From scaling to cross sections within GFMC

Noemi Rocco

UNIVERSITY OF SURREY

In collaboration with:
L. Alvarez-Ruso, A. Lovato, J. Nieves, A. Sobczyk

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Outline

- Scaling features of the Green’s Function Monte Carlo results

- Extending the applicability of non relativistic approaches in the quasi elastic region

- From the scaling analysis to cross sections

Introduction

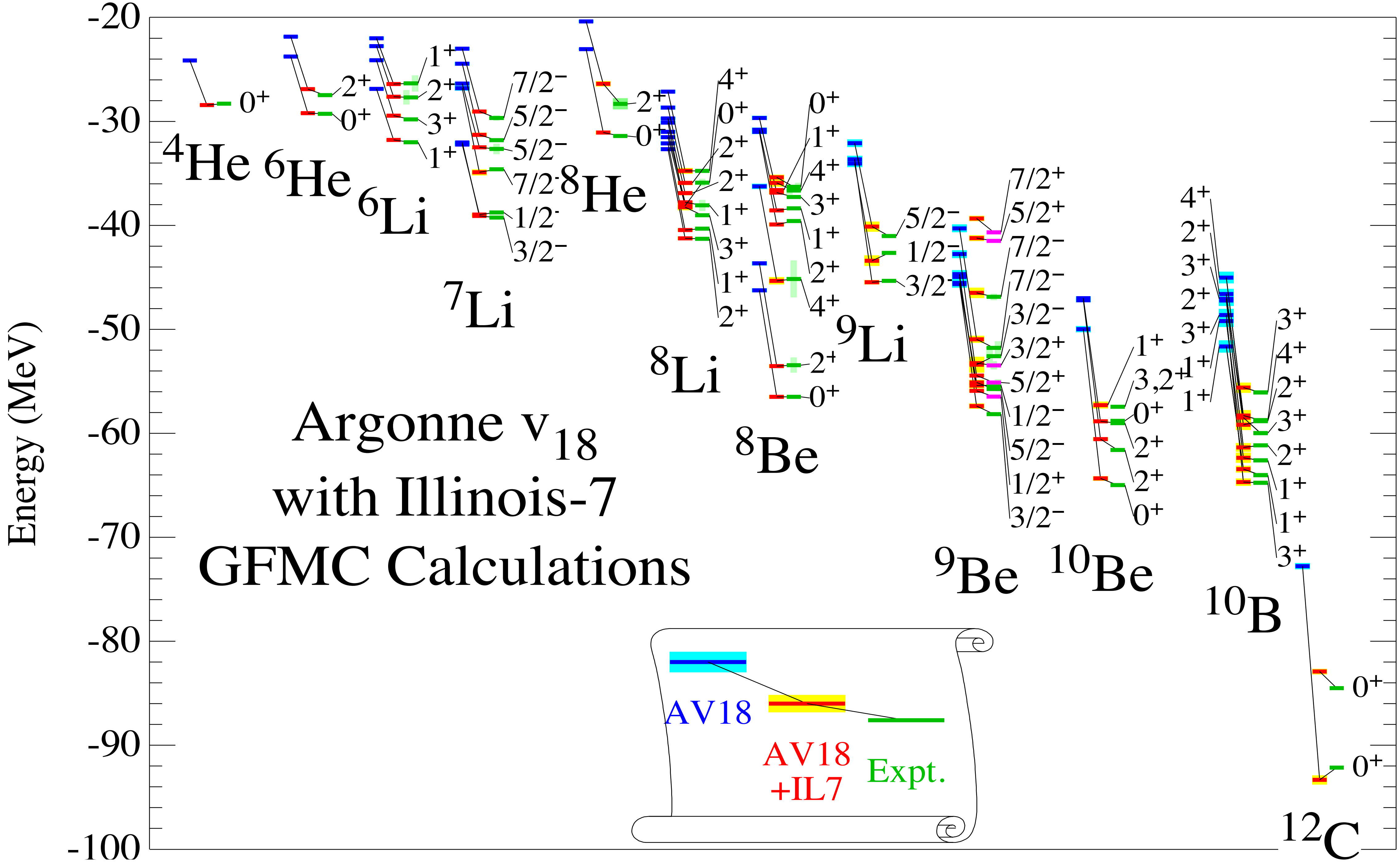
• The analysis of scaling properties of nuclear response functions is useful to unveil information on the underlying nuclear structure and dynamics; singling-out individual-nucleon interactions allows to highlight the many-body aspects of the calculation.

• The study of the behavior of the scaling functions obtained from the Green's Function Monte Carlo calculations, is aimed at elucidating the role of initial and final state correlations in the asymmetric shape of the scaling function.
Although this asymmetry is clearly visible in the experimental scaling functions, independent particle models largely fail to reproduce it.

• Valuable results for nuclear responses have been recently obtained using ab initio methods. These approaches allows for accurate calculations supplemented by theoretical uncertainties. Main limitations: non relativistic approximation, huge computational effort is needed to obtain cross section.

• Ab initio methods can provide strict benchmarks, valuable to constrain more approximate models in the limit of moderate momentum transfer

The Green's Function Monte Carlo approach



- Green's function Monte Carlo combined with a realistic nuclear hamiltonian reproduces the spectrum of ground- and excited states of light and medium heavy nuclei

The Green's Function Monte Carlo approach

- Accurate calculations of the electromagnetic responses of ^{4}He and ^{12}C have been recently performed within GFMC.

- Valuable information on the energy dependence of the response functions can be inferred from the their Laplace transforms

$$E_{\alpha,\beta}(\mathbf{q},\tau) = \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega,\mathbf{q}) \qquad R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle 0|J^\dagger_\alpha(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

- Using the completeness relation for the final states, we are left with ground-state expectations value

$$E_L(\mathbf{q},\tau) = \langle 0|\rho^*(\mathbf{q})e^{-(H-E_0)\tau}\rho(\mathbf{q})|0\rangle - |\langle 0|\rho(\mathbf{q})|0\rangle|^2 e^{-\omega_{el}\tau}$$

$$E_T(\mathbf{q},\tau) = \langle 0|\mathbf{j}^\dagger_T(\mathbf{q})e^{-(H-E_0)\tau}\mathbf{j}_T(\mathbf{q})|0\rangle - |\langle 0|\mathbf{j}_T(\mathbf{q})|0\rangle|^2 e^{-\omega_{el}\tau}$$

- Non relativistic reduction of charge and current operators

$$\rho_i(\mathbf{q}) = \Big[\frac{G_{E,i}(Q^2)}{\sqrt{1+Q^2/4m^2}} - i\frac{(2G_{M,i}(Q^2) - G_{E,i}(Q^2))}{4m^2}\mathbf{q}\cdot(\sigma\times\mathbf{p}_i)\Big]$$

$$\mathbf{j}^T_i(\mathbf{q}) = \Big[\frac{G_{E,i}(Q^2)}{m}\mathbf{p}^T_i - i\frac{G_{M,i}(Q^2)}{2m}\mathbf{q}\times\sigma\Big]$$

Relativistic corrections O(1/m^2)

Scaling in the Fermi gas model

• Scaling of the first kind: the nuclear electromagnetic responses divided by an appropriate function describing the single-nucleon physics no longer depend on the two variables ω and **q**, but only upon $\psi(\mathbf{q},\omega)$

Adimensional variables:

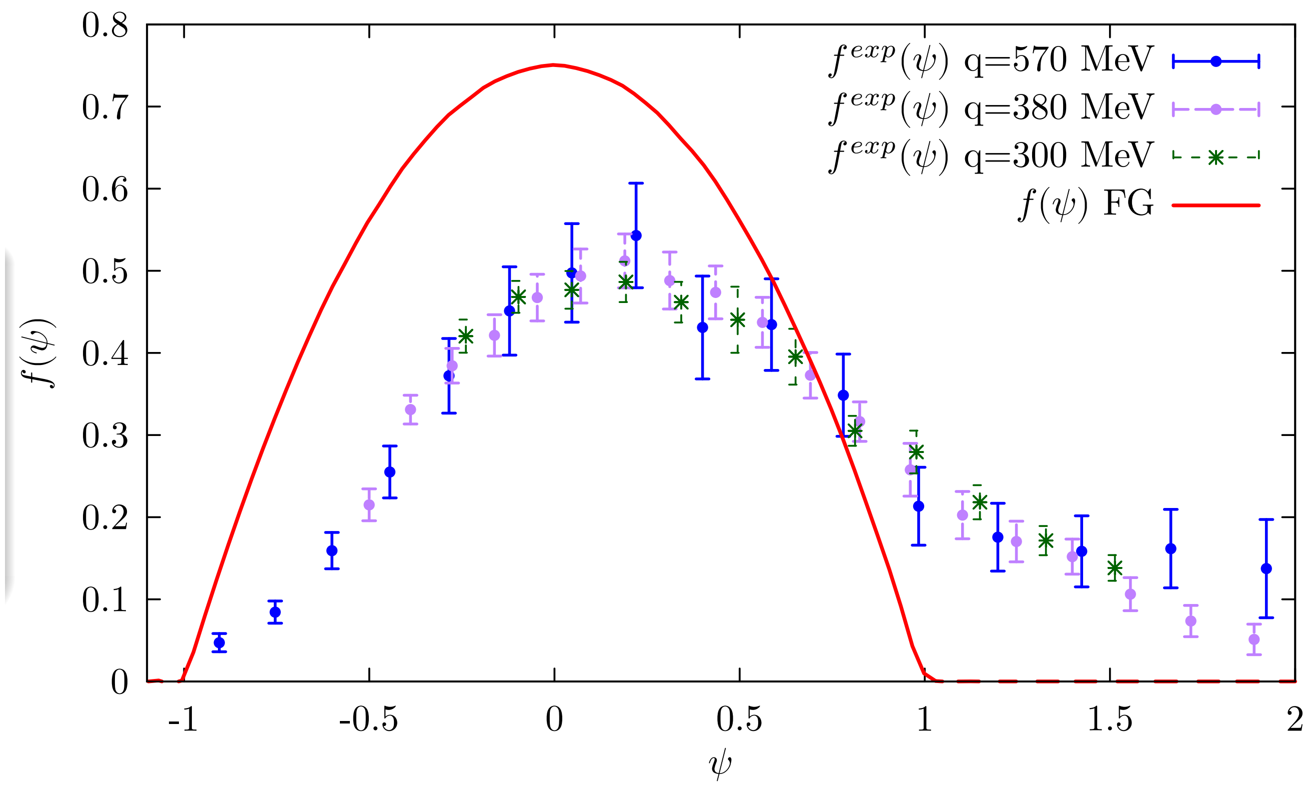
$$\lambda = \omega/2m$$
$$\kappa = |\mathbf{q}|/2m$$
$$\tau = \kappa^2 - \lambda^2$$
$$\eta_F = p_F/m$$
$$\xi_F = \sqrt{p_F^2 + m^2}/m - 1$$

