Joanna Sobczyk

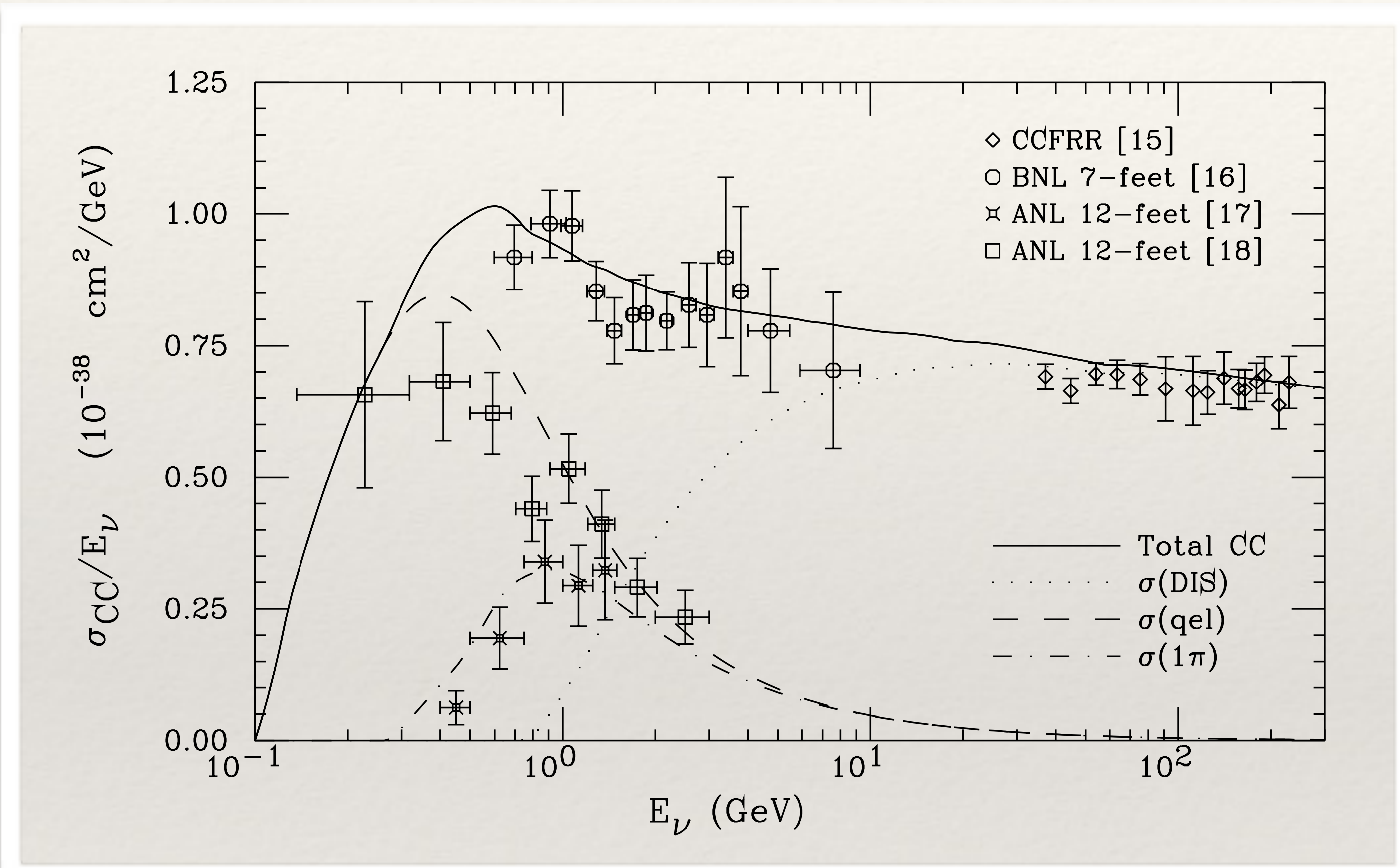
Semiphenomenological spectral functions in neutrino scattering

(J. Nieves and JS, Annals of Physics 383C (2017) pp. 455-496)

Outline

- Motivation
- Formalism
  - Semiphenomenological Spectral Functions
  - RPA & Spectral functions
- Results & comparisons:
  - low-energy processes: muon capture, pion capture, neutrino scattering
  - intermediate and high energy transfer
- Summary

Motivation



P. Lipari et al, Phys. Rev. Lett. 74 (1995) 4384

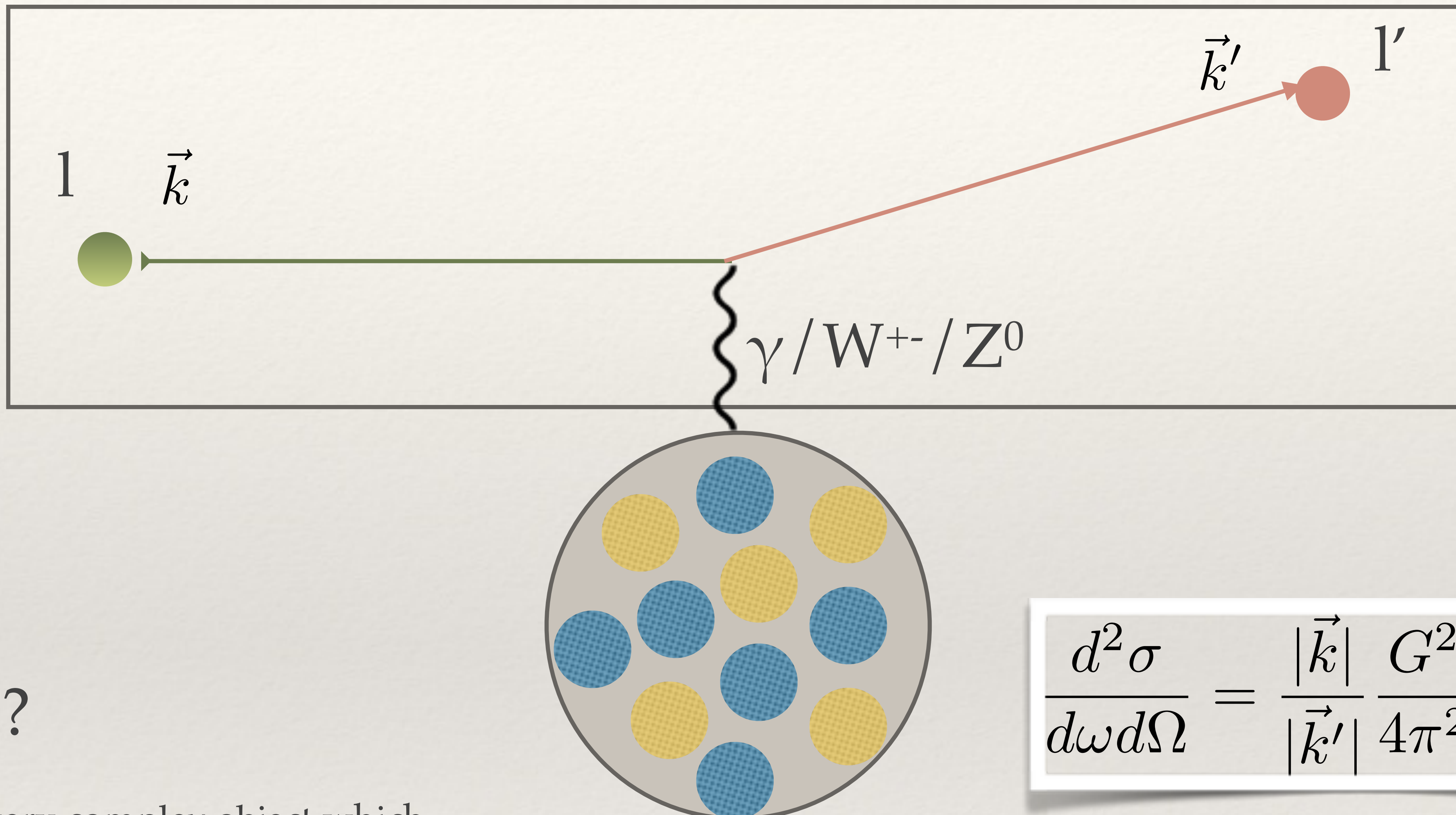
*"We will argue that it is possible to improve on the description of the cross sections (…) including a more careful treatment of the lowest multiplicity channels (**quasi–elastic scattering** and **single pion production**)"*

- We need to control nuclear effects in order to control the axial part of the interaction -> estimate neutrino cross-sections.
- Various neutrino experiments - different targets: ^{12}C, ^{16}O, ^{40}Ar, …
- Use models that work well for the electron scattering (control the vector part of the interaction)

Framework

$L^{\mu\nu}$

lepton tensor



$$\frac{d^2\sigma}{d\omega d\Omega} = \frac{|\vec{k}|}{|\vec{k}'|}\frac{G^2}{4\pi^2}L_{\mu\nu}W^{\mu\nu}$$

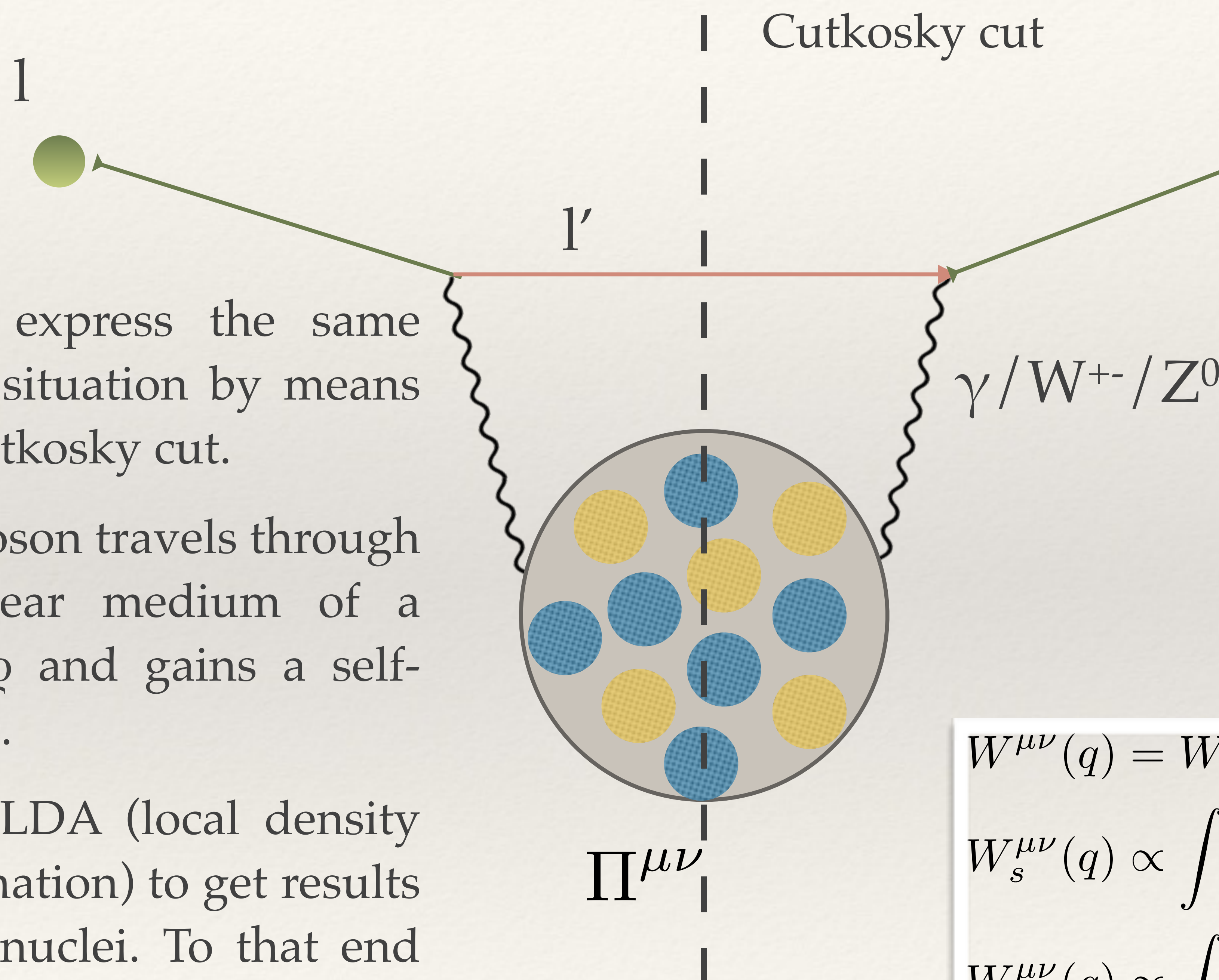
$W_{\mu\nu} = ?$

hadron tensor

very complex object which should account for many processes

We want to calculate inclusive cross section for neutrino scattering off the nucleus.

