THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Pion photo- and electroproduction and the chiral MAID interface

Stefan Scherer

Institute for Nuclear Physics, Johannes Gutenberg University Mainz

Uppsala University, 20 May 2016

work in collaboration with[1]

Marius Hilt, Björn C. Lehnhart, Lothar Tiator

[1] Phys. Rev. C **87**, 045204 (2013), Phys. Rev. C **88**, 055207 (2013)

1. **Introduction**

2. **Renormalization and power counting**

3. **Application to pion photo- and electroproduction**

4. **Summary and outlook**

1. Introduction

Effective field theory

> ... if one writes down the **most general possible Lagrangian**, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian **to any given order of perturbation theory**, the result will simply be the most general possible S−matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. ... [2]

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[2] S. Weinberg, Physica A **96**, 327 (1979)

... if we include in the Lagrangian all of the infinite number of interactions allowed by symmetries, then there will be a counterterm available to cancel every ultraviolet divergence. ... [3]

[3] S. Weinberg, *The Quantum Theory of Fields*, Vol. I, 1995, Chap. 12

	Fundamental theory	Effective field theory
	QCD	ChPT
dof	quarks & gluons	Goldstone bosons (+ other hadrons)
parameters	g_3 + quark masses	(∞ # of) LECs + quark masses

Simplified analogies between multipole expansion and EFT

Multipole expansion	EFT
$\vert\vec{x}\,\vert \gg R$	$q \ll \Lambda_\chi$
$\phi(\vec{x}) = \sum_{lm} \boxed{q_{lm}} \boxed{\frac{1}{2l+1}\frac{Y_{lm}(\theta,\phi)}{r^{l+1}}}$	$\mathcal{L}_{\text{eff}} = \sum_{lm} \boxed{c_{lm}} \boxed{\mathcal{L}_{lm}}$
multipole moment q_{lm}	LEC c_{lm}
$\frac{1}{2l+1}\frac{Y_{lm}(\theta,\phi)}{r^{l+1}}$	Structures $\mathcal{L}_{lm}$

- In principle, infinite number of terms.
  Actual calculation: Truncation at finite order.

- Systematic improvement possible.

Perturbative calculations in effective field theory require two main ingredients

1. **Knowledge of the most general effective Lagrangian**

   (a) **Mesonic ChPT [SU(3)×SU(3)]** [4] (π, K, η)

$$\underbrace{2}_{\mathcal{O}(q^2)} + \underbrace{10+2}_{\mathcal{O}(q^4)} + \underbrace{90+4+23}_{\mathcal{O}(q^6)} + \cdots$$

   - q: **Small quantity such as a pion mass**
   - **Even powers**
   - **Two-loop level**

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[4] Gasser, Leutwyler (1985), Fearing, Scherer (1996), Bijnens, Colangelo, Ecker (1999), Ebertshäuser, Fearing, Scherer (2002) Bijnens, Girlanda, Talavera (2002)

(b) Baryonic ChPT [SU(2)×SU(2)×U(1)] [5] (π, N)

$$\underbrace{\mathbf{2}}_{\mathcal{O}(q)} + \underbrace{\mathbf{7}}_{\mathcal{O}(q^2)} + \underbrace{\mathbf{23}}_{\mathcal{O}(q^3)} + \underbrace{\mathbf{118}}_{\mathcal{O}(q^4)} + \quad \ldots$$

- **Odd and even powers (spin)**

- **One-loop level**

Each term comes with an independent low-energy constant (LEC)

Lowest-order Lagrangians: F, $M^2 = 2B\hat{m}$, m, g_A
Higher-order Lagrangians: l_i, c_i, d_i, e_i, ...

[5] Gasser, Sainio, Švarc (1988), Bernard, Kaiser, Meißner (1995), Ecker, Mojžiš (1996), Fettes, Meißner, Mojžiš, Steininger (2000)

2. Consistent expansion scheme for observables

   (a) Tree-level diagrams, loop diagrams $\rightsquigarrow$ ultraviolet divergences, regularization (of infinities)

   (b) Renormalization condition

   (c) Power counting scheme for renormalized diagrams

   (d) Remove regularization

ChPT: Momentum and quark mass expansion at fixed ratio m_{quark}/q^2 [6]

[6] J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984)

2. Renormalization and power counting

- **Most general Lagrangian**

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi + \mathcal{L}_{\pi N} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \ldots$$

Basic Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\gamma_\mu\partial^\mu - \boxed{m}\right)\Psi - \frac{1}{2}\frac{\boxed{\mathrm{g}_A}}{F}\bar{\Psi}\gamma_\mu\gamma_5\tau^a\partial^\mu\pi^a\Psi + \cdots$$

m, g_A, and F denote the chiral limit of the physical nucleon mass, the axial-vector coupling constant, and the pion-decay constant, respectively

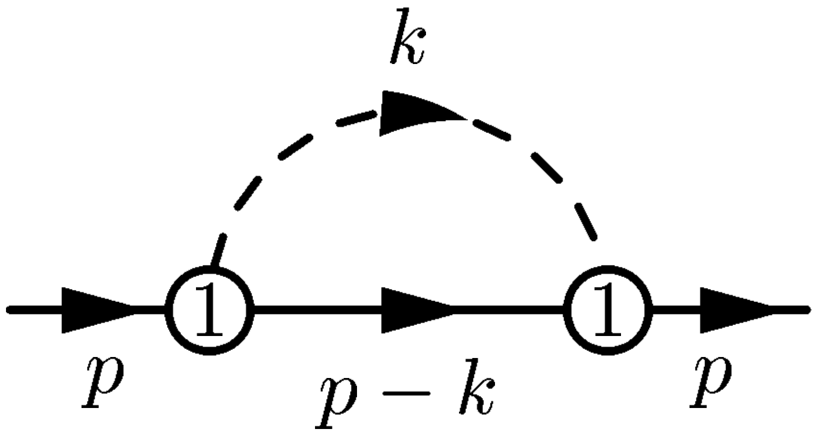
- **Power counting:** Associate chiral order D with a diagram
  - Square of the lowest-order pion mass:
    $M^2 = B(m_u + m_d) \sim \mathcal{O}(q^2)$
  - Nucleon mass in the chiral limit $m \sim \mathcal{O}(q^0)$
  - Loop integration in n dimensions $\sim \mathcal{O}(q^n)$
  - Vertex from $\mathcal{L}_\pi^{(2k)} \sim \mathcal{O}(q^{2k})$
  - Vertex from $\mathcal{L}_{\pi N}^{(k)} \sim \mathcal{O}(q^k)$
  - Nucleon propagator $\sim \mathcal{O}(q^{-1})$
  - Pion propagator $\sim \mathcal{O}(q^{-2})$

- Renormalization
  - Regularize (typically dimensional regularization)

$$
\begin{aligned}
I(M^2,\mu^2,n) &= \mu^{4-n}\int \frac{d^nk}{(2\pi)^n}\frac{i}{k^2-M^2+i0^+} \\
&= \frac{M^2}{16\pi^2}\left[R+\ln\left(\frac{M^2}{\mu^2}\right)\right]+O(n-4), \\
\boxed{\mathrm{R}} &= \frac{2}{n-4}-[\ln(4\pi)+\Gamma'(1)]-1 \to \boxed{\infty}
\end{aligned}
$$

  - Adjust counterterms such that they absorb all the divergences occurring in the calculation of loop diagrams
  - Renormalization prescription: Adjust finite pieces such that renormalized diagrams satisfy a given power counting

- **Example: Contribution to nucleon mass**



Goal: $D = n \cdot 1 - 2 \cdot 1 - 1 \cdot 1 + 2 \cdot 1 = n - 1$

$$\Sigma = -\frac{3g_{A0}^2}{4F_0^2}\left[(\not{p}+m)I_N + M^2(\not{p}+m)I_{N\pi}(-p,0) + \cdots\right]$$

Apply $\widetilde{\mathrm{MS}}$ renormalization scheme

$$\Sigma_r = -\frac{3g_{Ar}^2}{4F_r^2}[M^2(\not{p}+m) \underbrace{I_{N\pi}^r(-p,0)}_{= -\frac{1}{16\pi^2}+\ldots} + \ldots] = \boxed{\mathcal{O}(q^2)}$$

GSS [7]: It turns out that loops have a much more complicated low-energy structure if baryons are included. Because the nucleon mass m_N does not vanish in the chiral limit, the mass scale m (nucleon mass in the chiral limit) occurs in the effective Lagrangian $\mathcal{L}_{\pi N}^{(1)}$ .... This complicates life a lot.

