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Viterbi process for pairwise Markov models

Thursday, 17 August 2017 10:00 (30 minutes)

My talk is based on ongoing joint work with my supervisor Jüri Lember.

We consider a Markov chain $Z=\{Z_k\}_{k\geq 1}$ with product
state space $\mathcal{X}\times\mathcal{Y}$, where $\mathcal{Y}$ is
a finite set (state space) and $\mathcal{X}$ is an arbitrary
separable metric space (observation space). Thus, the process Z
decomposes as $Z=(X,Y)$, where $X=\{X_k\}_{k\geq 1}$ and $Y=\{Y_k\}_{k\geq 1}$ are random processes taking values in $\mathcal{X}$
and $\mathcal{Y}$, respectively. Following
cite{pairwise,pairwise2,pairwise3}, we call the process Z a
\textit{pairwise Markov model}. The process X is identified as an
observation process and the process Y, sometimes called the \textit{regime}, models the observations-driving hidden state sequence.
Therefore our general model contains many well-known stochastic
models as a special case: hidden Markov models, Markov
switching models, hidden Markov models with dependent noise and many
more. The \textit{segmentation} or \textit{path estimation} problem
consists of estimating the realization of $(Y_1,\ldots,Y_n)$ given a
realization $x_{1:n}$ of $(X_1,\ldots,X_n)$. A standard estimate is
any path $v_{1:n}\in\mathcal{Y}^n$ having maximum posterior
probability:
$$v_{1:n}=\mathrm{argmax}_{y_{1:n}} P(Y_{1:n}=y_{1:n}|X_{1:n}=x_{1:n}). Anysuchpathiscalled *Viterbi path* andweareinterestedinthebehaviourof v_{1:n}

We show that under some conditions the infinite Viterbi path indeed exists for almost every realization $x\_\{1:\backslash infty\}$ of X, thereby defining an infinite Viterbi decoding of X, called the *Viterbi process.* This is done trough construction of *barriers.* A barrier is a fixed-sized block in the observations $x\_\{1:n\}$ that fixes the Viterbi path up to itself: for every continuation of $x\_\{1:n\}$, the Viterbi path up to the barrier remains unchanged. Therefore, if almost every realization of X-process contains infinitely many barriers, then the Viterbi process exists.

Having infinitely many barriers is not necessary for existence of infinite Viterbi path, but the barrier-construction has several advantages. One of them is that it allows to construct the infinite path *piecewise*, meaning that to determine the first k elements $v\_\{1:k\}$ of the infinite path it suffices to observe $x\_\{1:n\}$ for n big enough. Barrier construction has another great advantage: namely, the process $(Z,V)=\backslash\{(Z\_k,V\_k)\backslash\}$*{k \geq 1}*, $where$ V= *\{V_k\}*$\{k\backslash geq1\}$ denotes the Viterbi process, is under certain conditions regenerative. This is can be proven by, roughly speaking, applying the Markov splitting method to construct regeneration times for Z which coincide with the occurrences of barriers. Regenerativity of
(Z,V)\$allowstoeasilyprovelimittheoremstounderstandtheasymptoticbehaviourofinferencesbasedonViterbi paths.Infact,inaspecialcaseofhiddenMarkovmodelthisregenerativepropertyhasalreadybeenknowntoholdandhasf

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