Framework



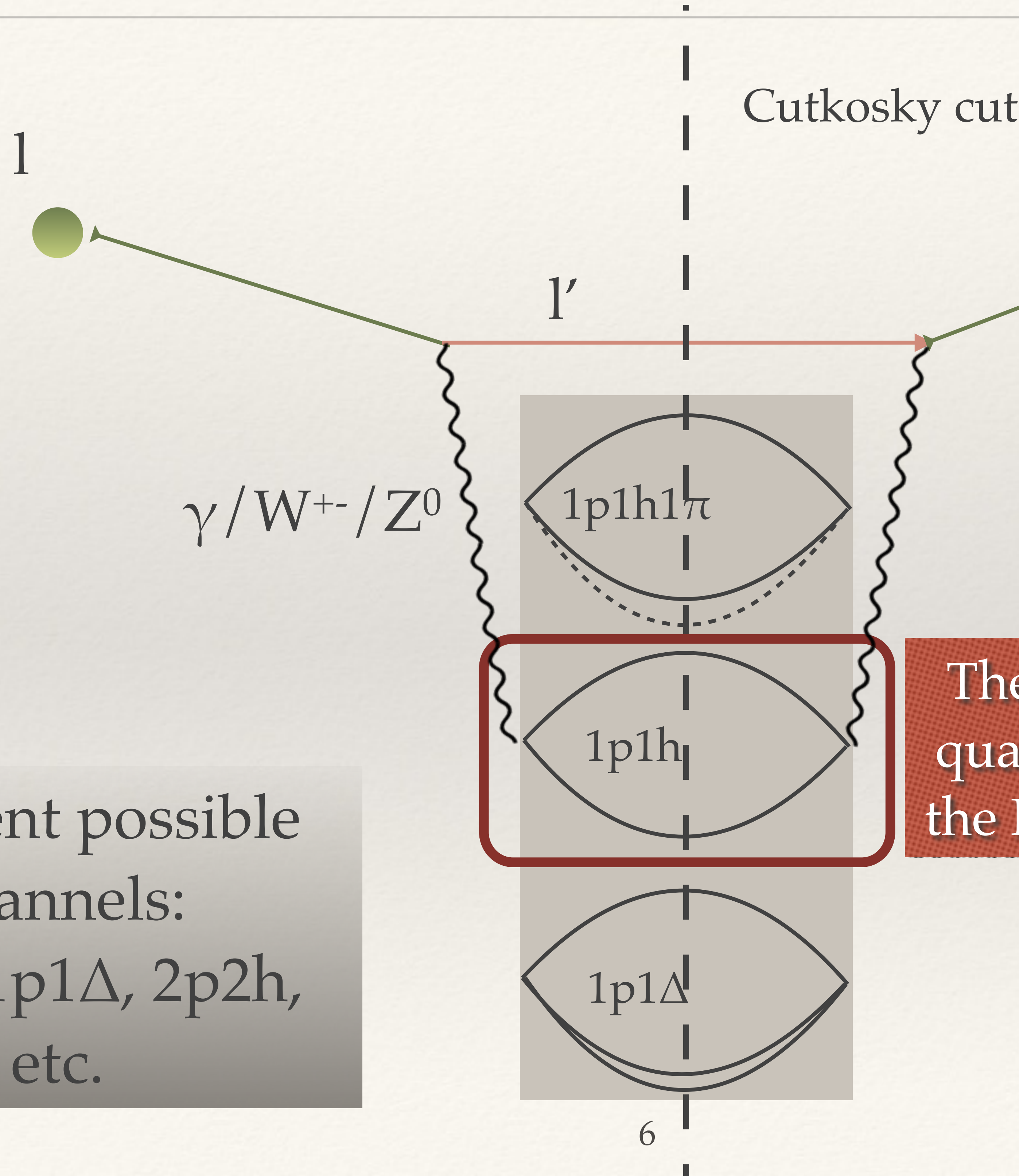
- We can express the same physical situation by means of the Cutkosky cut.
- Gauge boson travels through the nuclear medium of a density ϱ and gains a self-energy Π.
- Use the LDA (local density approximation) to get results in finite nuclei. To that end integrate over a nucleus density profile.

$$W^{\mu\nu}(q) = W_s^{\mu\nu}(q) + iW_a^{\mu\nu}(q)$$

$$W_s^{\mu\nu}(q) \propto \int \frac{d^3r}{2\pi} \mathrm{Im}(\Pi^{\mu\nu} + \Pi^{\nu\mu})$$

$$W_a^{\mu\nu}(q) \propto \int \frac{d^3r}{2\pi} \mathrm{Re}(\Pi^{\mu\nu} - \Pi^{\nu\mu})$$

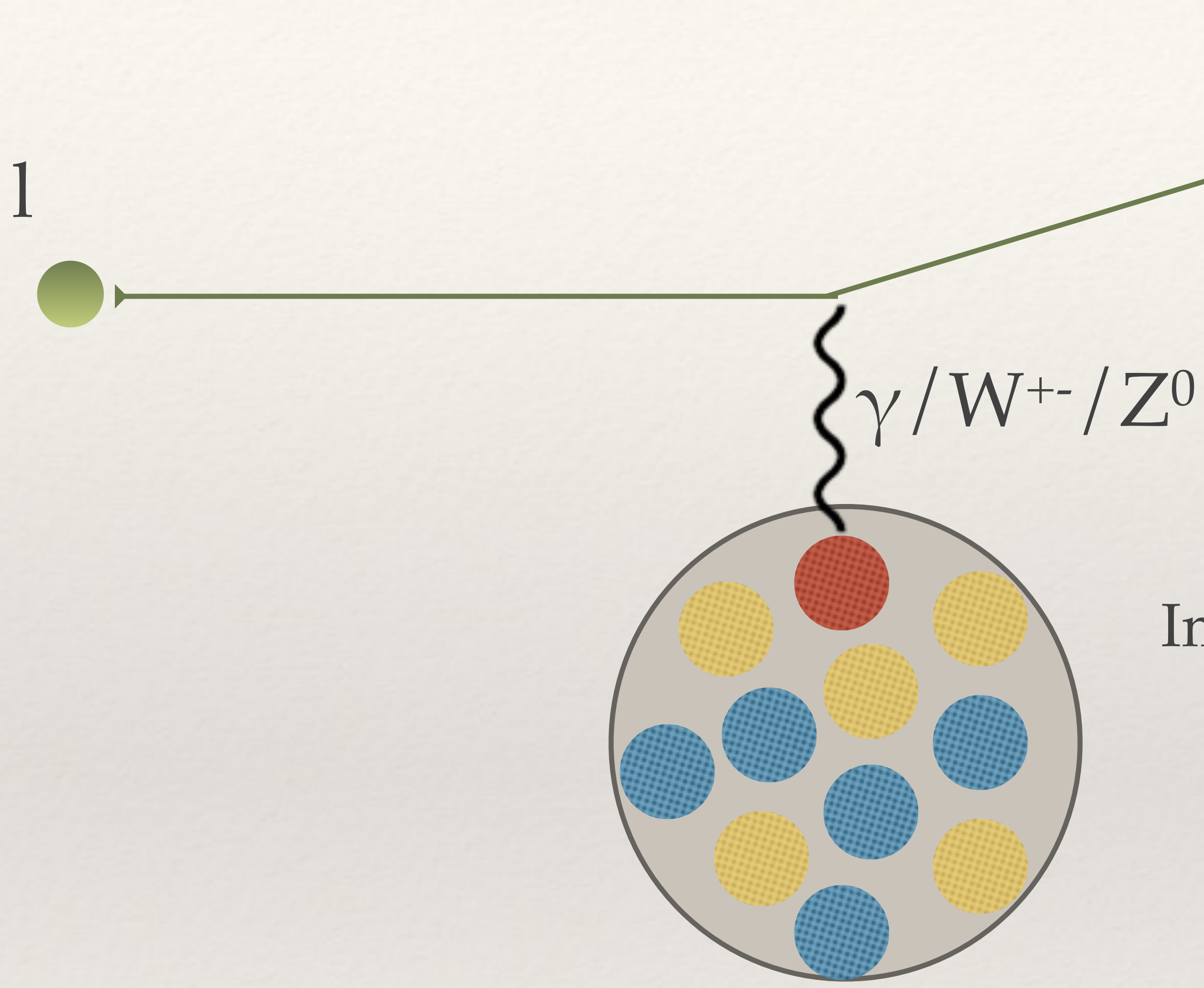
Framework - different channels



The main topic of this talk: quasi-elastic mechanism (in the Impulse Approximation)

Different possible channels: 1p1h, 1p1Δ, 2p2h, etc.

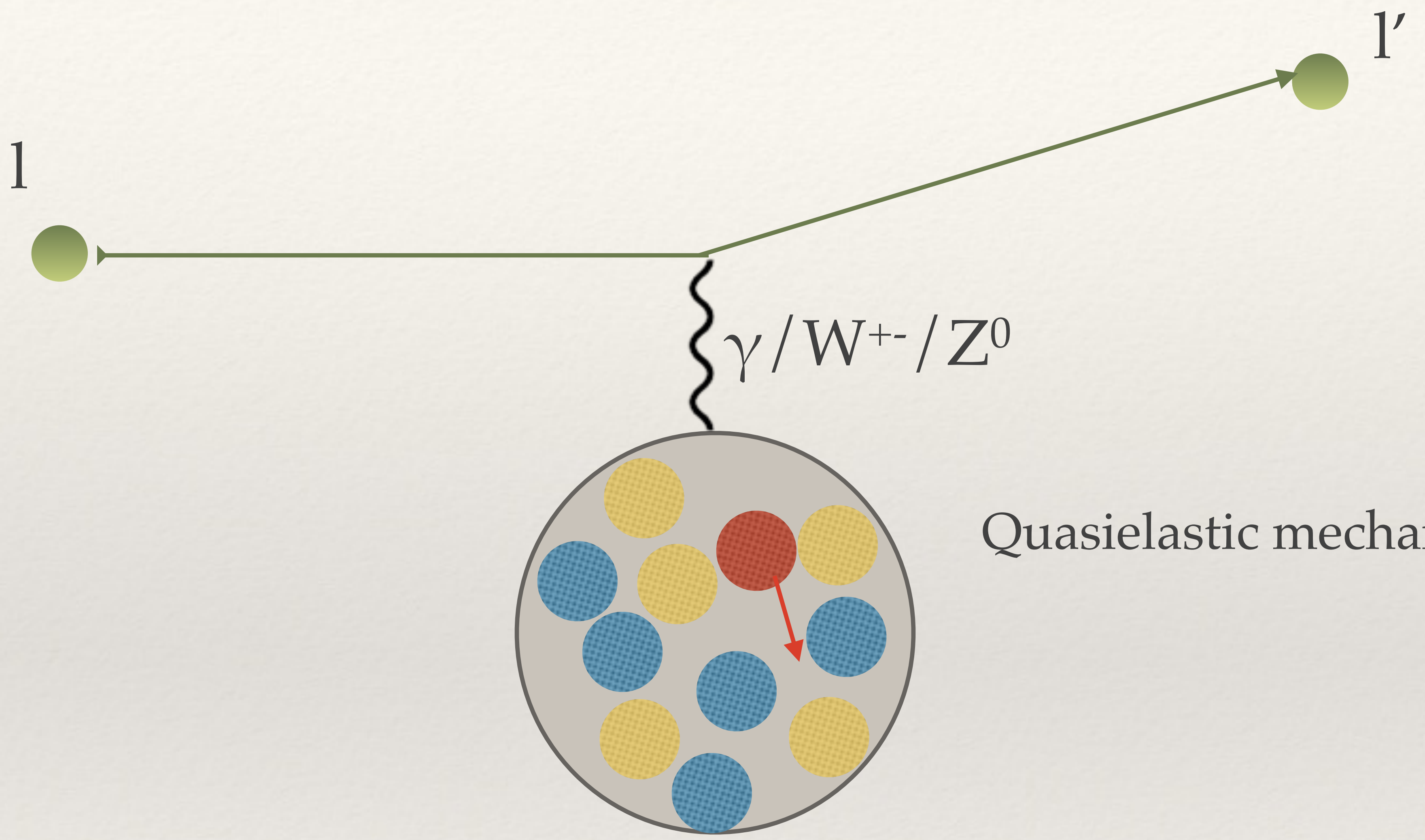
1p1h



Impulse Approximation

$\gamma/W^{+-}/Z^{0}$ absorbed
by one nucleon

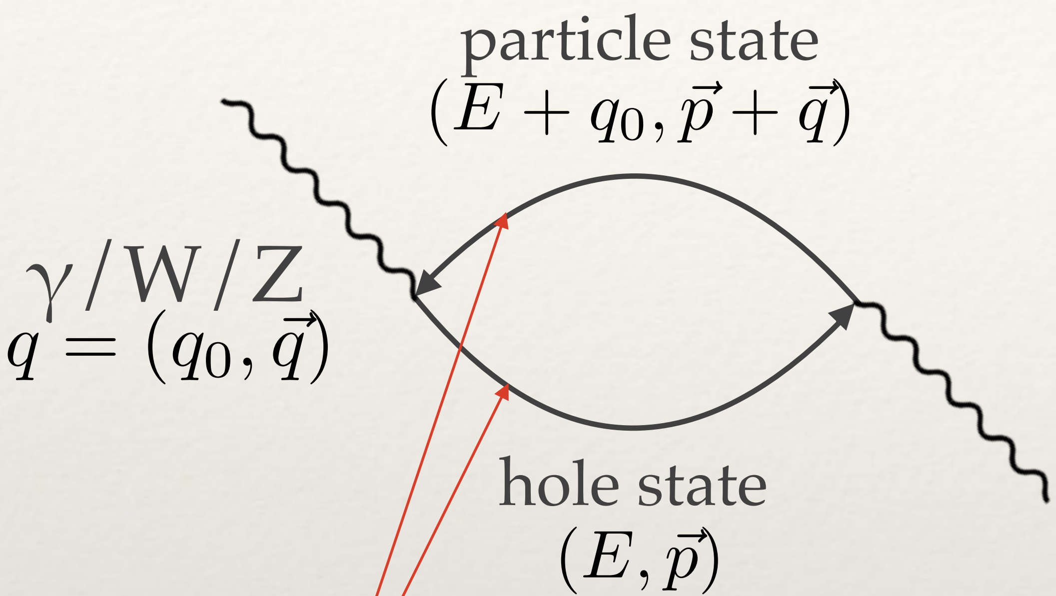
1p1h



Quasielastic mechanism

Our aim: describe properly the interaction taking into account that nucleons interact in the nuclear medium.

Formalism

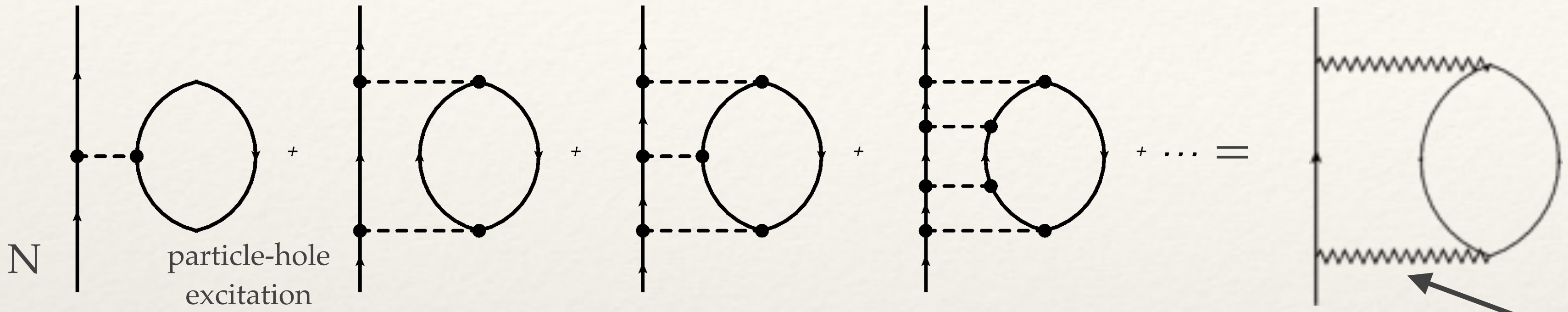


Each of nucleon lines is dressed using an in-medium effective interaction. This gives raise to the **nucleon self-energy Σ.**

$$U(q,\rho) = -2i \int \frac{d^4p}{(2\pi)^4} 2MG(p,\rho) 2MG(p+q,\rho)$$

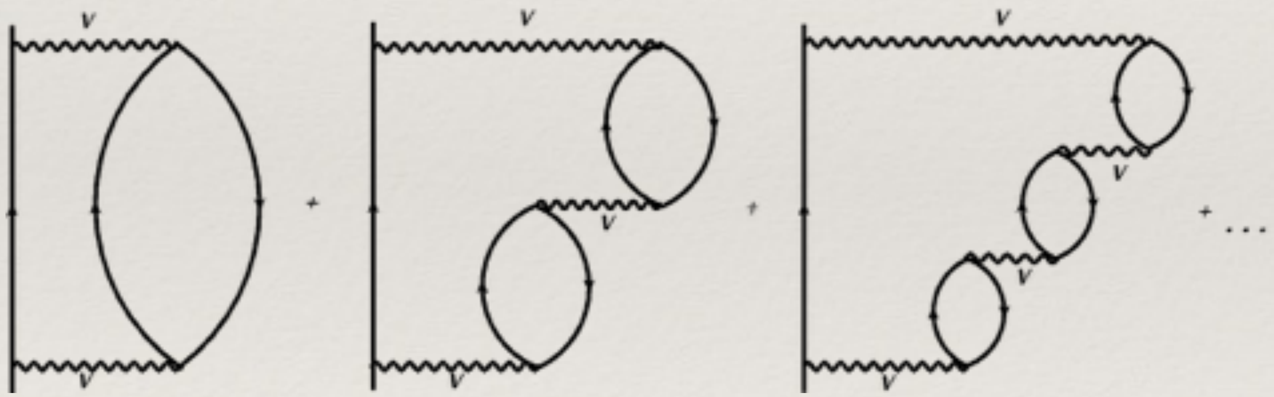
- Lindhard function (particle-hole propagator in the nuclear medium)
- **G(p,ϱ)** - Green's function (nucleon propagator). The nuclear effects are encoded in this object by means of the **nucleon self-energy Σ**.
- The vertices of interaction are omitted
- **Im U** = the nucleon-density response of the system

