Modeling neutrino-nucleus interaction at intermediate energies

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The 19th International Workshop on Neutrinos from Accelators (NuFact17), Uppsala, Sweden, 25-30 September, 2017
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I Introduction

II QE (mean-field vs plane waves)

III Low-excitation energies: CRPA

IV 2p2h processes

IV Single-pion production

VI Conclusions
What we know from \((e,e')\)

Figures by T. Van Cuyck
What we know from \((e,e')\):

**QE (SuSAv2):**
RGJ et al., PRC 90, 035501 (2014)

**MEC:** Megias et al., PRD 91, 073004 (2015)

**1pion:** RGJ et al., JPS Conf.Proc. 12, 010047 (2016); PRD 95, 113007 (2017)
Quasielastic scattering
Impulse approximation

\[ J_{\text{had}}^{\mu} = \langle N, A - 1 | \hat{O}_{\text{many-body}}^{\mu} | A \rangle \]

Impulse Approximation

\[ J_{\text{had}}^{\mu} = \sum_{i}^{A} \int d\mathbf{r} \bar{\psi}_{F}(\mathbf{r}) \hat{O}_{\text{one-body}}^{\mu} \psi_{B}(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}} \]

where \[ \hat{O}_{\text{one-body}}^{\mu} = F_{1}\gamma^{\mu} + i \frac{F_{2}}{2M_{N}} \sigma^{\mu\alpha} Q_{\alpha} \]
\[ J_{\text{had}}^\mu = \langle N, A - 1 | \hat{O}_{\text{many-body}}^\mu | A \rangle \]

**Impulse Approximation**

\[ J_{\text{had}}^\mu = \sum_i^A \int \text{d}r \overline{\Psi}_F(r) \hat{O}_{\text{one-body}}^\mu \Psi_B(r) e^{i \mathbf{q} \cdot \mathbf{r}} \]

where

\[ \hat{O}_{\text{one-body}}^\mu = F_1 \gamma^\mu + i \frac{F_2}{2M_N} \sigma^{\mu\alpha} Q_\alpha \]
I) Ground state nucleus is a shell model:

- The mean field and wave functions are calculated with a Hartree-Fock (HF) approximation using an effective Skyrme NN force (SkE2).
- Binding energies, Fermi motion, nuclear structure.

II) Continuum wave functions are calculated using the same mean-field potential:

- The residual nucleus influence the knocked out nucleon (distorted waves).
Mean-field vs plane waves

$^{12}$C(e,e$'$)


Green line: Relativistic Fermi Gas with Pauli blocking [Amaro et al., PRC 71, 065501 (2005); RGJ et al., PRC 90, 035501 (2014)]
Relativistic mean-field (RMF) model:

✔ Same philosophy: mean-field wave functions

✔ Completely different approach: fully relativistic (Dirac vs Schrödinger)

✔ Similar results: excellent agreement with QE data

✔ Same conclusions: mean-field wave functions in both initial and final nucleon are essential
Mean-field vs plane waves

Intermediate energies (typical QE regime)
Mean-field vs plane waves

RPWIA: Scattered nucleon wf is described as a Dirac plane wave.

\[ [-i \mathbf{\alpha} \cdot \nabla + V(r) + \beta(M + S(r))] \Psi_i(r) = E_i \Psi_i(r) \]

\[ J_{\text{had}}^\mu = \sum_i \int d\mathbf{r} \bar{\Psi}_F(r) \hat{\sigma}_{\text{one-body}}^{\mu} \Psi_B(r) e^{i q \cdot r} \]

Intermediate energies (typical QE regime)

440 MeV, 60°

4q ~ 401

560 MeV, 36°

q ~ 332

680 MeV, 36°

q ~ 402

961 MeV, 37.5°

q ~ 585
**Mean-field vs plane waves**

**RPWIA:** Scattered nucleon wf is described as a Dirac plane wave.

**RMF-FSI:** Scattered nucleon wf is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon wf.

\[
[i \alpha \cdot \nabla + V(r) + \beta (M + S(r))] \Psi_i(r) = E_i \Psi_i(r)
\]

\[
J_{\text{had}}^\mu = \sum_i \int dr \bar{\Psi}_F(r) \hat{G}_{\text{one-body}}^\mu \Psi_B(r) e^{iq \cdot r}
\]

Intermediate energies (typical QE regime)

- **440 MeV, 60°**
  - QE (RMF)
  - QE (RPWIA)
  - QE (RFG)

