Theoretical Studies of Hadronic Reactions with Vector Mesons

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HEF/KF seminar, 4th of May

PhD defence: 19th of May, 10:15, Polhemsalen

Content

- Introduction to effective field theories for pseudoscalar and vector mesons
- **Part I:** Formulation of an effective theory and first tree-level tests
- **Part II:** Reactions with an odd number of pions
- **Part III:** Beyond-tree-level calculations
- Summary and outlook

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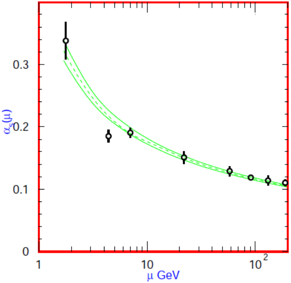

running coupling constant in QCD
$\hookrightarrow$ perturbation theory not applicable for low energies

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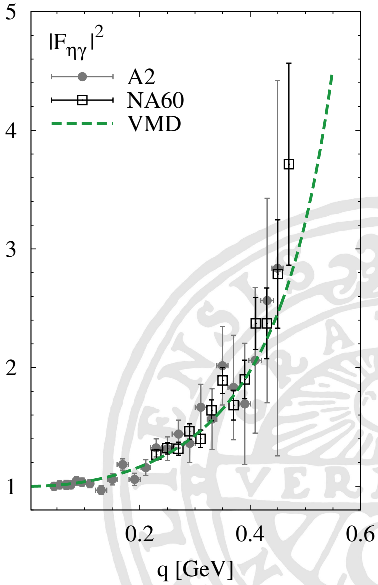

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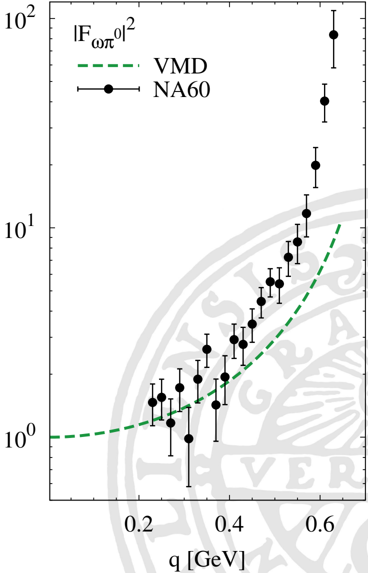
- systematic improvements are not possible
- can be successful anyway
- VMD: good agreement with data for some reactions, but fails for other reactions



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  $\hookrightarrow$ scale = region where EFT should be valid
- scale of an EFT should be separated from energy regions with particles not included in the EFT
- comparing scales $\rightarrow$ power counting scheme can be formulated
- degrees of freedom of an EFT might be different form the ones of the underlying theory

  $\hookrightarrow$ only degrees of freedom relevant at the scale of interest are used

  Example: rocket flying to the moon described as point-like object

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Formulating an EFT = identification of 3 parts:
scales, relevant degrees of freedom,
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Advantage compared to phenomenological models:

EFT is a systematic theory

$\hookrightarrow$ knowledge about theory-intrinsic errors

$\hookrightarrow$ possibility for systematic improvements

Chiral perturbation theory (χPT)

Confinement: quarks cannot be unbound at low energies
$\rightarrow$ bound in hadrons

$\hookrightarrow$ take lightest (pseudoscalar) mesons as relevant DOF: pions, kaons, η-meson (and η'-meson)

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$\Rightarrow$ **Aim:** find EFT for both light pseudoscalar and vector mesons
(ρ-, ω-, K^*-, ϕ-meson)

Towards an EFT for light pseudoscalar and vector mesons

Steps for formulating and testing an approach for an EFT:

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5.) ...