Semiphenomenological model for self-energy



NN interaction taken from NN scattering cross-section

Nucleon line dressed with in-medium interactions



In order to account for in-medium effects, we sum-up polarisation diagrams

We calculate **Im Σ(p,ϱ).** **Re Σ(p,ϱ)** obtained from dispersion relations (calculated up to a constant term)

P. Fernandez de Cordoba and E. Oset, Phys. Rev. C46, 1697 (1992)

Spectral functions

$$G(p,\rho) = \frac{1}{2M}\frac{1}{p^0 - \vec{p}^2/2M - \Sigma(p,\rho)}$$

- Green's function (nucleon propagator)
- **Σ(p,ϱ)** - nucleon self-energy contains information about nucleon's interaction with the nuclear medium

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Hole and particle spectral functions

$$S_{p/h}(E,\vec{p},\rho) = \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(p,\rho)}{(E-\vec{p}^2/2M - \mathrm{Re}\Sigma(p,\rho))^2 + (\mathrm{Im}\Sigma(p,\rho))^2}$$

chemical potential:

$$\mu = k_F^2/2M + \mathrm{Re}\Sigma(\mu, k_F)$$

$$S_h : \quad E < \mu$$

$$S_p : \quad E \geq \mu$$

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change of dispersion relation

nucleon's width

chemical potential:

$$\mu = k_F^2/2M + \mathrm{Re}\Sigma(\mu, k_F)$$

$$S_h : \quad E < \mu$$

$$S_p : \quad E \geq \mu$$

In the case when Σ(p,ϱ) = 0 we reduce it to a non-interacting system of nucleons (Local Fermi Gas)

Spectral functions

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$$\mathrm{Im}U(q,\rho) = -\frac{\Theta(q^0)}{4\pi^2}\int d^3p\int_{\mu-q^0}^{\mu} d\omega S_h(\omega,\vec{p})S_p(q^0+\omega,\vec{p}+\vec{q})$$

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$$W^{\mu\nu}(q) \propto -\frac{\Theta(q^0)}{4\pi^2}\int d^3r \int d^3p \int_{\mu-q^0}^{\mu} d\omega S_h(\omega,\vec{p})S_p(q^0+\omega,\vec{p}+\vec{q})A^{\mu\nu}(p,q)$$

(the particle SF or FSI)

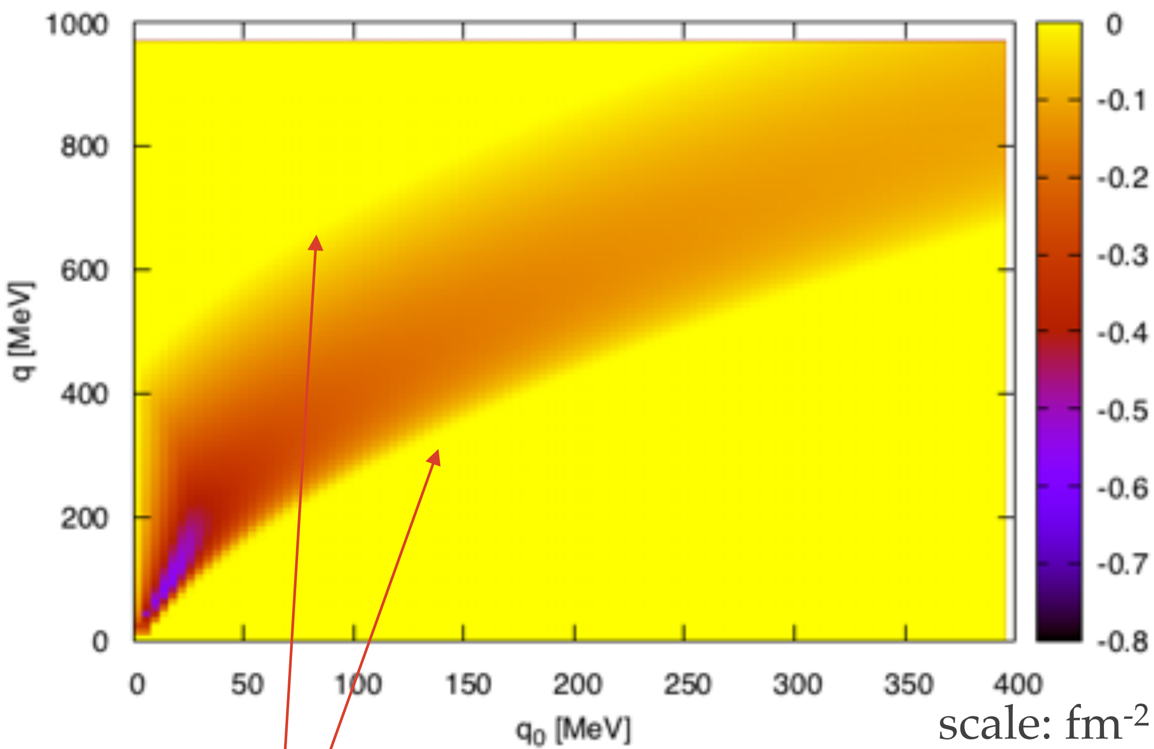
(the particle SF or FSI)

(hadron tensor for a single nucleon)

Effect of SFs

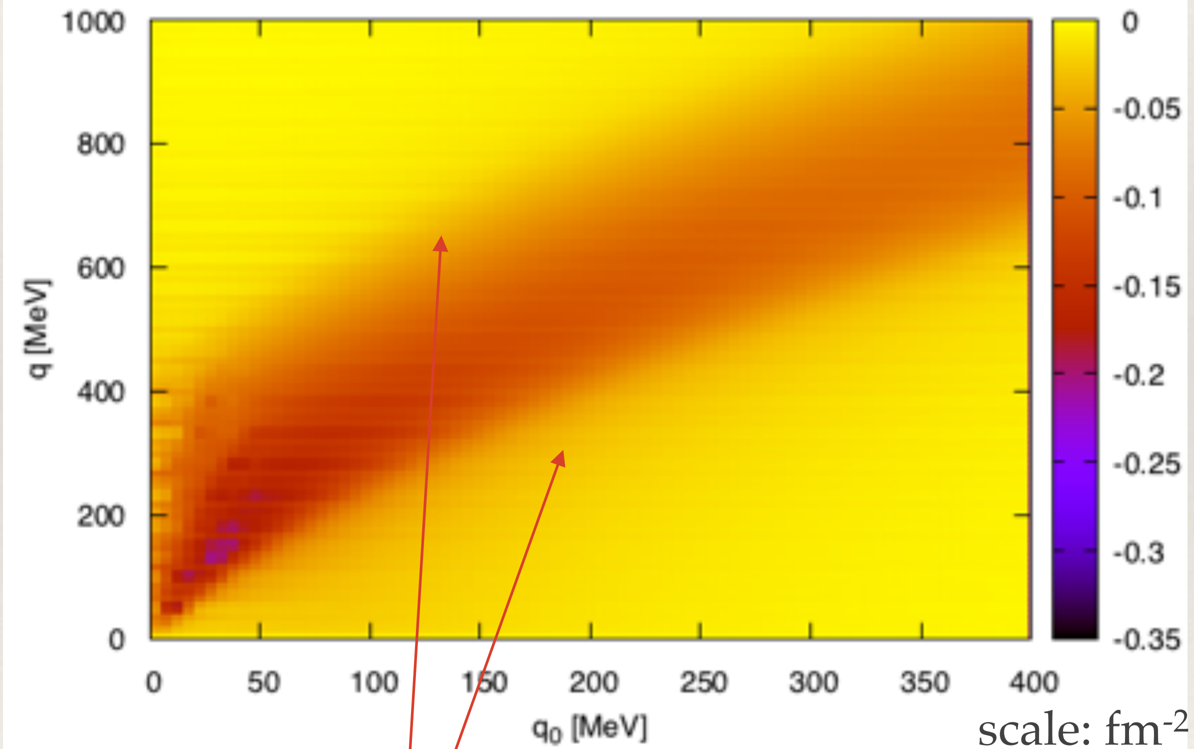
$$\mathrm{Im}U(q,\rho) = -\frac{\Theta(q^0)}{4\pi^2}\int d^3p \int_{\mu-q^0}^{\mu} d\omega S_h(\omega,\vec{p})S_p(q^0+\omega,\vec{p}+\vec{q})$$

ImU for the LFG, ϱ=0.09fm^{-3}



scale: fm^{-2}

ImU for SF, ϱ=0.09fm^{-3}



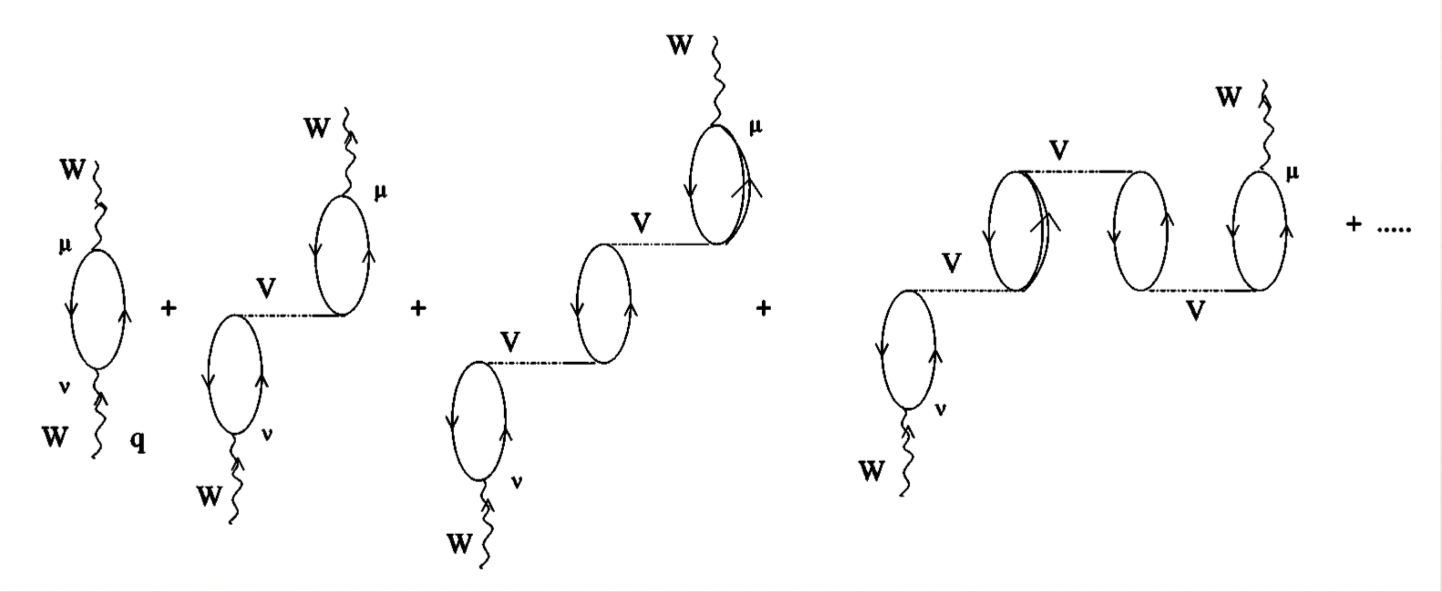
scale: fm^{-2}

very clear cut where Im U(q) = 0
(effect of Pauli blocking)

smeared+ visible quenching
(effect of Σ)

RPA & SF

- RPA important for low q (momentum transfer) - $W^{+/-}$ boson absorbed by the nucleus as a whole (account for some nuclear medium polarization effects sensitive to the collective degrees of freedom of the nucleus)
- sum of ph and Δh excitations
- (When introducing RPA with SF: RPA parameters were fixed for the LFG. That is why in the denominator of the RPA sum we keep $\mathrm{ReU}_{\mathrm{LFG}}$ instead of $\mathrm{ReU}_{\mathrm{SF}}$.)



$$\mathrm{Im}\bar{U}(q;\rho)\left[a\hat{q}_i\hat{q}_j + b\left(\delta_{ij} - \hat{q}_i\hat{q}_j\right)\right] \to$$
$$\mathrm{Im}\bar{U}(q;\rho)\left[a\frac{\hat{q}_i\hat{q}_j}{|1-U(q;\rho)V_l(q)|^2} + b\frac{\delta_{ij}-\hat{q}_i\hat{q}_j}{|1-U(q;\rho)V_t(q)|^2}\right]$$

Low-energy check

Nuclear effects are getting more pronounced at low energy-momentum transfers. The model should be checked in this regime.

→

(I) Radiative pion capture
(II) Muon capture
(III) Neutrino scattering

…we are at the verge of usability of the model!