- **560 MeV, 36°**
  - q ~ 332

- **680 MeV, 36°**
  - q ~ 401

- **961 MeV, 37.5°**
  - q ~ 585
Mean-field vs plane waves

**RPWIA**: Scattered nucleon wf is described as a Dirac plane wave.

\[ [-i \alpha \cdot \nabla + V(r) + \beta(M + S(r))] \Psi_i(r) = E_i \Psi_i(r) \]

\[ J_{\text{had}}^\mu = \sum_i^A \int \! dr \, \Psi_F(r) \hat{\mathcal{G}}_{\text{one-body}}^{\mu} \Psi_B(r) e^{i \mathbf{q} \cdot \mathbf{r}} \]

**RMF-FSI**: Scattered nucleon wf is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon wf.

Low energies

\[ \omega \approx 118 \]

\[ \omega \approx 297 \]

High energies

\[ \omega \approx 1317 \]

\[ \omega \approx 2235 \]
Long-range correlations: CRPA model

+ Long-range correlations between the nucleons are introduce through a continuum Random Phase Approximation (CRPA).

+ RPA equations are solved using a Green's function approach.

\[
\Pi^{(RPA)}(x_1, x_2; \omega) = \Pi^{(0)}(x_1, x_2; \omega) + \frac{1}{\hbar} \int dx \int dx' \Pi^{(0)}(x_1, x; \omega) \tilde{V}(x, x') \Pi^{(RPA)}(x', x_2; \omega)
\]

Excitations are obtained as linear combinations of different particle-hole configurations.

\[
|\Psi_{RPA}\rangle = \sum_c \left\{ X_{(\Psi, C)} \left| p^{-1} h \right\rangle - Y_{(\Psi, C)} \left| h^{-1} p \right\rangle \right\}
\]
Long-range correlations: CRPA model

\[ ^{12}\text{C}(\text{e}, \text{e}^{'}) \]

**HF (blue) vs CRPA (red)**

[V. Pandey PhD Thesis; PRC 92, 024606 (2015)]
Low-energy contributions in flux-folded XS

MicroBooNE flux-folded cross section.
Two-nucleon knockout processes

In our approach, two mechanisms give rise to the emission of two nucleons:

**Short-range correlations**

Log(Momentum distribution)

- Mean-field
- Fat tail: SRC!

**Meson-exchange currents**

![Diagram](image)

The same **mean-field** model is used to describe the bound and scattered nucleons:

![Diagram](image)
Short range correlations

\[
|\Psi\rangle = \frac{1}{\sqrt{N}} \hat{G} |\Phi\rangle \quad \text{with} \quad \hat{G} \approx \hat{S} \left( \prod_{i<j}^A \left[ 1 + \hat{l}(i,j) \right] \right)
\]

\[
\hat{l}(i,j) = -g_c(r_{ij}) + f_{\sigma\tau}(r_{ij}) (\vec{\sigma}_i \cdot \vec{\sigma}_j) (\vec{\tau}_i \cdot \vec{\tau}_j) + f_{t\tau}(r_{ij}) \hat{S}_{ij} (\vec{\tau}_i \cdot \vec{\tau}_j),
\]

The complexity induced by correlations is shifted from the wave functions to the operators.

[Graphs showing functions of r_{12} and k_{12}]

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Short range correlations

SRC affect the 1p1h and the 2p2h responses

**Meson-exchange currents**

Van Cuyck et al., PRC 95, 054611 (2017)

1p1h contributions (vector current)

2p2h contributions (vector current)

Axial current (1p1h and 2p2h)
Meson-exchange currents

Good agreement with other predictions [Amaro et al., NPA578, 365 (1994)]:

1p1h and 2p2h contributions to the longitudinal and transverse responses:
Meson-exchange currents

In spite of the complexity and ambiguities of the calculation, we find good agreement with other predictions (Amaro et al.): Exclusive, as well.

1p1h and 2p2h contributions to the longitudinal and transverse responses:

Figure 4.5: The $^{12}\text{C}(\nu_\mu, \mu^-N_aN_b)$ cross section ($N_a = p$, $N_b = p'$, n) at $\nu_\mu = 750$ MeV, $\epsilon_\mu = 550$ MeV, $\theta_\mu = 15^\circ$ and $T_p = 50$ MeV for in-plane kinematics. Left with SRCs, right with MECs, the bottom plot shows the ($\theta_a, \theta_b$) regions with $P_{12} < 300$ MeV/c.
Flux folded xs: MiniBooNE & T2K

MiniBooNE
(QE-like: QE+2p2h)

T2K
(inclusive: QE, 2p2h, pions, DIS)
Single-Pion Production on the nucleon

González-Jiménez et al., PRD 95, 113007 (2017)
Pion production is more important for DUNE

Electroweak single-pion production
Low-energy model

Resonances:
P33(1232), D13(1520), S11(1535), P11(1440)

ChPT background:

Low-energy model for pion-production on the nucleon:
ChPT background + resonances
The Problem

Unphysical predictions at large invariant masses.