Part I:

Formulation and first tree-level tests

Electromagnetic transitions in an effective chiral Lagrangian with the η' and light vector mesons

C. T., S. Leupold, and M.F.M. Lutz, *Eur. Phys. J., A48:190, 2012*

Basics of our approach

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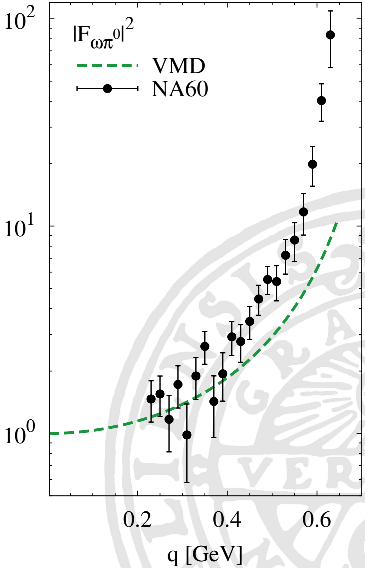
$\Rightarrow$ formulate power counting scheme including large-N_c behaviour already visible for $N_c = 3$ (*e.g.*, suppression of many-body forces)

$\hookrightarrow$ perform tree-level tests

Example: Decay $\omega \to \pi^0\mu^+\mu^-$

Recall: VMD fails to describe the experimental data



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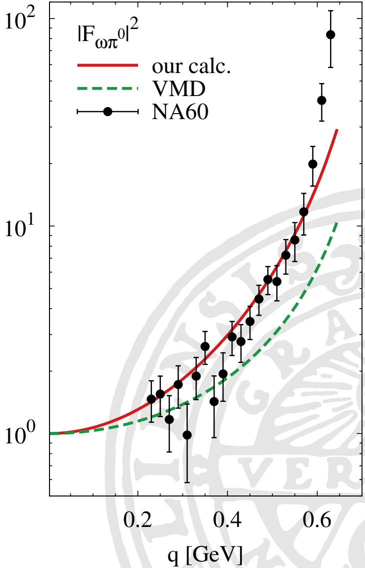

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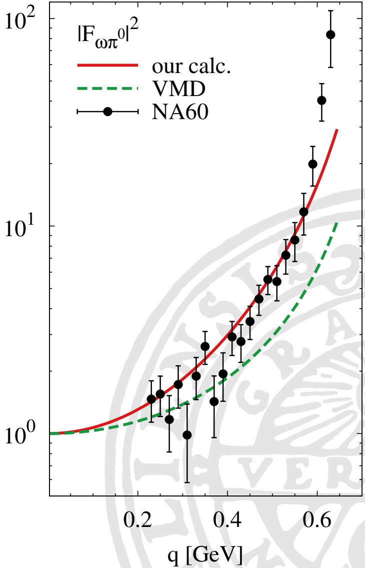

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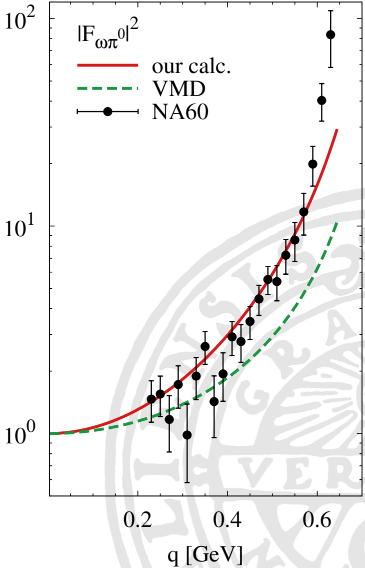
Tree-level results for other decays $V \to Pl^+l^-$ and $P \to Vl^+l^-$:

show fair agreement with available experimental data



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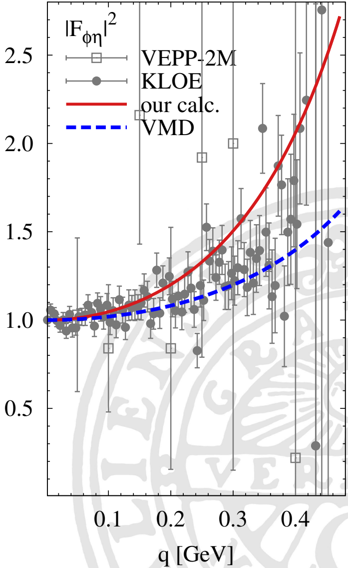

