Workshop on the generation of single-cycle pulses with Free-Electron Lasers,
16–17 May 2016, Minsk

STATISTICAL FLUCTUATIONS OF ELECTROMAGNETIC RADIATION IN SHORT-PULSE FREE-ELECTRON DEVICES

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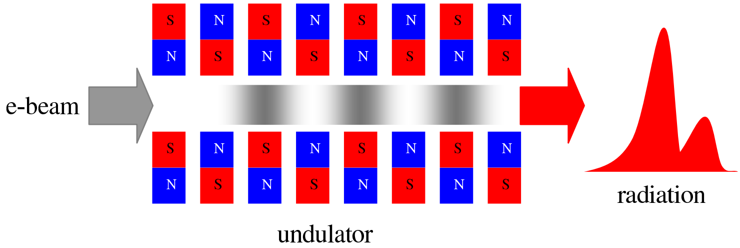
Statistical fluctuations in SASE FEL[1]

- Stochastic behavior of SASE
- Photon statistics
- Coherence properties
- Fluctuations of saturation length
- Statistics of the instantaneous radiation power, statistics of the finite-time integrals of the instantaneous power

[1] R. Bonifacio, et. al. Phys. Rev. Lett. 1994. Vol. 73. P. 70; E.L. Saldin, Opt. Commun. 1998. Vol. 148. P. 383–403; J. Andruszkov, et. al., Phys. Rev. Lett. 2000. Vol. 85. No. 18. P. 3825–3829; M.V. Yurkov, Nucl. Instrum. Methods A483 (2002) 51–56; V.A. Atvazyan,Nucl. Instrum. Methods (2003) 368–372; R. Bonifacio, F. Casagrande, Nucl. Instrum. Methods A 237 (1985) 168; Opt. Commun. 50 (1984) 251; E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Opt. Commun. 281 (2008) 1179–1188.

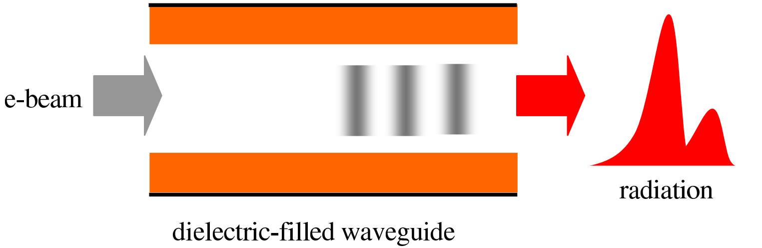
SASE FEL[2]



The increment of electron beam instability is proportional to $\sqrt[3]{\rho}$. Here, ρ is the electron beam density.

[2] A.M. Kondratenko, E.L. Saldin, Part. Accel. 1980. V. 10. P. 207–216; R. Bonifacio, C. Pellegrini, L. Narducci, Opt. Commun. 1984. V. 50. P. 373–378.

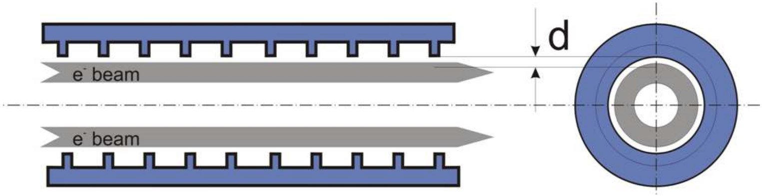
Cherenkov generators[3]



In the Cherenkov generators, the increment of electron beam instability is proportional to $\sqrt[3]{\rho}$.

[3] B.W.J. McNeil, G.R.M. Robb, and D.A. Jaroszynsky, Opt. Commun. 1999. V. 163. P. 203–207; S.M. Wiggins et al.,Phys. Rev. Lett. 2000. V. 84. N 1. P. 2393–2396.