Scaling function:

$$\psi = \frac{1}{\xi_F} \frac{\lambda - \tau}{\sqrt{(1+\lambda)\tau + \kappa\sqrt{\tau(1+\tau)}}}$$

In the FG the L and T responses have the same functional form :

$$R_{L,T} = (1-\psi^2)\theta(1-\psi^2) \times G_{L,T}$$



GFMC longitudinal and transverse scaling functions

- Non relativistic L and T scaling functions:

$$f_{L,T}(\psi^{nr}) = p_F \times \frac{R^{GFMC}_{L,T}}{G^{nr}_{L,T}}$$

$$\psi^{nr} = \frac{1}{\sqrt{2\xi_F^{nr}}}\left(\frac{\lambda}{\kappa} - \kappa\right)$$

- Scaling of the first kind is clearly visible when the effects of nuclear dynamics are singled out

- Asymmetric shape of the scaling functions is present for all the different **q** considered

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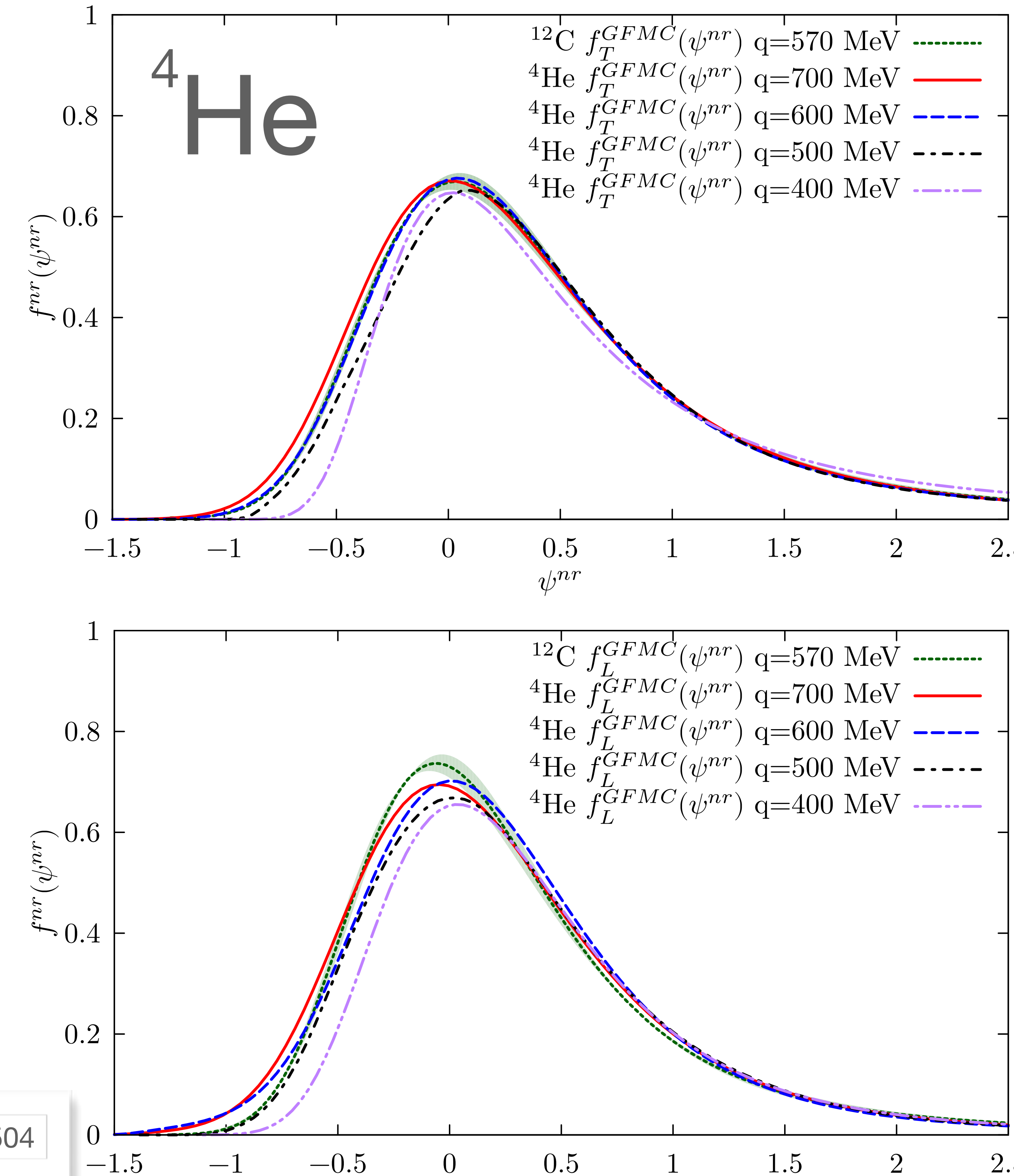
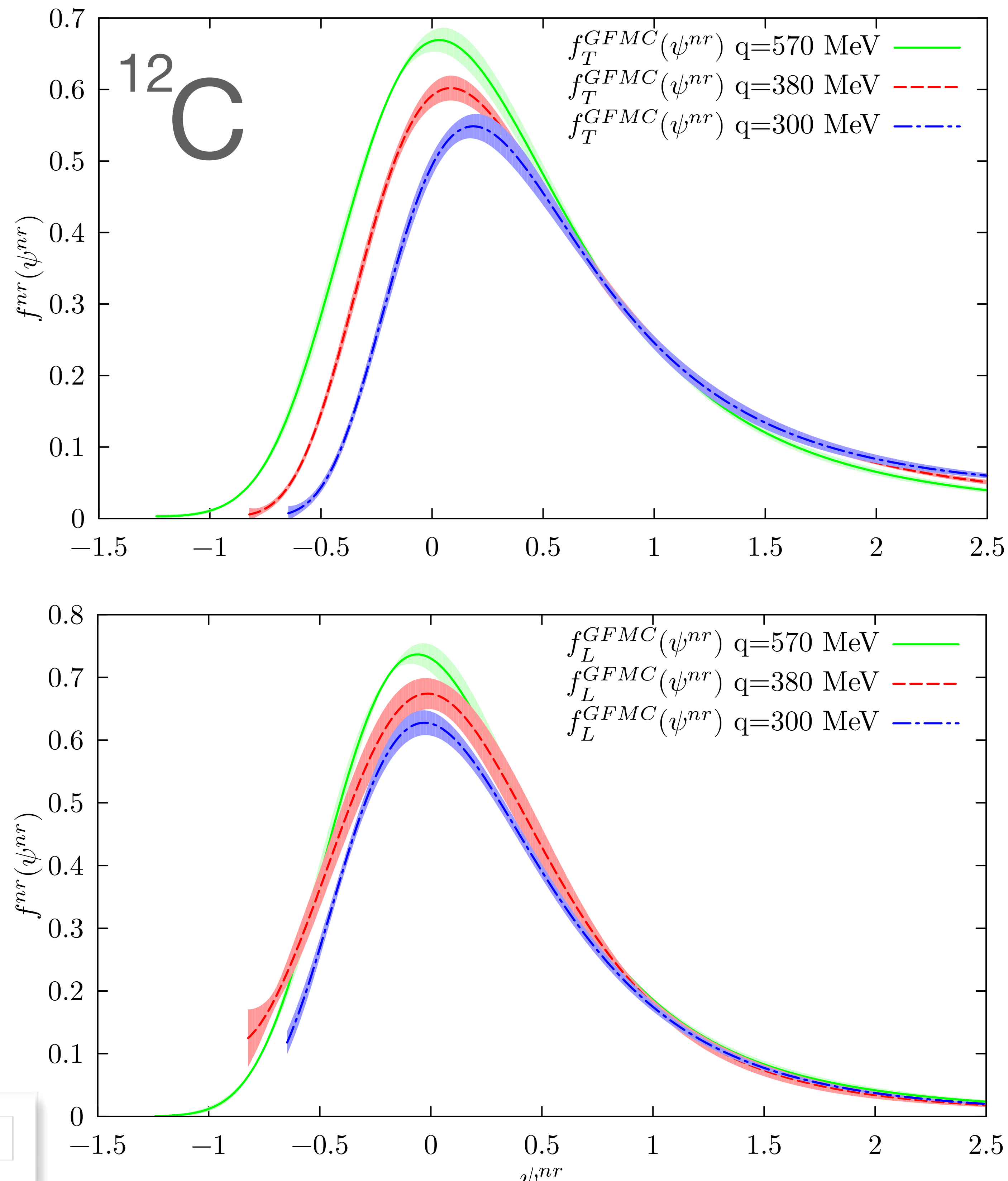


GFMC longitudinal and transverse scaling functions

- In the transverse channel the differences between the three curves for $\psi^{nr} < 0$ suggest that for the kinematical setups considered the scaling function can not be introduced for all values of ω

- The longitudinal response of ^{12}C is known to be affected by the elastic and low lying state transitions. In order to compare with quasi elastic data these are subtracted from the Euclidean response using the experimental excitation energies and transition form factors.

- In the longitudinal case, although theoretical results seem to indicate that first kind scaling occurs, the interpretation of the differences between the three curves is obscured by the residual effect of the low-lying transitions.



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GFMC longitudinal and transverse scaling functions

- This analysis suggests that scaling occurs in the GFMC calculations of the longitudinal and transverse response functions of ^{4}He and ^{12}C