[7] J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. **B307**, 779 (1988)

One possible solution: Extended on-mass-shell (EOMS) scheme[8]

Main idea: Perform additional subtractions such that renormalized diagrams satisfy the power counting

Motivation for this approach[9]

Terms violating the power counting are analytic in small quantities (and can thus be absorbed in a renormalization of counterterms)

- Example (chiral limit)

$$H(p^2, m^2; n) = -\int \frac{d^n k}{(2\pi)^n} \frac{i}{[(k-p)^2 - m^2 + i0^+][k^2 + i0^+]}$$

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[8] T. Fuchs, J. Gegelia, G. Japaridze, S. Scherer, Phys. Rev. D **68**, 056005 (2003)

[9] J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999)

Small quantity

$$\Delta = \frac{p^2 - m^2}{m^2} = \mathcal{O}(q)$$

We want the renormalized integral to be of order

$$D = n - 1 - 2 = n - 3$$

Result of integration

$$H \sim F(n, \Delta) + \Delta^{n-3} G(n, \Delta)$$

- F and G are hypergeometric functions
- analytic in Δ for arbitrary n

Observation[10]

F **corresponds to first expanding the integrand in small quantities and then performing the integration**

$\Rightarrow$ **Algorithm: Expand integrand in small quantities and subtract those (integrated) terms whose order is smaller than suggested by the power counting**

[10] J. Gegelia, G. Japaridze, K. S. Turashvili, Theor. Math. Phys. **101**, 1313 (1994)

Here:

$$H^{\mathrm{subtr}} = -\int \frac{d^n k}{(2\pi)^n} \frac{i}{(k^2 - 2k\cdot p + i0^+)(k^2 + i0^+)}\bigg|_{p^2=m^2}$$

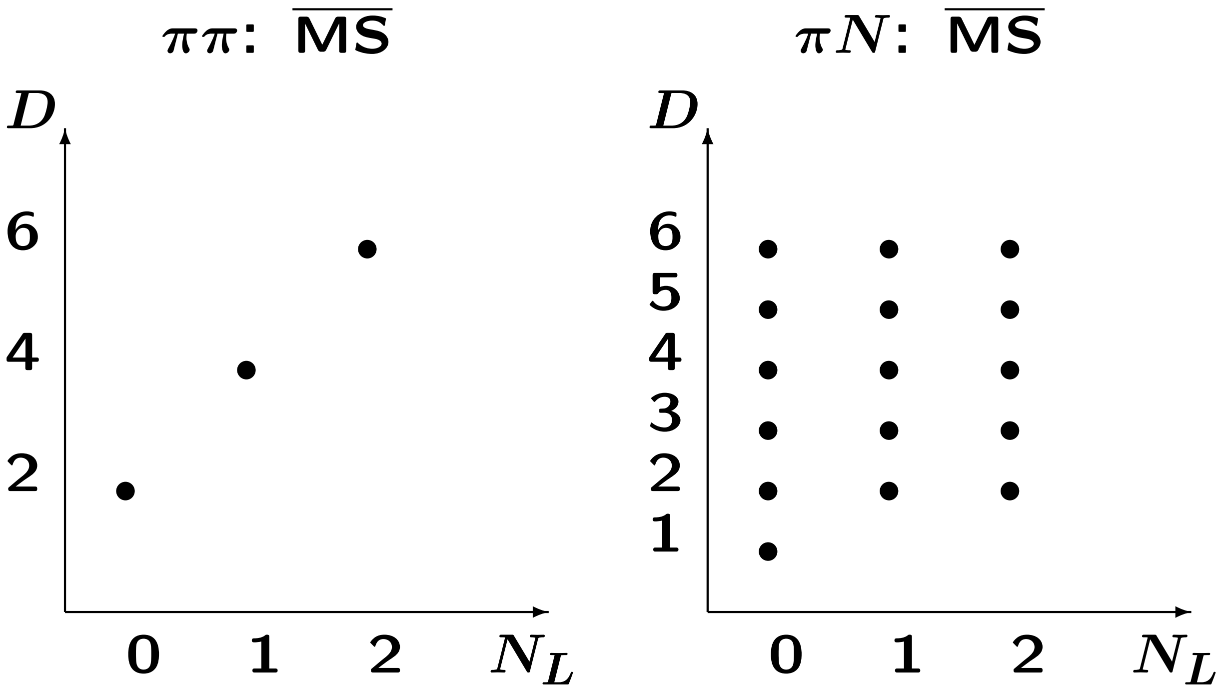
$$= -2\bar{\lambda} + \frac{1}{16\pi^2} + O(n-4)$$

where

$$\bar{\lambda} = \frac{m^{n-4}}{(4\pi)^2}\left\{\frac{1}{n-4} - \frac{1}{2}\left[\ln(4\pi) + \Gamma'(1) + 1\right]\right\}$$

$$H^R = H - H^{\mathrm{subtr}} = \mathcal{O}(q^{n-3})$$

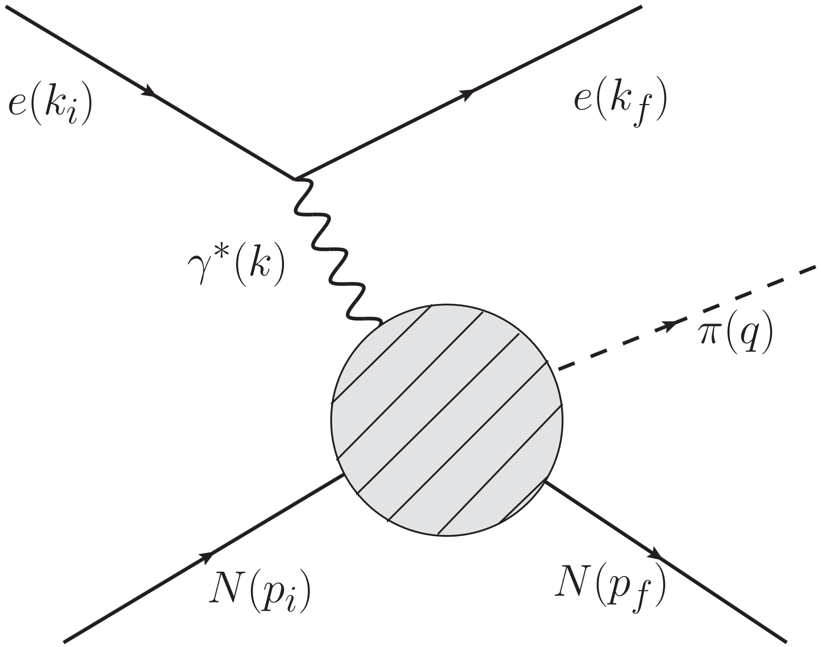
Chiral versus loop expansion



3. Application to pion photo- and electroproduction

$$e(k_i) + N(p_i) \to e(k_f) + N(p_f) + \pi(q)$$

One-photon-exchange approximation



Invariant amplitude

$$\mathcal{M} = \text{leptonic vertex} \times i\,\text{propagator} \times \text{hadronic vertex} = \epsilon_\mu \mathcal{M}^\mu$$

$$\epsilon_\mu = e\frac{\bar{u}(k_f)\gamma_\mu u(k_i)}{k^2}, \qquad \mathcal{M}^\mu = -ie\langle N(p_f), \pi(q)|J^\mu(0)|N(p_i)\rangle$$

Current conservation

$$k_\mu \mathcal{M}^\mu = 0$$

Parameterization in terms of six invariant amplitudes

$$\mathcal{M}^\mu = \bar{u}(p_f, s_f)\left(\sum_{i=1}^{6} \boxed{A_i(s,t,u)} M_i^\mu\right) u(p_i, s_i), \qquad u(p,s)\text{: Dirac spinor}$$

Mandelstam variables

$$\begin{aligned} s &= (p_i + k)^2, \\ t &= (p_i - q)^2, \\ u &= (p_i - p_f)^2, \\ s + t + u &= 2m_N^2 + M_\pi^2 - Q^2, \quad Q^2 = -k^2 \end{aligned}$$

Lorentz structures

$$
\begin{aligned}
M_1^\mu &= -\frac{i}{2}\gamma_5\left(\gamma^\mu \not{k} - \not{k}\gamma^\mu\right), \\
M_2^\mu &= 2i\gamma_5\left[P^\mu k\cdot\left(q-\frac{1}{2}k\right) - \left(q^\mu - \frac{1}{2}k^\mu\right)k\cdot P\right], \\
M_3^\mu &= -i\gamma_5\left(\gamma^\mu k\cdot q - \not{k}q^\mu\right), \\
M_4^\mu &= -2i\gamma_5\left(\gamma^\mu k\cdot P - \not{k}P^\mu\right) - 2m_N M_1^\mu, \\
M_5^\mu &= i\gamma_5\left(k^\mu k\cdot q - q^\mu k^2\right), \\
M_6^\mu &= -i\gamma_5\left(\not{k}k^\mu - \gamma^\mu k^2\right),
\end{aligned}
$$

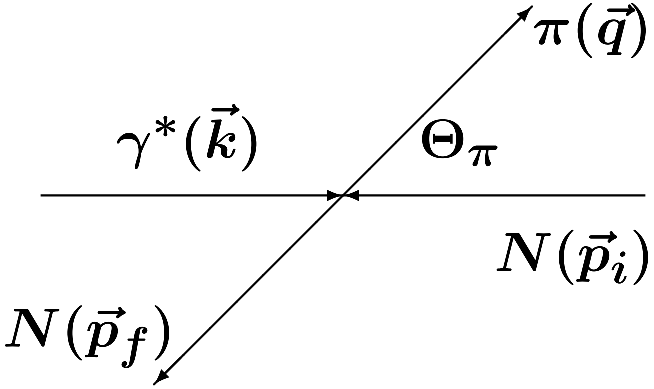
where

$$
P = \frac{1}{2}(p_i + p_f)
$$

Current conservation

$$
k_\mu M_i^\mu = 0, \qquad i = 1, \ldots, 6.
$$

cm frame: $\vec{k} = -\vec{p}_i$, $\vec{q} = -\vec{p}_f$



$$\mathcal{M} = \epsilon_\mu \bar{u}(p_f, s_f) \left(\sum_{i=1}^{6} A_i M_i^\mu \right) u(p_i, s_i) = \frac{4\pi W}{m_N} \chi_f^\dagger \boxed{\mathcal{F}} \chi_i$$

