Towards Model-Independent Tests of WIMP Dark Matter

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**Outline**

**Introduction**
- Standard WIMP searches
- Direct detection
- Indirect detection (at neutrino telescopes)

**Main focus**
- Non-relativistic effective theory of WIMP-nucleon interactions
- Predictions:
  - a) Overview of selected results
  - b) DAMA vs null results
  - c) WIMP capture and annihilation in the Earth
Weakly Interacting Massive Particles (WIMPs)

- Most studied candidate for dark matter, and a testable scenario!

**Expected mass:** $m_{\text{WIMP}} \sim 1 \text{ GeV} - 100 \text{ TeV}

**WIMP interactions:**

- Annihilation
- Scattering
- Production
WIMP detection strategies

- Direct detection
- Indirect detection
- Collider searches
WIMP detection strategies

- Direct detection
- Indirect detection
- Collider searches

WIMP → SM

Direct detection

WIMP → WIMP
WIMP detection strategies

- Direct detection
- Indirect detection
- Collider searches

WIMP  SM

?
Motivation and strategy:

Physical observable: rate of dark matter-nucleus scattering events in terrestrial detectors:

\[
\frac{dR}{dE_{nr}} = \frac{\rho_{dm}}{m_\chi m_T} \int_{|v|>v_{\text{min}}} d^3v |v| f(v) \frac{d\sigma}{dE_{nr}}
\]
Dark matter direct detection

- **Modulation:** the Earth’s orbit inclination induces an annual modulation in the rate of recoil events

\[ \mathcal{A}(E_-, E_+) = \frac{1}{E_+ - E_-} \frac{1}{2} \left[ \mathcal{R}(E_-, E_+) \bigg|_{\text{June 1st}} - \mathcal{R}(E_-, E_+) \bigg|_{\text{Dec 1st}} \right] \]

- **Kinematics:**
  a) For \( m_\chi \sim 100 \) GeV, one expects a flux of \( \sim 7 \times 10^4 \) cm\(^{-2}\) s\(^{-1}\)
  b) Expected recoil energy, \( E_R = (2\mu_T^2 v^2 / m_T) \cos^2 \theta \sim \mathcal{O}(10) \) keV
Local dark matter density in 5 steps

- Assume a mass model for the Milky Way: halo, stellar disk, bulge
- Calculate the observables: rotation curves, surface density, velocity dispersion of stars, weak lensing optical depth, etc...
- Compare predictions with astronomical observations: the Bayesian approach has proven to be a powerful tool for this
- Extract preferred regions in parameter space, e.g. credible regions
- Translate them into an estimate for the local dark matter density, e.g. posterior PDF
Local dark matter density: Bayesian analysis

Catena & Ullio 2010
Local dark matter velocity distribution in 5 + 3 steps

- Simplifying assumption: spherically symmetric galactic gravitational potential

- Use Eddington’s inversion formula to relate the local dark matter velocity distribution to the parameters of the assumed mass model

- From the posterior PDF of the model parameters, obtain the posterior PDF of local dark matter velocity distribution at sampled velocities
Local dark matter velocity distribution: Bayesian analysis

Bozorgnia, Catena and Schwetz 2014

The graph shows the distribution of dark matter velocities, with the velocity $u$ in km/s on the x-axis and the function $g_\chi(u)$ in units of $\text{GeV/cm}^3 \cdot (\text{s/km})^2$ on the y-axis. The plot includes a shaded region indicating the uncertainty range, with a peak at around $u = 300$ km/s.
**Standard paradigm**: spin-independent and spin-dependent dark matter-nucleon interactions

\[
\frac{d\sigma_T}{dE_{nr}} = \frac{m_T}{2\pi v^2} \frac{1}{(2j_X + 1)(2J + 1)} \sum_{\text{spins}} \left| \langle F \sum_{i=1}^{A} e^{-i\mathbf{q} \cdot \mathbf{r}_i} (\mathcal{H}_{SI} + \mathcal{H}_{SD}) | I \rangle \right|^2
\]
Spin-independent interaction $\mathcal{H}_{SI}$

- **Scalar/Scalar coupling:**
  \[ \mathcal{L}_{SS} = \frac{1}{\Lambda^3} \sum_q C_q^{SS} \bar{\chi} \chi m_q \bar{q} q \]

- **S-matrix element:**
  \[
  \langle f | i S | i \rangle = -i \bar{u}_\chi(p') u_\chi(p) \int d^4 x \ e^{i q x} \langle N' | \sum_q c_q \bar{q}(x) q(x) | N \rangle \\
  \simeq -i (2\pi)^4 \delta^4(q - k' + k) \xi^{\dagger}_\tau \xi_\chi \xi^{\dagger}_N (b_0 + b_1 \tau_3) \xi_N
  \]