Figure: The model overshoots inclusive electron-proton scattering data.
The Problem

**W values? We don't know...**

- Fermi motion
- Flux-folding

Therefore, we need reliable predictions in:

- the **resonance region** $W < 2$ GeV,
- the **high-energy** energy region $W > 2$ GeV
Regge approach for the vector amplitudes.

We use the approach of Guidal, Laget, and Vanderhaeghen [NPA627, 645 (1997)], originally developed for pion photoproduction ($Q^2 = 0$):

1) Feynman *meson-exchange diagrams* are reggeized.

\[ \frac{1}{t - m_{\pi}^2} \]

The pion propagator is replaced by the Regge trajectory of the pion family

\[ \mathcal{P}_\pi(t, s) = -\alpha'_\pi \varphi_\pi(t) \Gamma[-\alpha_\pi(t)](\alpha'_\pi s)^{\alpha_\pi(t)} \]
Regge approach for the vector amplitudes.

We use the approach of Guidal, Laget, and Vanderhaeghen [NPA627, 645 (1997)], originally developed for pion photoproduction ($Q^2 = 0$):

1) Feynman **meson-exchange diagrams** are reggeized.

2) s-channel, u-channel, and contact term diagrams are included to keep Conservation of Vector Current.

\[
\frac{1}{t - m_{\pi}^2}
\]

The pion propagator is replace by the Regge trajectory of the pion family

\[
\mathcal{P}_\pi(t, s) = -\alpha'_\pi \phi_\pi(t) \Gamma[-\alpha_\pi(t)](\alpha'_\pi s)^{\alpha_\pi(t)}
\]
**Figure:** High-energy model (red lines), low-energy model (blue lines) and electron-induced single-pion production data.
Regge approach for the axial amplitudes.

We need meson exchange diagrams to apply the reggeization procedure of the current.

Effective rho-exchange diagrams. This allows us to consider the rho-exchange as the main Regge trajectory in the axial current.

\[
\mathcal{O}_{CT_{\rho}}^{\mu} = i \mathcal{I} \frac{m_{\rho}^2}{m_{\rho}^2 - t} F_{A_{\rho,\pi}}(Q^2) \frac{1}{\sqrt{2} f_{\pi}} 
\times \left( \gamma^{\mu} + i \frac{\kappa_{\rho}}{2M} \sigma_{\mu\nu} K_{t,\nu} \right). 
\]

We consider \( \kappa_{\rho} = 0 \) so that the low-energy model amplitude is recovered.

The propagator of the rho is replaced by the Regge trajectory of the rho family:

\[
\mathcal{P}_{\rho}(t, s) = -\alpha_{\rho}' \varphi_{\rho}(t) \Gamma[1 - \alpha_{\rho}(t)](\alpha_{\rho}' s)^{\alpha_{\rho}(t)-1}
\]
High-energy model

Regge approach for the axial amplitudes.

We need meson exchange diagrams to apply the reggeization procedure of the current.

Effective rho-exchange diagrams. This allows us to consider the rho-exchange as the main Regge trajectory in the axial current.

\[ \mathcal{O}_{CT}^{\mu} = i \mathcal{I} \frac{m_\rho^2}{m_\rho^2 - t} F_{A\rho\pi}(Q^2) \frac{1}{\sqrt{2} f_\pi} \times \left( \gamma^\mu + i \frac{\kappa_\rho}{2M} \sigma^{\mu\nu} K_{t,\nu} \right). \]

We consider \( \kappa_\rho = 0 \) so that the low-energy model amplitude is recovered.
“Reggeizing” the ChPT background:

\[ \mathcal{O}_{ReChi,V}^{\mu} = \mathcal{O}_{ChPT,V}^{\mu} \mathcal{P}_\pi(t, s)(t - m_\pi^2) \]

**high-energy model**: ReChi (from Reggeized ChPT background)

**low-energy model** (only the ChPT background)

The pion propagator is replaced by the **Regge propagator** of the pion trajectory.