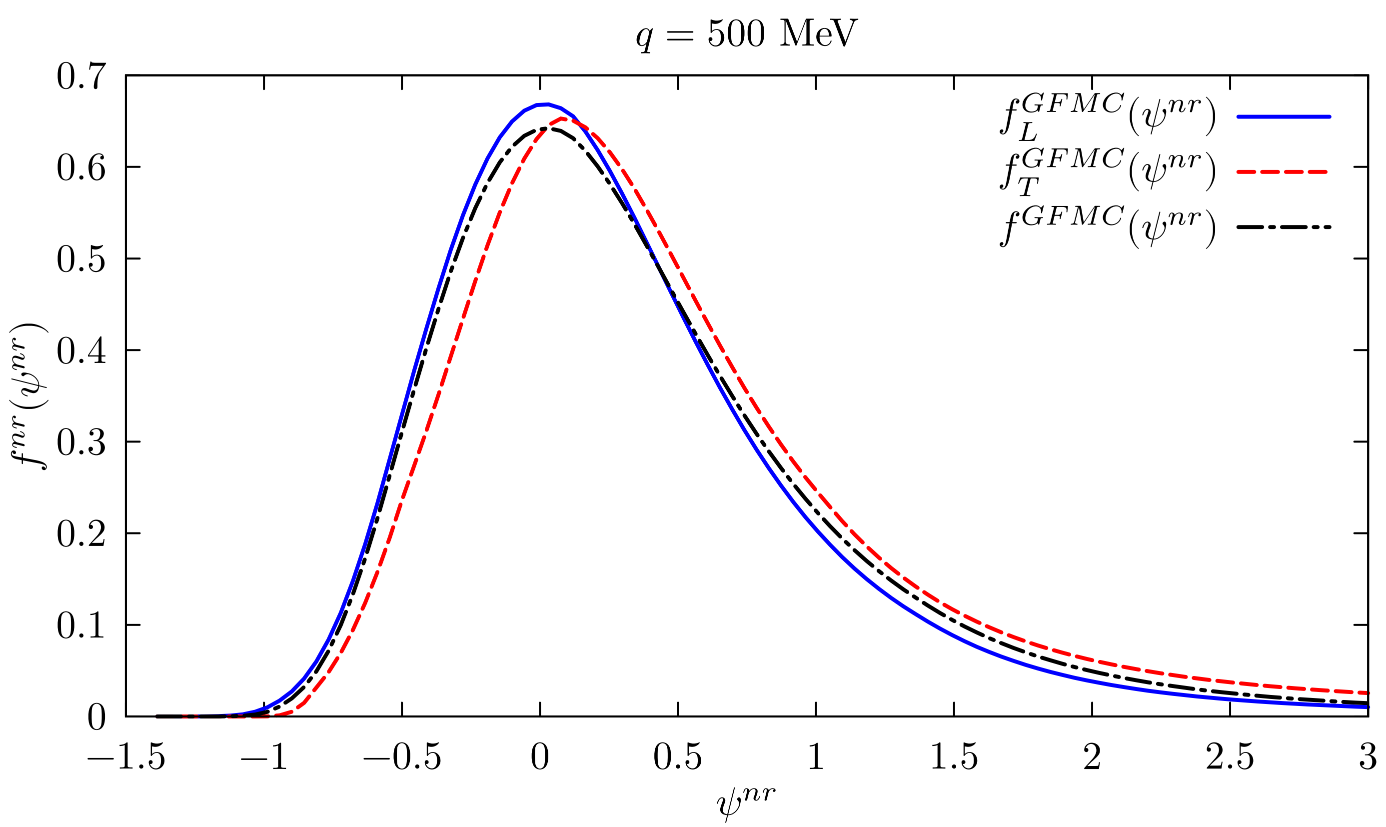
- Comparing the definition of the response functions and the one of the corresponding prefactors, in the limit of large momentum transfer, a novel interpretation of the scaling function emerges….

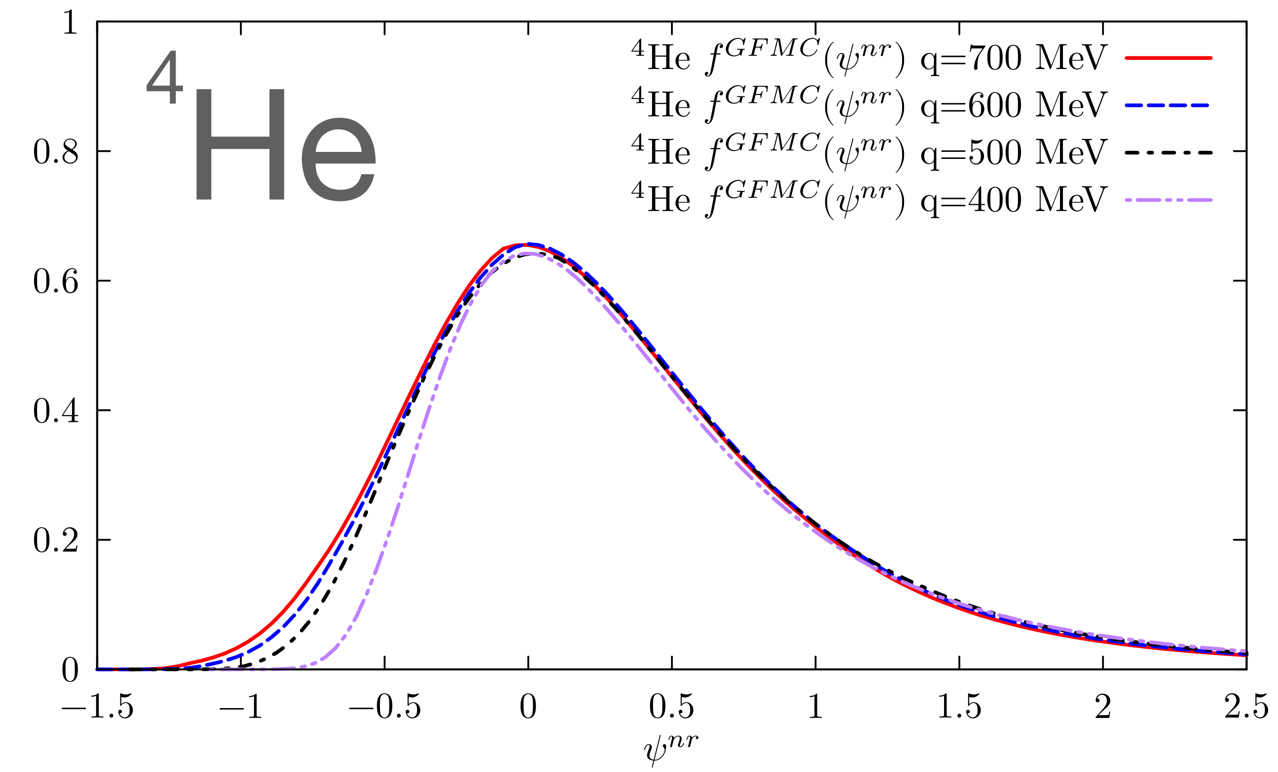
$$f_{p(n)} \propto 2\kappa \times \sum_f \langle 0|\rho^\dagger_{p(n)}(\mathbf{q})|f\rangle\langle f|\rho_{p(n)}(\mathbf{q})|0\rangle\delta(E_0+\omega-E_f)$$

$$\rho_{p(n)} = \sum_i e^{i\mathbf{q}\cdot\mathbf{r}}\frac{(1\pm\tau_{i,z})}{2}$$

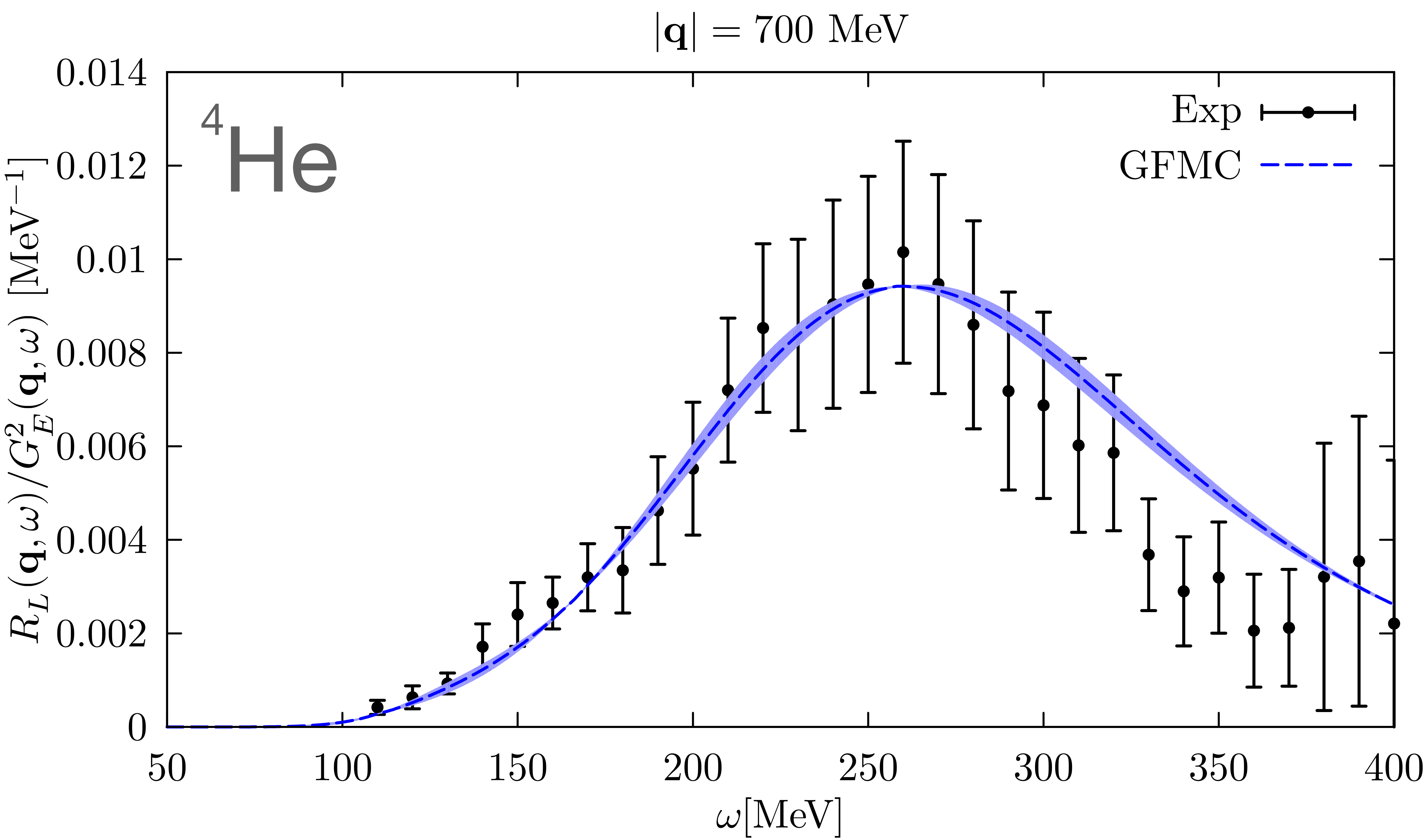
proton (neutron) density operator

…in terms of the proton (neutron) response function





Limits of applicability of non relativistic approximation



- Goal: extend the range of applicability of non relativistic calculations in the quasi-elastic region .

Relativistic aspects of nuclear dynamics

- One manifestation of the importance of relativistic effects is the frame dependence that occurs in the non relativistic calculations at high **q**.

- A genuine relativistic calculation can be performed in any frame and lead to the same LAB frame result. This is not true for the non relativistic case.

