A proposal of a New Charged Lepton Flavor Violation Experoment: $\mu^- e^- \rightarrow e^- e^-$ in muonic atom

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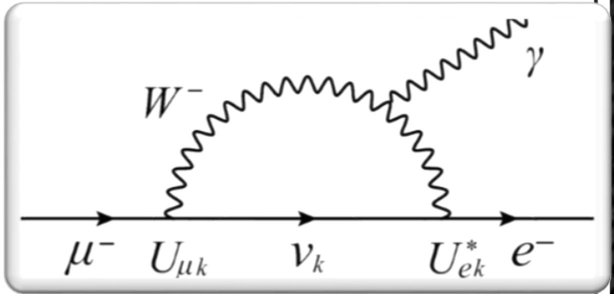
M. Koike, Y. Kuno, J. S, M.Yamanaka, Phys. Rev. Lett. 105, 121601

Y. Uesaka, Y. Kuno, J. S, T. Sato, M.Yamanaka., Phys. Rev. D93 076006

+ arXiv: 1710?????

Introduction

In Standard Model (SM)



Charged Lepton Flavor Violation (cLFV) via neutrino oscillation

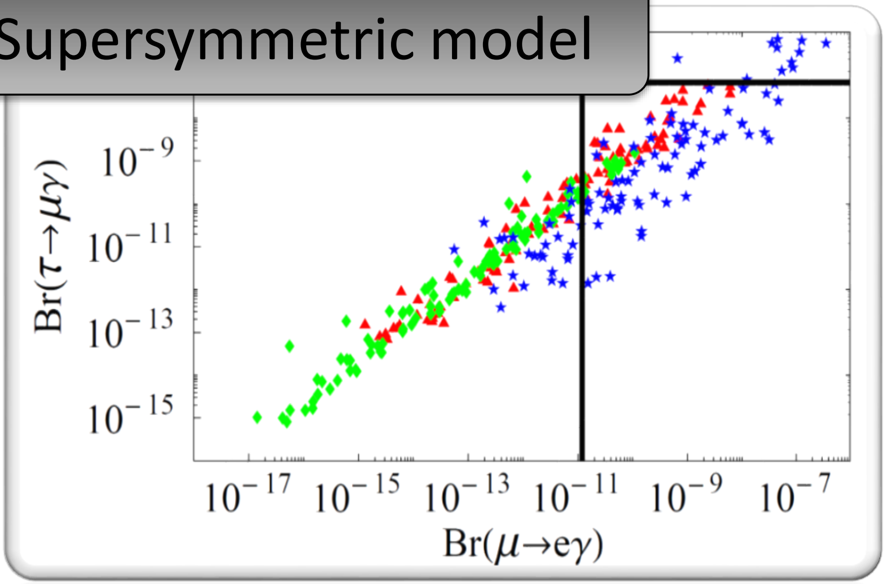
But … $\mathrm{BR}(\mu \to e\gamma) \sim \left(\frac{\delta m_\nu^2}{m_W^2}\right)^2 < 10^{-54}$ ⇨ Forever invisible

Discovery of the cLFV signal

⇨ One of the evidence for beyond the SM

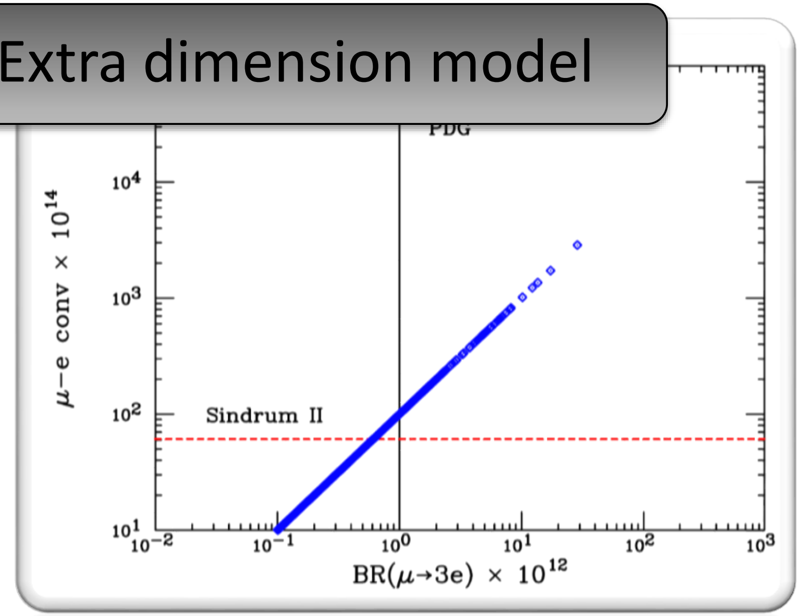
Introduction

Supersymmetric model



[M.~Raidal et al., Eur. Phys. J. C 57 (2008)]

Extra dimension model



[K.~Agashe, et al., Phys. Rev. D 74 (2006)]

Comparing LFV signals and their rates

⇒ Discrimination of new physics models

⇒ Prove for structure of new physics

Desire for many detectable cLFV processes

Introduction

New idea for cLFV search

$$\mu^- e^- \longrightarrow e^- e^- \text{ in muonic atom}$$

- What is target ?

  Flavor violation between μ and e

- What is advantage ?

  - Sensitive to both photonic dipole cLFV operator and 4-Fermi contact cLFV operator

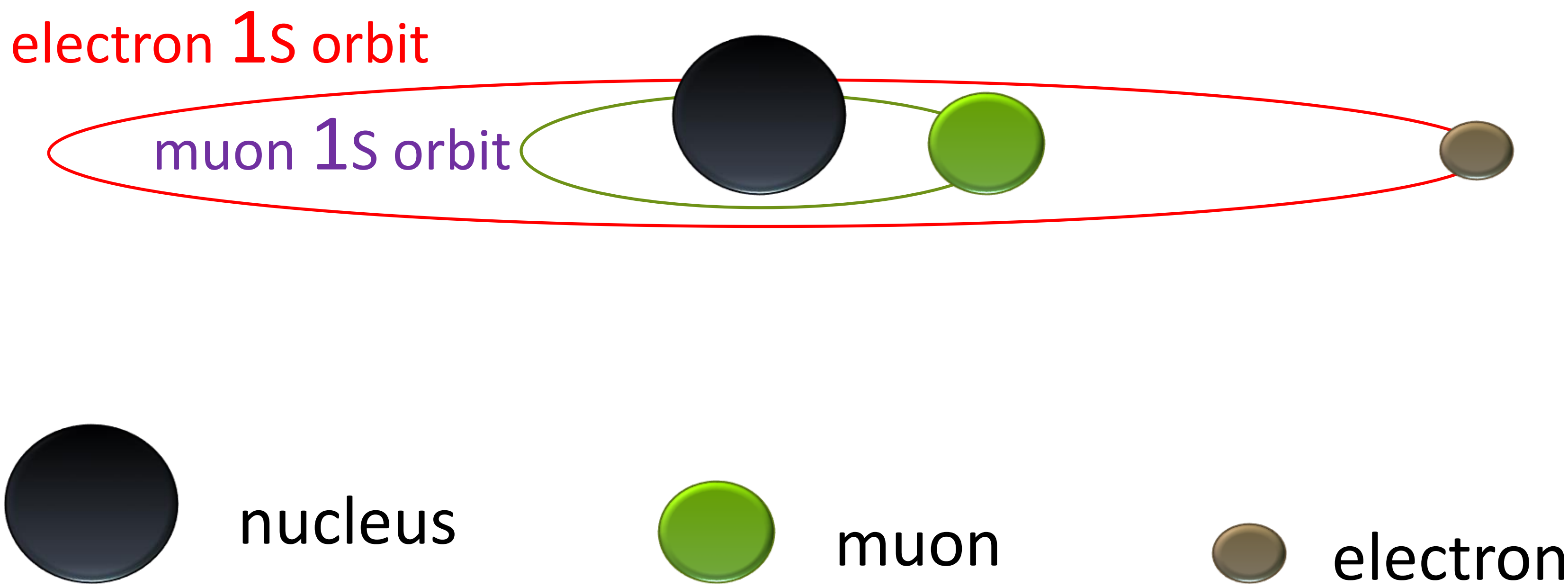
  - Clean signal [back-to-back dielectron]

Basic Idea (PRL)

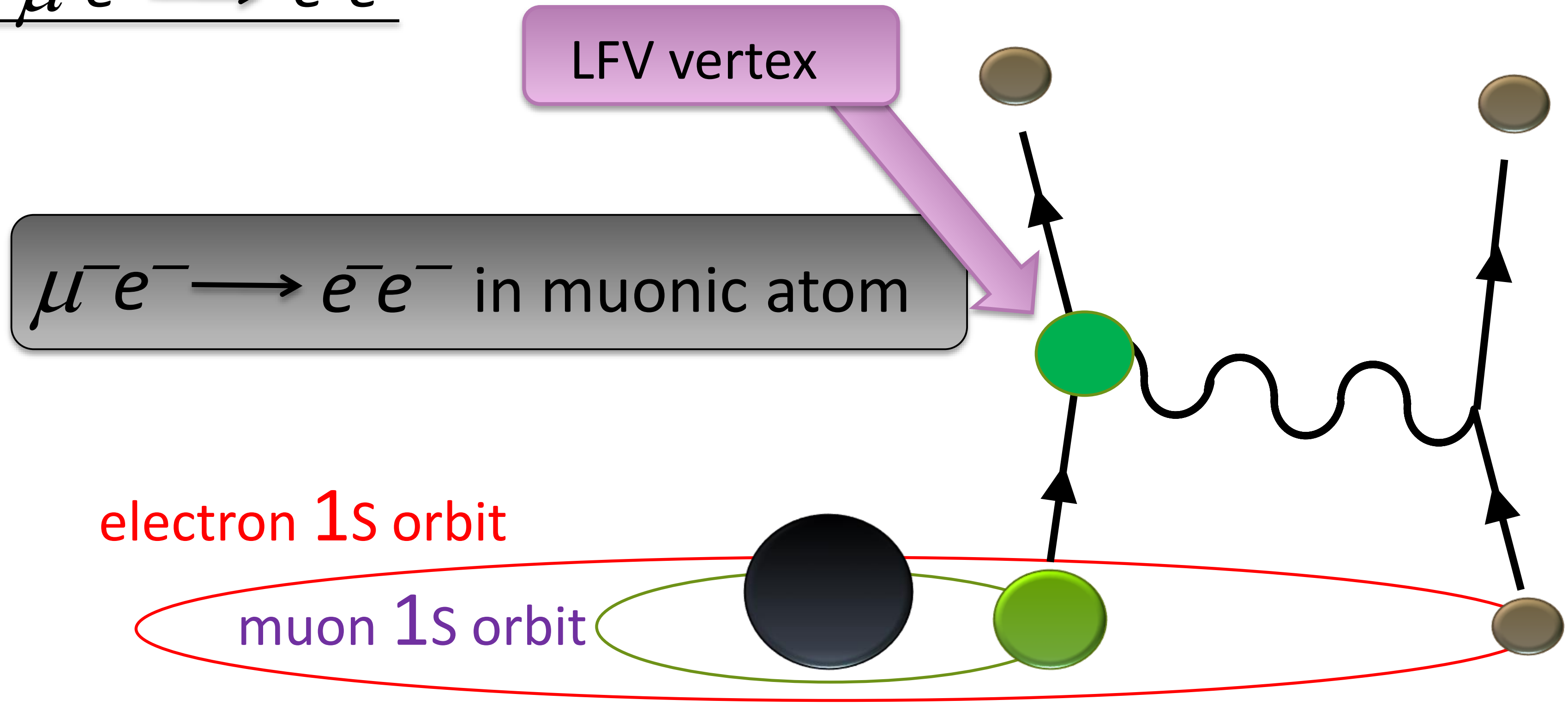
$$\mu^- e^- \longrightarrow e^- e^-$$

$\mu\, e \longrightarrow e\, e$

Muonic atom



$\mu\, e \longrightarrow e\, e$

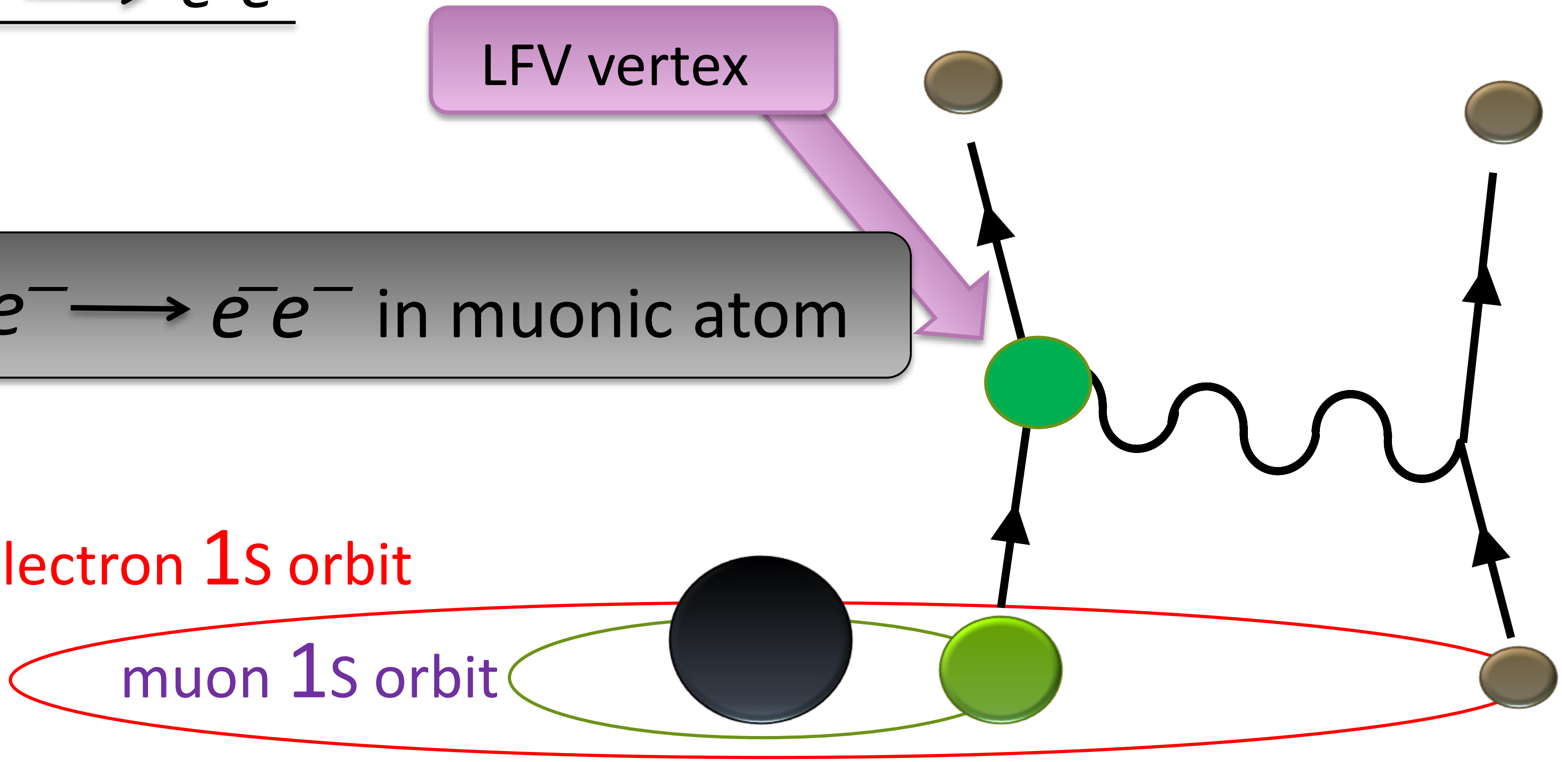


Interaction rate

$$\Gamma(\mu^- e^- \to e^- e^-; Z) = 2\sigma v_{\mathrm{rel}} \left| \psi_{1\mathrm{S}}^{(\mathrm{e})}(0; Z-1) \right|^2$$

$\mu\, e \longrightarrow e\, e$

$\mu^- e^- \longrightarrow e^- e^-$ in muonic atom

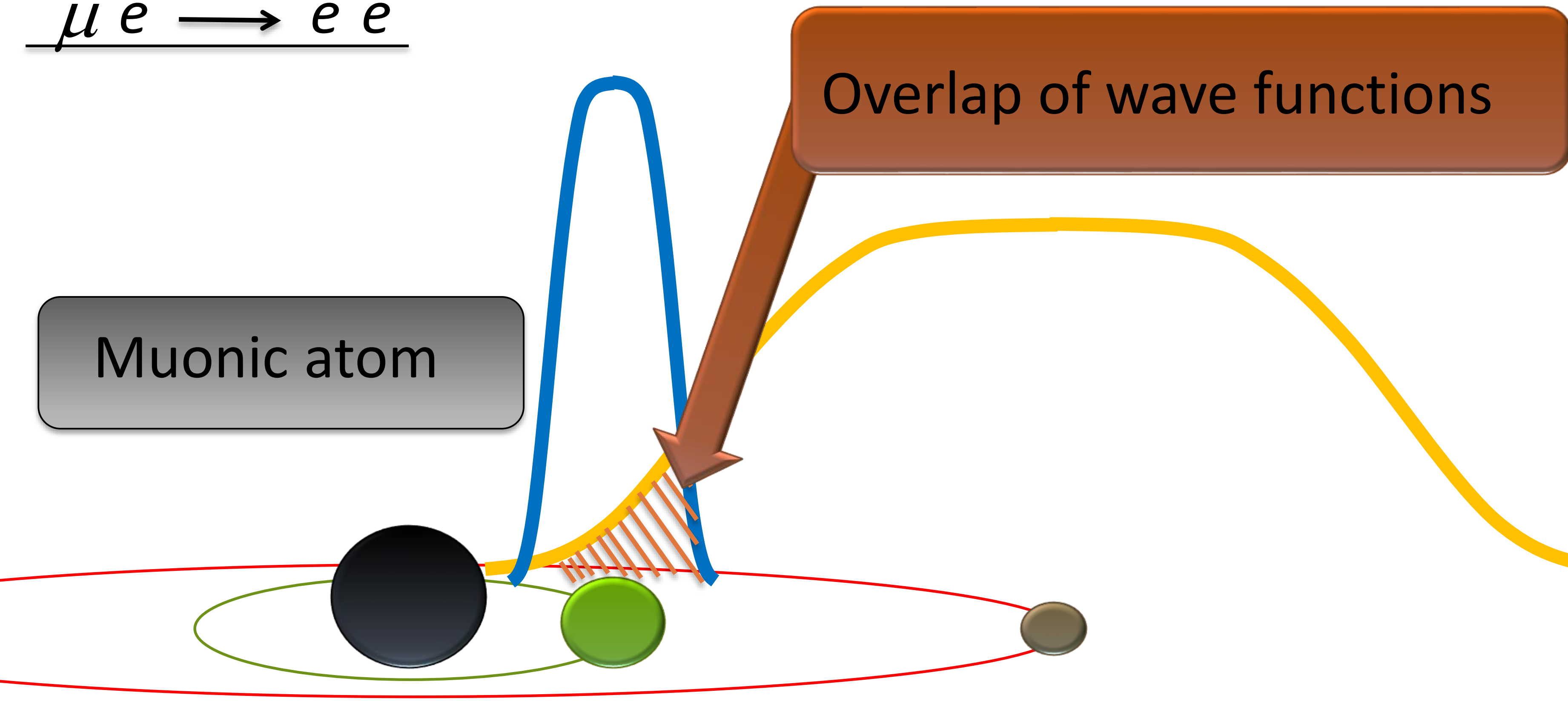


Interaction rate

$$\Gamma(\mu^- e^- \to e^- e^-; Z) = 2\sigma v_{\mathrm{rel}} \left|\psi^{(e)}_{1S}(0; Z-1)\right|^2$$

