Latest Sterile Neutrino Results from MINOS+ & Combined Analysis with Daya Bay and Bugey-3

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(for the MINOS+ collaboration)

NuFact Conference, Uppsala University

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Overview

- The MINOS & MINOS+ experiments
- Beyond three flavours
  - 3+1 sterile model
- Searching for sterile ν's in MINOS & MINOS+
  - ν_μ disappearance
  - $\nu_\mu \rightarrow \nu_e$ appearance
- Combined analysis with Daya Bay & Bugey-3

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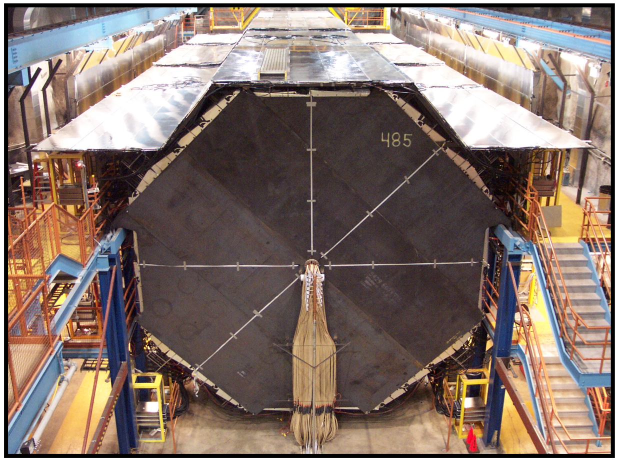
The MINOS/MINOS+ Experiments

Far Detector
(Soudan mine)



735 km from beam target
5.4 kton mass

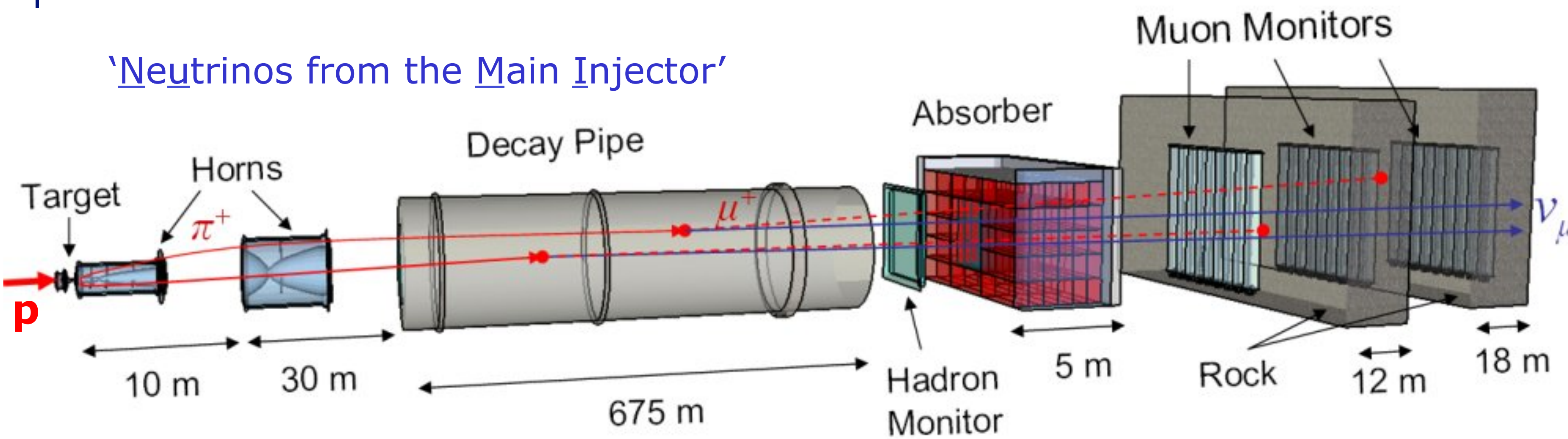
Near Detector
(Fermi Laboratory)

1 km from beam target
1 kton mass

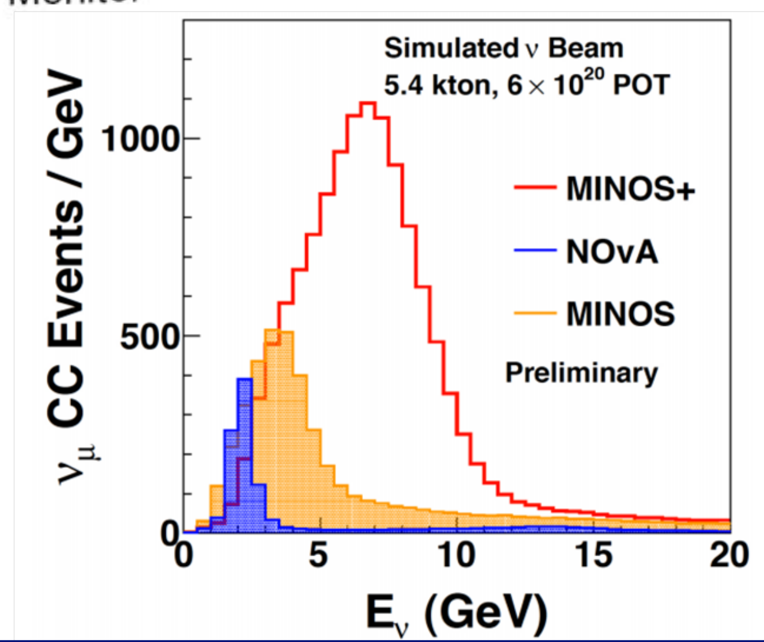
- The MINOS (2005-12) and MINOS+ (2013-16) experiments represent more than a decade of long-baseline neutrino physics.
  - Precision measurements of standard three-flavour oscillations.
  - Searches for new phenomena beyond standard oscillations.
- Experiment is now over, but data analysis continues. Latest results are based on a combined analysis of MINOS with ~50% of MINOS+ data.

The NuMI Accelerator Beam

'<u>N</u>e<u>u</u>trinos from the <u>M</u>ain <u>I</u>njector'

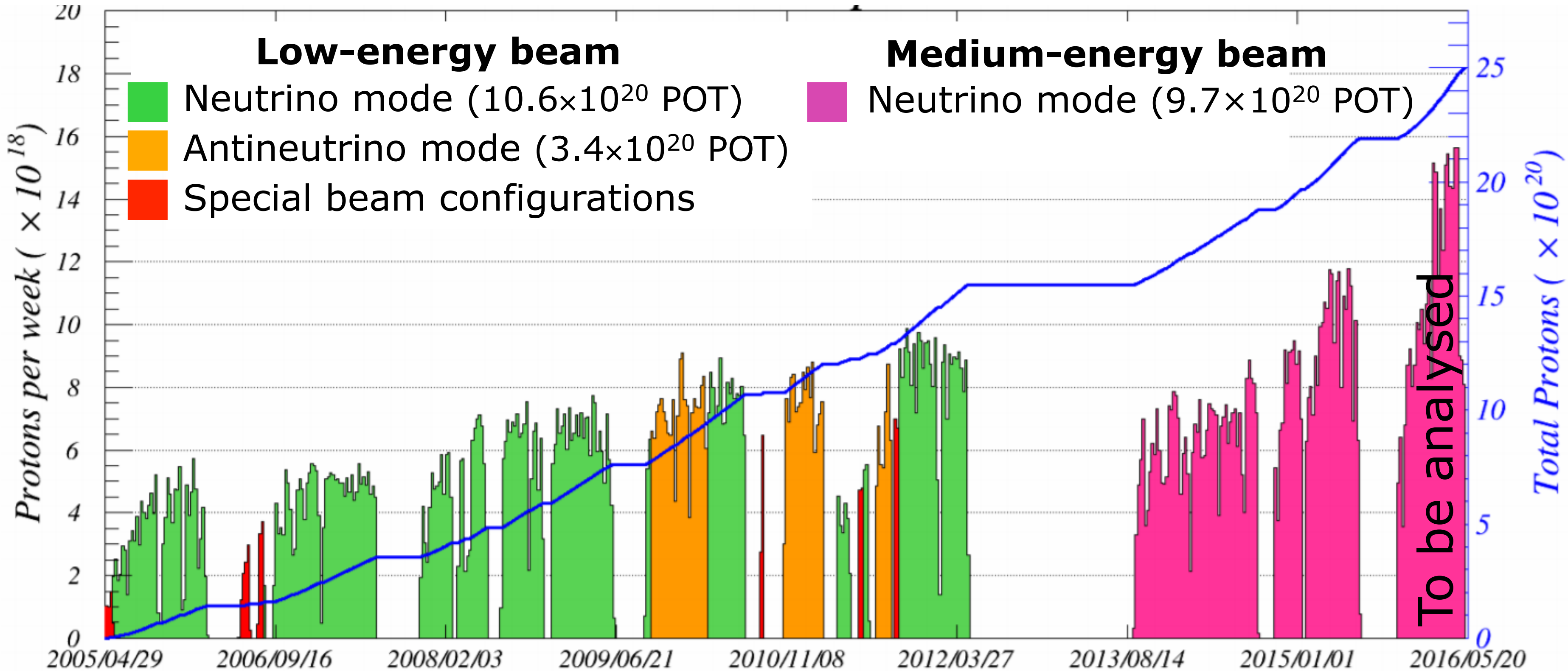


- The MINOS/MINOS+ detectors were located **on-axis** in the NuMI beam, resulting in a wide-band spectrum.
- MINOS operated using a **low-energy** beam configuration.
  - Both neutrino and anti-neutrino data sets were collected.
- MINOS+ coincided with the NOvA era, and ran in a **medium-energy** beam.



MINOS & MINOS+ Data

- MINOS & MINOS+ collected >25×10^{20} POT accelerator neutrino data during 11 years of operation.

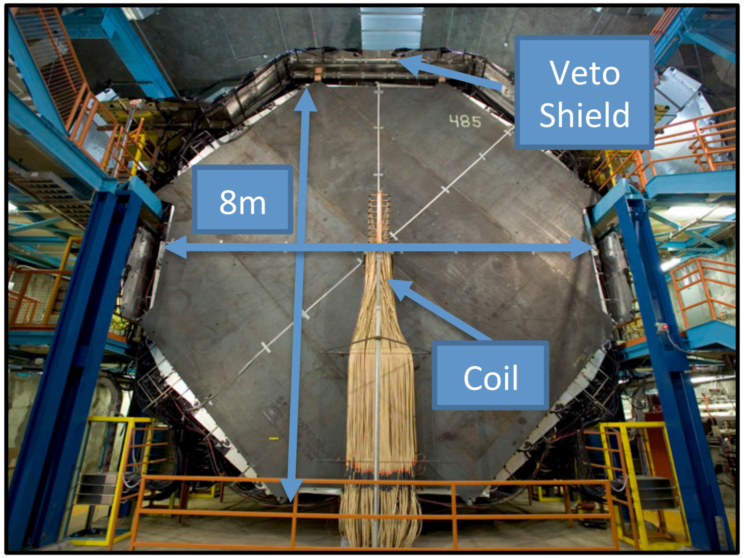


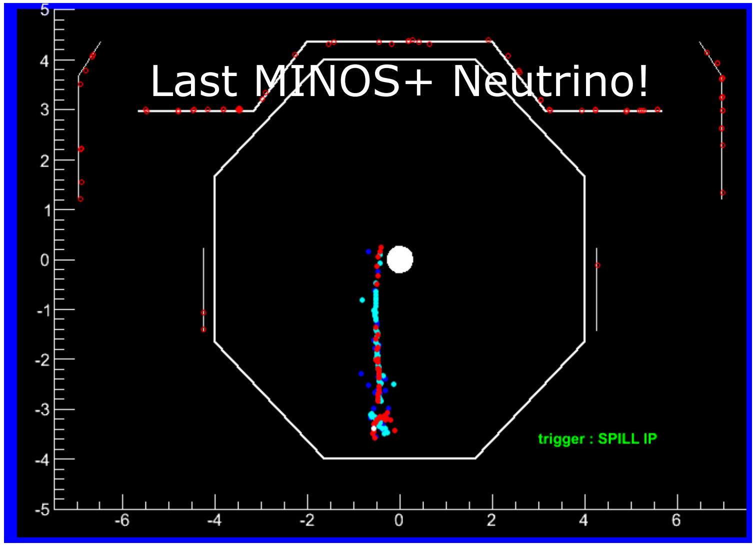
MINOS (2005-12)

MINOS+ (2013-16)

5.8×10^{20} POT analysed so far

The MINOS Detectors

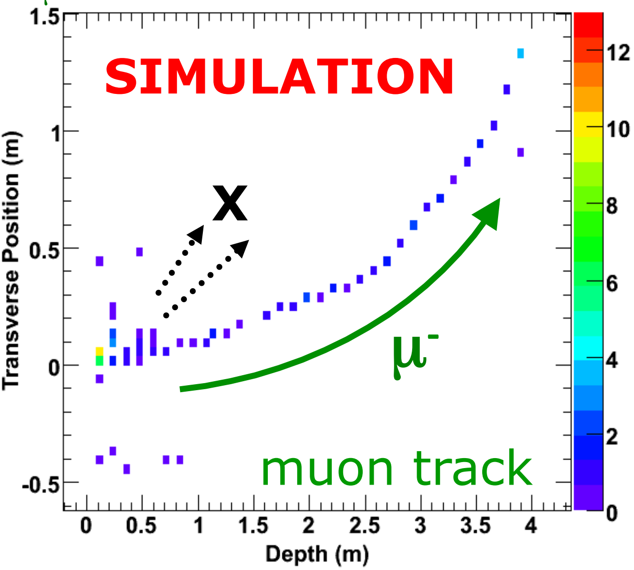




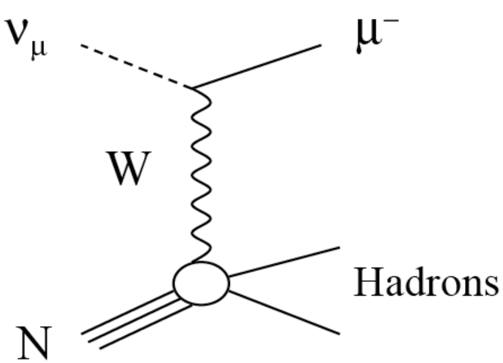
- MINOS/MINOS+ Near and Far Detectors were **functionally similar**.
  - Segmented, sampling, tracking steel/scintillator calorimeters.
  - Magnetised with ~1.2T field for charge-sign determination.
- Each detector measured energy spectrum and flavour composition of NuMI beam.
  - ν_μ CC, ν_e CC and NC interactions were identified and measured using event topology and calorimetry.
- Neutrino oscillations studied by combining information from both detectors.
  - Cancellation of systematics.
- Far Detector also collected 60 kton-years atmospheric neutrino data.

Neutrino Interactions

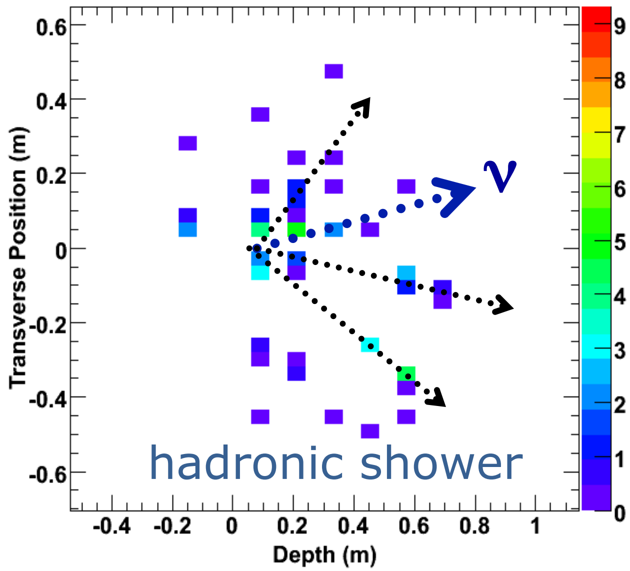
ν_μ Charged Current (CC)



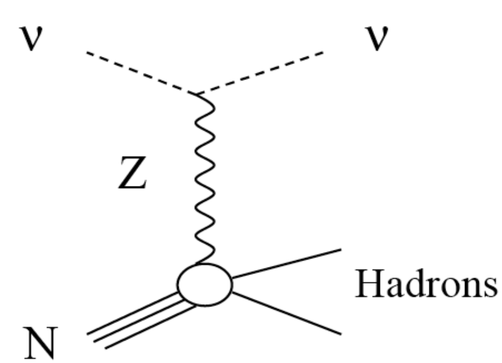
$$\nu_\mu + N \rightarrow \mu^- + X$$



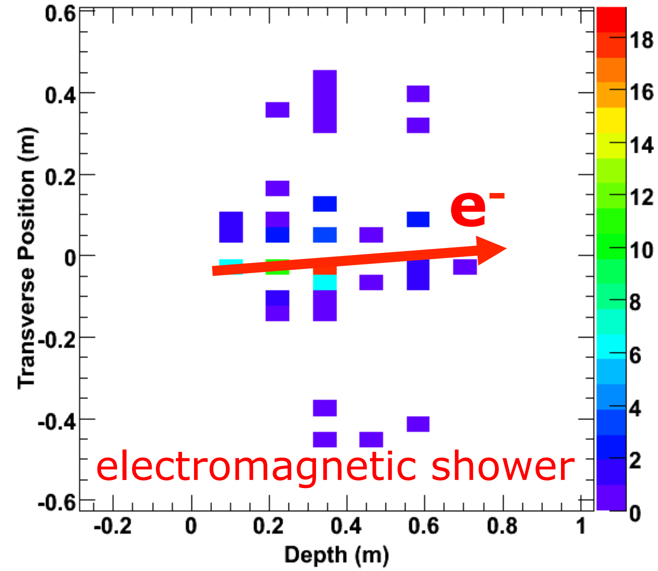
Neutral Current (NC)