- An appropriate choice of the reference frame can minimize the error introduced by using non relativistic kinematics

- We applied a procedure to the GFMC results in order to reduce the frame dependence in the quasi elastic peak region

PHYSICAL REVIEW C **72**, 011002(R) (2005)

Improved (e, e') response functions at intermediate momentum transfers: The ^{3}He case

Victor D. Efros,[1,*] Winfried Leidemann,[1] Giuseppina Orlandini,[1] and Edward L. Tomusiak[2]

Relativistic aspects of nuclear dynamics

- In the one photon exchange approximation, the inclusive electron scattering cross section in the LAB frame is given by

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{|\mathbf{q}|^4} R_L(\mathbf{q},\omega) + \left(\frac{Q^2}{2|\mathbf{q}|^2} + \tan^2\frac{\theta}{2} \right) R_T(\mathbf{q},\omega) \right]$$

- One may define related responses $R_L^{fr}(\mathbf{q},\omega)$ in a reference frame obtained by boosting the LAB frame along **q**.

$$R_L^{fr} = \sum_f \left| \langle \psi_i | \sum_j \rho_j(\mathbf{q}^{fr},\omega^{fr}) | \psi_f \rangle \right|^2 \delta(E_f^{fr} - E_i^{fr} - \omega^{fr})$$

- The delta function can be written in terms of the center of mass and internal energies of the initial and final states as

$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) \approx \delta[e_f^{fr} + (P_f^{fr})^2/(2M_T) - e_i^{fr} - (P_i^{fr})^2/(2M_T) - \omega^{fr}] \equiv \delta[e_f^{fr} - e_f^{nr}(q^{fr},\omega^{fr})]$$

internal energies of the final and initial state

Relativistic aspects of nuclear dynamics

- The LAB frame responses can be expressed in terms of $R_L^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr}),\ R_T^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr})$ using

$$R_L(\mathbf{q}, \omega) = \frac{\mathbf{q}^2}{(\mathbf{q}^{fr})^2} \frac{E_i^{fr}}{M_0} R_L^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr}) \qquad R_T(\mathbf{q}, \omega) = \frac{E_i^{fr}}{M_0} R_T^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr})$$

- The variables in the new reference frame can be obtained via

$$q^{fr} = \gamma(q - \beta\omega),\ \omega^{fr} = \gamma(\omega - \beta q),\ P_i^{fr} = -\beta\gamma M_0,\ E_i^{fr} = \gamma M_0$$

- In the relativistic case, the responses computed in different reference frames would lead to the same result for $R_L(\mathbf{q}, \omega),\ R_T(\mathbf{q}, \omega)$. However this is not the case when the non relativistic kinematics is used

- We will analyze the results obtained in four different reference frames:
LAB, Anti LAB, Breit, Active-Nucleon Breit

Relativistic aspects of nuclear dynamics

LAB:

$$P_i^{fr} = 0$$
$$P_f^{fr} = \mathbf{q}^{fr}$$
$$p_{Nf}^{fr} = \mathbf{q}$$
$$\mathbf{q}^{fr} = \mathbf{q}$$
$$\beta = \frac{q}{M_0 + \omega}$$

- In the LAB frame, the momentum of the active nucleon is the largest

Anti-LAB:

$$P_i^{fr} = -\mathbf{q}^{ALAB}$$
$$P_f^{fr} = 0$$
$$p_{Nf}^{fr} = \frac{A-1}{A}\mathbf{q}^{ALAB}$$
$$\beta = \frac{q^{ALAB}}{M_0 + \omega}$$

- The momentum of the active nucleon is $\approx q$

Breit:

$$P_i^{fr} = -\frac{\mathbf{q}^B}{2}$$
$$P_f^{fr} = \frac{\mathbf{q}^B}{2}$$
$$p_{Nf}^{fr} = \frac{2A-1}{2A}\mathbf{q}^B$$
$$\beta = \frac{q^B}{2M_0 + \omega}$$

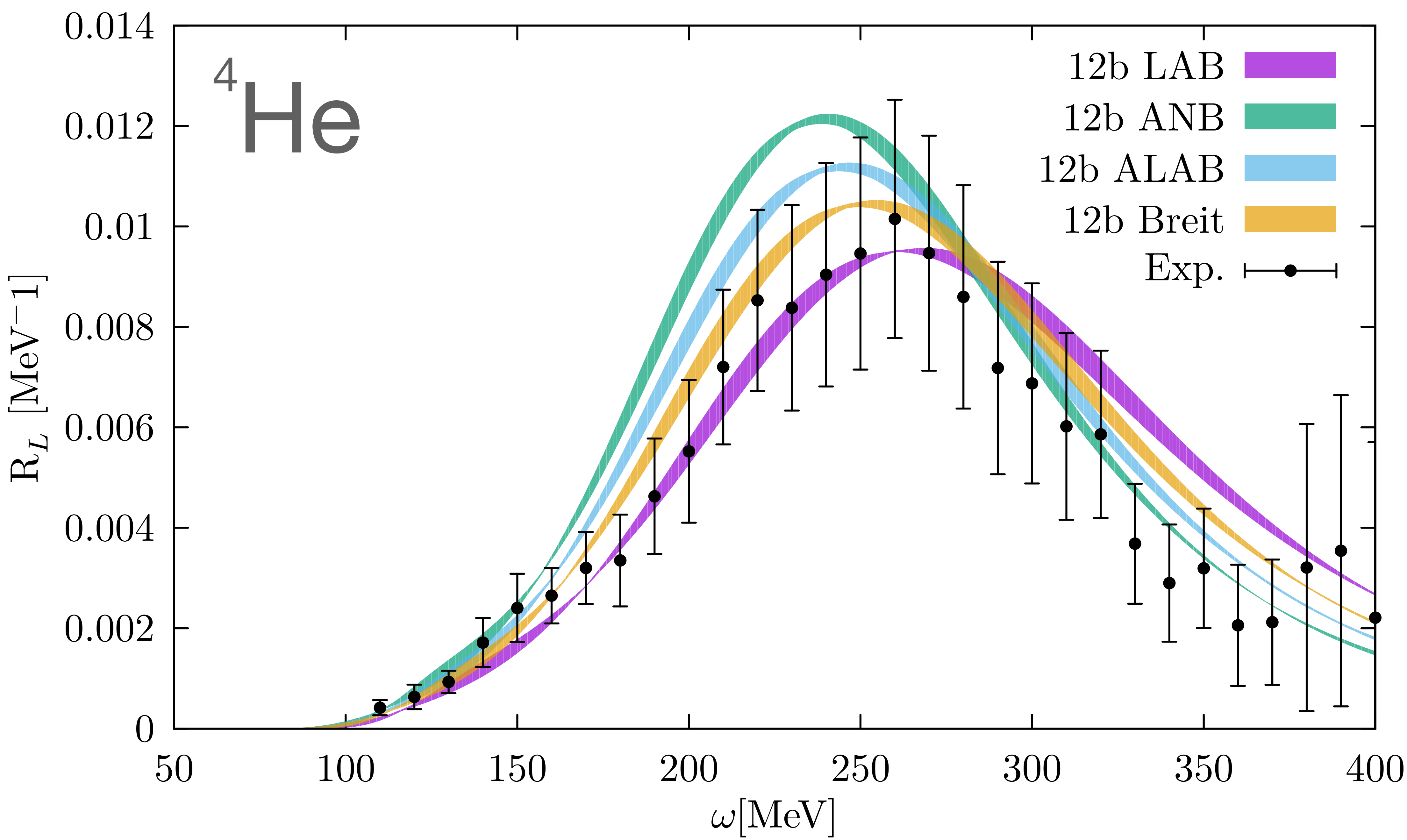
- The Breit frame minimizes the sum of the center of mass kinetic energies of the initial and final state

Active nucleon Breit:

$$P_i^{fr} = -\frac{A\mathbf{q}^{ANB}}{2}$$
$$P_f^{fr} = -\frac{(A-2)\mathbf{q}^{ANB}}{2}$$
$$p_{Nf}^{fr} = \frac{\mathbf{q}^{ANB}}{2}$$
$$\beta = \frac{q^{ANB}}{2M_0/A + \omega}$$

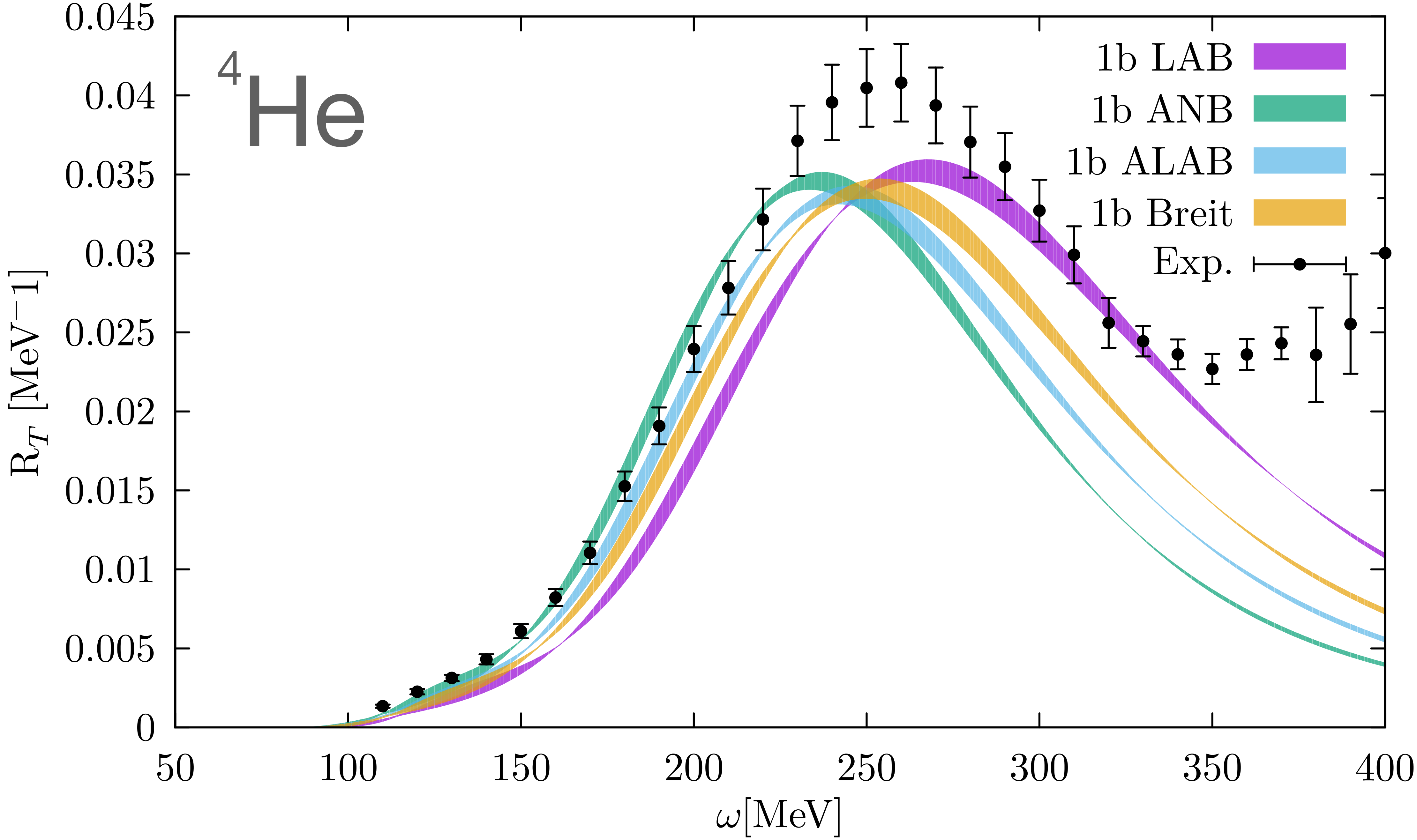
- ω^{ANB} at the QE peak is 0. This applies both to the relativistic and non relativistic case

Relativistic aspects of nuclear dynamics



- Longitudinal responses of ^{4}He for |q|=700 MeV in the four different reference frames. The curves show differences in both peak positions and heights.

