

# Viterbi process for pairwise Markov models

Thursday, 17 August 2017 10:00 (30 minutes)

My talk is based on ongoing joint work with my supervisor Jüri Lember.

We consider a Markov chain  $Z = \{Z_k\}_{k \geq 1}$  with product state space  $\mathcal{X} \times \mathcal{Y}$ , where  $\mathcal{Y}$  is a finite set (state space) and  $\mathcal{X}$  is an arbitrary separable metric space (observation space). Thus, the process  $Z$  decomposes as  $Z = (X, Y)$ , where  $X = \{X_k\}_{k \geq 1}$  and  $Y = \{Y_k\}_{k \geq 1}$  are random processes taking values in  $\mathcal{X}$

and  $\mathcal{Y}$ , respectively. Following

cite{pairwise,pairwise2,pairwise3}, we call the process  $Z$  a

\textit{pairwise Markov model}. The process  $X$  is identified as an

observation process and the process  $Y$ , sometimes called the \textit{regime}, models the observations-driving hidden state sequence.

Therefore our general model contains many well-known stochastic

models as a special case: hidden Markov models, Markov

switching models, hidden Markov models with dependent noise and many

more. The \textit{segmentation} or \textit{path estimation} problem

consists of estimating the realization of  $(Y_1, \dots, Y_n)$  given a

realization  $x_{1:n}$  of  $(X_1, \dots, X_n)$ . A standard estimate is

any path  $v_{1:n} \in \mathcal{Y}^n$  having maximum posterior

probability:

$v_{1:n} = \operatorname{argmax}_{y_{1:n}} P(Y_{1:n} = y_{1:n} | X_{1:n} = x_{1:n})$ . Any such path is called *Viterbi path* and we are interested in the behaviour of  $v_{1:n}$ .

We show that under some conditions the infinite Viterbi path indeed exists for almost every realization  $x_{1:\infty}$  of  $X$ , thereby defining an infinite Viterbi decoding of  $X$ , called the *Viterbi process*. This is done through construction of *barriers*. A barrier is a fixed-sized block in the observations  $x_{1:n}$  that fixes the Viterbi path up to itself: for every continuation of  $x_{1:n}$ , the Viterbi path up to the barrier remains unchanged. Therefore, if almost every realization of  $X$ -process contains infinitely many barriers, then the Viterbi process exists.

Having infinitely many barriers is not necessary for existence of infinite Viterbi path, but the barrier-construction has several advantages. One of them is that it allows to construct the infinite path *piecewise*, meaning that to determine the first  $k$  elements  $v_{1:k}$  of the infinite path it suffices to observe  $x_{1:n}$  for  $n$  big enough. Barrier construction has another great advantage: namely, the process  $(Z, V) = \{(Z_k, V_k)\}_{k \geq 1}$ , where  $V = \{V_k\}_{k \geq 1}$  denotes the Viterbi process, is under certain conditions regenerative. This can be proven by, roughly speaking, applying the Markov splitting method to construct regeneration times for  $Z$  which coincide with the occurrences of barriers. Regenerativity of  $(Z, V)$  allows to easily prove limit theorems to understand the asymptotic behaviour of inferences based on Viterbi paths. In fact, in a special case of hidden Markov model this regenerative property has already been known to hold and has f

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