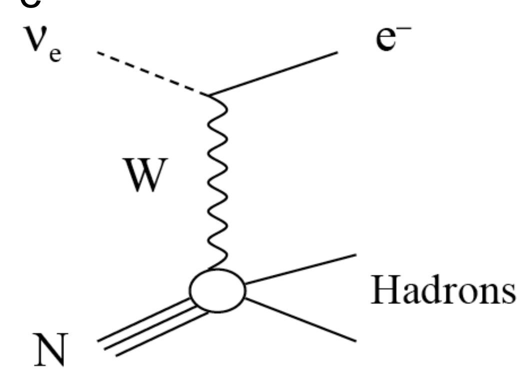
$$\nu + N \rightarrow \nu + X$$



ν_e Charged Current (CC)

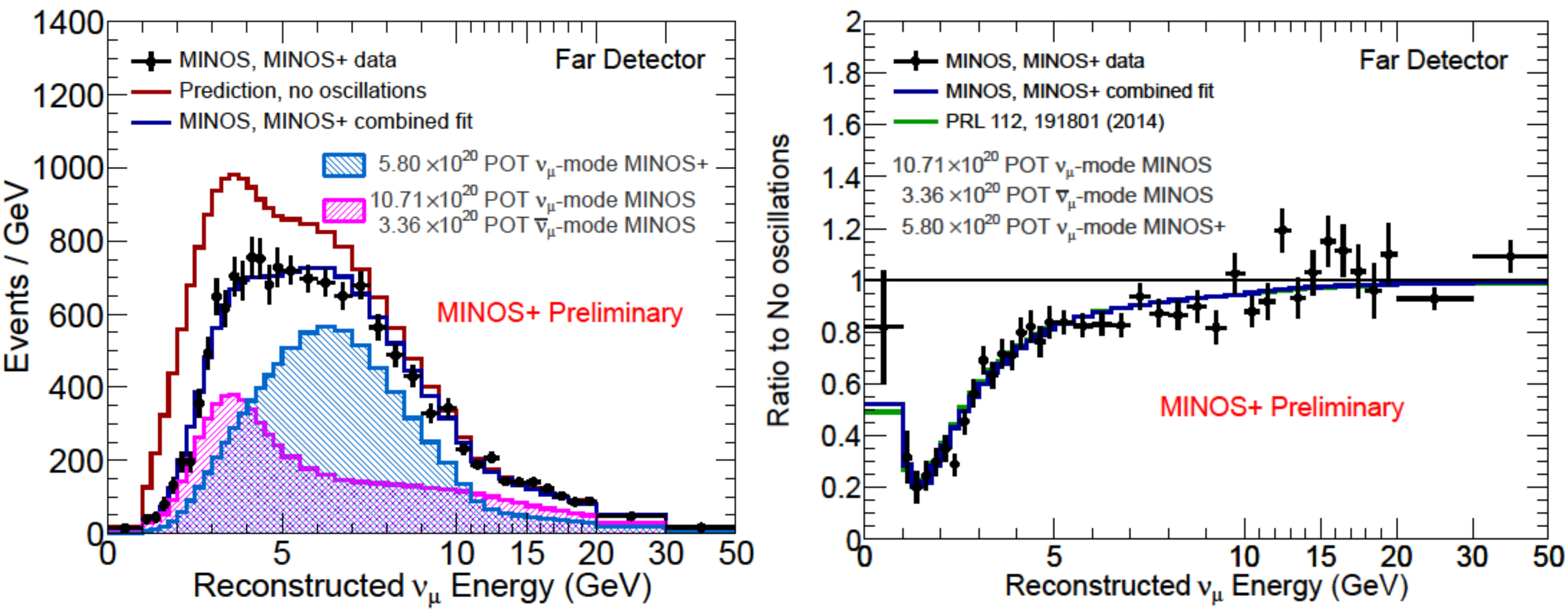


$$\nu_e + N \rightarrow e^- + X$$



Standard Oscillations

- Latest standard oscillation results are based on a combined analysis of accelerator and atmospheric data from MINOS and MINOS+.
  - Neutrinos, antineutrinos, ν_μ disappearance, $\nu_\mu \rightarrow \nu_e$ appearance.
  - Analysis includes 48 kton-years atmospheric neutrino data.
- Observed data are well-described by three-flavour neutrino oscillations.



Standard Oscillations

- Analysis of three-flavour oscillations yields precision measurements of Δm^2_{32} and $\sin^2\theta_{23}$ parameters:

Normal Hierarchy:

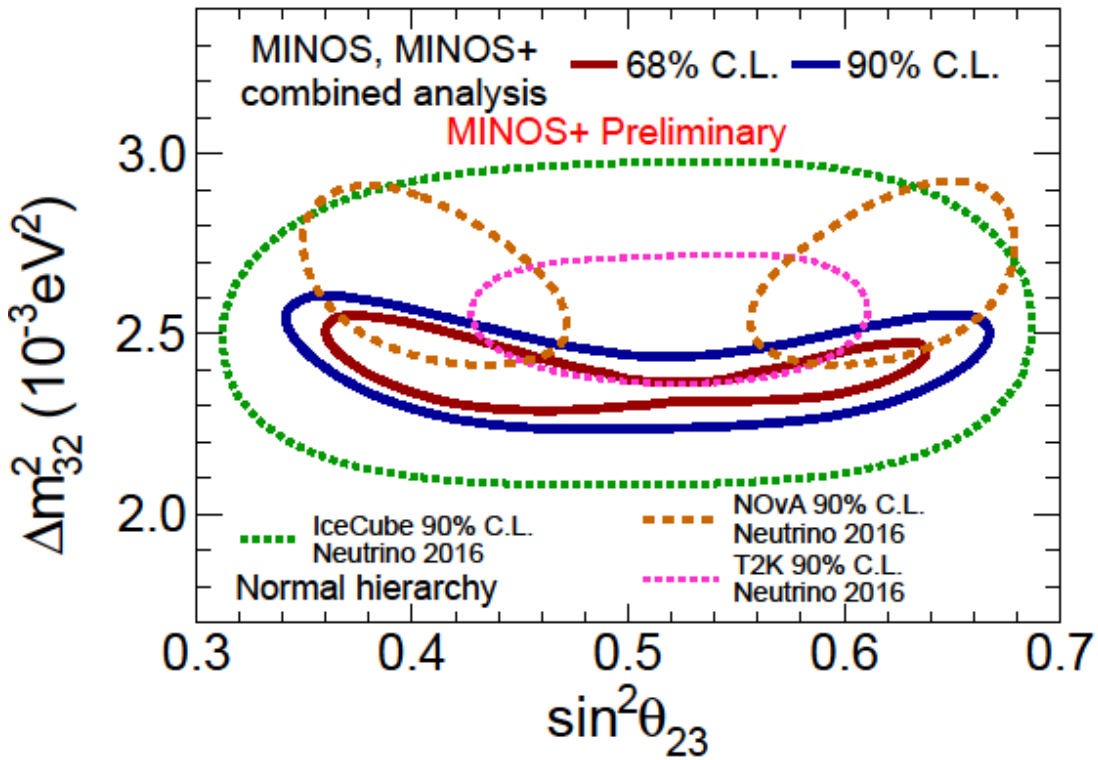
$$\Delta m^2_{32} = 2.42^{+0.09}_{-0.09} \times 10^{-3}\mathrm{eV}^2$$

$$0.35 < \sin^2\theta_{23} < 0.65 \;\; (90\% \text{ C.L.})$$

Inverted Hierarchy:

$$\Delta m^2_{32} = -2.48^{+0.09}_{-0.11} \times 10^{-3}\mathrm{eV}^2$$

$$0.35 < \sin^2\theta_{23} < 0.66 \;\; (90\% \text{ C.L.})$$

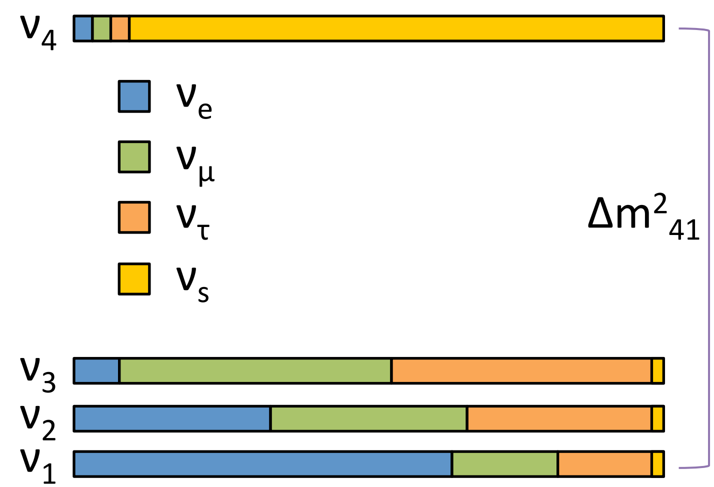


- The data from MINOS+ improve the standard oscillation measurement, but also significantly enhance searches for new physics.

Sterile Neutrino Oscillations

- The wideband L/E coverage of MINOS and MINOS+ generates strong sensitivity to oscillations involving sterile neutrinos.

- The MINOS/MINOS+ data have been analysed using a "3+1" model of sterile neutrinos:
  - ➢ 3 active flavours (ν_e, ν_μ, ν_τ).
  - ➢ Add 1 sterile flavour (ν_S) and 1 extra mass state (ν_4).
  - ⇒ 4 × 4 neutrino mixing matrix.



$$\left(\begin{array}{cccc} \multicolumn{3}{c}{\boxed{\quad U_{PMNS} \quad}} & \begin{array}{c} U_{e4} \\ U_{\mu 4} \\ U_{\tau 4} \end{array} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{array} \right)$$

- Neutrino oscillations are described by 12 parameters [3-flavour, 4-flavour]:
  - Mass splittings: Δm^2_{32}, Δm^2_{21}, Δm^2_{41}
  - Mixing angles: θ_{12}, θ_{23}, θ_{13}, θ_{14}, θ_{24}, θ_{34}
  - CP-violating phases: δ_{13}, δ_{14}, δ_{24}
  - ⇒ 6 new oscillation parameters.

Sterile Neutrino Signatures

- The combined data from MINOS and MINOS+ are sensitive to the third mass splitting and all three additional mixing angles:

(1) ν_μ disappearance analysis:

- ➢ Search for presence of additional oscillations in ν_μ CC spectrum due to third mass splitting.
  - ★ Predominantly sensitive to Δm^2_{41} and θ_{24}.
- ➢ Search for anomalous disappearance in spectrum of NC events arising from $\nu_\mu \rightarrow \nu_s$ oscillations.
  - ★ Additional sensitivity to θ_{24}, plus some sensitivity to θ_{34}.

(2) $\nu_\mu \rightarrow \nu_e$ appearance analysis:

- ➢ Search for anomalous $\nu_\mu \rightarrow \nu_e$ appearance in ν_e CC spectrum at energies above three-flavour oscillations.
  - ★ Predominantly sensitive to θ_{14} and θ_{24}.

Sterile Neutrino Signatures

- Sterile neutrino oscillations can occur in both MINOS detectors.

  In the case of ν_μ disappearance:

  ➢ Small Δm^2_{41} ($>\Delta m^2_{32}$) (10^{-3} – 10^{-1} eV^2)

  Far Detector: additional oscillations above 3-flavour oscillation maximum.

  Near Detector: no effect.

  ➢ Medium Δm^2_{41} (10^{-1} – 1 eV^2)

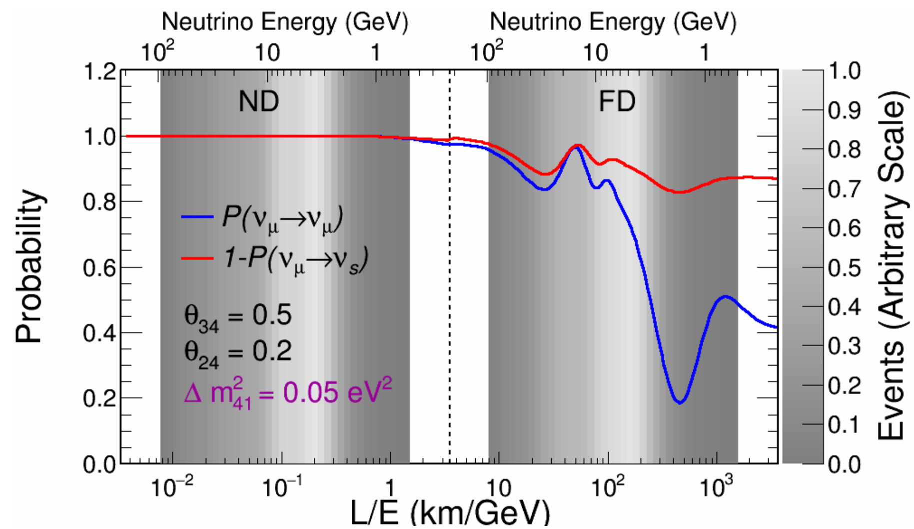
  Far Detector: oscillations become rapid and average out, causing a constant depletion ("counting experiment").

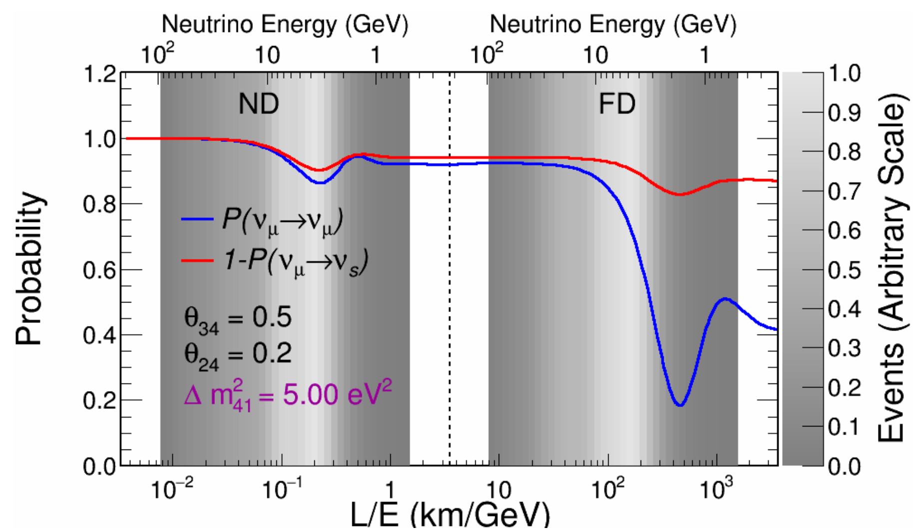
  Near Detector: no effect.

  ➢ Large Δm^2_{41} (1 – 10^2 eV^2)

  Far Detector: constant depletion.

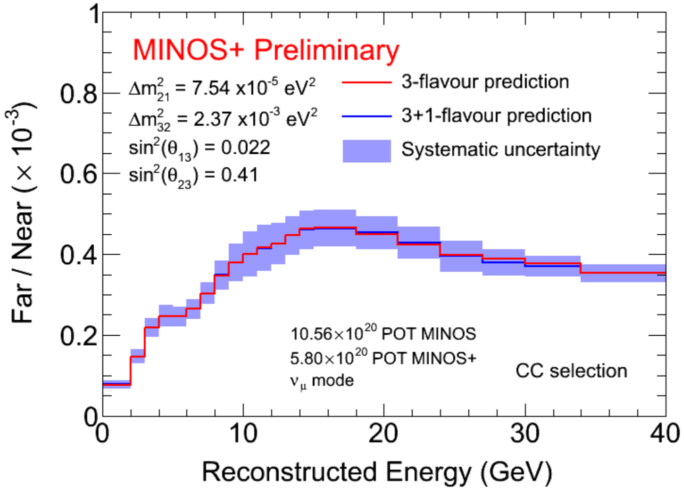
  Near Detector: oscillations.





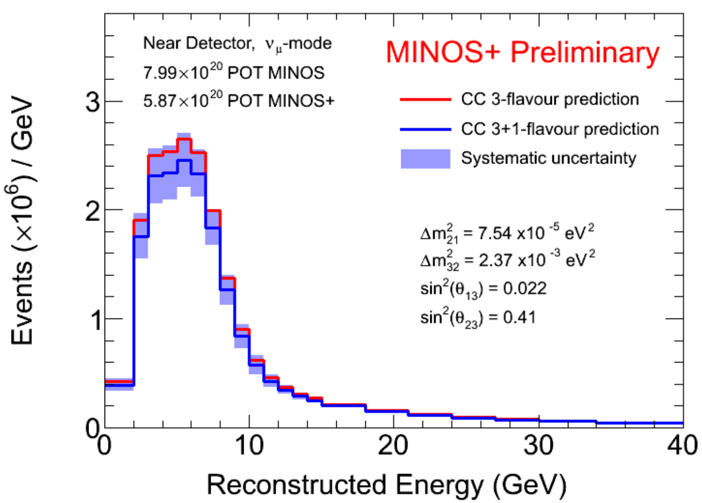
ν_μ Disappearance Analysis

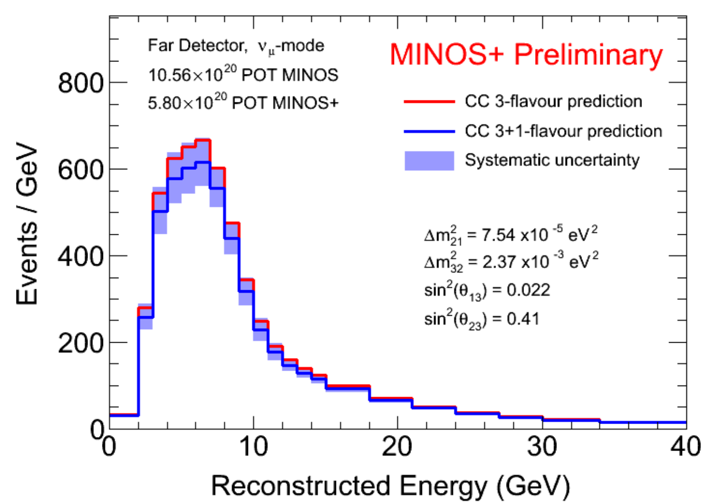
Far/Near Ratio



- Previous MINOS sterile analysis* based on ratio of Near and Far energy spectra.
  - ➢ Many systematics cancel in this ratio.
- But Far/Near ratio method has limitations:
  - ➢ Uncertainty dominated by Far statistics.
  - ➢ High-Δm^2_{41} oscillations cancel in ratio.
- For combined MINOS/MINOS+ analysis, have now developed a **two-detector fit**.

