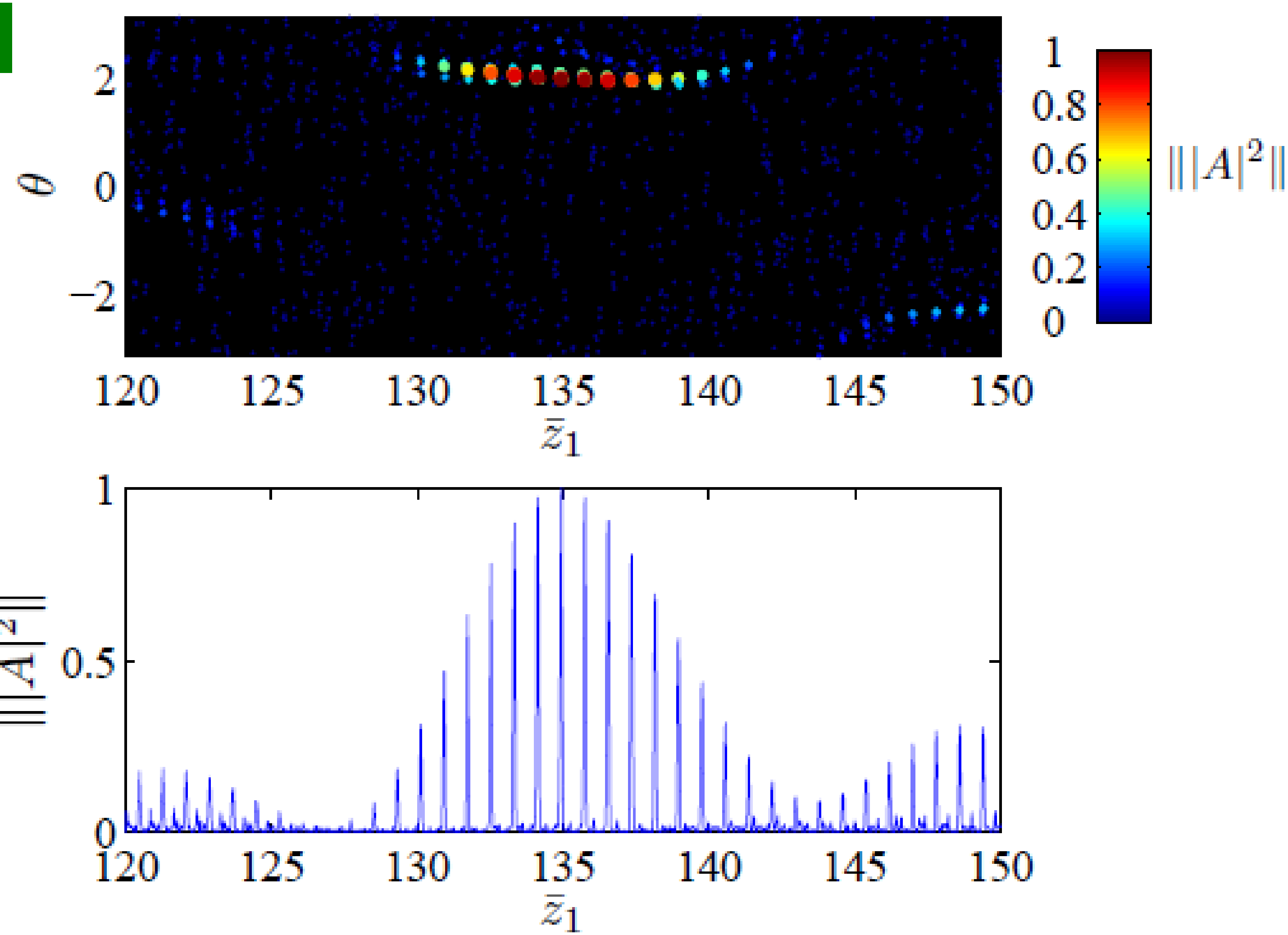
$\Rightarrow \tau_p \approx 440$ as : simulation ≈ 400as

XUV Output Comparison



Phase Coherence Between Spikes

Mode-Locked



X-ray Parameters

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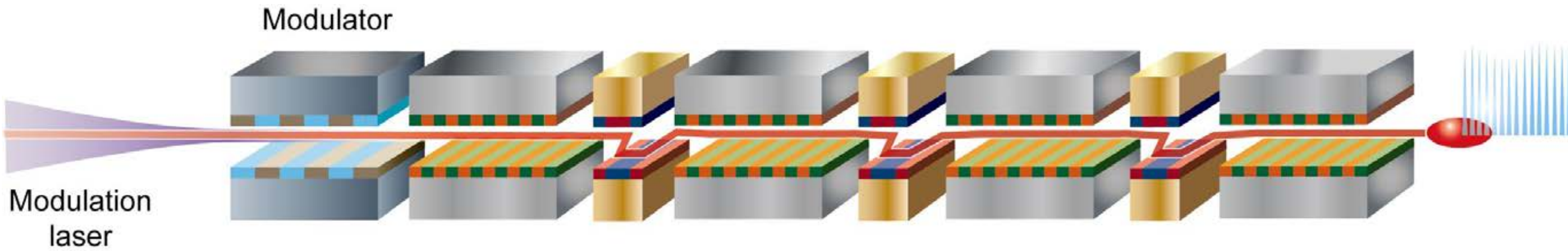
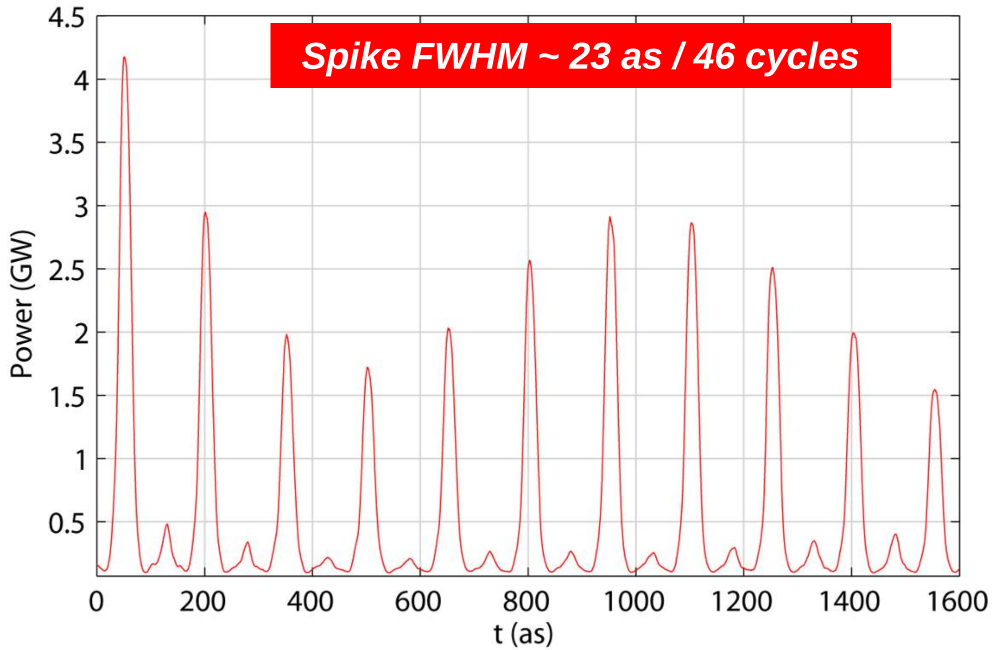
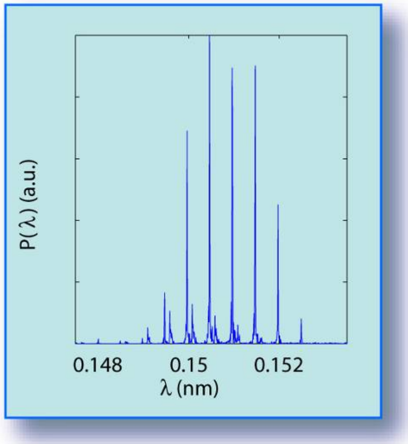
Mode-locked X-ray SASE FEL amplifier