Overlap of wave function of μ and e

$\underline{\mu\, e \longrightarrow e\, e}$

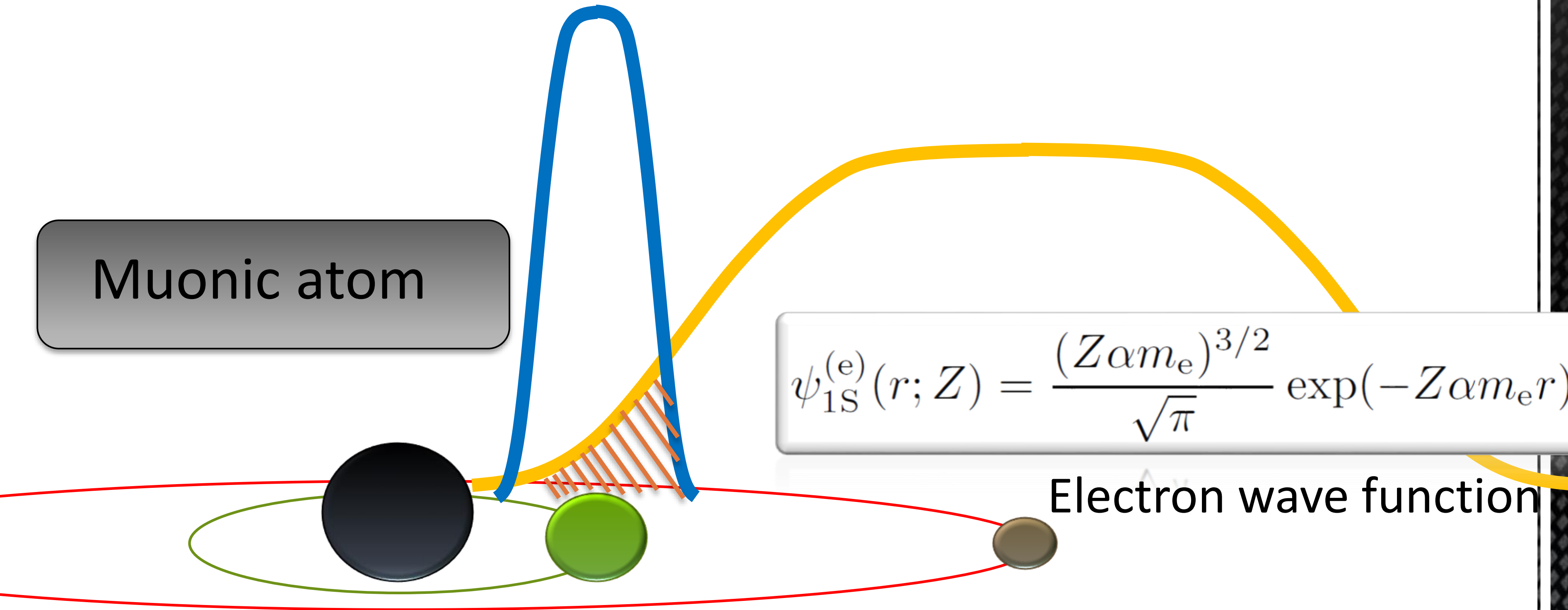


Approximation

Muon localization at nucleus position $\left(\because\ m_e \ll m_\mu \right)$

$\Rightarrow$ Overlap = electron wave function at nucleus

$\mu\, e \longrightarrow e\, e$



Muonic atom

$$\psi^{(\mathrm{e})}_{1\mathrm{S}}(r;Z) = \frac{(Z\alpha m_{\mathrm{e}})^{3/2}}{\sqrt{\pi}}\exp(-Z\alpha m_{\mathrm{e}} r)$$

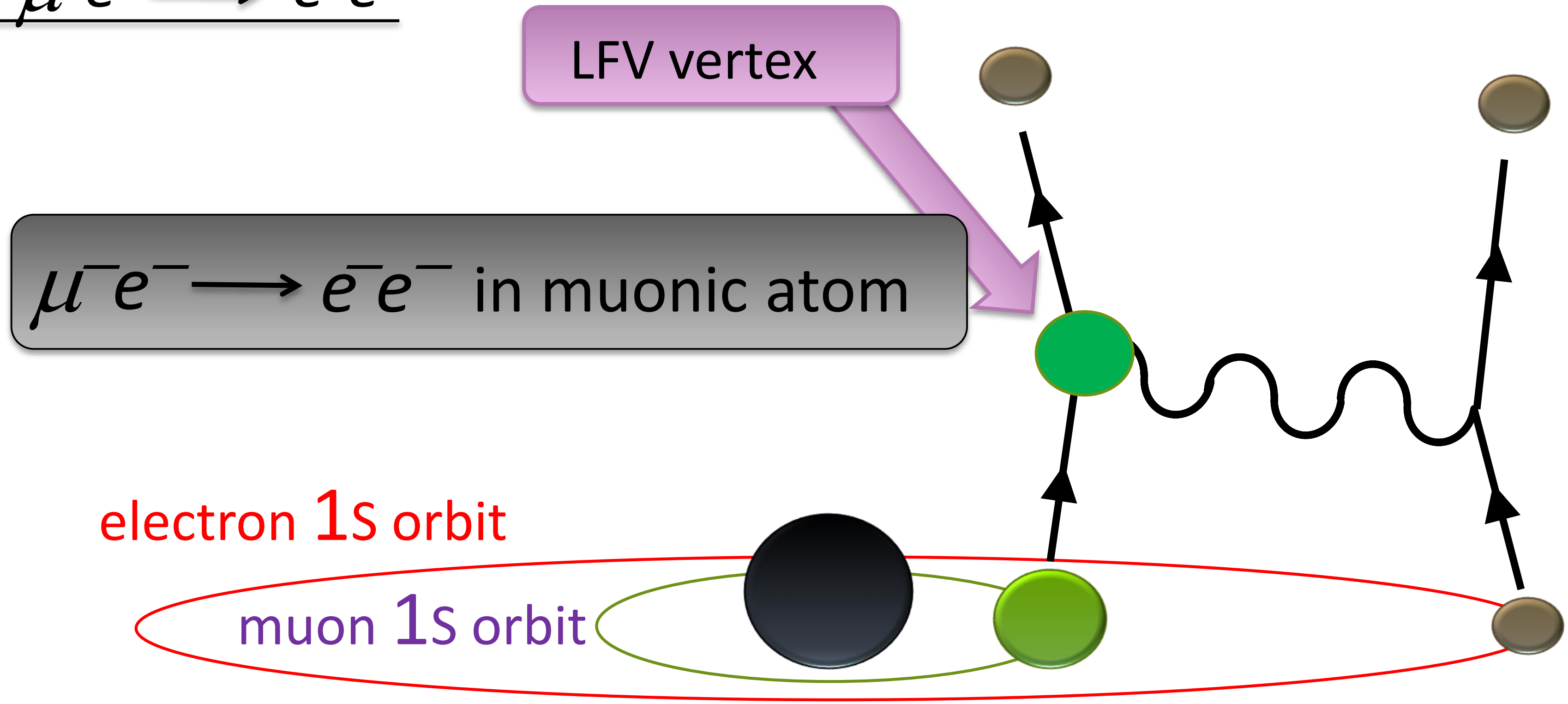
Electron wave function

r : radial coordinate (distance from nucleus)

Z : atomic number of nucleus in muonic atom

Overlap of wave functions $\left|\psi^{(\mathrm{e})}_{1\mathrm{S}}(0;Z-1)\right|^2$

$\mu\, e \longrightarrow e\; e$



Interaction rate

$$\Gamma(\mu^- e^- \to e^- e^-; Z) = 2\sigma v_{\rm rel} |\psi_{1S}^{(e)}(0; Z-1)|^2$$

Cross section for elemental interaction

$\mu\, e \longrightarrow e\, e$

Effective Lagrangian [Y. Kuno and Y. Okada Rev. Mod. Phys. **73** (2001)]

$$\mathcal{L}_{\mu^- e^- \to e^- e^-} = -\frac{4G_{\mathrm{F}}}{\sqrt{2}} \Big[m_\mu A_{\mathrm{R}}\, \overline{\mu_{\mathrm{R}}} \sigma^{\mu\nu} e_{\mathrm{L}} F_{\mu\nu} + m_\mu A_{\mathrm{L}}\, \overline{\mu_{\mathrm{L}}} \sigma^{\mu\nu} e_{\mathrm{R}} F_{\mu\nu}$$
$$+ g_1 \left(\overline{\mu_{\mathrm{R}}} e_{\mathrm{L}}\right)\left(\overline{e_{\mathrm{R}}} e_{\mathrm{L}}\right) + g_2 \left(\overline{\mu_{\mathrm{L}}} e_{\mathrm{R}}\right)\left(\overline{e_{\mathrm{L}}} e_{\mathrm{R}}\right)$$
$$+ g_3 \left(\overline{\mu_{\mathrm{R}}} \gamma^\mu e_{\mathrm{R}}\right)\left(\overline{e_{\mathrm{R}}} \gamma_\mu e_{\mathrm{R}}\right) + g_4 \left(\overline{\mu_{\mathrm{L}}} \gamma^\mu e_{\mathrm{L}}\right)\left(\overline{e_{\mathrm{L}}} \gamma_\mu e_{\mathrm{L}}\right)$$
$$+ g_5 \left(\overline{\mu_{\mathrm{R}}} \gamma^\mu e_{\mathrm{R}}\right)\left(\overline{e_{\mathrm{L}}} \gamma_\mu e_{\mathrm{L}}\right) + g_6 \left(\overline{\mu_{\mathrm{L}}} \gamma^\mu e_{\mathrm{L}}\right)\left(\overline{e_{\mathrm{R}}} \gamma_\mu e_{\mathrm{R}}\right) + (\mathrm{H.c.}) \Big]$$

cLFV effective coupling constant

A_{R} A_{L} g_1 g_2 g_3 g_4 g_5 g_6

→ Sensitive to the structure of new physics

Precise Estimate (PRD +) 1

Total Rate

Precise Estimate

- High Z nucleus is preferable
  - ⇒ All leptons are under strong Coulomb potentail

    Distorted wave function
- Out-going electron is very energetic >>m_e

  & High Z means high velocity for bound leptons
  - ⇒ Relativistic treatment
- Nuclei is not a point charge
  - ⇒ Numerical solution for Wave fns , integration for amplitude

⇒ Solve Dirac Eq. , integrate Wave funs, numerically

For trial, uniform charge density is assumed (not so important)

$$
\begin{aligned}
V(r) &= -\frac{Z\alpha}{R}\left(\frac{3}{2}-\frac{1}{2}\frac{r^2}{R^2}\right) \quad \text{for} \quad r<R \\
&= -\frac{Z\alpha}{r} \quad \text{for} \quad r>R \qquad\qquad R = 1.2A^{1/3}\text{fm}
\end{aligned}
$$

Calculating method

Decay rate Γ

$$\Gamma = 2\pi \sum_{f} \sum_{\bar{i}} \delta(E_f - E_i) \left| \left\langle \psi_e^{s_1}(\boldsymbol{p}_1)\psi_e^{s_2}(\boldsymbol{p}_2) \middle| H \middle| \psi_\mu^{s_\mu}(1s)\psi_e^{s_e}(1s) \right\rangle \right|^2$$

use partial wave expansion to express the distortion

$$\psi_e^s(\boldsymbol{p}) = \sum_{\kappa,\mu,m} 4\pi\, i^{l_\kappa} (l_\kappa, m, 1/2, s | j_\kappa, \mu) Y_{l_\kappa,m}^*(\hat{p}) e^{-i\delta_\kappa} \psi_p^{\kappa,\mu}$$

get radial functions by solving Dirac eq. numerically

$$\frac{dg_\kappa(r)}{dr} + \frac{1+\kappa}{r} g_\kappa(r) - \big(E + m + e\phi(r)\big) f_\kappa(r) = 0$$

$$\frac{df_\kappa(r)}{dr} + \frac{1-\kappa}{r} f_\kappa(r) + \big(E - m + e\phi(r)\big) g_\kappa(r) = 0$$

$$\psi(\boldsymbol{r}) = \begin{pmatrix} g_\kappa(r)\chi_\kappa^\mu(\hat{r}) \\ i f_\kappa(r)\chi_{-\kappa}^\mu(\hat{r}) \end{pmatrix}$$

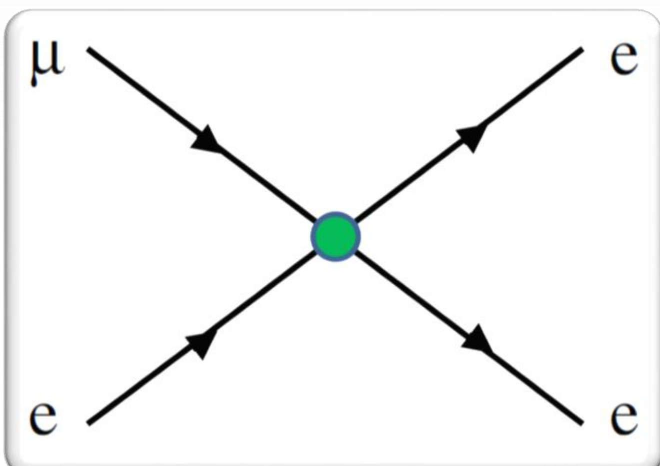
ϕ : nuclear Coulomb potential

$\mu\, e \longrightarrow e\, e$

Effective Lagrangian

[Y. Kuno and Y. Okada Rev. Mod. Phys. **73** (2001)]

$$
\begin{aligned}
\mathcal{L}_{\mu^- e^- \to e^- e^-} = -\frac{4G_F}{\sqrt{2}} \Big[& m_\mu A_R \, \overline{\mu_R} \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \, \overline{\mu_L} \sigma^{\mu\nu} e_R F_{\mu\nu} \\
& + g_1 (\overline{\mu_R} e_L)(\overline{e_R} e_L) + g_2 (\overline{\mu_L} e_R)(\overline{e_L} e_R) \\
& + g_3 (\overline{\mu_R} \gamma^\mu e_R)(\overline{e_R} \gamma_\mu e_R) + g_4 (\overline{\mu_L} \gamma^\mu e_L)(\overline{e_L} \gamma_\mu e_L) \\
& + g_5 (\overline{\mu_R} \gamma^\mu e_R)(\overline{e_L} \gamma_\mu e_L) + g_6 (\overline{\mu_L} \gamma^\mu e_L)(\overline{e_R} \gamma_\mu e_R) + (\text{H.c.}) \Big]
\end{aligned}
$$



4-Fermi interaction type

Reaction rate for contact interactions

- ☑ Reaction rate for contact ints.

$$\Gamma = 2\pi \sum_f \sum_i \delta(E_f - E_i) \left| \langle \psi^e_{\vec{p}_1} \psi^e_{\vec{p}_2} | \mathcal{L}_I | \psi^\mu_B \psi^e_B \rangle \right|^2$$

$$= \frac{G_F^2}{\pi^3} \int dE_{p_1} |\boldsymbol{p}_1||\boldsymbol{p}_2| \sum_{J,\kappa_1,\kappa_2} (2J+1)(2j_{\kappa_1}+1)(2j_{\kappa_2}+1)$$

$$\times \left| \sum_{i=1}^{6} g_i W_i(J, \kappa_1, \kappa_2, E_{p_1}) \right|^2$$

(Amplitude)^2 up to J

κ : total angular momentum for scattered electron

- ☑ cLFV interactions Physics !!
- ☑ Overlap of wave functions

High angular momentum is very important

TABLE I. The convergence of the partial wave expansion of Γ/Γ_0.

nuclei	$\|\kappa\| \leq 1$	$\|\kappa\| \leq 5$	$\|\kappa\| \leq 10$	$\|\kappa\| \leq 20$
^{40}Ca	0.141	0.847	1.11	1.15
^{120}Sn	0.731	2.17	2.21	2.21
^{208}Pb	2.89	6.94	6.96	6.96

Γ_0 : previous calculation

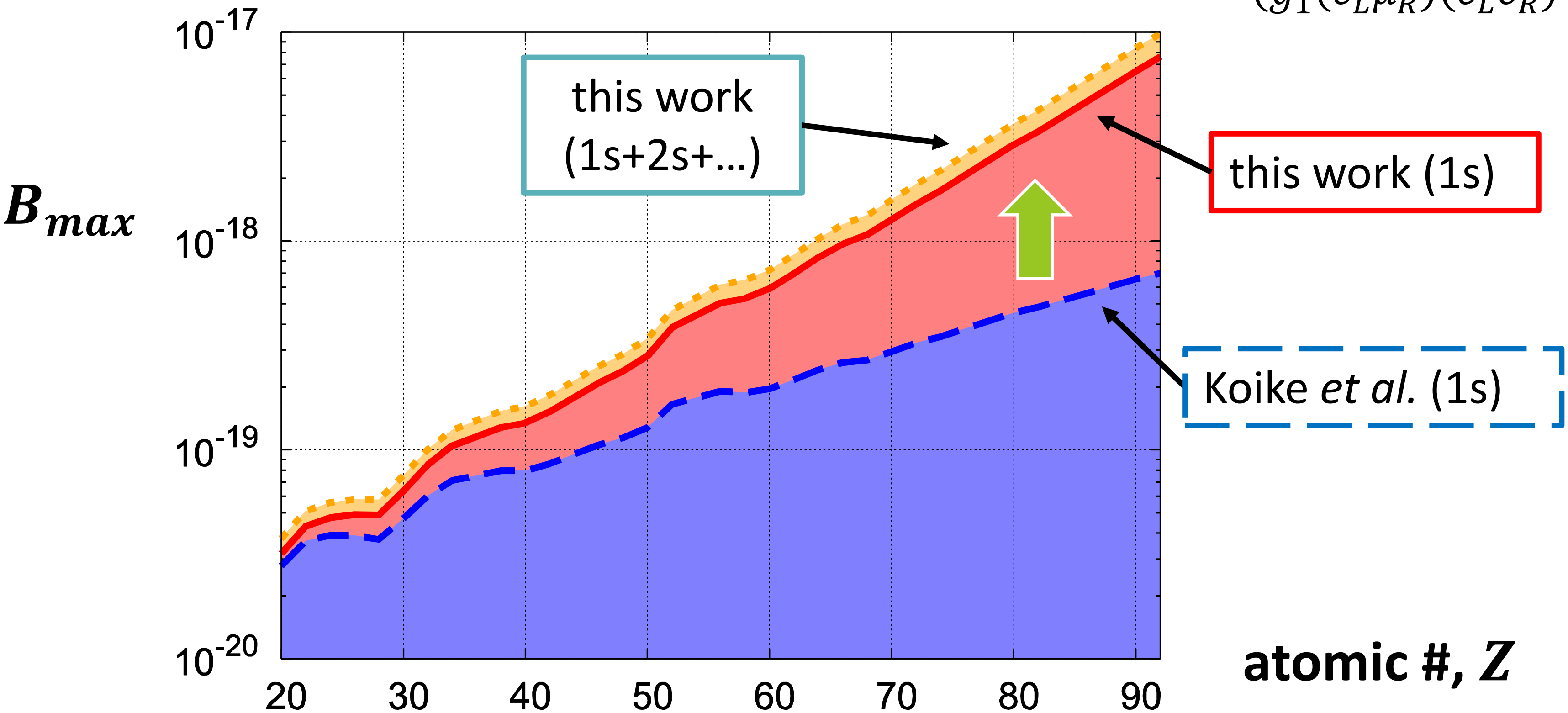
$$\Gamma(\mu^- e^- \to e^- e^-; Z) = 2\sigma v_{\mathrm{rel}} \left| \psi_{1S}^{(e)}(0; Z-1) \right|^2$$

Upper limits of BR (contact process)

$\boxed{BR(\mu^+ \to e^+e^-e^+) < 1.0 \times 10^{-12}}$ ⇒ $\boxed{BR(\mu^-e^- \to e^-e^-) < B_{max}}$

(SINDRUM, 1988)

$(g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R))$

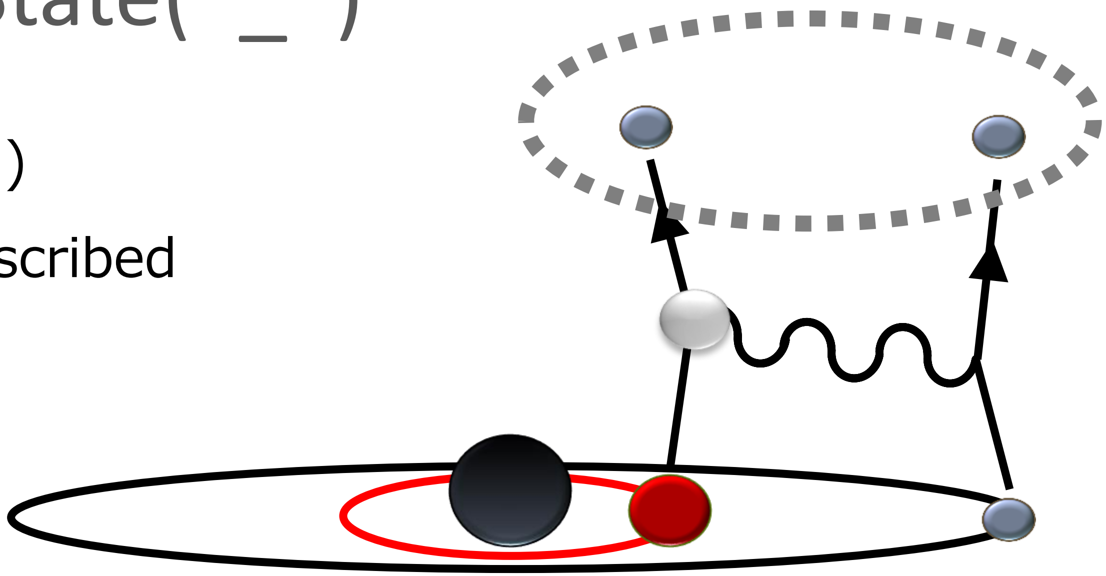


needed # of muonic atoms ($Z = 82$)

2.1×10^{18} ⇒ $\boxed{3.0 \times 10^{17}}$

Precise Estimate

Dirac Eq. : Final State(^_^)



- ☑ Previous calculation(1)