- χ: Pauli spinor, $W = \sqrt{s}$

- Gauge transformation ($\rightsquigarrow$ longitudinal multipoles)

$$a^\mu = \epsilon^\mu - k^\mu \frac{\epsilon_0}{k_0}$$

Six CGLN amplitudes[11]

$$\mathcal{F} = i\vec{\sigma}\cdot\vec{a}_{\perp}\boxed{\mathcal{F}_1(W,\Theta_\pi,Q^2)} + \vec{\sigma}\cdot\hat{q}\,\vec{\sigma}\cdot\hat{k}\times\vec{a}_{\perp}\mathcal{F}_2$$
$$+ i\vec{\sigma}\cdot\hat{k}\,\hat{q}\cdot\vec{a}_{\perp}\mathcal{F}_3 + i\vec{\sigma}\cdot\hat{q}\,\hat{q}\cdot\vec{a}_{\perp}\mathcal{F}_4$$
$$+ i\vec{\sigma}\cdot\hat{k}\,\hat{k}\cdot\vec{a}_{\parallel}\mathcal{F}_5 + i\vec{\sigma}\cdot\hat{q}\,\hat{k}\cdot\vec{a}_{\parallel}\mathcal{F}_6$$

Multipole expansion of $\mathcal{F}_i$ in terms of Legendre polynomials

$$\mathcal{F}_1 = \sum_{l=0}^{\infty}\left\{[lM_{l+} + E_{l+}]P'_{l+1}(x) + [(l+1)M_{l-} + E_{l-}]P'_{l-1}(x)\right\}, \quad \ldots$$

$$x = \cos\Theta_\pi = \hat{q}\cdot\hat{k}$$

$$E_{l\pm}, M_{l\pm}, L_{l\pm}: \quad \text{functions of } W \text{ and } Q^2$$

Total angular momentum of final state: $j = l \pm \frac{1}{2}$

Reduced multipole

$$\overline{\mathcal{M}}_{l\pm} = \frac{\mathcal{M}_{l\pm}}{|\vec{q}|^l}$$

[11] Chew, Goldberger, Nambu, Low

Isospin decomposition: four physical channels

$$\begin{aligned}
A_i(\gamma^{(*)}p \to n\pi^+) &= \sqrt{2}\left(A_i^{(-)} + A_i^{(0)}\right), \\
A_i(\gamma^{(*)}p \to p\pi^0) &= A_i^{(+)} + A_i^{(0)}, \\
A_i(\gamma^{(*)}n \to p\pi^-) &= -\sqrt{2}\left(A_i^{(-)} - A_i^{(0)}\right), \\
A_i(\gamma^{(*)}n \to n\pi^0) &= A_i^{(+)} - A_i^{(0)},
\end{aligned}$$

expressed in terms of three isospin amplitudes (0), (+), and (−)

1. **Number of diagrams**

   - **$\mathcal{O}(q^3)$: 15 tree-level diagrams + 50 one-loop diagrams**

   - **$\mathcal{O}(q^4)$: 20 tree-level diagrams + 85 one-loop diagrams**

2. **Calculate loop contributions numerically using CAS MATHEMATICA with FeynCalc and LoopTools packages**

3. **Checks: Current conservation and crossing symmetry**

4. **LECs from other processes (mesonic and baryonic Lagrangians)**

LEC	Source
l_3	$M_\pi = 134.977$ MeV
l_4, l_6	pion form factor
c_1	proton mass $m_p = 938.272$ MeV
c_2, c_3, c_4	pion-nucleon scattering
c_6, c_7	magnetic moment of proton ($\mu_p = 2.793$) and neutron ($\mu_n = -1.913$)
d_6, d_7, e_{54}, e_{74}	world data for nucleon electromagnetic form factors ($Q^2 < 0.3$ GeV^2)
d_{16}	axial-vector coupling constant $g_A = 1.2695$
d_{18}	pion-nucleon coupling
d_{22}	axial radius of the nucleon $\langle r_A^2 \rangle = 12/M_A^2$, $M_A = 1.026$ GeV

l_i: $\mathcal{L}_\pi^{(4)}$,

c_i: $\mathcal{L}_{\pi N}^{(2)}$, d_i: $\mathcal{L}_{\pi N}^{(3)}$, e_i: $\mathcal{L}_{\pi N}^{(4)}$

5. Analytic expressions for the contact diagrams

(a) 4 LECs at $\mathcal{O}(q^3)$

isospin

$$\mathcal{L}_{\pi N}^{(3)} = \frac{d_8}{2m}\left(i\bar{\Psi}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left(\tilde{f}_{\mu\nu}^{+}u_\alpha\right)D_\beta\Psi + \mathrm{H.c.}\right) \qquad (+)$$

$$+\frac{d_9}{2m}\left(i\bar{\Psi}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left(f_{\mu\nu}^{+} + 2v_{\mu\nu}^{(s)}\right)u_\alpha D_\beta\Psi + \mathrm{H.c.}\right) \qquad (0)$$

$$-\frac{d_{20}}{8m^2}\left(i\bar{\Psi}\gamma^\mu\gamma_5\left[\tilde{f}_{\mu\nu}^{+}, u_\lambda\right]D^{\lambda\nu}\Psi + \mathrm{H.c.}\right) \qquad (-)$$

$$+\,i\frac{d_{21}}{2}\bar{\Psi}\gamma^\mu\gamma_5\left[\tilde{f}_{\mu\nu}^{+}, u^\nu\right]\Psi \qquad (-)$$

Structures contribute to photoproduction, no free parameters for electroproduction

(b) 15 LECs at $\mathcal{O}(q^4)$

$$\mathcal{L}_{\pi N}^{(4)} = -\frac{e_{48}}{4m}\left(i\bar{\Psi}\mathrm{Tr}\left(f_{\lambda\mu}^{+} + 2v_{\lambda\mu}^{(s)}\right) h_{\nu}^{\lambda}\gamma_5\gamma^{\mu}D^{\nu}\Psi + \mathrm{H.c.}\right)$$
$$+ 14 \text{ more terms}$$

- photoproduction

isospin channel	(0)	$(+)$	$(-)$
# LECs	5	5	1

- electroproduction

isospin channel	(0)	$(+)$	$(-)$
# LECs	2	2	0

6. Web interface chiral MAID

[http://www.kph.uni-mainz.de/MAID/chiralmaid/]

M A I D

Photo- and Electroproduction of Pions, Etas and Kaons on the Nucleon

Institut für Kernphysik, Universität Mainz

Mainz, Germany

MAID2007	unitary isobar model for (e,e'p)
DMT2001	dynamical model for (e,e'p)
KAON-MAID	isobar model for (e,e'K)
ETA-MAID	isobar model for (e,e'h) reggeized isobar model for (g,h)
ChiralMAID NEW	chiral perturbation theory approach for (e,e'p)
2-PION-MAID	isobar model for (g,pp)
archive	MAID2000 MAID2003 DMT2001original ETAprime2003

Back to Theory Group Homepage

Chiral MAID

Chiral MAID info and updates (please read first)

Pion Photo- and Electroproduction on the Nucleon in Relativistic Chiral Perturbation Theory

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M. Hilt, B.C. Lehnhart, S. Scherer, L. Tiator
Phys. Rev. C88 (2013) 5, 055207

- Electromagnetic Multipoles ($E_{l\pm}$, $M_{l\pm}$, $L_{l\pm}$, $S_{l\pm}$)
- Amplitudes (F_1,...,F_6, H_1,...,H_6, A_1,...,A_6)
- Differential Cross Sections (ds_T, ds_L, ds_{LT}, ds_{TT}, ...)
- 5-fold Diff. Cross Section (d^5s, G, ds^v= ds_T+e ds_L+e ds_{TT} cos 2f + ...)
- Total Cross Sections (s_T , s_L , $s_{LT'}$, $s_{TT'}$, ...)
- Transverse Polarization Observables (ds/dW, T, S, P, E, F, G, H, ...)

External services:
MAID Homepage MAID2003 DMT2001 KAON-MAID ETA-MAID2000 ETA-MAID2003 ETA'-MAID

A1 kinematics calculator for electroproduction (Java)
SAID Partial-Wave Analyses

Multipoles

The multipoles can be given in 4 unique sets of isospin or charge channels (click here for a larger image):

$$\left(A_p^{1/2}, A_n^{1/2}, A^{3/2}\right), \left(A^{1/2}, A^0, A^{3/2}\right), \left(A^0, A^+, A^-\right), \left(A_{\pi^+ n}, A_{\pi^- p}, A_{\pi^0 p}, A_{\pi^0 n}\right)$$

$$A_{\pi^+ n} = \sqrt{2}\left(A^- + A^0\right) = \sqrt{2}\left(A_p^{1/2} - \tfrac{1}{3}A^{3/2}\right) = \sqrt{2}\left(A^0 + \tfrac{1}{3}A^{1/2} - \tfrac{1}{3}A^{3/2}\right)$$

$$A_{\pi^- p} = -\sqrt{2}\left(A^- - A^0\right) = \sqrt{2}\left(A_n^{1/2} + \tfrac{1}{3}A^{3/2}\right) = \sqrt{2}\left(A^0 - \tfrac{1}{3}A^{1/2} + \tfrac{1}{3}A^{3/2}\right)$$

$$A_{\pi^0 p} = A^+ + A^0 = A_p^{1/2} + \tfrac{2}{3}A^{3/2} = A^0 + \tfrac{1}{3}A^{1/2} + \tfrac{2}{3}A^{3/2}$$

$$A_{\pi^0 n} = A^+ - A^0 = -A_n^{1/2} + \tfrac{2}{3}A^{3/2} = -A^0 + \tfrac{1}{3}A^{1/2} + \tfrac{2}{3}A^{3/2}$$