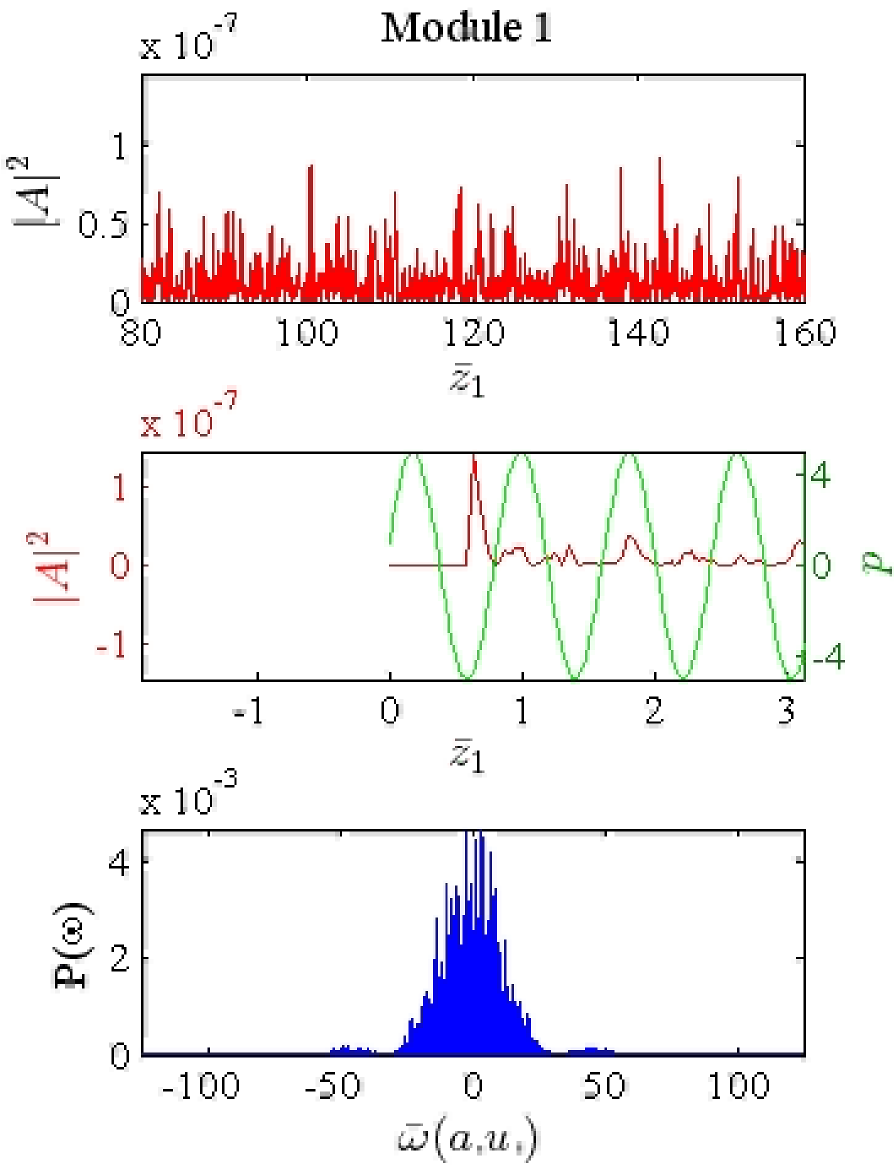


PRL 100, 203901 (2008) PHYSICAL REVIEW LETTERS week ending 23 MAY 2008

Mode Locking in a Free-Electron Laser Amplifier

N. R. Thompson[1,2,*] and B. W. J. McNeil[1,†]

Modelocked Amplifier FEL: Animation



Averaged vs Non-averaged FEL Equations

Averaged 1D FEL Equations

$$\frac{\partial \theta_j}{\partial \bar{z}} = p_j,$$
$$\frac{\partial p_j}{\partial \bar{z}} = -(A(\bar{z},\bar{z}_1)\exp[i\theta_j] + c.c.)$$
$$\left(\frac{\partial}{\partial \bar{z}} + \frac{\partial}{\partial \bar{z}_1}\right) A(\bar{z},\bar{z}_1) = \chi(\bar{z}_1)\langle \exp[-i\theta]\rangle \equiv b(\bar{z}_1)$$

equal charge weighting over one wavelength

Radiation field averaged over one period

electron phases (positions) averaged over one period

Field and electrons 'sampled' once per radiation period. Structure on smaller scale not revealed. Minimum sample rate is:

$\Delta t_s = f_r^{-1}$ Nyquist freq. $f_{\mathrm{Ny}} = \dfrac{1}{2\Delta t_s} = \dfrac{f_r}{2}$

=> From Nyquist theorem, frequency range that can be simulated without aliasing is: $\dfrac{f_r}{2} < f < \dfrac{3f_r}{2}$

Non-Averaged 1D FEL Equations

$$\frac{d\bar{z}_{1_j}}{d\bar{z}} = 2\rho p_j \qquad (8)$$
$$\frac{dp_j}{d\bar{z}} = -\left(A(\bar{z},\bar{z}_{1_j})\exp\left(i\frac{\bar{z}_{1_j}}{2\rho}\right) + \text{c.c.}\right) \qquad (9)$$
$$\left(\frac{\partial}{\partial \bar{z}} + \frac{\partial}{\partial \bar{z}_1}\right) A(\bar{z},\bar{z}_1) = \frac{1}{\bar{n}_\parallel}\sum_{j=1}^{N} \chi_j \delta(\bar{z}_1 - \bar{z}_{1_j}) \times \exp\left(-i\frac{\bar{z}_1}{2\rho}\right) \qquad (7)$$