  Final electrons are described by plane wave

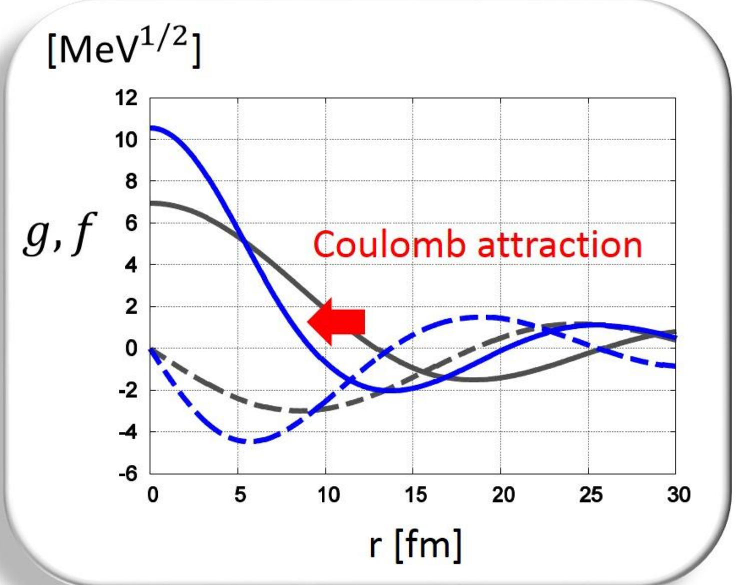
Though Out-going electrons are highly relativistic and its wave functions are distorted by nuclear Coulomb potential, especially for high Z

- ☑ Improvement for scattered electrons

  Plane wave → Coulomb scattering wave function of Dirac Eq. with finite size nucleus

Precise Estimate

Dirac Eq. : Final State



$$\psi_\kappa(\vec{r}) = \begin{pmatrix} g_\kappa(r)\chi_\kappa(\hat{r}) \\ i f_\kappa(r)\chi_{-\kappa}(\hat{r}) \end{pmatrix}$$

solid line : large component, g

dotted line : small component, f

(black line : w.f. plane wave)

- Wave fun. near the center position becomes larger, which leads enhancement of the overlap and hence reaction rate

Muon is located at r<O(10) fm

☑ Improvement for scattered electrons

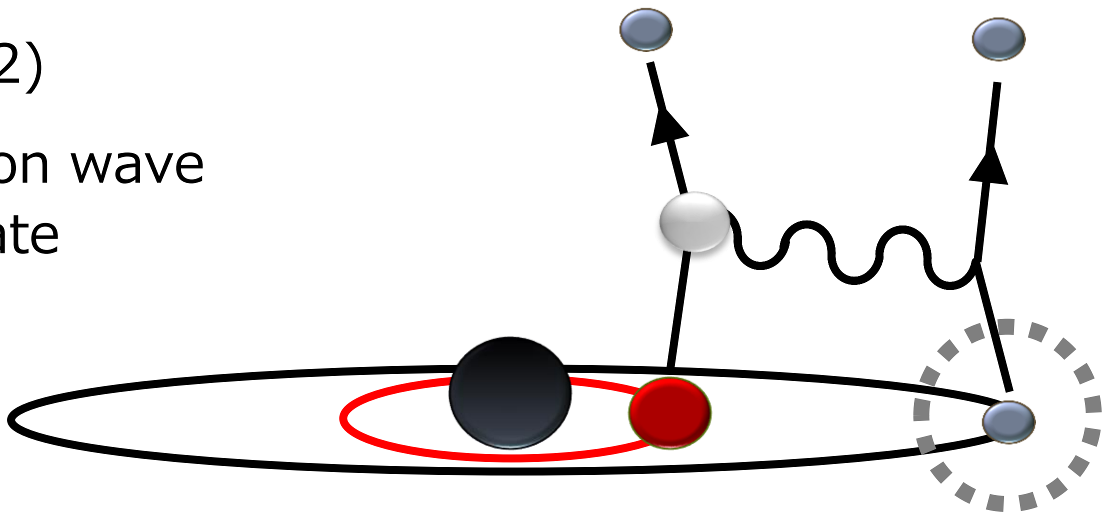
Plane wave → Coulomb scattering wave function of Dirac Eq. with finite size nucleus

Precise Estimate

Dirac Eq. : Bound electron(^_^)

☑ Previous calculation (2)

Non-relativistic electron wave function for bound state



The bound electron is a relativistic Dirac particle

+

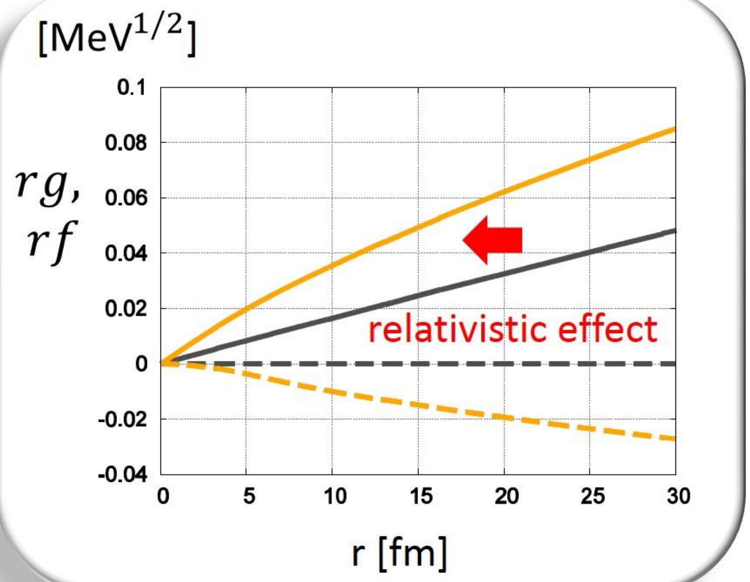
Electron orbit radius ≫ Nucleus size

☑ Improvement for bound electron

NR wave function → Wave function of Dirac particle in point Coulomb potential

Precise Estimate

Dirac Eq. : Bound electron



$$\psi_\kappa(\vec{r}) = \begin{pmatrix} g_\kappa(r)\chi_\kappa(\hat{r}) \\ if_\kappa(r)\chi_{-\kappa}(\hat{r}) \end{pmatrix}$$

solid line : large component, g

dotted line : small component, f

(black line : g: Non-Rel , f=0)

Muon is located at r<O(10) fm

☑ More attracted, leading enhancement of the overlap

☑ Improvement for bound electron

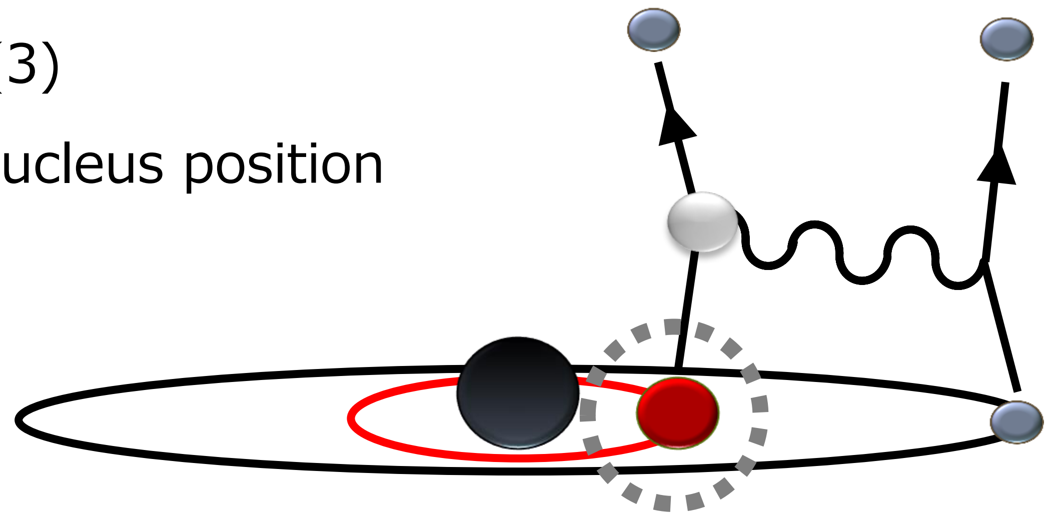
NR wave function → Wave function of Dirac particle in point Coulomb potential

Precise Estimate

Dirac Eq. : Bound muon(T _ T)

☑ Previous Calculation (3)

Muon is localized at nucleus position



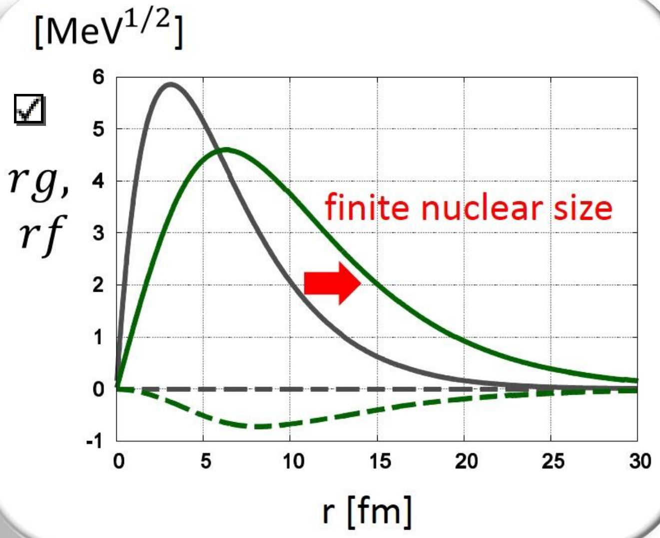
Muon is not at center but spread around nucleus

☑ Improvement for bound muon

Localized muon wave function → Wave function of Dirac particle in Coulomb potential by finite size nucleus

Precise Estimate

Dirac Eq. : Bound muon



$$\psi_\kappa(\vec{r}) = \begin{pmatrix} g_\kappa(r)\chi_\kappa(\hat{r}) \\ if_\kappa(r)\chi_{-\kappa}(\hat{r}) \end{pmatrix}$$

solid line : large component, g

dotted line : small component, f

(black line : w.f. used in previous work)

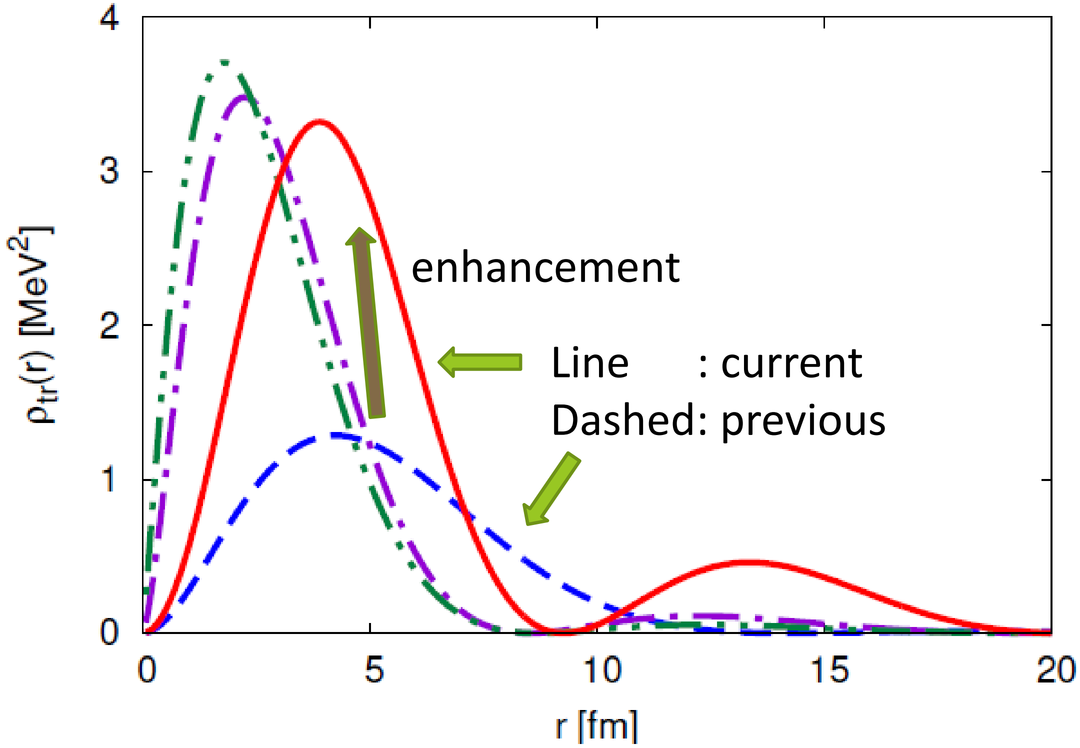
☑ Density near the center position becomes smaller, leading decline of the overlap and hence the reaction rate

☑ Improvement for bound muon

Localized muon wave function → Wave function of Dirac particle in Coulomb potential by finite size nucleus

Overlap of wave functions

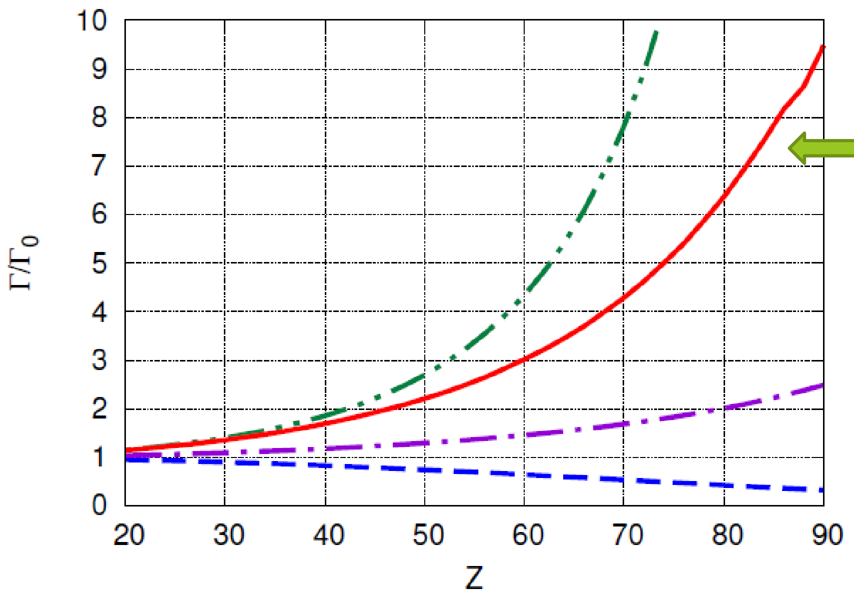
$$\rho_{\rm tr}(r) = g_{p_1}^{-1}(r) g_{\mu}^{-1}(r) g_{p_2}^{-1}(r) g_e^{-1}(r)$$



$$A = 208 \text{ if } Z = 82$$

Not only 1S but also 2S, …, contribute

nuclei	c [fm]	z [fm]	Γ/Γ_0 (only $1S$)	$1S + 2S + \cdots$
^{40}Ca	3.51(7)	0.563	1.15	1.35
^{120}Sn	5.315(25)	0.576(11)	2.21	2.67
^{208}Pb	6.624(35)	0.549(8)	6.96	8.78



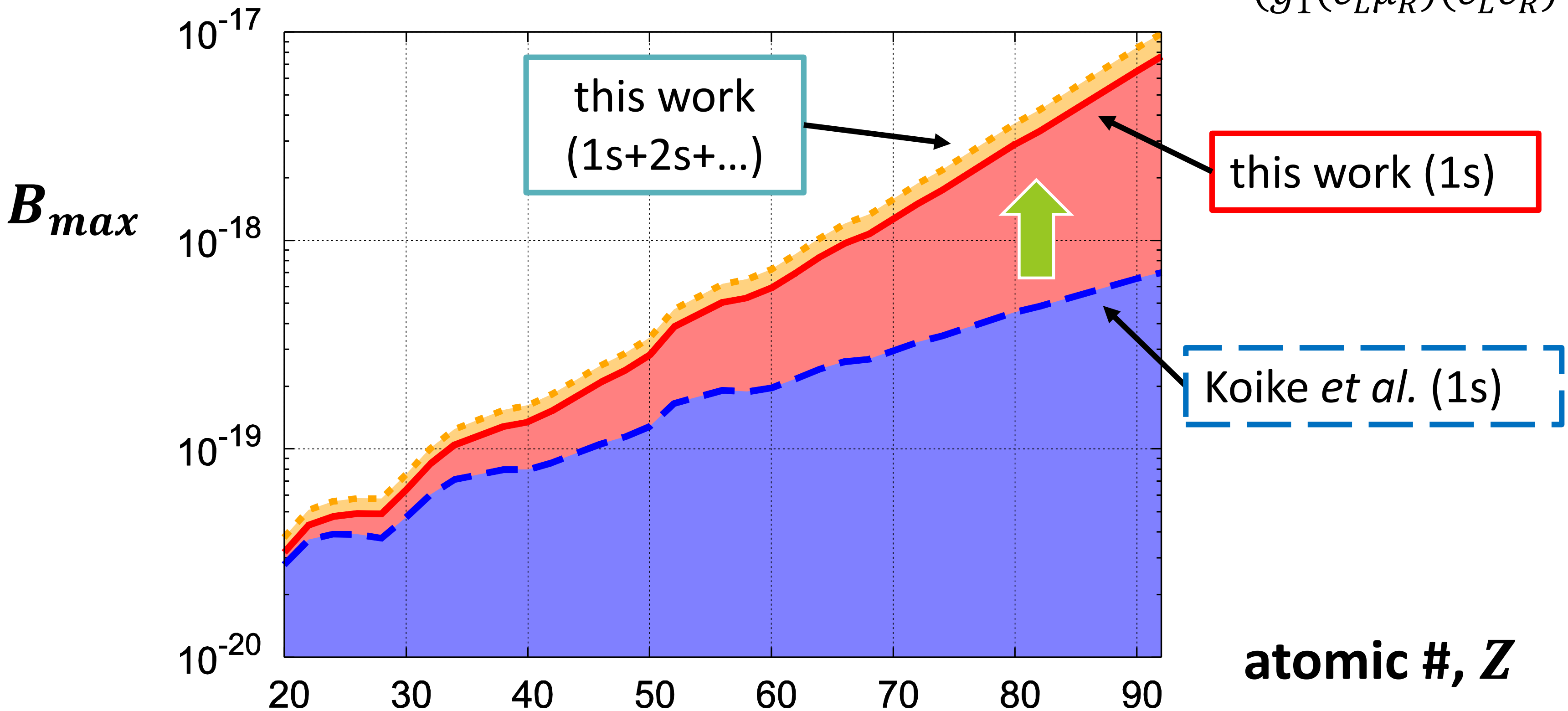
Rate : much larger

Upper limits of BR (contact process)

$\boxed{BR(\mu^+ \to e^+ e^- e^+) < 1.0 \times 10^{-12}}$ ⇒ $\boxed{BR(\mu^- e^- \to e^- e^-) < B_{max}}$

(SINDRUM, 1988)

$(g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R))$



needed # of muonic atoms ($Z = 82$)