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Radiative pion capture & muon capture

$$(A_Z - \pi^-)_{\text{bound}} \to \gamma + X$$

$$(A_Z - \mu^-)^{1s}_{\text{bound}} \to \bar{\nu} + X$$

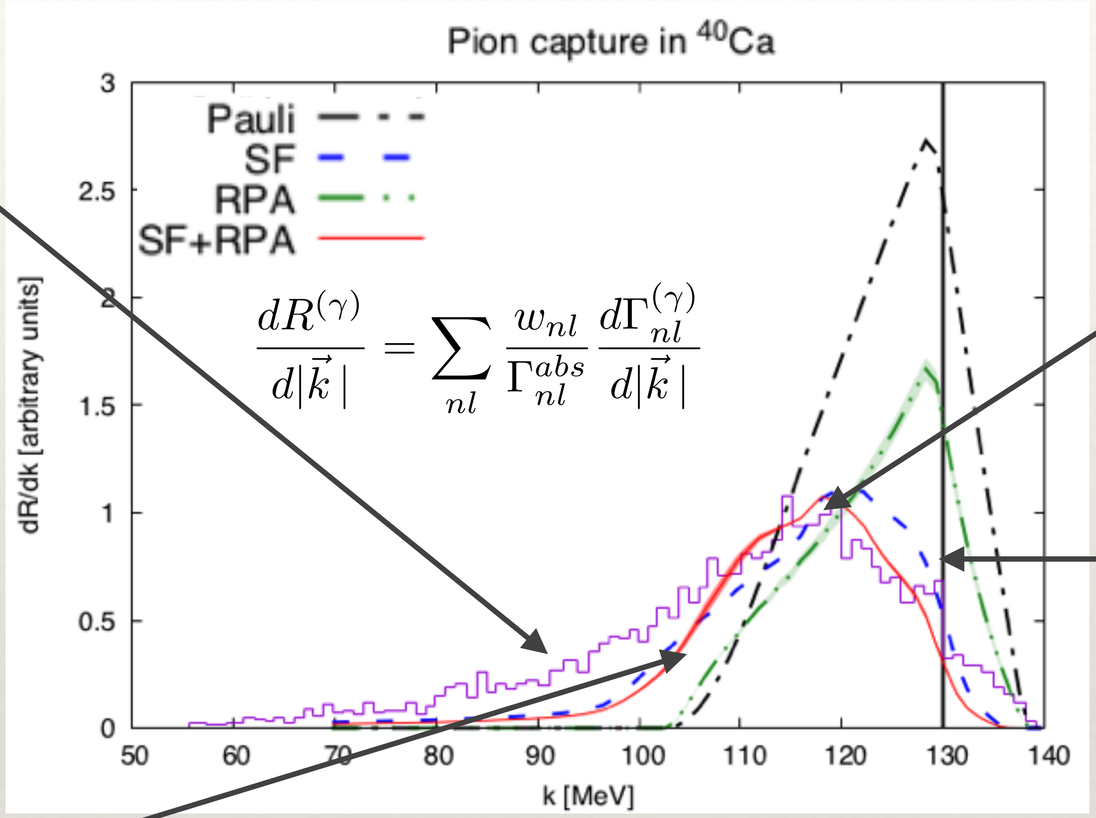
These nuclear systems are not stable because μ^-, π^- are much heavier than e^- and their wave function overlaps with the nucleus.

1. Calculate the ϱ-dependant decay width of μ^-, π^- in the nuclear medium $\hat{\Gamma}(q, \rho_n(r), \rho_p(r))$
2. Calculate the μ^-, π^- wave functions $\phi(r)$ for a given nucleus.
3. $\Gamma = \int d^3r |\phi(r)|^2 \hat{\Gamma}(q, \rho_n(r), \rho_p(r))$

Radiative pion capture

SF with respect to the LFG: shifts the peak, causes quenching and spreading to higher energy transfers

We are missing some strength at higher energy transfer (because the 2p2h contribution is not included).



Photon energy distributions (arbitrary units) from pion capture. Theoretical SF+RPA curves were adjusted to data in the peak, other curves (Pauli, RPA, SF) were scaled by the same factor.

We get the right position of the quasielastic peak

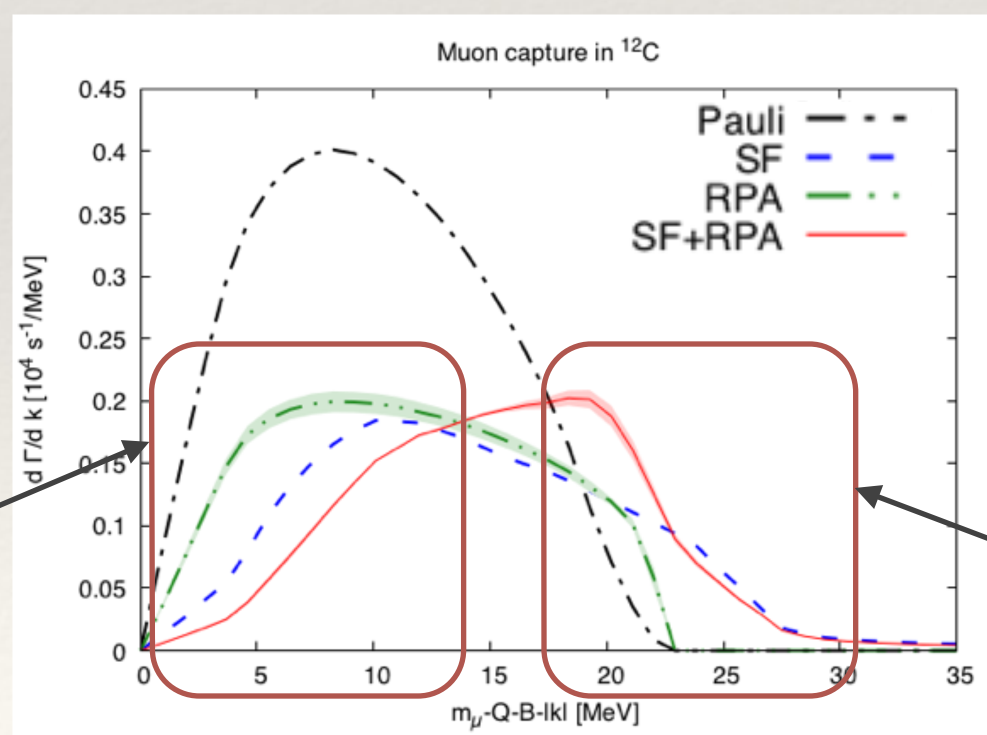
Cut above energies where there are discrete transitions

Muon capture

40-50% quenching

Nucleus	Pauli (10^4 s^{-1})	RPA (10^4 s^{-1})	SF (10^4 s^{-1})	SF+RPA (10^4 s^{-1})	Exp. (10^4 s^{-1})
^{12}C	5.76	3.37 ± 0.16	3.22	3.19 ± 0.06	3.79 ± 0.03
^{16}O	18.7	10.9 ± 0.4	10.6	10.3 ± 0.2	10.24 ± 0.06
^{18}O	13.8	8.2 ± 0.4	7.0	8.7 ± 0.1	8.80 ± 0.15
^{23}Na	64.5	37.0 ± 1.5	30.9	34.3 ± 0.4	37.73 ± 0.14
^{40}Ca	498	272 ± 11	242	242 ± 6	252.5 ± 0.6

- Calculated for a variety of (almost) symmetric nuclei.
- There is not much difference between **SF and SF+RPA** in the integrated width

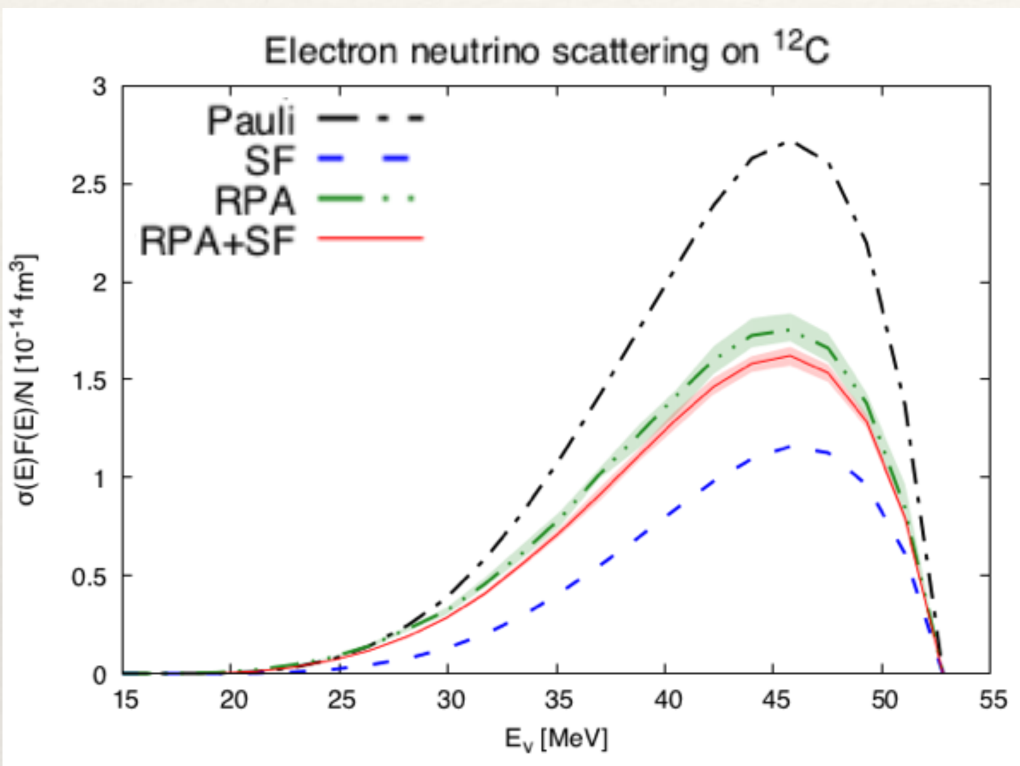


RPA causes quenching

RPA causes enhancement

Neutrino scattering at low energies

- Comparison with LSND data & other theoretical approaches

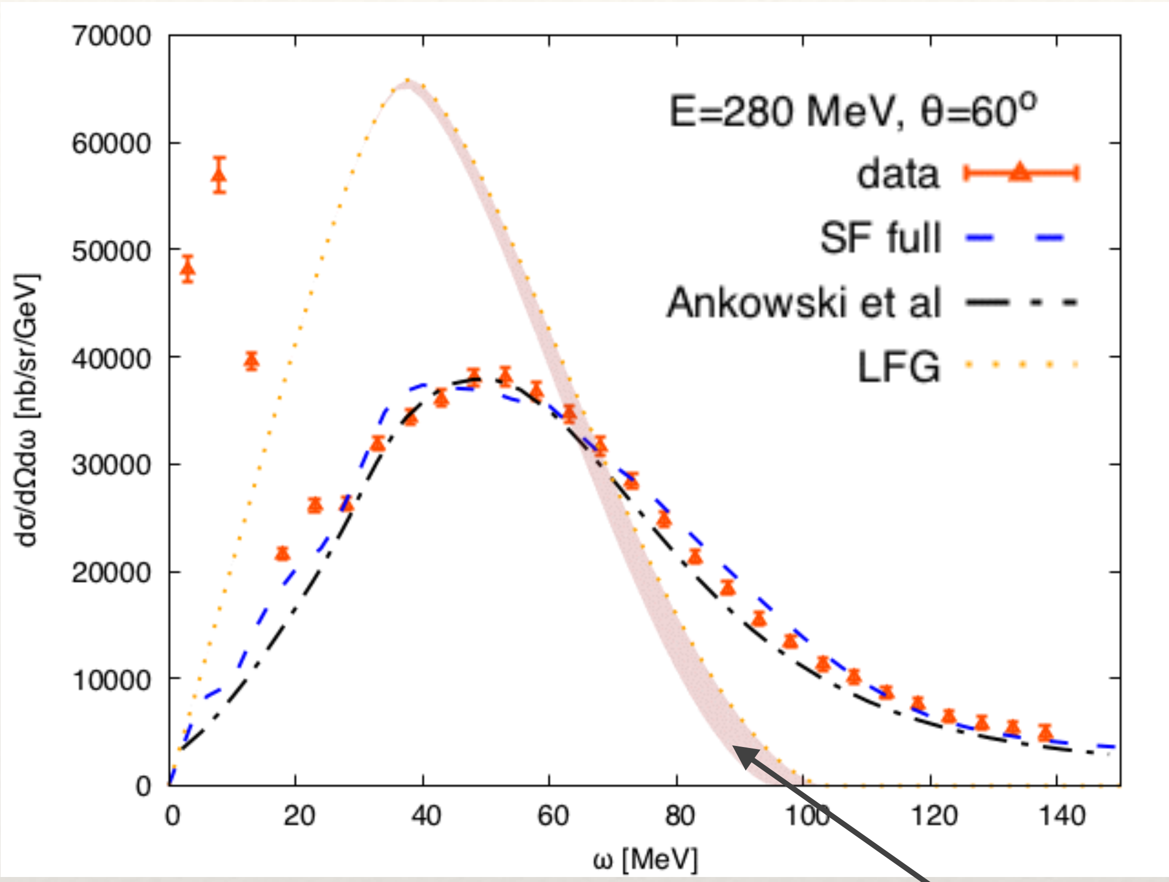


Neutrino cross sections convoluted with the LSND flux

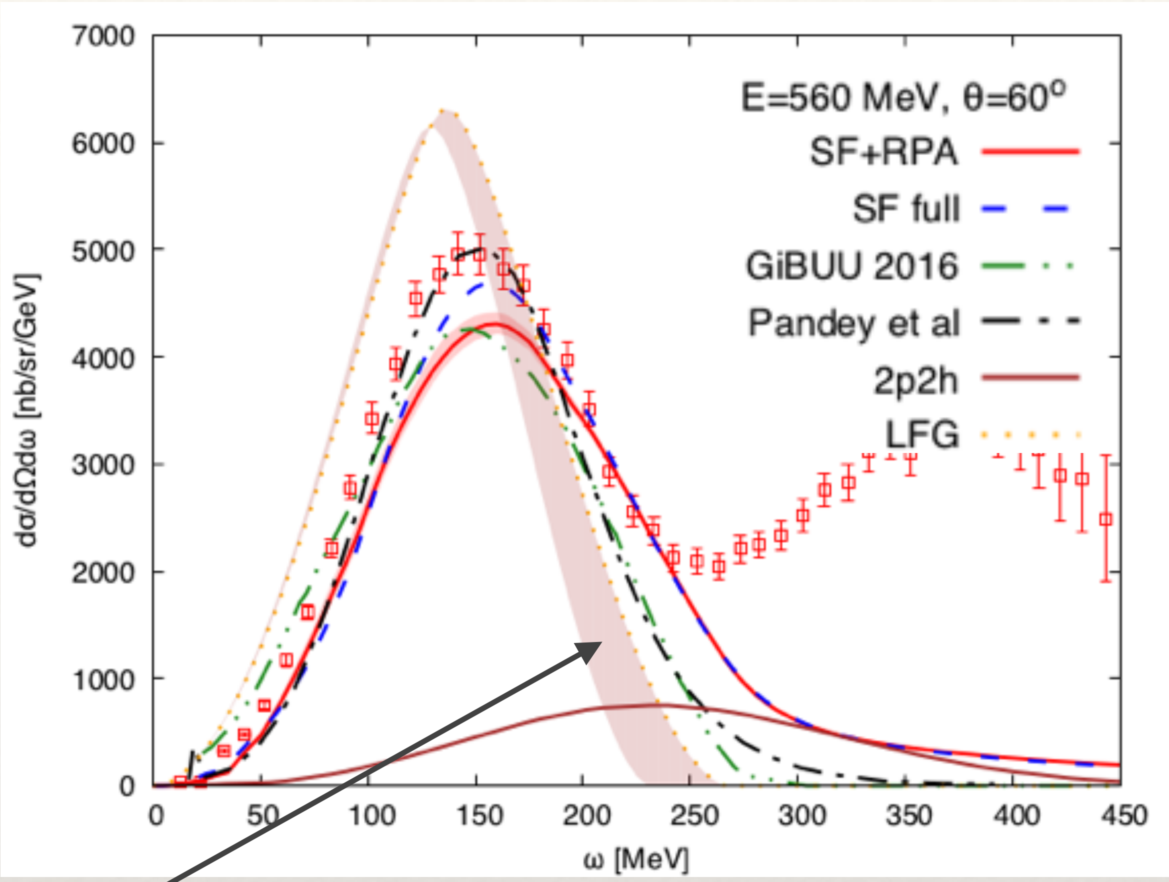
A. C. Hayes and I. S. Towner, Phys. Rev. C61, 044603 (2000)
C. Volpe, N. Auerbach, G. Colo, T. Suzuki, and N. Van Giai, Phys.Rev. C62, 015501 (2000)
E. Kolbe, K. Langanke, G. Martinez-Pinedo, and P. Vogel, J. Phys. G29, 2569 (2003).