Further details can be found in D. Drechsel and L. Tiator, J. Phys. G 18 (1992) 449-497. (scanned version)

Type of the multipoles: ○ (p(1/2), n(1/2), 3/2) ○ (1/2, 0, 3/2) ○ (0, +, -) ◉ charge channels

Choose pion angular momentum l : [0 ▾] ☑ El+ ☐ El- ☐ Ml+ ☐ Ml- ☐ Ll+ ☐ Ll- ☐ Sl+ ☐ Sl-

Reduced multipoles: ☐

Choose kinematical variables
choose an independent (running) variable: ○ Q^2 ◉ W
choose values for Q^2, W, step size and maximum value:

Q^2 $(GeV/c)^2$	W (MeV)	increment	upper value	click here	
0	1074	1	1090	Calculate	Reset

$O(q^3)$ (all couplings in GeV^{-2})

0	+	-	
d_9	d_8	d_{20}	d_{21}
-1.216	-1.092	4.337	-4.260

$O(q^4)$ (all couplings in GeV^{-3})

Isospin 0

e_{48}	e_{49}	e_{50}	e_{51}	e_{52}	e_{53}	e_{112}
5.235	0.925	2.205	6.629	-4.103	-2.654	9.342

Isospin +

e_{67}	e_{68}	e_{69}	e_{71}	e_{72}	e_{73}	e_{113}
-8.269	-0.925	-1.035	-4.352	10.539	2.120	-13.745

Isospin -

e_{70}
3.910

Remark: $O(q^3)$ refers to corresponding LECs (this is not a $O(q^3)$ calculation)

C h M A I D 2 0 1 2

M. Hilt, S. Scherer, L. Tiator
Institut fuer Kernphysik, Universitaet Mainz

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Pion angular momentum l= 0

All multipoles are given in 10^-3/Mpi+

Q^2 = .000 (GeV/c)^2

****

.3028	1.2695	.0924	13.2100				e,gA,F[GeV],gpiN=gA*mp/F
-1.0920	-1.2160	4.3370	-4.2600				d8,d9,d20,d21 [GeV^-2]
5.2350	.9250	2.2050	6.6290	-4.1030	-2.6540		e48,e49,e50,e51,e52,e53 [GeV^-3]
-8.2690	-.9250	-1.0350	3.9100	-4.3520	10.5390	2.1200	e67,e68,e69,e70,e71,e72,e73 [GeV^-3]
9.3420	-13.7450						e112,e113 [GeV^-3]

W	E0+(pi0_p)		E0+(pi0_n)		E0+(pi+_n)		E0+(pi-_p)		E(lab)	q(cm)
(MeV)	Re	Im	Re	Im	Re	Im	Re	Im	(MeV)	(MeV)
1074.00	-1.0608	.0000	2.8400	.0000	27.1931	.0000	-32.7097	.0000	145.54	13.44
1075.00	-.9960	.0000	2.8898	.0000	26.9933	.0000	-32.4886	.0000	146.69	20.45
1076.00	-.9210	.0000	2.9504	.0000	26.7940	.0000	-32.2689	.0000	147.84	25.64
1077.00	-.8301	.0000	3.0279	.0000	26.5939	.0000	-32.0500	.0000	148.98	29.96
1078.00	-.7093	.0000	3.1376	.0000	26.3903	.0000	-31.8308	.0000	150.13	33.75
1079.00	-.4769	.0000	3.3685	.0000	26.1676	.0000	-31.6058	.0000	151.28	37.18
1080.00	-.3758	.3249	3.4564	.3534	25.9705	-.0617	-31.3900	.0215	152.43	40.33
1081.00	-.3959	.4764	3.4121	.5183	25.7986	-.0924	-31.1839	.0331	153.58	43.27
1082.00	-.4162	.5891	3.3672	.6412	25.6292	-.1166	-30.9798	.0430	154.74	46.03
1083.00	-.4367	.6826	3.3218	.7433	25.4625	-.1378	-30.7777	.0521	155.89	48.66
1084.00	-.4573	.7641	3.2758	.8323	25.2982	-.1573	-30.5776	.0609	157.04	51.16
1085.00	-.4780	.8371	3.2293	.9121	25.1364	-.1757	-30.3793	.0696	158.20	53.55
1086.00	-.4989	.9035	3.1822	.9849	24.9770	-.1933	-30.1829	.0782	159.36	55.86
1087.00	-.5199	.9649	3.1346	1.0523	24.8200	-.2104	-29.9883	.0868	160.52	58.08
1088.00	-.5410	1.0221	3.0864	1.1151	24.6654	-.2270	-29.7954	.0955	161.67	60.24
1089.00	-.5623	1.0758	3.0377	1.1741	24.5131	-.2433	-29.6042	.1043	162.83	62.33
1090.00	-.5837	1.1264	2.9884	1.2299	24.3630	-.2593	-29.4147	.1131	164.00	64.36

