Amplitude-dependent tune-shift compensation method using Hamiltonians, Lie Algebra and Normal Forms

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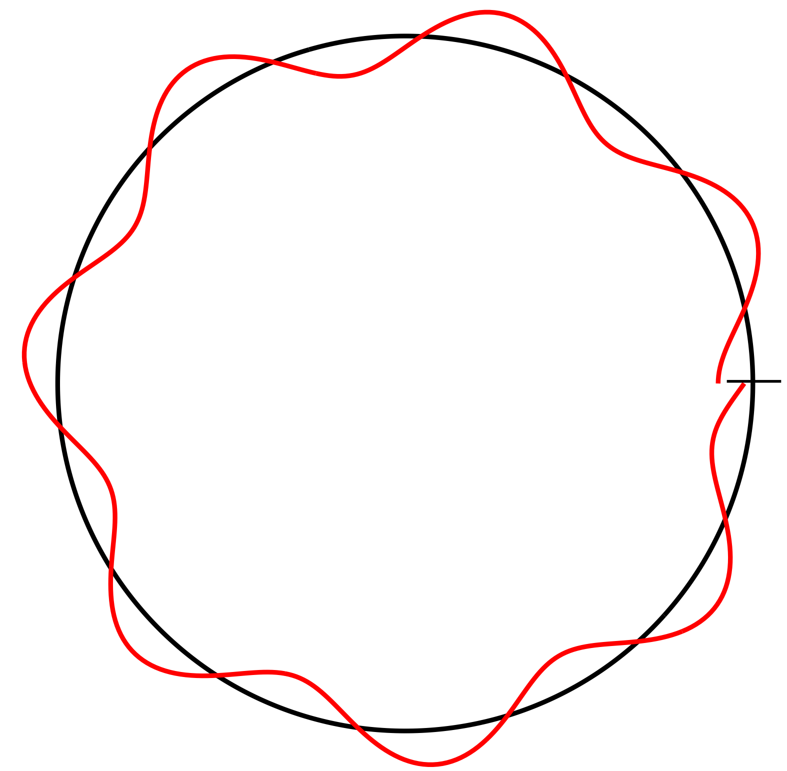
FREIA-meeting, Nov 28, 2016

Outline

- Introduction
- Transfer maps
- Hamiltonians and Lie Algebra
- Normal Forms
- Example: Tune-shift compensation
- Simulation results
- Conclusions

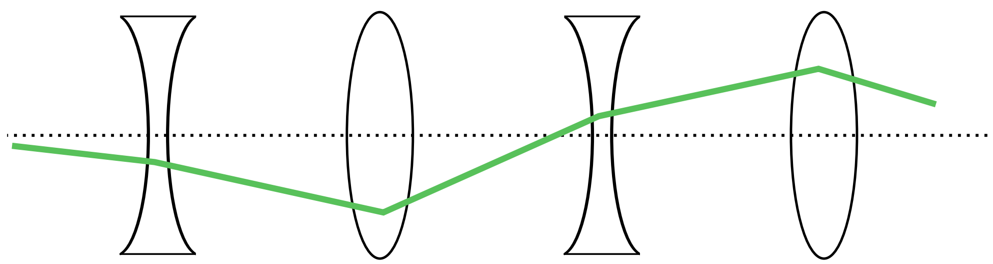
Introduction



Particles oscillate around design orbit. Number of oscillations is the **tune** of accelerator.

E.g. Qx = 7.**23** integer and **fraction** part, latter is important for beam stability.

Tune is a design parameter and depends on the optics of the accelerator, i.e. the spacing and strengths of the **quadrupoles**:



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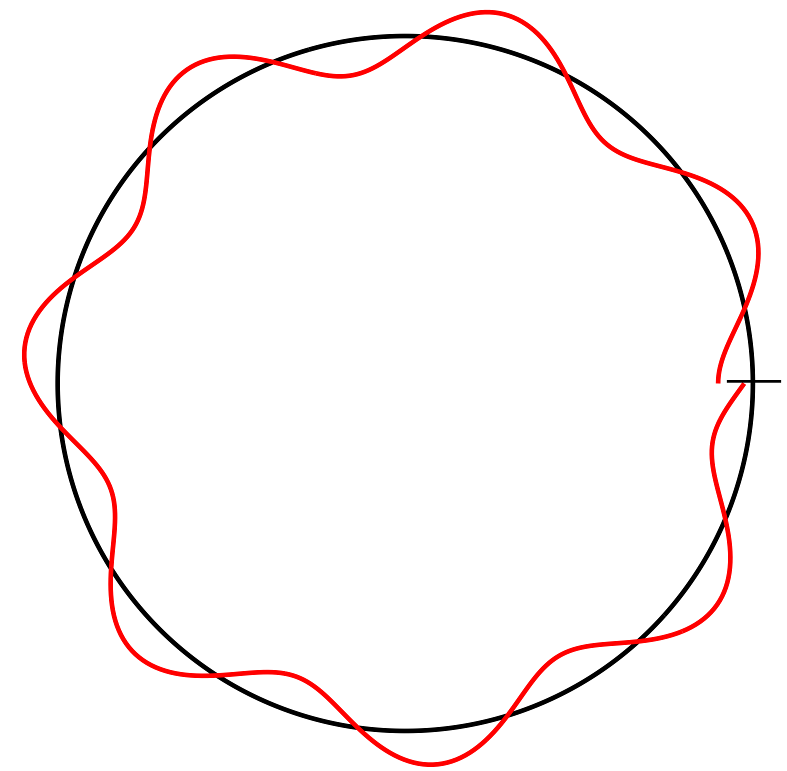

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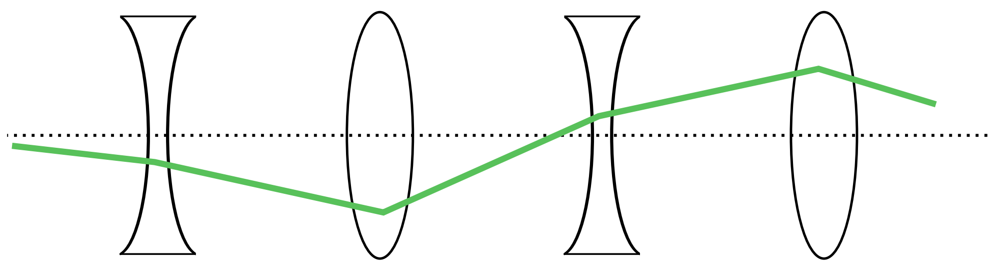
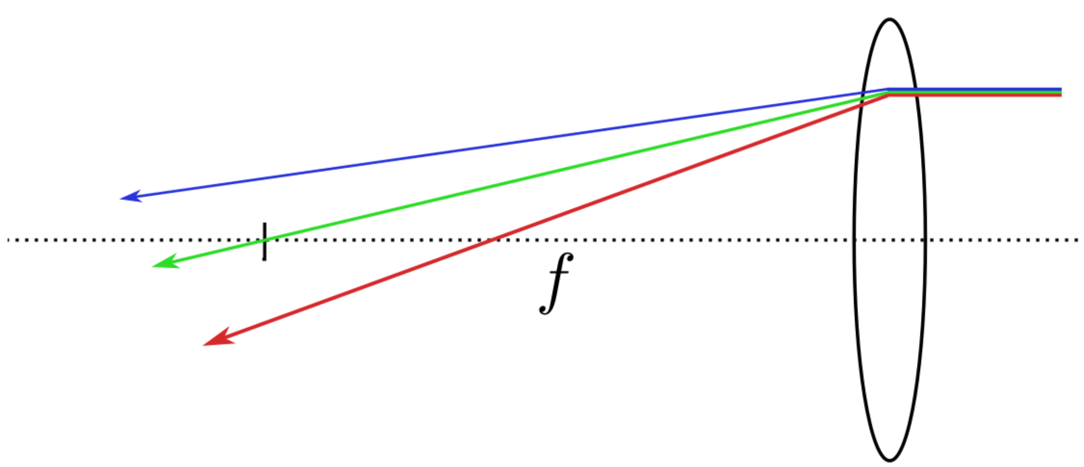
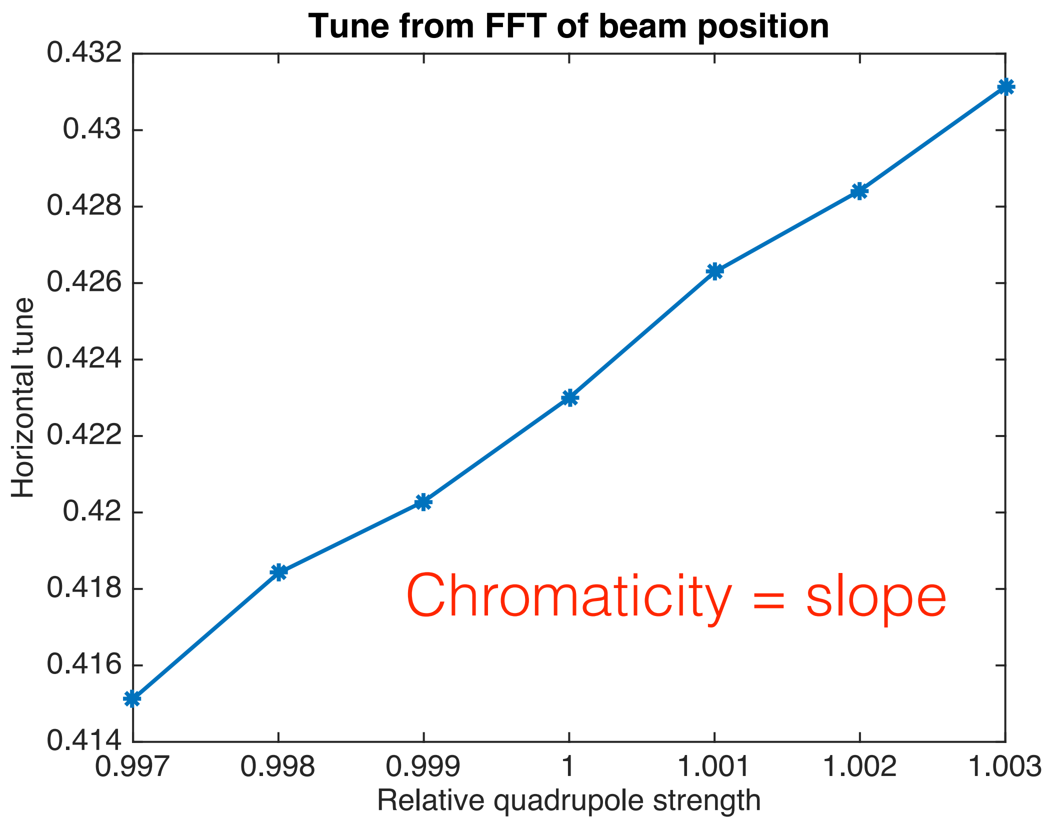


Chromaticity: tune is energy-dependent. Since a beam has an energy distribution we have a tune distribution - or a "tune spread".