	Pauli	RPA	SF	SF+RPA	SM	SM	CRPA	Experiment		
								LSND	LSND	LSND
$\bar{\sigma}(\nu_\mu, \mu^-)$	23.1	13.2 ± 0.7	12.2	9.7 ± 0.3	13.2	15.2	19.2	$8.3 \pm 0.7 \pm 1.6$	$11.2 \pm 0.3 \pm 1.8$	$10.6 \pm 0.3 \pm 1.8$
								KARMEN	LSND	LAMPF
$\bar{\sigma}(\nu_e, e^-)$	0.200	0.143 ± 0.006	0.086	0.138 ± 0.004	0.12	0.16	0.15	$0.15 \pm 0.01 \pm 0.01$	0.15 ± 0.01	0.141 ± 0.023

Results - electron scattering



A. M. Ankowski et al., Phys. Rev. D91, 033005 (2015)



V. Pandey et al., Phys. Rev. C92, 024606 (2015)

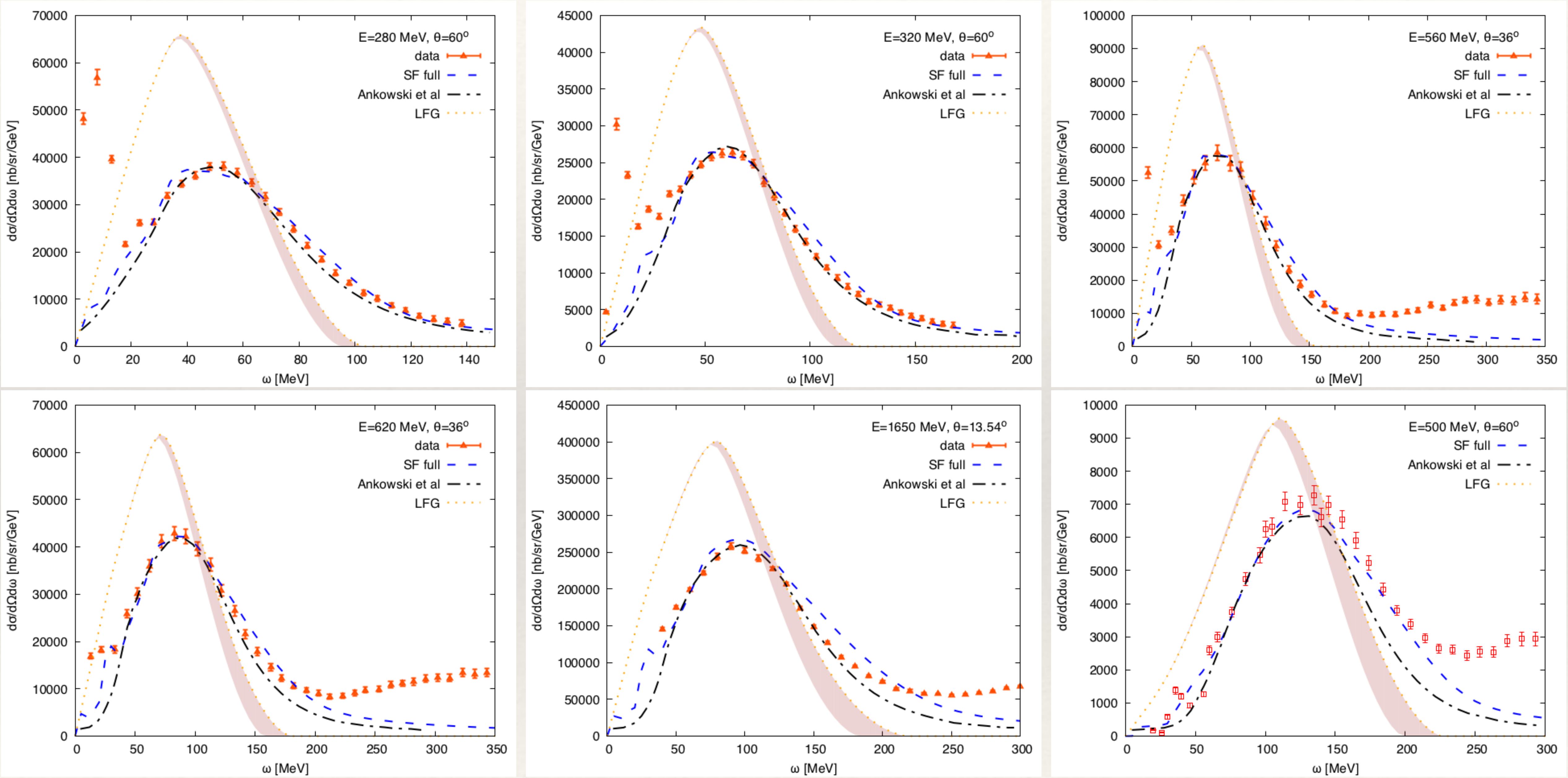
relativistic effect

Non relativistic kinematics does not cause any problems.

Non relativistic kinematics causes a broadening of the peak

Problem: relativistic energies

Results - electron scattering



Neutrino scattering - results

Muon neutrino/antineutrino

		$\sigma(\nu_\mu + {}^{16}\mathrm{O} \to \mu^- + X)\ [10^{-40}\mathrm{cm}^2]$		
		Non-relativistic	Relativistic	SF
500 MeV	Pauli	625	580	494
	RPA	520 ± 40	470 ± 40	445 ± 27
375 MeV	Pauli	443	418	328
	RPA	329 ± 24	308 ± 22	274 ± 14
250 MeV	Pauli	199	192	132
	RPA	123 ± 7	118 ± 7	101 ± 5
		$\sigma(\bar\nu_\mu + {}^{16}\mathrm{O} \to \mu^+ + X)\ [10^{-40}\mathrm{cm}^2]$		
		Non-relativistic	Relativistic	SF
500 MeV	Pauli	143.8	134.4	118.9
	RPA	106.3 ± 1.9	98.5 ± 1.9	105.6 ± 1.5
375 MeV	Pauli	99.8	94.1	78.2
	RPA	71.6 ± 1.4	66.9 ± 1.3	68.6 ± 1.2
250 MeV	Pauli	51.5	49.0	37.6
	RPA	34.3 ± 0.8	32.5 ± 0.8	31.0 ± 0.7

Electron neutrino/antineutrino

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310 MeV	Pauli	370	350	271
	RPA	259 ± 18	244 ± 16	219 ± 11
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130 MeV	Pauli	44.6	43.1	28.3
	RPA	25.6 ± 1.2	24.8 ± 1.1	23.2 ± 0.8
		$\sigma(\bar\nu_e + {}^{16}\mathrm{O} \to e^+ + X)\ [10^{-40}\mathrm{cm}^2]$		
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310 MeV	Pauli	81.6	77.3	63.1
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relativistic effect:
<10% difference

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<10% difference

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Electron neutrino/antineutrino

40% reduction

17% reduction

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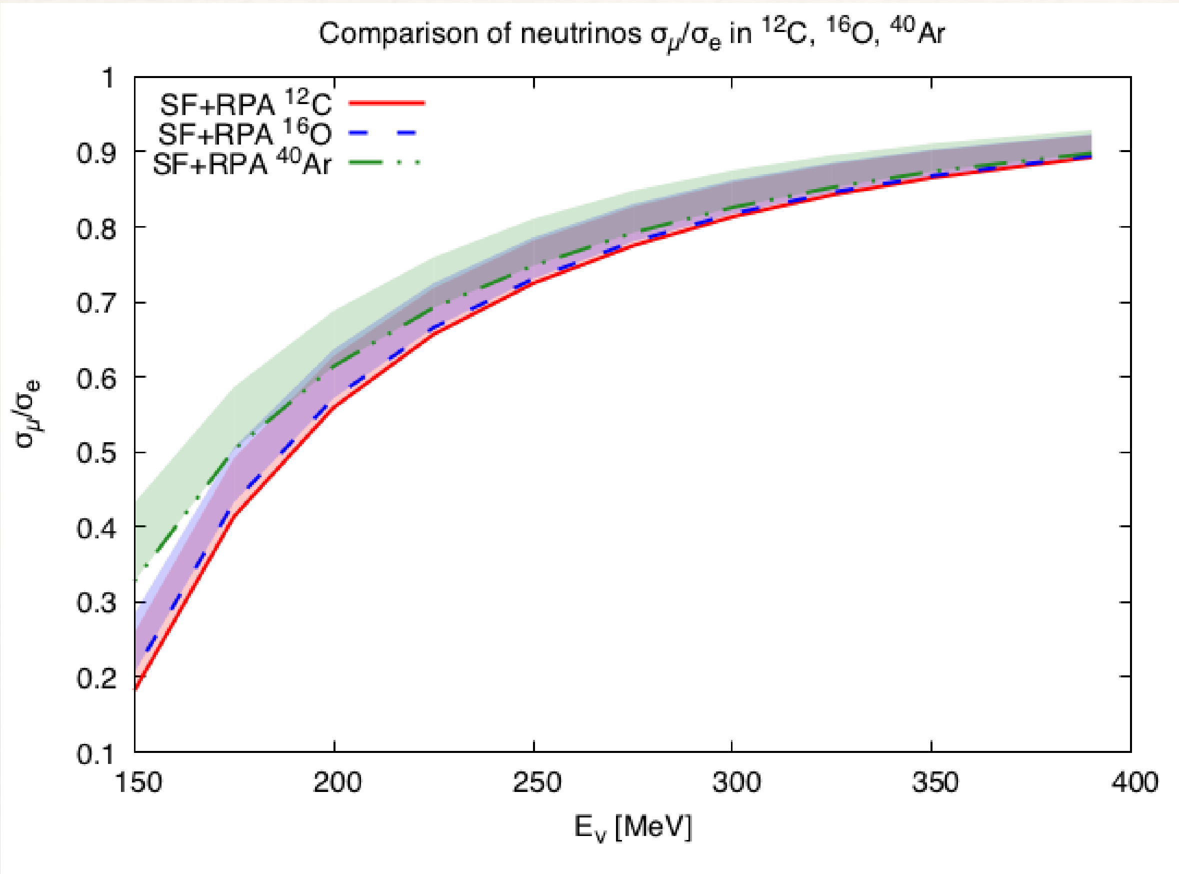
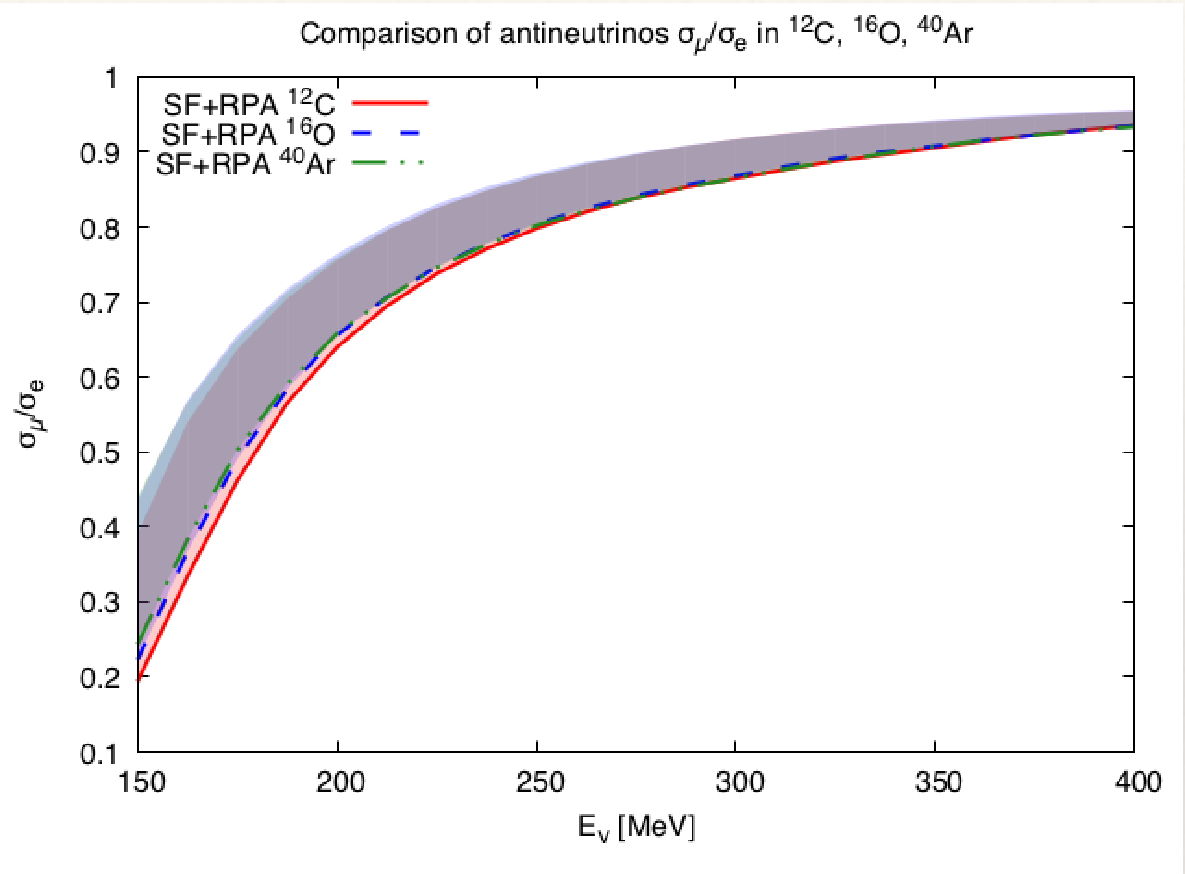
Ratio σ(μ)/σ(e)