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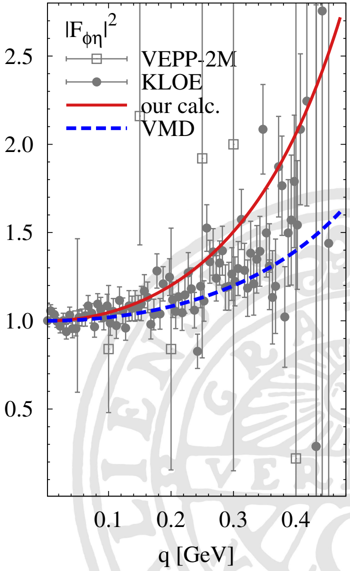
$\hookrightarrow$ **Can an NLO calculation improve our description?**



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Part II:

Reactions with an odd number of pions

Reactions with pions and vector mesons in the sector of odd intrinsic parity

C. T., B. Strandberg, S. Leupold, and F. Eichstädt, *Eur. Phys. J., A49:116, 2013*

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  $\hookrightarrow$ necessary for reactions with contributions from both pure χPT and involving vector mesons

  $\hookrightarrow$ relative signs connect to whether "$\pm$(vector-meson field)" and "$\pm$(pseudoscalar-meson field)" produce mesons

Relative signs between Lagrangians (I)

Consider system of three Lagrangians:

$$\mathcal{L}_1 = c_1 \text{``}PGG\text{''},\ \mathcal{L}_2 = c_2 \text{``}VG\text{''},\ \mathcal{L}_3 = c_3 \text{``}PVV\text{''}$$

with
- pseudoscalar-meson-like particle P,
- photon-like particle G,
- and vector-meson-like particle V.

Relative signs between Lagrangians (II)

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- $|c_2|$ can be determined by comparing results for $V \to G^* \to l^+ l^-$ into a dilepton $l^+ l^-$

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Relative signs between Lagrangians (III)

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- $|c_3|$ can be determined by comparing results for $V \to V'P$
  $\hookrightarrow$ sign relative to c_1 has physical significance for processes with interference between $\mathcal{L}_1$ and $\mathcal{L}_3$
  [*e.g.*, $P \to GG$ and $P \to V_1^* V_2^* \to GG$]

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  $\hookrightarrow$ rephrase question:

  Does $\mathcal{L}_1 + \mathcal{L}_3$ or $\mathcal{L}_1 - \mathcal{L}_3$ describe the right physics?

Relative signs between Lagrangians (IV)

Same problem for interaction parts of Lagrangians for pure χPT and involving vector mesons:

Does $\mathcal{L}^{\text{int}}_{\chi\text{PT}} + \mathcal{L}^{\text{int}}_{\text{vec}}$ or $\mathcal{L}^{\text{int}}_{\chi\text{PT}} - \mathcal{L}^{\text{int}}_{\text{vec}}$ describe the right physics (*or none*)?

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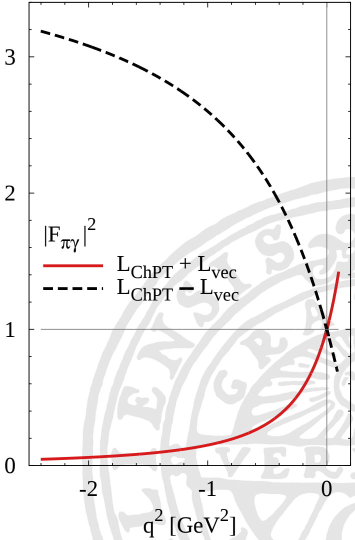

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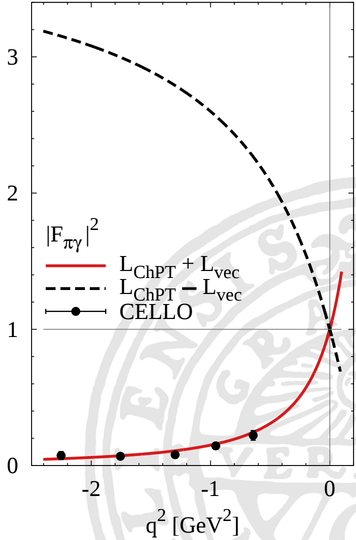