$\boxed{2.1 \times 10^{18}}$ ⇒ $\boxed{3.0 \times 10^{17}}$

$\mu\, e \longrightarrow e\, e$

Effective Lagrangian

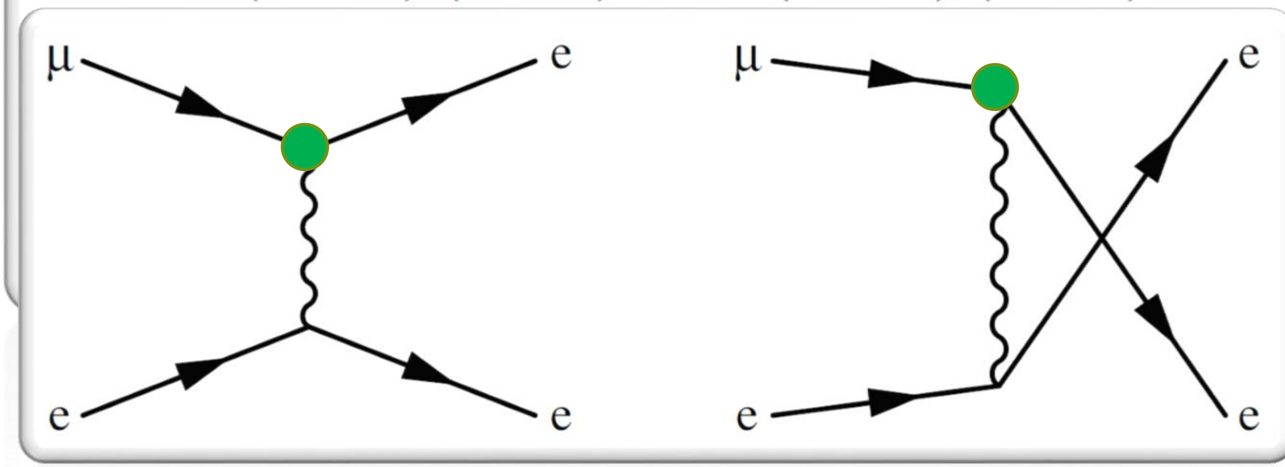
[Y. Kuno and Y. Okada Rev. Mod. Phys. **73** (2001)]

$$\mathcal{L}_{\mu^- e^- \to e^- e^-} = -\frac{4G_F}{\sqrt{2}}\Big[\boxed{m_\mu A_R\, \overline{\mu_R}\sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L\, \overline{\mu_L}\sigma^{\mu\nu} e_R F_{\mu\nu}}$$

$$+ g_1 (\overline{\mu_R} e_L)(\overline{e_R} e_L) + g_2 (\overline{\mu_L} e_R)(\overline{e_L} e_R)$$

$$\ldots \mu e_L)$$

$$\ldots \mu e_R) + (\mathrm{H.c.})\Big]$$



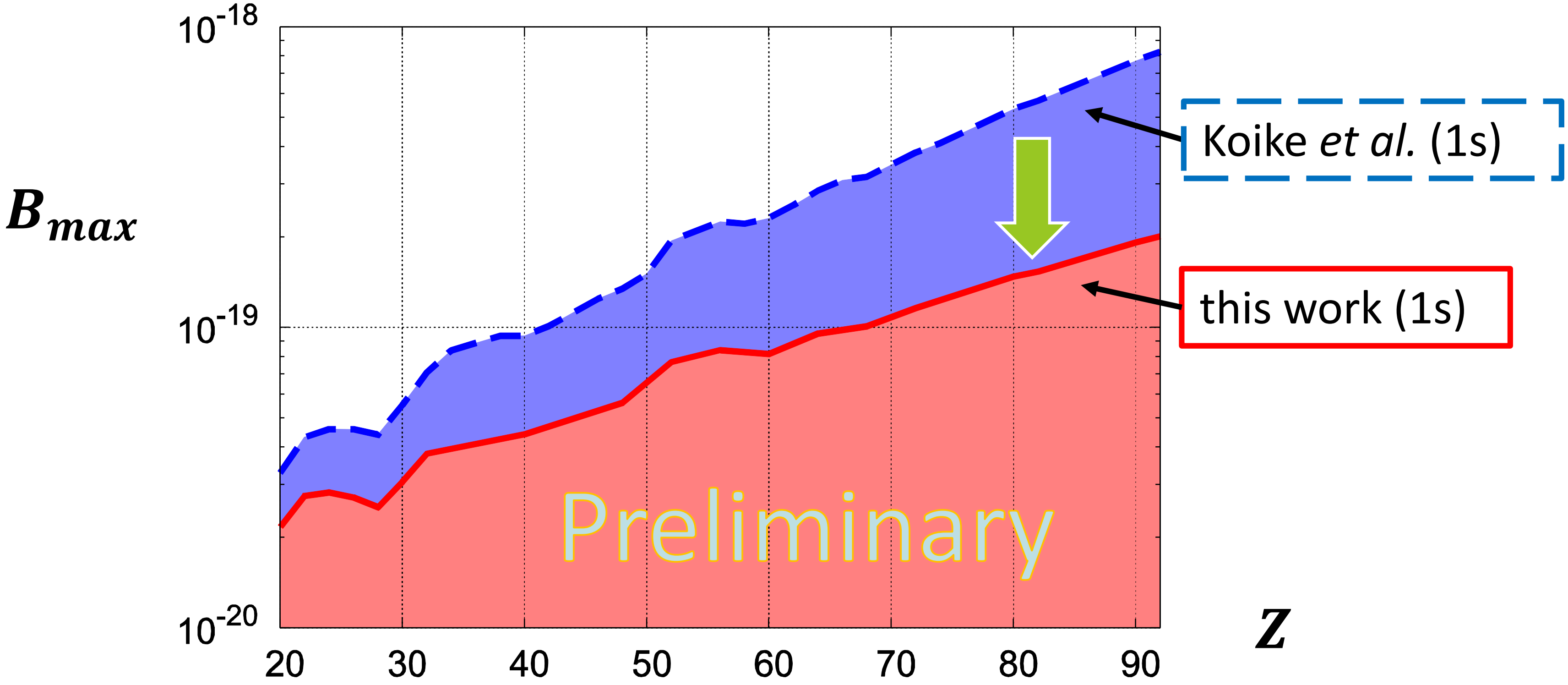
Photonic interaction type

Upper limits of BR (photonic process)

$\boxed{BR(\mu^+ \to e^+\gamma) < 5.7 \times 10^{-13}}$ ➡ $\boxed{BR(\mu^- e^- \to e^- e^-) < B_{max}}$

(MEG, 2013)

$\left(g_L \overline{e_L} \sigma^{\mu\nu} \mu_R F_{\mu\nu}\right)$

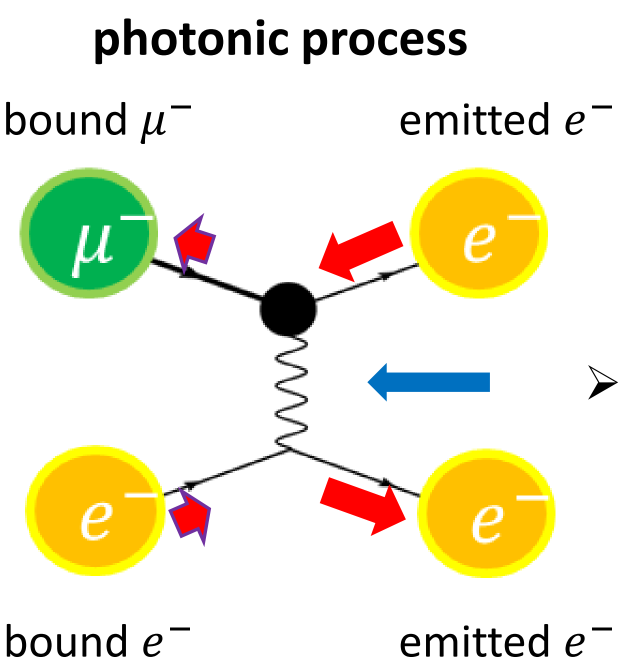


needed # of muonic atoms ($Z = 82$)

$\boxed{1.8 \times 10^{18}}$ ➡ $\boxed{6.6 \times 10^{18}}$

Suppresion factor for photonic interaction
Phase shift effect of distortion

(makes a momentum of e^- larger effectively)



- No effect on photon by coulomb force
- No distortion for photon propergater
- momentum transfers to bound leptons make overlap integrals smaller

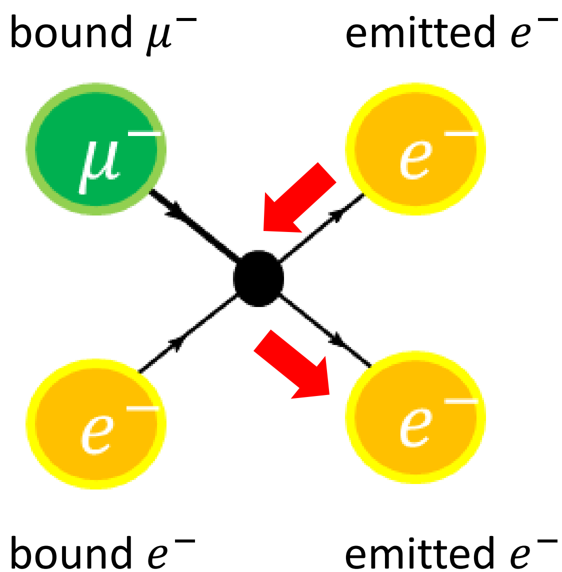
Totally (combined with the effect to enhance the value near the origin),

suppressed…

Phase shift effect of distortion

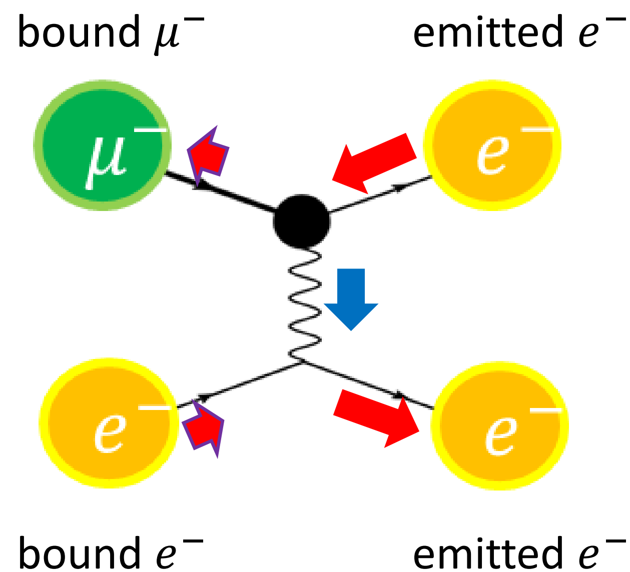
(makes a momentum of e^- larger effectively)

contact process



- no momentum mismatches

photonic process



- momentum transfers to bound leptons make overlap integrals smaller

Totally (combined with the effect to enhance the value near the origin),

enhanced !!

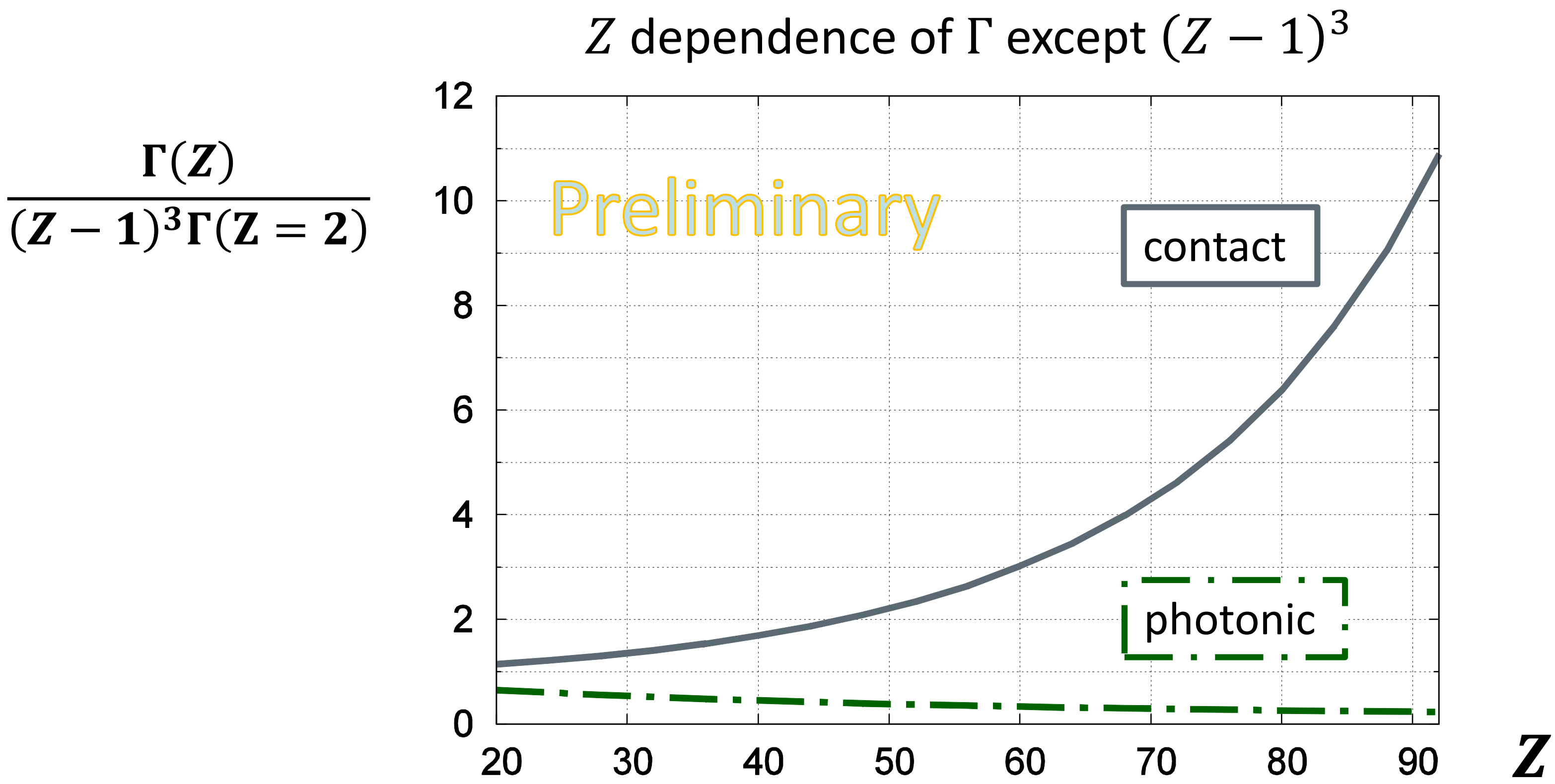
suppressed…

Precise Estimate (PRD +) 1

Interaction type dependence
How to discriminate ?

Discriminating method 1

~ atomic # dependence of decay rates ~



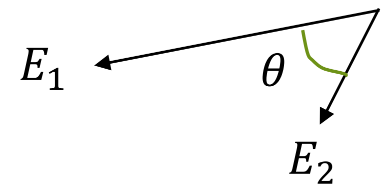
- The Z dependences are different between interactions.
- Compared to $(Z-1)^3$, that of contact process is larger, while that of photonic process is smaller.

Discriminating method 2

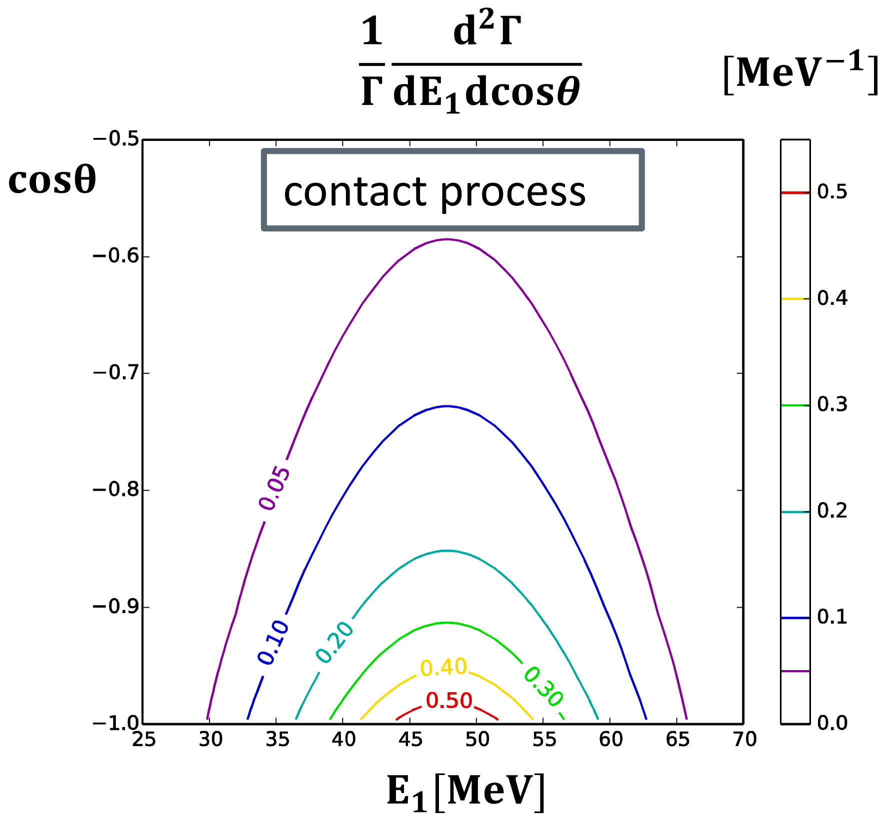
~ energy and angular distributions ~

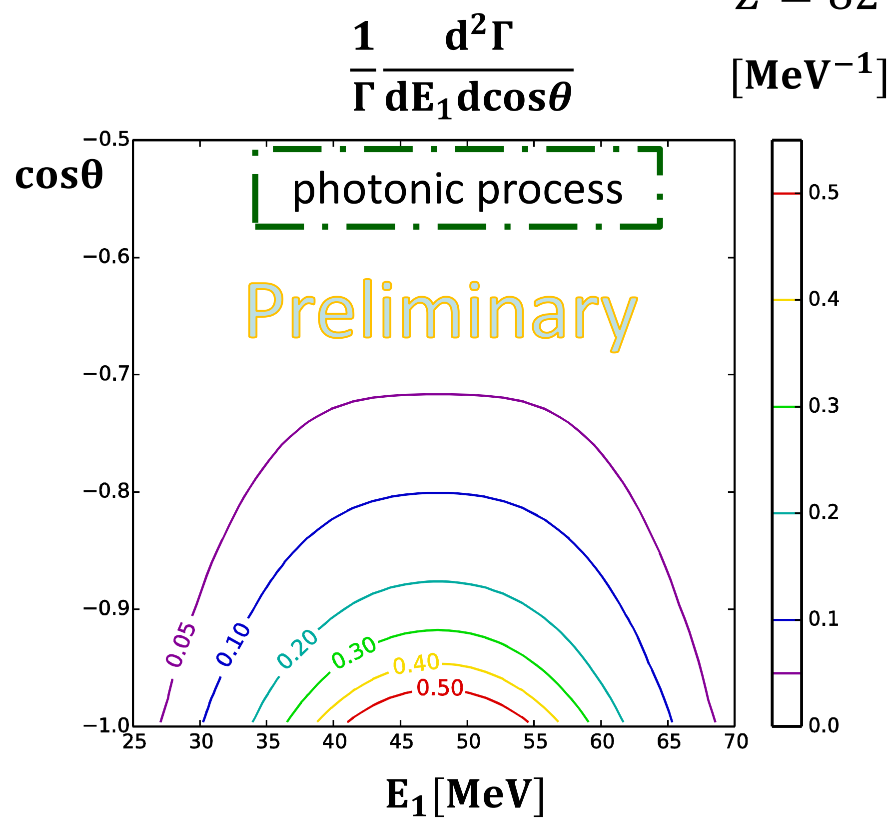
E_1 : energy of an emitted electron

θ : angle between two emitted electrons



$Z = 82$





Summary

Summary

- New LFV process $\mu^- e^- \longrightarrow e^- e^-$ in muonic atom

- Clean signal (back to back electron with $E_e \cong m_\mu/2$)

- Interaction rate $\Gamma(\mu^- e^- \to e^- e^-; Z) \sim (Z-1)^3$

⇨ Advantage : Large nucleus

- Discrimination among interactions : possible

Making use of Z dependence, energy spectrum, position dependence

Summary

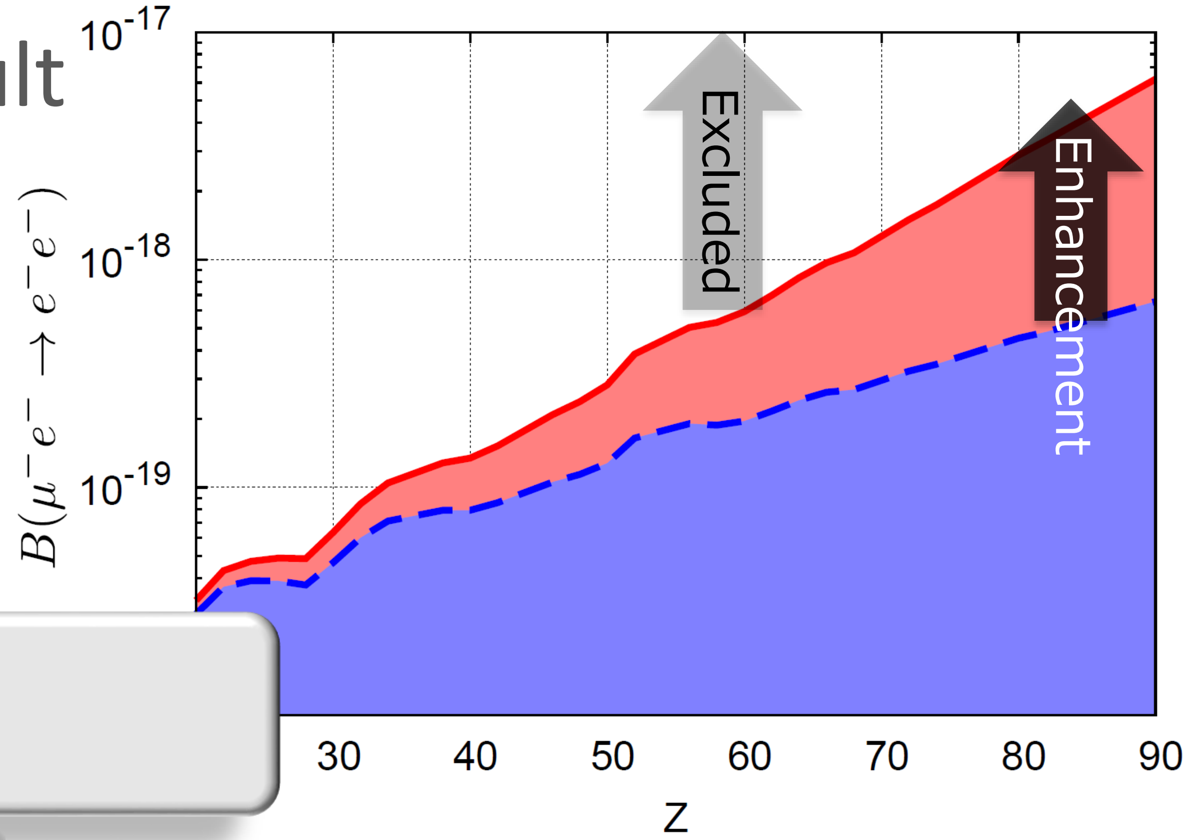
- Detectable in on-going or future experiments