Stability and tune-shifts

The tune cannot be an integer since oscillations would amplify each turn.

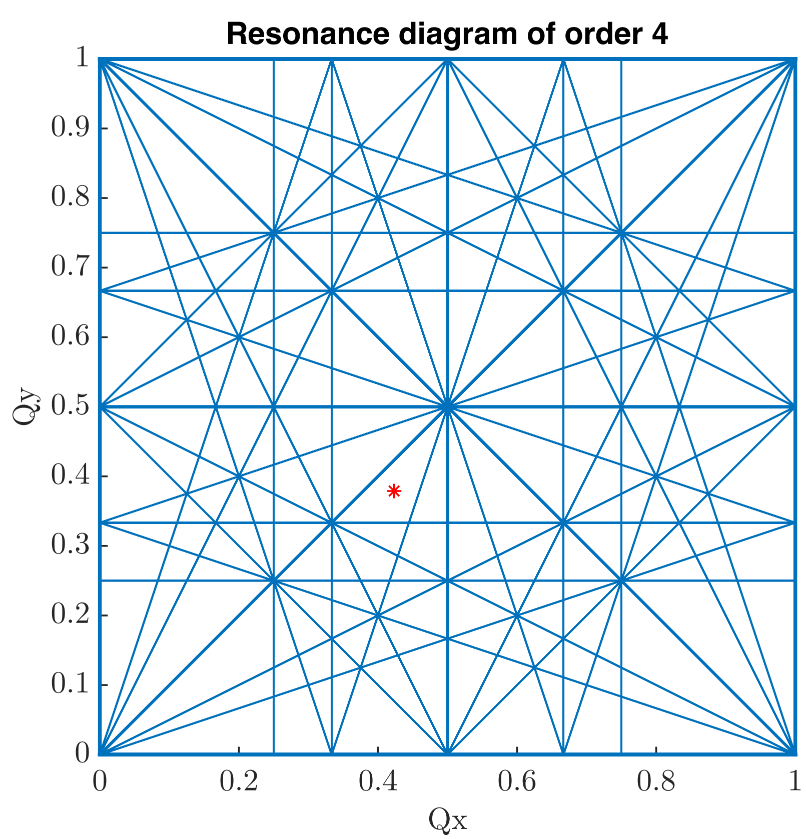
Higher order resonances require that no perturbations affect the coherence over a number of turns. Number of turns gives the order of the resonance.

General **resonance condition**:

$$mQ_x + nQ_y = k$$

where *m*, *n* and k are integers.

All resonance lines up to 4th order



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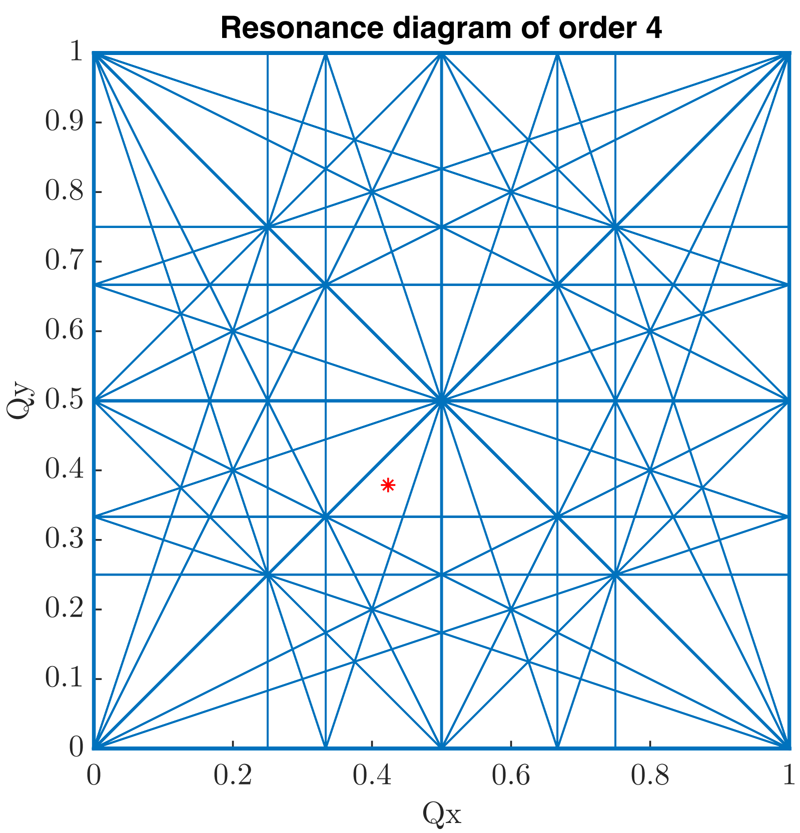

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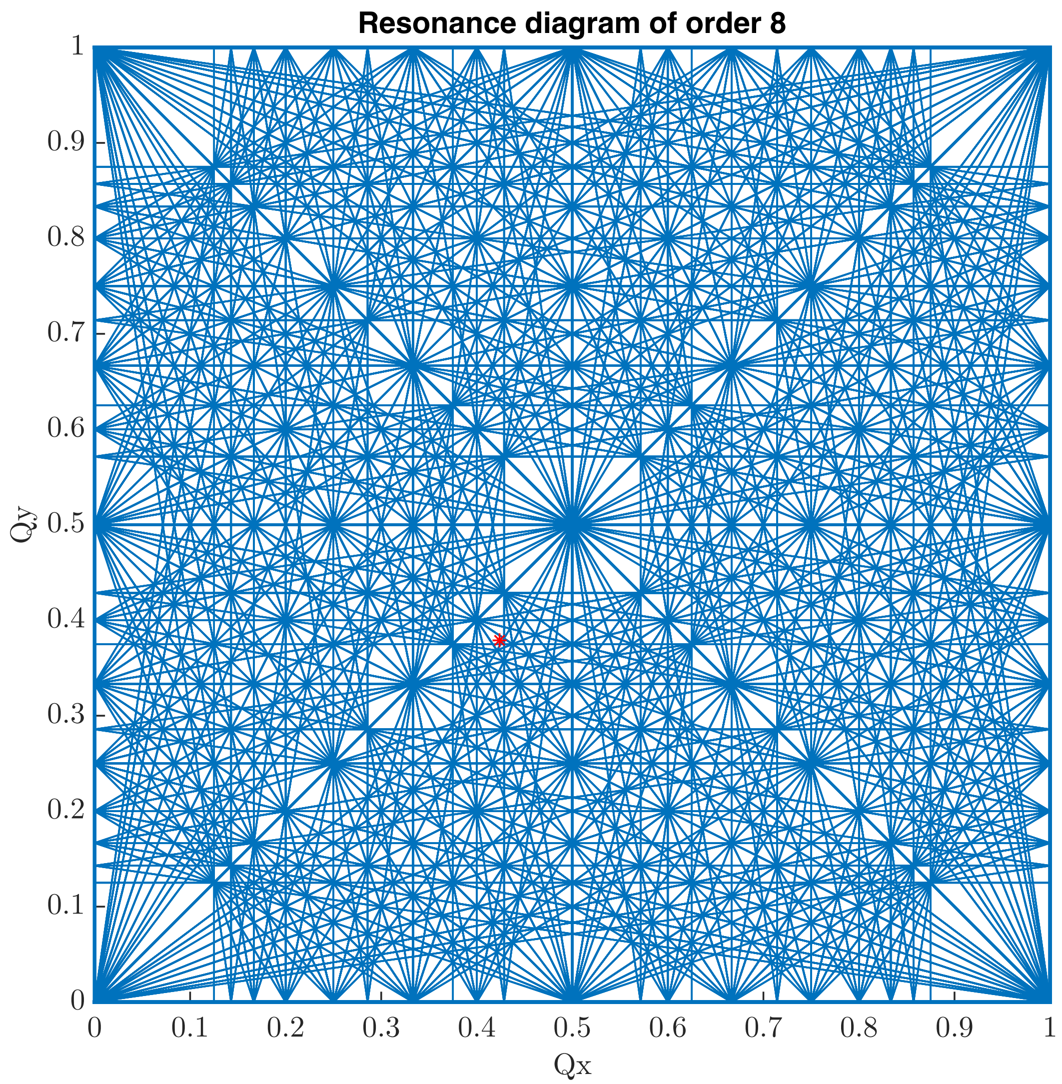
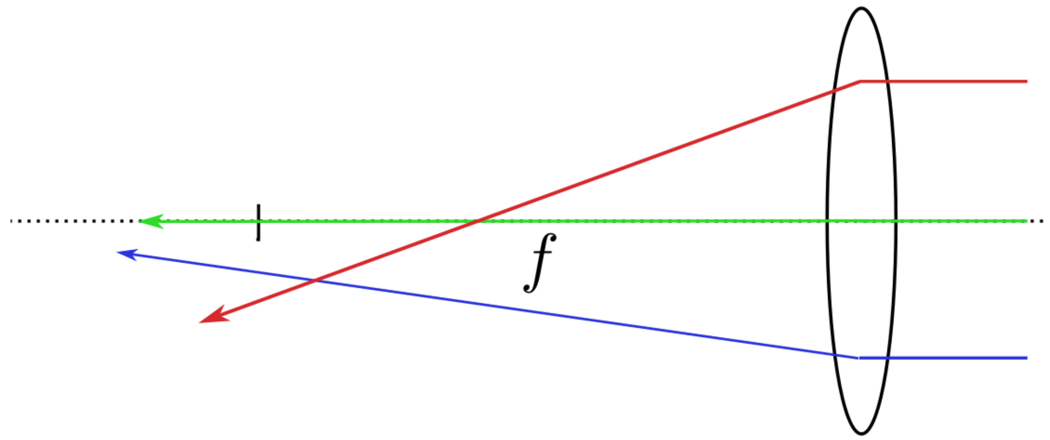
All resonance lines up to 8th order



Amplitude-dependent tune-shifts

We can compensate chromaticity by inserting sextupoles in dispersive sections.