Fits to available experimental data

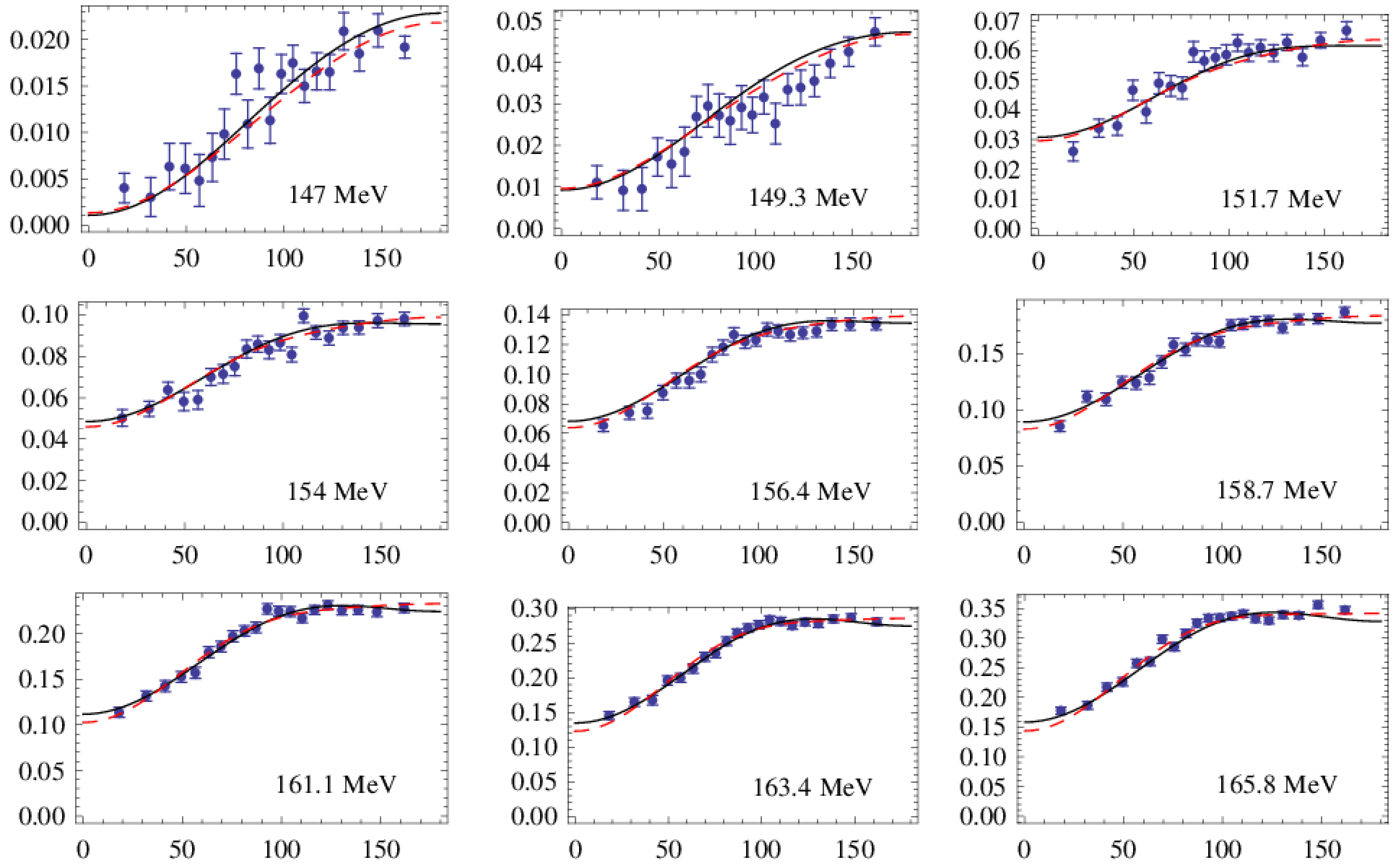
1. $\gamma + p \rightarrow p + \pi^0$

2. $\gamma^* + p \rightarrow p + \pi^0$

3. $\gamma + p \rightarrow n + \pi^+$ and $\gamma + n \rightarrow p + \pi^-$

4. $\gamma^{(*)} + p \rightarrow n + \pi^+$

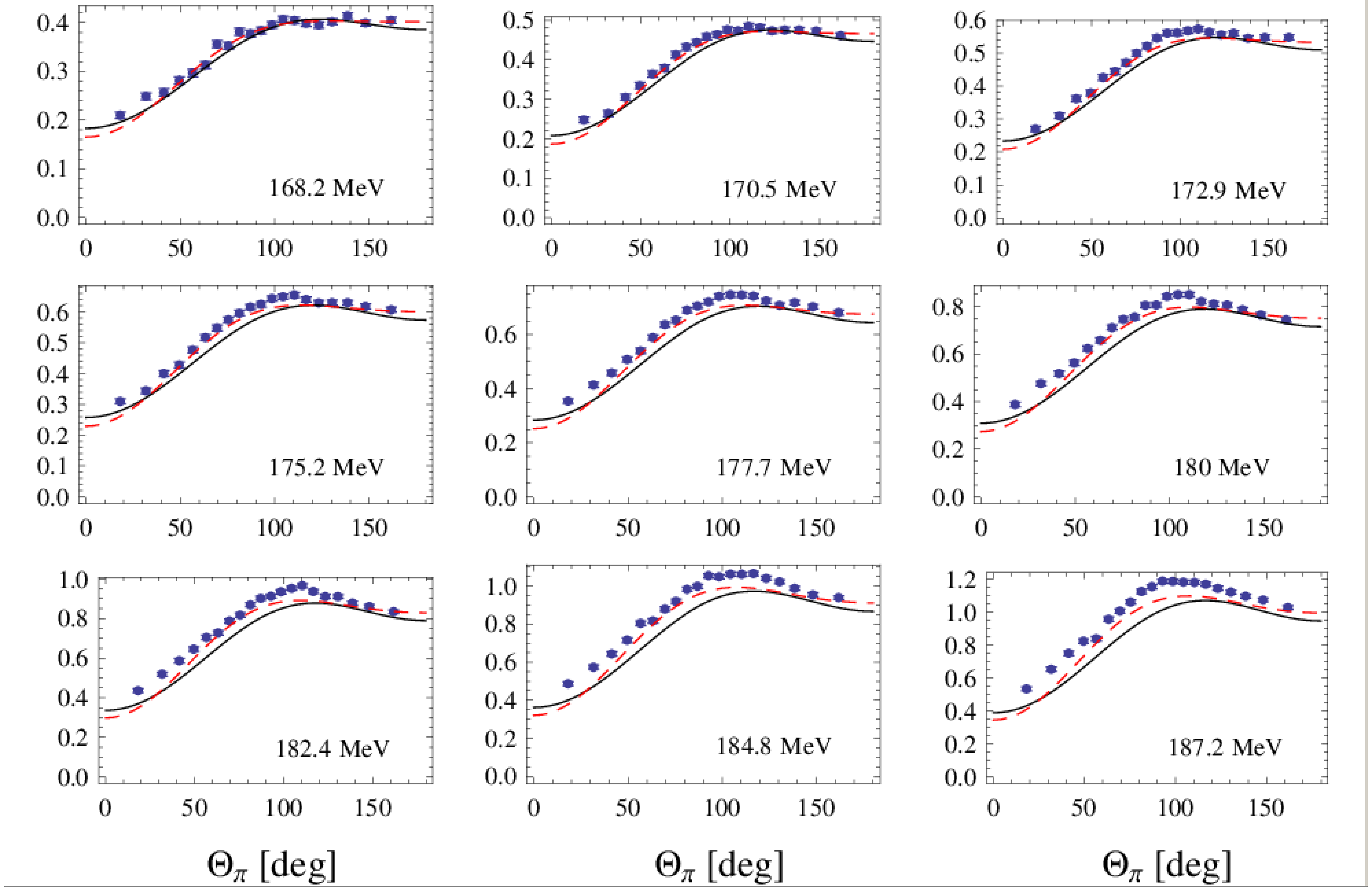
Differential cross sections $d\sigma/d\Omega_\pi$ in μb/sr for $\gamma + p \to p + \pi^0$ [12]



Solid: RChPT, dashed HBChPT

[12] Data taken from D. Hornidge et al., Phys. Rev. Lett. **111**, 062004 (2013)

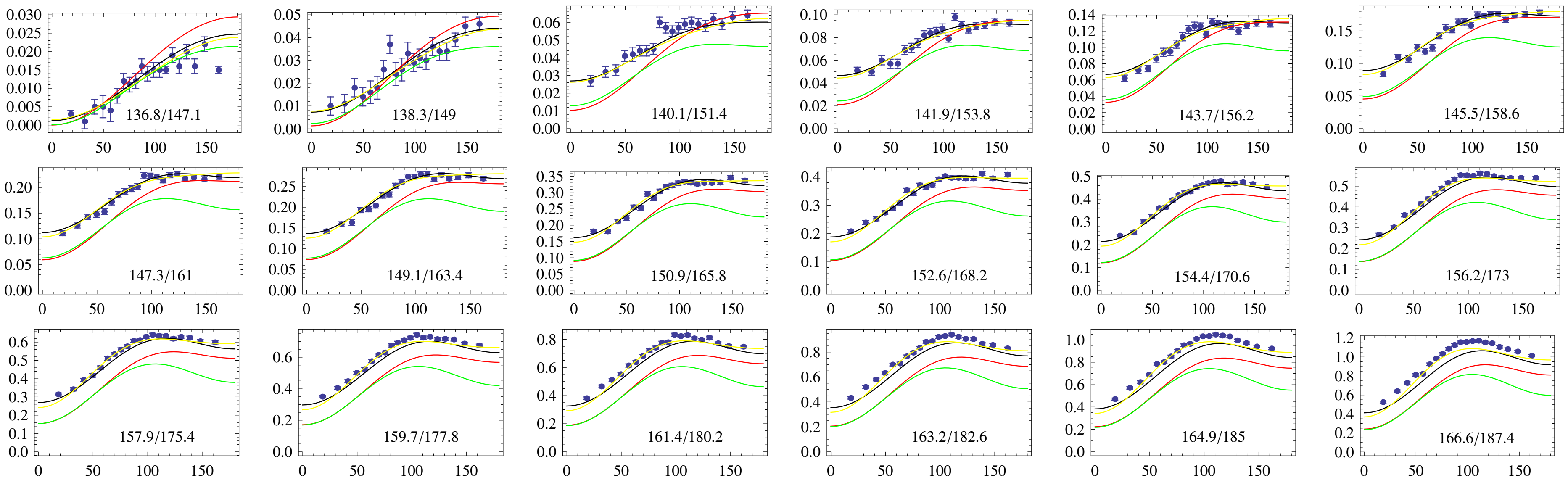
Differential cross sections $d\sigma/d\Omega_\pi$ in μb/sr for $\gamma + p \to p + \pi^0$ [13]



Solid: RChPT, dashed HBChPT

[13] Data taken from D. Hornidge et al., Phys. Rev. Lett. **111**, 062004 (2013)

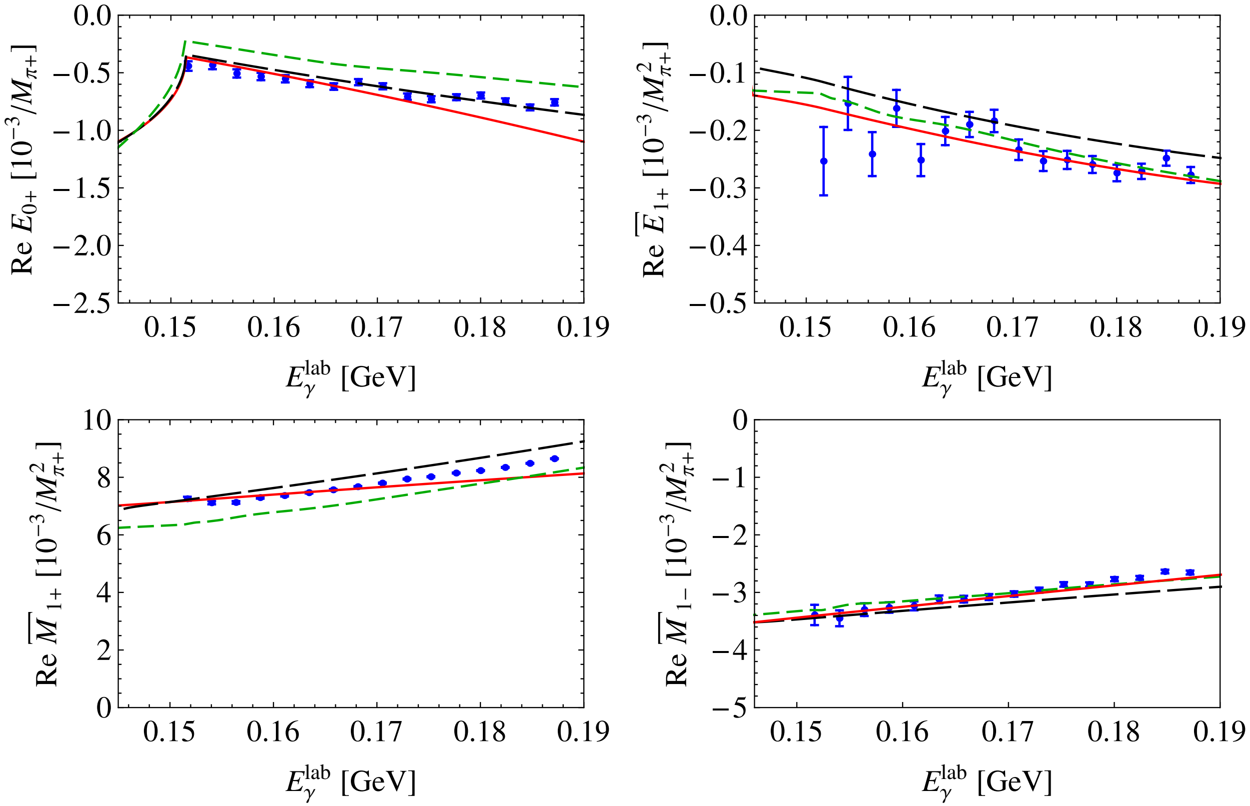
Comparison of $\mathcal{O}(q^3)$ with $\mathcal{O}(q^4)$ [14]