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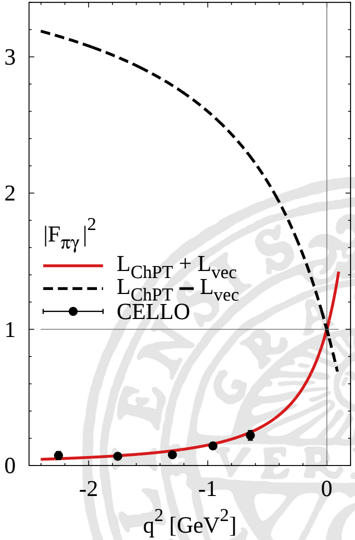
$\Rightarrow$ $\mathcal{L}^{\text{int}}_{\chi\text{PT}} + \mathcal{L}^{\text{int}}_{\text{vec}}$ should be used

$\hookrightarrow$ use result to calculate other reactions at tree level: good agreement with available experimental data



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Part III:

Beyond-tree-level calculations

Renormalisation of the low-energy constants of chiral perturbation theory from loops with dynamical vector mesons

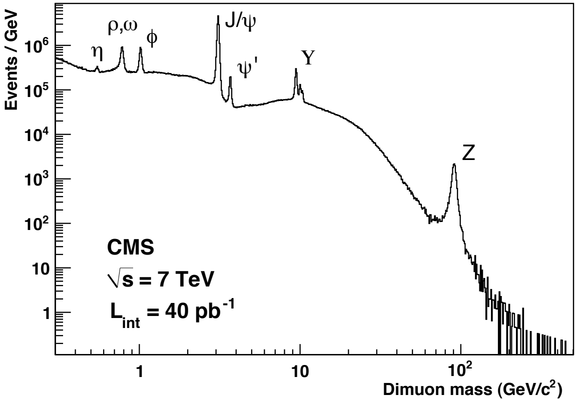
C. T. and S. Leupold, *accepted by Phys. Rev. D, arXiv:1603.05524 [hep-ph]*

Contributions of loops with dynamical vector mesons to masses and decay constants of pseudoscalar mesons and their quark mass dependence

C. T. and S. Leupold, *submitted to Phys. Rev. D, arXiv:1604.01682 [hep-ph]*

Example: Calculation of masses (I)

How can one 'see' masses?



Example:

η-meson production in proton-proton collision

$\Rightarrow$ peak in dimuon ($\mu^+\mu^-$) spectrum

$\hookrightarrow$ peak position $\hat{=}$ mass of η-meson $= 0.548$ GeV$/c^2$

CMS muon results with data from 2010,
https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsMUO#Invariant_mass_spectra_of_opposi

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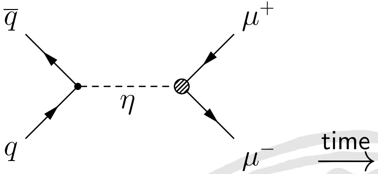

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$\hookrightarrow$ corresponds to bare mass m_{bare},
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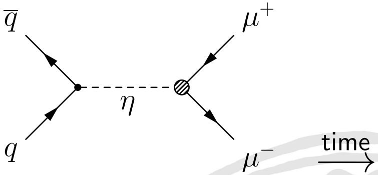

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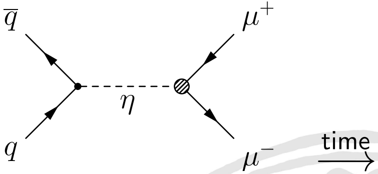
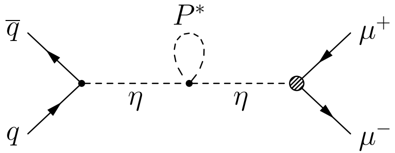


Additionally, one-loop diagrams:



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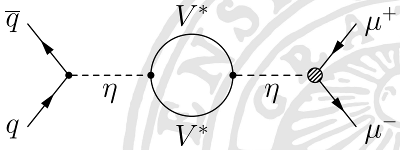