Relativistic aspects of nuclear dynamics



- Transverse responses of ^{4}He for |q|=700 MeV in the four different reference frames. The curves show differences in both peak positions and heights.

Two-fragment model

- The frame dependence can be drastically reduced if one assumes a two-body breakup model with relativistic kinematics to determine the input to the non relativistic dynamics calculation

- The two-fragment model assumes a quasi elastic knock-out of a nucleon such that the residual nucleus remains in its lowest energy state

- We have to deal with a two-body problem where the center of mass and relative momentum of the two fragments are given by

$$p^{fr} = \mu\left(\frac{p_N^{fr}}{m_N} - \frac{p_X^{fr}}{M_X}\right)$$

$$P_f^{fr} = p_N^{fr} + p_X^{fr}$$

where

$$\mu = \frac{m_N M_X}{m_N + M_X}$$

- The value of $\mathbf{p}^{fr}$ can be obtained from : $\omega^{fr} = E_f^{fr} - E_i^{fr}$

where

$$E_f^{fr} = \sqrt{m_N^2 + [\mathbf{p}^{fr} + \mu/M_X \mathbf{P}_f^{fr}]^2} + \sqrt{M_X^2 + [\mathbf{p}^{fr} - \mu/m_N \mathbf{P}_f^{fr}]^2}$$

Two-fragment model

- We use the relative momentum, derived in a relativistically correct way, to compute the non relativistic internal energy:

$$e_f^{fr} = (p^{fr})^2/(2\mu)$$

- The energy conserving delta function entering the definition of the responses reads

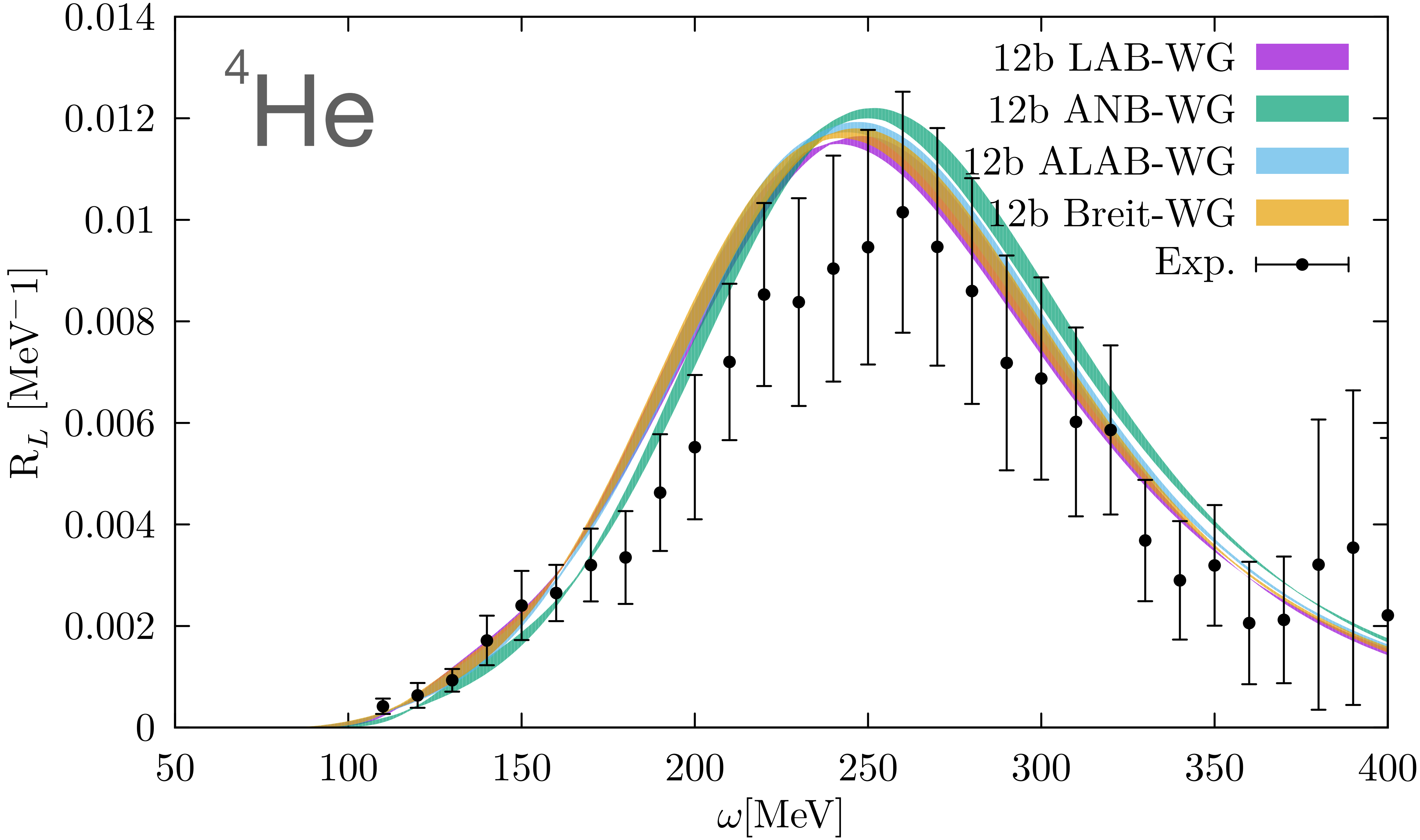
$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) = \delta(F(e_f^{fr}) - \omega^{fr}) = \Big(\frac{\partial F^{fr}}{\partial e_f^{fr}}\Big)^{-1} \delta[e_f^{fr} - e_f^{rel}(q^{fr}, \omega^f)]$$

with

$$\delta(F(e_f^{fr}) - \omega^{fr}) = \frac{p^{fr}}{\mu}\Big(\frac{\partial E_f}{\partial p^{fr}}\Big)^{-1}$$

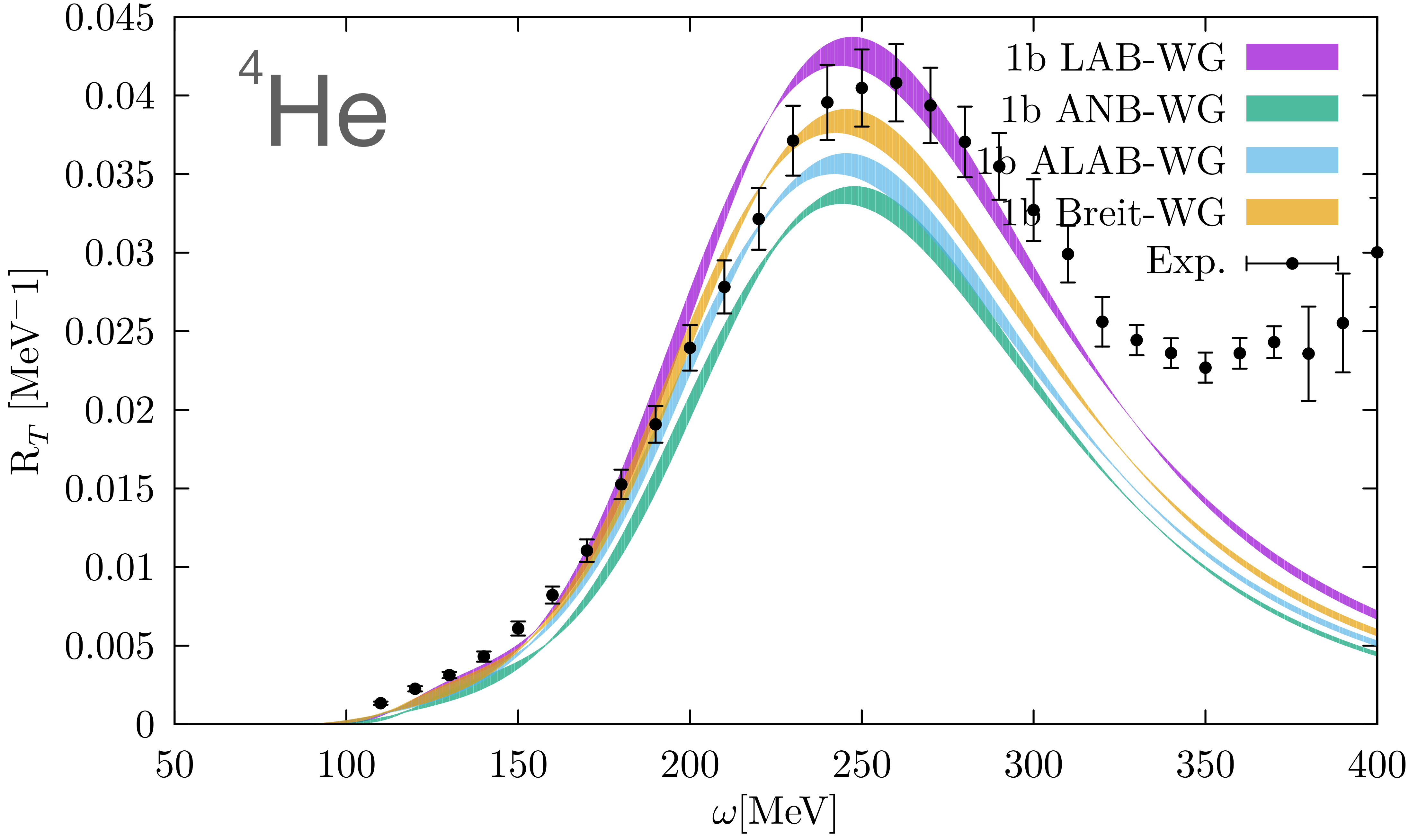
- The two-body model is only used to determine the kinematic input of the calculation.