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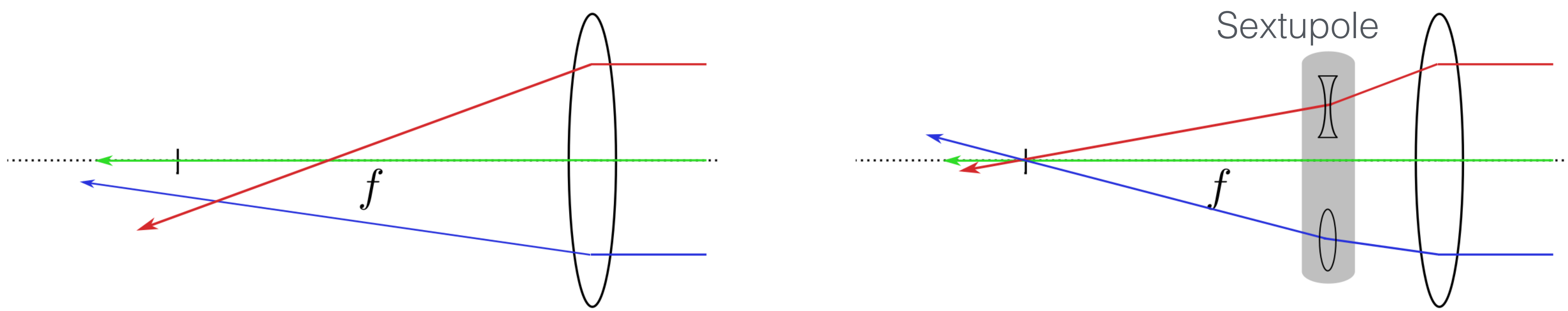

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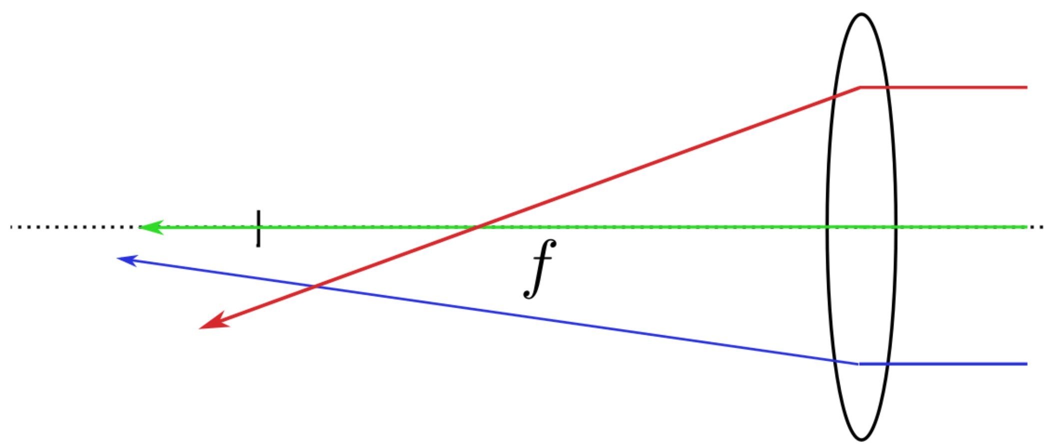
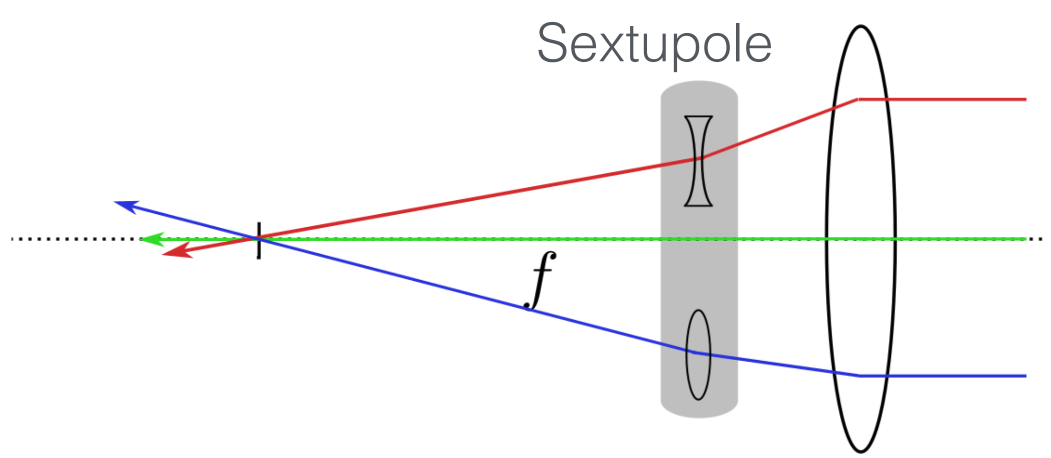


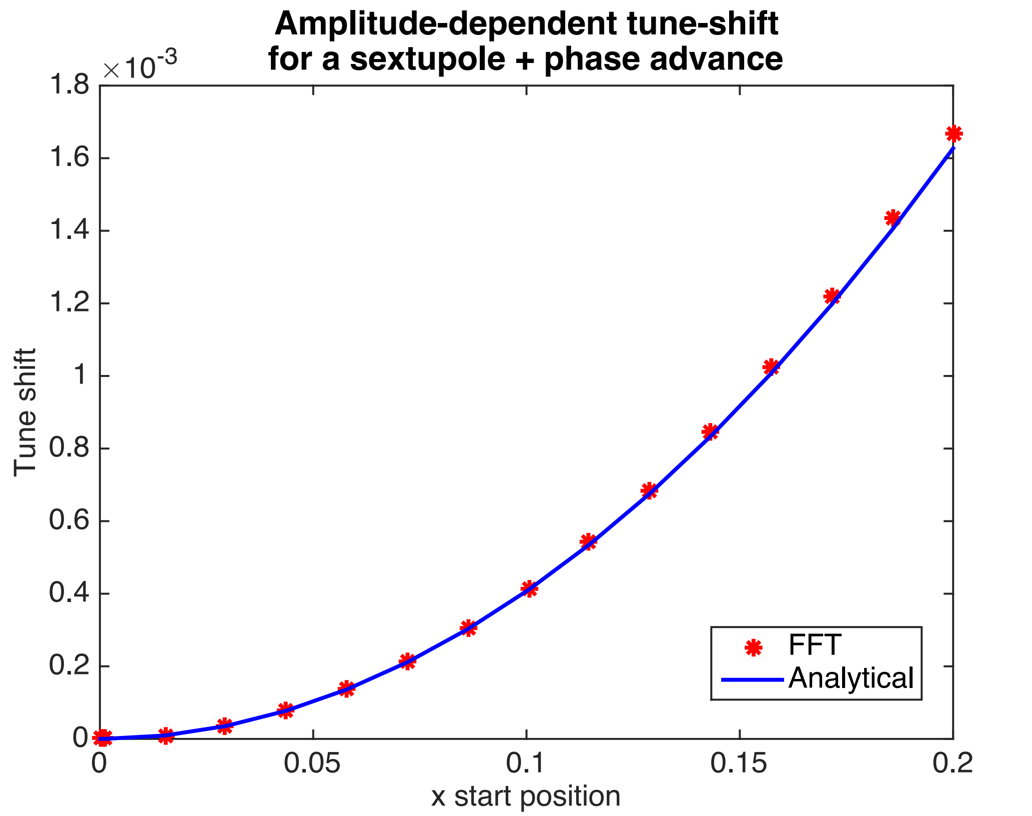


No chromaticity is nice but sextupoles are nonlinear elements and they in turn introduce another type of tune-shift: **amplitude dependent**.

Tune-shift is proportional to the action:

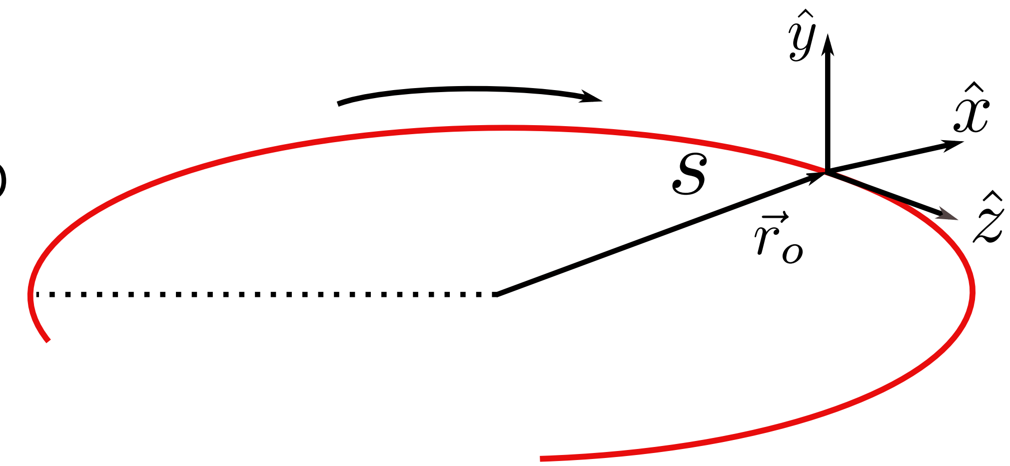
$$J_x = \frac{x^2 + x'^2}{2}$$

Particles oscillating with larger amplitudes are more susceptible to tune-shifts and may be lost due to resonances => limits **dynamic aperture**



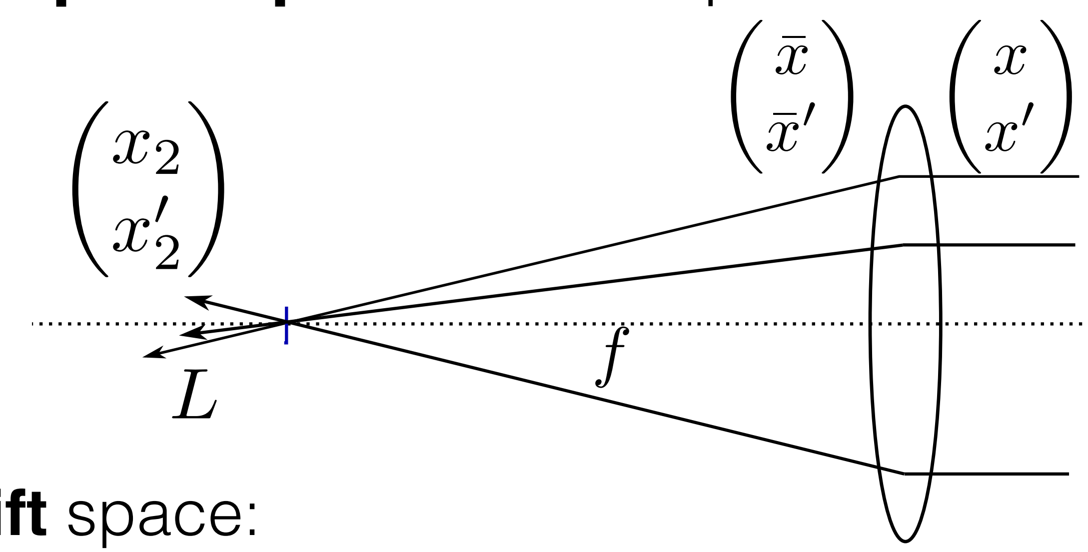
Transfer maps

Transfer map: describes how the particle moves or rather how to map the incoming coordinates to outgoing coordinates.



Maps can describe: a single element, a cell, the whole accelerator (full turn map).