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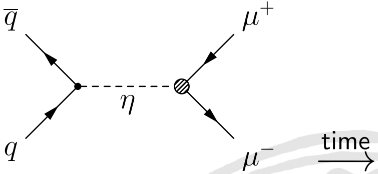

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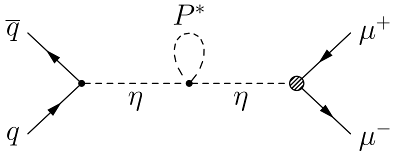

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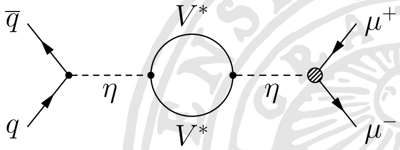

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(and diagrams with more loops).

Example: Calculation of masses (III)

Can use such collisions to define the physical (measured) mass m_η :

$$m_\eta = \underbrace{\begin{pmatrix}\text{contributions from}\\ \text{tree-level diagrams}\end{pmatrix}}_{= m_{\text{bare}}} + \underbrace{\begin{pmatrix}\text{contributions from}\\ \text{loop diagrams}\end{pmatrix}}_{=: m_{\text{loop}}}$$

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Problem: loop contribution is infinite

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m_{bare} just a mathematical construct, not an observable quantity

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Can use the **'effective action'** Z to calculate observables, e.g., masses and cross sections

$\hookrightarrow$ depends on Lagrangian and, thus, all Feynman diagrams

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$\hookrightarrow$ **(1)** need to identify the kind of infinities, give them a well-defined mathematical meaning ('regularisation')

(2) find terms to cancel infinities ('renormalisation')

$\hookrightarrow$ different choices of regularisation and renormalisation possible, but physics is independent of these choices

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Dimensional regularisation: $4+2\varepsilon$ dimensions (ε arbitrary)

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In the following: pion masses as example

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Mass M of a particle is defined as the pole of its propagator Δ: $\boxed{\Delta(p^2 = M^2)^{-1} \equiv 0}$

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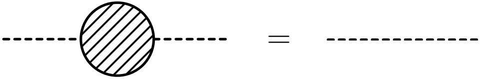

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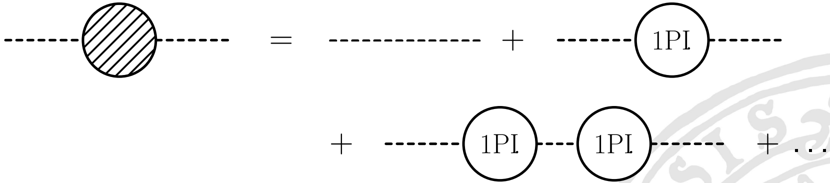


$\hookrightarrow$ **in LO:** $M =: \mathring{M} \sim m_{u/d}^2$

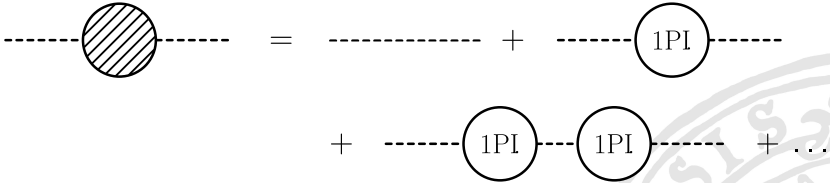
$\hookrightarrow$ **in higher orders:** include one-particle-irreducible (1PI) diagrams

$\rightarrow$ sum over all 1PI diagrams $= -i\,\Sigma$

self energy

Mass calculation in detail (II)

Mass M of a particle is defined as the pole of its propagator Δ: $\boxed{\Delta(p^2 = M^2)^{-1} \equiv 0}$



$\Rightarrow$ propagator can be rewritten as a geometric series yielding

$$[\Delta(p^2)]^{-1} = p^2 - \mathring{M}^2 - \Sigma(p^2) + i0^+$$

$\Rightarrow$ mass given by the mass equation $\boxed{M^2 - \mathring{M}^2 - \Sigma(M^2) = 0}$