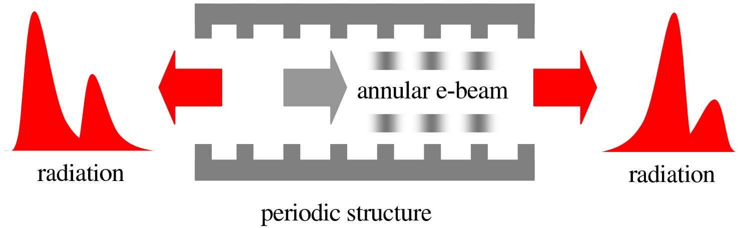
Cherenkov generators with periodic structures[4]



When an electron passes near the surface of the diffraction grating at the distance less then $d < \frac{\lambda\beta\gamma}{4\pi}$, it effectively excites an electromagnetic wave.
In TWT and BWO regimes, the increment of electron beam instability is proportional to $\sqrt[3]{\rho}$.

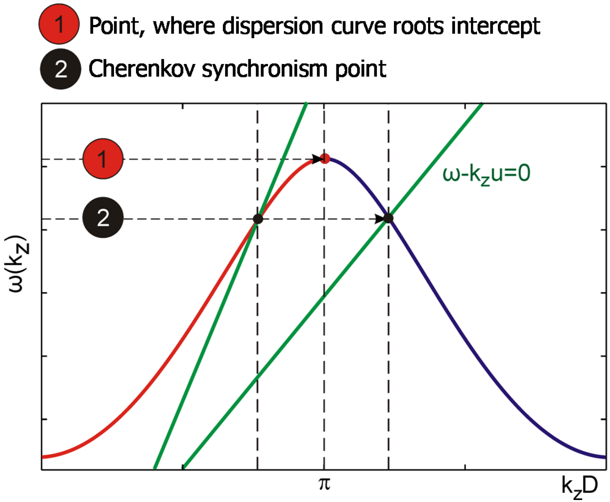
[4] A.A. Elchaninov, et al., JETP Lett. 2003. V. 77. N 6. P. 266–269; S.D. Korovin et al., Phys. Rev. E. 2006. V. 74. P. 016501.

Two waves

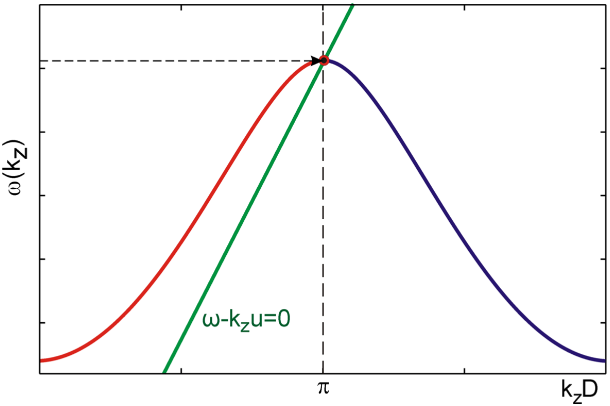


As a result of diffraction, two waves are excited in one dimensional periodic structure.

Cherenkov synchronism



Interception of roots

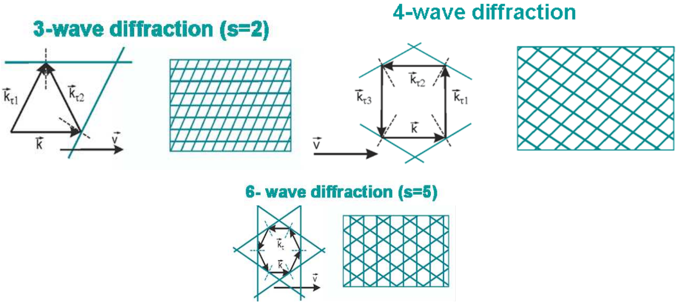


New law of instability[5]

- The electron-beam instability increment is proportional to $\sqrt[4]{\rho}$
- It is possible to create X-ray free electron laser at $j = 10^8 \mathrm{A/cm}^2$ instead of $j = 10^{13} \mathrm{A/cm}^2$ (G. Kurizki, M. Strauss, I. Oreg, N. Rostoker, Phys. Rev. A35 (1987) 3427)
- The concept of volume free electron laser operating in different spectral ranges (microwave,teraherz, optical,x-ray) emerged.

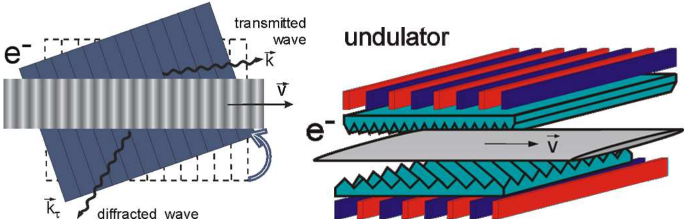
[5] V. Baryshevsky, I. Feranchuk, Phys. Lett. 102A. 1984. P. 141; V.G. Baryshevsky, Dokl. Akad. Nauk SSSR 299 (1988) 1363.