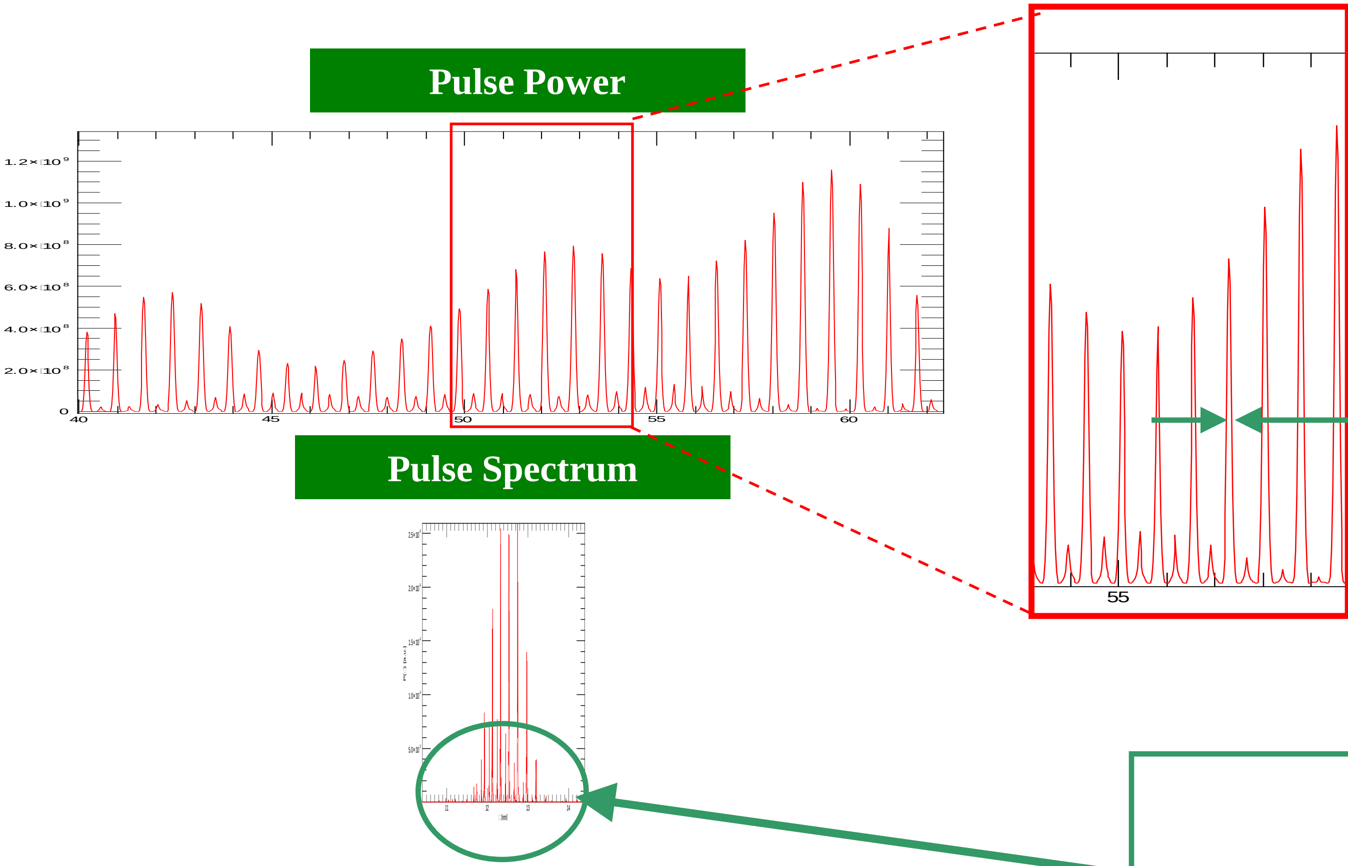
non-averaged field

particles have individual charge weightings

particles have individual positions

Can describe wider frequency range and sub-wavelength structure

Recap of full 3D (Averaged Code) results @ 12.4nm



Spike width
FWHM = ***400as***
(~10 optical cycles)

$N_{modes} \sim 8$:

Equivalent Non-Averaged Code Result

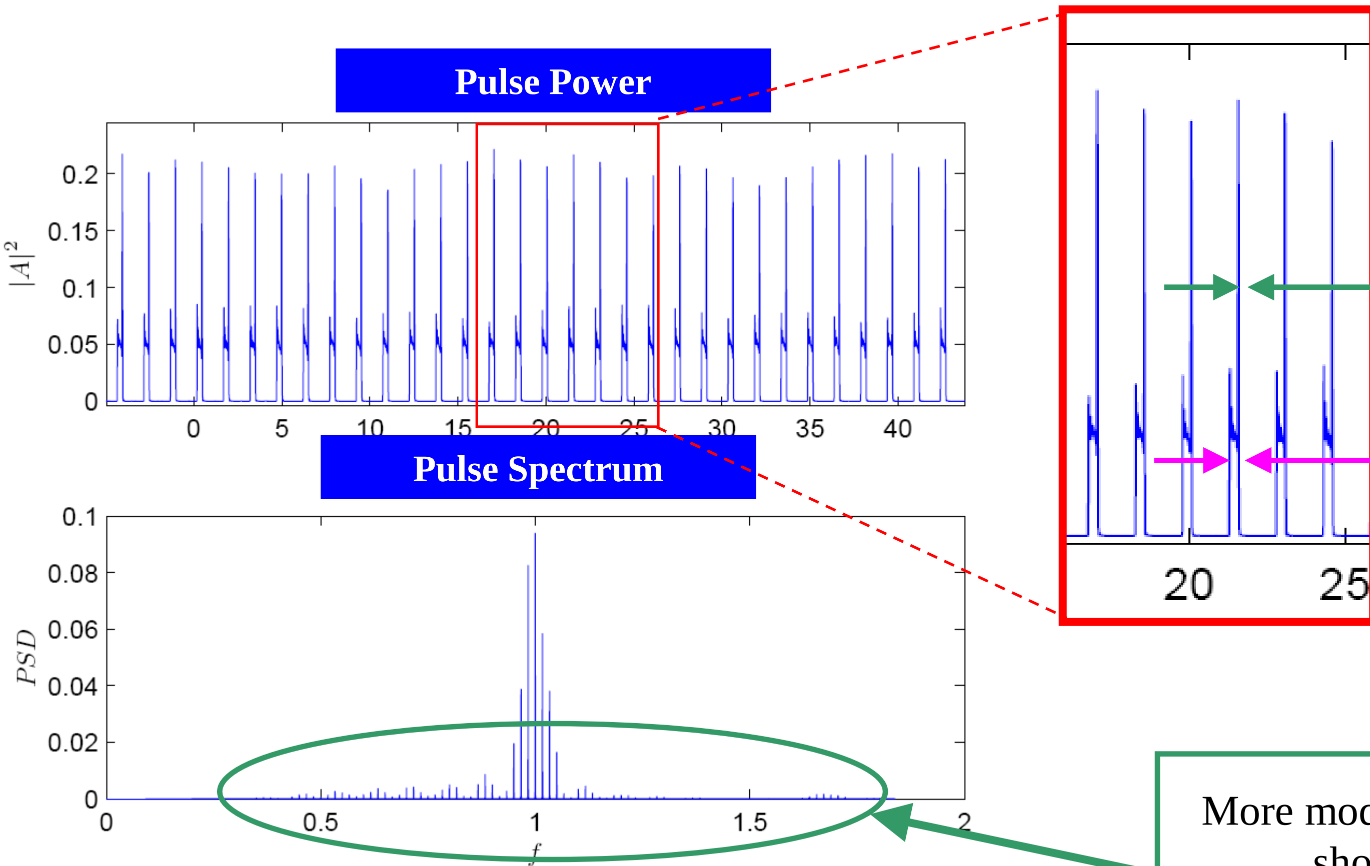


Figure 2: Simulation result from one-dimensional non-averaged CSE code, for parameters equivalent to the 12.4 nm Genesis 1.3 mode-locking simulations. The FWHM spike length scales to 57 as.

If scale to 0.15nm,
FWHM ~ 700 zs

Spike width
FWHM = ***57as !***
(~1.4 optical cycles)

450 as:
same as Genesis
@12.4nm

More modes now, therefore shorter spikes:

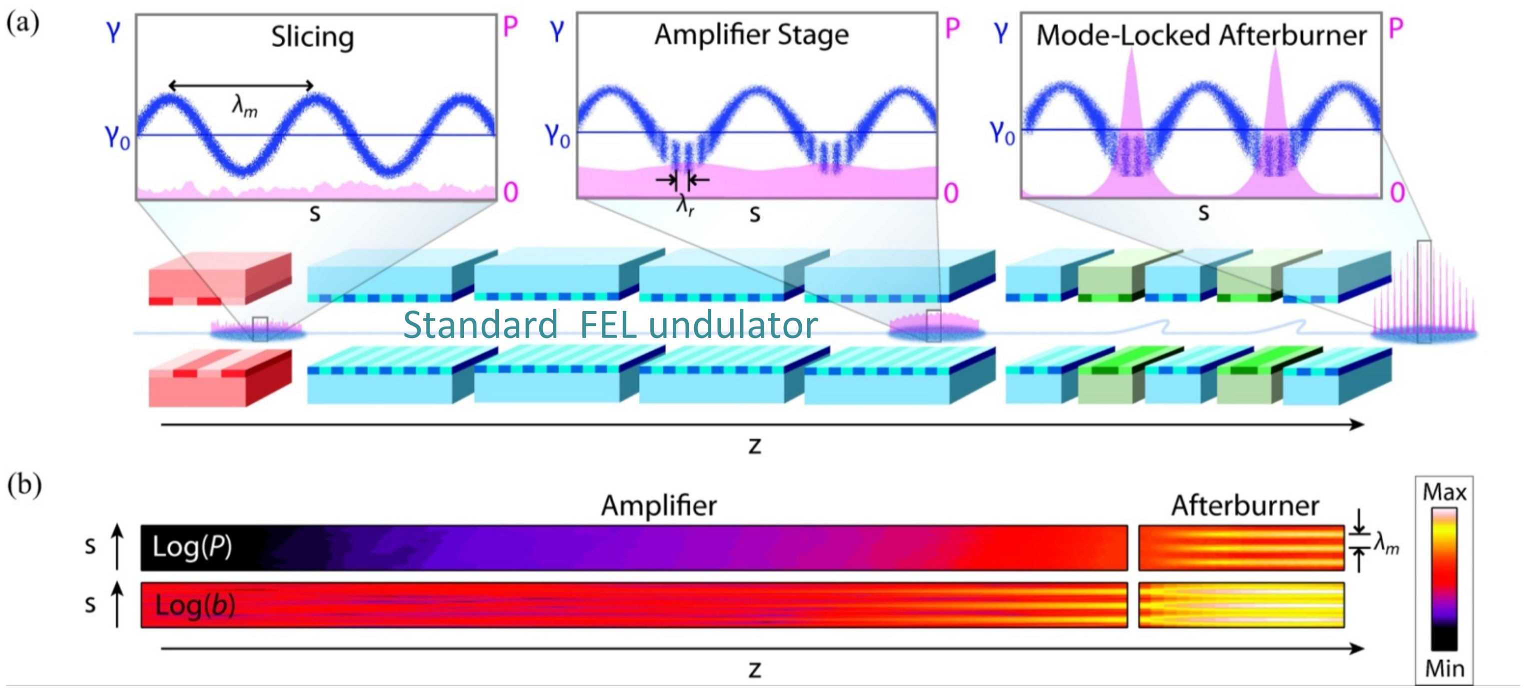
$$\tau_p \approx 0.5/\sqrt{N_0} f_m$$

New concept: mode-locked afterburner FEL

- Afterburner is a continuation of the ML-FEL concept, capable of generating similar output – difference is in how it’s applied:
- The afterburner uses:
  - a standard undulator line for amplification
  - then only a short ‘mode-locked’ section for emission (exponential growth means the majority of FEL emission is in the last gain length)
- So the afterburner can be
  - a relatively small addition to existing FELs
  - optimised for shortest pulses.

Mode-locked afterburner FEL

- Modulate the electron beam properties prior to a standard FEL amplifier
- No structure in radiation (‘P’ below) within standard undulator
- But there is a few-cycle pulse train structure in electron micro-bunching (‘*b*’ below)
- The ‘afterburner’ section converts structure in bunching to radiation



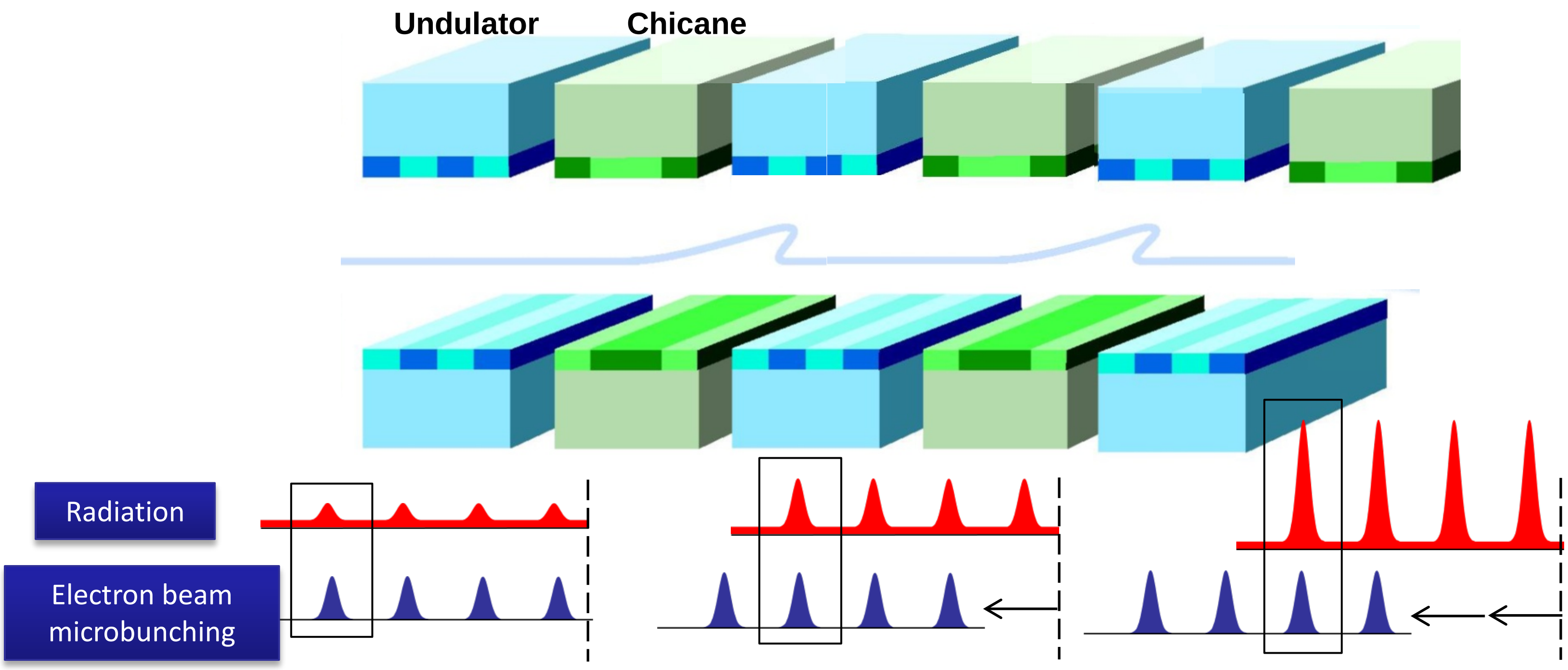
PRL 110, 104801 (2013) PHYSICAL REVIEW LETTERS week ending 8 MARCH 2013

Few-Cycle Pulse Generation in an X-Ray Free-Electron Laser

D. J. Dunning,[1,2,*] B. W. J. McNeil,[2,†] and N. R. Thompson[1,2,‡]
[1]ASTeC, STFC Daresbury Laboratory and Cockcroft Institute, Warrington WA4 4AD, United Kingdom
[2]Department of Physics, SUPA, University of Strathclyde, Glasgow G4 0NG, United Kingdom
(Received 31 August 2012; published 5 March 2013)