- **Underlying non-relativistic Hamiltonian**
  \[
  \mathcal{H}_{SI} = \sum_{\tau=0,1} b_\tau \mathbb{1}_\chi \mathbb{1}_N t^\tau \equiv \sum_{\tau=0,1} c_1^{\dagger} \mathbb{1}_\chi N t^\tau
  \]
Spin-dependent interaction $\mathcal{H}_{SD}$

- **Axial-Vector/Axial-Vector:**
  \[ \mathcal{L}_{AA} = \frac{1}{\Lambda^2} \sum_q C_q^{AA} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu \gamma_5 q \]

- **S-matrix element:**
  \[ \langle f | iS | i \rangle = -i \bar{u}_X(p') \gamma_\mu \gamma_5 u_X(p) \int d^4x e^{iqx} \langle N' | \sum_q c_q \bar{q}(x) \gamma^\mu \gamma_5 q(x) | N \rangle \]
  \[ \simeq -i (2\pi)^4 \delta^4(q - k' + k) \xi_X^\dagger \sigma_X \xi_X \cdot \xi_N^\dagger (a_0 + a_1 \tau_3) \sigma_N \xi_N \]

- **Underlying non-relativistic Hamiltonian**
  \[ \mathcal{H}_{SD} = \sum_{\tau=0,1} a_\tau \sigma_X \cdot \sigma_N t^\tau \equiv \sum_{\tau=0,1} c_4^\tau \hat{S}_X \cdot \hat{S}_N t^\tau \]
Conflicting experimental results

Billard et al. 2014
Indirect Detection

- **WIMP searches at neutrino telescopes**

- They search for neutrinos produced by the annihilation of dark matter particles bound to the Sun/Earth
Indirect Detection

- **Physical observable:** flux of neutrinos produced by dark matter annihilation in the Sun/Earth

\[
\frac{d\Phi_\nu}{dE_\nu} = \frac{\Gamma_a}{4\pi D^2} \sum_f B^f_\chi \frac{dN^f_\nu}{dE_\nu}
\]

\[
\Gamma_a = \Gamma_a [\rho_{dm}, f(v), d\sigma/dE_{nr}]
\]

- More specifically, \(\Gamma_a\) depends on

\[
\frac{dC}{dV} = \int_0^\infty du \frac{\rho_{dm} \langle f(u) \rangle}{u} \sum_T n_T w^2 \Theta \int dE_{nr} \frac{d\sigma_T}{dE_{nr}}
\]
WIMP capture in the Earth

Catena 2016

\[ c_1^0 \neq 0 \]

\[ c \begin{align*} & 10^5 \\ & 10^7 \\ & 10^9 \\ & 10^{11} \end{align*} \]

\[ m_\chi \begin{align*} & 10^1 \\ & 10^2 \\ & 10^3 \end{align*} \]

\begin{align*} & 16^\text{O} \\ & 28^\text{Si} \\ & 24^\text{Mg} \\ & 56^\text{Fe} \\ & 40^\text{Ca} \\ & 31^\text{P} \\ & 23^\text{Na} \\ & 32^\text{S} \\ & 58^\text{Ni} \\ & 27^\text{Al} \\ & 52^\text{Cr} \end{align*} \]

Total
IceCube search for WIMP annihilations in the Earth

IceCube collaboration 2016

$\Gamma_A (s^{-1})$

IC86–I sensitivity for $\chi\chi \rightarrow W^+ W^- \ or \ \tau^+ \tau^-$
IC86–I upper limit for $\chi\chi \rightarrow W^+ W^- \ or \ \tau^+ \tau^-$
AMANDA upper limit for $\chi\chi \rightarrow W^+ W^- \ or \ \tau^+ \tau^-$

$m_\chi (GeV)$
Standard paradigm based upon SI and SD interactions

- Is it complete? No
- Is it favoured by observations? No

- **Conflicting data** and the **increase in sensitivity** due to already operating ton-scale and km$^3$ detectors motivate the exploration of more general approaches
Effective Theory (ET) of WIMP-nucleon interactions

Fitzpatrick et al. 2013

- **Separation of scales:** \(|q|/m_V \ll 1\), where \(m_V\) is the mediator mass