A linear map can be represented by matrix, e.g. a **quadrupole** or drift space:



$$\begin{pmatrix} \bar{x} \\ \bar{x}' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$

$$M_Q = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Drift space:

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{x}' \end{pmatrix} \qquad M_D = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

Transfer map for quadrupole followed by drift space:

$$M = M_D M_Q$$

Normalized phase space

Parametrization of transfer matrix:

$$M = \begin{pmatrix} \sqrt{\beta} & 0 \\ -\frac{\alpha}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\alpha}{\sqrt{\beta}} & \sqrt{\beta} \end{pmatrix}$$

$$M = A^{-1}RA$$

A particle under a linear transfer map trace out ellipses in phase-space. If we transfer into normalized phase space we get circles instead described by the rotation matrix R. The angle μ is called the **phase advance**.

We can write:

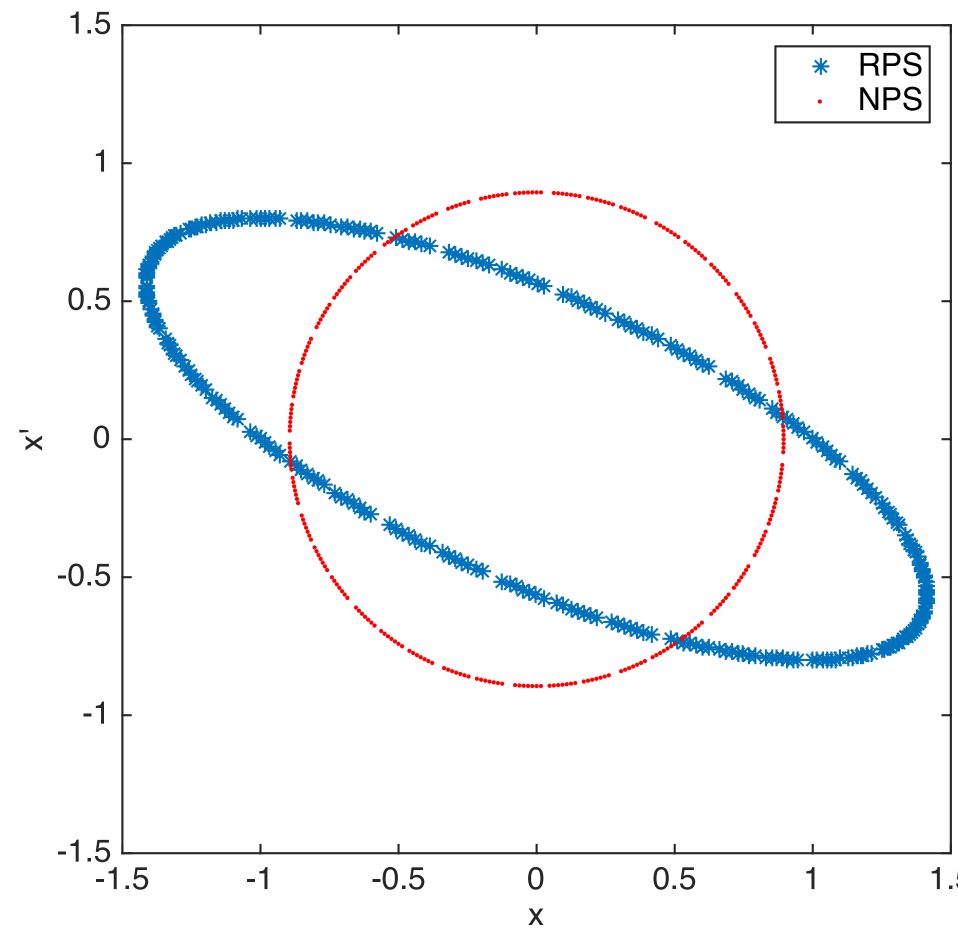
$$\begin{pmatrix} \tilde{x} \\ \tilde{x}' \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\alpha}{\sqrt{\beta}} & \sqrt{\beta} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$

The action *J* is an invariant of the motion:

$$J = \frac{\tilde{x} + \tilde{x}'}{2}$$

Poincaré section:

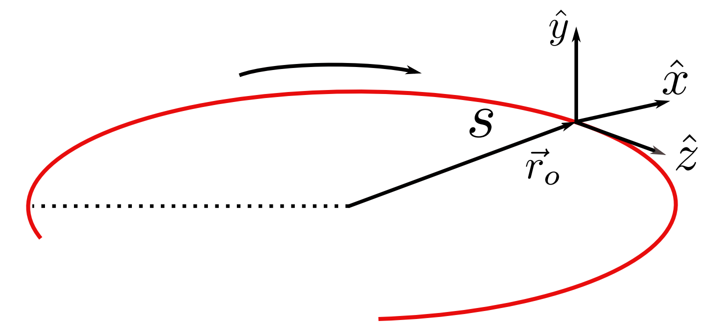
$$\begin{pmatrix} x \\ x \end{pmatrix}_{n+1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_n$$



Hamiltonians

A Hamiltonian *H* together with Hamiltons equations describes a particle trajectory.

$$\frac{dx}{ds} = \frac{\partial H}{\partial x'} \quad ; \quad \frac{dx'}{ds} = -\frac{\partial H}{\partial x}$$



Or expressed using the Poisson bracket:

$$[f, g] = \frac{\partial f}{\partial x}\frac{\partial g}{\partial x'} - \frac{\partial f}{\partial x'}\frac{\partial g}{\partial x}$$

Then Hamilton's equations can be written as:

$$\frac{dx}{ds} = [-H, x] \quad ; \quad \frac{dx'}{ds} = [-H, x']$$

Ex: Hamiltonians for sextupole and octupole (thin elements):

$$H_{\text{sext}} = \frac{k_2}{3!}(x^3 - 3xy^2)$$

Third order

$$H_{\text{oct}} = \frac{k_3}{4!}(x^4 - 6x^2y^2 + y^4)$$

Fourth order

Nonlinear maps

The **Lie operator**

$$:f:g = [f,g] = \frac{\partial f}{\partial x}\frac{\partial g}{\partial x'} - \frac{\partial f}{\partial x'}\frac{\partial g}{\partial x}$$

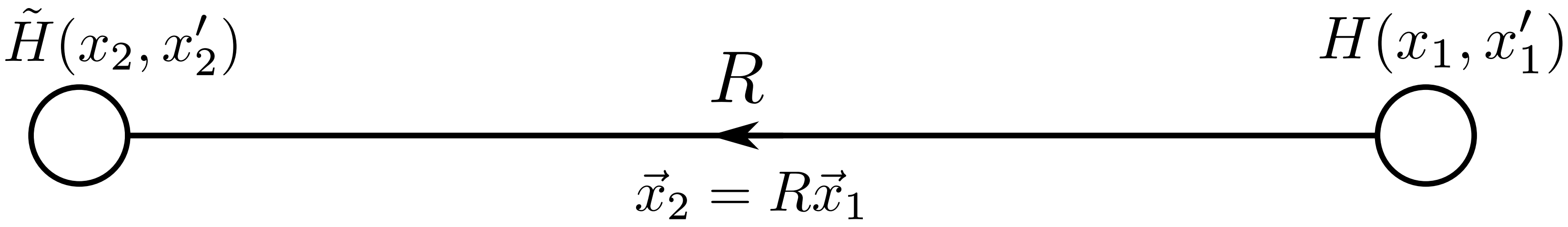
The Lie operator *f* on *g* is the Poisson bracket.

We can can calculate the change of a particle passing through an element with Hamiltonian *H* by a **Lie transformation** of the coordinate function:

$$\bar{x} = \mathrm{e}^{-:H:}x = x - [H,x] + \frac{1}{2!}[H,[H,x]] + \ldots$$

Which essentially is a Taylor map. The Lie transformation maps incoming coordinates to outgoing coordinates for a nonlinear element described by Hamiltonian *H.*

Lie Algebra



Similarity transformation:

$$
\begin{aligned}
\mathcal{M} &= R\mathrm{e}^{:-H(\vec{x}_1):} \\
&= \underline{R\mathrm{e}^{:-H(\vec{x}_1):}R^{-1}}R \\
&= \mathrm{e}^{:-H(R\vec{x}_1):}R \\
&= \mathrm{e}^{:-H(\vec{x}_2):}R
\end{aligned}
$$

We can move the Hamiltonian to another location via the similarity transformation.

We can transform the operator by transforming the generator.

Campbell-Baker-Hausdorff formula

$$\mathrm{e}^{:H_A:}\mathrm{e}^{:H_B:} = \mathrm{e}^{:H:}$$

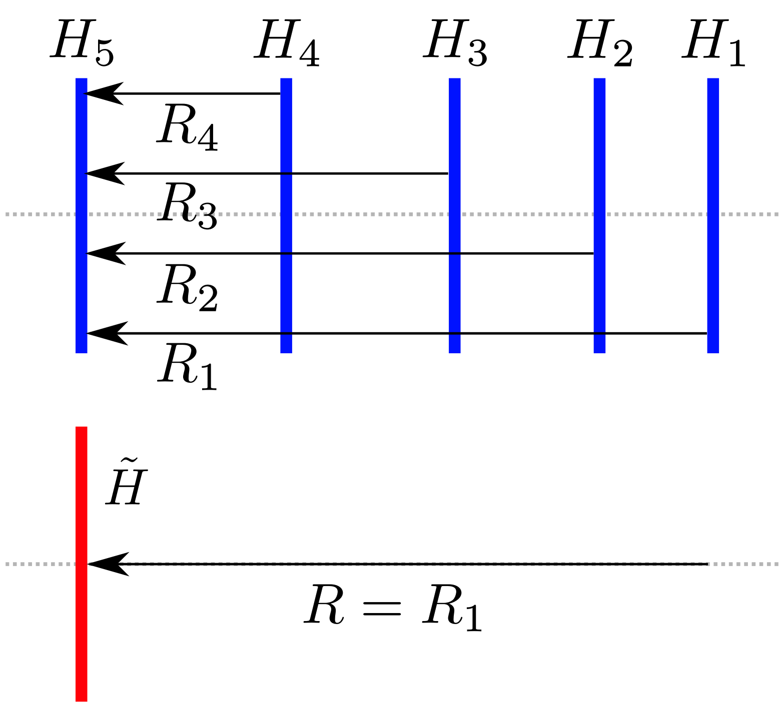
CBH tells us how to concatenate Hamiltonians

where

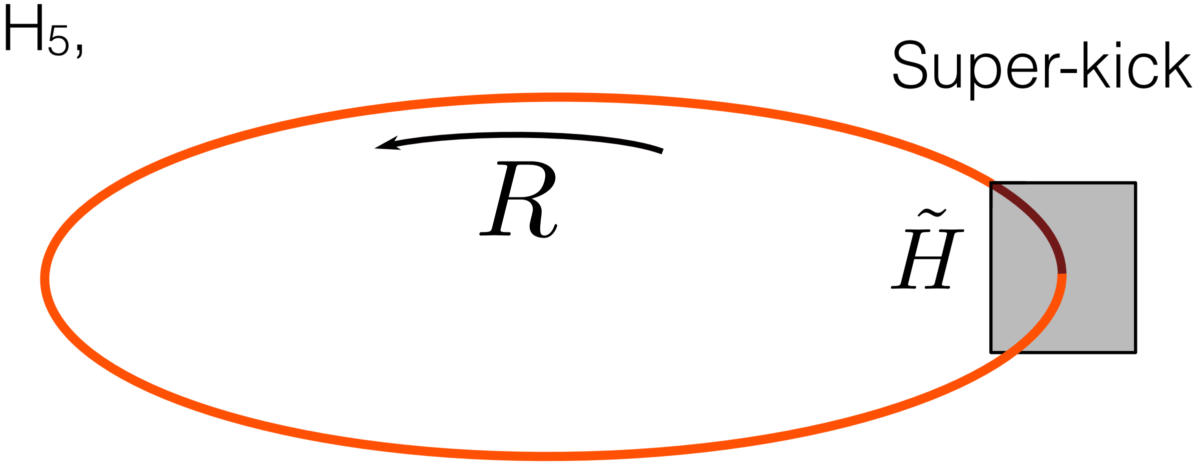
$$H = H_A + H_B + \frac{1}{2}[H_A, H_B] + \frac{1}{12}\Big[H_A - H_B, [H_A, H_B]\Big] + \dots$$