\[ \mathcal{O}_{ReChi,A}^{\mu} = \mathcal{O}_{ChPT,A}^{\mu} \mathcal{P}_\rho(t, s)(t - m_\rho^2) \]
FIG. 21. (Color online) Different model predictions for the differential cross section $d\sigma/(dQ^2dW)$, for the channel $p(\nu_\mu, \mu^- \pi^+)p$. The incoming neutrino energy is fixed to $E_\nu = 10$ GeV.
Hybrid model: results

$W < 1.4 \text{ GeV}$

$\nu_\mu + p \rightarrow \mu^- + \pi^+ + p$

$\sigma \left(10^{-39} \text{ cm}^2\right)$

$E_\nu (\text{GeV})$

$\nu_\mu + n \rightarrow \mu^- + \pi^+ + n$

$\nu_\mu + n \rightarrow \mu^- + \pi^0 + p$

$\nu_\mu + n \rightarrow \mu^- + \pi^0 + p$

No cut in $W$

$\nu_\mu + p \rightarrow \mu^- + \pi^+ + p$

$\sigma \left(10^{-39} \text{ cm}^2\right)$

$E_\nu (\text{GeV})$

$\nu_\mu + n \rightarrow \mu^- + \pi^+ + n$

$\nu_\mu + n \rightarrow \mu^- + \pi^0 + n$

$\nu_\mu + n \rightarrow \mu^- + \pi^0 + p$

$\sigma \left(10^{-39} \text{ cm}^2\right)$
Electroweak one-pion production on nuclei
**Relativistic mean field model**

**Relativistic Impulse Approximation**

Plane waves (for the moment...)

\[ J_{\text{had}}^\mu = \sum_i \int \dd r \overline{\psi}_F(r) \phi_i^*(r) \hat{\Omega}_{\text{one-body}}^\mu(r) \psi_B(r) e^{i q \cdot r} \]

8-fold differential cross section:

\[
\frac{d^8 \sigma}{d\varepsilon d\Omega_f dE_\pi d\Omega_{\pi} d\Omega_N} = \frac{m_i m_f}{(2\pi)^8} \frac{M_N p_N k_\pi}{E_N f_{\text{rec}}} \frac{k_f}{\varepsilon} \sum_{fi} |M_{fi}|^2
\]

Computationally very demanding
MiniBooNE neutrino CC 1pion+.  

MINERvA antineutrino CC 1pion0.  

MiniBooNE neutrino CC 1pion+. 

MINERvA antineutrino CC 1pion0.
Conclusions

✔ **QE scattering:** Mean-field wave functions in both bound and scattered nucleon are important: long tails in inclusive cross sections, redistribution of the strength, position of the peak.

✔ **2p2h** is induced by two mechanisms, **SRC and MEC**. Preferably, mean-field wave functions.

✔ **Single-pion production:** Low-energy models should not be used in high-W regions. We propose to combine the low-energy model with a Regge-based approach: **Hybrid model**.

Future: Lot of work to do...
The end...

Thanks for your attention
Backup slides
Regularization of the residual interaction:

\[ V(Q^2) \rightarrow V(Q^2 = 0) \frac{1}{(1 + \frac{Q^2}{\Lambda^2})^2} \]
Relativistic corrections at higher energies (J. Jeschonnek and T. Donnelly, PRC57, 2438 (1998)):

**Shift:**

\[ \lambda \rightarrow \lambda(\lambda + 1) \]

\[ \lambda = \frac{\omega}{2M_N} \]

- The outgoing nucleon obtains the correct relativistic momentum
- Shifts the QE peak to the right relativistic position

**Boost:**

\[ R_{CC}^N(q, \omega) \rightarrow \frac{q^2}{q^2 - \omega^2} R_{CC}^N(q, \omega), \]

\[ R_{LL}^A(q, \omega) \rightarrow \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R_{LL}^A(q, \omega), \]

\[ R_{TV}^N(q, \omega) \rightarrow \frac{q^2 - \omega^2}{q^2} R_{TV}^N(q, \omega), \]

\[ R_{TV}^A(q, \omega) \rightarrow \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R_{TV}^A(q, \omega), \]

\[ R_{TV}^{2A}(q, \omega) \rightarrow \sqrt{\frac{q^2 - \omega^2}{q^2}} \sqrt{1 + \frac{q^2 - \omega^2}{4m^2}} R_{TV}^{2A}(q, \omega). \]
Folding procedure

- A limitation of RPA formalism at lower energies:
  - energy position of the giant resonances is generally well predicted
  - width is underestimated
  - height is overestimated

\[ R'(q,\omega') = \int_{-\infty}^{\infty} d\omega R(q,\omega)L(\omega,\omega') \]

\[ L(\omega,\omega') = \frac{1}{2\pi} \left[ \frac{\Gamma}{(\omega - \omega')^2 + (\Gamma/2)^2} \right] \]

\[ \Gamma = 3 \text{ MeV} \]
Relativistic mean-field model

RMF model provides a microscopic description of the ground state of finite nuclei which is consistent with Quantum Mechanic, Special Relativity and symmetries of strong interaction.