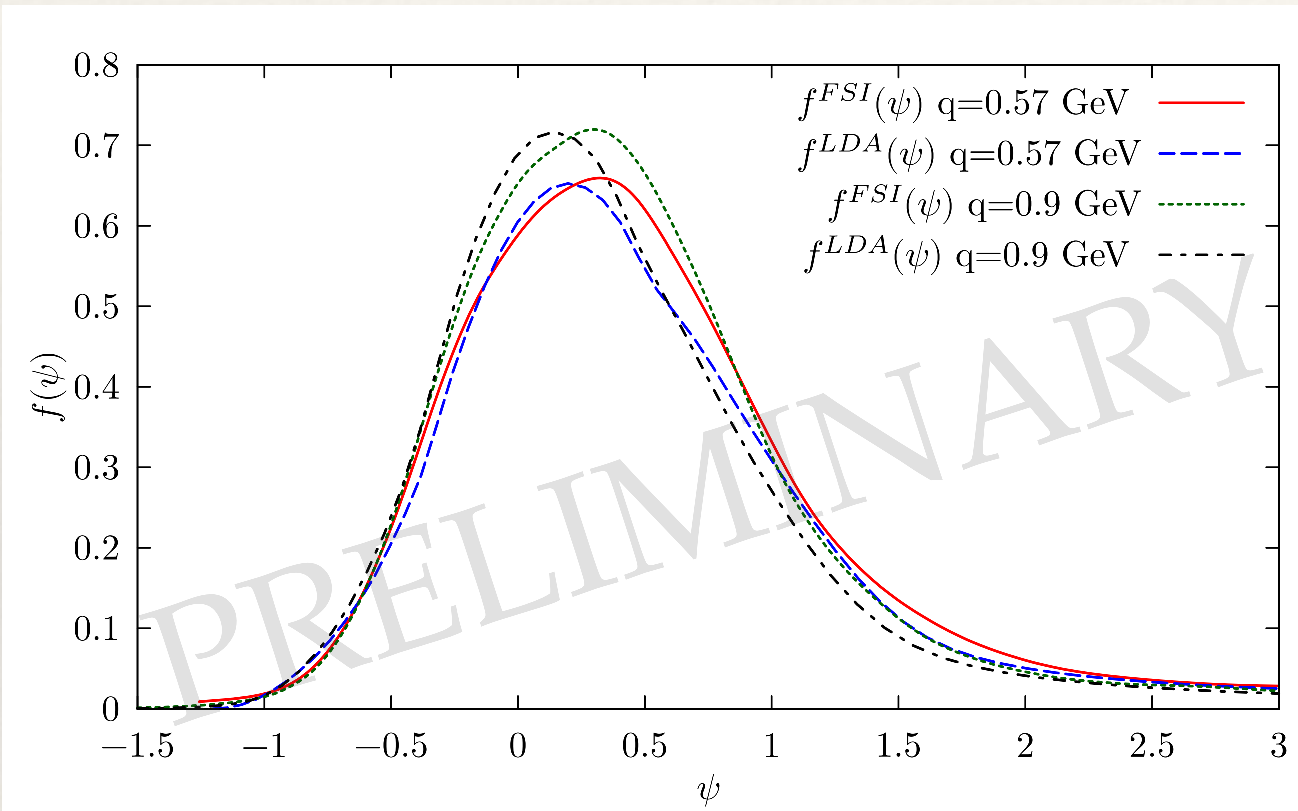




Nuclear effects **do not cancel out** when we take the ratio σ(μ)/σ(e) $\equiv \sigma(\nu_\mu + {}^A Z \rightarrow \mu^- + X)/\sigma(\nu_e + {}^A Z \rightarrow e^- + X)$

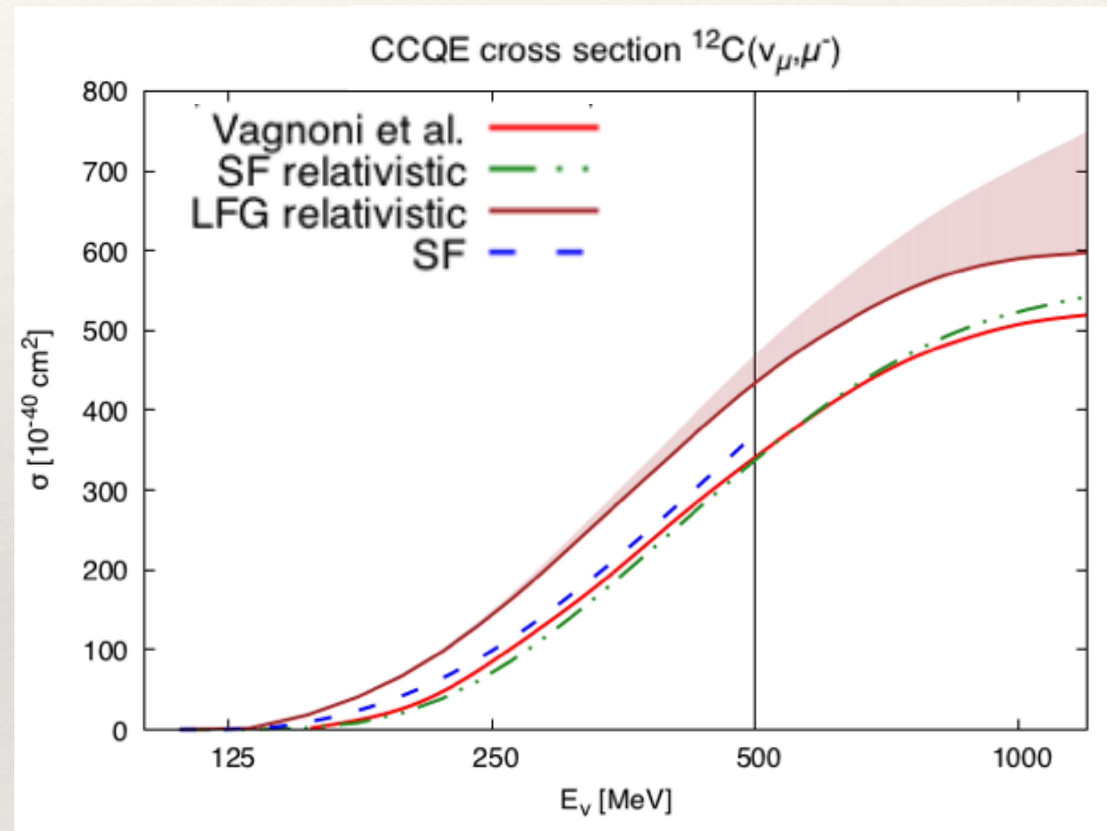
Neutrino scattering: relativistic model

(I) Use relativistic approximation for SFs



A. Lovato, J. Nieves, N. Rocco, JS, *work in progress*

(II) Use the PWIA (neglect FSI)

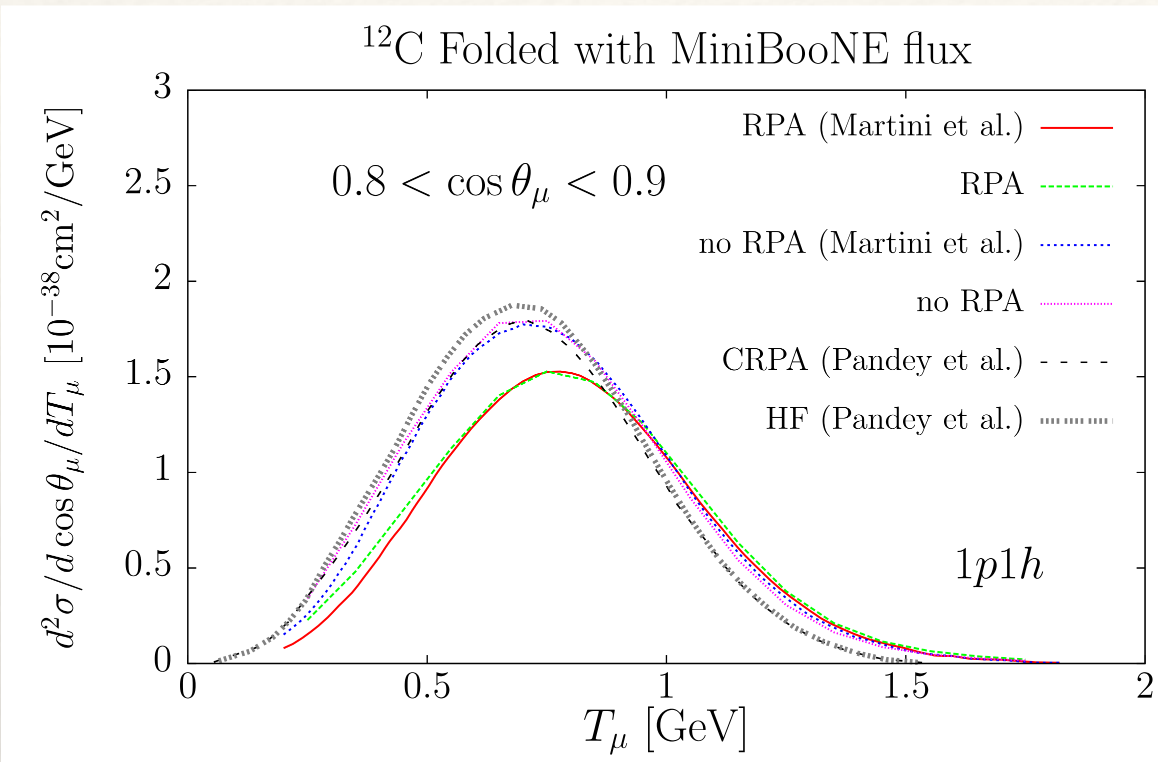


E. Vagnoni, O. Benhar, and D. Meloni, Phys. Rev. Lett. 118, 142502 (2017)

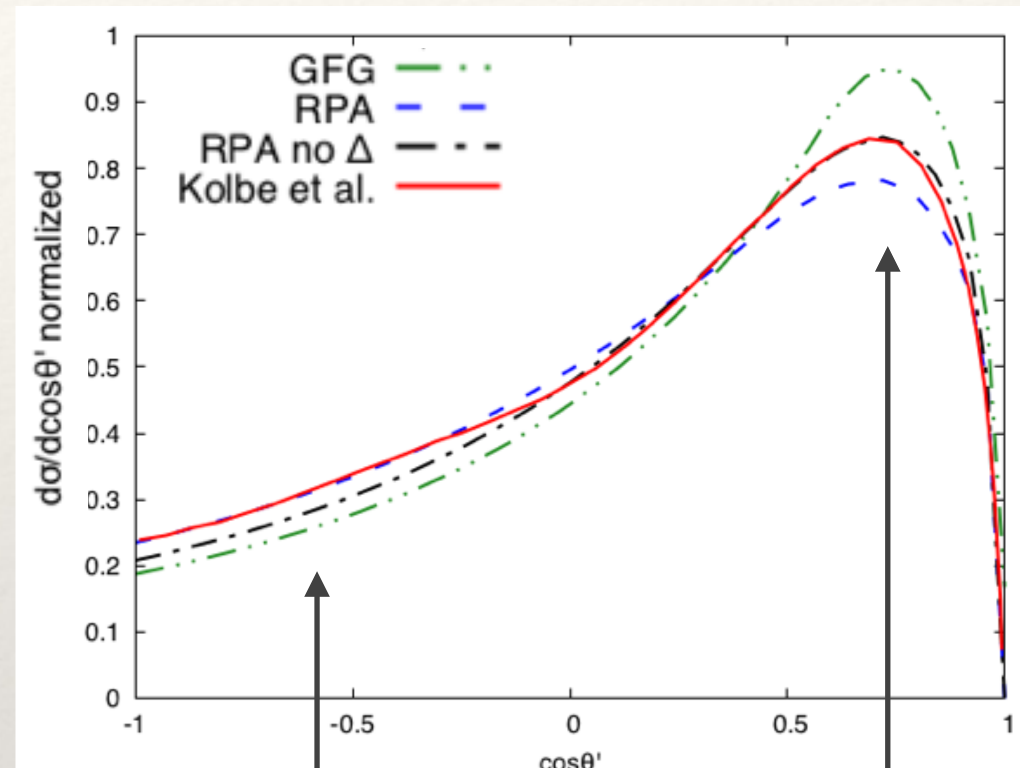
Scaling function: comparison with the model of Benhar SF+FSI

P. Fernandez de Cordoba et al., Nucl. Phys. A611, 514 (1996)

RPA: comparison



M. Martini, M. Ericson, and G. Chanfray, Phys.Rev. C84, 055502 (2011)
V. Pandey et al., Phys. Rev. C94, 054609 (2016)



E. Kolbe et al. J. Phys. G29, 2569 (2003)

enhancement for high scattering angles (wrt GFG)

decrease for low scattering angles (wrt GFG)

We sum up ph-ph and Δh excitation

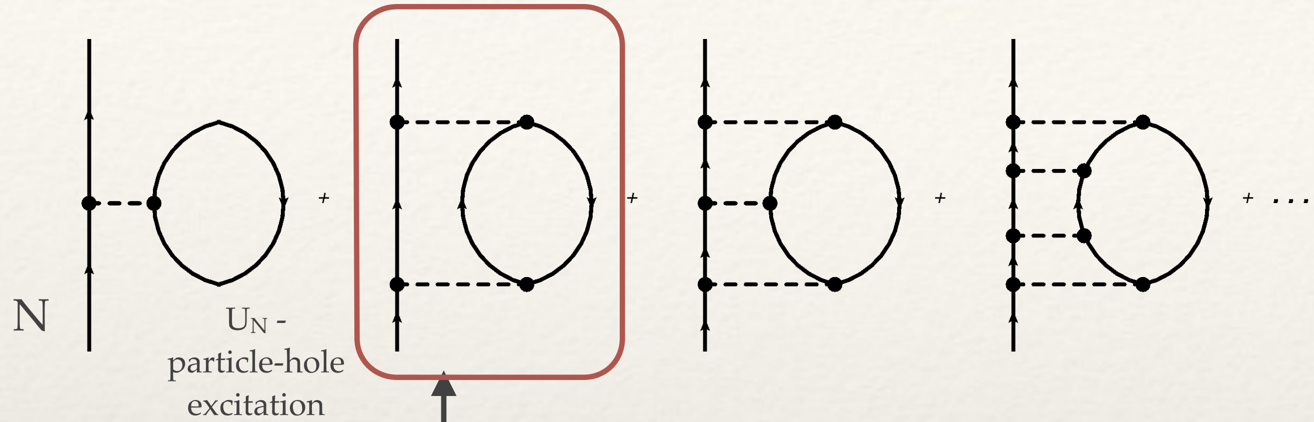
Summary

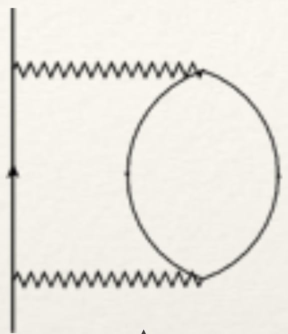
- We presented a model for inclusive neutrino-nucleus scattering which accounts both for RPA effects and introduces nucleon spectral functions. Model has been checked for low-energy processes (pion, muon capture, neutrino scattering).
- The model can be used for a variety of nuclei (LDA).
- The main problem in the description of the nuclear system: **non relativistic physics.** There are two possible ways out:
  - neglect FSI (use only the hole SF)
  - use of a “relativised” model for the particle SF
- A comparison with other approaches has been performed (electron scattering - very good agreement with the data; neutrino scattering, RPA effects).