Multiwave diffraction



The increment of electron beam instability relates to the electron density ρ as $\sqrt[3+s]{\rho}$, where s is the number of extra waves produced through diffraction. The interaction length could be much shorter!!!

Frequency tuning[6]



Rotation of the diffraction grating allows VFEL lasing frequency to be tuned

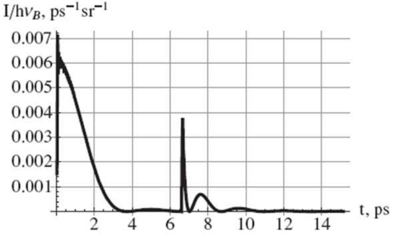
[6]Gurinovich, I. Ilienko, A. Lobko, V. Moroz, P. Sofronov, and V. Stolyarsky, NIM A 483 (2002) 21

VFEL experimental history[7]

- 2001
  First lasing of volume free electron laser in mm-wavelength range. Demonstration of validity of VFEL principles and possibility for frequency tuning at constant electron energy
- 2004—2015
  VFEL prototype with volume photonic crystal made from metallic threads and foils

[7] V.G. Baryshevsky, K.G. Batrakov, A.A. Gurinovich, I. Ilienko, A. Lobko, V. Moroz, P. Sofronov, and V. Stolyarsky, NIM A 483 (2002) 21;V.G. Baryshevsky, A.A. Gurinovich, NIM 252B (2006) 91; Free electron laser conference, FEL2006, FEL2007, FEL2008, FEL2009, FEL2010; Sher Alam, M. O. Rahman, C. Bentley, and M. Ando, *Proceeding of the Second Asian Particle Accelerator Conference*, THP069 277–280, Beijing, China, 15 September 2001.

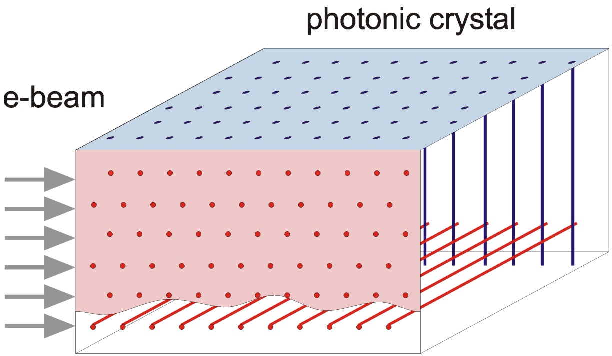
Spontaneous quasi-Cherenkov (parametric) radiation[8]



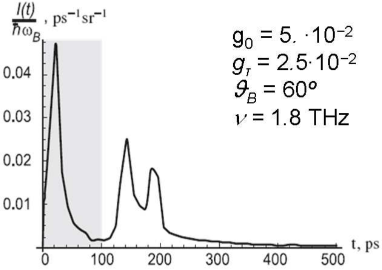
Parametric X-ray radiation in a LiF crystal.
$[L = 1\ \mathrm{mm},\ \theta = 1/\gamma = 5 \cdot 10^{-5}\ \mathrm{rad},\ \beta_1 = -1,\ \theta_B = 80^o]$.

[8] V.G. Baryshevsky, A.A. Gurinovich, NIM 252B (2006) 91; S.V. Anishchenko, V.G. Baryshevsky, A.A. Gurinovich, Journal of Nanophotonics 6 (2012) 061714

Diffraction grating



Spontaneous emission. T-rays



Calculated dependency of the quasi-Cherenkov radiation intensity on time in case of two-beam diffraction in the photonic crystal with period $d = 0.17$ mm, thickness $L = 3$ cm, and elcetron beam Lorenz-factor $\gamma = 2 \cdot 10^3$

Coherent spontaneous emission[9]. T-rays

Estimations for SwissFEL

- Frequency: $\nu = 1$ THz
- Photon per particle (spontaneous emission): 10^{-3}
- Charge per bunch: $N_e = 1.25 \cdot 10^9$ electrons
- Electron energy: $E_e = 5.8$ GeV
- Bunch length: $t_b = 30$ fs (rms)

Output parameters

- Photon number: $N_{ph} = 10^{-3} N_e^2 = 1.6 \cdot 10^{15}$
- Energy per pulse: $N_{ph} h\nu = 1$ mJ
- Crystal thickness: $L = 10$ cm!!!