→ Discussion with COMET people

Unfortunately not a discovery mode …., though

Backup (photonic dipole Interaction)

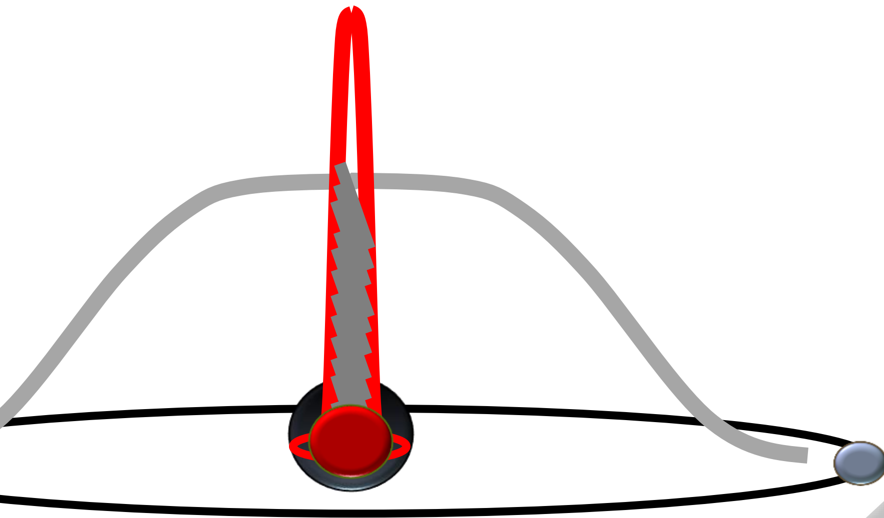
Numerical result



Upper bound
with $\mu \to 3e$ current limit

- ☑ Large enhancement of the reaction rate by the improvements
- ☑ Especially the improvements of electron wave functions
- ☑ Example: upper bound for pb nucleus ~3.5×10^{-18}

The reason of enhancement



- ☑ Wave function of initial electron approaches to nucleus

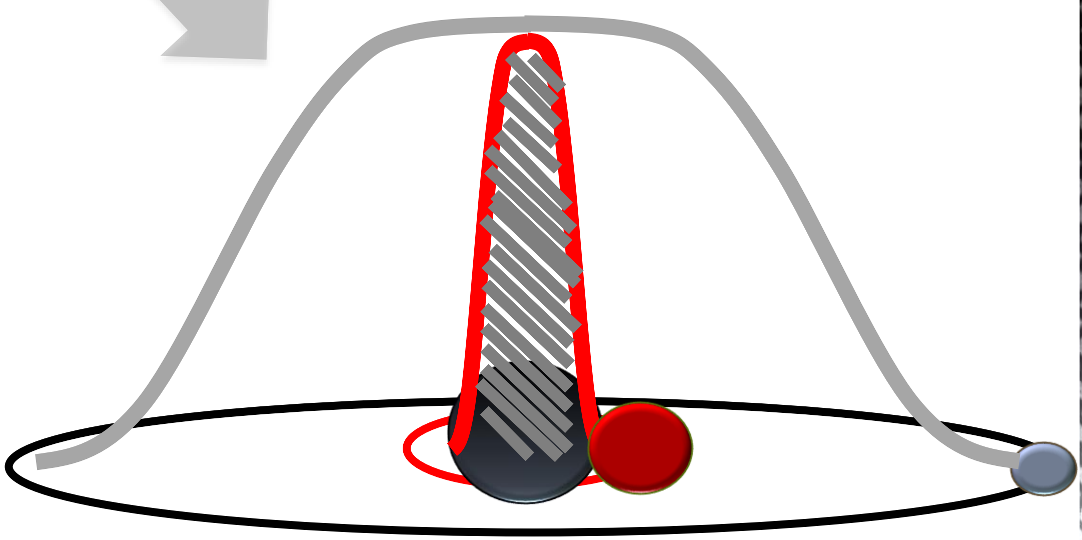
  Increase of overlap

- ☑ Wave function density of muon at nucleus becomes smaller

  Decrease of overlap

- ☑ Wave functions of final electron also approach to nucleus

  Increase of overlap



As a total

Enhancement

Cross section of cLFV elemental process

☑ cLFV effective Lagrangian

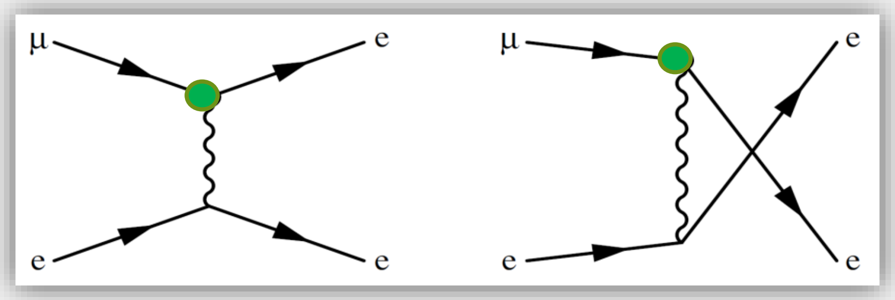
$$\mathcal{L}_{\mu^-\mathrm{e}^-\to\mathrm{e}^-\mathrm{e}^-} = -\frac{4G_\mathrm{F}}{\sqrt{2}}\Big[m_\mu A_\mathrm{R}\,\overline{\mu_\mathrm{R}}\sigma^{\mu\nu}e_\mathrm{L}F_{\mu\nu} + m_\mu A_\mathrm{L}\,\overline{\mu_\mathrm{L}}\sigma^{\mu\nu}e_\mathrm{R}F_{\mu\nu}$$

$$+ g_1\left(\overline{\mu_\mathrm{R}}e_\mathrm{L}\right)\left(\overline{e_\mathrm{R}}e_\mathrm{L}\right) + g_2\left(\overline{\mu_\mathrm{L}}e_\mathrm{R}\right)\left(\overline{e_\mathrm{L}}e_\mathrm{R}\right)$$

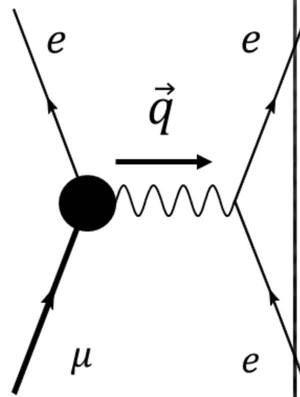
$$+ g_3\left(\overline{\mu_\mathrm{R}}\gamma^\mu e_\mathrm{R}\right)\left(\overline{e_\mathrm{R}}\gamma_\mu e_\mathrm{R}\right) + g_4\left(\overline{\mu_\mathrm{L}}\gamma^\mu e_\mathrm{L}\right)\left(\overline{e_\mathrm{L}}\gamma_\mu e_\mathrm{L}\right)$$

$$+ g_5\left(\overline{\mu_\mathrm{R}}\gamma^\mu e_\mathrm{R}\right)\left(\overline{e_\mathrm{L}}\gamma_\mu e_\mathrm{L}\right) + g_6\left(\overline{\mu_\mathrm{L}}\gamma^\mu e_\mathrm{L}\right)\left(\overline{e_\mathrm{R}}\gamma_\mu e_\mathrm{R}\right) + (\mathrm{H.c.})\Big]$$

Dipole photonic interaction



Photonic interaction: Long-range force



Photon propagator : infrared divergent

$$q_0 = p_1 - E_\mu = E_e - p_2$$

$$\mathcal{M}\left(p_1, s_1; p_2, s_2; 1S, s_\mu; n, s_e\right) \equiv \langle e(p_1, s_1); e(p_2, s_2)|H|\mu(1S, s_\mu); e(n, s_e)\rangle$$

$$\propto \int \mathrm{d}^3x \mathrm{d}^3x' \boxed{\int \frac{\mathrm{d}^3q}{(2\pi)^3} \frac{q_\nu e^{-i\vec{q}\cdot(\vec{x}-\vec{x'})}}{q_0^2 - |\vec{q}|^2 + i\epsilon}}$$

$$\times \overline{\psi_e^{p_1,s_1}}(\vec{x})\sigma^{\mu\nu}\psi_\mu^{1S,s_\mu}(\vec{x})\overline{\psi_e^{p_2,s_2}}\left(\vec{x'}\right)\gamma_\mu\psi_e^{n,s_e}\left(\vec{x'}\right) - (1 \leftrightarrow 2)$$

Scattered electrons (Coulomb distorted) Bound leptons

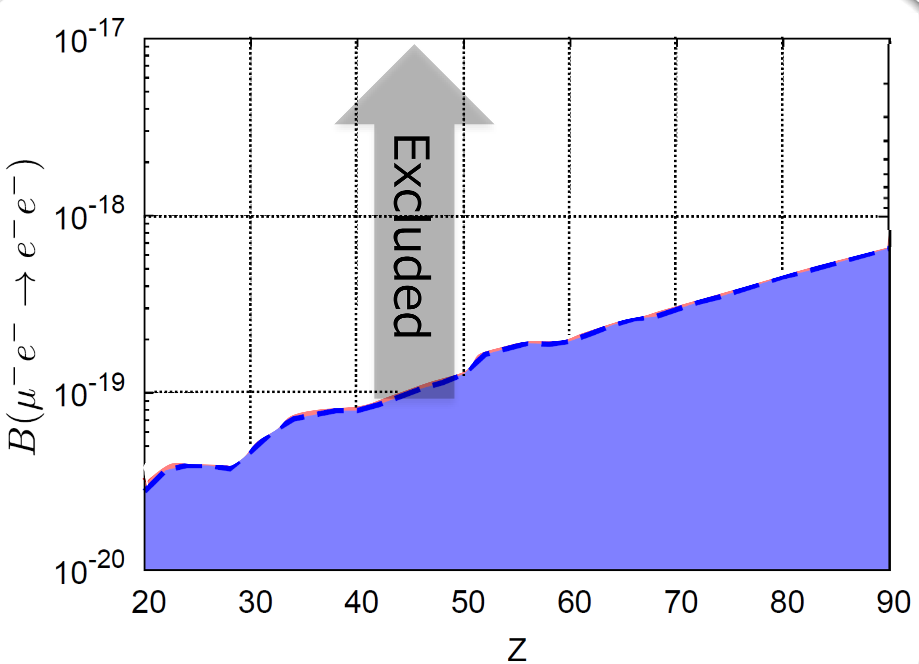
Infrared divergent , especially for high Z

Backup slides

Numerical result (previous work)

- ☑ The new process and $\mu \to 3e$ are described by same operator
- ☑ Relation between upper bounds of the new process and $\mu \to 3e$

$$\frac{\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-)}{\mathrm{Br}(\mu^+ \to \mathrm{e}^+ \mathrm{e}^+ \mathrm{e}^-)} \lesssim 192\pi (Z-1)^3 \alpha^3 \left(\frac{m_\mathrm{e}}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} ,$$



- ☑ Large enhancement by large nucleus charge
- ☑ Example: Upper bound for Pb nucleus ~ 5×10^{-19}

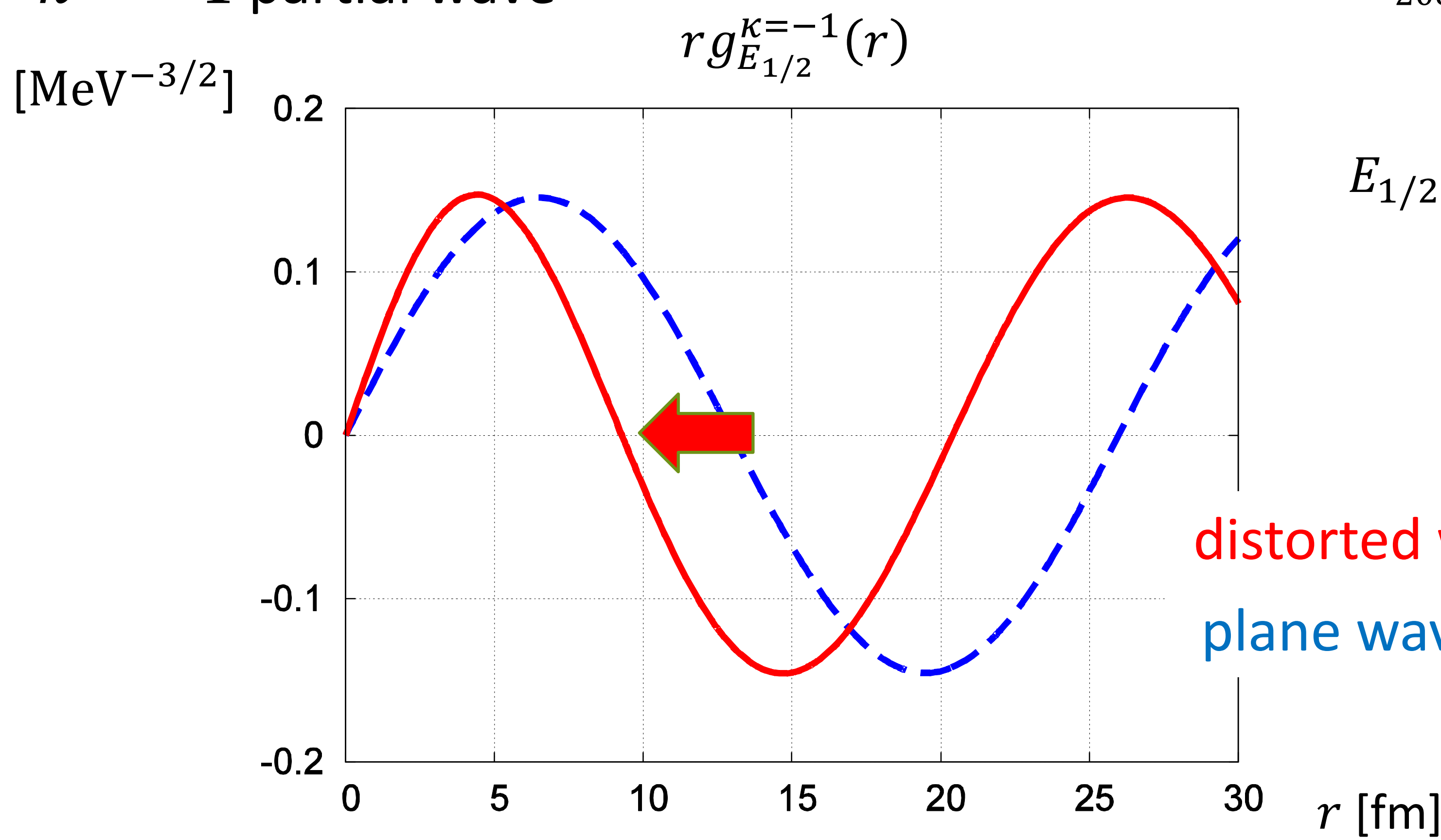
Distortion of emitted electrons

- $\kappa = -1$ partial wave

^{208}Pb case

$(Z = 82)$

$E_{1/2} \approx 48\mathrm{MeV}$



what the distortion makes

1. enhanced value near the origin
2. phase shift to boost momentum effectively

overlap of w.f.

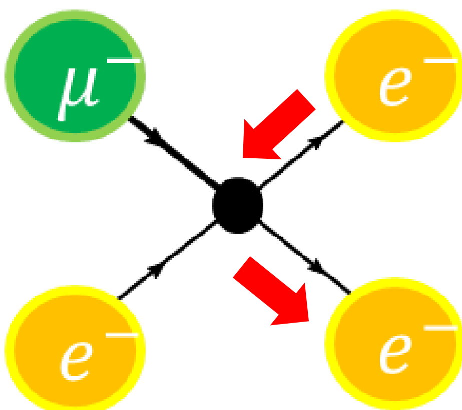
complicated a little...

Phase shift effect of distortion

(makes a momentum of e^- larger effectively)

contact process

bound μ^- emitted e^-

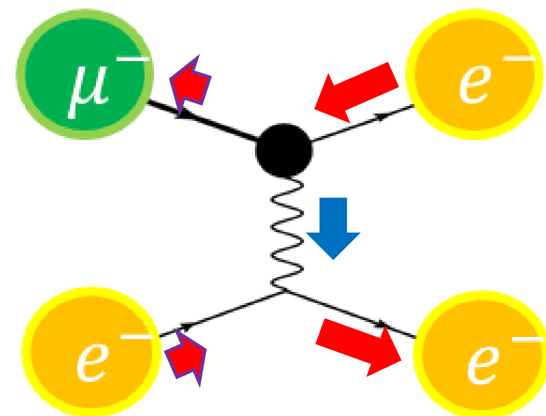


bound e^- emitted e^-

- no momentum mismatches

photonic process

bound μ^- emitted e^-



bound e^- emitted e^-

- <u>momentum transfers to</u> bound leptons make overlap integrals smaller

Totally (combined with the effect to enhance the value near the origin),

enhanced !!

suppressed…

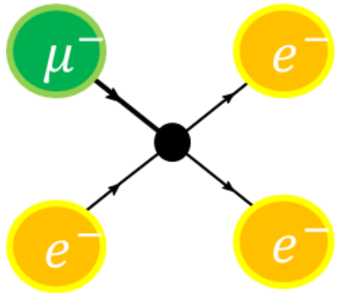
Model-discriminating power

After finding CLFV transition,
"which CLFV interaction exists" would be important.

Here, only 2 simple models will be considerd.

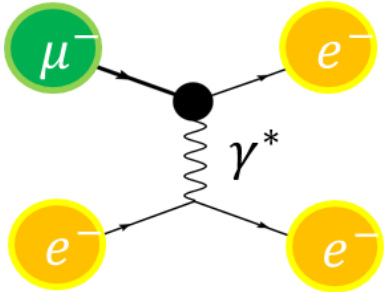
model 1 : contact type

$$\mathcal{L}_I = g_1(\overline{e}_L \mu_R)(\overline{e}_L e_R)$$



model 2 : photonic type

$$\mathcal{L}_I = g_R \overline{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu}$$



Summary

- $\mu^- e^- \rightarrow e^- e^-$ process in a muonic atom
  - ✓ interesting candidate for CLFV search
  - ✓ Our finding
    - Distortion of emitted electrons
    - Relativistic treatment of a bound electron

are important in calculating decay rates.

⇩ Distortion makes difference between 2 processes.