Mode-locked afterburner FEL

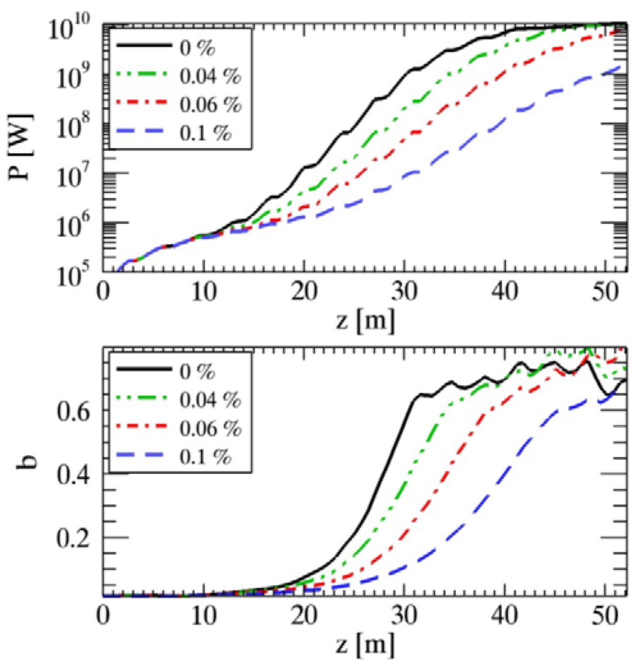
- Figure below shows how pulse-train structure in micro-bunching is converted into the radiation.
- Radiation aligned with micro-bunching spike is amplified, then slips ahead to next micro-bunching spike for further amplification (and so on)
- Result is amplification with retention of the few-cycle structure



Simulation: Beam Modulation in Amplifier

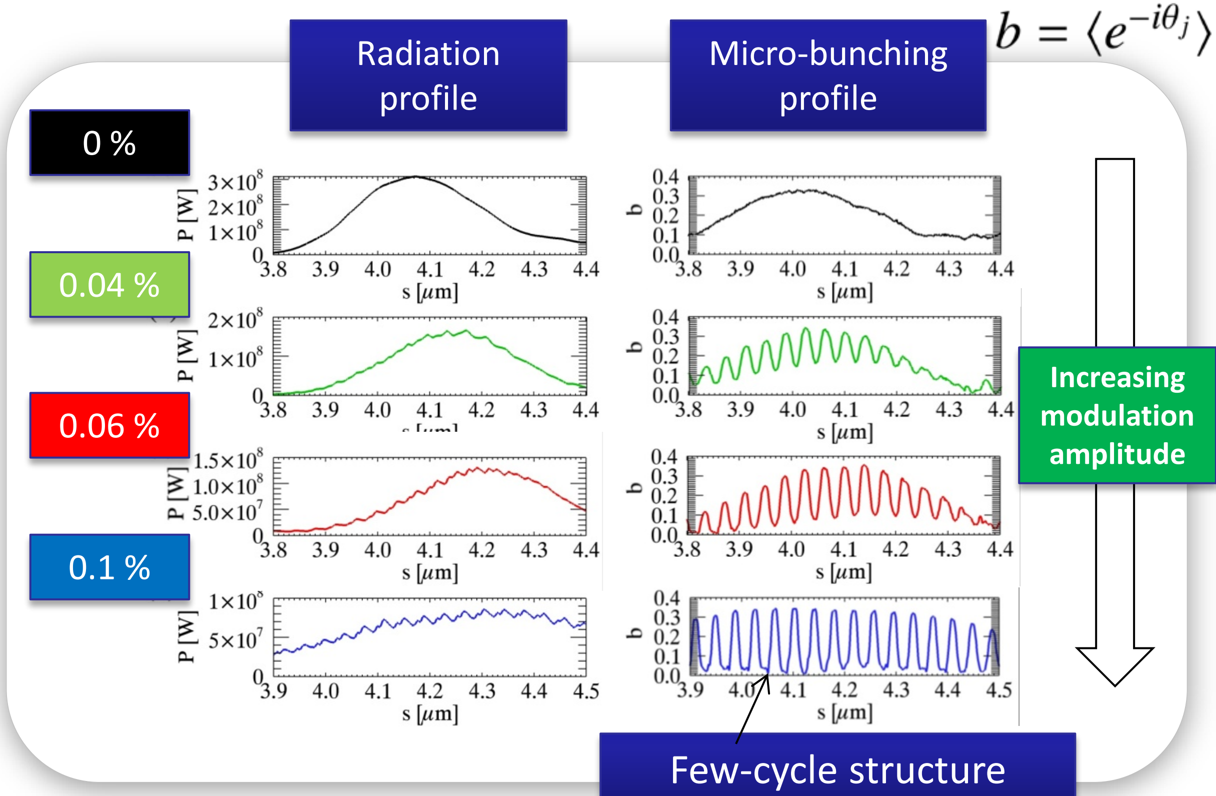
- Soft x-ray FEL at 1.24 nm. Starts from noise.
- Applied sinusoidal energy modulation, period ~30xλ_r (=40nm), and varied the modulation amplitude.
- Amplification rate reduces with increasing modulation amplitude.
- Only minor changes in radiation profile – increased l_c + 'ripple'
- ***Generates well-defined comb structure in e-beam micro-bunching***.

Parameter	Soft x ray
Amplifier stage	
Electron beam energy [GeV]	2.25
Peak current [kA]	1.1
ρ parameter	1.6×10^{-3}
Normalized emittance [mm-mrad]	0.3
rms energy spread, σ_γ/γ_0	0.007%
Undulator period, λ_u [cm]	3.2
Undulator periods per module	78
Resonant wavelength, λ_r [nm]	1.24
Modulation period, λ_m [nm]	38.44



Maximum radiation power (top) and electron microbunching (bottom) with distance through FEL amplifier

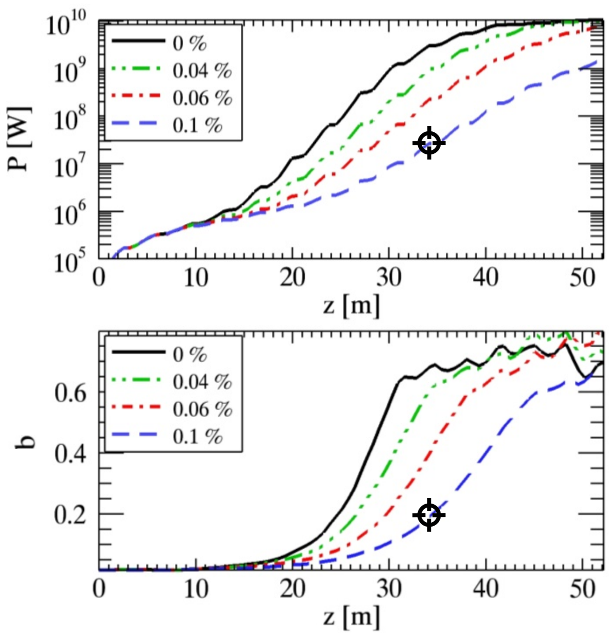
$$b = \langle e^{-i\theta_j} \rangle$$



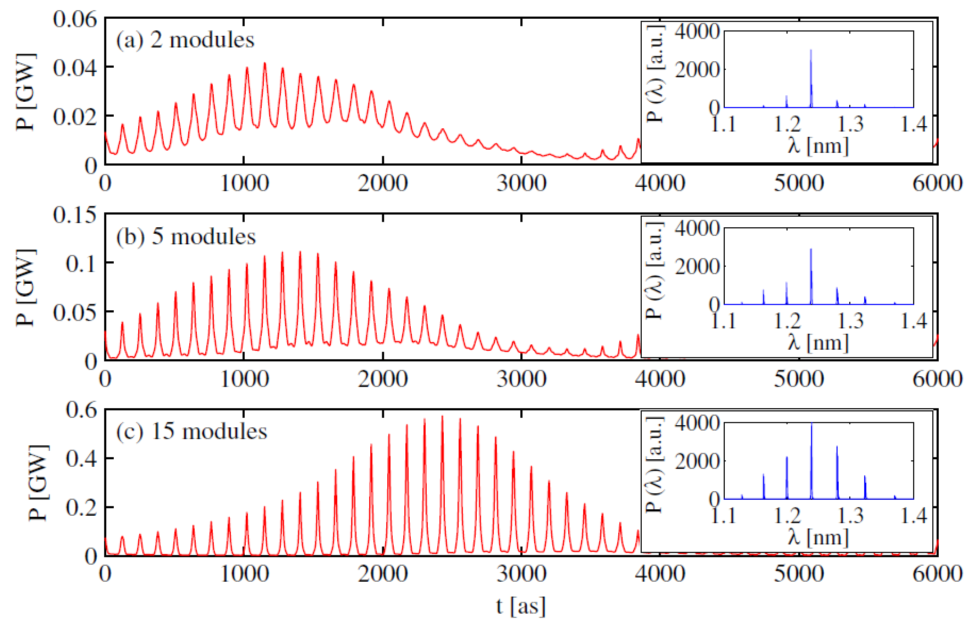
Simulation: into the Mode-locked Afterburner