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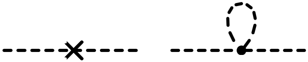
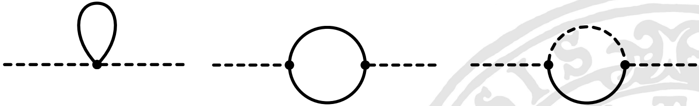
(I) pure χPT: 

(II) loops with vector mesons: 

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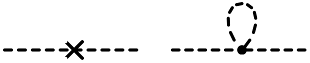

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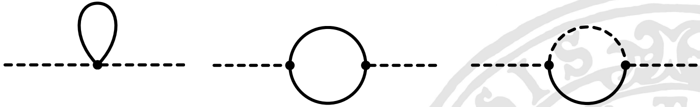

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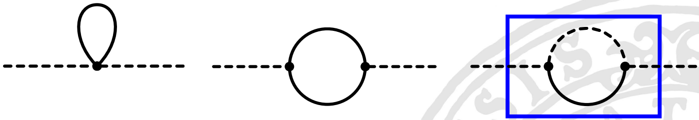

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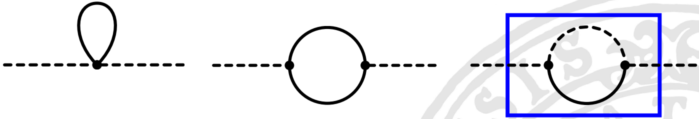

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$\hookrightarrow$ done similarly for decay constants

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First comparison for pion masses (I)

Compare calculation with full and point-like vector-meson propagator:

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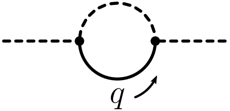
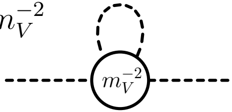


$\Leftrightarrow$

m_V^{-2}



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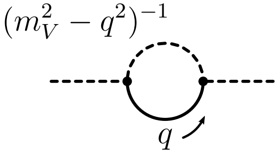

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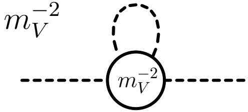

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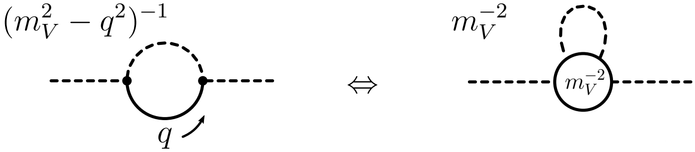

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$\hookrightarrow$ define ΔI and ΔT as normalised contribution to the pion mass for full and point-like propagator

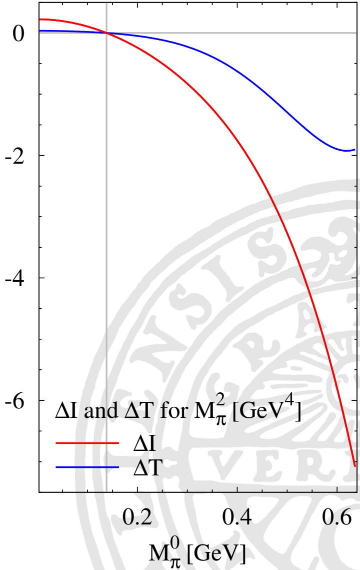
- as a function of the LO pion mass $\mathring{M}_\pi$
- in comparison to a reference point: $\Delta T(M_\pi^{\text{exp}}) = \Delta I(M_\pi^{\text{exp}}) = 0$

First comparison for pion masses (II)

$\Delta I \hat{=}$ full propagator,
$\Delta T \hat{=}$ point-like prop.