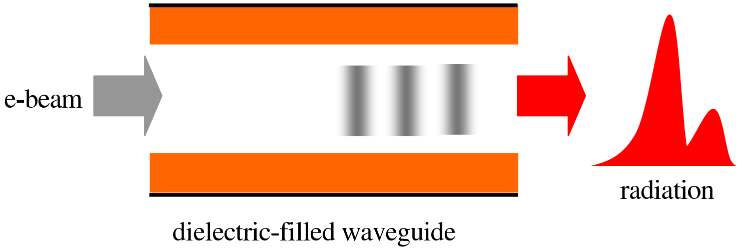
[9] V.G. Baryshevsky, A.A. Gurinovich, Nucl. Instrum. Methods B. 355. P. 69–75.

Fluctuations of Cherenkov and quasi-Cherenkov Superradiance [10]

- Spontaneous emission
- Coherent spontaneous emission
- Induced radiation
- **Cooperative radiation (superradiance)** $(\lambda_{rad} \ll L_{front} \ll L_{bunch} \ll L_{crystal})$

[10] S.V. Anishchenko, V.G. Baryshevsky, arXiv: 1605.04331v1 (2016); S.V. Anishchenko, V.G. Baryshevsky, Nucl. Instrum. Methods B. 2015. Vol. 355. P. 76–80

Cherenkov superradiance. Statement of the problem



The spread of peak power— ? Phasing time — ?

Nonlinear theory of Cherenkov superradiance

Input parameters

- Dimensionless bunch length $\xi = C\beta L \frac{\sqrt[3]{v_0^2 v_{gr}}}{v_0 - v_{gr}}$, where $C = \left(\frac{eIZ}{2mc^2\gamma_0^3}\right)^{1/3}$ — the Pierce parameter
- Nonlinear coefficient $\nu = 2C\gamma_0^2 \sqrt[3]{\frac{v_{gr}}{v_0}}$
- Energy spread $\sigma = \frac{C\Delta\gamma_\alpha}{\gamma_0^3} \sqrt[3]{\frac{v_0}{v_{gr}}}$
- Electron number N_e

Output parameters

- Conversion ratio $\eta = \frac{v_{gr}}{v_0} P_0 = \frac{v_{gr}}{v_0} \frac{\nu |F_{peak}|^2}{8}\Big|_{z=0}$, where $F = \frac{eE_0}{\gamma_0^3 mc^2 \beta C^2} \frac{\sqrt[3]{v_0^2 v_{gr}}}{v_{gr}}$ — dimensionless field strength
- Dimensionless phasing time $T_0 = C\beta t_0 \sqrt[3]{v_0^2 v_{gr}}$

Nonlinear theory of Cherenkov superradiance

- Particles' equations

$$\frac{d^2\theta_\alpha}{d\tau^2} = -\left(1 + \nu\frac{d\theta}{d\tau}\right)^{3/2}\mathrm{Re}(Fe^{i\theta_\alpha}) \qquad (1)$$