Two-Detector Fit





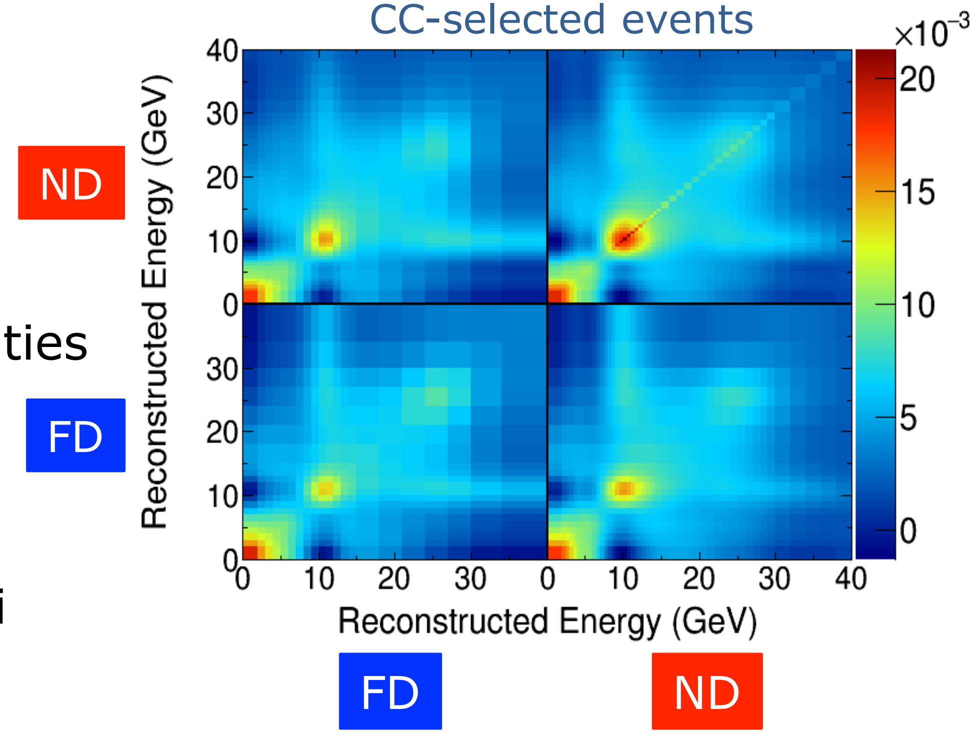
Input oscillation parameters:
θ_{24} = 0.2;
Δm^2_{41} = 80 eV^2

* P. Adamson *et al.*, Phys. Rev. Lett. 117, 151803 (2016)

Two-Detector Fit

- Combine ν_μ CC and NC data from MINOS (neutrino-mode) and MINOS+ into single analysis, using simultaneous two-detector fit.
- Treatment of 3+1 oscillation parameters same as previous MINOS analysis:
  - ➢ Fitted: Δm^2_{41}, Δm^2_{32}, θ_{23}, θ_{24}, θ_{34}.
  - ➢ Set to zero: θ_{14}, δ_{13}, δ_{14}, δ_{24}.
  - ➢ Global best-fits: Δm^2_{21}, θ_{12}, θ_{13}.
- Statistical and systematic uncertainties enter fit via covariance matrices.
  - ➢ Have incorporated 44 sources of systematic uncertainty.
  - ➢ In particular, now utilise a-priori flux prediction from Minerva*.
  - ➢ Many uncertainties cancel via matrix cross-terms.

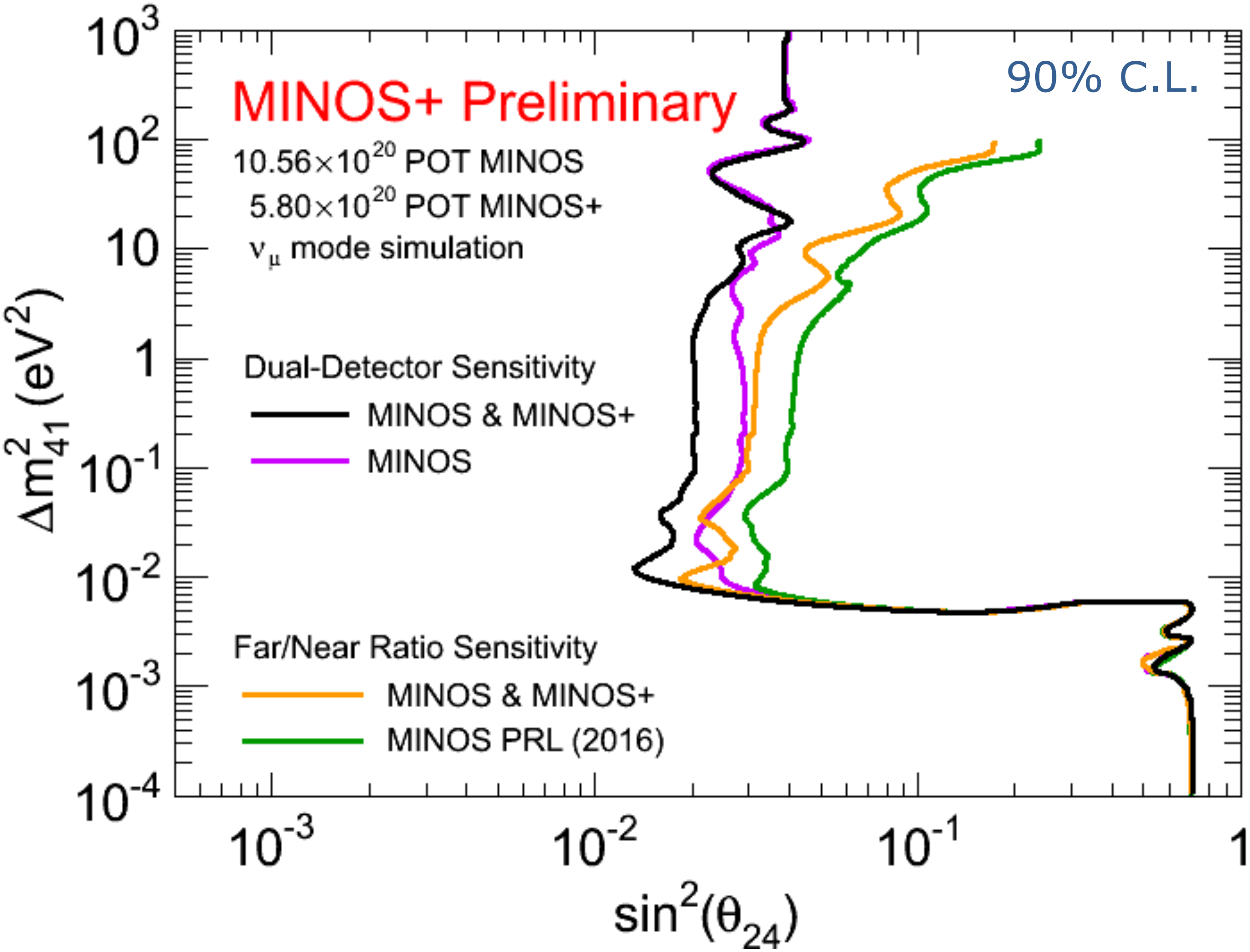
Full covariance matrix for CC-selected events



$$\chi^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}(O_i - E_i)^T\left[V^{-1}\right]_{ij}(O_j - E_j)$$

* L. Aliaga *et al.*, Phys. Rev. D 94, 092005 (2016)

Sterile Neutrino Sensitivity



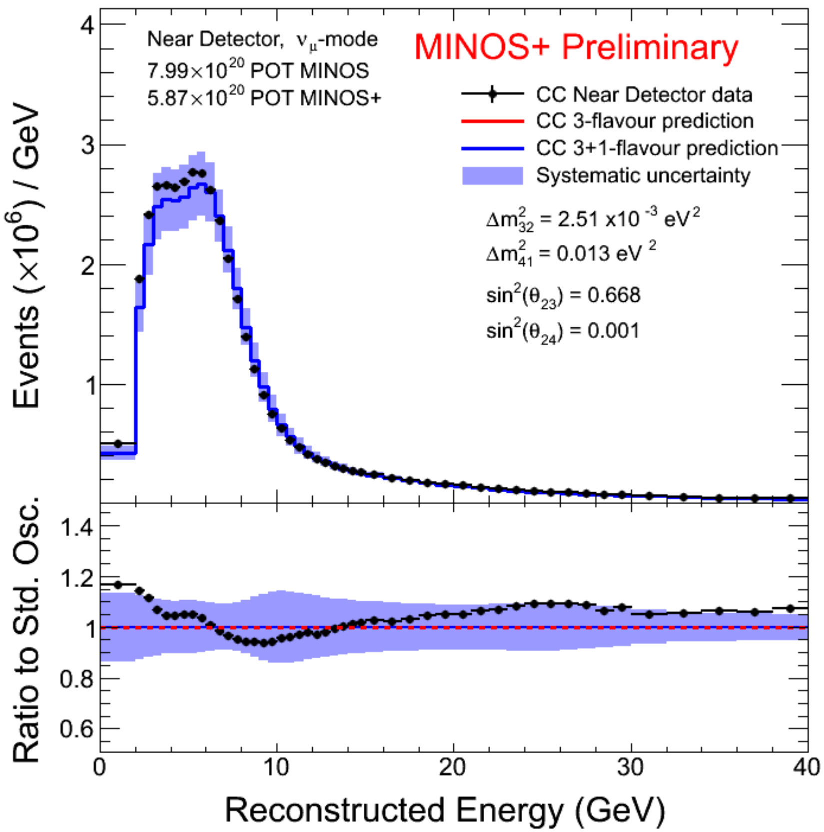
Addition of MINOS+ data and use of new fitting method yield significant improvement in sensitivity compared with previous MINOS analysis.

MINOS PRL (2016) : P. Adamson *et al.*, Phys. Rev. Lett. 117, 151803 (2016)