Thank you for your attention!

Back up

SF





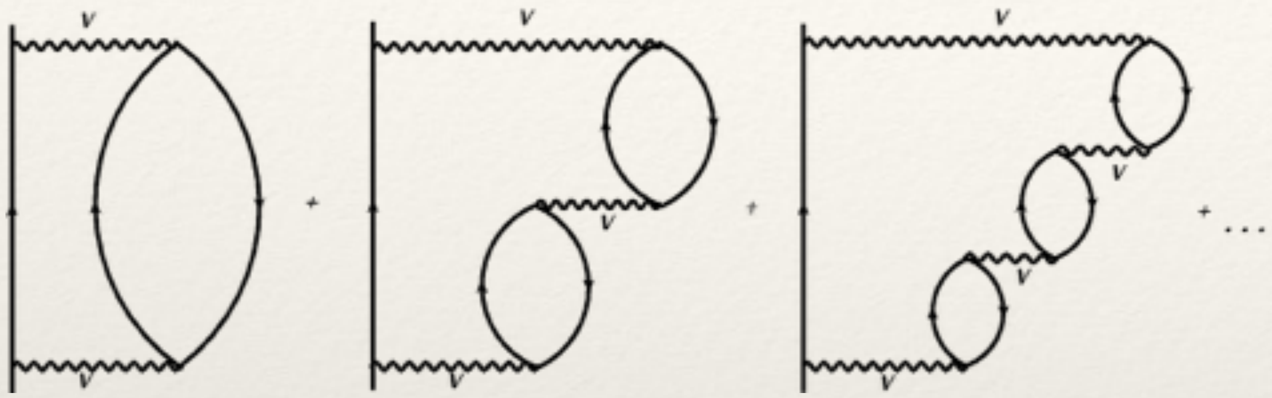
$$-i\Sigma(k)=\int\frac{d^4q}{(2\pi)^4}\,i\left\{\frac{1-n(\mathbf{k}-\mathbf{q})}{k^0-q^0-\varepsilon(\mathbf{k}-\mathbf{q})+i\epsilon}+\frac{n(\mathbf{k}-\mathbf{q})}{k^0-q^0-\varepsilon(\mathbf{k}-\mathbf{q})-i\epsilon}\right\}(-i)V(q)(-i)V(q)iU_N(q)$$

$$\mathrm{Im}\Sigma(k)=\int\frac{d^3q}{(2\pi)^3}\left\{\left[1-n(\mathbf{k}-\mathbf{q})\right]\Theta(k^0-\varepsilon(\mathbf{k}-\mathbf{q}))-n(\mathbf{k}-\mathbf{q})\Theta(\varepsilon(\mathbf{k}-\mathbf{q})-k^0)\right\}$$
$$\times\,\mathrm{Im}U_N(q)\overline{\sum}\sum|t|^2\Big|_{q^0=k^0-\varepsilon(\mathbf{k}-\mathbf{q})},$$

$$\mathrm{Re}\Sigma(\omega,k)=-\frac{1}{\pi}P\int_{\varepsilon_F}^{\infty}d\omega'\frac{\mathrm{Im}\Sigma(\omega',k)}{\omega-\omega'}+\frac{1}{\pi}P\int_{-\infty}^{\varepsilon_F}d\omega'\frac{\mathrm{Im}\Sigma(\omega',k)}{\omega-\omega'}$$

$$\overline{\sum}\sum|t|^2\to\frac{\pi s}{M^4}\sigma_{\mathrm{elas}}\simeq\frac{4\pi}{M^2}\sigma_{\mathrm{elas}}$$

SF - polarization effects



V - spin-isospin effective interaction

$$V(q) = \left[V_l(q)\hat{q}_i\hat{q}_j + V_t(q)(\delta_{ij} - \hat{q}_i\hat{q}_j)\right]\sigma_i\sigma_j\vec{\tau}\vec{\tau}$$

only this channel taken into account

$$\boxed{U_N(q) \rightarrow \frac{U_N(q)}{1 - V_t(q)U_N(q)}}$$

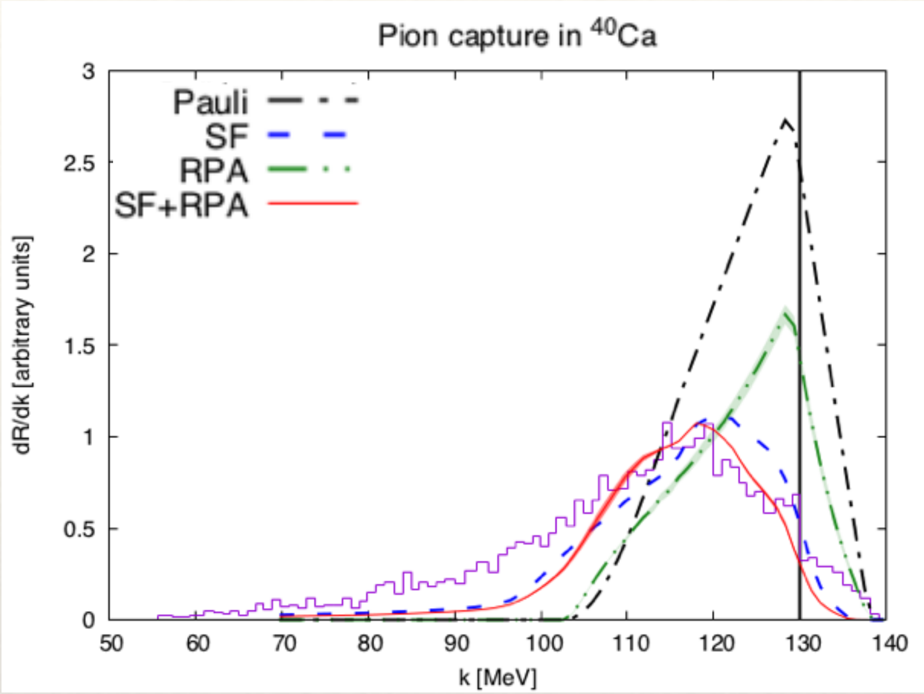
this is the result of the sum of ph excitations

Radiative pion capture

$$\frac{dR^{(\gamma)}}{d|\vec{k}\,|} = \sum_{nl} \frac{w_{nl}}{\Gamma^{abs}_{nl}} \frac{d\Gamma^{(\gamma)}_{nl}}{d|\vec{k}\,|}$$

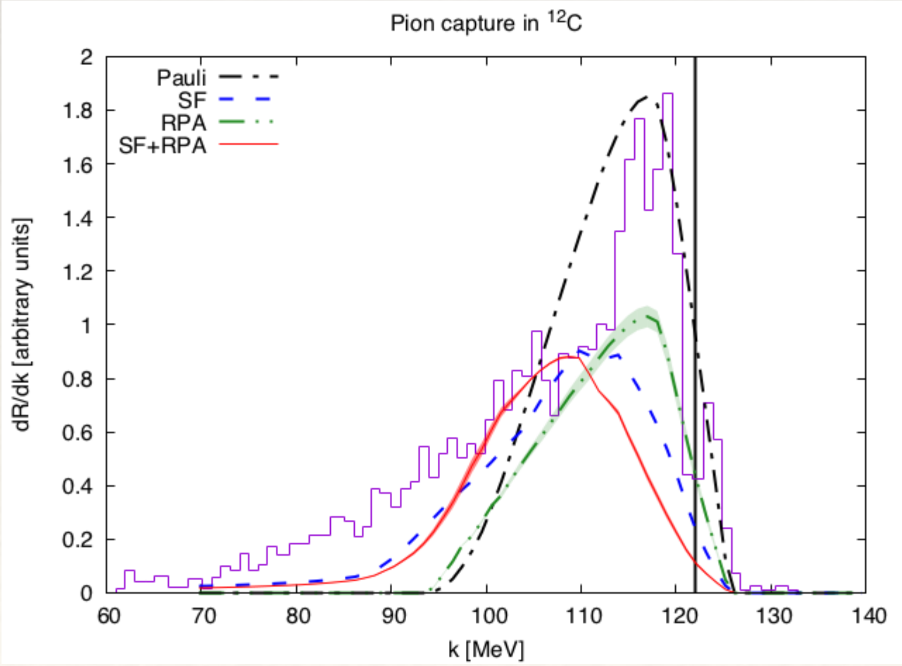
Γ^{abs}_{nl} total pion absorption width from the orbit nl

w_{nl} absorption probability from nl pionic level



Nucleus	nl	w_{nl}	Γ^{abs}_{nl} [keV]	Pauli [eV]	RPA [eV]	SF [eV]	SF+RPA [eV]
^{12}C	1s	0.1	3.14 ± 0.14	88.9	48.3 ± 2.1	58.6	50.6 ± 1.3
	2p	0.9	0.00136 ± 0.00020	18.3×10^{-3}	$(11.1 \pm 0.4) \times 10^{-3}$	12.2×10^{-3}	$(11.1 \pm 0.2) \times 10^{-3}$
^{40}Ca	2p	0.7	1.59 ± 0.02	41.5	24.3 ± 0.9	23.9	21.5 ± 0.5
	3d	0.3	0.0007 ± 0.0003	20.9×10^{-3}	$(13.8 \pm 0.4) \times 10^{-3}$	11.7×10^{-3}	$(11.1 \pm 0.1) \times 10^{-3}$

Within the SF+RPA scheme, we obtain ratios $R^{(\gamma)}$ of (0.9±0.1)% and (1.4±0.2)% for carbon and calcium, respectively. The experimental values for these ratios are (1.92 ± 0.20)% for ^{12}C and (1.94 ± 0.18)% for ^{40}Ca.



RPA

$$V = c_0\Big\{f_0(\rho) + f_0'(\rho)\vec{\tau}_1\cdot\vec{\tau}_2 + g_0(\rho)\vec{\sigma}_1\cdot\vec{\sigma}_2 + \boxed{g_0'(\rho)\left(\vec{\sigma}_1\cdot\vec{\sigma}_2\right)\left(\vec{\tau}_1\cdot\vec{\tau}_2\right)}\Big\}$$

$$f_i(\rho(r)) = \frac{\rho(r)}{\rho(0)}f_i^{(in)} + \left(1-\frac{\rho(r)}{\rho(0)}\right)f_i^{(ex)}$$

$$\vec{\tau}_1\cdot\vec{\tau}_2\sum_{i,j}\sigma^i\sigma^j V_{ij}^{\sigma\tau}$$

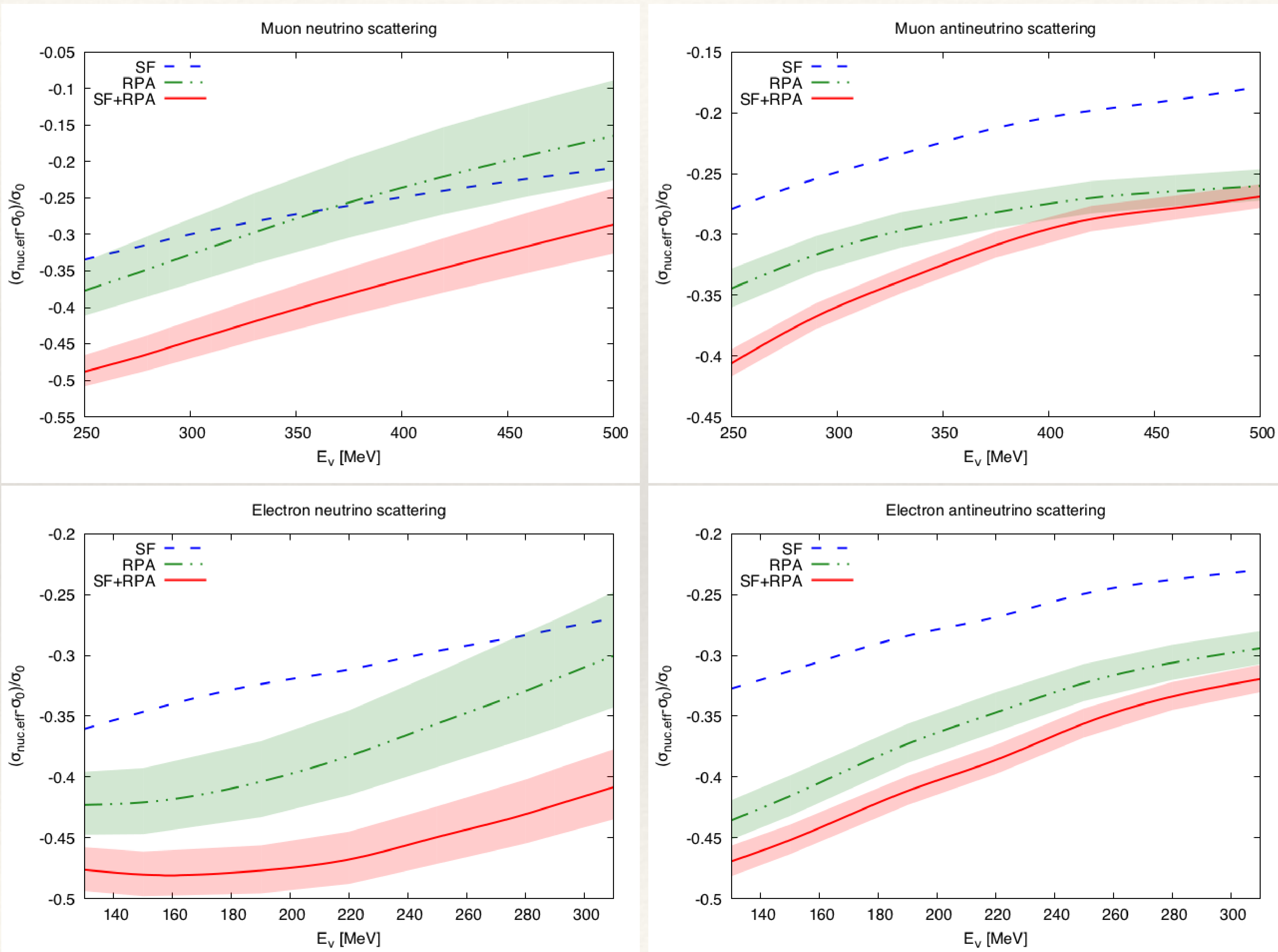
Effective spin-isospin potential - Magdal-Lindau potential

$$V_l(q) = \frac{f^2}{m_\pi^2}\left\{\left(\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2-q^2}\right)^2\frac{\vec{q}^{\,2}}{q^2-m_\pi^2}+g'\right\}$$