- Initial conditions

$$\frac{d\theta_\alpha}{d\tau}|_{\tau=0} = 0 \qquad (2)$$

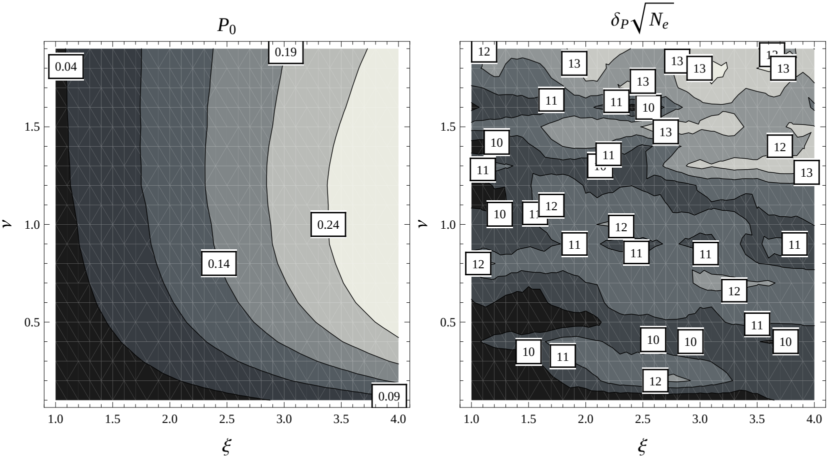
- Field equations

$$\frac{\partial F}{\partial \tau} - \frac{\partial F}{\partial z} = \frac{2}{N_l}\sum e^{-i\theta_\alpha} \qquad (3)$$

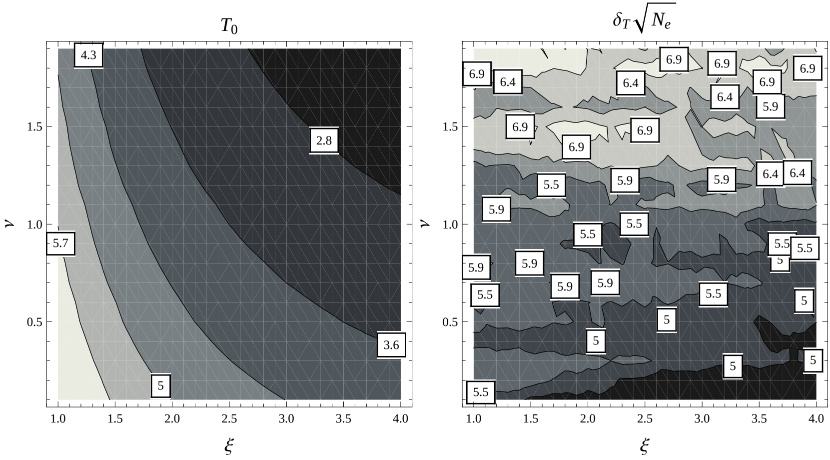
- Boundary conditions

$$F(\xi, \tau) = 0. \qquad (4)$$

Conversion coefficient: Shot noise



Phasing time: Shot noise

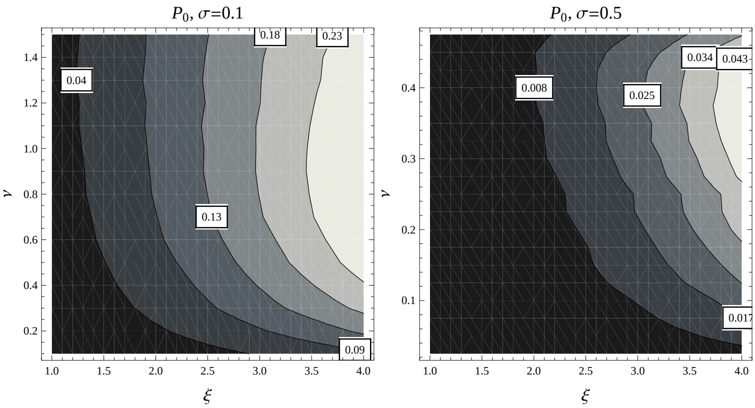


Estimations ($\nu = 1.0$)

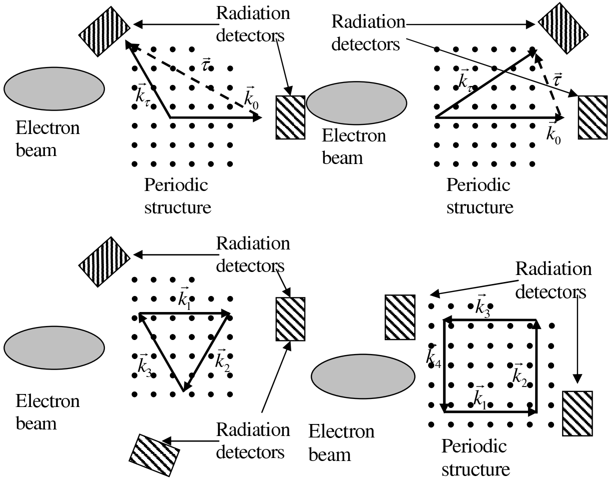
for SwissFEL

- Electrons per bunch:
  $N_e = 1.25 \cdot 10^9$
- Spread of peak power:
  $\delta_P \sim 11/\sqrt{N_e} = 3.1 \cdot 10^{-4}$
- Spread of phasing time:
  $\delta_T \sim 6/\sqrt{N_e} = 1.7 \cdot 10^{-4}$