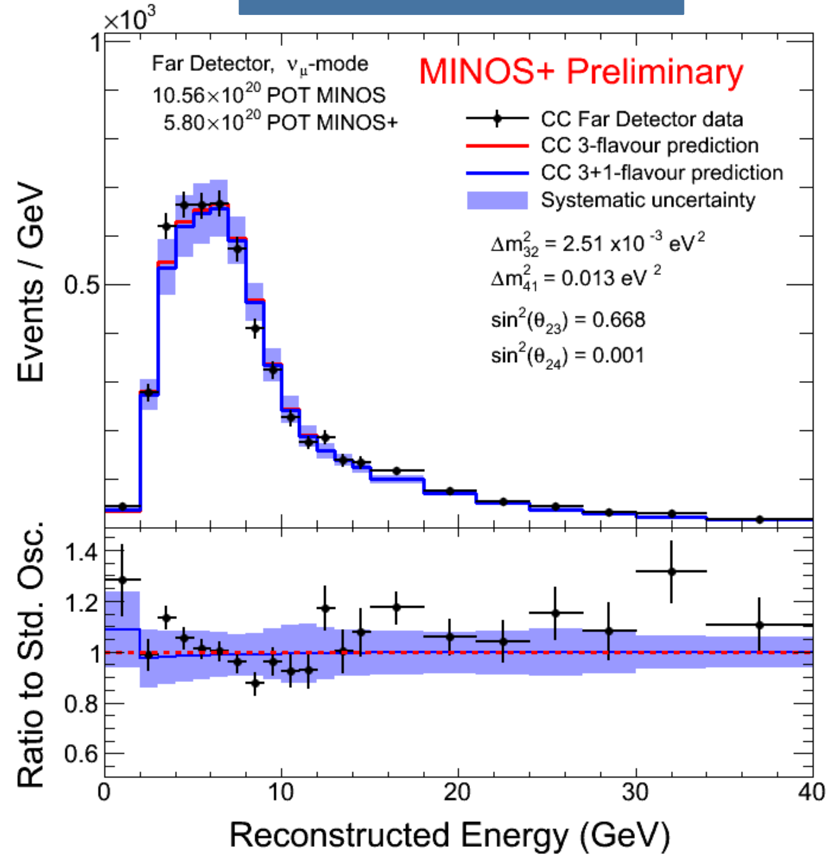
Observed Energy Spectra

CC-selected events

NEAR DETECTOR



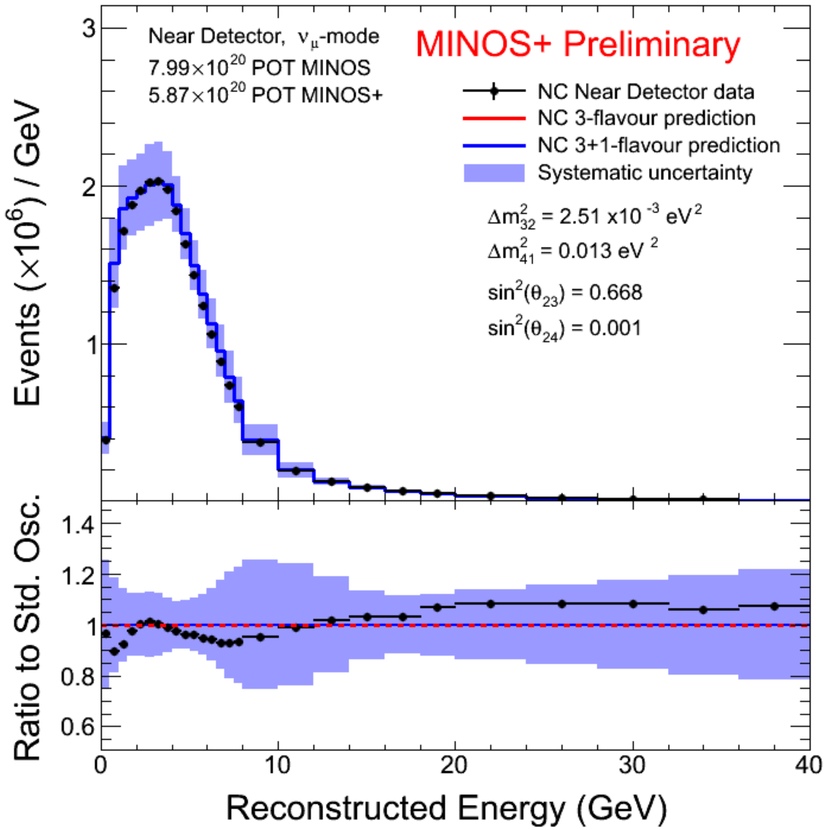
FAR DETECTOR



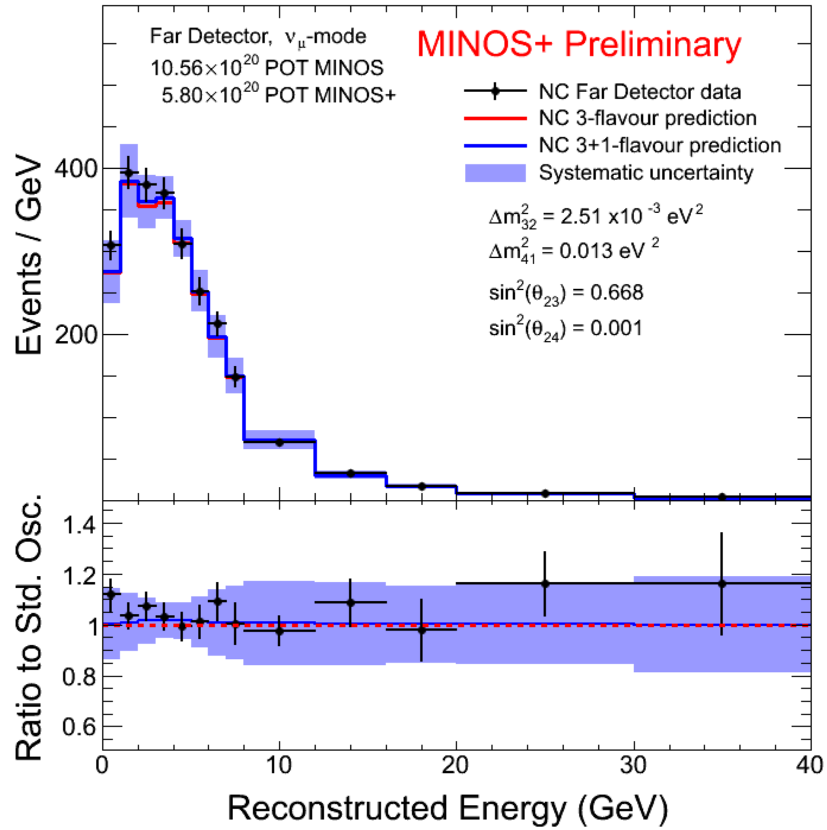
Observed Energy Spectra

NC-selected events

NEAR DETECTOR



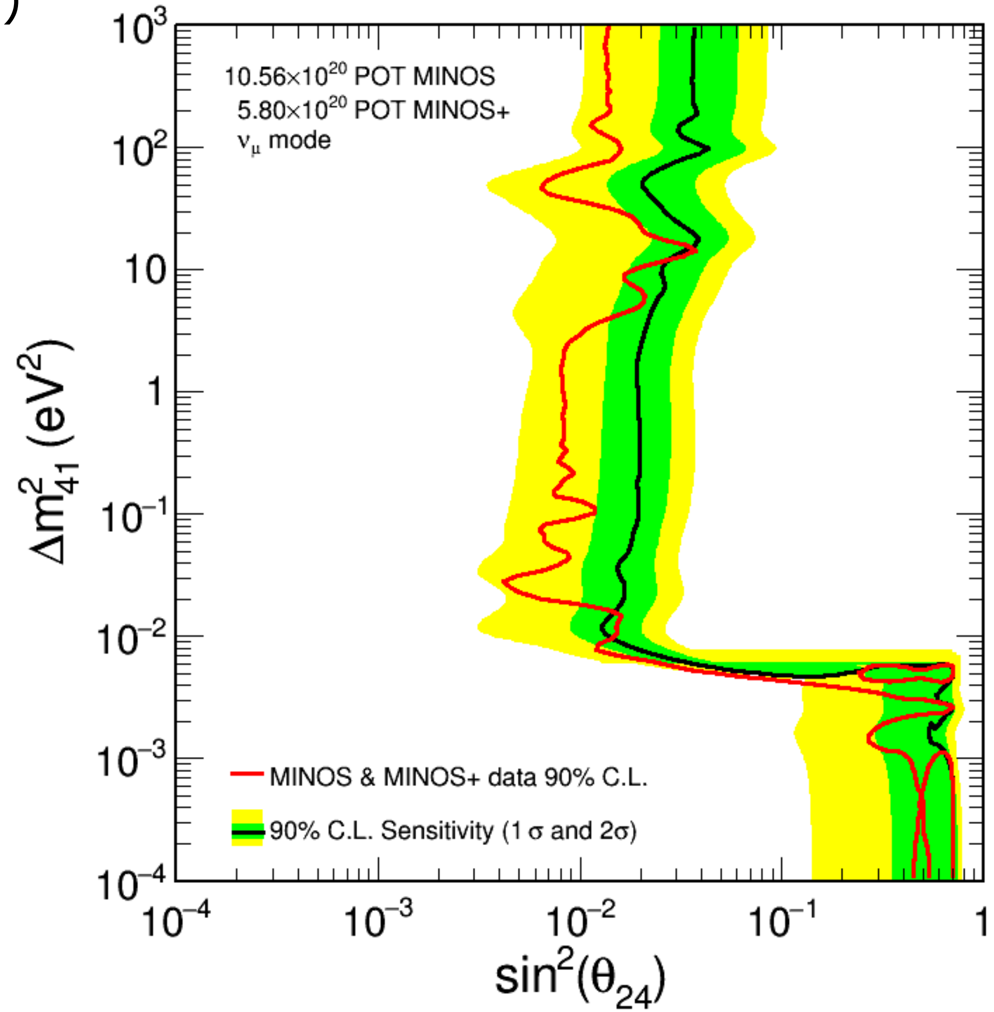
FAR DETECTOR



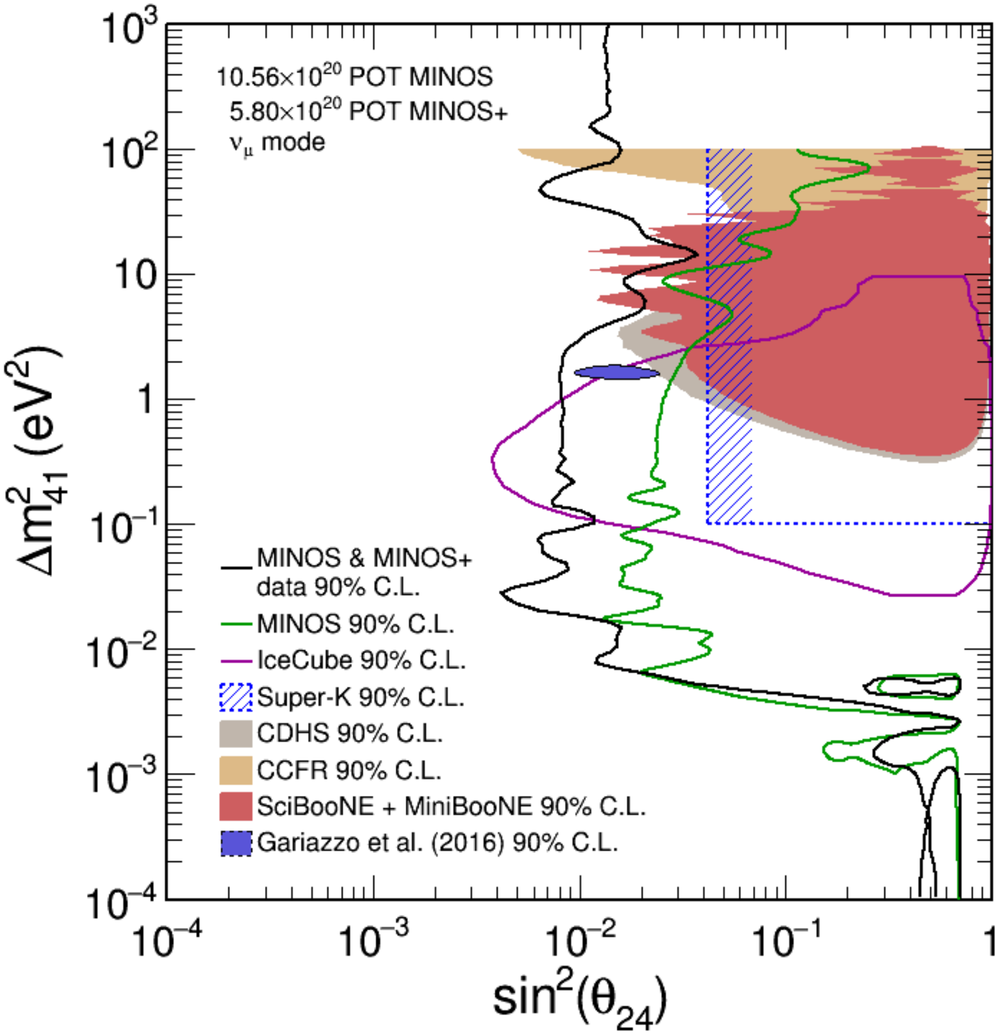
Exclusion Contours

- Confidence limits in (Δm^2_{41}, θ_{24}) are constructed using the Feldman-Cousins method.
  - <u>Note</u>: χ^2 is minimised with respect to Δm^2_{32}, θ_{23} and θ_{34} in each bin of this 2D space.
- A strong exclusion limit on the mixing angle θ_{24} is obtained over several decades in Δm^2_{41}.
- The exclusion limit calculated using the observed data falls within $\pm 2\sigma$ sensitivity band.
- Obtain the following 1D limits at Δm^2_{41}=0.5eV^2:

$$\sin^2\theta_{24} < 0.005 \text{ (90\% C.L.)}$$
$$\sin^2\theta_{34} < 0.16 \text{ (90\% C.L.)}$$



Comparison with Other Experiments



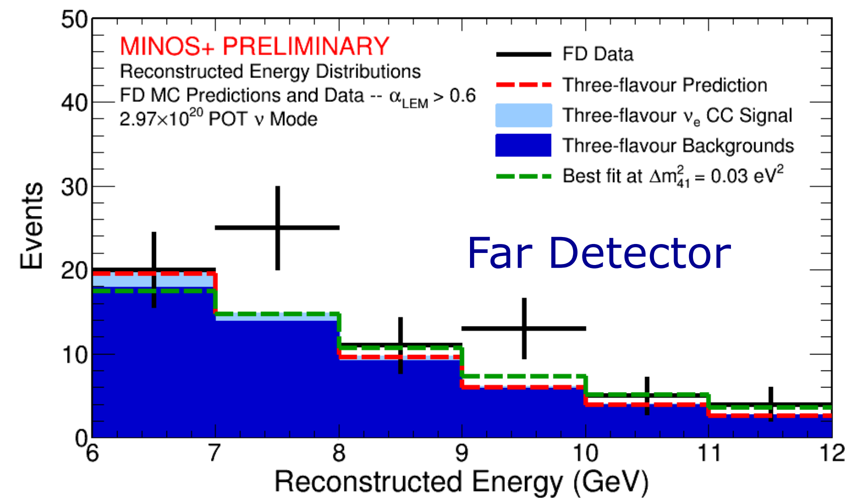
- New MINOS & MINOS+ limit improves upon the previous MINOS analysis.
  - Limit on θ_{24} is world-leading for much of Δm^2_{41} range.
- Results increase tension with with hints from global fits*.
  - e.g. fit from Gariazzo et al. is displayed in (Δm^2_{41}, θ_{24}) parameter space by setting $|U_{e4}|^2$=0.023.

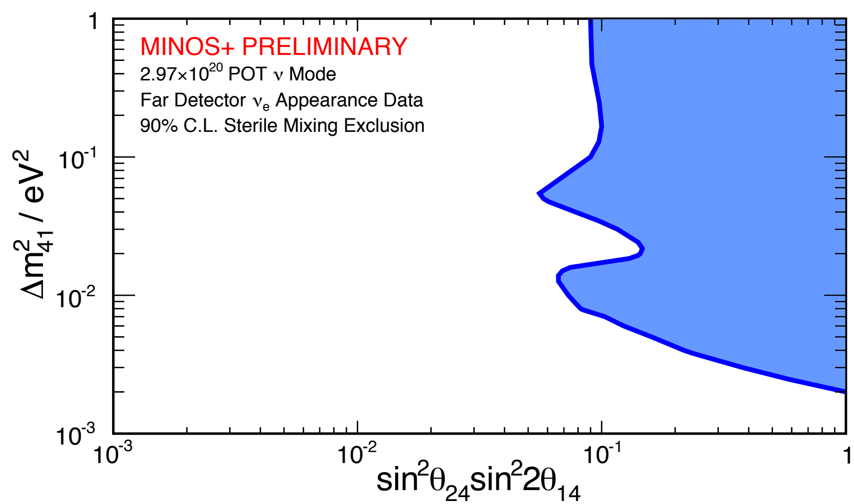
(This fit doesn't include data from MINOS or IceCube)

* S. Gariazzo, C. Giunti, M. Laveder, Y.F. Li, E.M. Zavanin, J. Phys. G43, 033001 (2016)

$\nu_\mu \rightarrow \nu_e$ Appearance Analysis

- A sterile-driven $\nu_\mu \rightarrow \nu_e$ appearance has also been performed using 3×10^{20} POT of MINOS+ data.
- Search for anomalous appearance in 6-12 GeV energy region.
  - ➢ Away from standard oscillations.
- Near Detector is used to produce Far Detector prediction.
  - ➢ Expect 56.7 events, observe 78.
  - ➢ 2.3σ excess.
- Exclusion contours in $\sin^2\theta_{24}\,\sin^2 2\theta_{14}$ calculated using Feldman-Cousins method.
- This analysis is based on one third of the available data from MINOS+. More to come!

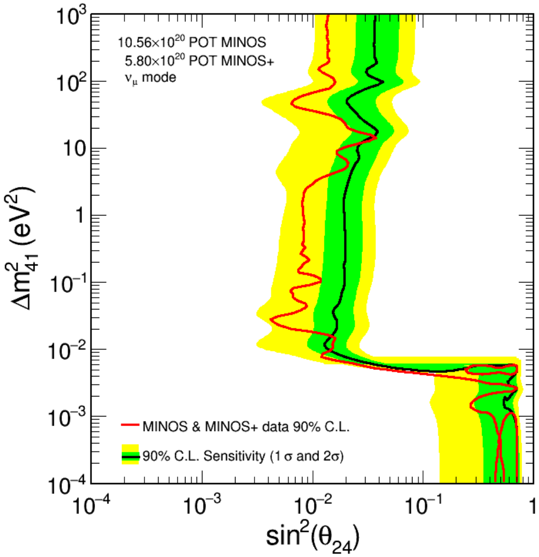




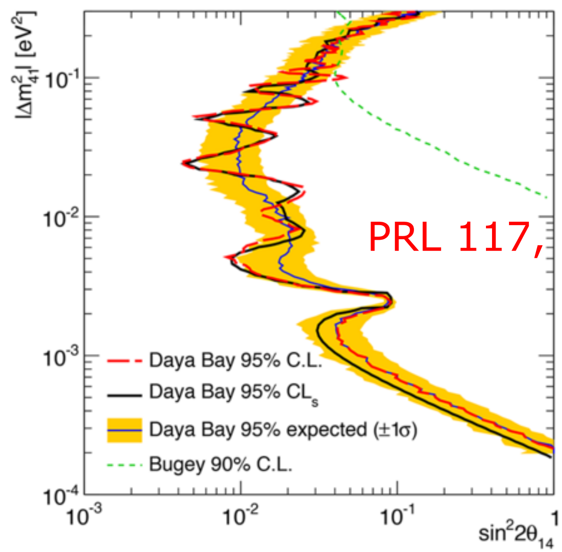
Combining with Daya Bay & Bugey-3

- Can probe $\nu_\mu \rightarrow \nu_e$ appearance hints from experiments such as LSND and MiniBooNE by combining long-baseline ν_μ disappearance data with reactor ν_e disappearance data.

MINOS/MINOS+



Daya Bay (& Bugey-3)



PRL 117, 151802 (2016)

$$|U_{\mu 4}|^2 = \sin^2\theta_{24}\cos^2\theta_{14}$$

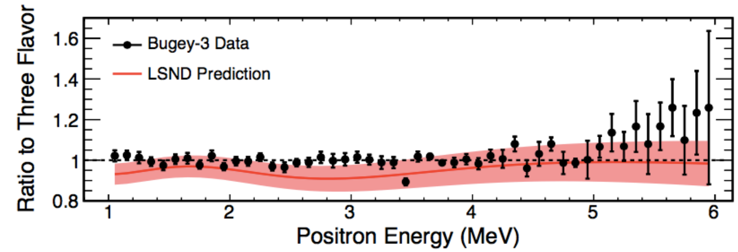
$$|U_{e4}|^2 = \sin^2\theta_{14}$$

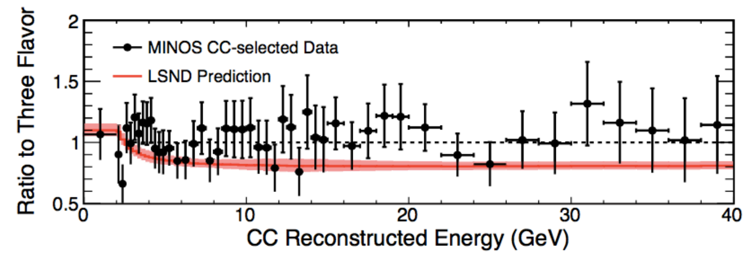
$$\Rightarrow 4|U_{\mu 4}|^2|U_{e4}|^2 = \sin^2\theta_{24}\sin^2 2\theta_{14} \equiv \sin^2 2\theta_{\mu e}$$

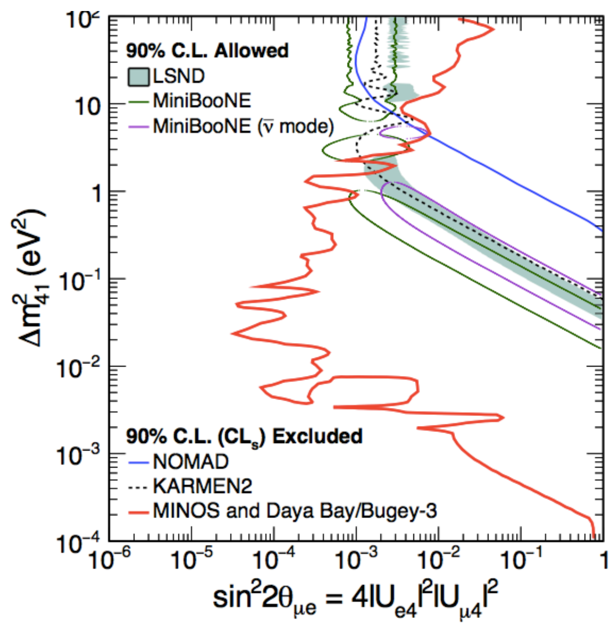
Combined Analysis

- In 2016, MINOS and Daya Bay published a combined sterile result, with inclusion of Bugey-3 data*.
- Details of reactor data:
  - Daya Bay [8AD data set, 404 days]:
    Baselines: 520m, 570m, 1590m
    Sensitivity: $\Delta m^2_{41} \sim 10^{-3} - 10^{-1}\ \text{eV}^2$
  - Bugey-3 [Nucl Phys B434, 503 (1995)]:
    Baselines: 15m, 40m, 95m
    Sensitivity: $\Delta m^2_{41} \sim 10^{-1} - 10\ \text{eV}^2$
- Combined analysis yielded strong exclusion limits on $\sin^2 2\theta_{\mu e}$.
- Have now updated combined analysis to include **MINOS+ data**.