The starting point is a Lorentz covariant Lagrangian density

\[ \mathcal{L} = \overline{\Psi} \left( i \gamma_\mu \partial^\mu - M \right) \Psi + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - U(\sigma) \]

- \[ \frac{1}{4} \Omega_\mu \nu \Omega^{\mu \nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} R_\mu \nu R^{\mu \nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} F_\mu \nu F^{\mu \nu} \]

- \[ g_\sigma \overline{\Psi} \sigma \Psi - g_\omega \overline{\Psi} \gamma_\mu \omega^\mu \Psi - g_\rho \overline{\Psi} \gamma_\mu \tau \rho^\mu \Psi - g_e \frac{1 + \tau_3}{2} \overline{\Psi} \gamma_\mu A^\mu \Psi . \]

where

\[ \Omega^{\mu \nu} = \partial^\mu \omega_\nu - \partial^\nu \omega_\mu , \]

\[ R^{\mu \nu} = \partial^\mu \rho_\nu - \partial^\nu \rho_\mu , \]

\[ F^{\mu \nu} = \partial^\mu A_\nu - \partial^\nu A_\mu . \]

\[ U(\sigma) = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 \]

**Main approximations:**

1) Mean-field approximation:

\[ \omega_\mu \rightarrow \langle \omega_\mu \rangle \quad \sigma \rightarrow \langle \sigma \rangle \quad \rho_\mu \rightarrow \langle \rho_\mu \rangle \]

2) Static limit:

\[ \partial^0 \omega_0 = \partial^0 \rho_0 = \partial^0 \sigma = 0 \quad \omega_\mu = \delta_\mu_0 \omega_0 , \quad \rho_\mu = \delta_\mu_0 \rho_0 \]

3) Spherical symmetry for finite nuclei:

\[ \omega_0 = \omega_0(r) \quad \rho_0 = \rho_0(r) \quad \sigma = \sigma(r) \]
Relativistic mean-field model

Dirac equation for nucleons (eq. of motion for the barionic fields):

\[ [ -i \mathbf{\alpha} \cdot \nabla + V(r) + \beta (M + S(r)) ] \Psi_i(r) = E_i \Psi_i(r) \]

where the scalar (S) and vector (V) potential are given by:

\[ S(r) = g_\sigma \sigma(r) \],
\[ V(r) = g_\omega \omega^0(r) + g_\rho \tau_3 \rho_3^0(r) + e \frac{1 + \tau_3}{2} A^0(r) \]

Eqs. of motion for the mesons and the photon:

\[ \begin{align*}
[ -\nabla^2 + m_\sigma^2 ] \sigma(r) &= -g_\sigma \rho_s(r) - g_2 \sigma^2(r) - g_3 \sigma^3(r), \\
[ -\nabla^2 + m_\omega^2 ] \omega^0(r) &= -g_\omega \rho_B(r), \\
[ -\nabla^2 + m_\rho^2 ] \rho_3^0(r) &= -g_\rho \rho_\rho(r), \\
-\nabla^2 A^0 &= e \rho_c,
\end{align*} \]

Solution of the couple equations for the fields in a self-consistent way.

Current densities

\[ \begin{align*}
\rho_s(r) &= \sum_i \overline{\Psi}_i(r) \Psi_i(r), \\
\rho_B(r) &= \sum_i \Psi_i(r) \Psi_i(r), \\
\rho_\rho(r) &= \sum_i \Psi_i(r) \tau_3 \Psi_i(r), \\
\rho_c(r) &= \sum_i \Psi_i(r) \frac{1 + \tau_3}{2} \Psi_i(r)
\end{align*} \]
Relativistic mean-field model

In general, the parameters are fit to reproduce some general properties of some closed shell spherical nuclei and nuclear matter.