Two-fragment model



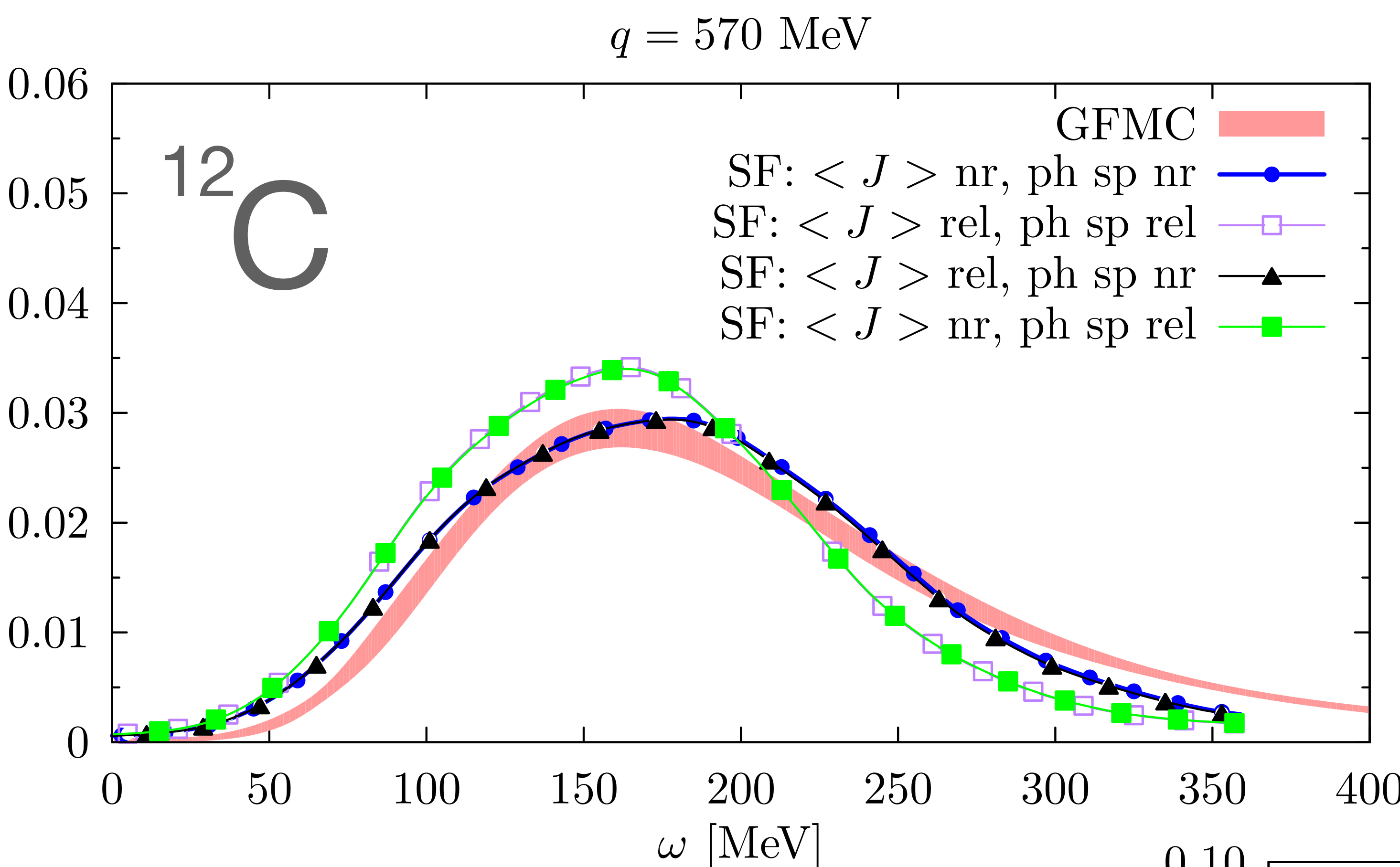
- Longitudinal responses of ^{4}He for |q|=700 MeV in the four different reference frames. The different curves are almost identical.

Two-fragment model



- Transverse responses of ^{4}He for |q|=700 MeV in the four different reference frames. The different curves show different heights, relativistic effects in the current operator?!

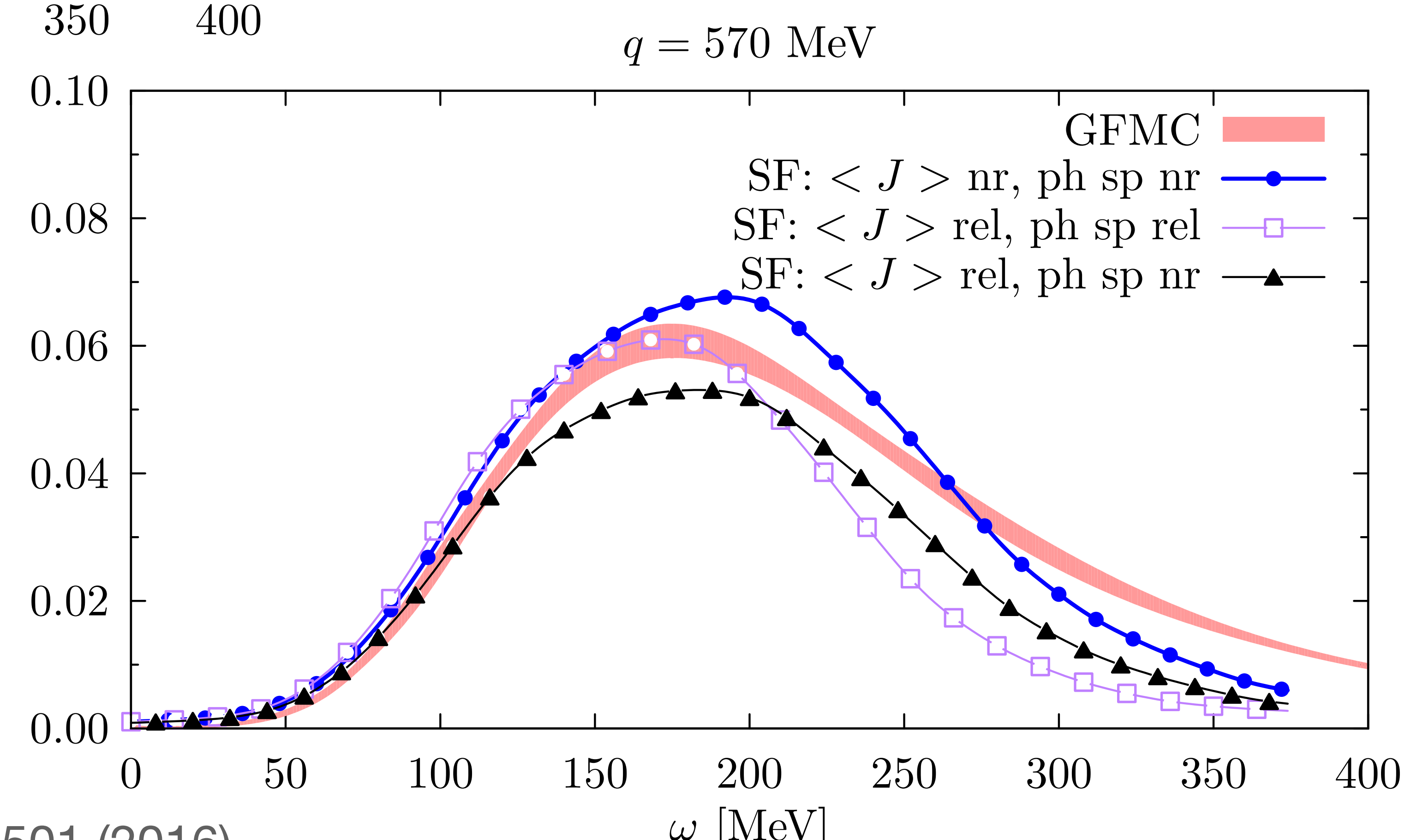
Comparison with the SF results



$$R^{\alpha\beta} = \int dE d\mathbf{k} P(\mathbf{k}, E) \sum_i \langle k | j_i^{\alpha\dagger} | k+q \rangle \langle k+q | j_i^{\beta} | k \rangle$$
$$\times \frac{m^2}{E(k)E(k+q)} \delta(\omega - E + m - E(k+q))$$

phase space

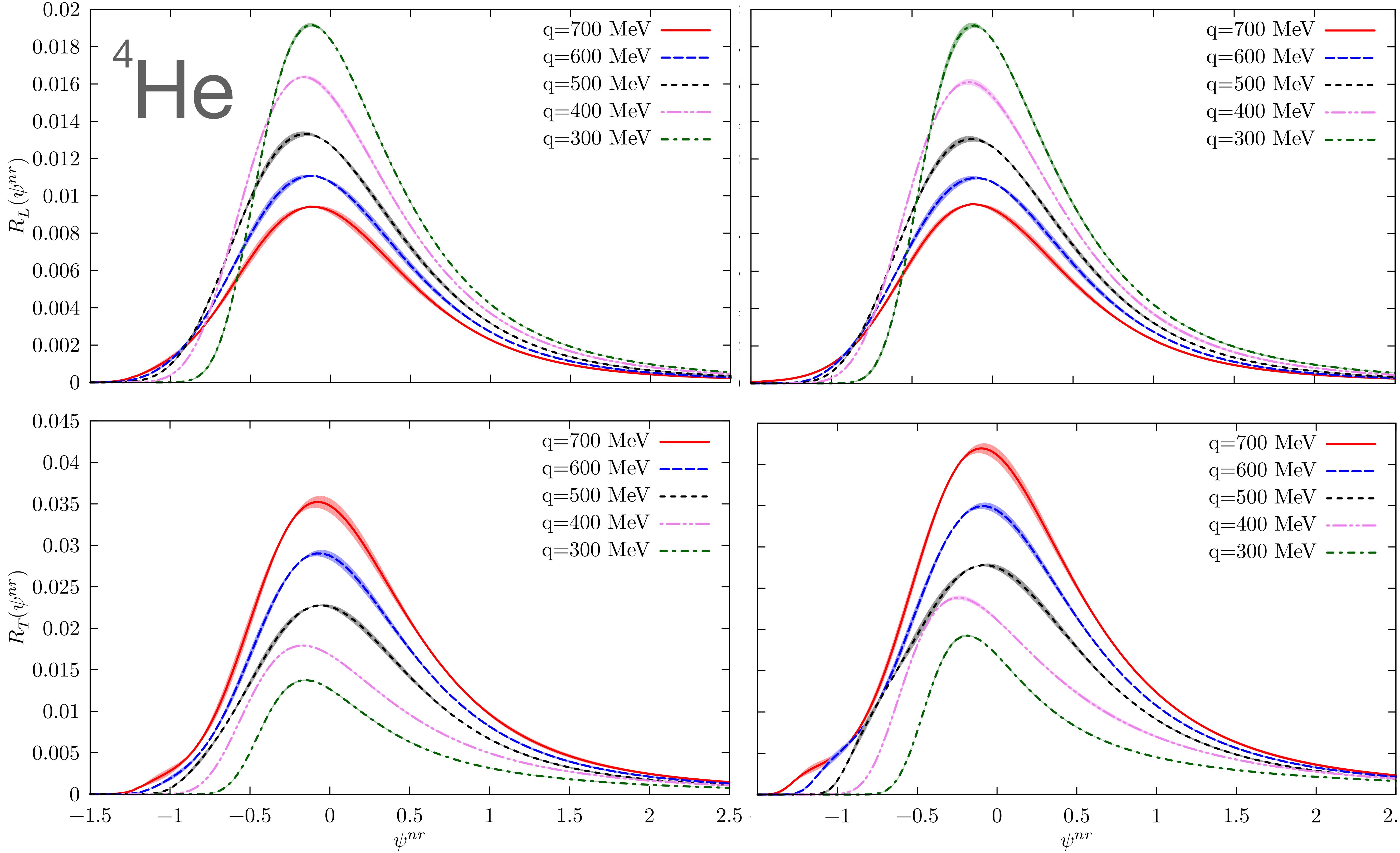
- The hybrid calculations show that relativistic corrections in the current operator lead to a reduction of the strength while using relativistic kinematics enhances it
- In the transverse might be important to include higher order contributions