* MINOS: P. Adamson et al, PRL 117, 151803 (2016)
Daya Bay: F. P. An et al.,PRL 117, 151802 (2016)
Combination: P. Adamson et al.,PRL 117, 151801 (2016)

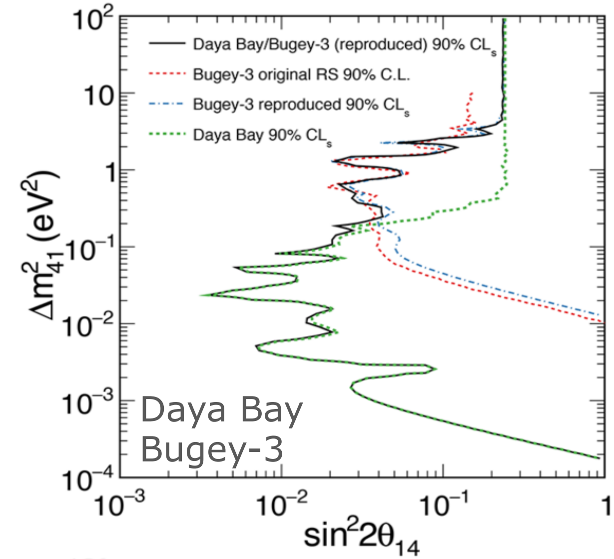


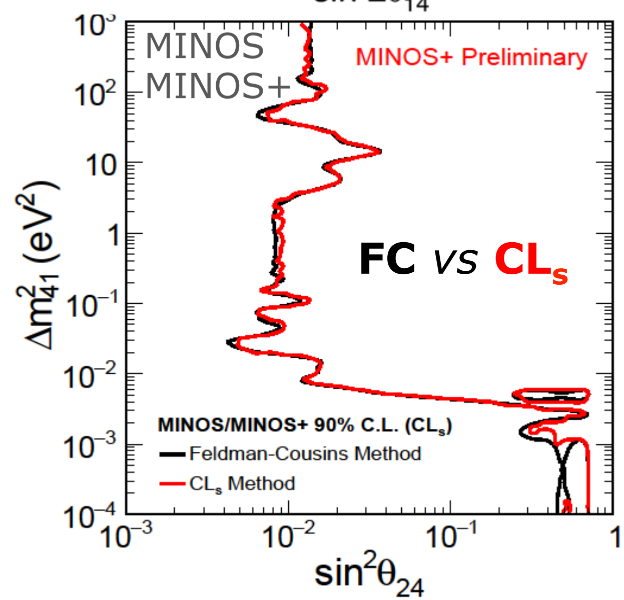




Combined Analysis

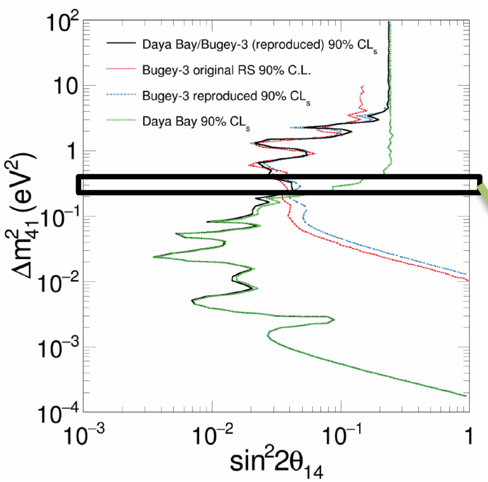
- Combined analysis uses **CL_s method** to calculate joint confidence limits.
  - Problem: while joint likelihood surface is straight forward to compute, Feldman-Cousins correction is onerous.
    - Would involve combined fits with Δm^2_{41}, $\sin^2 2\theta_{14}$, $\sin^2\theta_{24}$ all free.
    - Difficult without joint fit framework.
  - CL_s method provides a solution.
    - Each CL_s value is calculated with Δm^2_{41}, $\sin^2 2\theta_{14}$, $\sin^2\theta_{24}$ fixed.
- When the MINOS/MINOS+, Daya Bay and Bugey-3 limits are individually re-calculated using CL_s method, resulting contours agree well with Feldman-Cousins method.



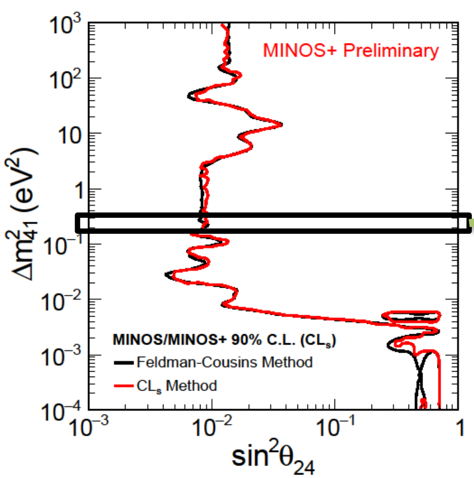


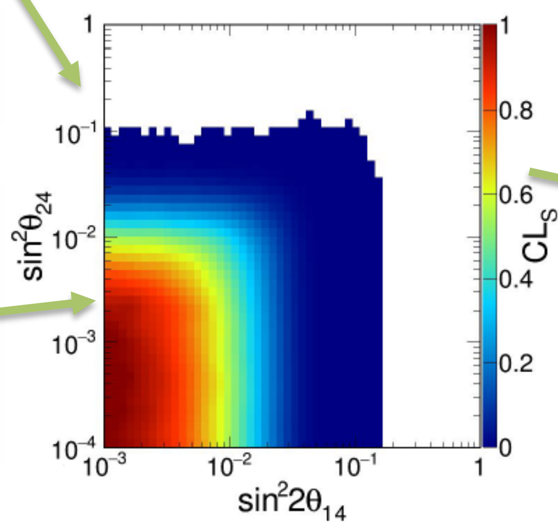
Combined Analysis

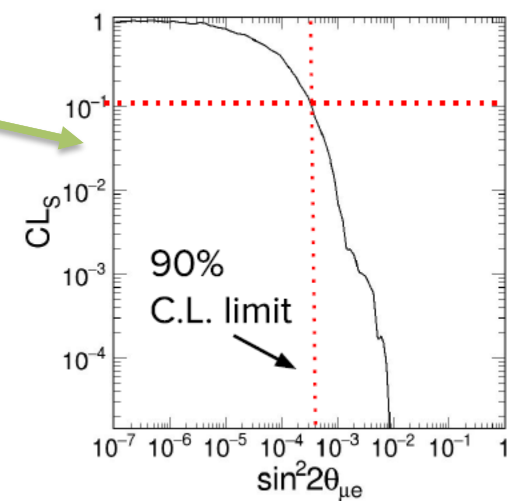
- Likelihood surfaces from each experiment have a shared y-axis (Δm^2_{41}), but different x-axes ($\sin^2 2\theta_{14}$ *vs* $\sin^2\theta_{24}$).



- Combined analysis proceeds as follows:
  - ➢ For each row in Δm^2_{41}, calculate CLs for all 2D combinations of ($\sin^2 2\theta_{14}$, $\sin^2\theta_{24}$).
  - ➢ Convert this into a 1D CL_s distribution as a function of $\sin^2 2\theta_{\mu e} = \sin^2 2\theta_{14} \sin^2\theta_{24}$.
  - ➢ Read off 90% C.L. in $\sin^2 2\theta_{\mu e}$ for this Δm^2_{41}.

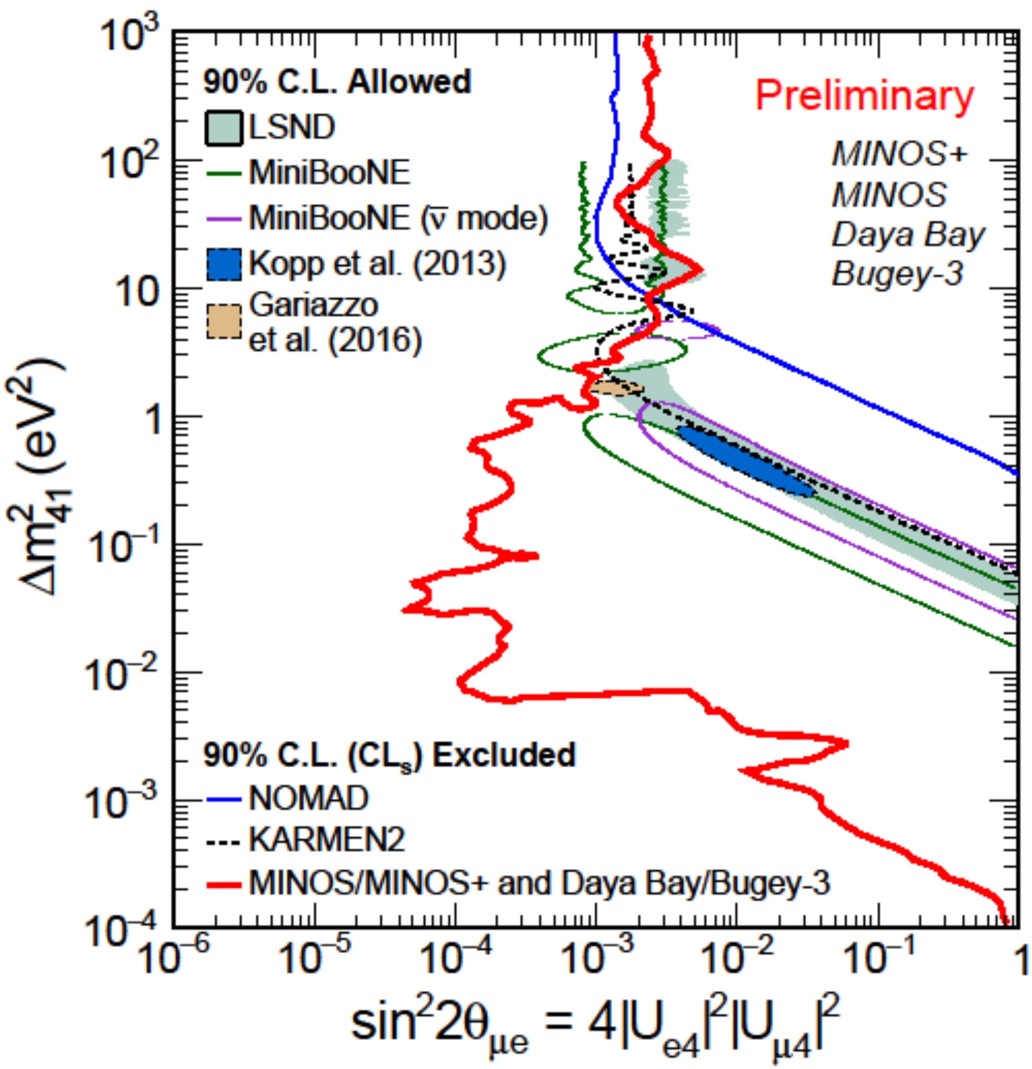






New Combined Result

- New preliminary result from the ongoing collaboration between MINOS/MINOS+ and Daya Bay (with the inclusion of Bugey-3).
- No evidence for 3+1 sterile neutrino oscillations.
- Strong exclusion limits on $\sin^2 2\theta_{\mu e}$ are obtained for a wide range of Δm^2_{41}.

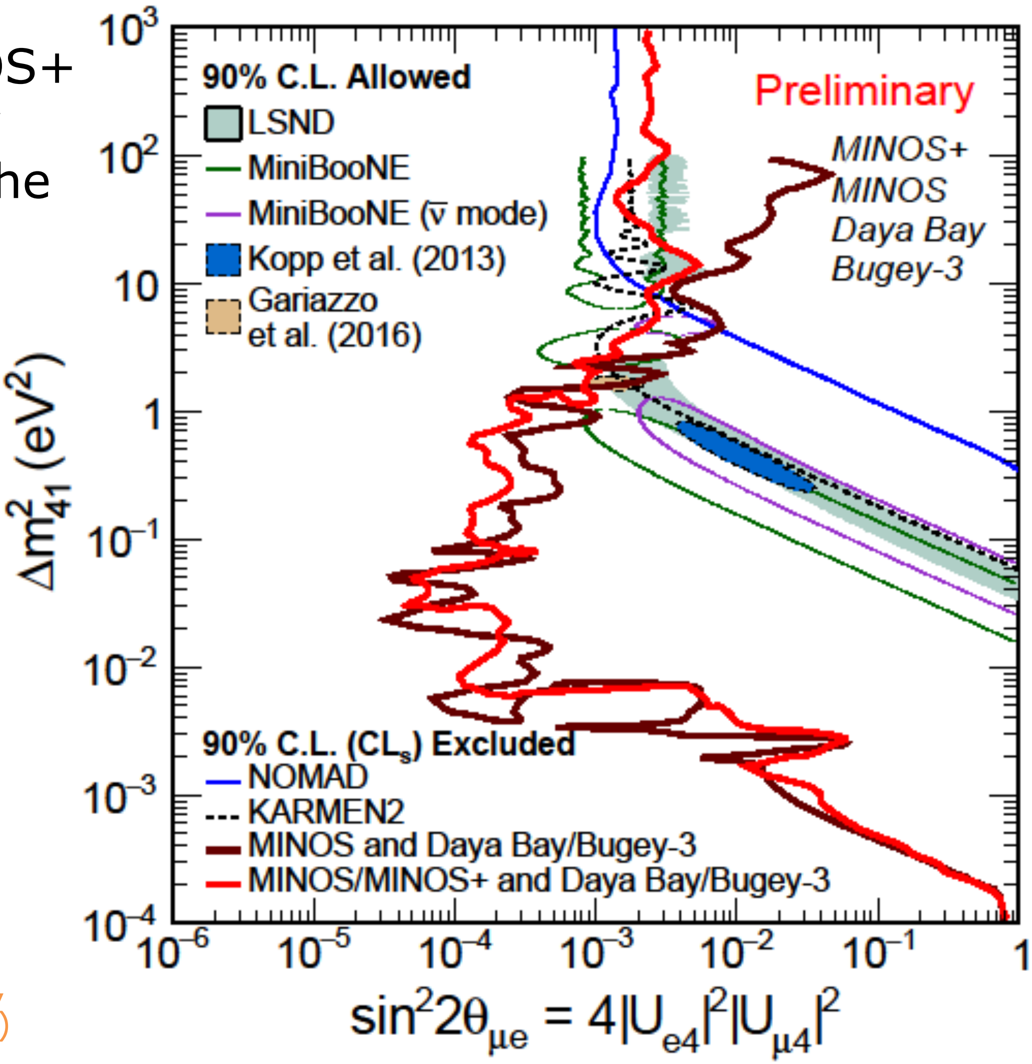


J. Kopp, P. Machado, M. Maltoni, T. Schwetz, JHEP 1305:050 (2013)

S. Gariazzo, C. Giunti, M. Laveder, Y.F. Li, E.M. Zavanin, J.Phys. G43 033001 (2016)

New Combined Result

- As expected, the new MINOS+ two-detector fit significantly improves the constraint in the region $\Delta m^2_{41} > 10\text{eV}^2$.
- A new combined analysis with a larger data set from Daya Bay is planned for the future.



P. Adamson et al., Phys. Rev. Lett. 117, 151801 (2016)

J. Kopp, P. Machado, M. Maltoni, T. Schwetz, JHEP 1305:050 (2013)

S. Gariazzo, C. Giunti, M. Laveder, Y.F. Li, E.M. Zavanin, J.Phys. G43 033001 (2016)

Summary

- MINOS/MINOS+ ν_μ disappearance analysis has set strong limits on the θ_{24} mixing angle over several decades in Δm^2_{41}.
  - ➢ Exclusion contours enhanced by two-detector fit method.
- New preliminary combined fit with data from Daya Bay & Bugey-3.
  - ➢ Further increases tension between sterile neutrino results from appearance and disappearance.
- More sterile neutrino results to come:
  - ➢ New ν_μ disappearance and ν_e appearance results using complete MINOS+ data set.
  - ➢ New anti-ν_μ disappearance analysis.
  - ➢ Updated combined analysis with Daya Bay & Bugey-3.
- Watch this space!

BACKUP

Event Selection

◆ ν_μ disappearance analysis selects two event topologies:

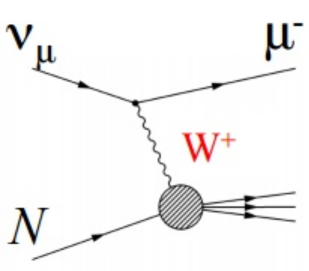
(1) ν_μ CC interactions:

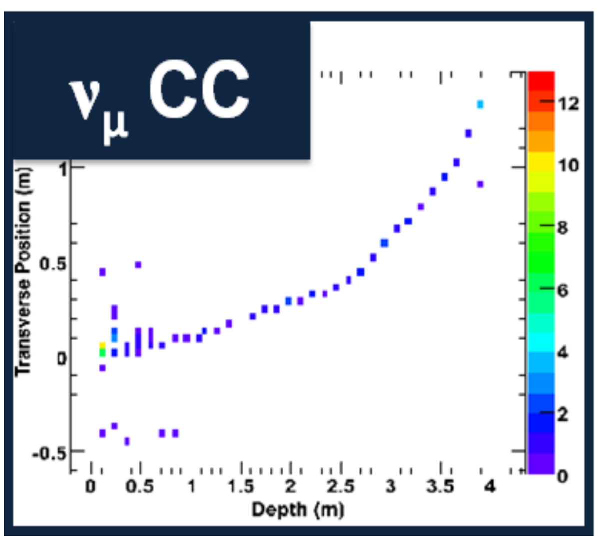
➢ Distinguished by presence of reconstructed muon track.

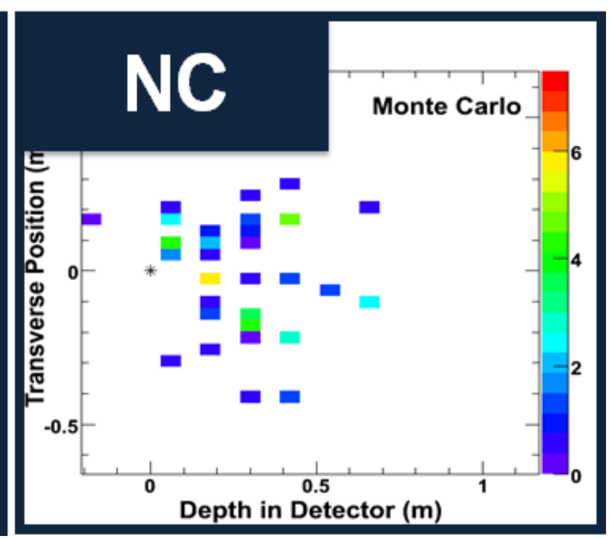
(2) NC interactions:

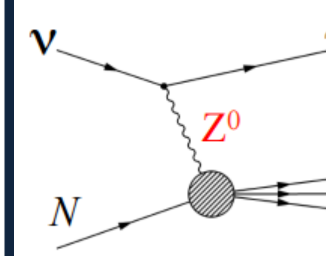
➢ Distinguished by presence of hadronic shower and no muon track.

Note: ν_e CC and ν_τ CC events typically enter as small backgrounds (usually appears shower-like).