Conversion coefficient: Energy spread

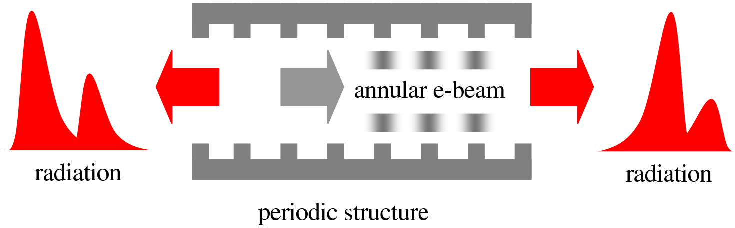


Quasi-Cherenkov radiation[11]: Diffraction geometries



[11] Anishchenko S.V., Baryshevsky V.G. Nucl. Instrum. Methods B, doi: 10.1016/j.nimb.2015.03.054

Quasi-Cherenkov superradiance. Statement of the problem



The spread of peak power— ?

Nonlinear theory of quasi-Cherenkov superradiance

- Particles' equations

$$\frac{d^2\theta_\alpha}{d\tau^2} = -\left(1+\nu\frac{d\theta_\alpha}{d\tau}\right)^{3/2}\mathrm{Re}(F_0 e^{i\theta_\alpha}). \tag{5}$$

- Initial conditions

$$\frac{d\theta_\alpha}{d\tau}|_{\tau=0} = 0 \tag{6}$$

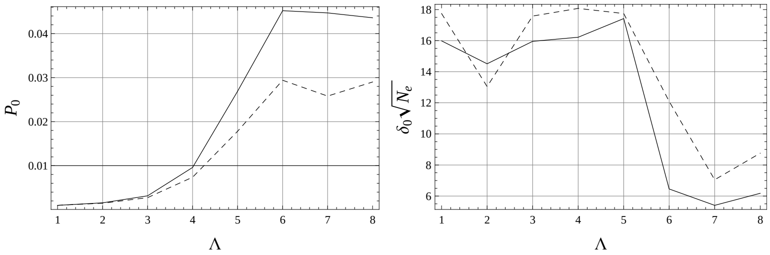
- Field equations

$$\begin{aligned}\frac{\partial F_0}{\partial \tau} + \frac{\partial F_0}{\partial z} + i\chi F_\tau &= -\frac{2}{N_\lambda}\sum_\alpha e^{-i\theta_\alpha},\\ \frac{\partial F_\tau}{\partial \tau} - \frac{\partial F_\tau}{\partial z} + i\chi F_0 &= 0.\end{aligned} \tag{7}$$

- Boundary conditions

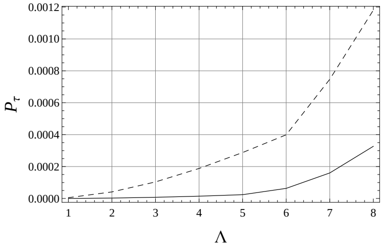
$$\begin{aligned}F_0(0,\tau) &= 0,\\ F_\tau(\Lambda,\tau) &= 0.\end{aligned} \tag{8}$$

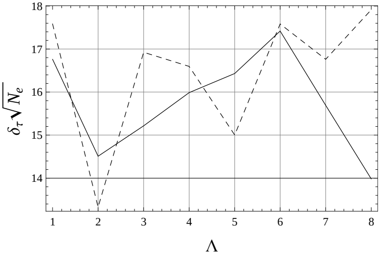
Quasi-Cherenkov superradiance in forward direction



Solid curve — $\chi = 0.1$, dashed curve — $\chi = 0.4$ [$\nu = 1.0$ and $\xi = 1.0$].