$$V_t(q) = \frac{f^2}{m_\pi^2}\left\{C_\rho\left(\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2-q^2}\right)^2\frac{\vec{q}^{\,2}}{q^2-m_\rho^2}+g'\right\}$$

we explicitly take into account π and ϱ meson exchange

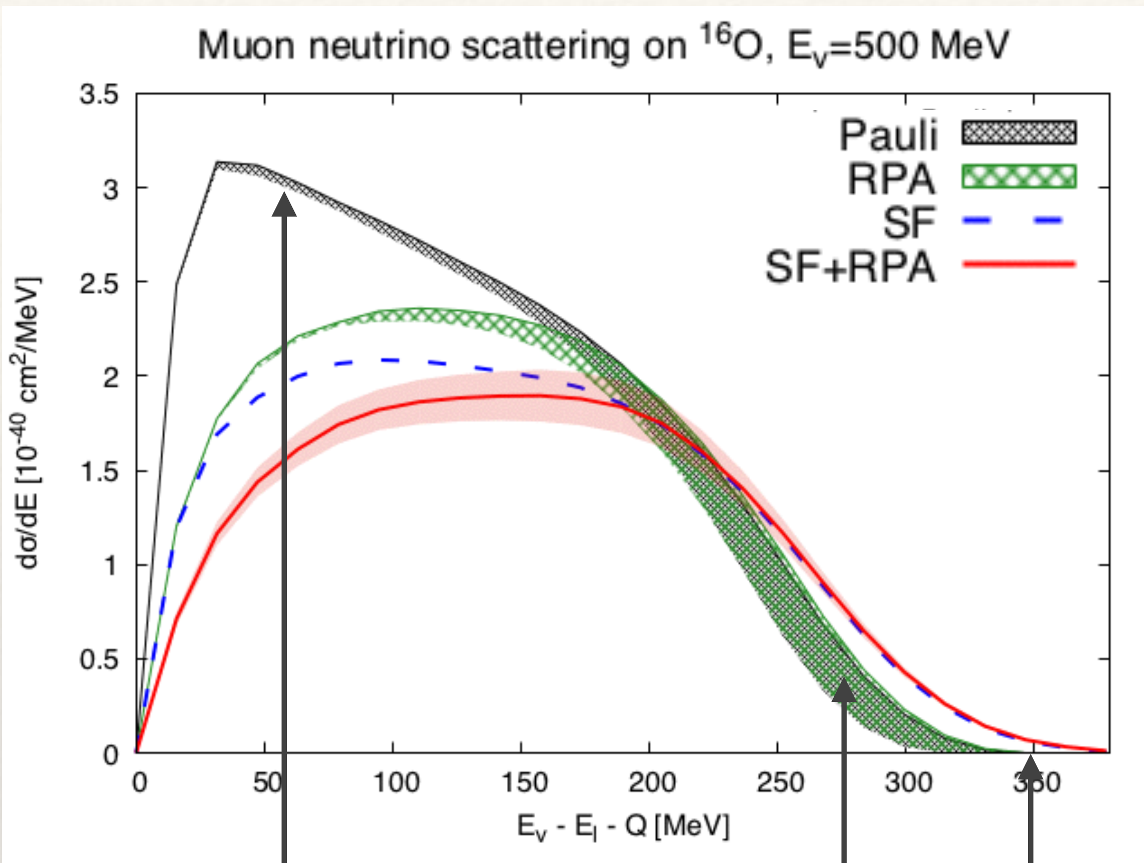
Nuclear effects: energy dependence

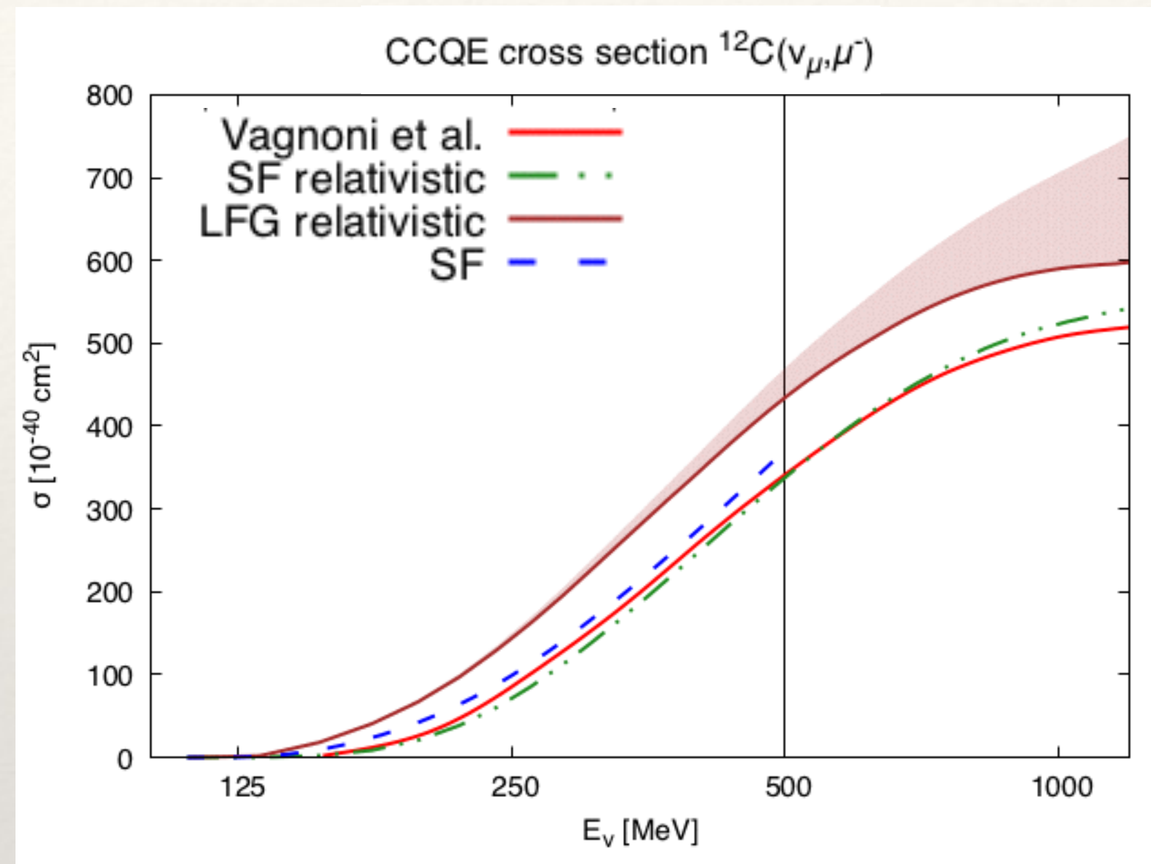


$(\sigma_{nuc.eff}-\sigma_0)/\sigma_0$

σ_0 - cross section without nuclear effects (LFG)

Results - neutrino scattering





E. Vagnoni, O. Benhar, and D. Meloni, Phys. Rev. Lett. 118, 142502 (2017)

spread to higher energies

quenching of the QE peak

relativistic effects

Relativistic case (I)

- It is possible to use some approximations in order to make the model applicable for higher energy-momentum transfers.

(I) Use the PWIA (neglect FSI = outcoming particle is free)

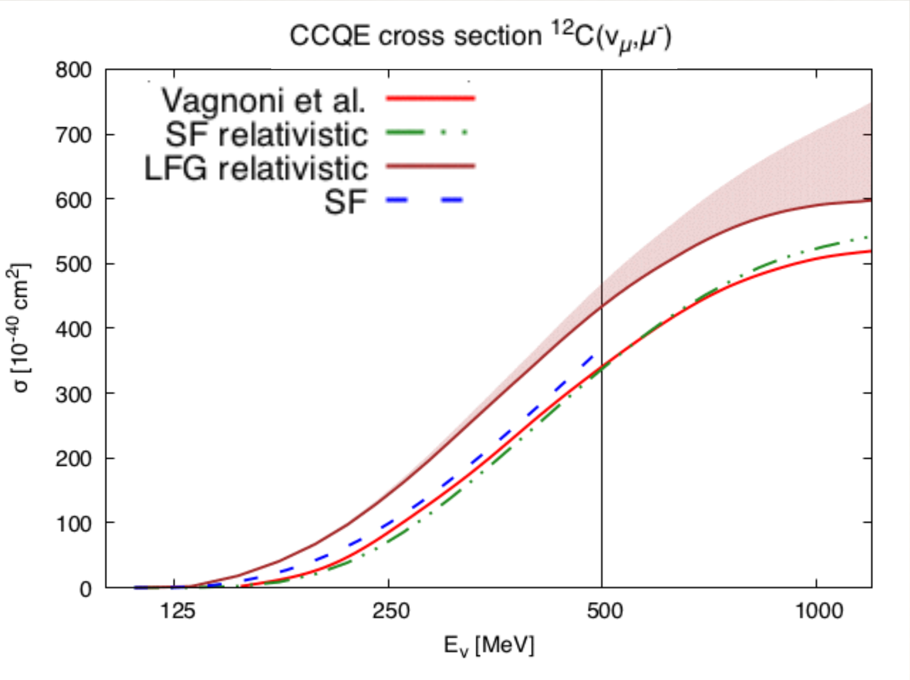
$$S_h(\mathbf{p}, E) = \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(\mathbf{p}, \widehat{E})}{\left(E - \mathbf{p}^2/2m - \mathrm{Re}\Sigma(\mathbf{p}, \widehat{E}) - \boxed{C\rho}\right)^2 + \mathrm{Im}\Sigma(\mathbf{p}, \widehat{E}\,)^2}$$

$$\hat{E} = E - \hat{\mathcal{C}} \qquad \mu = \frac{p_F^2}{2m} + \widehat{\mathcal{C}}$$

$$\widehat{\mathcal{C}} = \mathrm{Re}\Sigma(p_F^2/2m, p_F) + C\rho$$

P. Fernandez de Cordoba et al., Nucl. Phys. A611, 514 (1996)

In the model that we use, ReΣ is known up to a constant term. We introduce a ϱ-dependent parameter which we adjust to get a proper binding energy.



E. Vagnoni, O. Benhar, and D. Meloni, Phys. Rev. Lett. 118, 142502 (2017)

Relativistic case (II)

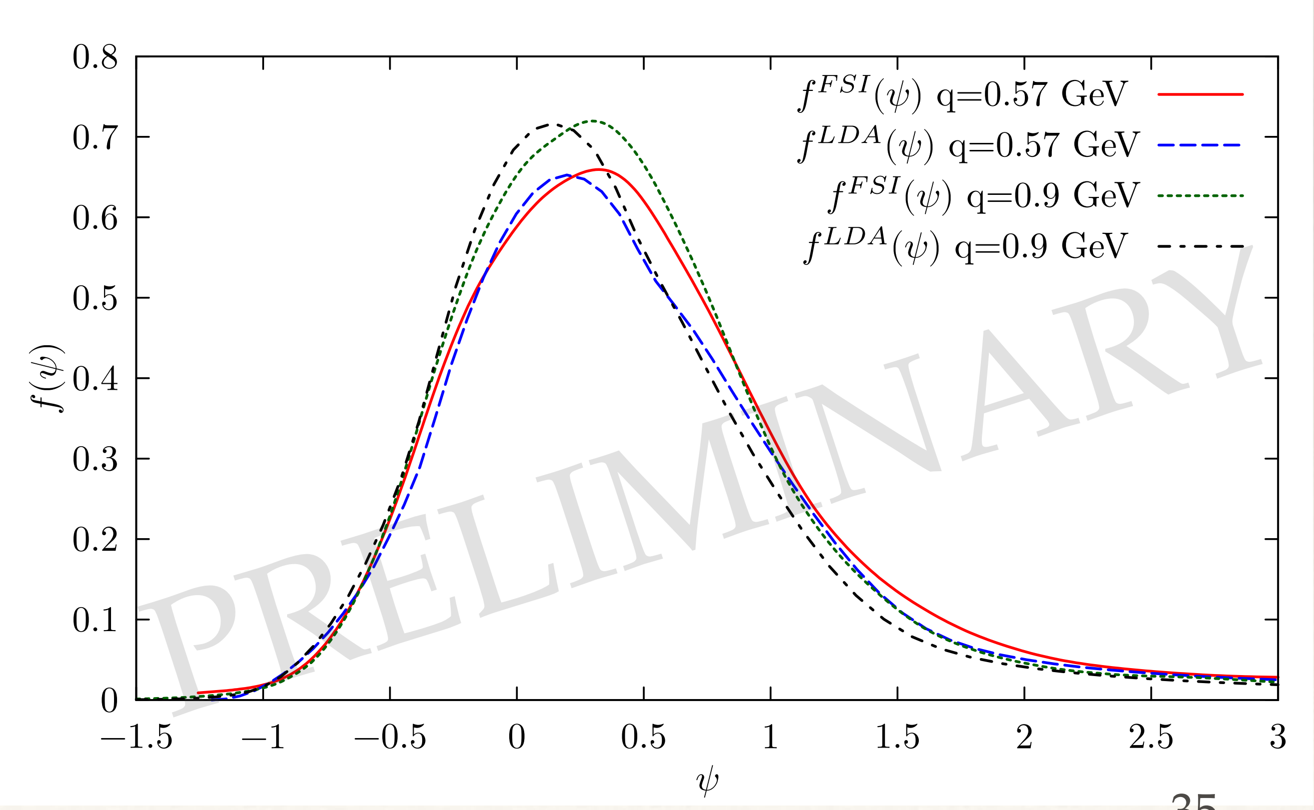
(II) Use relativistic approximation for SFs:

$$S_{p,h}(\mathbf{p},E) = \mp\frac{1}{\pi}\frac{\frac{m}{\epsilon(\mathbf{p})}\mathrm{Im}\Sigma(\mathbf{p},E)}{\left(E-\epsilon(\mathbf{p})-\frac{m}{\epsilon(\mathbf{p})}\mathrm{Re}\Sigma(\mathbf{p},E)\right)^2+\left(\frac{m}{\epsilon(\mathbf{p})}\mathrm{Im}\Sigma(\mathbf{p},E)\right)^2}$$

$$\epsilon(\mathbf{p}) = \sqrt{m^2+\mathbf{p}^2}$$

P. Fernandez de Cordoba et al., Nucl. Phys. A611, 514 (1996)

$$f_{LDA}(\psi) \propto \mathrm{S}_{LDA}(\mathbf{q},\omega) = \frac{\Theta(\omega)}{4\pi^3}\int d^3r\int d^3p\int_{\mu-\omega}^{\mu} dE\,\boxed{\frac{m}{\epsilon(\mathbf{p})}\frac{m}{\epsilon(\mathbf{p}+\mathbf{q})}}S_h(\mathbf{p},E)S_p(\mathbf{p}+\mathbf{q},E+\omega)$$



Scaling function: comparison with the model of Benhar SF +FSI