- ■ contact process : decay rate Enhanced (7 times in $Z = 82$)
- ■ photonic process : decay rate suppressed (1/4 times in $Z = 82$)

◆ How to identify interaction types, found by this analyses

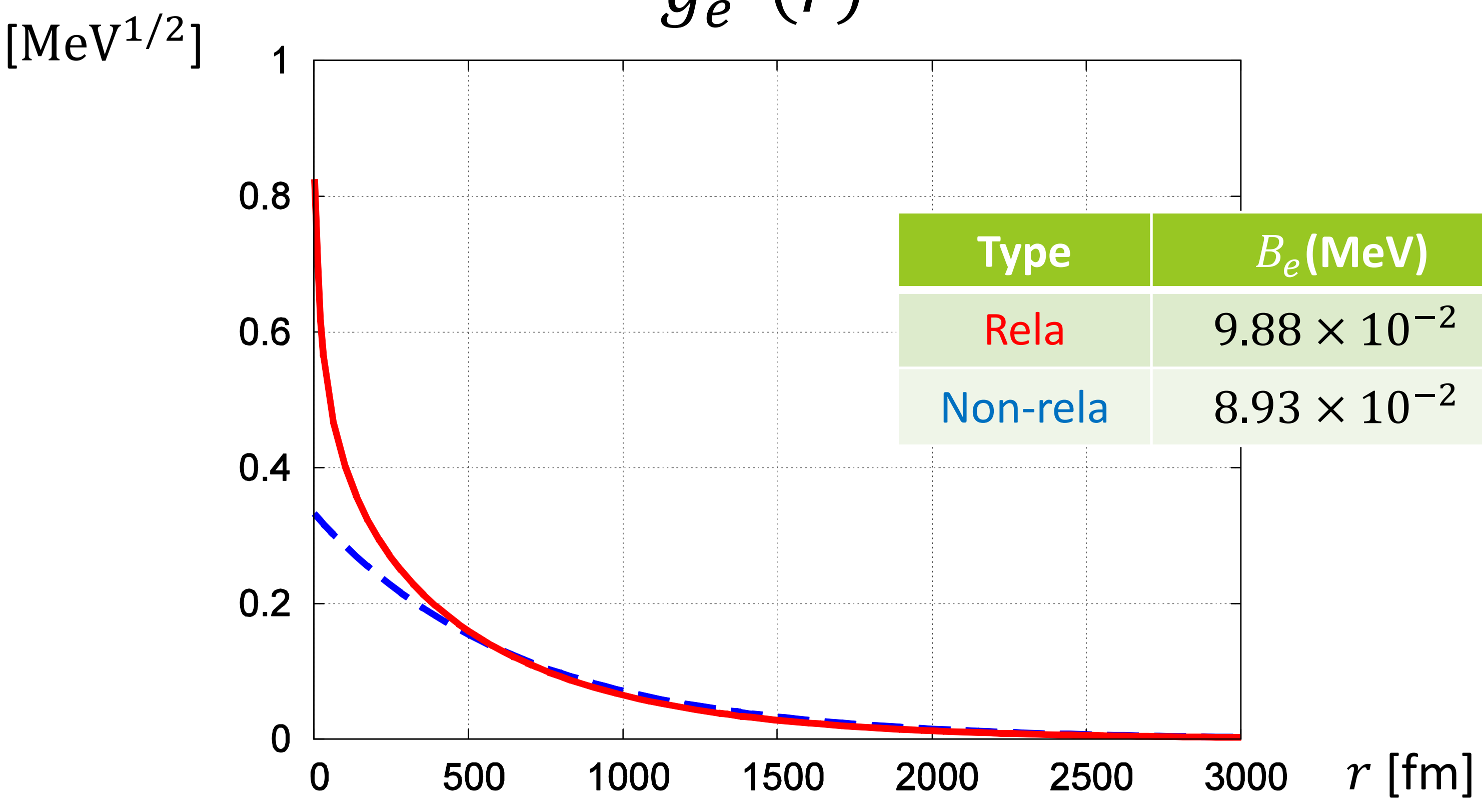
- ✓ atomic # dependence of the decay rate
- ✓ energy and angular distributions of emitted electrons

Radial functions (bound e^-)

^{208}Pb case $\quad Z = 81$

(considering μ^- screening)

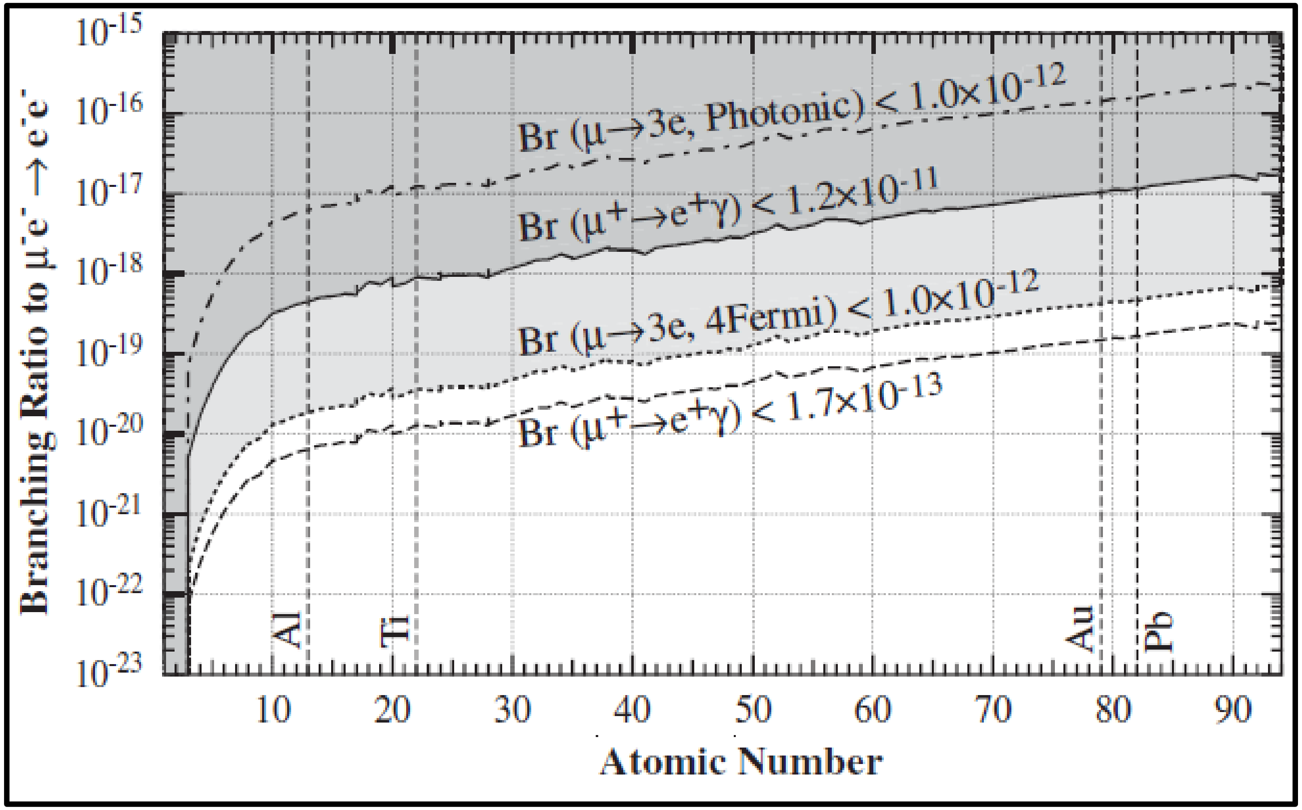
$g_e^{1s}(r)$



Type	B_e(MeV)
Rela	9.88×10^{-2}
Non-rela	8.93×10^{-2}

Relativity enhances the value near the origin.

Upper limits of $\mathrm{Br}(\mu^- e^- \to e^- e^-)$



✓ μeee interaction

$$\mathrm{Br}(\mu^+ \to e^+ e^- e^+) < 1.0 \times 10^{-12}$$

$$\Rightarrow \mathrm{Br}(\mu^- e^- \to e^- e^-) < 4.5 \times 10^{-19}$$

for Pb ($Z = 82$)

✓ $\mu e \gamma$ interaction

$$\mathrm{Br}(\mu^+ \to e^+ \gamma) < 5.7 \times 10^{-13}$$

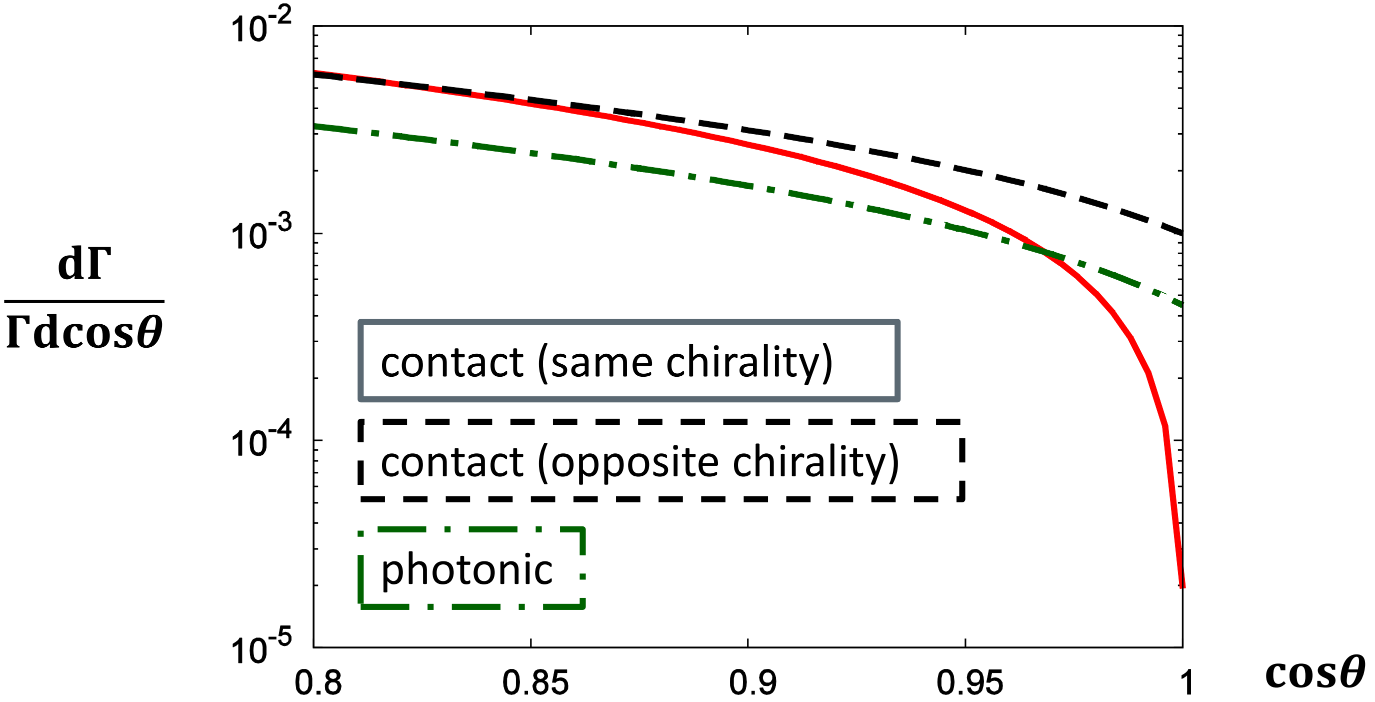
$$\Rightarrow \mathrm{Br}(\mu^- e^- \to e^- e^-) < 5.7 \times 10^{-19}$$

for Pb ($Z = 82$)

Discriminating method 2

~ energy and angular distributions ~

angular distributions ($\cos\theta \approx 1$)



- g_5 has larger tail than g_1 due to Pauli principle.

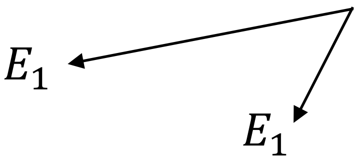
Energy and angular disribution (g1) $g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R)$

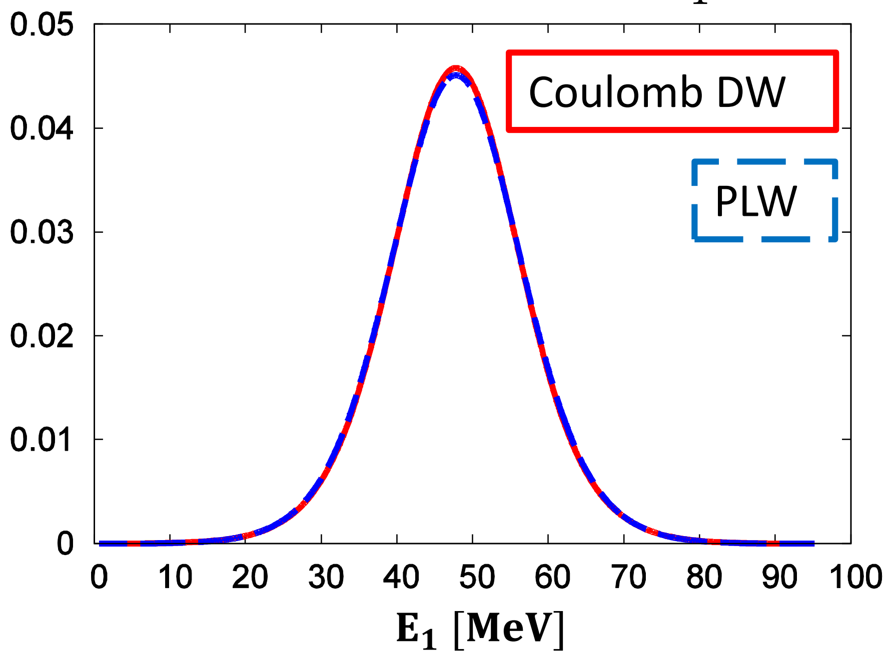
Muon is not point-like →distribution is spread

$Z = 82$

E_1 : energy of scattered electrion

$\frac{1}{\Gamma}\frac{d\Gamma}{dE_1}$ [MeV^{-1}]

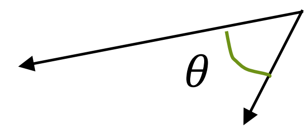


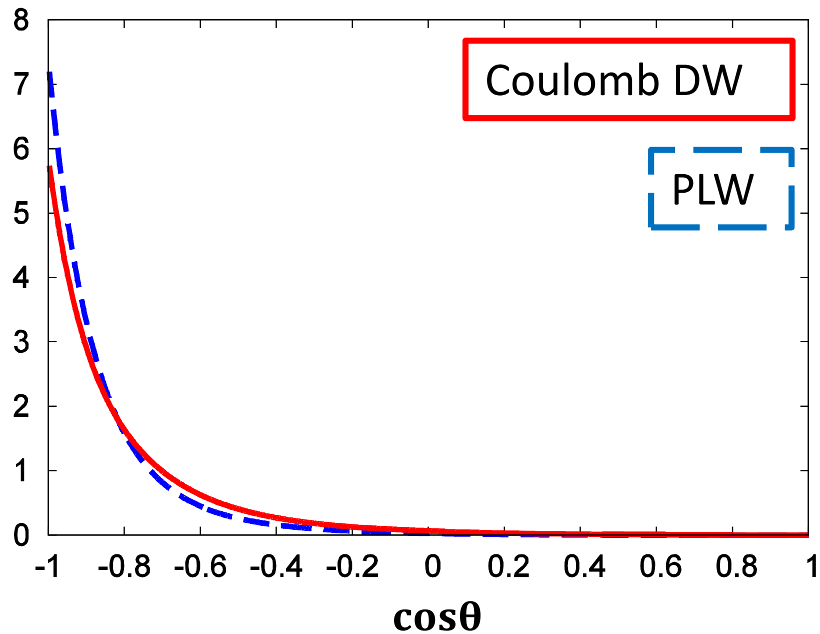


- $m_e \leq E_1 \leq m_\mu - B_\mu - B_e$
- Peak at half value

θ : angle between scattered electrons

$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta}$





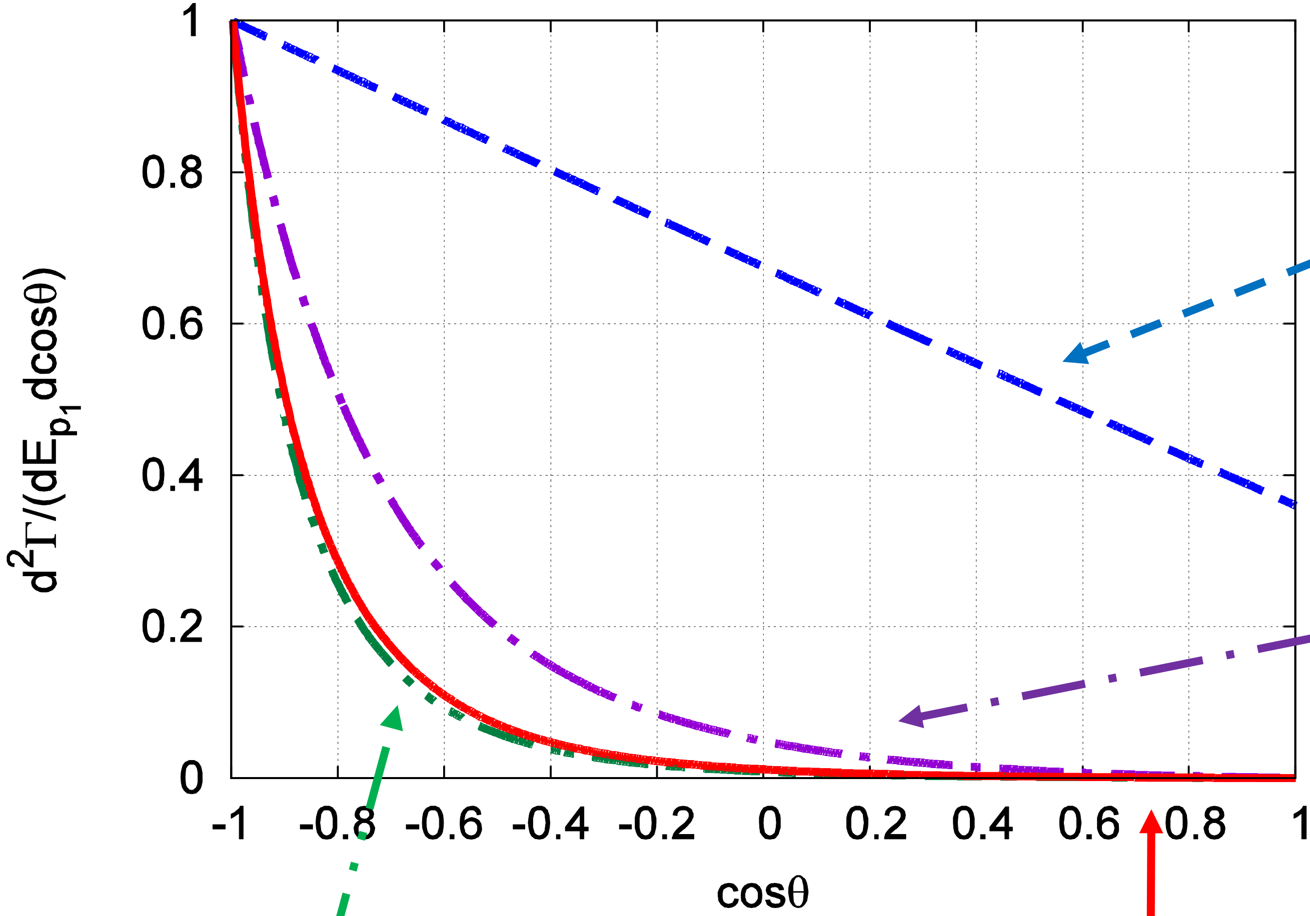
- Almost back to back

Energy dependence of angular distribution (g1)

$g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R)$

(Normalized by the maximum value)

$Z = 82$



$E_{p_1} = 0.57\text{MeV}$

$E_{p_1} = 23.0\text{MeV}$

$E_{p_1} = 47.7\text{MeV}$

total

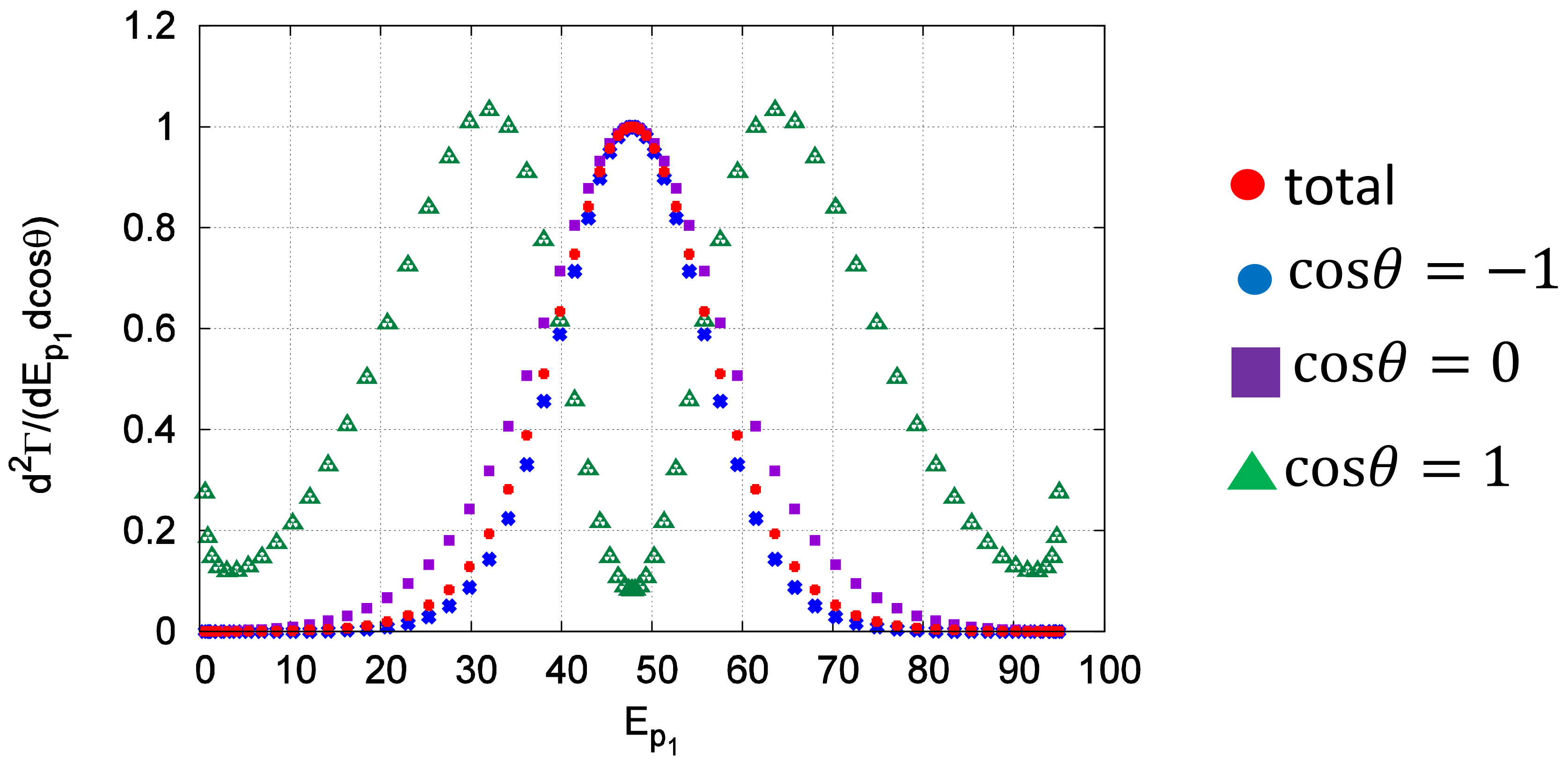
At half value, almost back-to-back

Angular dependence of energy distribution(g1)

$$g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R)$$

$Z = 82$

(Normalized by the muximum value)