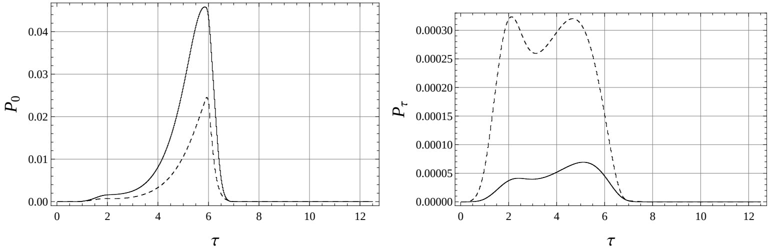
Quasi-Cherenkov superradiance in backward direction





Solid curve — $\chi = 0.1$, dashed curve — $\chi = 0.4$ [$\nu = 1.0$ and $\xi = 1.0$].

Quasi-Cherenkov superradiance at saturation



Solid curve — $\chi = 0.1$, dashed curve — $\chi = 0.4$ [$\nu = 1.0$ and $\xi = 1.0$].

Estimations

Input parameters

- Electrons per bunch: $N_e = 10^9$
- Beam power: $P_b = 20$ GW
- $\nu = 1.0$
- $\xi = 1$
- $\Lambda = 6$

Output parameters

- Radiation power: $P_{rad} = 1$ GW
- $\delta_P = 1.9 \cdot 10^{-4}$

Conclusions

- A detailed numerical analysis is given for cooperative Cherenkov and quasi-Cherenkov radiation in the presense of shot noise and energy spread.
- Using femtosecond electron bunches it is possible to create quasi-Cherenkov THz radiation source with 1 mJ energy per pulse
- Typical relative rms deviation of radiated power and phasing time in superradiant Cherenkov generators with femtosecond electron bunches reach $\delta_{T,P} \sim 6/\sqrt{N_e} \sim 2 \cdot 10^{-4}$ $(N_e \sim 10^9)$.

Thank you for the attention!

Quasi-cherenkov radiation: nonlinear theory

- Particles' equations

$$\frac{dp_{z\alpha}}{dt} = 2\theta Re\Big(E_0 e^{-i(t-k_{0z}z_\alpha+\phi_\alpha)}\Big),$$

- Field equations

$$\frac{\partial E_0}{\partial t} + \gamma_0 \frac{\partial E_0}{\partial z} + \frac{i\chi_0}{2}E_0 + \frac{i\chi_\tau}{2}E_\tau = -\sum_j \frac{\theta\chi_{b\alpha}}{2}\frac{e^{i(t-k_{0z}z_\alpha+\phi_\alpha)}}{N_l},$$

$$\frac{\partial E_\tau}{\partial t} + \gamma_\tau \frac{\partial E_\tau}{\partial z} + \frac{i\chi_0}{2}E_\tau + \frac{i\chi_\tau}{2}E_0 = 0.$$

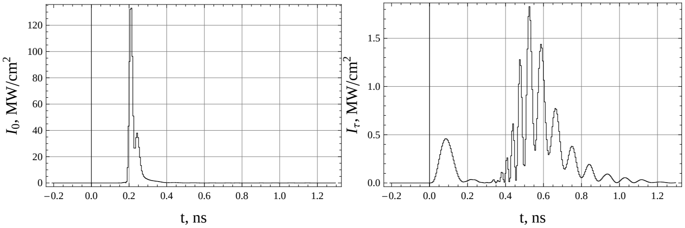
Quasi-cherenkov radiation: parameters

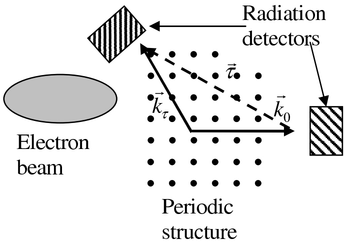
- Crystal length $L_c = 6$ cm
- Bunch length $L_b = 0.6$ cm
- Relativistic factor $\gamma = 3.0$
- Dielectric susceptibilities $\chi_0 = 0.1$, $\chi_\tau = 0.05$
- Current density $j = 10.0$ kA/cm^2
- Frequency $\frac{\omega}{2\pi}$=0.1 THz

Current densities

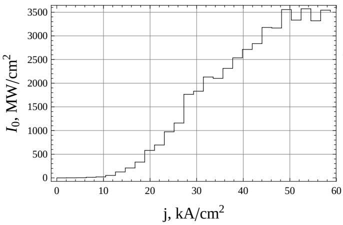
- Limitations
  $jt_{pulse} < 4.5 \cdot 10^{-5}$ C/cm^2
- Estimations
  $j = 30$ kA/cm^2, $t_{pulse} = 1$ ns.
  $jt_{pulse} < 3.0 \cdot 10^{-5}$ C/cm^2