Moving all elements to reference point

By iterative usage of the similarity transform and CBH we can represent the whole beam line as a **linear map** + a **nonlinear kick**.



First move H_4 and concatenate with H_5, then move H_3 etc.



We have written a code that can represent polynomials of (x, x', y, y'), and concatenate the Hamiltonians consistently up to 5th order. But to see what resonances and tune-shifts we get we need to transform our effective Hamiltonian into a **normal form**, which will be explained next.

Normal forms

We can propagate a Hamiltonian by propagating its coefficients

$$H^{(1)} = h_i^{(1)} x_i = h_i^{(1)} R_{ij}^{-1} y_j = \tilde{h}^{(1)} y_j$$

$$\tilde{h}^{(1)} = \left(R^{-1}\right)^T h^{(1)} = S^{(1)} h^{(1)}$$

Linear transform:

$$\vec{y} = R\vec{x}$$

To write a map M on its normal form we need to find K and C such that:

$$\mathcal{M} = \mathrm{e}^{:-H:} R = \mathrm{e}^{:-K:} \mathrm{e}^{:-C:} R \mathrm{e}^{:K:}$$

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We can re-write as

$$\mathrm{e}^{:-H:} \boxed{R \mathrm{e}^{:-K:} R^{-1}} = \mathrm{e}^{:-K:} \mathrm{e}^{:-C:}$$

A similarity transform! We get:

$$\mathrm{e}^{:-H:} \mathrm{e}^{:-SK:} = \mathrm{e}^{:-K:} \mathrm{e}^{:-C:}$$

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A similarity transform! We get:

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This we can write order-by-order:

$$H = H^{(3)} + H^{(4)} + H^{(5)}$$

$$K = K^{(3)} + K^{(4)} + K^{(5)}$$

$$C = C^{(3)} + C^{(4)} + C^{(5)}$$

$$SK = S^{(3)} K^{(3)} + S^{(4)} K^{(4)} + S^{(5)} K^{(5)}$$

Normal forms cont'd

We solve order-by-order

$$e^{:-H:}e^{:-SK:} = e^{:-K:}e^{:-C:}$$

$$e^{:-H^{(3)}:}e^{:-S^{(3)}K^{(3)}:} = e^{:-K^{(3)}:}e^{:-C^{(3)}:}$$

$$H = H_A + H_B + \frac{1}{2}\left[H_A, H_B\right] + \frac{1}{12}\Big[H_A - H_B, \left[H_A, H_B\right]\Big] + \ldots$$

From CBH we get:

$$H^{(3)} + S^{(3)}K^{(3)} = K^{(3)} + C^{(3)} + \text{higher orders}$$

Since $C^{(3)} = 0$ (no tune-shift term of third order) we can write

$$K^{(3)} = (1 - S^{(3)})^{-1}H^{(3)}$$

Normal forms cont'd

We solve order-by-order $\quad \mathrm{e}^{:-H:}\mathrm{e}^{:-SK:} = \mathrm{e}^{:-K:}\mathrm{e}^{:-C:}$

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Keeping all order up to fourth order:

$$H^{(4)} + S^{(4)}K^{(4)} + \frac{1}{2}\Big[H^{(3)}, S^{(3)}K^{(3)}\Big] = K^{(4)} + C^{(4)} + \text{higher orders}$$

We solve for $C^{(4)}$ and $K^{(4)}$:

$$(1 - S^{(4)})K^{(4)} + C^{(4)} = H^{(4)} + \frac{1}{2}\Big[H^{(3)}, S^{(3)}K^{(3)}\Big]$$

In fourth order we have nonzero tune-shift polynomial

Compensating the tune-shift

$$(1 - S^{(4)})K^{(4)} + C^{(4)} = H^{(4)} + \frac{1}{2}\left[H^{(3)}, S^{(3)}K^{(3)}\right]$$

We cannot invert (1 - $S^{(4)}$) because it has 3 zero eigenvalues. But $S^{(4)}$ is constructed from a pure rotation matrix *R* and these zero eigenvalues corresponds to eigenvector monomials:

$$(x^2 + x'^2)^2 \quad (y^2 + y'^2)^2 \quad (x^2 + x'^2)(y^2 + y'^2)$$

which are proportional to:

$$J_x^2, \quad J_y^2, \quad J_x J_y$$

We invert (1 - $S^{(4)}$) by SVD and construct a projector from the eigenvectors corresponding to the zero eigenvalues, i.e. a null space projector:

$$U \Lambda V^T = (1 - S^{(4)})^{-1} \qquad \mathrm{Pr} = \sum_{\mathrm{eig}=0} \frac{|V >< U|}{< V|U >}$$

Then we get $C^{(4)}$ by projecting RHS onto null space:

$$C^{(4)} = \mathrm{Pr}\left\{H^{(4)} + \frac{1}{2}\left[H^{(3)}, S^{(3)}K^{(3)}\right]\right\}$$

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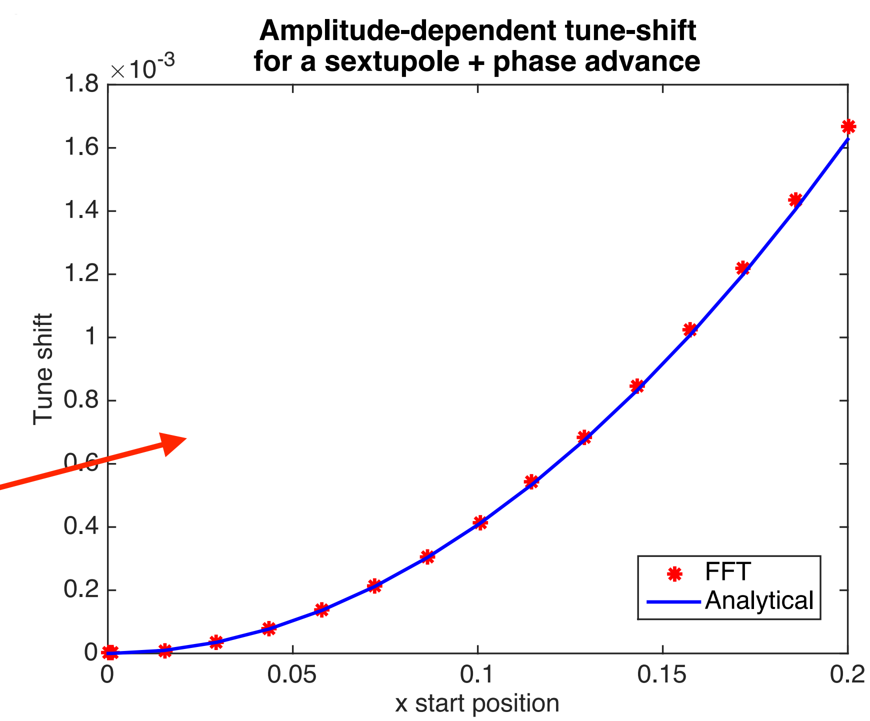

Compensating the tune-shift

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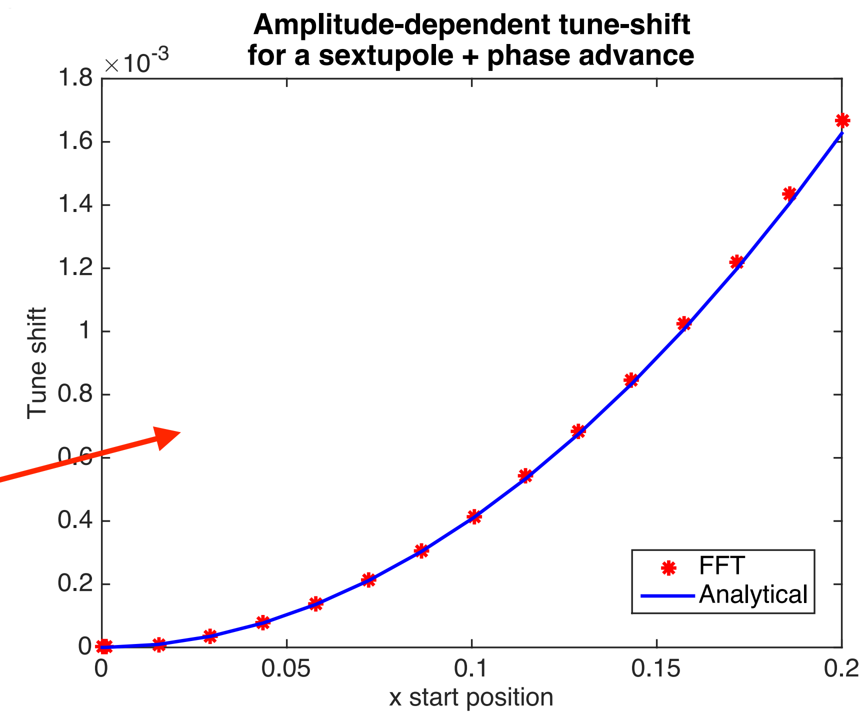


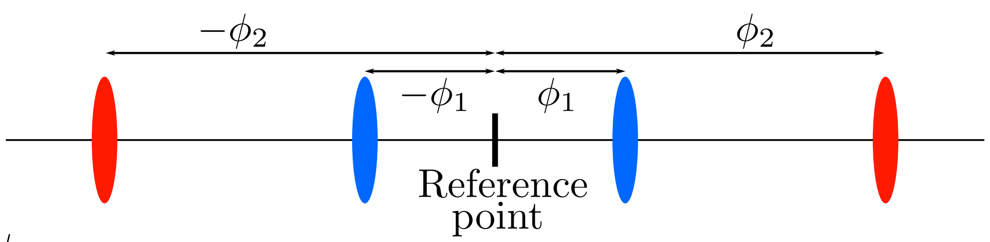
Adding octupoles only contribute linearly to fourth order:

$$C^{(4)} = \mathrm{Pr}\left\{\tilde{H}^{(4)} + H^{(4)} + \frac{1}{2}\left[H^{(3)}, S^{(3)}K^{(3)}\right]\right\}$$

To compensate tune-shift: set octuple strengths such RHS = 0.