$\hookrightarrow$ differences are clearly visible for $\overset{\circ}{M}_\pi \gtrsim 200\,\text{MeV}$

$\hookrightarrow$ **indicates:** important to include vector mesons as active degrees of freedom



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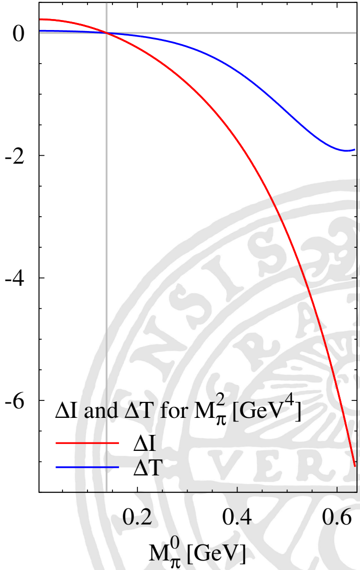
$\hookrightarrow$ similar for masses and decay constants of (other) pseudo-scalar mesons



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$\hookrightarrow$ we adjust parameters such that there is no difference between the two calculation at chiral order Q^4:

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$\hookrightarrow$ how important are these higher order contributions?

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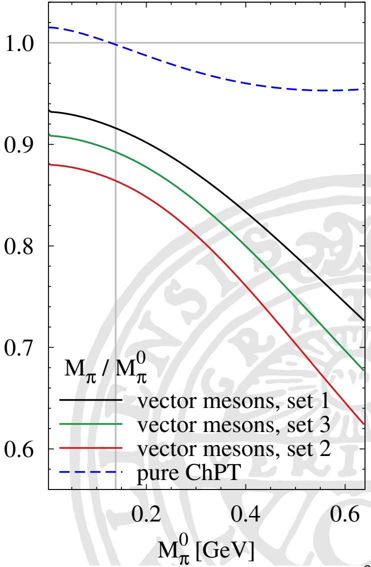

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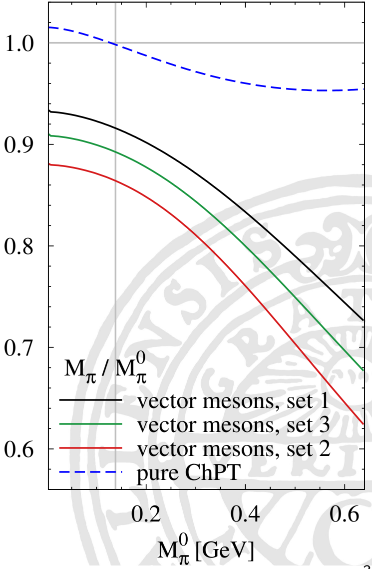

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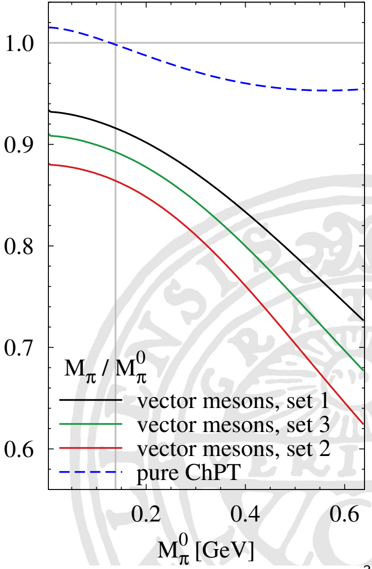

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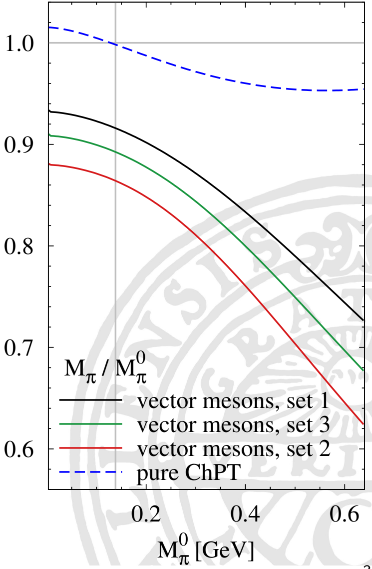

[similar for other masses/decay const.]

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  $\hookrightarrow$ for reactions which also have contributions from pure χPT
  $\Rightarrow$ justifies further investigations into this approach
- feasibility study and calculations for pseudoscalar properties were preformed at one-loop level including vector mesons
  $\hookrightarrow$ Lagrangian with reduced number of interaction terms used
  $\hookrightarrow$ results indicate importance of including vector mesons as active degrees of freedom
  $\Rightarrow$ provide foundations for calculations beyond tree level

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