Parameters for the NLSH model (fitted to the mean charge radius, binding energy and neutron radius of the $^{16}\text{O}$, $^{40}\text{Ca}$, $^{90}\text{Zr}$, $^{116}\text{Sr}$, $^{124}\text{Sn}$ and $^{208}\text{Pb}$.

<table>
<thead>
<tr>
<th>$M_N$</th>
<th>$m_\sigma$</th>
<th>$m_\omega$</th>
<th>$m_\rho$</th>
<th>$g_\sigma$</th>
<th>$g_\omega$</th>
<th>$g_\rho$</th>
<th>$g_2$</th>
<th>$g_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>939.0</td>
<td>526.059</td>
<td>783.0</td>
<td>763.0</td>
<td>10.444</td>
<td>12.945</td>
<td>4.3830</td>
<td>-6.9099</td>
<td>-15.8337</td>
</tr>
</tbody>
</table>

\[
-i\alpha \cdot \nabla + V(r) + \beta(M + S(r))\Psi_i(r) = E_i\Psi_i(r)
\]

\[
\Psi_{k}^{m_j}(r) = \begin{pmatrix}
g_k(r)\varphi_k^{m_j}(\Omega_T) \\
if_k(r)\varphi_{-k}^{m_j}(\Omega_T)
\end{pmatrix},
\]

\[
\varphi_k^{m_j}(\Omega_T) = \sum_{m_\ell s} \langle \ell m_\ell \frac{1}{2}s | j m_j \rangle Y_{\ell m_\ell}(\Omega_T) \chi^s
\]
Back slides: isospin coefficients and resonances parameters

Table: Isospin coefficients for the CC reaction.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\Delta P$</th>
<th>$C\Delta P$</th>
<th>$NP$</th>
<th>$CNP$</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \rightarrow \pi^+ + p$</td>
<td>$\sqrt{3}/2$</td>
<td>$\sqrt{1}/6$</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$n \rightarrow \pi^0 + p$</td>
<td>$-\sqrt{1}/3$</td>
<td>$\sqrt{1}/3$</td>
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<td>0</td>
<td>$-1$</td>
</tr>
<tr>
<td>$n \rightarrow \pi^- + n$</td>
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<td>$\sqrt{1}/6$</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>$-\sqrt{1}/3$</td>
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<td>1</td>
<td>0</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

Table: Isospin coefficients for the neutral current (EM and WNC) reactions.

<table>
<thead>
<tr>
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<th>$\Delta P$</th>
<th>$C\Delta P$</th>
<th>$NP$</th>
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<td>$-\sqrt{1}/2$</td>
<td>$-\sqrt{1}/2$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table: quantum numbers and other parameters of the nucleon resonances.

<table>
<thead>
<tr>
<th></th>
<th>$I$</th>
<th>$S$</th>
<th>$P$</th>
<th>$M_R$</th>
<th>$\pi N$-br</th>
<th>$\Gamma_{width}^{exp}$</th>
<th>$f_{\pi NR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{33}$</td>
<td>$3/2$</td>
<td>$3/2$</td>
<td>+</td>
<td>$1232$</td>
<td>$100%$</td>
<td>120</td>
<td>2.18</td>
</tr>
<tr>
<td>$D_{13}$</td>
<td>$1/2$</td>
<td>$3/2$</td>
<td>−</td>
<td>$1515$</td>
<td>$60%$</td>
<td>115</td>
<td>1.62</td>
</tr>
<tr>
<td>$P_{11}$</td>
<td>$1/2$</td>
<td>$1/2$</td>
<td>+</td>
<td>$1430$</td>
<td>$65%$</td>
<td>350</td>
<td>0.391</td>
</tr>
<tr>
<td>$S_{11}$</td>
<td>$1/2$</td>
<td>$1/2$</td>
<td>−</td>
<td>$1535$</td>
<td>$45%$</td>
<td>150</td>
<td>0.16</td>
</tr>
</tbody>
</table>

R. González-Jiménez    Ghent University
Medium modifications of the Delta

**Delta propagator:**

\[
S_{\Delta, \alpha \beta} = \frac{-(K_{\Delta} + M_{\Delta})}{K_{\Delta}^2 - M_N^2 + iM_{\Delta} \Gamma_{\text{width}}} \left( g_{\alpha \beta} - \frac{1}{3} \gamma_{\alpha} \gamma_{\beta} - \frac{2}{3M_{\Delta}^2} K_{\Delta, \alpha} K_{\Delta, \beta} - \frac{2}{3M_{\Delta}} (\gamma_{\alpha} K_{\Delta, \beta} - K_{\Delta, \alpha} \gamma_{\beta}) \right)
\]