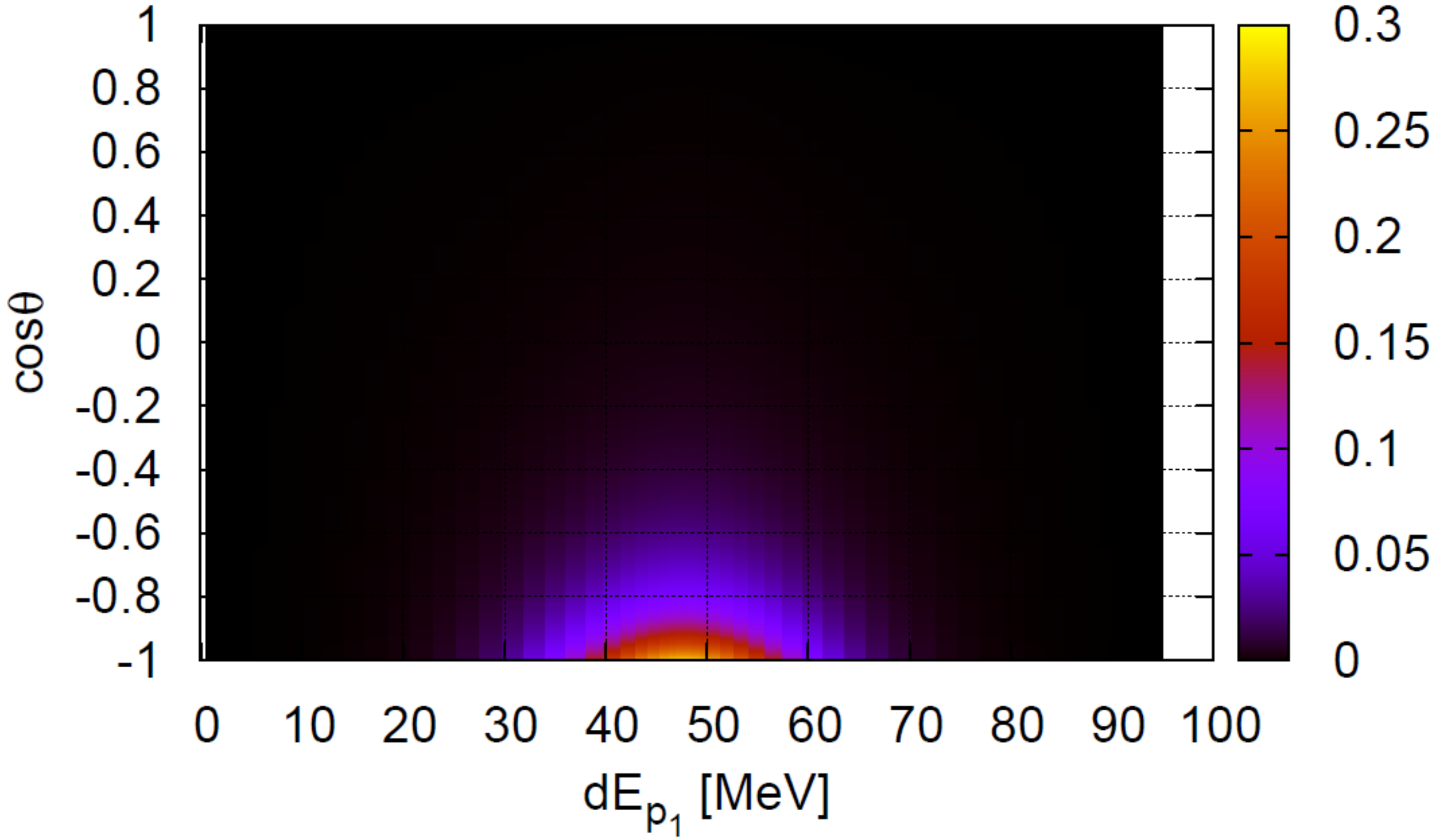


Angle and energy distribution(g1)

$g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R)$

$Z = 82$

“3D” plot

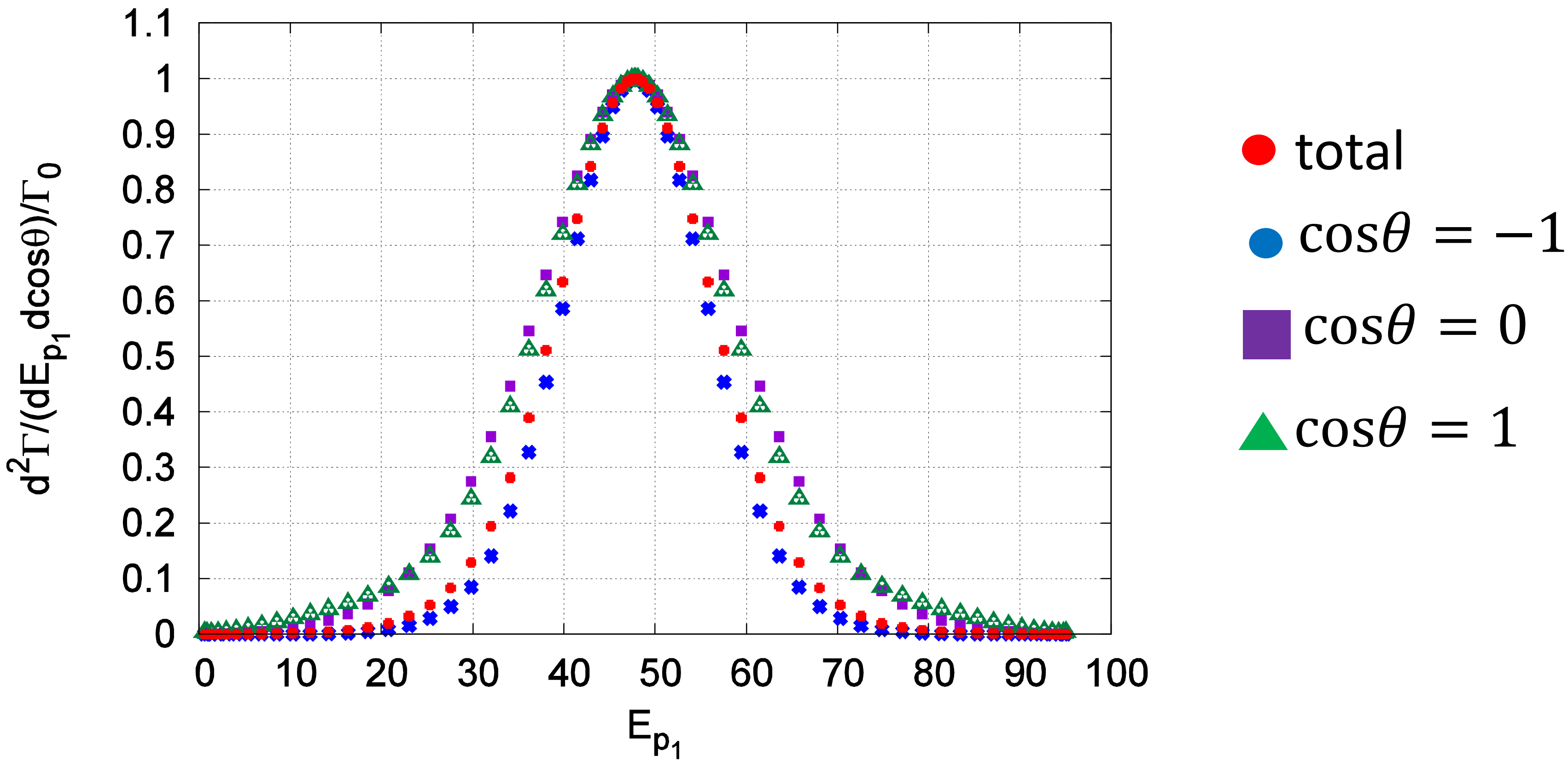


Angular dependence of energy distribution(g5)

$$g_5\left(\overline{e_R}\gamma_\mu\mu_R\right)\left(\overline{e_L}\gamma^\mu e_L\right)$$

$Z = 82$

(Normalized by the maximum value)

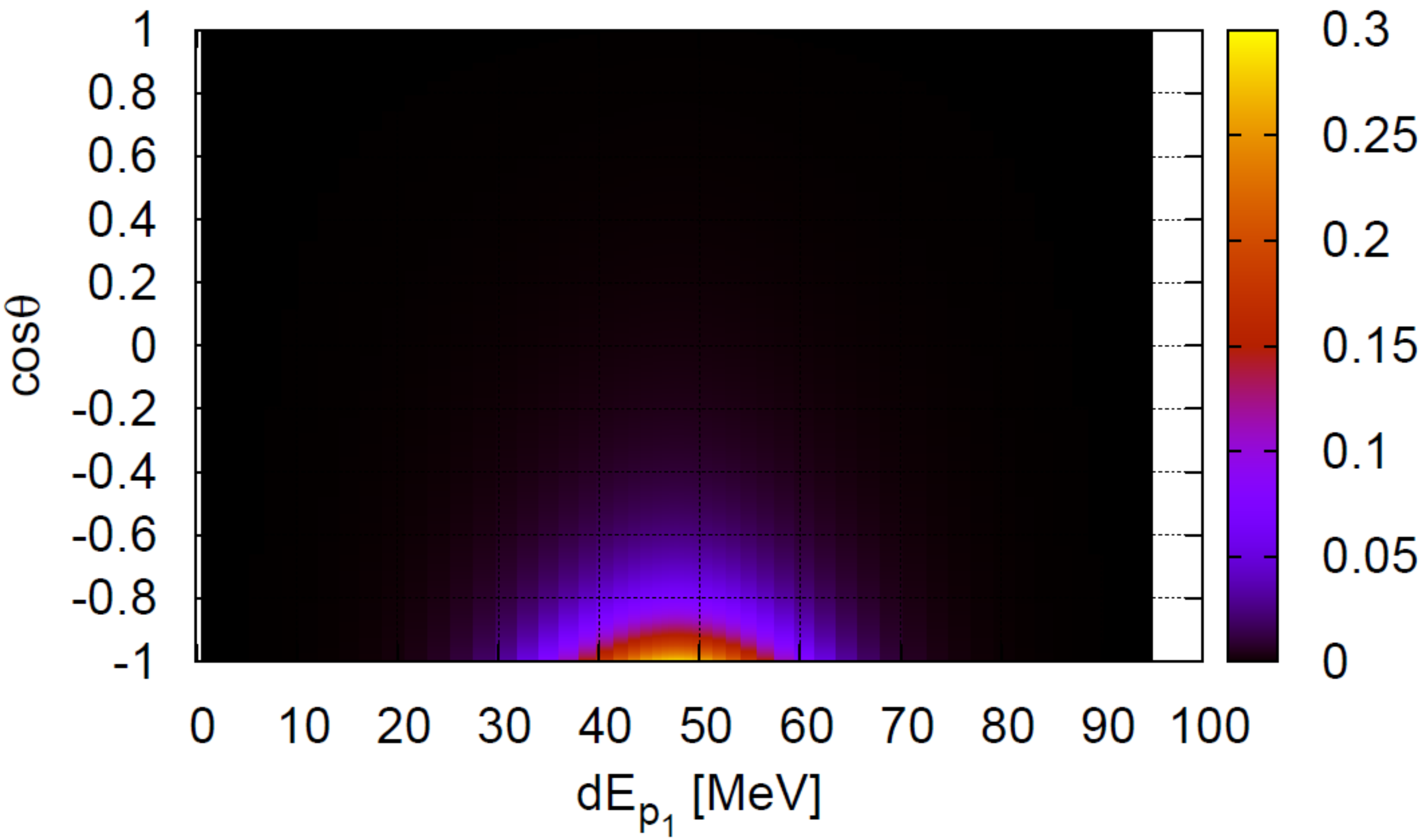


Angle and energy distribution(g5)

$$g_5(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_L}\gamma^\mu e_L)$$

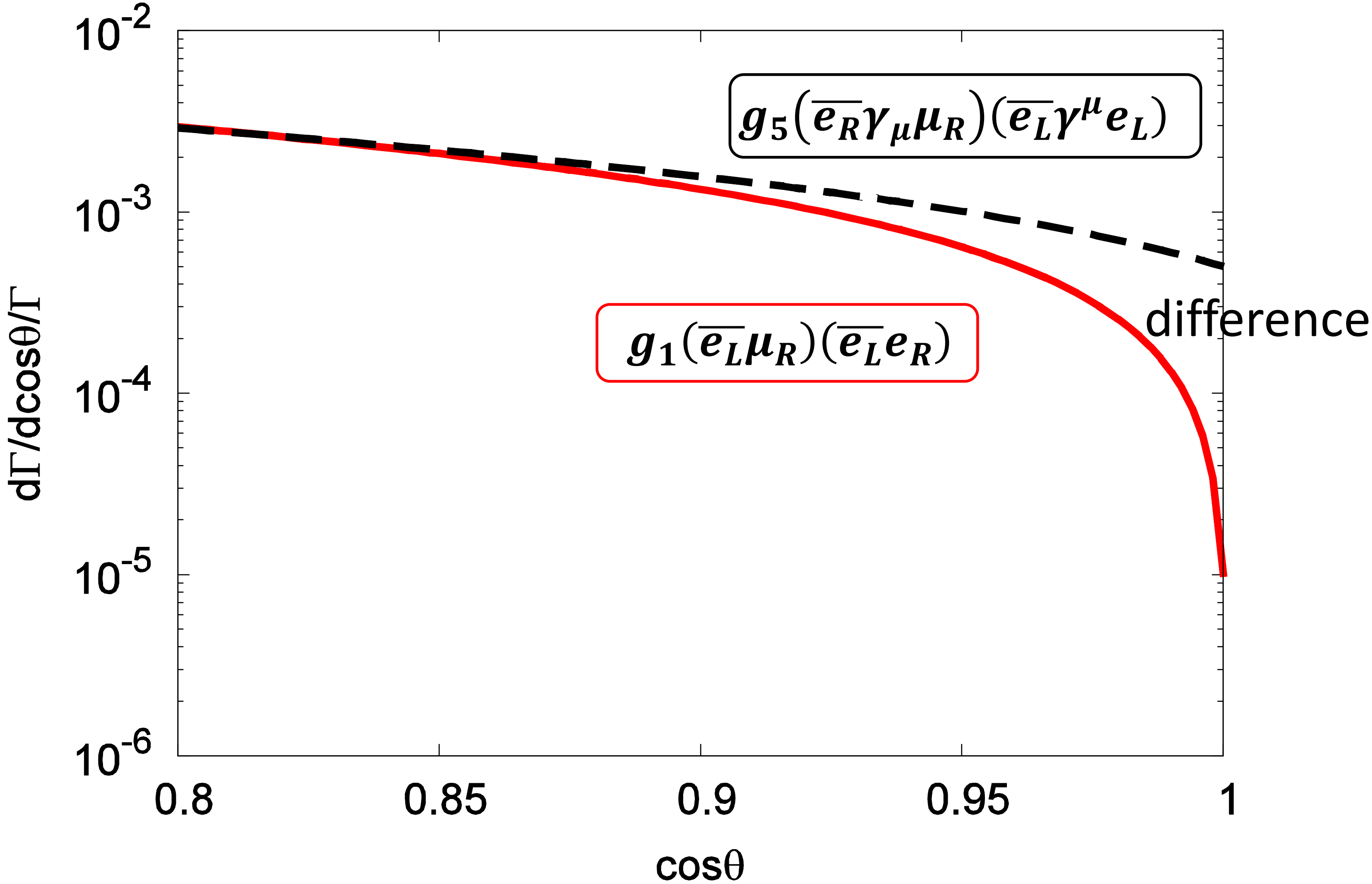
$Z = 82$

"3D" plot



Angular distribution(forward)

$Z = 82$



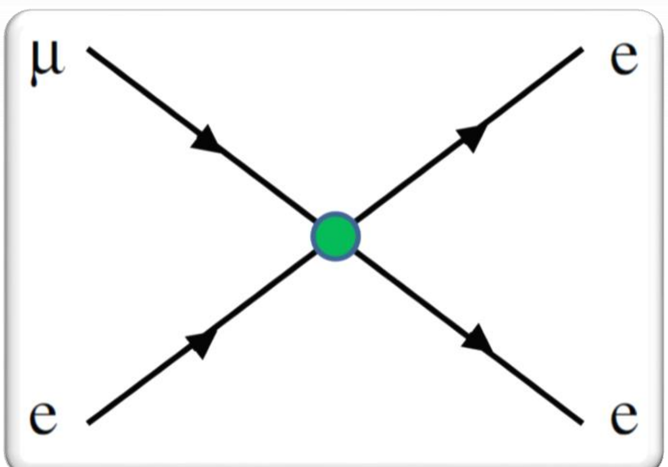
Electron emission with same chirality to same direction is suppressed by Pauli principle

$\mu\, e \longrightarrow e\, e$

Effective Lagrangian

[Y. Kuno and Y. Okada Rev. Mod. Phys. **73** (2001)]

$$
\begin{aligned}
\mathcal{L}_{\mu^- e^- \to e^- e^-} = -\frac{4G_F}{\sqrt{2}} \Big[& m_\mu A_R\, \overline{\mu_R} \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L\, \overline{\mu_L} \sigma^{\mu\nu} e_R F_{\mu\nu} \\
& + g_1 (\overline{\mu_R} e_L)(\overline{e_R} e_L) + g_2 (\overline{\mu_L} e_R)(\overline{e_L} e_R) \\
& + g_3 (\overline{\mu_R} \gamma^\mu e_R)(\overline{e_R} \gamma_\mu e_R) + g_4 (\overline{\mu_L} \gamma^\mu e_L)(\overline{e_L} \gamma_\mu e_L) \\
& + g_5 (\overline{\mu_R} \gamma^\mu e_R)(\overline{e_L} \gamma_\mu e_L) + g_6 (\overline{\mu_L} \gamma^\mu e_L)(\overline{e_R} \gamma_\mu e_R) + (\text{H.c.}) \Big]
\end{aligned}
$$



4-Fermi interaction type

$\mu\, e \longrightarrow e\, e$

4-Fermi interaction dominant case

Branching ratio

$$\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-)$$
$$= 24\pi (Z-1)^3 \alpha^3 \left(\frac{m_\mathrm{e}}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G$$
$$= (3.31 \times 10^{-12})(Z-1)^3 (\tilde{\tau}_\mu / \tau_\mu) G$$

τ_μ Lifetime of free muon (2.197×10^{-6} s)

$\tilde{\tau}_\mu$ Lifetime of bound muon $\begin{pmatrix} 2.19 \times 10^{-6}\ \mathrm{s} & \text{for } {}^{1}\mathrm{H} \\ (7-8) \times 10^{-8}\ \mathrm{s} & \text{for } {}^{238}\mathrm{U} \end{pmatrix}$

$\underline{\mu\ e \longrightarrow e\ e}$

4-Fermi interaction dominant case

Branching ratio

$$
\begin{aligned}
\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-) &\equiv \tilde{\tau}_\mu \Gamma(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-) \\
&= 24\pi (Z-1)^3 \alpha^3 \left(\frac{m_\mathrm{e}}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G \\
&= (3.31 \times 10^{-12})(Z-1)^3 (\tilde{\tau}_\mu / \tau_\mu) G
\end{aligned}
$$

$$G \equiv G_{12} + 16 G_{34} + 4 G_{56} + 8 G'_{14} + 8 G'_{23} - 8 G'_{56}$$

$$G_{ij} \equiv |g_i|^2 + |g_j|^2$$

$$G'_{ij} \equiv \mathrm{Re}\,(g_i^* g_j)$$

$\mu\, e \longrightarrow e\, e$

4-Fermi interaction dominant case

Branching ratio
($\mu \longrightarrow e\ e\ e$)

$$\mathrm{Br}(\mu^+ \to \mathrm{e}^+\mathrm{e}^+\mathrm{e}^-) = \frac{1}{8}(G_{12} + 16G_{34} + 8G_{56})$$

Comparison two BRs $\Rightarrow$ probe for CP violating

$$G \equiv G_{12} + 16G_{34} + 4G_{56} + 8G'_{14} \quad 8G'_{23} - 8G'_{56}$$

$$G_{ij} \equiv |g_i|^2 + |g_j|^2$$

$$G'_{ij} \equiv \mathrm{Re}\,(g_i^* g_j)$$

$\mu\, e \longrightarrow e\, e$

4-Fermi interaction dominant case

Branching ratio

$$\mathrm{Br}(\mu^- e^- \to e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-)$$
$$= 24\pi (Z-1)^3 \alpha^3 \left(\frac{m_e}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G$$
$$= (3.31 \times 10^{-12})(Z-1)^3 (\tilde{\tau}_\mu / \tau_\mu) G$$

Enhancement factor from overlap of wave functions

(∵ Positive charge attracts muon and electron toward the nucleus position.)