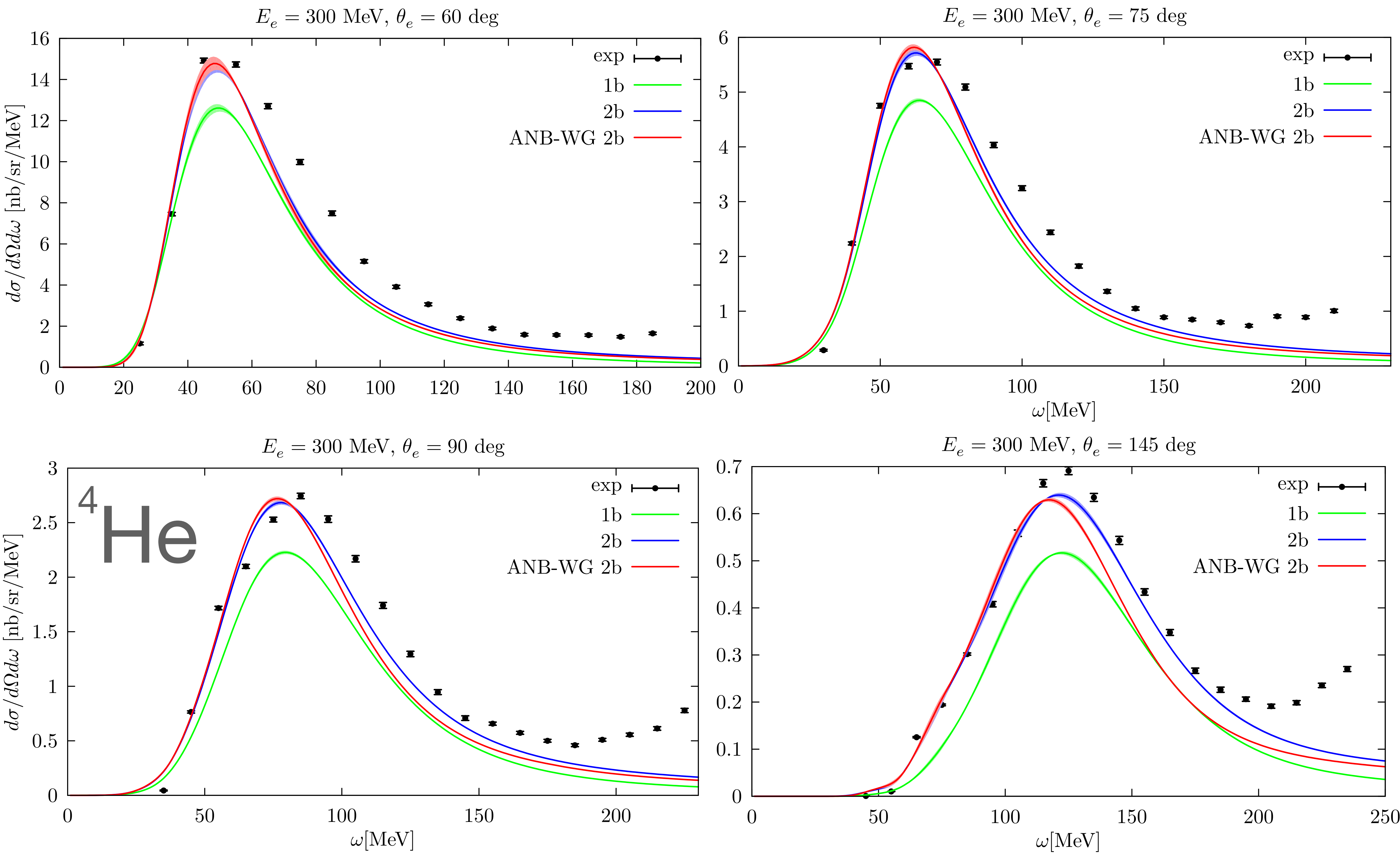
★ N.R, A. Lovato, and O. Benhar PRC 94, 065501 (2016)

Scaling as a tool to interpolate the responses

- 1 body contribution
- 1+2 body contribution



We use the interpolation to obtain cross sections



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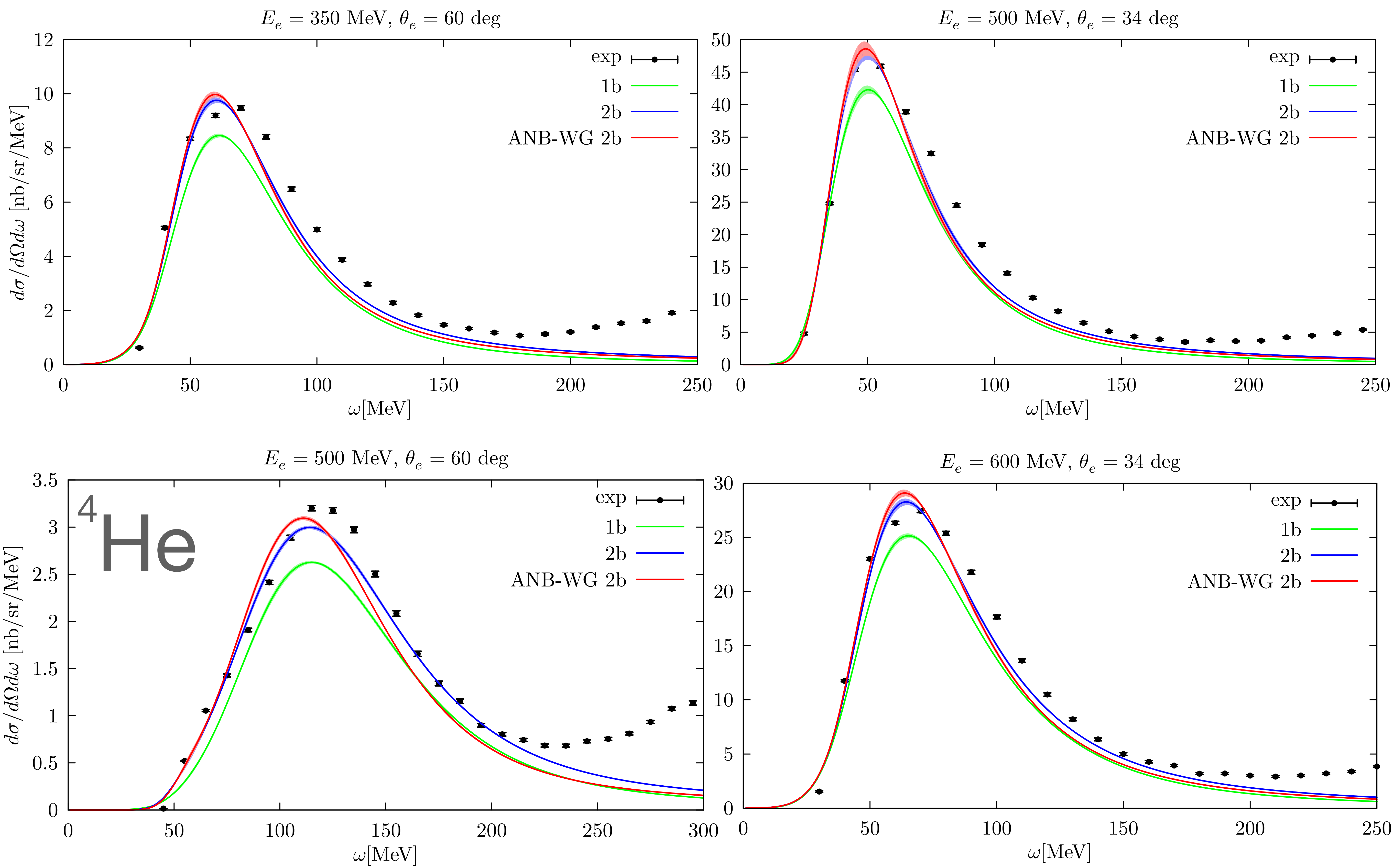
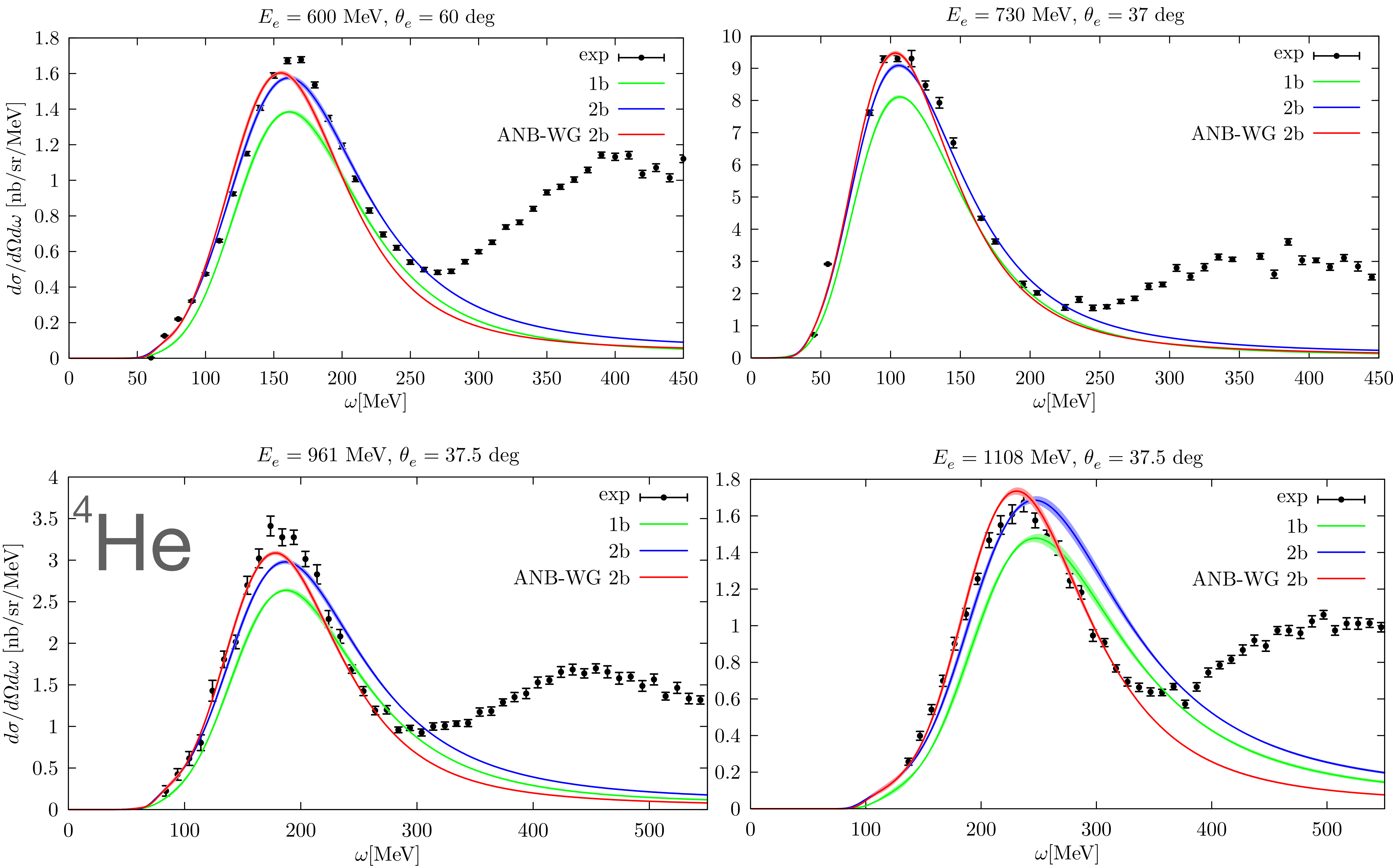