with the energy dependent Delta width:

\[
\Gamma_{\text{width}}(W) = \frac{1}{12\pi} \frac{(f_{\Delta N})^2}{m_\pi^2 W} (\rho_{\pi, cm})^3 (M + E_{N, cm})
\]

\[
\Gamma_{\text{width}}^{\text{free}} \rightarrow \Gamma_{\text{width}}^{\text{in-medium}} = \Gamma_{\text{Pauli}} - 2\Im(\Sigma_{\Delta}), \quad M_{\Delta}^{\text{free}} \rightarrow M_{\Delta}^{\text{in-medium}} = M_{\Delta}^{\text{free}} + \Im(\Sigma_{\Delta}).
\]

\[\Gamma_{\text{Pauli}}: \text{some nucleons from } \Delta \text{-decay are Pauli blocked (the } \Delta \text{-decay width decreases).}\]

\[\Im(\Sigma_{\Delta}) \quad \text{and } \Re(\Sigma_{\Delta}) \text{ is given in terms of the nuclear density } \rho:\]

\[
\begin{align*}
-\Im(\Sigma_{\Delta}) &= C_{QE} (\rho/\rho_0)^\alpha + C_{A2} (\rho/\rho_0)^\beta + C_{A3} (\rho/\rho_0)^\gamma, \\
\Re(\Sigma_{\Delta}) &= 40 \text{ MeV} (\rho/\rho_0).
\end{align*}
\]

We modify the free $\Delta_\pi N$-decay constant ($f_{\Delta \pi N}$) to take into account the $E$-dependent medium modification of the $\Delta$ width:

\[
f_{\Delta \pi N}^{\text{in-medium}}(W) = f_{\Delta \pi N} \sqrt{\frac{\Gamma_{\text{Pauli}} + 2C_{QE} (\rho/\rho_0)^\alpha}{\Gamma_{\text{width}}^{\text{free}}}}
\]
Medium modifications of the Delta

\[ -\mathcal{G}(\Sigma_\Delta) = C_{QE} \left( \rho / \rho_0 \right)^\alpha + C_{A2} \left( \rho / \rho_0 \right)^\beta + C_{A3} \left( \rho / \rho_0 \right)^\gamma \]

Each contribution corresponds to a different process:

- **QE** \( \Rightarrow \Delta N \rightarrow \pi NN \) (still one pion in the final state)
- **A2** \( \Rightarrow \Delta N \rightarrow NN \) (no pions in the final state)
- **A3** \( \Rightarrow \Delta NN \rightarrow NNN \) (no pions in the final state)

We modify the free Delta decay constant to take into account the E-dependent medium modification of the Delta-width

\[ \Gamma_{\Delta N}^\alpha = \frac{f_{\pi N\Delta}}{m_\pi} P_\pi^\alpha \]

\[ f_{\Delta N}^{\text{in-medium}}(W) = f_{\Delta N} \sqrt{\frac{\Gamma_{\text{Pauli}} + 2 C_{QE} \left( \rho / \rho_0 \right)^\alpha}{\Gamma_{\text{free width}}}} \]

Interferences

\[ J^\nu = \langle J_{\Delta P}^\nu \rangle + \langle J_{CT,V}^\nu \rangle + \langle J_{CT,A}^\nu \rangle + \langle J_{NP}^\nu \rangle + \langle J_{CNP}^\nu \rangle + \langle J_{PF}^\nu \rangle + \langle J_{PP}^\nu \rangle \]

Physical Review D 93, 014016 (2016)

Watson’s theorem and the \( N\Delta(1232) \) axial transition

L. Alvarez-Ruso, E. Hernández, J. Nieves, and M. J. Vicente Vacas

We present a new determination of the \( N\Delta \) axial form factors from neutrino induced pion production data. For this purpose, the model of Hernandez \textit{et al.} [Phys. Rev. D 76, 033005 (2007)] is improved by partially restoring unitarity. This is accomplished by imposing Watson’s theorem on the dominant vector and axial multipoles. As a consequence, a larger \( C_5^A(0) \), in good agreement with the prediction from the off-diagonal Goldberger-Treiman relation, is now obtained.
Other results

FIG. 5. T2K flux-folded inclusive CC double-differential cross sections per target nucleon on $^{12}$C plotted as a function of muon momentum $p_\mu$, for different bins of $\cos \theta_\mu$. CRPA (solid curves) and HF (dashed-curves) are compared with T2K measurements of [12].
The Problem

The pathologies come from the **resonances** and **background terms**

Electron scattering

WNC neutrino scattering
Why does this happen?