Black: RChPT $\mathcal{O}(q^4)$, red RChPT $\mathcal{O}(q^3)$, yellow HChPT $\mathcal{O}(q^4)$, green RChPT + vector mesons $\mathcal{O}(q^3)$

[14] Data taken from D. Hornidge et al., Phys. Rev. Lett. **111**, 062004 (2013)

S- and reduced P-wave multipoles for $\gamma + p \to p + \pi^0$



Red RChPT; green DMT model[15]; black Gasparyan & Lutz[16]; data from Hornidge et al. (2013)

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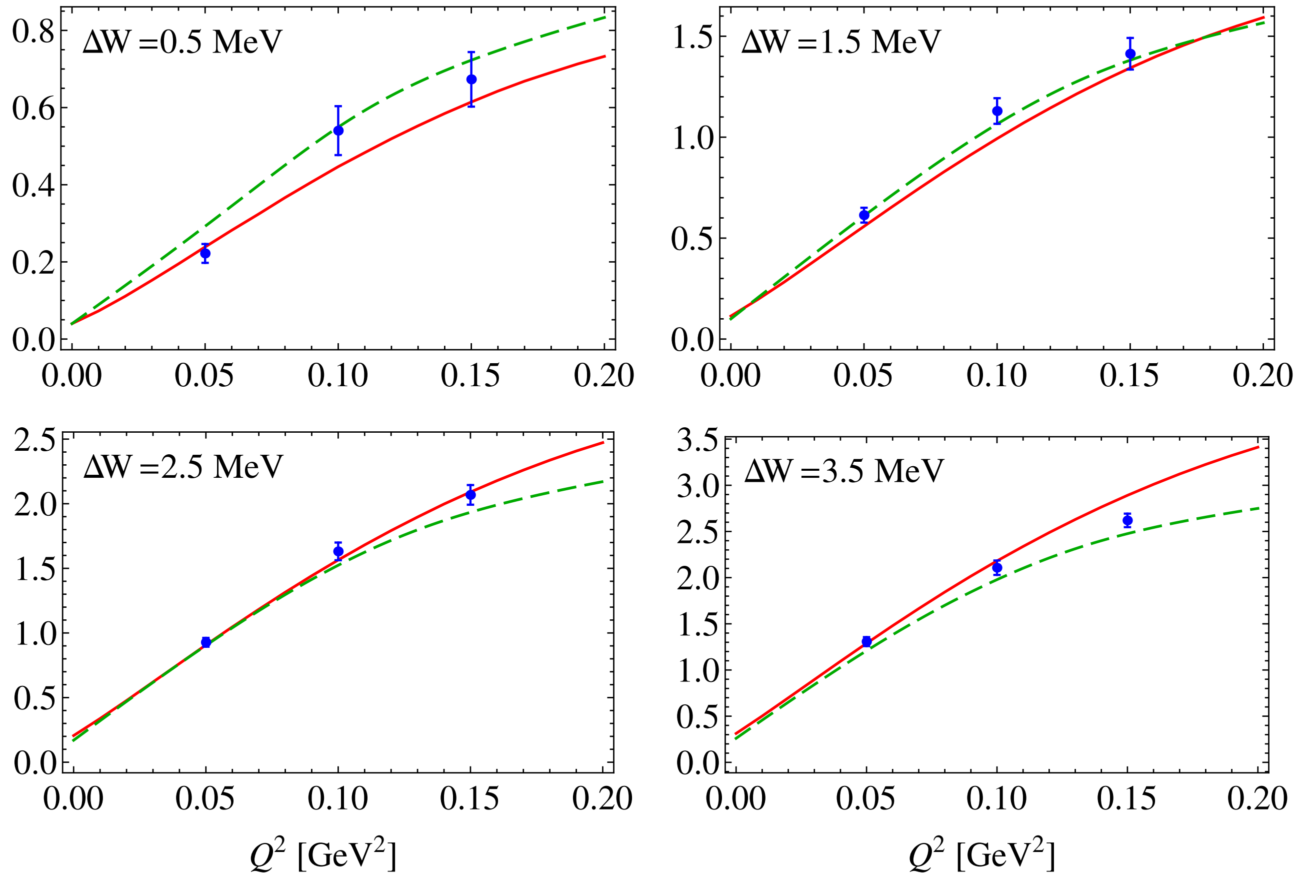
[15] S. S. Kamalov et al., Phys. Rev. C **64**, 032201 (2001)

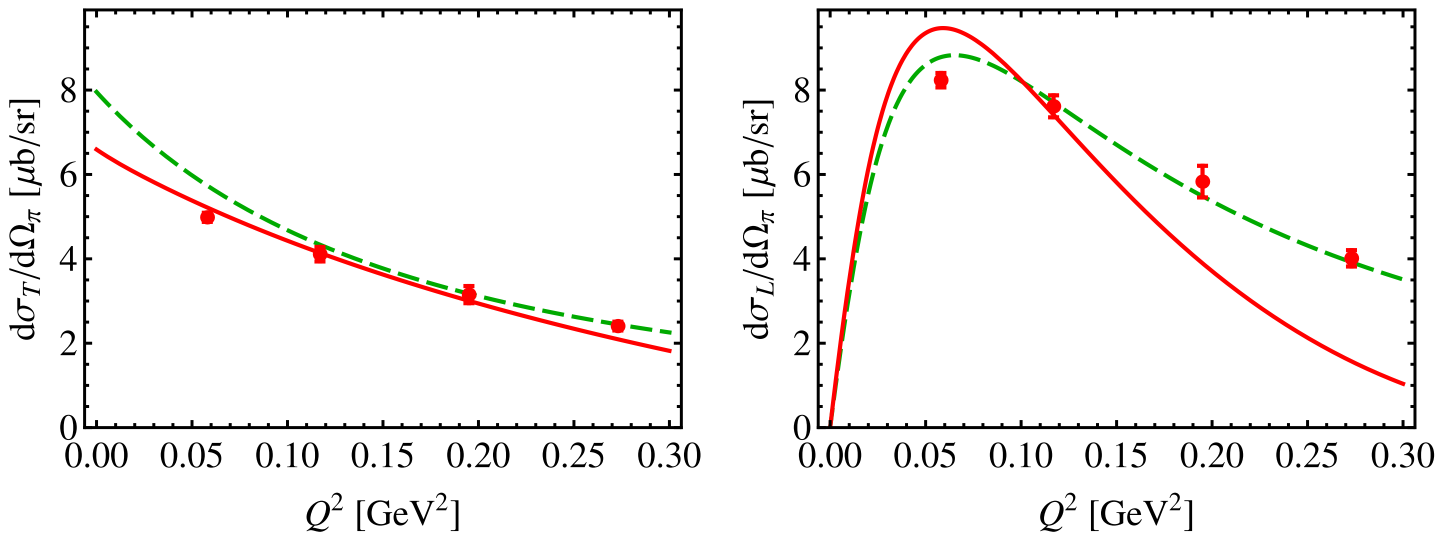
[16] A. Gasparyan and M. F. M. Lutz, Nucl. Phys. **A848**, 126 (2010)

Total cross sections for $\gamma^* + p \to p + \pi^0$ in μb



red RChPT;
green DMT model;
data from Merkel et al. 2009, 2011

Differential cross sections as a function of Q^2 for $\gamma^* + p \to n + \pi^+$ at $W = 1125$ MeV and $\Theta_\pi = 0°$.



red RChPT; green DMT model;
data from Baumann (PhD thesis, JGU, 2005)

from fits with all data

Isospin channel	LEC	Value
0	d_9 [GeV^{-2}]	-1.22 ± 0.12
0	e_{48} [GeV^{-3}]	5.2 ± 1.4
0	e_{49} [GeV^{-3}]	0.9 ± 2.6
0	e_{50} [GeV^{-3}]	2.2 ± 0.8
0	e_{51} [GeV^{-3}]	6.6 ± 3.6
0	e^*_{52} [GeV^{-3}]	-4.1
0	e^*_{53} [GeV^{-3}]	-2.7
0	e_{112} [GeV^{-3}]	9.3 ± 1.6
+	d_8 [GeV^{-2}]	-1.09 ± 0.12
+	e_{67} [GeV^{-3}]	-8.3 ± 1.5
+	e_{68} [GeV^{-3}]	-0.9 ± 2.6
+	e_{69} [GeV^{-3}]	-1.0 ± 2.2
+	e_{71} [GeV^{-3}]	-4.4 ± 3.7
+	e^*_{72} [GeV^{-3}]	10.5
+	e^*_{73} [GeV^{-3}]	2.1
+	e_{113} [GeV^{-3}]	-13.7 ± 2.6
−	d_{20} [GeV^{-2}]	4.34 ± 0.08
−	d_{21} [GeV^{-2}]	-3.1 ± 0.1
−	e_{70} [GeV^{-3}]	3.9 ± 0.3

4. Summary and outlook

- Baryonic ChPT: Renormalization condition $\leftrightarrow$ consistent power counting

- Example: EOMS renormalization (manifestly Lorentz-invariant)

- Application to pion photo- and electroproduction

- 20 tree-level diagrams + 85 one-loop diagrams

- Chiral MAID interface

- **Inclusion of heavy degrees of freedom (vector mesons, axial vector mesons, $\boldsymbol{\Delta}$ [17])**