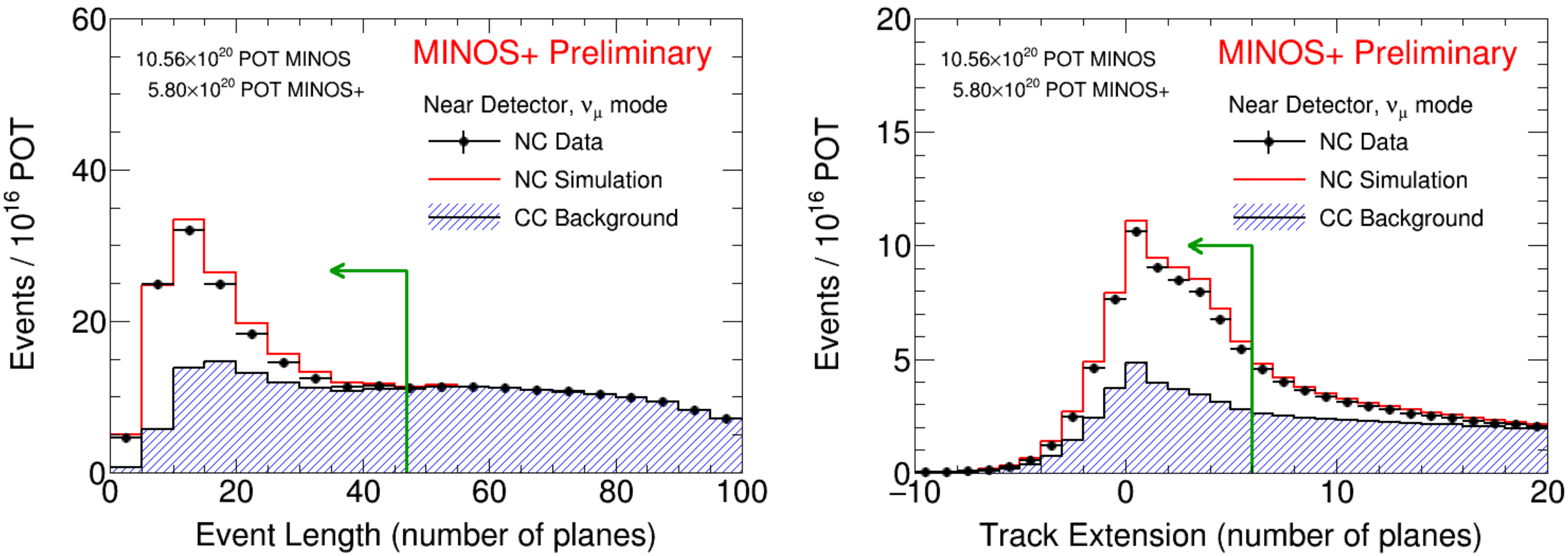






Selection of NC-like Events

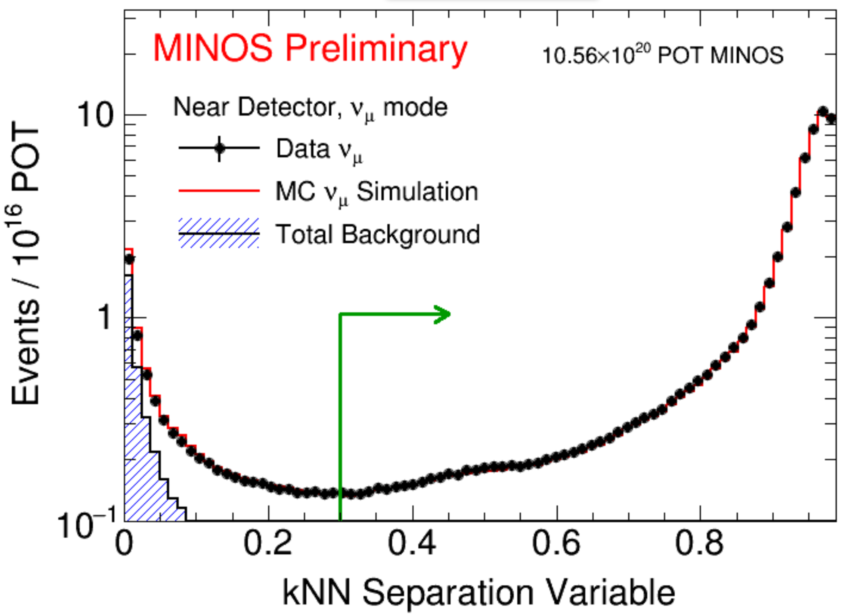
- Two main selection criteria are used to separate NC interactions from the dominant background of ν_μ CC interactions:
  - Event length.
  - Extension of reconstructed track beyond hadronic shower.
- In each case, the selection variables are sensitive to the presence of minimally-ionising muon tracks produced by ν_μ CC interactions.



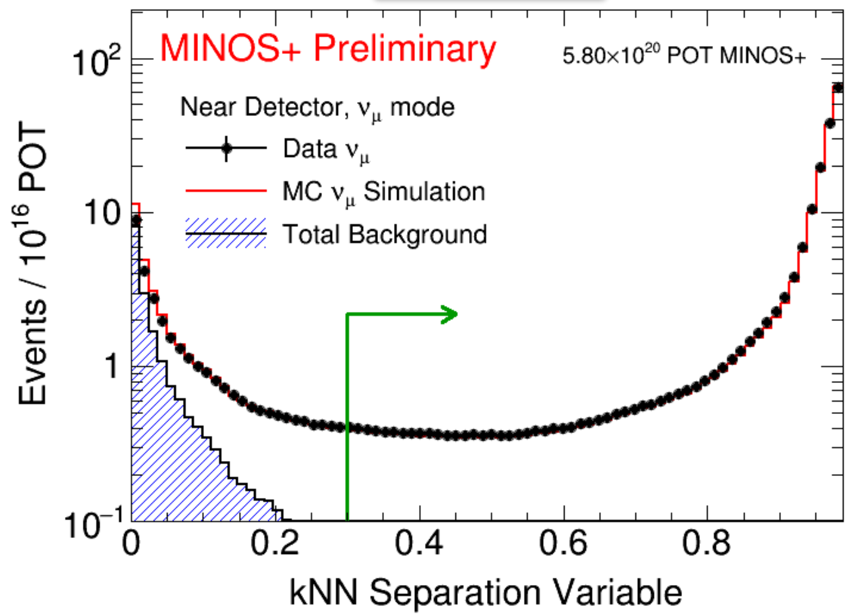
Selection of CC-like Events

- ν_μ CC interactions are identified using a multivariate kNN algorithm, which takes the following inputs:
  - ➢ Track length
  - ➢ Transverse profile
  - ➢ Mean dE/dx
  - ➢ Energy loss fluctuations
- Inputs are designed to identify characteristic properties of the muon.

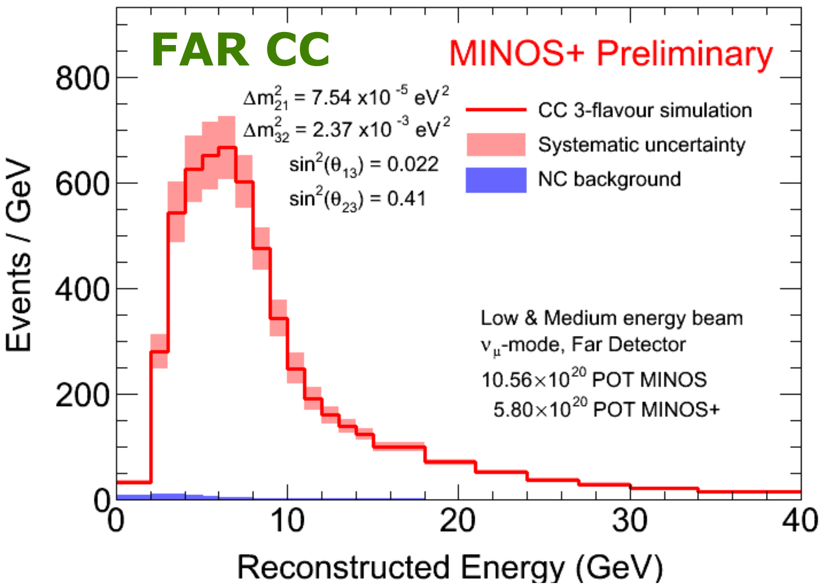
MINOS

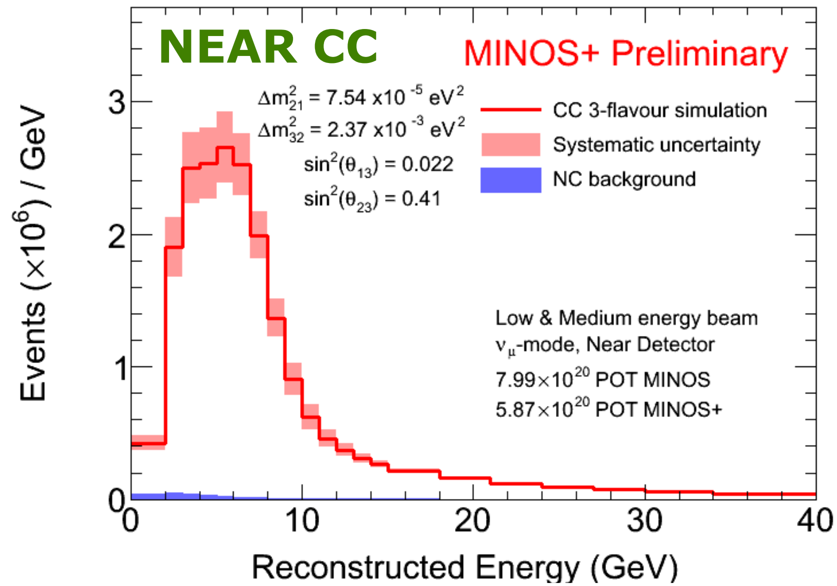


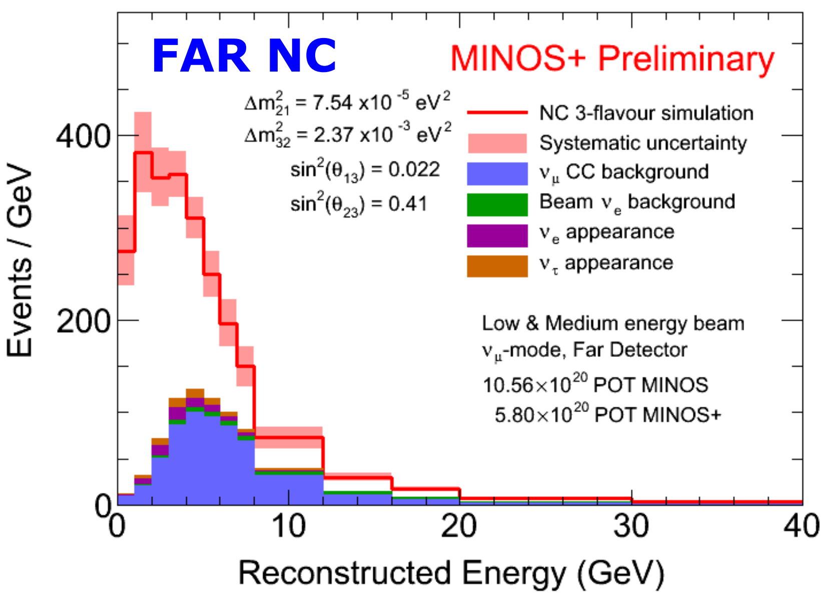
MINOS+

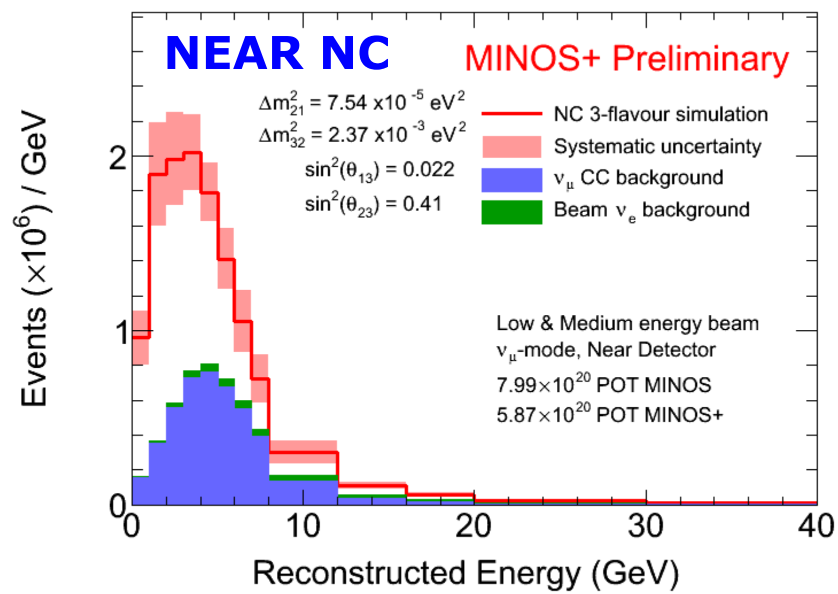


Event Spectra









Systematic Uncertainties

- Consider 44 sources of systematic uncertainty in a variety of categories:
  - Beam focusing
  - Hadron production
  - Beam focusing
  - X-sections **[largest for CC]**
  - Backgrounds
  - Energy scale **[largest for NC]**
  - Normalisation
  - ND acceptance & reconstruction

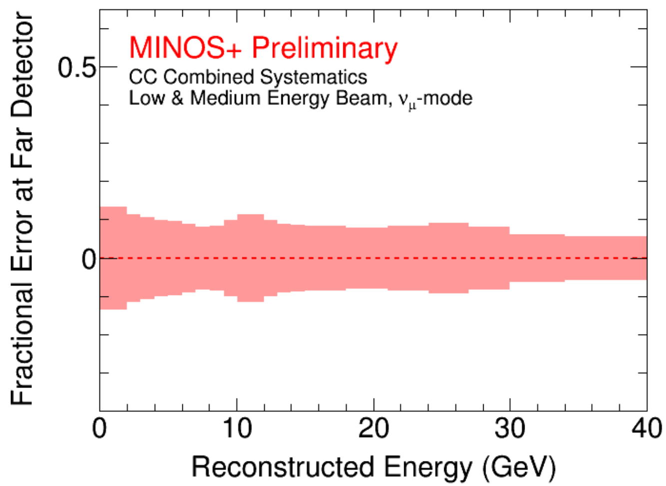
Note: focusing parameters are incorporated into fit as nuisance parameters.

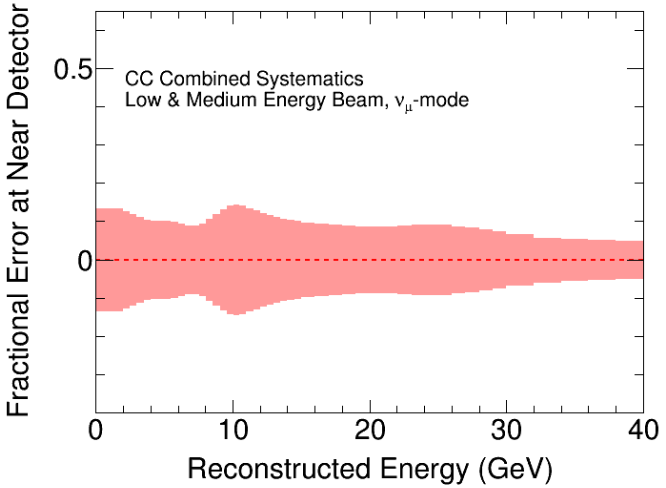
Sources of Systematic Uncertainty:

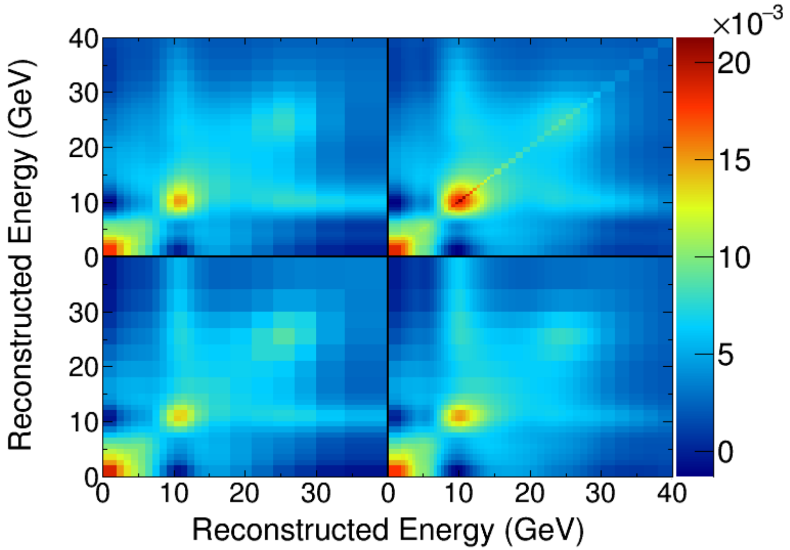
Uncertainty source	Maximum uncertainty (%) ND CC	FD CC	ND NC	FD NC
Hadron production	7%	7%	7%	7%
Cross-sections	10%	10%	11%	13%
Backgrounds	1%	1%	10%	5%
Energy scale	10%	8%	20%	18%
Other	6%	3%	6%	3%
Total	15%	12%	25%	20%

$$\chi^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}(o_i - e_i)^T[V^{-1}]_{ij}(o_j - e_j)$$