Cross channels:

\[ \mathcal{A}(t, s) = \sum_{\ell} (2\ell + 1) \, A_\ell(t) \, P_\ell(z_t) \]

\[ P_\ell(z_t) \xrightarrow{s \to \infty} (2s)^\ell \]

Direct channels:

\[ \mathcal{A}(s, t) = \sum_{\ell} (2\ell + 1) \, A_\ell(s) \, P_\ell(z_s) \]

\[ A_\ell(s) \sim \left( \frac{s - 4m^2}{2} \right)^\ell \]

\[ z_t \equiv \cos \theta_t = 1 + \frac{2s}{t - 4m^2} \]

\[ z_s \equiv \cos \theta_s = 1 + \frac{2t}{s - 4m^2} \]

Behavior at threshold (barrier factor).
Feynman diagrams provide the right behavior at threshold but not at high s.
Based on unitarity, causality and crossing symmetry, Regge Theory predicts the following **high energy** \((s \to \infty)\) **behavior** for the invariant amplitude:

\[
A(s,t) \sim \beta(t) \ s^{\alpha(t)}
\]

Regge theory does not predict the **t-dependence** of the amplitude.

For that, one needs a model.

\[\alpha(t): \text{Families or Regge trajectories}\]
Regge Theory

\[ A(t, s) = \sum_{\ell} (2\ell + 1) A_\ell(t) P_\ell(z_t) \]

\[ z_t \equiv \cos \theta_t = 1 + \frac{2s}{t - 4m^2} \]

\[ L=0 \quad + \quad L=1 \quad + \quad ... \]

\[ \frac{\chi^2}{m^2 - t} P_\ell(z_t) \xrightarrow{s \to \infty} (2s)^\ell \]
Regge Theory

\[
\mathcal{M}(s, t) = -\frac{1}{2i} \oint_{C_1} d\lambda \frac{2\lambda + 1}{\sin(\pi\lambda)} \mathcal{M}_\lambda(t) P_\lambda(-\cos \theta_t)
\]

\[
\mathcal{M}_{\text{Regge}}^\zeta(s, t) = C \sum_i \left( \frac{s}{s_0} \right)^{\alpha_i^\zeta(t)} \frac{\beta_i^\zeta(t)}{\sin(\pi \alpha_i^\zeta(t))} \frac{1 + \zeta e^{-i\pi \alpha_i^\zeta(t)}}{2} \frac{1}{\Gamma\left(\alpha_i^\zeta(t) + 1\right)}.
\]
ReChi model: One free parameter in the boson-nucleon-nucleon vertex

\[ G_A(Q^2, s(u)) = g_A \left( 1 + \frac{Q^2}{\Lambda_{Apn}^* [s(u)]^2} \right)^{-2} \]

\[ \Lambda_{Apn}^*(s) = \Lambda_{Apn} + (\Lambda_A^A - \Lambda_{Apn}) \left( 1 - \frac{M^2}{s} \right) \]

\[ \Lambda_A^A = (7.20 \pm 0.90^{+1.32}_{-1.32}) \text{ GeV} !!! \]

NuWro: Based on DIS formalism and PYTHIA for hadronization.

Antineutrino cross section is \(~2\) the neutrino one:

\[ \bar{\nu} + uud \rightarrow \mu^+ + \bar{u}d + uud, \]

\[ \nu + uud \rightarrow \mu^- + \bar{u}d + uud. \]
Hybrid model

1) Regularizing the behavior of resonances (u- and s-channel contributions): we multiply the resonance amplitude by a dipole-Gaussian form factor

\[ F(s, u) = F(s) + F(u) - F(s)F(u) \]

\[ F(s) = \exp \left( \frac{-(s - M_R^2)^2}{\lambda_R^4} \right) \frac{\lambda_R^4}{(s - M_R^2)^2 + \lambda_R^4} \]

2) Gradually replacing the ChPT background by the High-energy (ReChi) model: we use a phenomenological transition function

\[ \bar{\Omega} = \cos^2 \phi(W) \mathcal{O}_{ChPT} + \sin^2 \phi(W) \mathcal{O}_{ReChi} \]

\[ \phi(W) = \frac{\pi}{2} \left( 1 - \frac{1}{1 + \exp \left( \frac{W - W_0}{L} \right)} \right), \quad W_0 = 1.7 \text{ GeV} \]

\[ L = 100 \text{ MeV} \]