- **Basic symmetries:** Galilean and translational invariance

- **Four “degrees of freedom”:**
  - Consider the scattering \(\chi(p) + N(k) \rightarrow \chi(p') + N(k')\)
  - Momentum conservation \(\rightarrow M(p, k, q)\)
  - Galilean invariance \(\rightarrow M(v = p/m_\chi - k/m_N, q)\)
  - In general, \(M = M(v, q, S_\chi, S_N)\)
  - We therefore identify four basic operators

\[
i\hat{q} \quad \hat{v}^\perp = \hat{v} + \frac{\hat{q}}{2\mu_N} \quad \hat{S}_\chi \quad \hat{S}_N
\]
The most general Hamiltonian density is therefore a power series in \( \hat{q} \).
Each term is Galilean invariant and constructed from the four basic operators:

\[
\hat{H}(r) = \sum_{\tau=0,1} \sum_k c^\tau_k \hat{O}_k(r) t^\tau
\]

a) \( t^0 = 1, \ t^1 = \tau_3 \)

b) \( c^p_k = (c^0_k + c^1_k)/2 \) and \( c^n_k = (c^0_k - c^1_k)/2 \)
Dark matter-nucleon interaction operators

\[ \hat{O}_1 = \mathbb{1}_N \chi \]
\[ \hat{O}_3 = i \hat{S}_N \cdot \left( \frac{\hat{q}}{m_N} \times \hat{v} \right) \]
\[ \hat{O}_4 = \hat{S}_X \cdot \hat{S}_N \]
\[ \hat{O}_5 = i \hat{S}_X \cdot \left( \frac{\hat{q}}{m_N} \times \hat{v} \right) \]
\[ \hat{O}_6 = \left( \hat{S}_X \cdot \frac{\hat{q}}{m_N} \right) \left( \hat{S}_N \cdot \frac{\hat{q}}{m_N} \right) \]
\[ \hat{O}_7 = \hat{S}_N \cdot \hat{v} \]
\[ \hat{O}_8 = \hat{S}_X \cdot \hat{v} \]
\[ \hat{O}_9 = i \hat{S}_X \cdot \left( \hat{S}_N \times \frac{\hat{q}}{m_N} \right) \]
\[ \hat{O}_{10} = i \hat{S}_N \cdot \frac{\hat{q}}{m_N} \]
\[ \hat{O}_{11} = i \hat{S}_X \cdot \frac{\hat{q}}{m_N} \]
\[ \hat{O}_{12} = \hat{S}_X \cdot \left( \hat{S}_N \times \hat{v} \right) \]
\[ \hat{O}_{13} = i \left( \hat{S}_X \cdot \hat{v} \right) \left( \hat{S}_N \cdot \frac{\hat{q}}{m_N} \right) \]
\[ \hat{O}_{14} = i \left( \hat{S}_X \cdot \frac{\hat{q}}{m_N} \right) \left( \hat{S}_N \cdot \hat{v} \right) \]
\[ \hat{O}_{15} = - \left( \hat{S}_X \cdot \frac{\hat{q}}{m_N} \right) \left[ \left( \hat{S}_N \times \hat{v} \right) \cdot \frac{\hat{q}}{m_N} \right] \]
Dark matter-nucleus scattering cross-section:

\[ \frac{d\sigma_T}{dE_{nr}} \propto \sum_{\text{spins}} \left| \langle F | \sum_{i=1}^{A} \int dr \ e^{-i\mathbf{q} \cdot \mathbf{r}} \hat{H}_i(r) | I \rangle \right|^2 \]

In the ET framework, it depends on 28 coupling constants and 8 nuclear response functions.

Nuclear response functions for 16 elements in the Sun: R. Catena & B. Schwabe 2015
Direct detection

- Current experiments place limits on commonly neglected WIMP-nucleon interaction operators that are comparable with those on the strength of the SD interaction
- Destructive operator interference effects can weaken standard direct detection exclusion limits by up to one order of magnitude in the coupling constants
- The interpretation of direct detection experiments can be significantly biased if WIMPs interact via neglected interactions
- New ring-like features are expected in the angular distribution of nuclear recoil events

Indirect detection

- WIMP capture and annihilation in the Sun and Earth need to be revisited
DAMA confronts null searches

- Is there a linear combination of $\hat{O}_k$ such that DAMA can be reconciled with null searches?