- Used 0.1% modulation amplitude which gave strong micro-bunching structure.
- We want FEL amplification to continue into the mode-locked afterburner - extract before saturation.
- Choose 8-period undulators and set chicanes appropriately
- Pulse train emerges above the amplifier radiation within 15 modules (length of afterburner = 7 m) .
- Generates **9 as/2 cycle** (rms) pulses separated by 124 as, at ~0.6GW.

Parameter	Soft x ray
Amplifier stage	
Electron beam energy [GeV]	2.25
Peak current [kA]	1.1
ρ parameter	1.6×10^{-3}
Normalized emittance [mm-mrad]	0.3
rms energy spread, σ_γ/γ_0	0.007%
Undulator period, λ_u [cm]	3.2
Undulator periods per module	78
Resonant wavelength, λ_r [nm]	1.24
Modulation period, λ_m [nm]	38.44
Modulation amplitude, γ_m/γ_0	0.1%
Extraction point [m]	34.1
Mode-locked afterburner	
Undulator periods per module	8
Chicane delays [nm]	28.52
No. of undulator-chicane modules	~15



Extraction point for mode-locked afterburner



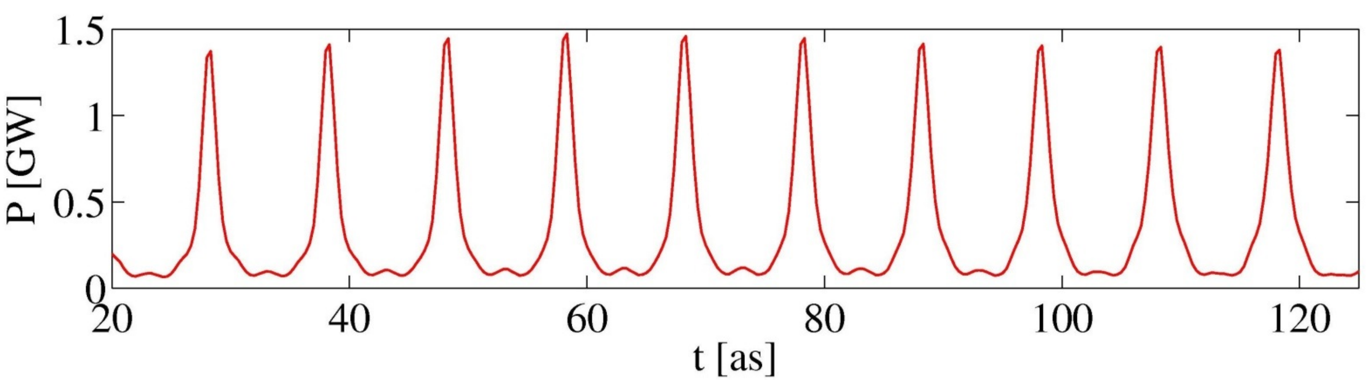
9as/ 2 cycle (rms) 0.6 GW pulses

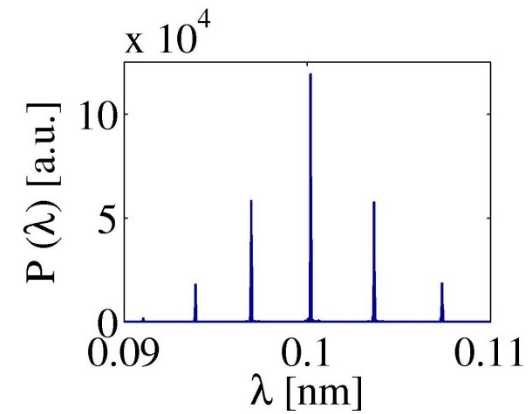
Hard X-ray simulation

- A hard x-ray case of resonant FEL wavelength 0.1 nm was also simulated, with the aim of demonstrating shorter pulse generation.
- Used parameters similar to the SACLA facility.
- Aiming for shortest pulses so used 8-period undulator modules in afterburner and 3nm modulation period ($30x\lambda_r$).
- The results show pulse durations of **700 zs / 2 cycle** (rms) at 1.3 GW.
- Future FELs at shorter wavelength could allow shorter still.
- We note for all these results that the spectrum is a set of discrete modes under a broad-bandwidth envelope – increased by ~2 orders of magnitude over SASE

Hard x-ray 0.1 nm example

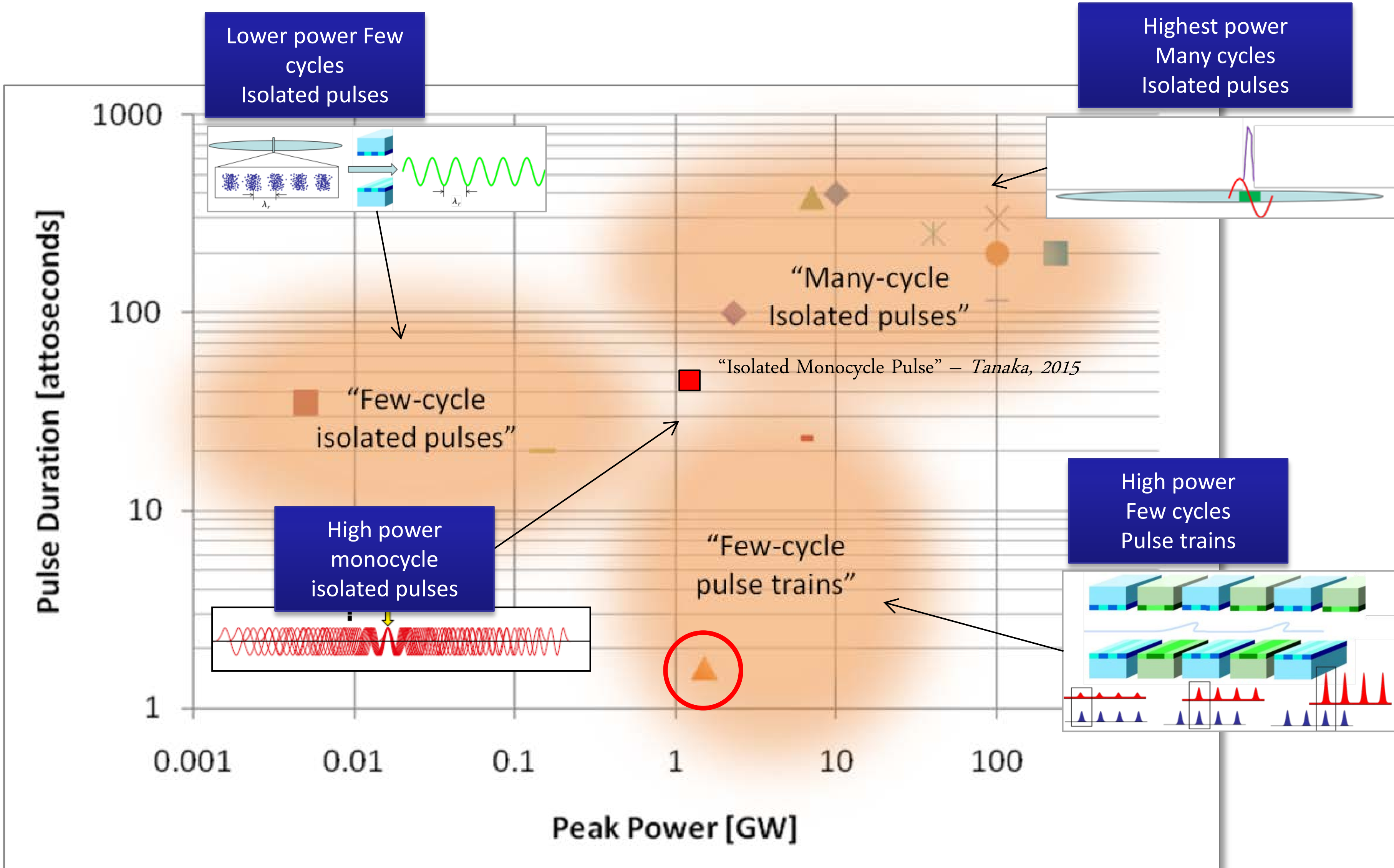
Parameter	Hard x ray
Amplifier stage	
Electron beam energy [GeV]	8.5
Peak current [kA]	2.6
ρ parameter	6×10^{-4}
Normalized emittance [mm-mrad]	0.3
rms energy spread, σ_γ/γ_0	0.006%
Undulator period, λ_u [cm]	1.8
Undulator periods per module	277
Resonant wavelength, λ_r [nm]	0.1
Modulation period, λ_m [nm]	3
Modulation amplitude, γ_m/γ_0	0.06%
Extraction point [m]	36.0
Mode-locked afterburner	
Undulator periods per module	8
Chicane delays [nm]	2.2
No. of undulator-chicane modules	~40