- **New data [18] $\rightsquigarrow$ reanalysis of LECs**

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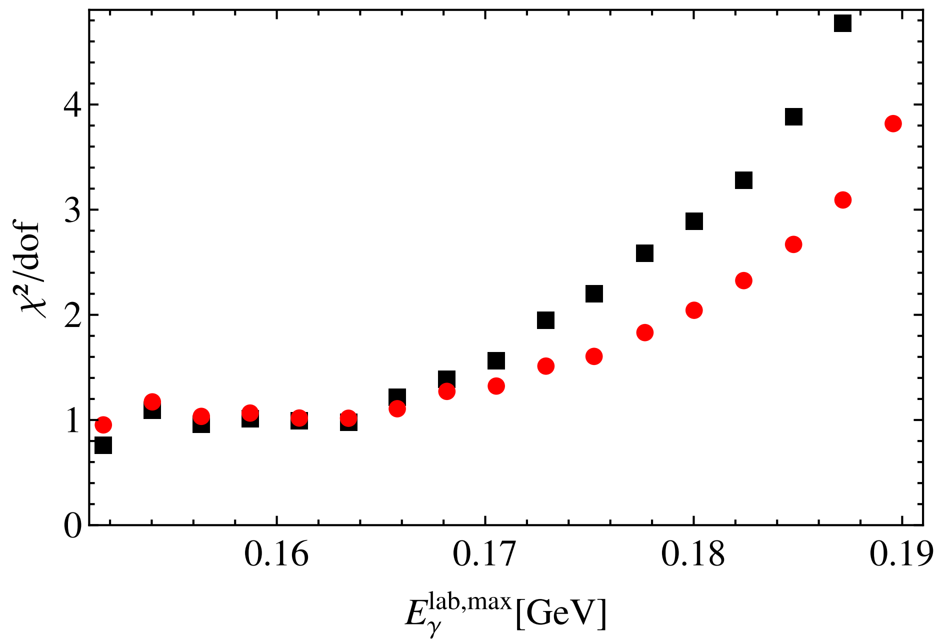
[17] A. N. Hiller Blin, T. Ledwig, and M. J. V. Vacas, Phys. Lett. B **747** (2015), 217, see talks by A. N. Hiller Blin and L. Cawthorne

[18] K. Chirapatpimol *et al.*, $\boldsymbol{p(e,e'p)\pi^0}$, Phys. Rev. Lett. **114**, 192503 (2015), I. Fricic, $\boldsymbol{p(e,e'\pi^+)n}$, PhD thesis, University of Zagreb, 2015

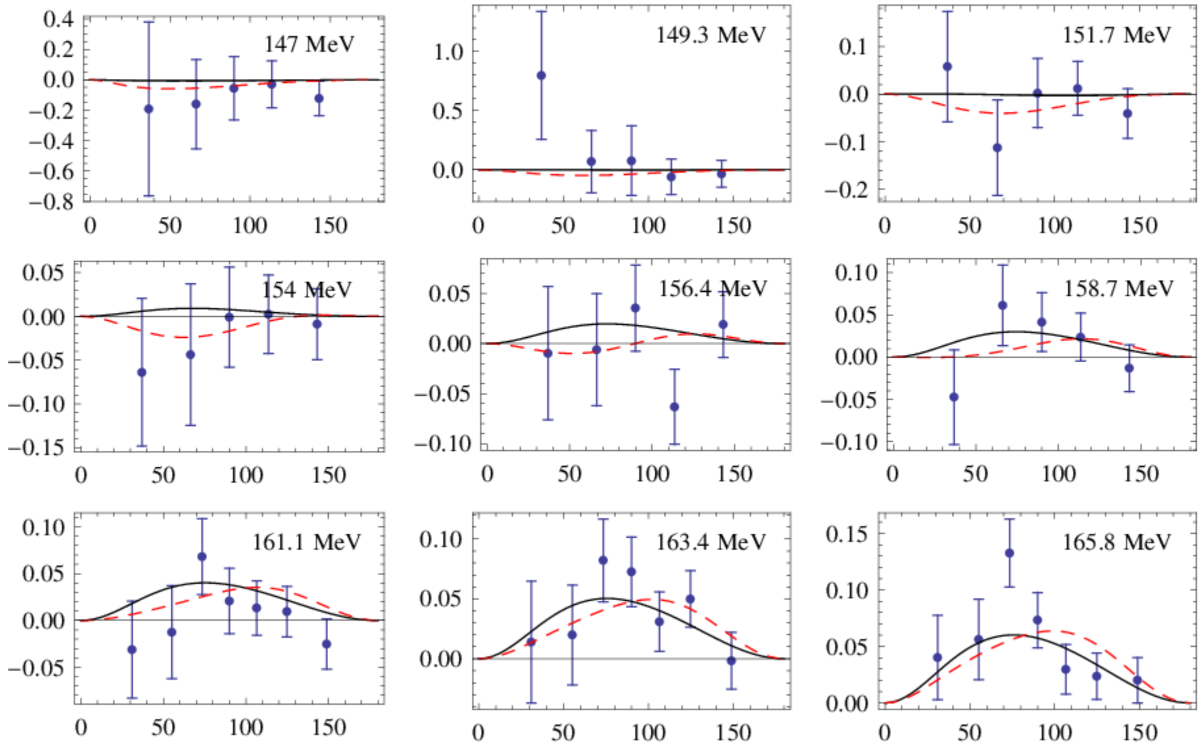
Thank You!

$\chi^2_{\mathbf{red}}$ as a function of the fitted energy range: RBChPT vs. HBChPT



Photon asymmetries for $\gamma + p \to p + \pi^0$ [19]

Solid: RChPT, dashed HBChPT



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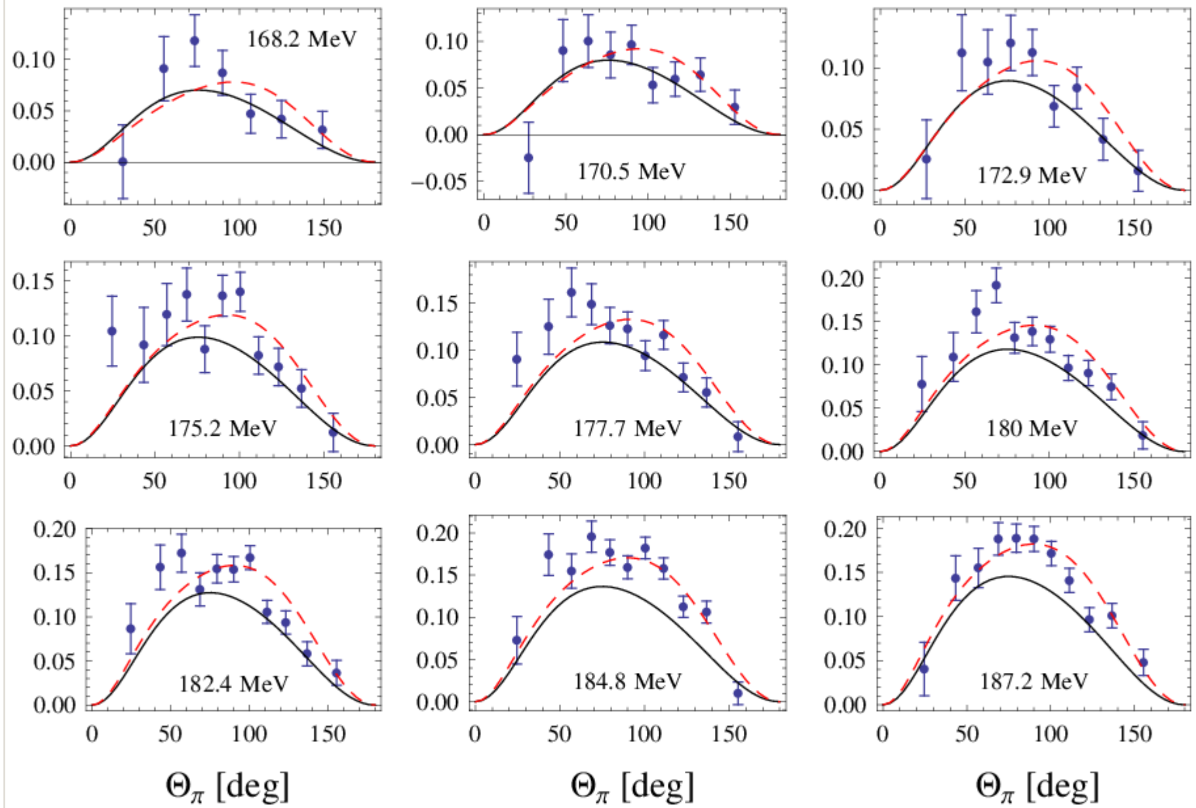
[19] Data taken from D. Hornidge et al., Phys. Rev. Lett. **111**, 062004 (2013)

Photon asymmetries for $\gamma + p \to p + \pi^0$ [20]



Solid: RChPT, dashed: HBChPT

[20] Data taken from D. Hornidge et al., Phys. Rev. Lett. **111**, 062004 (2013)