- In the ET framework, this question can be reformulated in terms of intersection of ellipsoids
DAMA confronts null searches II

Catena, Ibarra and Wild 2016

\[
\begin{align*}
&c^T A_{\text{exp-1}}^{95\%-u.l.} (m_\chi) c < 1 \\
&\Rightarrow c^T A_{\text{exp-2}}^{95\%-u.l.} (m_\chi) c < 1 \\
&\Rightarrow c^T A_{\text{DAMA}[E_-,E_+]}^{n_\sigma} (m_\chi) c > 1
\end{align*}
\]
DAMA confronts null searches III

Catena, Ibarra and S. Wild 2016

DM with non-zero spin

\[ Q_{Na} = 0.3, \ Q_I = 0.09 \]

DM with zero spin

\[ Q_{Na} = 0.3, \ Q_I = 0.09 \]

DM with non-zero spin

\[ Q_{Na} = 0.4, \ Q_I = 0.05 \]

DM with zero spin

\[ Q_{Na} = 0.4, \ Q_I = 0.05 \]
WIMP annihilation in the Earth revisited

Catena 2016

\[ c_{11}^0 \neq 0 \]

\[ c_{12}^0 \neq 0 \]

\[ m_\chi \text{ [GeV]} \]

\[ c_{11} \text{, } c_{12} \text{, } m_{\text{V}} \]

- IC-79 Sun (W^+W^-/\tau^+\tau^-)
- SK 2015 (\tau^+\tau^-)
- IC-86 Earth (W^+W^-/\tau^+\tau^-)
- LUX 2013
Connection to Simplified Models for dark matter

Dent et al. 2015

<table>
<thead>
<tr>
<th>WIMP spin</th>
<th>Mediator spin</th>
<th>( \mathcal{L} ) terms</th>
<th>leading NR operator</th>
<th>Eqv. ( M_m )</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( h_1, g_1 )</td>
<td>( \mathcal{O}_1 )</td>
<td>13 TeV</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( h_2, g_1 )</td>
<td>( \mathcal{O}_{10} )</td>
<td>14 GeV</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>( h_4, g_4 )</td>
<td>( \mathcal{O}_{10} )</td>
<td>8 GeV</td>
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<tr>
<td>0</td>
<td>1/2(^\dagger)</td>
<td>( y_1 )</td>
<td>( \mathcal{O}_1 )</td>
<td>3.2 PeV</td>
</tr>
<tr>
<td>0</td>
<td>1/2(^\dagger)</td>
<td>( y_2 )</td>
<td>( \mathcal{O}_1 )</td>
<td>3.2 PeV</td>
</tr>
<tr>
<td>0</td>
<td>1/2(^\dagger)</td>
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<td>( \mathcal{O}_{10} )</td>
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</tr>
<tr>
<td>1/2</td>
<td>0</td>
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<td>( \mathcal{O}_1 )</td>
<td>12.7 TeV</td>
</tr>
<tr>
<td>1/2</td>
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<td>( \mathcal{O}_{10} )</td>
<td>293 GeV</td>
</tr>
<tr>
<td>1/2</td>
<td>0</td>
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<td>( \mathcal{O}_{11} )</td>
<td>14 GeV</td>
</tr>
<tr>
<td>1/2</td>
<td>0</td>
<td>( h_2, \lambda_2 )</td>
<td>( \mathcal{O}_6 )</td>
<td>1.9 GeV</td>
</tr>
<tr>
<td>1/2</td>
<td>1</td>
<td>( h_3, \lambda_3 )</td>
<td>( \mathcal{O}_1 )</td>
<td>6.3 TeV</td>
</tr>
<tr>
<td>1/2</td>
<td>1</td>
<td>( h_4, \lambda_3 )</td>
<td>( \mathcal{O}_9 )</td>
<td>6.4 GeV</td>
</tr>
<tr>
<td>1/2</td>
<td>1</td>
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<td>( \mathcal{O}_8 )</td>
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</tr>
<tr>
<td>1/2</td>
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<td>( \mathcal{O}_4 )</td>
<td>135 GeV</td>
</tr>
<tr>
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<td>( l_1 )</td>
<td>( \mathcal{O}_1 )</td>
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</tr>
<tr>
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<td>( \mathcal{O}_1 )</td>
<td>5.5 TeV</td>
</tr>
<tr>
<td>1/2</td>
<td>1(^\dagger)</td>
<td>( d_1 )</td>
<td>( \mathcal{O}_1 )</td>
<td>5.9 TeV</td>
</tr>
<tr>
<td>1/2</td>
<td>1(^\dagger)</td>
<td>( d_2 )</td>
<td>( \mathcal{O}_1 )</td>
<td>6.7 TeV</td>
</tr>
</tbody>
</table>

▶ Non standard operators are the leading interaction in 6 additional cases if dark matter has spin 1
WIMP-nucleon interactions have been systematically classified in terms of Galilean invariant operators.

It is the first step towards model-independent direct (and indirect) tests of WIMP dark matter.

In the talk I have reviewed the phenomenology of this general theoretical framework.

**Selected results:**

a) Direct detection exclusion limits need to be reconsidered in the ET framework

b) The same is true for the capture and annihilation of WIMPs in the Sun and Earth

c) DAMA remains incompatible with the null searches