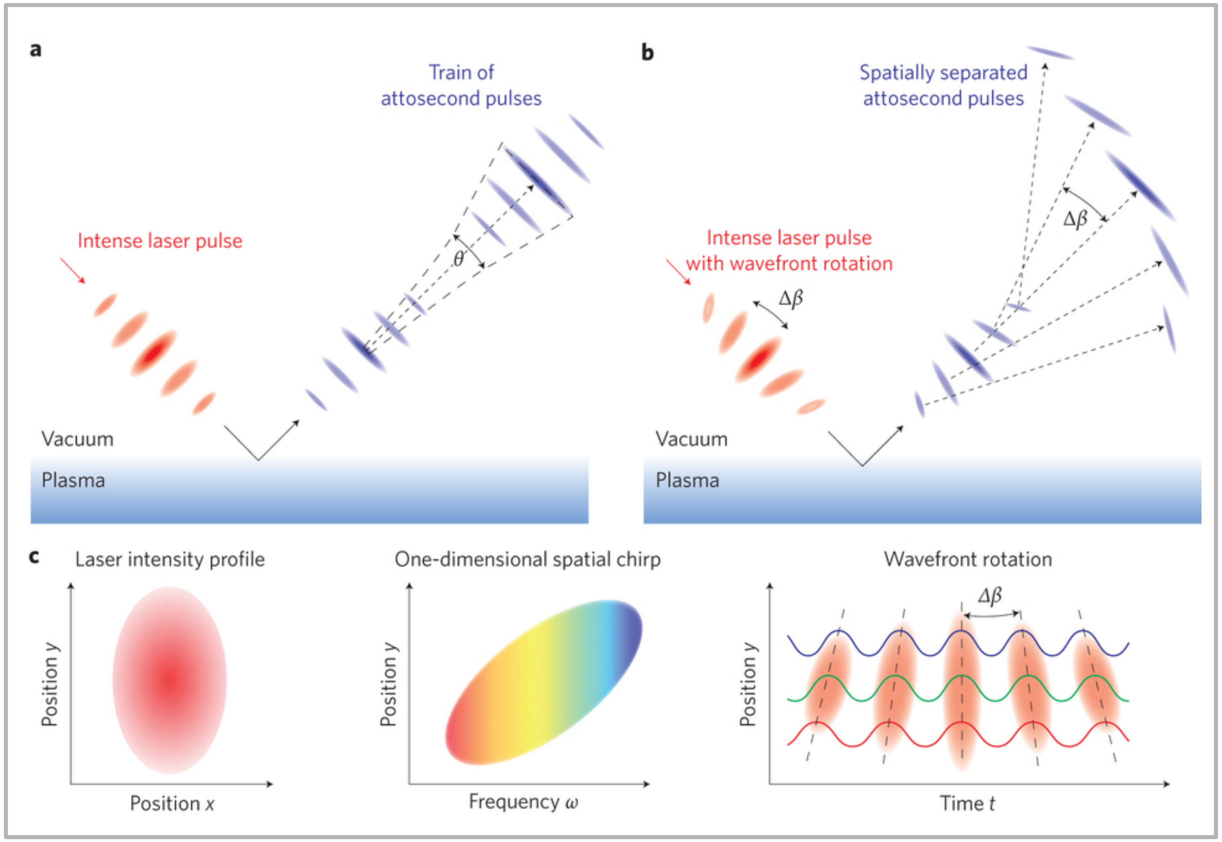
~700 zs / 2 cycle (rms) 1.2 GW pulses

Comparison with other FEL short pulse techniques



What next: Isolated Pulses?

- Can we generate an isolated pulse? Trains are OK for some applications but isolated pulses more useful.
  - Borrow attosecond lighthouse concept by applying a wavefront rotation along the pulse train. *Maybe this could be done using transverse gradient undulators....?*

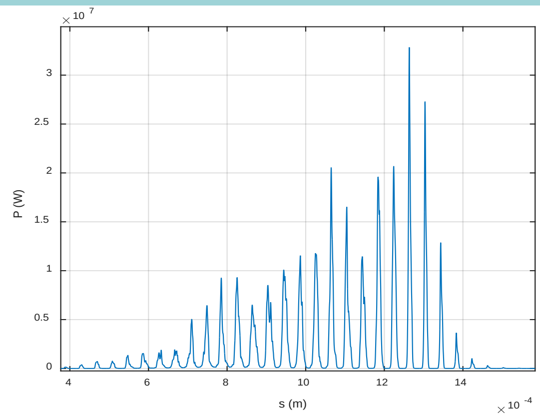


"*Attosecond lighthouses from plasma mirrors*", Jonathan A Wheeler *et al*, **Nature Photonics** 6, 829–33 (2012)