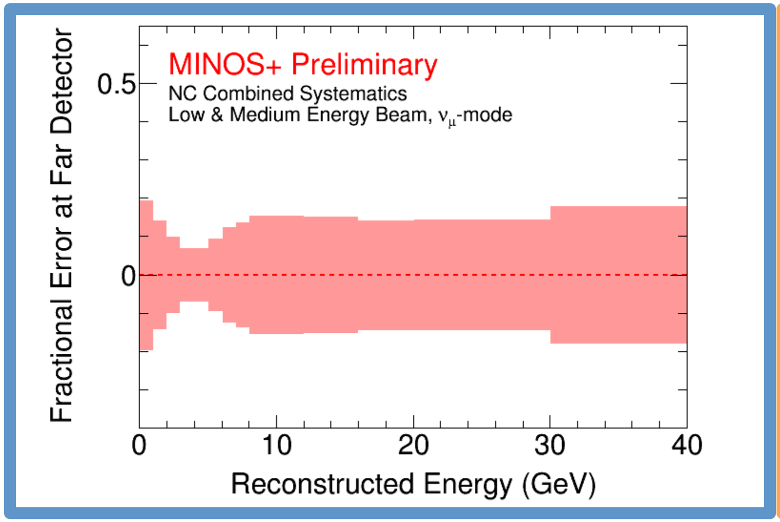
CC Systematics

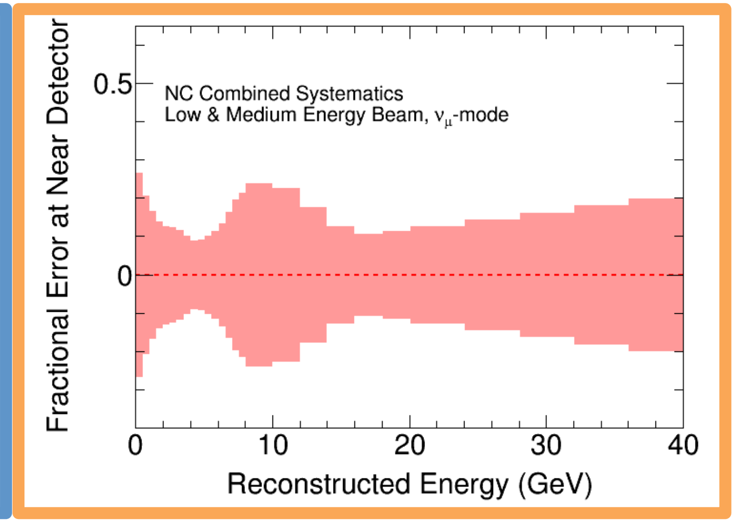




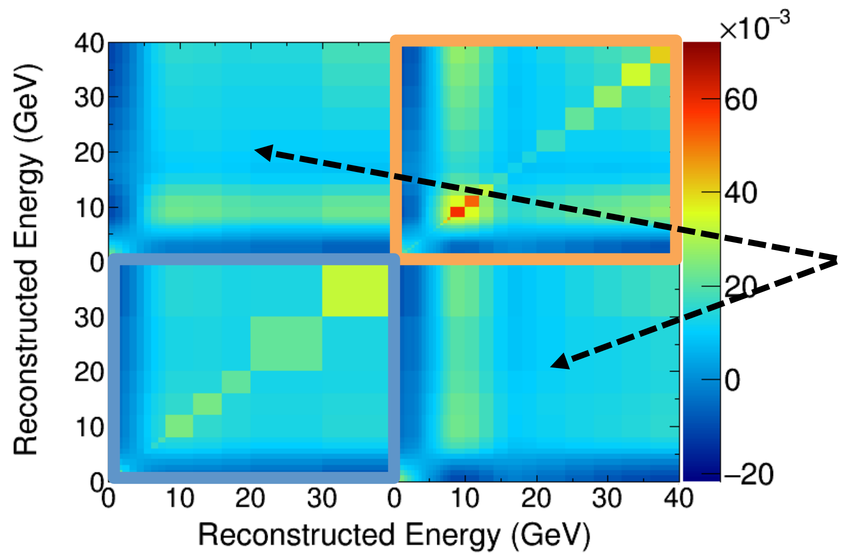


NC Systematics





Diagonal components form the bands above



Correlations between the detectors

3+1 Sterile Neutrino Model

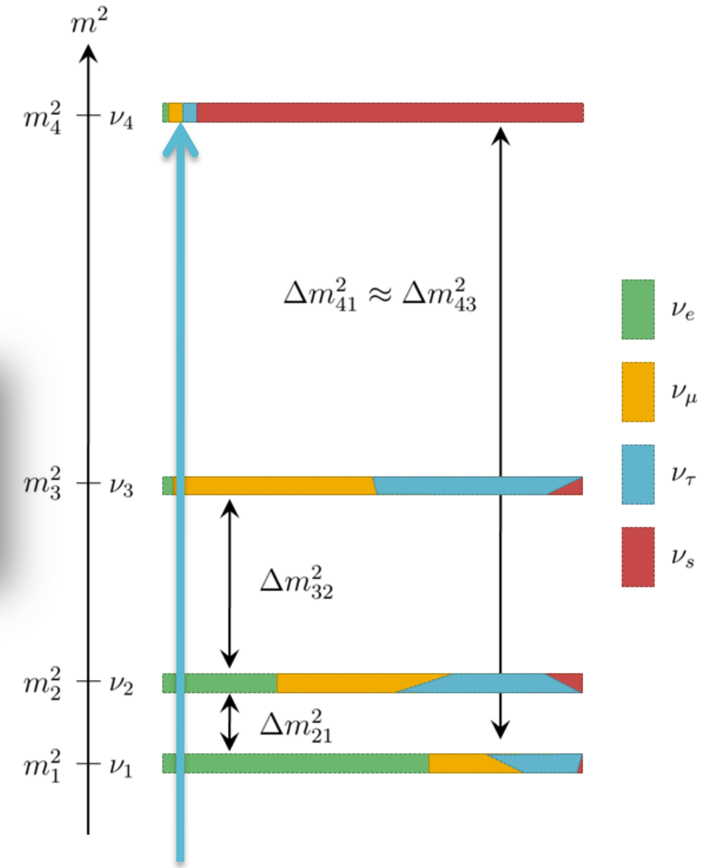
$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

$$\begin{aligned} P(\nu_\mu \to \nu_\mu) \approx 1 &- 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2)\sin^2\Delta_{31} \\ &- 4|U_{\mu 4}|^2|U_{\mu 3}|^2\sin^2\Delta_{43} \\ &- 4|U_{\mu 4}|^2(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2)\sin^2\Delta_{41} \end{aligned}$$

For $U = R_{34}R_{24}R_{14}R_{23}R_{13}R_{12}$

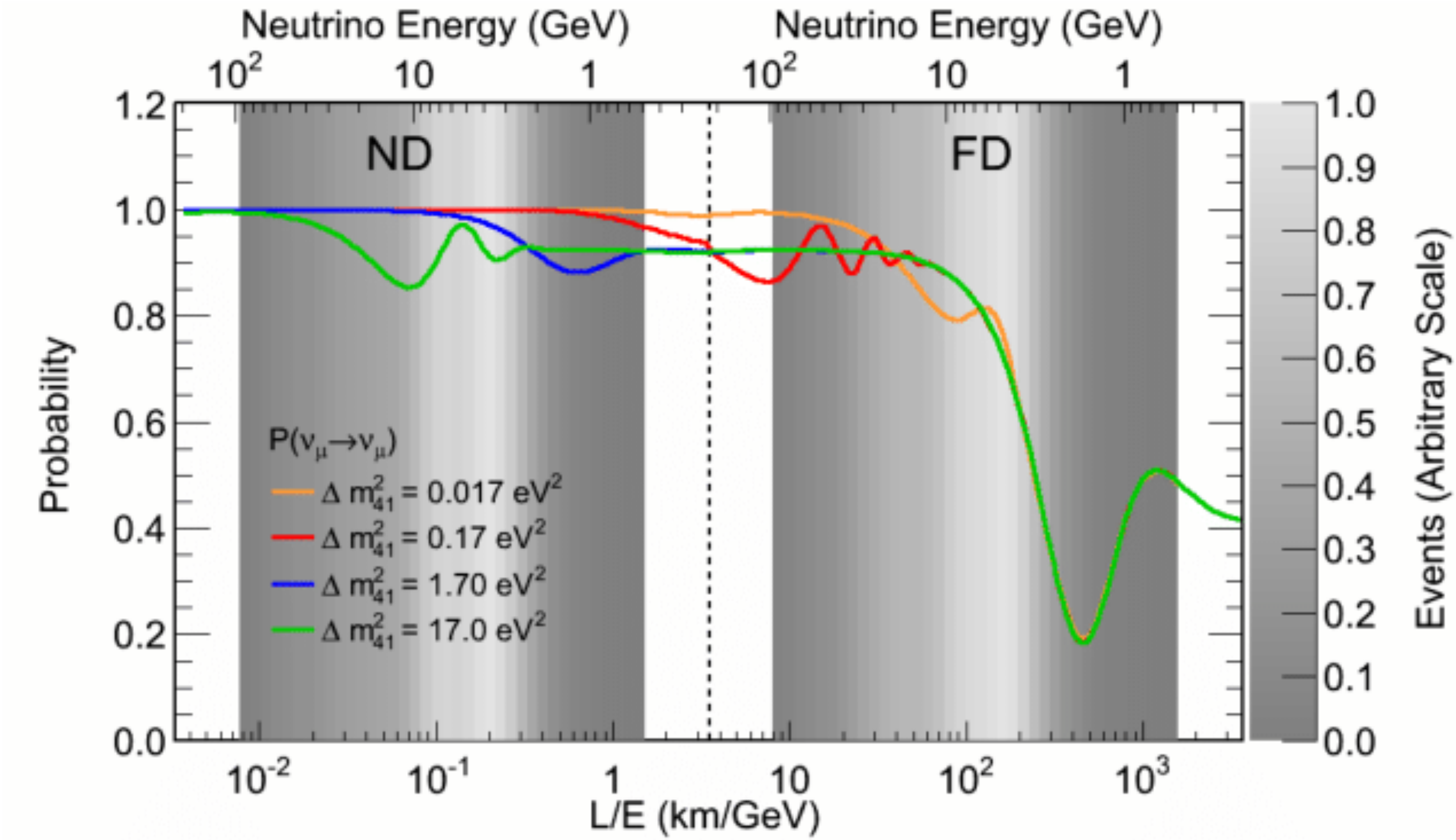
$$\boxed{|U_{\mu 4}|^2 = \cos^2\theta_{14}\sin^2\theta_{24}}$$

$$\begin{aligned} |U_{\mu 3}|^2 = &\sin^2\theta_{13}\sin^2\theta_{14}\sin^2\theta_{24} \\ &+ \cos^2\theta_{13}\sin^2\theta_{23}\cos^2\theta_{24} \\ &- \frac{1}{2}\cos(\delta_{13} - \delta_{14} + \delta_{24})\sin 2\theta_{13}\sin\theta_{23}\sin\theta_{14}\sin 2\theta_{24} \end{aligned}$$



Dominant observable in MINOS+

Sterile Neutrino Oscillations

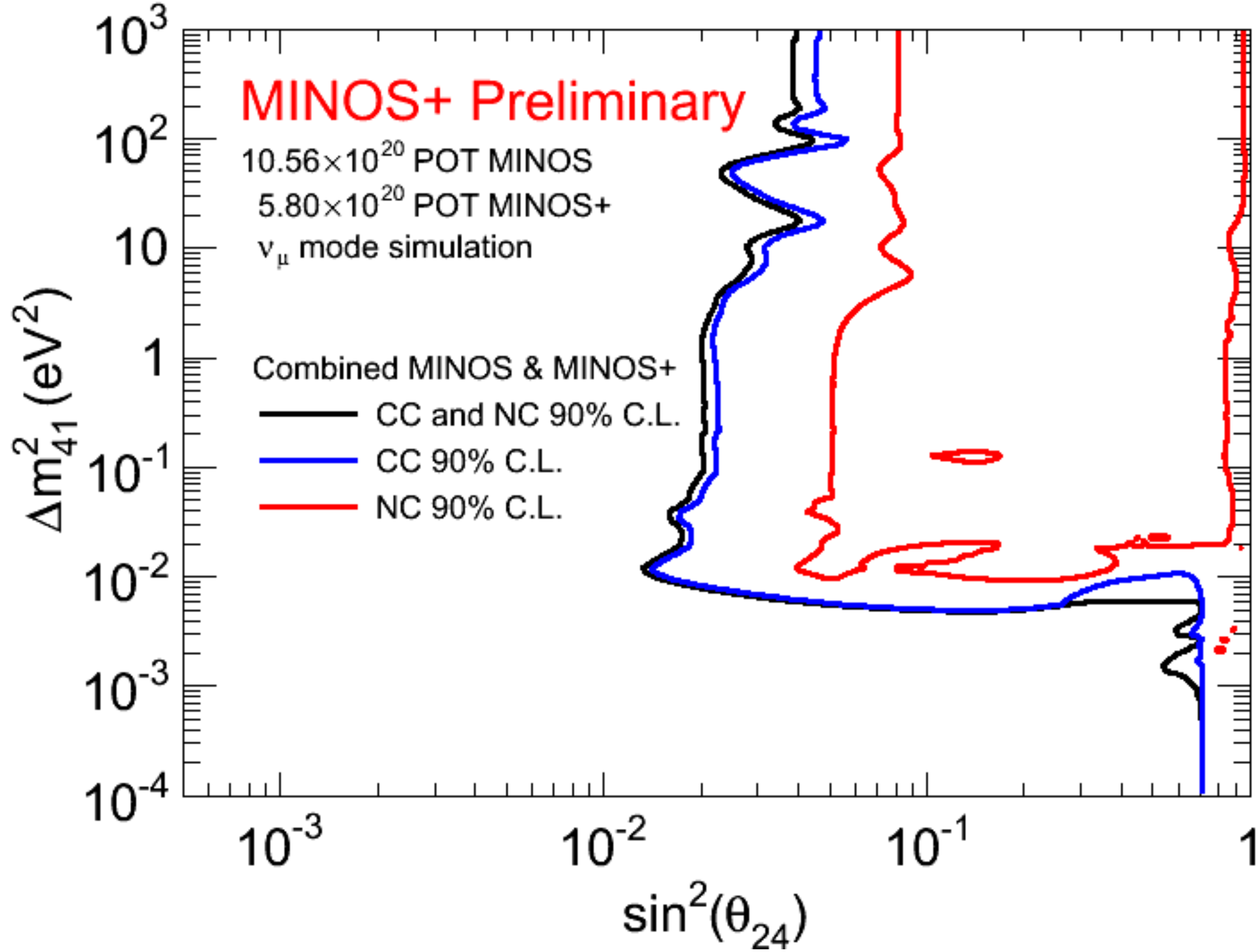


Smaller Δm^2_{41} – Distortions in FD above oscillation maximum

Larger Δm^2_{41} – Rapid oscillations in FD & Distortions in ND

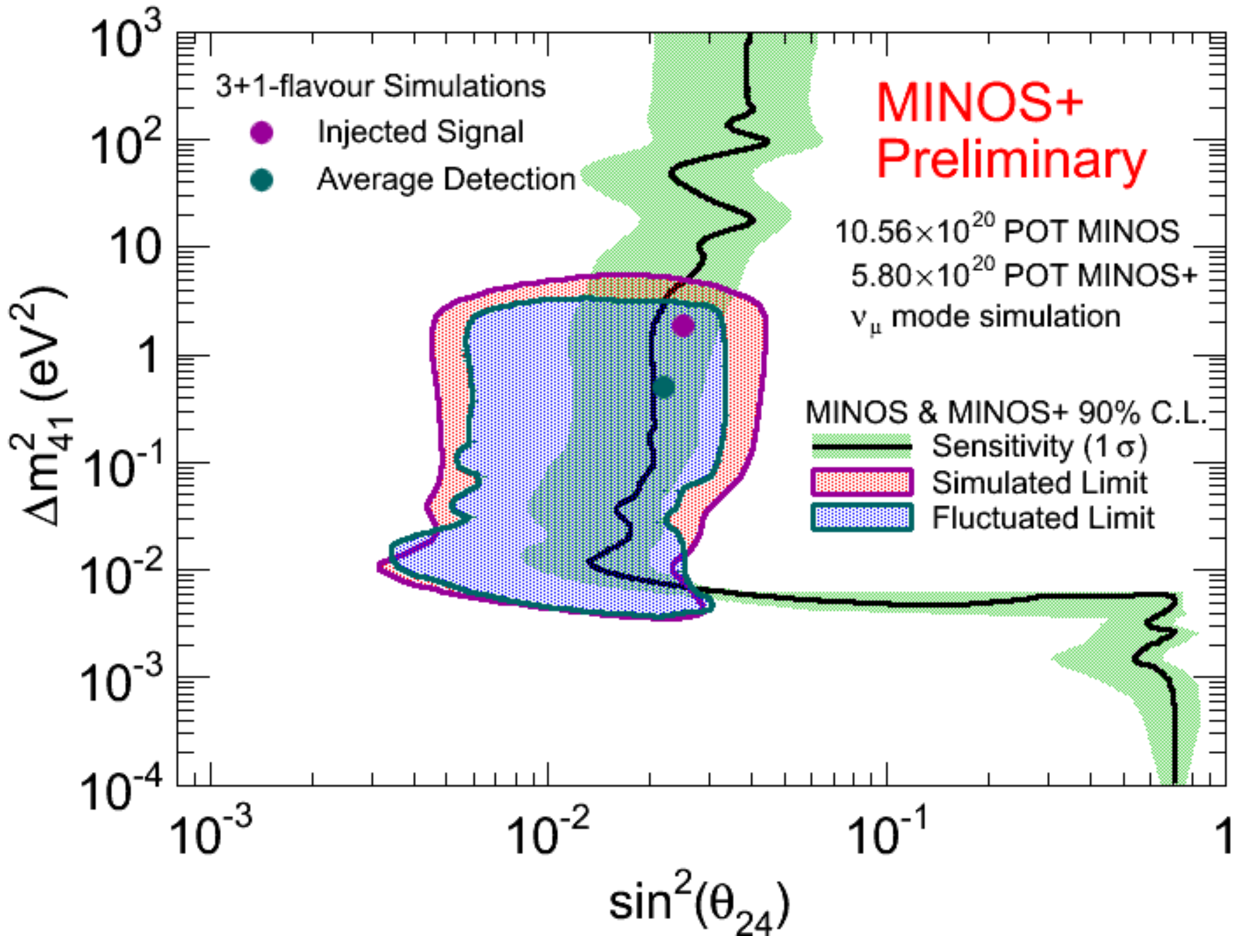
Sterile Sensitivity

◆ Relative contributions of CC and NC events in Δm^2_{41}-θ_{24} sensitivity:



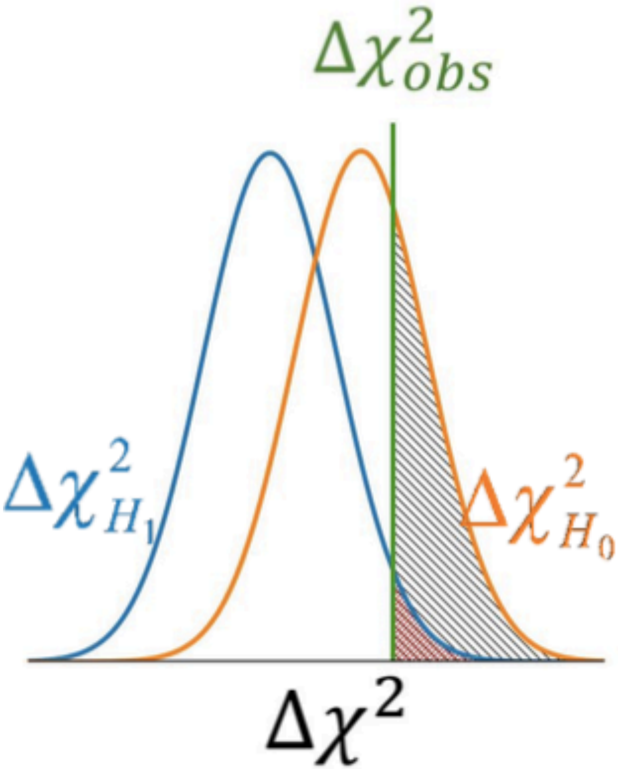
Signal Injection Test

- A signal injection test was performed for input parameters consistent with recent global best fit results: θ_{24}=0.15 ; Δm^2_{41}=1.65eV^2.
- An allowed region is visible with or without systematic fluctuations.