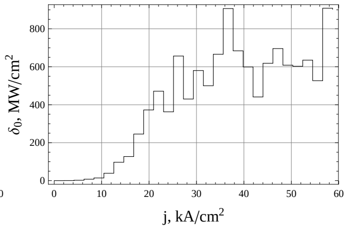
The Bragg case

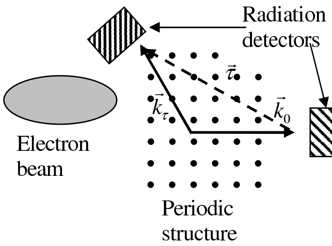




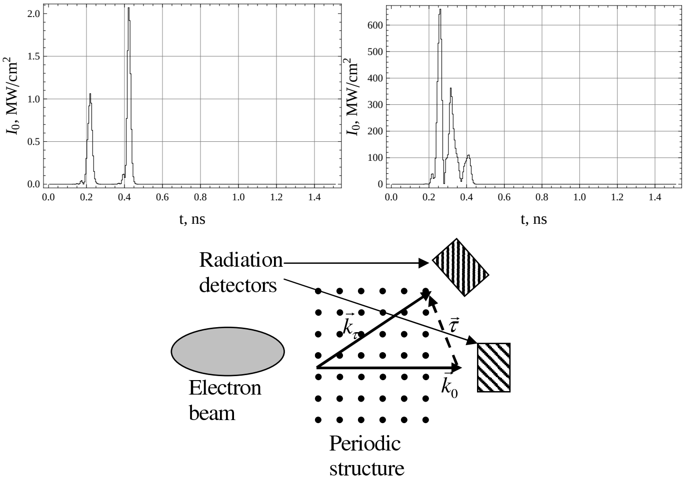
Fluctuations (the Bragg case)



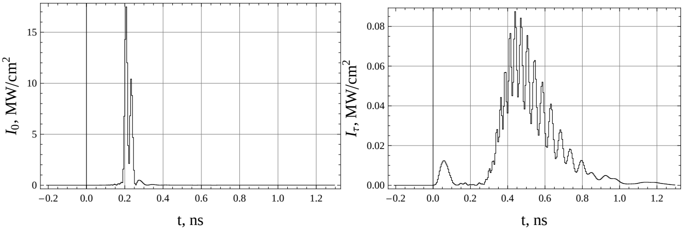


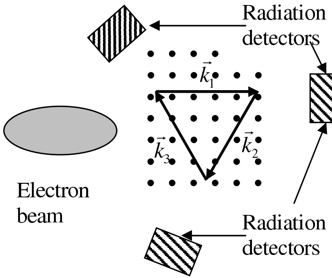


Shot noise (the Laue case)

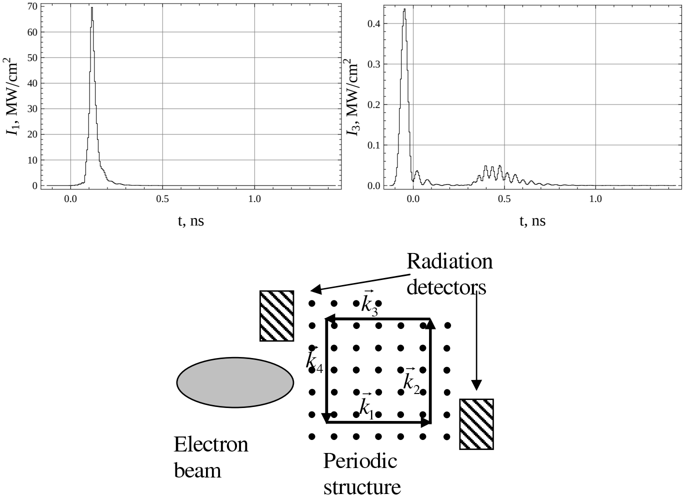


Three-wave diffraction





Four-wave diffraction



Current densities

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- Estimations
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  $jt_{pulse} < 3.0 \cdot 10^{-5}$ C/cm^2