We use the interpolation to obtain cross sections



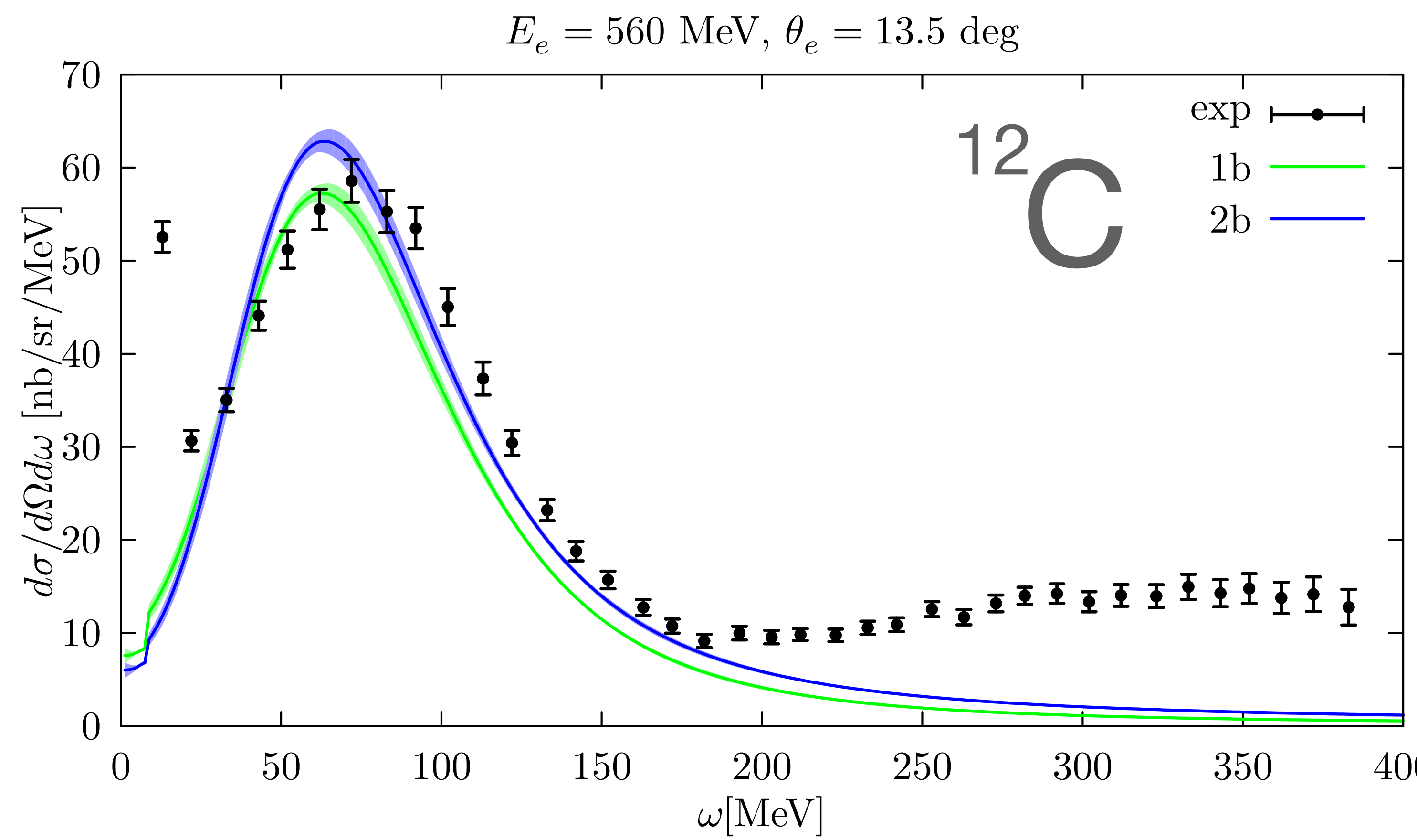
Prospects….

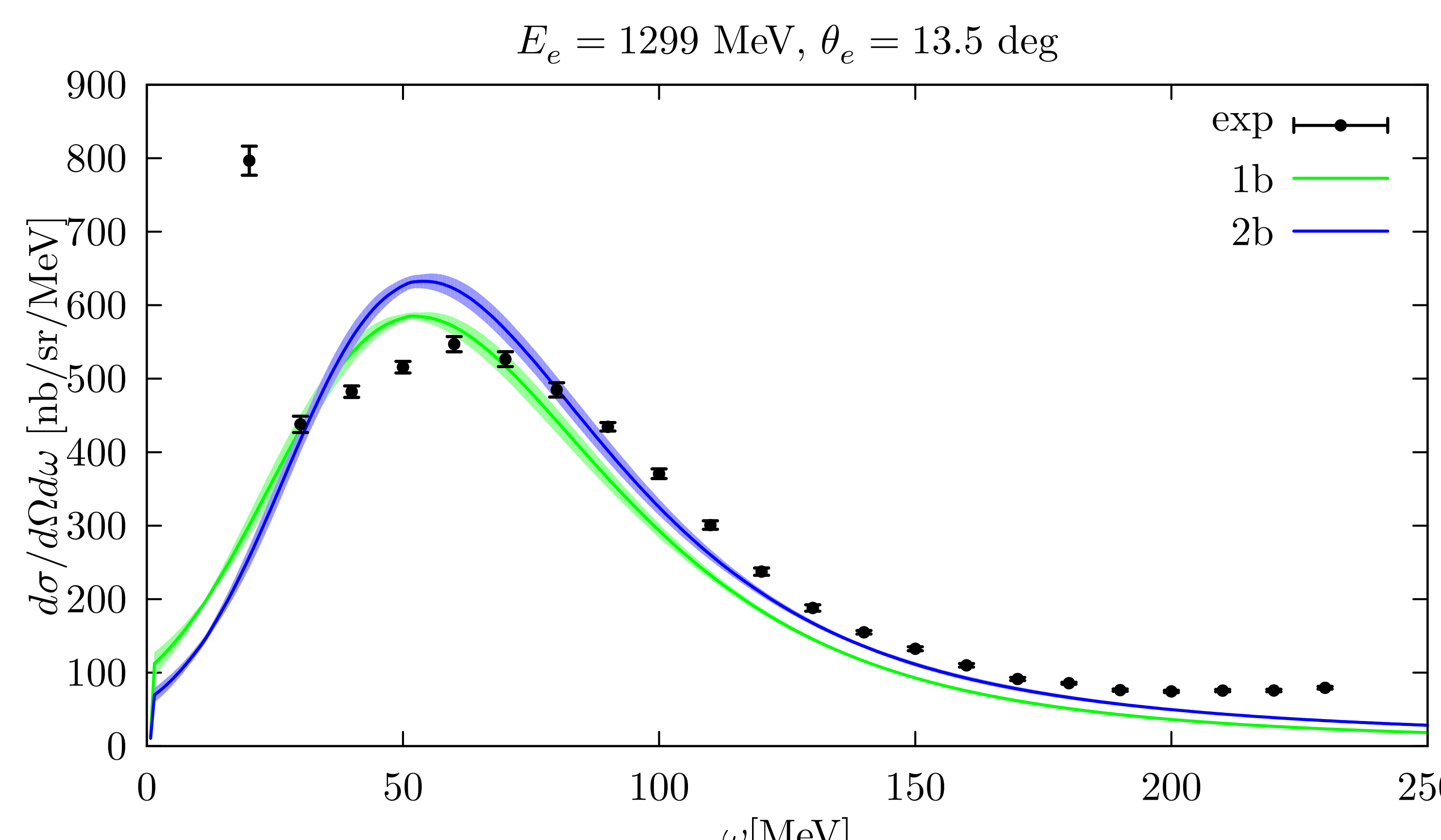
⋆ In collaboration with A. Lovato….

- We obtained the double differential cross sections for ^{12}C but the situation is more controversial than in the ^{4}He case.

- Right now we used only 3 kinematical setup, corresponding to |q|= 300, 380, 570 MeV. Results for the inversion of the Euclidean response at |q|=500, 700 MeV have just been obtained. This will improve the reliability of the interpolations.

- A deeper understanding of how to treat (subtract or not and how to do it) the low-lying excitations and the elastic state is very important.





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Few things that might be interesting for neutrino experiments….

⋆ In collaboration with A. Lovato, A. Sobczyk, and J. Nieves….

• We used the novel interpretation of the scaling function we provided to compare two different approaches for the Spectral Function calculations (CBF+FSI and semi-phenomenological).
We further investigated the origin of scaling and the presence of asymmetry by means of a toy model.

⋆ In collaboration with A. Sobczyk….

• We recently studied the quasi-elastic weak production of Λ and Σ hyperons from nucleons and nuclei induced by antineutrinos in the intermediate energy region.

• The final state interaction effects due to hyperon nucleon scattering have been estimated with the help of a Monte Carlo code for propagation of hyperons in the nuclear medium using as input the scarce available experimental cross sections for the hyperon-nucleon scattering cross sections.

⋆ In collaboration with C. Barbieri….

• We obtained the momentum distribution and spectral function of closed shell nuclei within the Self Consistent Green's Function approach. We exploited the Impulse Approximation to compute the double differential cross sections of ^{4}He and ^{16}O in a variety of kinematical setups

• We are working on the extension the calculation for open shell nuclei (the goal is ^{40}Ar) using the Gorkov formalism.

Thank you!