Multipoles

The multipoles can be given in 4 unique sets of isospin or charge channels (click here for a larger image):

$$\left(A_p^{1/2}, A_n^{1/2}, A^{3/2}\right), \left(A^{1/2}, A^0, A^{3/2}\right), \left(A^0, A^+, A^-\right), \left(A_{\pi^+n}, A_{\pi^-p}, A_{\pi^0p}, A_{\pi^0n}\right)$$

$$\begin{aligned}
A_{\pi^+n} &= \sqrt{2}\left(A^- + A^0\right) = \sqrt{2}\left(A_p^{1/2} - \tfrac{1}{3}A^{3/2}\right) = \sqrt{2}\left(A^0 + \tfrac{1}{3}A^{1/2} - \tfrac{1}{3}A^{3/2}\right)\\
A_{\pi^-p} &= -\sqrt{2}\left(A^- - A^0\right) = \sqrt{2}\left(A_n^{1/2} + \tfrac{1}{3}A^{3/2}\right) = \sqrt{2}\left(A^0 - \tfrac{1}{3}A^{1/2} + \tfrac{1}{3}A^{3/2}\right)\\
A_{\pi^0p} &= A^+ + A^0 = A_p^{1/2} + \tfrac{2}{3}A^{3/2} = A^0 + \tfrac{1}{3}A^{1/2} + \tfrac{2}{3}A^{3/2}\\
A_{\pi^0n} &= A^+ - A^0 = -A_n^{1/2} + \tfrac{2}{3}A^{3/2} = -A^0 + \tfrac{1}{3}A^{1/2} + \tfrac{2}{3}A^{3/2}
\end{aligned}$$

Further details can be found in D. Drechsel and L. Tiator, J. Phys. G 18 (1992) 449-497. (scanned version)

Type of the multipoles: ○ (p(1/2), n(1/2), 3/2) ○ (1/2, 0, 3/2) ○ (0, +, -) ◉ charge channels

Choose pion angular momentum l : 1 ☑ El+ ☐ El- ☐ Ml+ ☐ Ml- ☑ Ll+ ☐ Ll- ☐ Sl+ ☐ Sl-

Reduced multipoles: ☐

Choose kinematical variables

choose an independent (running) variable: ◉ Q^2 ○ W

choose values for Q^2, W, step size and maximum value:

Q^2 $(GeV/c)^2$	W (MeV)	increment	upper value	click here	
0	1080	0.01	0.1	Calculate	Reset

Change of model parameters:

Multipoles

```
               C h M A I D  2 0 1 2
            M. Hilt, S. Scherer, L. Tiator
        Institut fuer Kernphysik, Universitaet Mainz
      *************************************************

   Pion angular momentum l= 1

   All multipoles are given in 10^-3/Mpi+

                W = 1080.000 (MeV)
                ******************
  .3028  1.2695   .0924 13.2100                                   e,gA,F[GeV],gpiN=gA*mp/F
-1.0920 -1.2160  4.3370 -4.2600                                   d8,d9,d20,d21 [GeV^-2]
 5.2350   .9250  2.2050  6.6290 -4.1030 -2.6540                   e48,e49,e50,e51,e52,e53 [GeV^-3]
-8.2690  -.9250 -1.0350  3.9100 -4.3520 10.5390  2.1200           e67,e68,e69,e70,e71,e72,e73 [GeV^-3]
 9.3420-13.7450                                                   e112,e113 [GeV^-3]
```

Q^2 (GeV/c)^2	E1+(pi0_p) Re	Im	E1+(pi0_n) Re	Im	E1+(pi+_n) Re	Im	E1+(pi-_p) Re	Im	E(lab) (MeV)	q(cm) (MeV)
.00	-.0479	-.0001	-.0177	-.0002	1.4327	.0001	-1.4755	-.0001		
.01	-.0583	-.0001	-.0171	-.0002	1.4017	.0001	-1.4600	-.0001		
.02	-.0673	-.0001	-.0149	-.0001	1.3364	.0001	-1.4106	-.0001		
.03	-.0754	-.0001	-.0115	-.0001	1.2631	.0001	-1.3536	.0000		
.04	-.0831	-.0001	-.0071	-.0001	1.1903	.0001	-1.2977	.0000		
.05	-.0903	-.0001	-.0020	-.0001	1.1208	.0001	-1.2457	.0000		
.06	-.0973	-.0001	.0039	-.0001	1.0555	.0001	-1.1985	.0000		
.07	-.1040	-.0001	.0104	-.0001	.9944	.0001	-1.1561	.0000		
.08	-.1106	-.0001	.0174	-.0001	.9372	.0001	-1.1183	.0000		
.09	-.1171	-.0001	.0250	-.0001	.8836	.0001	-1.0845	.0000		
.10	-.1234	-.0001	.0331	-.0001	.8332	.0001	-1.0545	.0000		

Q^2 (GeV/c)^2	L1+(pi0_p) Re	Im	L1+(pi0_n) Re	Im	L1+(pi+_n) Re	Im	L1+(pi-_p) Re	Im	E(lab) (MeV)	q(cm) (MeV)
.00	-.0393	-.0001	-.0168	-.0001	.7782	.0000	-.8101	.0000		
.01	-.0440	-.0001	-.0169	-.0001	.6088	.0000	-.6471	.0000		
.02	-.0468	.0000	-.0162	-.0001	.4781	.0000	-.5214	.0000		
.03	-.0486	.0000	-.0151	.0000	.3797	.0000	-.4271	.0000		
.04	-.0496	.0000	-.0138	.0000	.3047	.0000	-.3554	.0000		
.05	-.0501	.0000	-.0124	.0000	.2465	.0000	-.2998	.0000		

Coincidence cross sections σ_0, σ_{TT}, and σ_{LT} in μb/sr and beam asymmetry $A_{LT'}$ in $\%$ at constant $Q^2 = 0.05$ GeV^2, $\Theta_\pi = 90^\circ$, $\Phi_\pi = 90^\circ$, and $\epsilon = 0.93$ as a function of ΔW above threshold.[21] Solid (red): RChPT, dotted (black): HBChPT (Bernard et al., 1996) dashed (green): DMT

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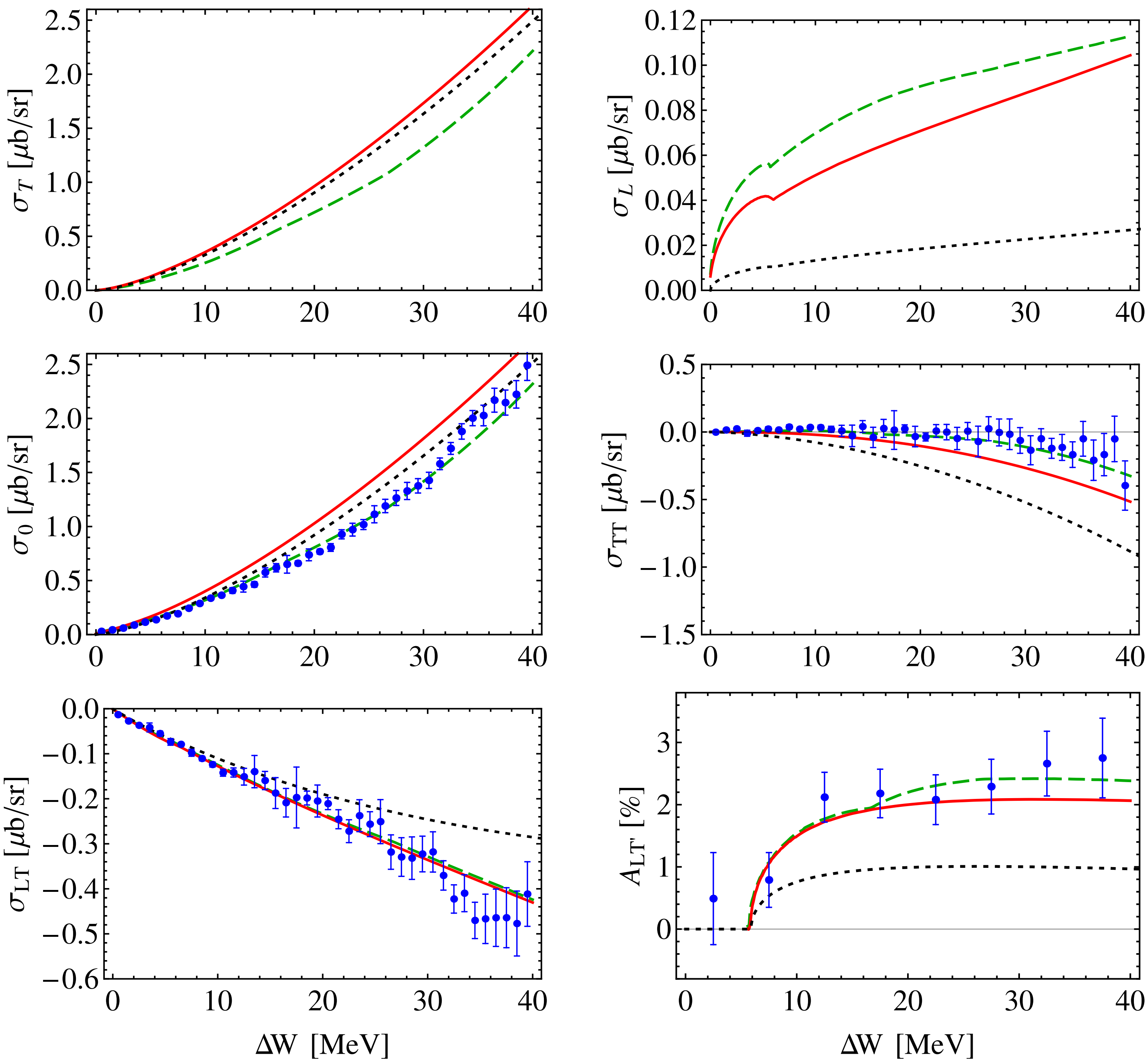
[21] Data taken from M. Weis et al., Eur. Phys. J. A **38**, 27 (2008).



σ_T [μb/sr]
σ_L [μb/sr]
σ_0 [μb/sr]
σ_{TT} [μb/sr]
σ_{LT} [μb/sr]
$A_{\mathrm{LT'}}$ [%]
ΔW [MeV]
ΔW [MeV]