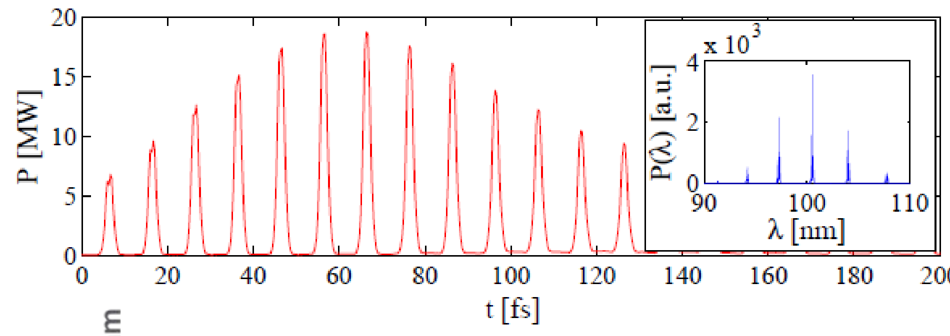
What Next: Experiments

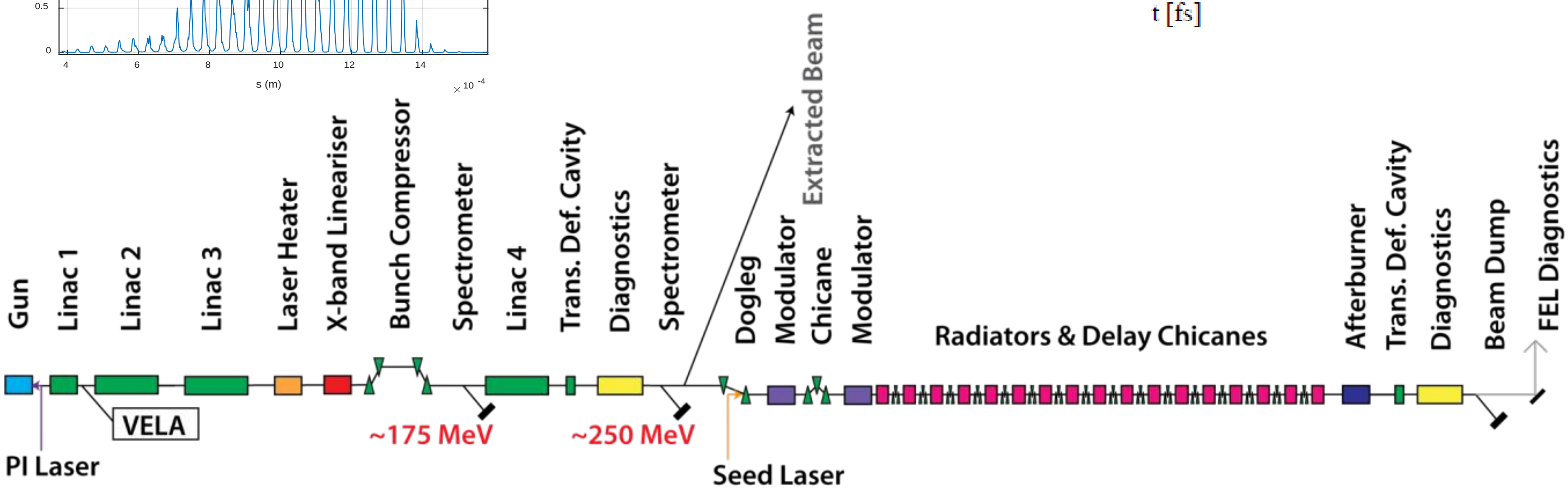
- We are building CLARA, a 250MeV FEL test facility at Daresbury. The FEL lattice is designed for testing Mode-Locking and Mode-Locked Afterburner, amongst other concepts.

266nm Mode-Locking
13 fs / 15 cycle FWHM Pulse Duration



100nm Mode-Locked Afterburner
1.6 fs / 5 cycle FWHM Pulse Duration

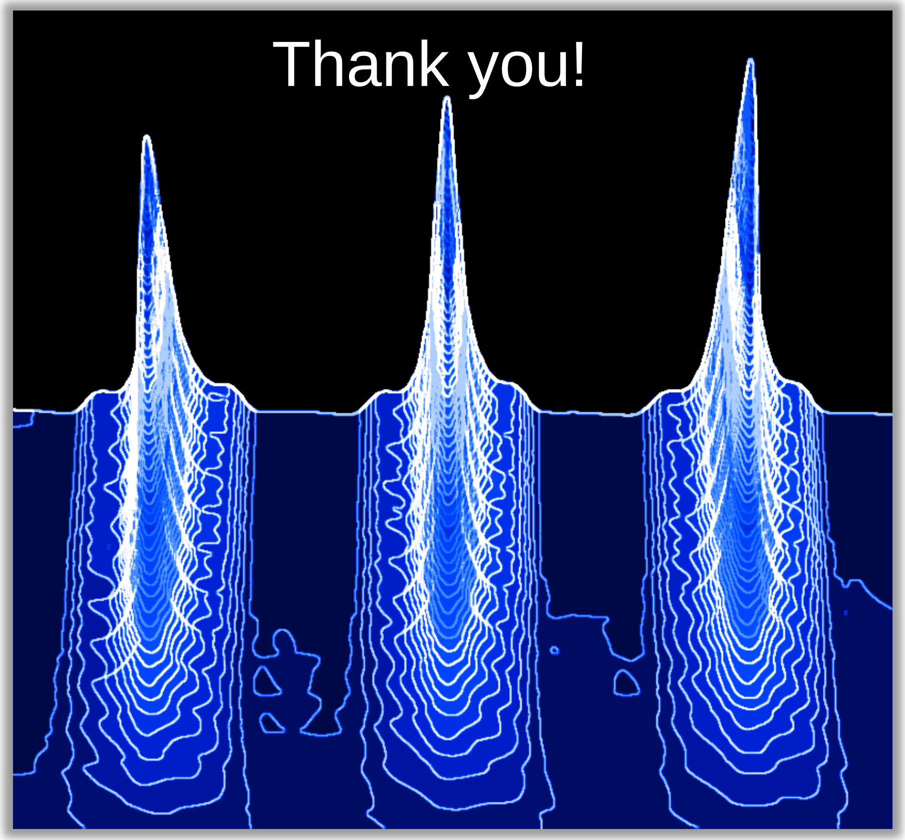




***Phase 1*, 50 MeV, 2016**

***Phase 2*, 250 MeV, 2018**

***Phase 3*, 100nm SASE, 2020**


Thank you!

…extra material

Locking the Generated Modes

- The effect of the energy modulation is to produce a gain modulation. The FEL is a coupled system so this drives a modulation in the bunching parameter.
- In a simple model, add a modulation to the bunching with period equal to the total delay *s*

$$b(\bar{z}_1) \;=\; b_0(\bar{z}_1)\left[1+\cos\left(\frac{2\pi\bar{z}_1}{\bar{s}_1}\right)\right] \;=\; b_0+\frac{b_0}{2}\left[e^{i\frac{2\pi}{\bar{s}_1}\bar{z}_1}+e^{-i\frac{2\pi}{\bar{s}_1}\bar{z}_1}\right]$$

- The Fourier transform of the bunching is then

$$\tilde{b}(\bar{\omega})=\tilde{b}_0(\bar{\omega})+\tfrac{1}{2}\tilde{b}_0(\bar{\omega}-\tfrac{2\pi}{\bar{s}_1})+\tfrac{1}{2}\tilde{b}_0(\bar{\omega}+\tfrac{2\pi}{\bar{s}_1})$$

- And the spontaneous emission spectrum becomes

$$\begin{aligned}|\tilde{A}(\bar{\omega})|^2 \;&=\; |\tilde{b}_0(\bar{\omega})+\tfrac{1}{2}\tilde{b}_0(\bar{\omega}-\tfrac{2\pi}{\bar{s}_1})+\tfrac{1}{2}\tilde{b}_0(\bar{\omega}+\tfrac{2\pi}{\bar{s}_1})|^2\\ &\quad\times\;\bar{l}^2\mathrm{sinc}^2\left(\frac{\bar{\omega}\bar{l}}{2}\right)\frac{1-\cos(N\bar{\omega}\bar{s}_1)}{1-\cos(\bar{\omega}\bar{s}_1)}.\end{aligned}$$

- So the field at frequency ω is driven by bunching at the frequency ω ***but also by the bunching at frequencies of the neighbouring modes ω ± 2π/s***
- Therefore, through the beam bunching, a coupling develops between neighbouring modes and they lock in phase.