⇒ Notable advantage for heavy nuclei

$\mu\, e \longrightarrow e\, e$

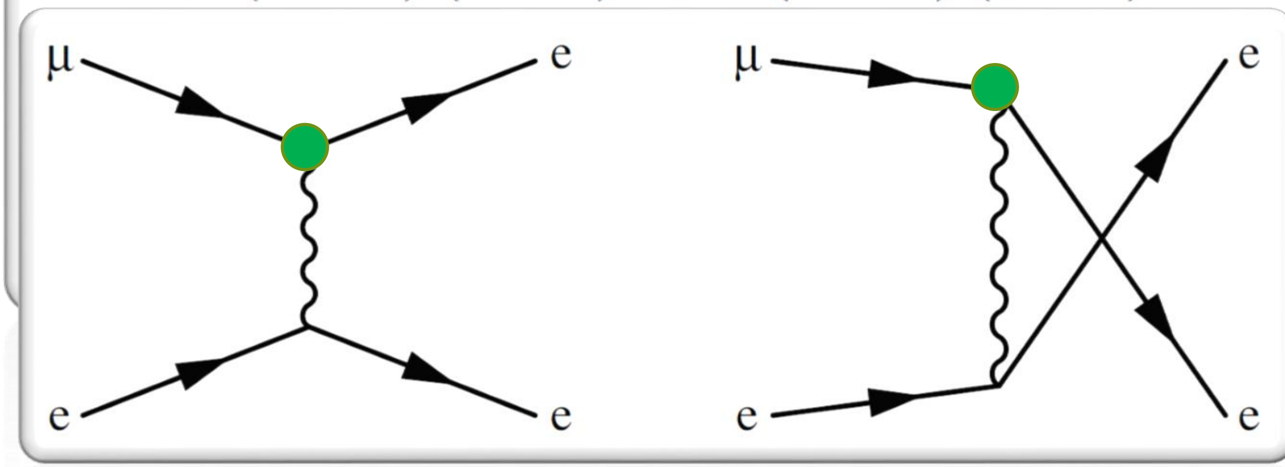
Effective Lagrangian [Y. Kuno and Y. Okada Rev. Mod. Phys. **73** (2001)]

$$\mathcal{L}_{\mu^- e^- \to e^- e^-} = -\frac{4G_{\mathrm{F}}}{\sqrt{2}}\Big[m_\mu A_{\mathrm{R}}\, \overline{\mu_{\mathrm{R}}}\sigma^{\mu\nu} e_{\mathrm{L}} F_{\mu\nu} + m_\mu A_{\mathrm{L}}\, \overline{\mu_{\mathrm{L}}}\sigma^{\mu\nu} e_{\mathrm{R}} F_{\mu\nu}$$

$$+ g_1 (\overline{\mu_{\mathrm{R}}} e_{\mathrm{L}})(\overline{e_{\mathrm{R}}} e_{\mathrm{L}}) + g_2 (\overline{\mu_{\mathrm{L}}} e_{\mathrm{R}})(\overline{e_{\mathrm{L}}} e_{\mathrm{R}})$$

$$\ldots \mu e_{\mathrm{L}})$$

$$\ldots \mu e_{\mathrm{R}}) + (\mathrm{H.c.})\Big]$$



Photonic interaction type

$\mu\ e \longrightarrow e\ e$

Photonic interaction dominant case

Branching ratio

$$\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-)$$
$$= 1536\pi^2 (Z-1)^3 \alpha^4 (|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2) \frac{m_\mathrm{e}}{m_\mu} \frac{\tilde{\tau}_\mu}{\tau_\mu}$$
$$= 2.08 \times 10^{-9} (Z-1)^3 (|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2)(\tilde{\tau}_\mu / \tau_\mu)$$

Photon propagator
in non-relativistic limit

$$\frac{1}{q^2} \simeq \frac{1}{m_\mu m_e}$$

Enhancement factor
compared with 4-Fermi case

$$\frac{m_\mu^2}{m_e^2}$$

$\mu\ e \longrightarrow e\ e$

Case : same order cLFV coupling

$$A_{L(R)} \cong g_i \quad \left(i = 1, 2, ...6 \right)$$

Ratio between photonic and 4-Fermi cross section

$$\sigma v_{\text{photonic}} \sim \alpha \frac{m_\mu^2}{m_e^2} \times \sigma v_{\text{4Fermi}} \sim 10^3 \times \sigma v_{\text{4Fermi}}$$

One of the distinct features for the process

Discovery reach
with the naïve estimate

Discovery reach

How to get upper limit for BR($\mu^- e^- \longrightarrow e^- e^-$)

Calculate ratio of the BR to other limited cLFV BR

4-Fermi interaction dominant case

$$\frac{\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-)}{\mathrm{Br}(\mu^+ \to \mathrm{e}^+ \mathrm{e}^+ \mathrm{e}^-)}$$

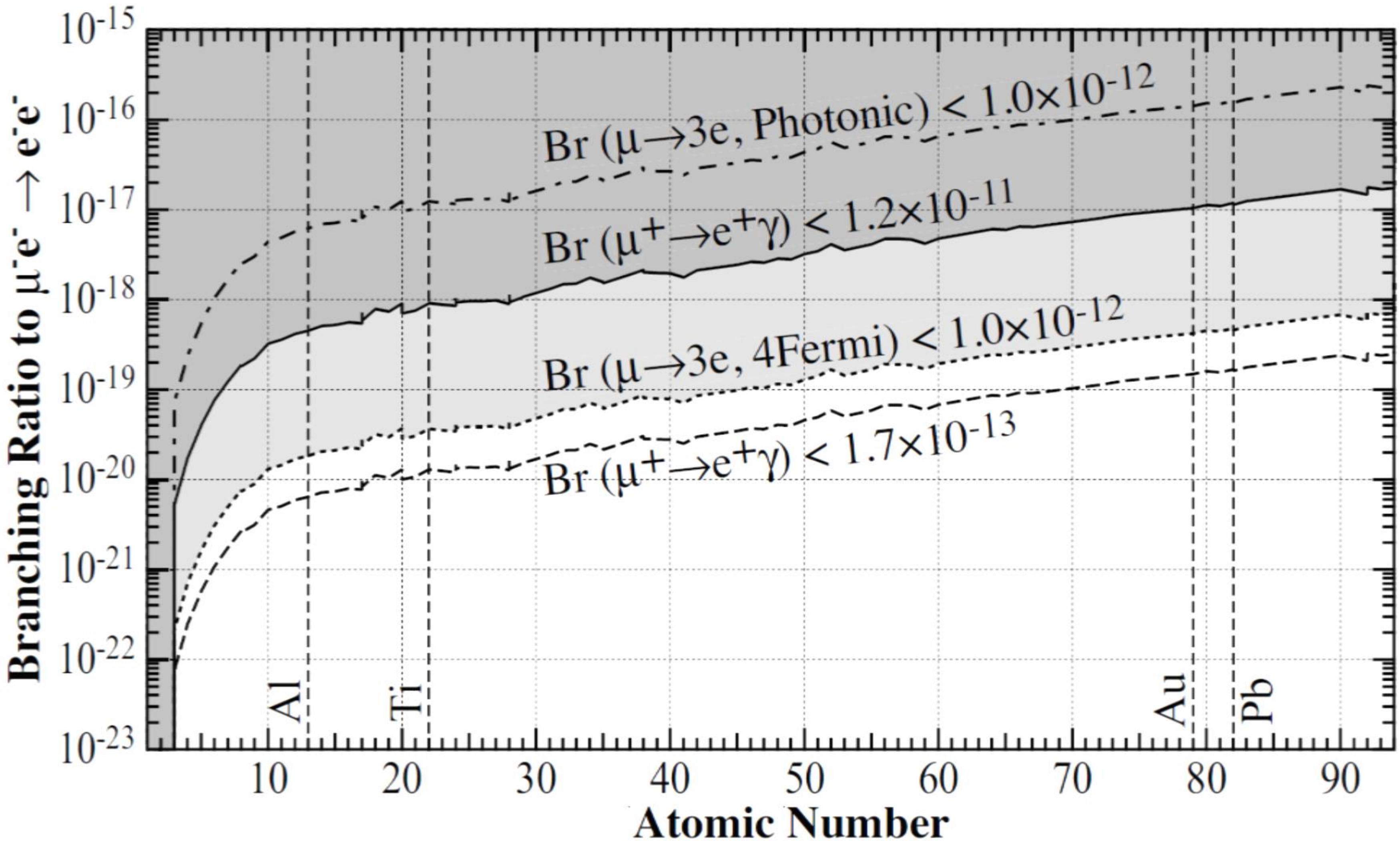
Photonic interaction dominant case

$$\frac{\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-)}{\mathrm{Br}(\mu^+ \to \mathrm{e}^+ \mathrm{e}^+ \mathrm{e}^-)}$$

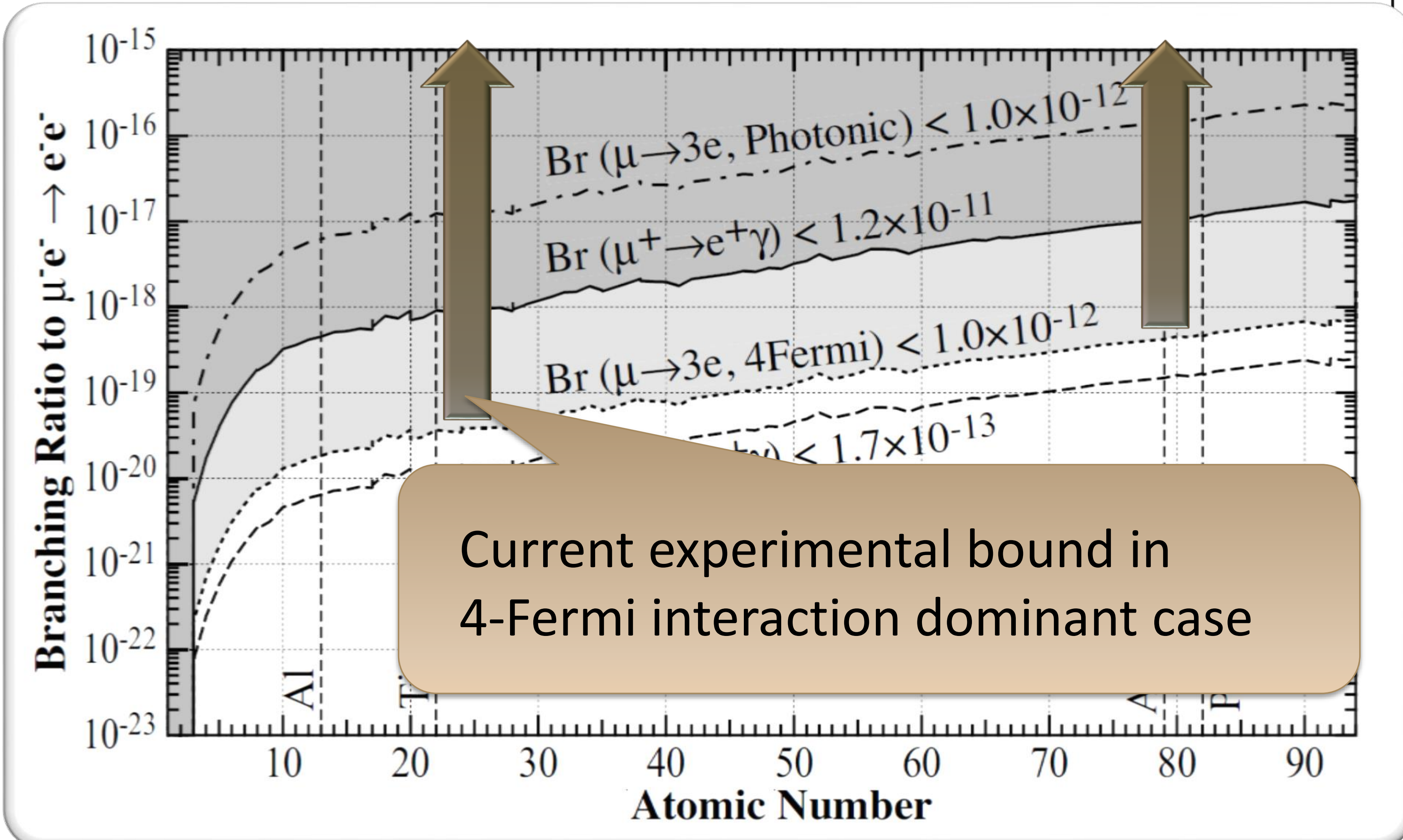
$$\frac{\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-)}{\mathrm{Br}(\mu^+ \to \mathrm{e}^+ \gamma)}$$

[These ratios are independent on cLFV effective coupling]

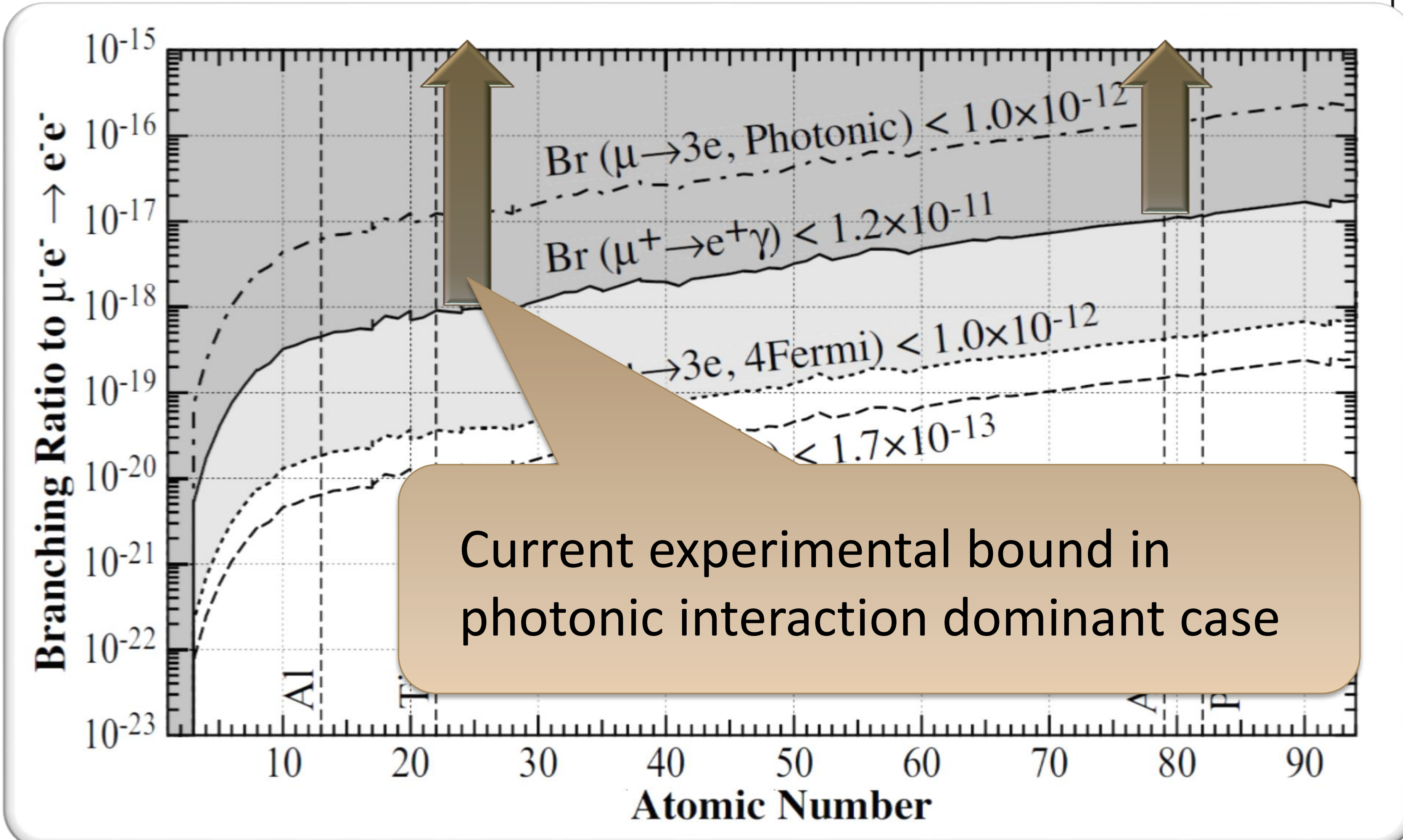
Discovery reach



Discovery reach



Discovery reach



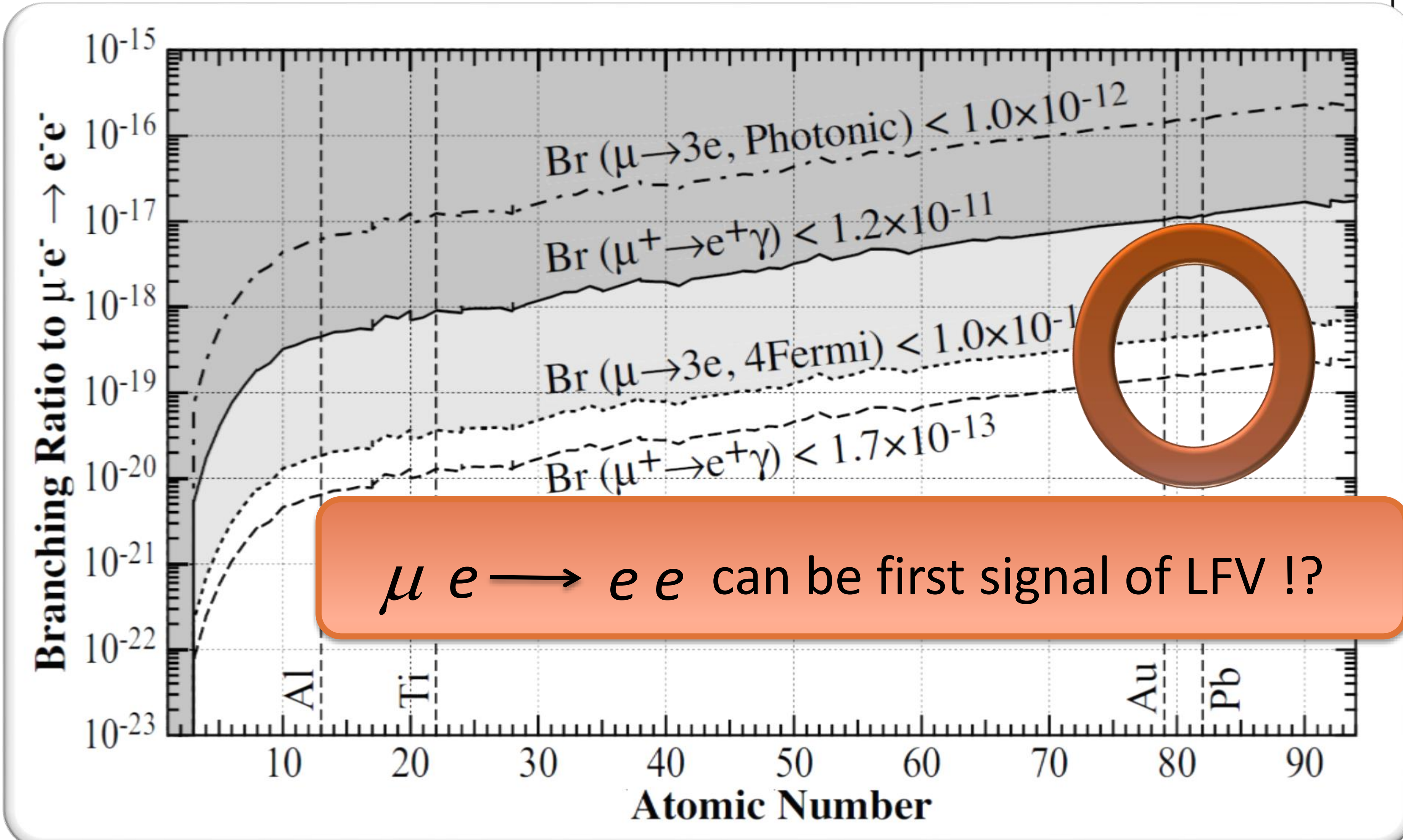
Discovery reach

Collaboration	Searching for	Intensity
MEG	$\mu \to e\gamma$	$10^{7.5}\ \mu/s$
MUSIC	$\mu \to 3e$	$10^{8}\ \mu/s$
COMET	$\mu^- N \to e^- N$	$10^{11}\ \mu/s$
Mu2E (E973)	$\mu^- N \to e^- N$	$10^{11}\ \mu/s$
PRISM	$\mu^- N \to e^- N$	$10^{12}\ \mu/s$

For run-time 1 year $\sim 3 \times 10^{7}$s

10^{18}-10^{19} muon at COMET experiment

Discovery reach



Discovery reach

Project	Intensity Reach
COMET / PRISM	$10^{18} - 10^{19}$ μ/year
ν factories	10^{21} μ/year

With these number of muons the process will be seen !!