CL$_s$ Method

- For each pair of ($\sin^2 2\theta_{14}$, Δm^2_{41}), is the 4ν (H_1) model much worse than the 3ν (H_0) model?



$$\Delta\chi^2 = \chi^2_{H_1} - \chi^2_{H_0}$$

$$\mathrm{CL_b} = P\left(\Delta\chi^2 \geq \Delta\chi^2_{obs} \middle| 3\nu\right) =$$

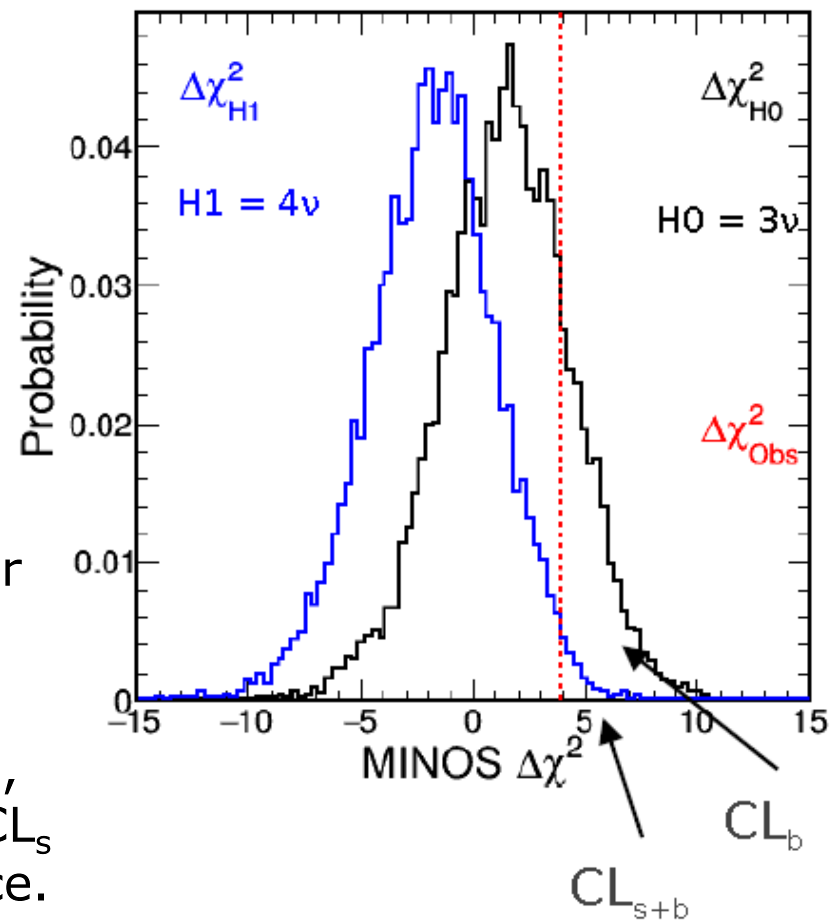
$$\mathrm{CL_{s+b}} = P\left(\Delta\chi^2 \geq \Delta\chi^2_{obs} \middle| 4\nu\right) =$$

$$\mathrm{CL_s} = \frac{\mathrm{CL_{s+b}}}{\mathrm{CL_b}} = \quad / $$

Point is excluded at (1-α) CL$_s$, if CL$_s$ < α

CL_s Method in MINOS+

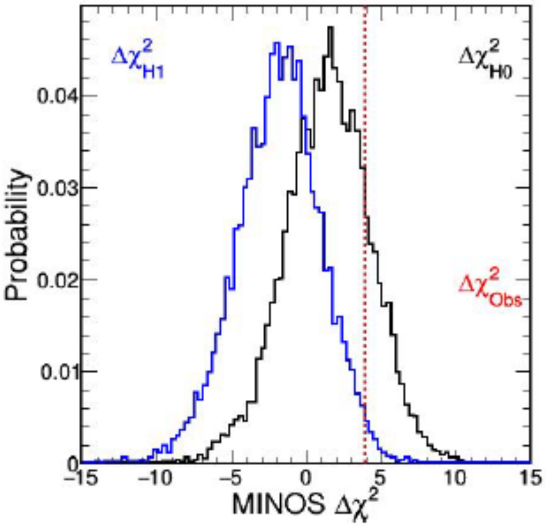
- For each (Δm^2_{41}, θ_{24}) point:
  - ➢ Generate 3-flavour pseudo experiments using PDG oscillation parameters.
  - ➢ Generate 4-flavour pseudo experiments using the current (Δm^2_{41}, θ_{24}) point.
  - ➢ Fit each fake experiment to both the 3-flavour and 4-flavour hypotheses to build the $\Delta\chi^2$ distributions.
  - ➢ Use generated $\Delta\chi^2$ distributions, along with $\Delta\chi^2_{Obs}$, to calculate CL_s for this point in parameter space.
  - ➢ 4-flavour hypothesis is excluded at (1-α) C.L. if $CL_s < \alpha$.

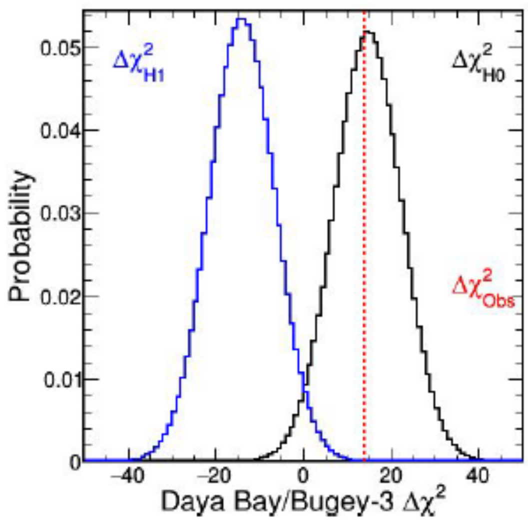


$CL_s = CL_{s+b}/CL_b$

Combined Analysis

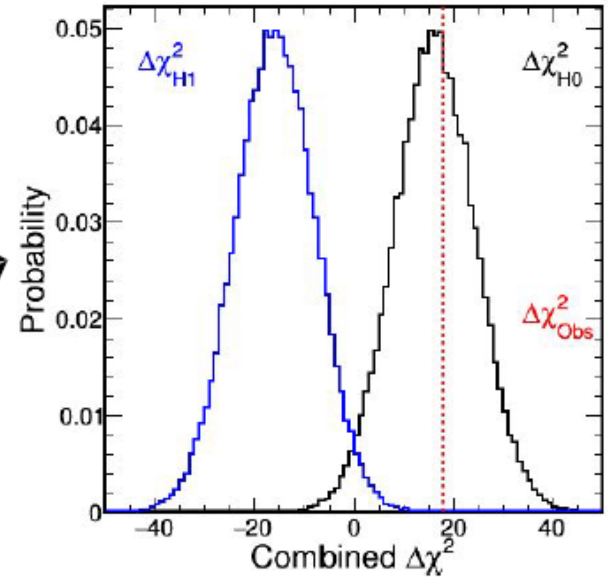
For each point in the parameter space, combine $\Delta\chi^2$ distributions





Draw MINOS $\Delta\chi^2$ values from fake experiments.

Draw Daya Bay/Bugey-3 $\Delta\chi^2$ values from Gaussian distributions.

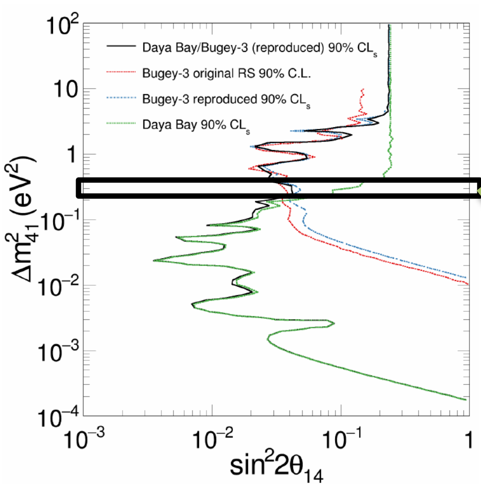


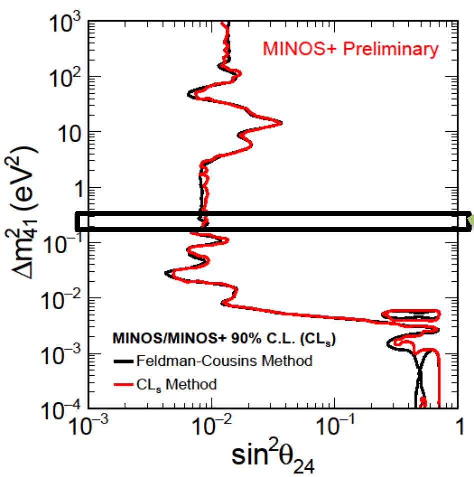
MINOS and Daya Bay/Bugey-3 have uncorrelated systematics so:

$$\Delta\chi^2_{combo} = \Delta\chi^2_{DB} + \Delta\chi^2_{MINOS}$$

Combined Analysis

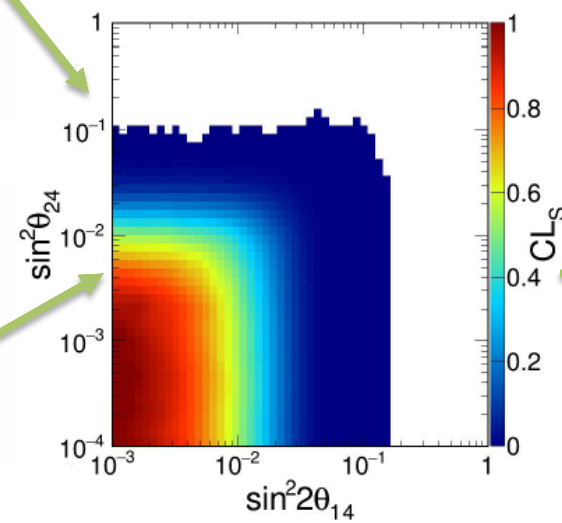
Step 1: For each row of fixed Δm^2_{41}, compute the combined limit in the appearance parameter space

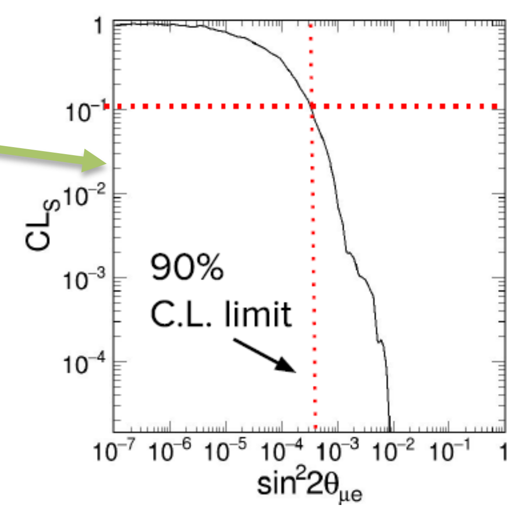




Step 2: For each fixed Δm^2_{41}, calculate CL_s for all combinations in the 2D space ($\sin^2 2\theta_{14}$, $\sin^2\theta_{24}$)

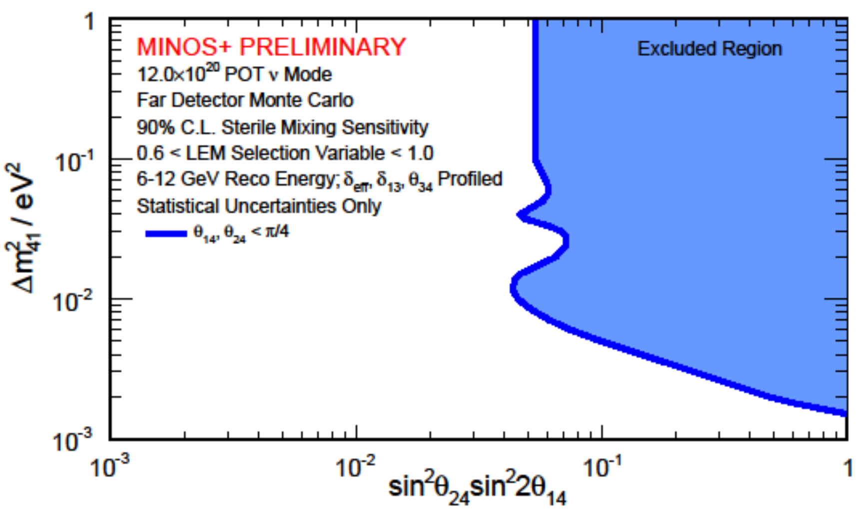
Step 3: Convert CL_s from a surface in the 2D space ($\sin^2 2\theta_{14}$, $\sin^2\theta_{24}$) to a 1D space in $\sin^2 2\theta_{\mu e}$.

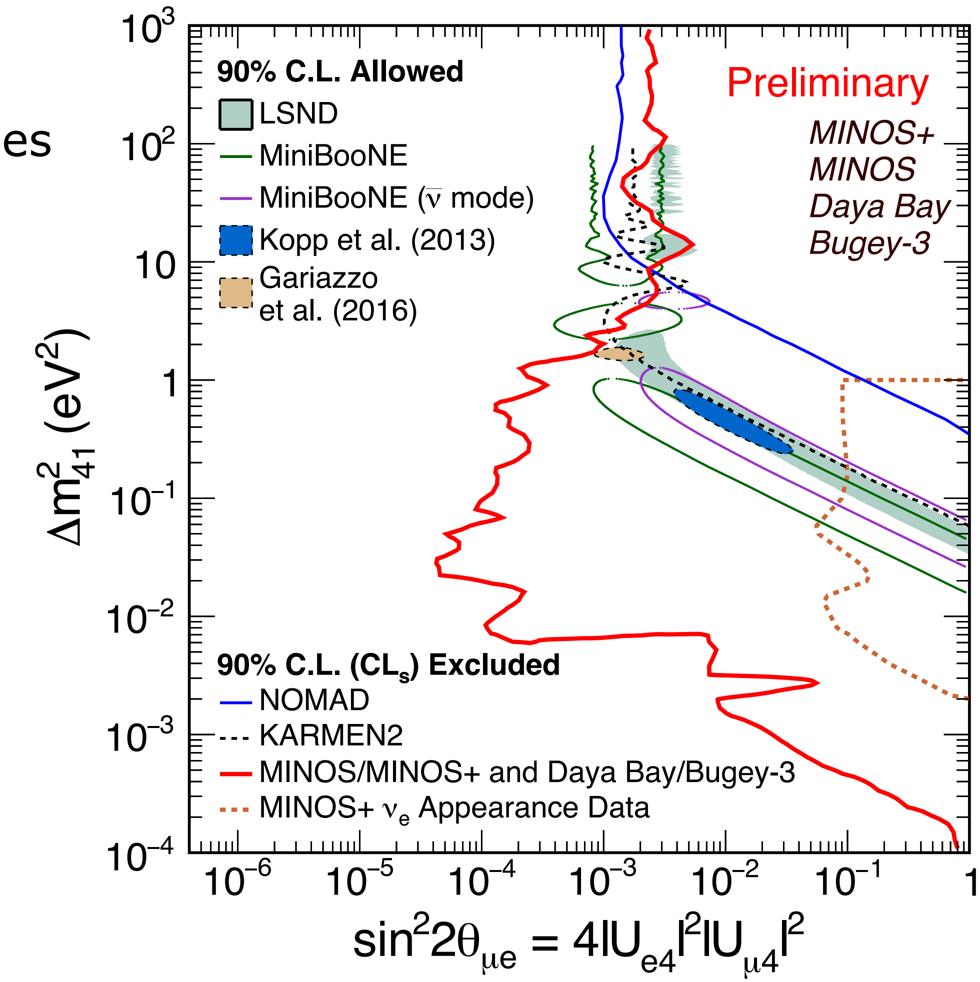




Electron Neutrino Appearance

- $\nu_\mu \rightarrow \nu_e$ appearance channel provides an independent exclusion contour.





Muon Antineutrino Sensitivity

- In the MINOS data, can select anti-ν_μ CC interactions from:
  - ➢ The antineutrino component of the neutrino-mode beam.
  - ➢ The antineutrino beam.