Optimum placement of octuples

We start with four octuples (horizontal motion only) and write the Hamiltonians in action-angle variables:



$$
\begin{aligned}
\tilde{H} &= k(x\cos\phi + x'\sin\phi)^4 + k(x\cos\phi - x'\sin\phi)^4 \\
&= k\left[x^4\cos^4\phi + 4x^3x'\cos^3\phi\sin\phi + 6x^2x'^2\cos^2\phi\sin^2\phi\right. \\
&\quad \left.+4xx'^3\cos\phi\sin^3\phi + x'^4\sin^4\phi\right] \\
&+ k\left[x^4\cos^4\phi - 4x^3x'\cos^3\phi\sin\phi + 6x^2x'^2\cos^2\phi\sin^2\phi\right. \\
&\quad \left.-4xx'^3\cos\phi\sin^3\phi + x'^4\sin^4\phi\right] \\
&= 2k\left\{x^4\cos^4\phi + 6x^2x'^2\cos^2\phi\sin\phi + x'^4\sin^4\phi\right\}
\end{aligned}
$$

Move via similarity transform

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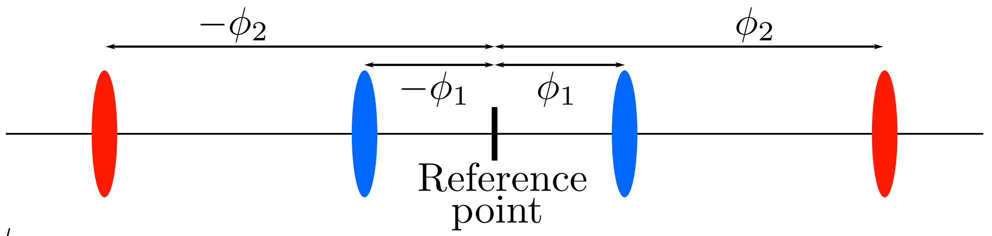

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&\quad \left. +4xx'^3\cos\phi\sin^3\phi + x'^4\sin^4\phi\right] \\
&+ k\left[x^4\cos^4\phi - 4x^3x'\cos^3\phi\sin\phi + 6x^2x'^2\cos^2\phi\sin^2\phi \right.\\
&\quad \left. -4xx'^3\cos\phi\sin^3\phi + x'^4\sin^4\phi\right] \\
&= 2k\left\{x^4\cos^4\phi + 6x^2x'^2\cos^2\phi\sin\phi + x'^4\sin^4\phi\right\}
\end{aligned}
$$

Move via similarity transform

Short-hand notation: $c_1 = \cos\phi_1 \quad s_1 = \sin\phi_1 \quad \text{etc.}$

Move all four octupoles to reference point:

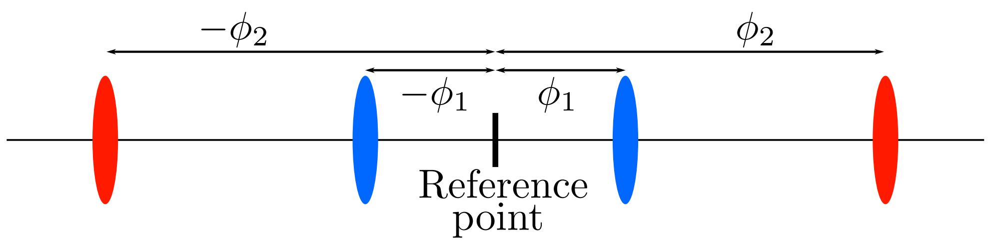
$$
\begin{aligned}
\bar{H} &= 2k_1\left[x^4c_1^4 + 6x^2x'^2c_1^2s_1^2 + x'^4s_1^4\right] + 2k_2\left[x^4c_2^4 + 6x^2x'^2c_2^2s_2^2 + x'^4s_2^4\right] \\
&= 2x^4(k_1c_1^4 + k_2c_2^4) + 12x^2x'^2(k_1c_1^2s_1^2 + k_2c_2^2s_2^2) + 2x'^4(k_1s_1^4 + k_2s_2^4)
\end{aligned}
$$

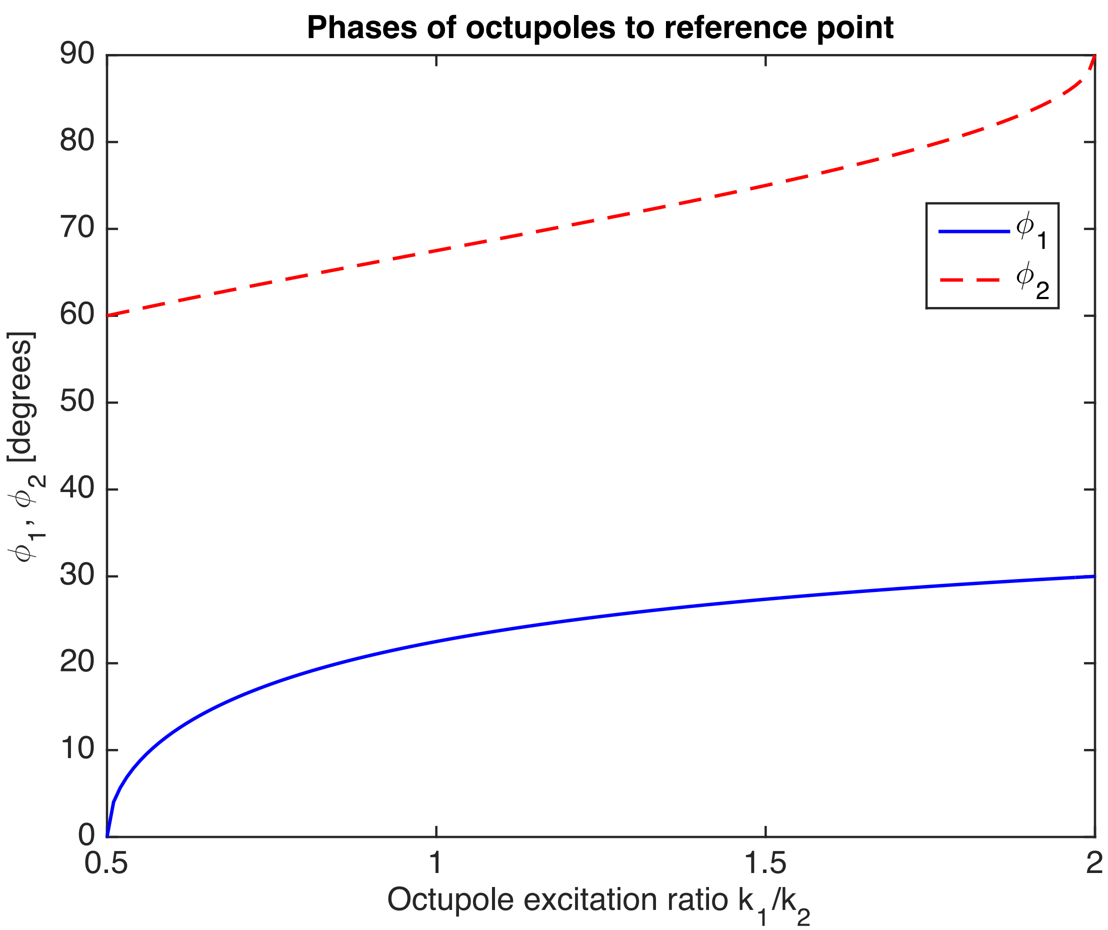
Terms with x^3x' and xx'^3 etc. cancel because symmetry => do not drive resonances.

Optimum placement of octuples cont'd

In order to compensate the amplitude-dependent tune-shift we need terms containing: $(x^2+x'^2)^2 \quad (y^2+y'^2)^2 \quad (x^2+x'^2)(y^2+y'^2)$

This gives us a relation between k_1/k_2 and the phase advances:

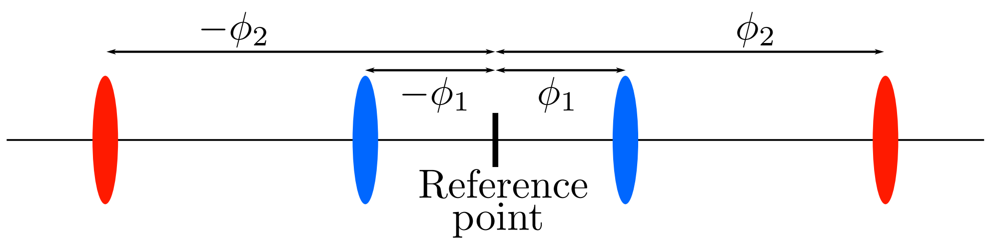


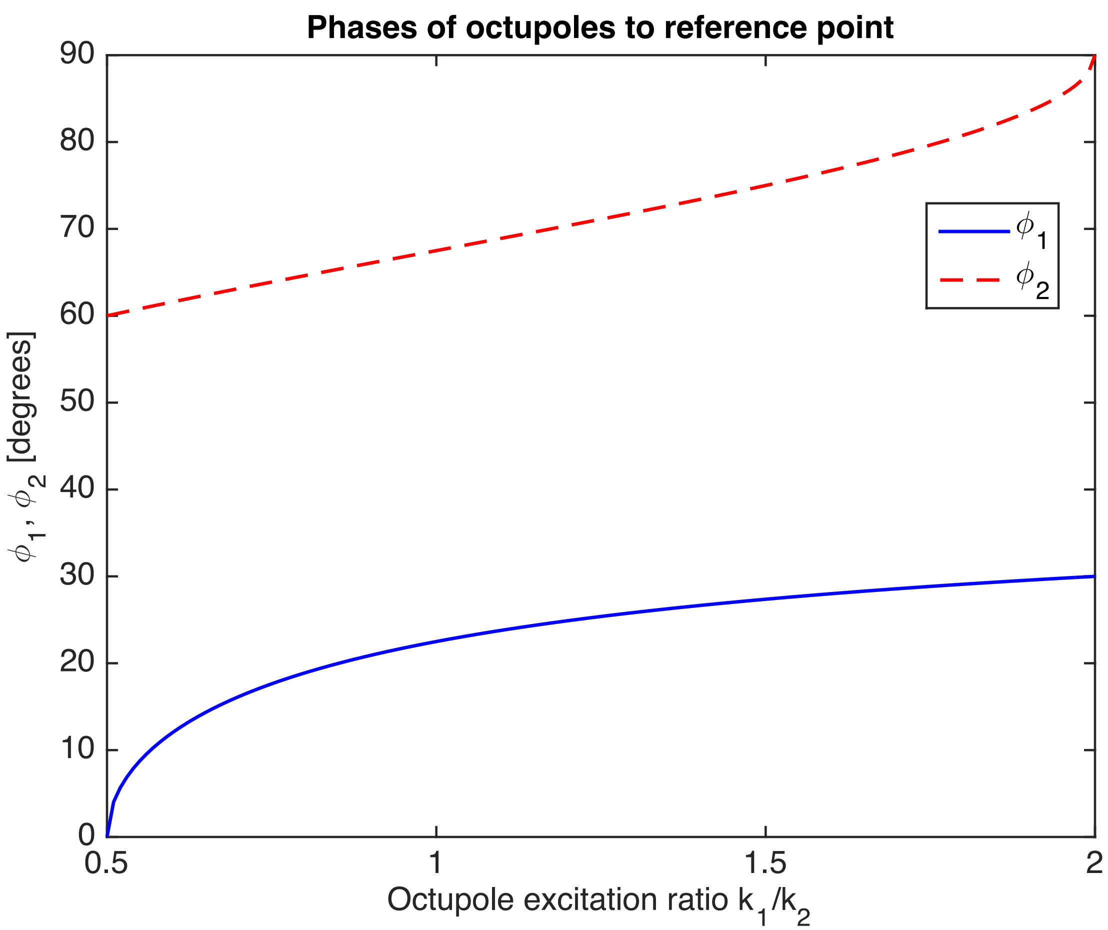


Optimum placement of octuples cont'd

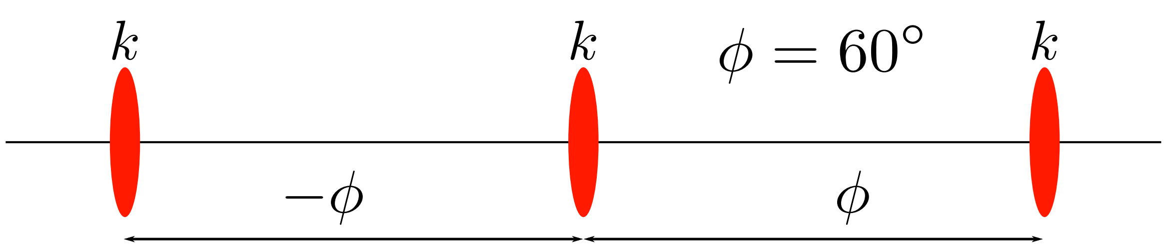
In order to compensate the amplitude-dependent tune-shift we need terms containing: $(x^2 + x'^2)^2 \quad (y^2 + y'^2)^2 \quad (x^2 + x'^2)(y^2 + y'^2)$

This gives us a relation between k_1/k_2 and the phase advances:





There is a solution with three equally powered octupoles and 60 degrees phase advance:



Optimum placement of octuples cont'd

The 4D Hamiltonian for an octupole in real phase space:

$$H = k\left(\beta_x^2\tilde{x}^4 - 6\beta_x\beta_y\tilde{x}^2\tilde{y}^2 + \beta_y^2\tilde{y}^4\right) = k_x\tilde{x}^4 - 6k_{xy}\tilde{x}^2\tilde{y}^2 + k_y\tilde{y}^4$$

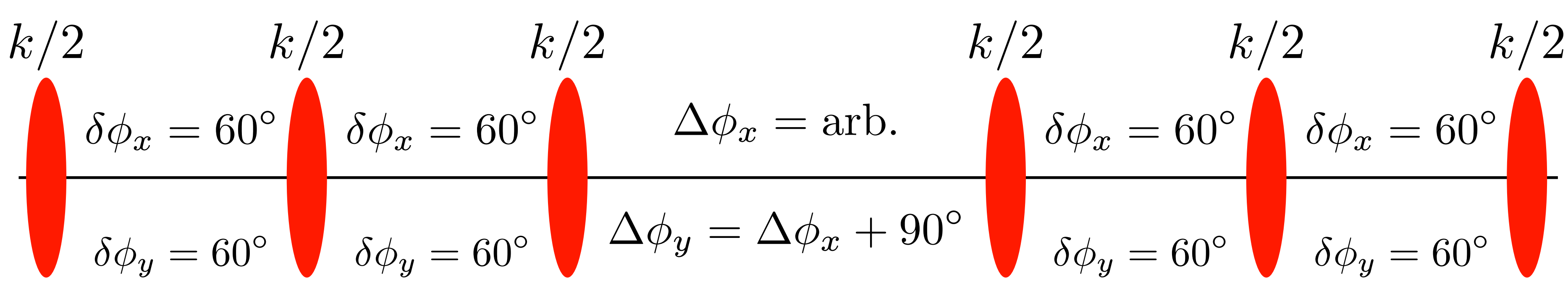
$$x = \sqrt{\beta_x}\tilde{x}$$

$$y = \sqrt{\beta_y}\tilde{y}$$

Carrying out the same procedure as before (action-angle variables etc.) for the triplet we get:

$$\tilde{H} = \frac{9}{2}\left[k_x J_x^2 + k_y J_y^2 - 4k_{xy}J_xJ_y - 2k_{xy}J_xJ_y\cos(2\psi_x - 2\psi_y)\right]$$

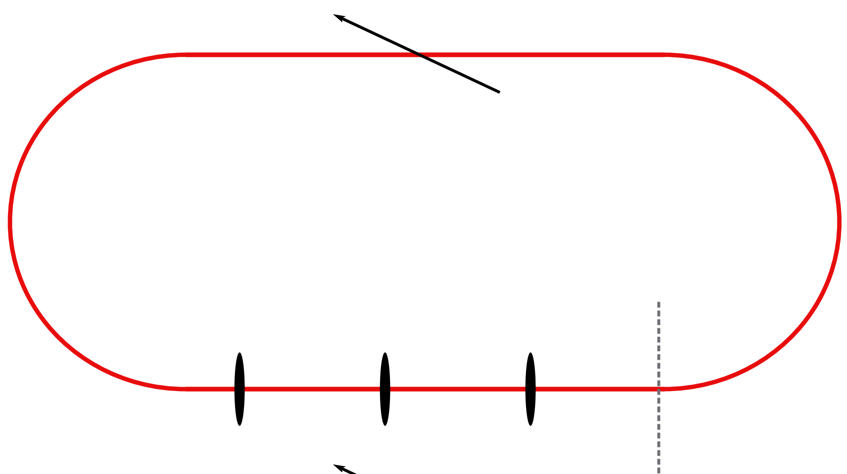
This drives the $2Q_x$ - $2Q_y$ resonance. In 2D we see that this setup cancel all resonances except oneWe can solve this by adding another triplet, i.e. a ”six-pack”:



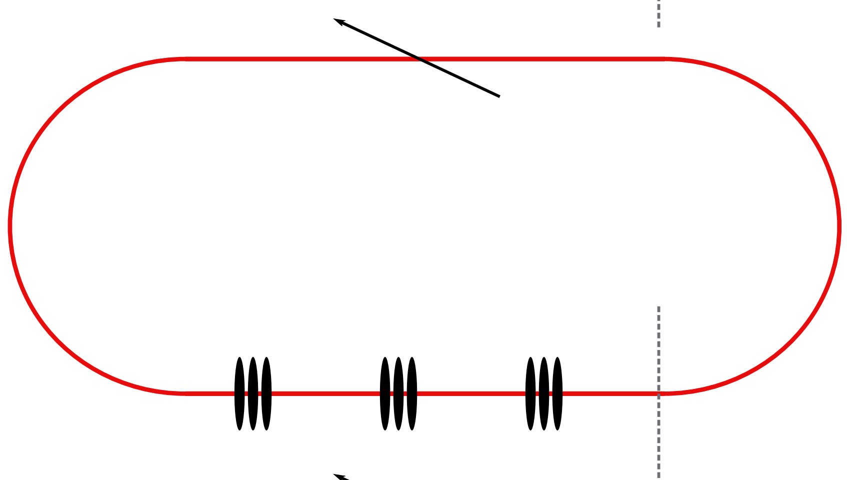
This setup can cancel a given tune-shift term without driving any fourth order resonances! In order to cancel all three tune-shift terms we need three six-packs.

Simulation: Octupoles + phase advance

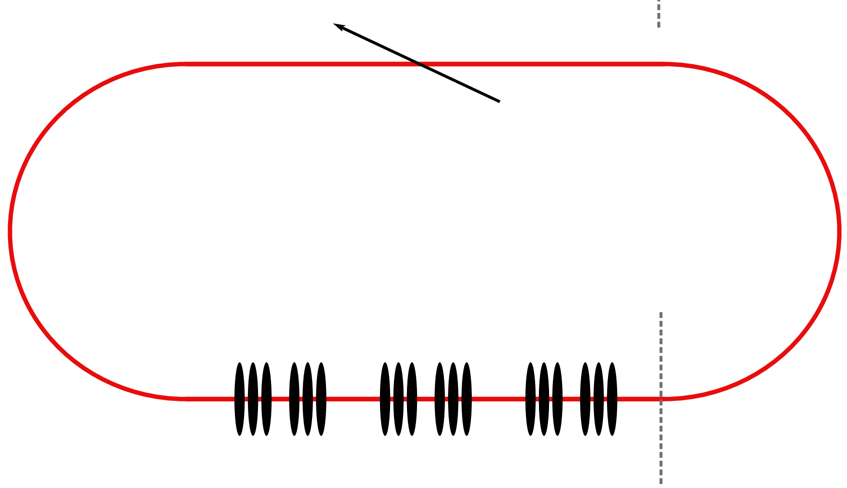
A simple setup with three setups of octupoles + a phase advance:



3 octupoles



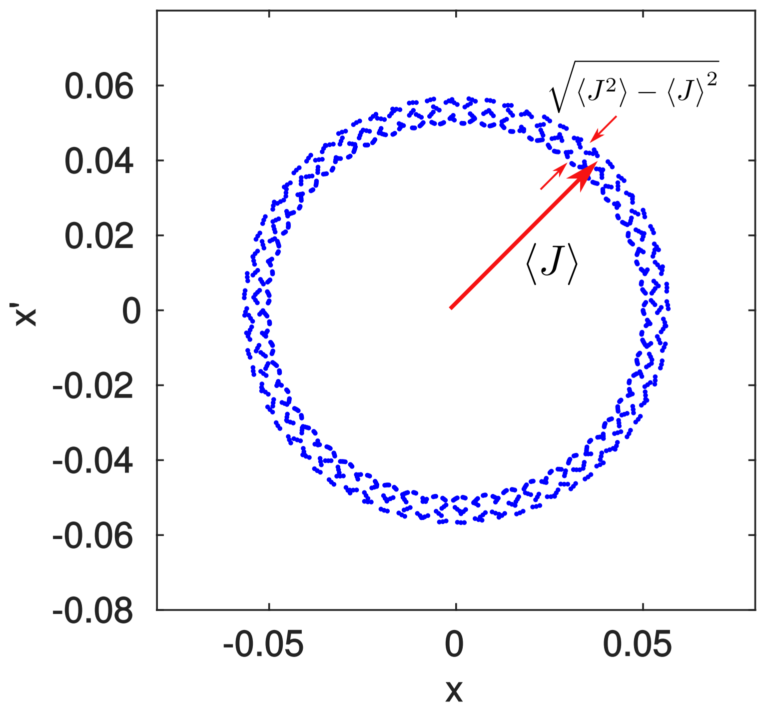
3 triplets



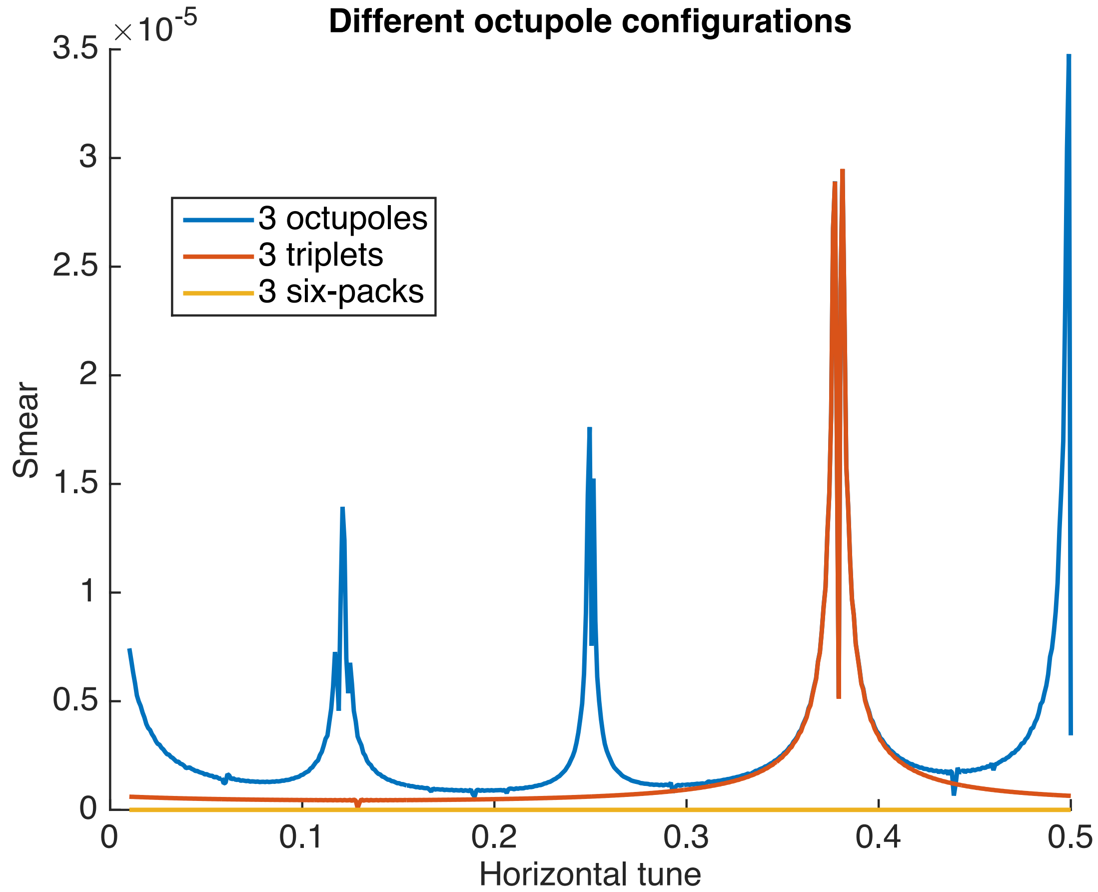
3 six-packs

Smear:

$$\sigma_J = \sqrt{\frac{\langle J^2 \rangle - \langle J \rangle^2}{\langle J \rangle^2}}$$

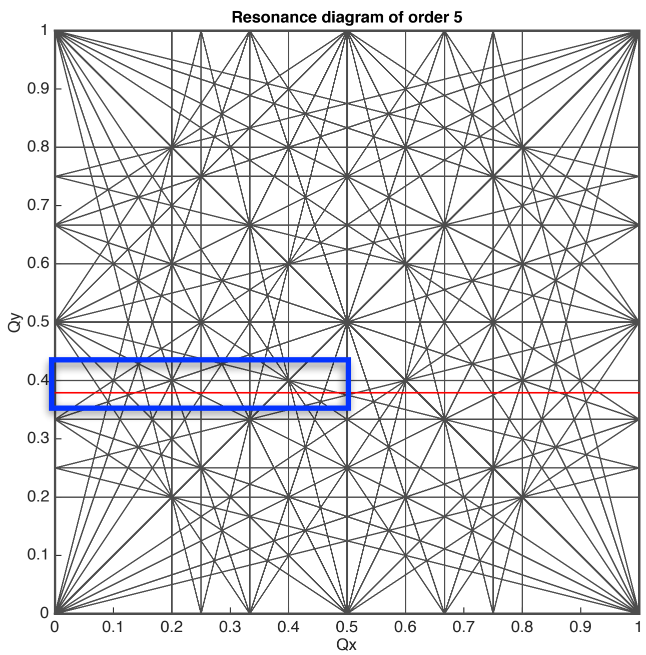


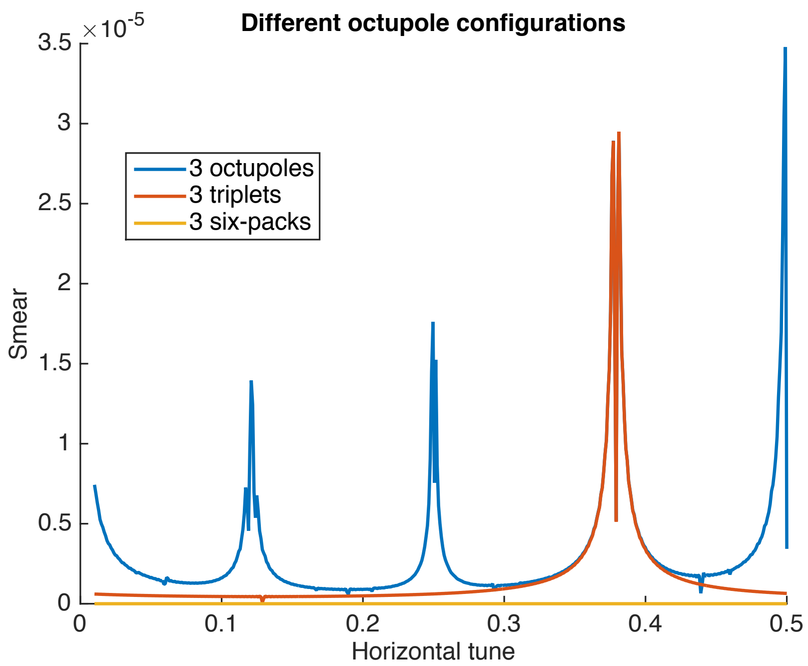
Smear plots to see resonances



Resonances

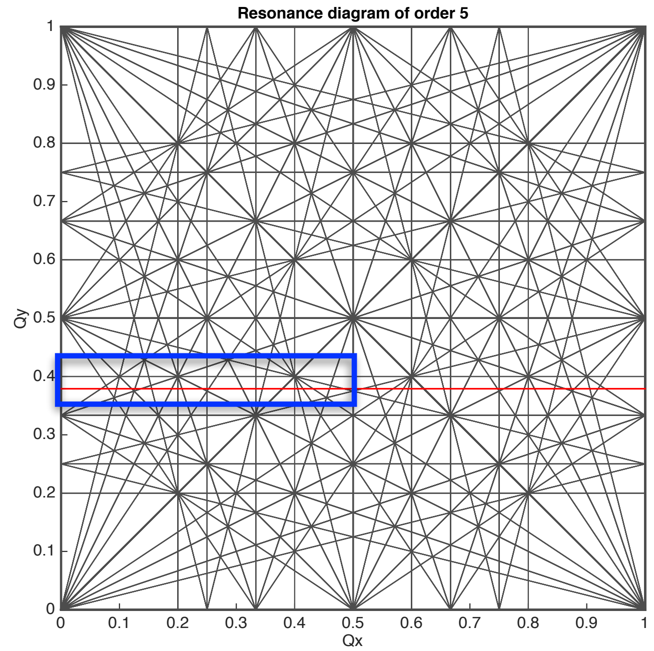
Plot smear on top of tune diagram to identify resonances

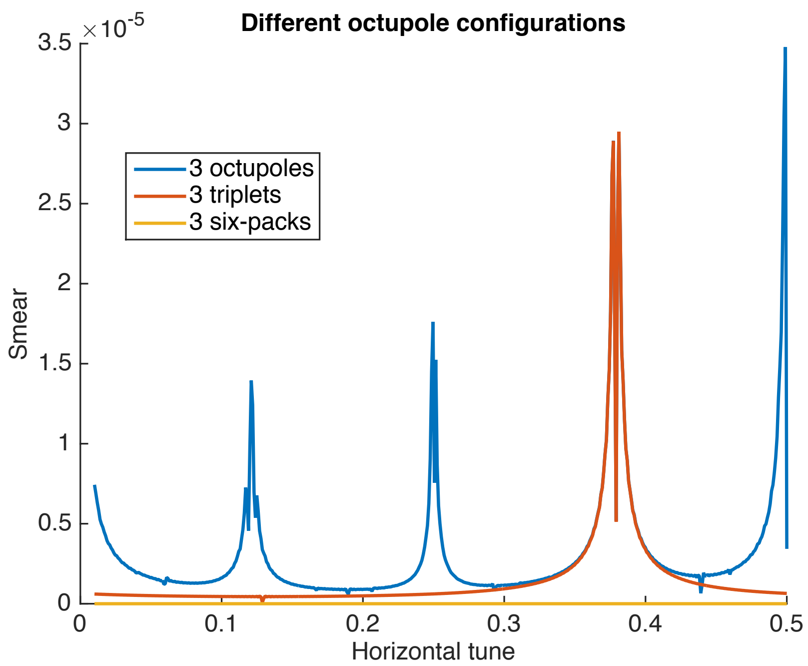


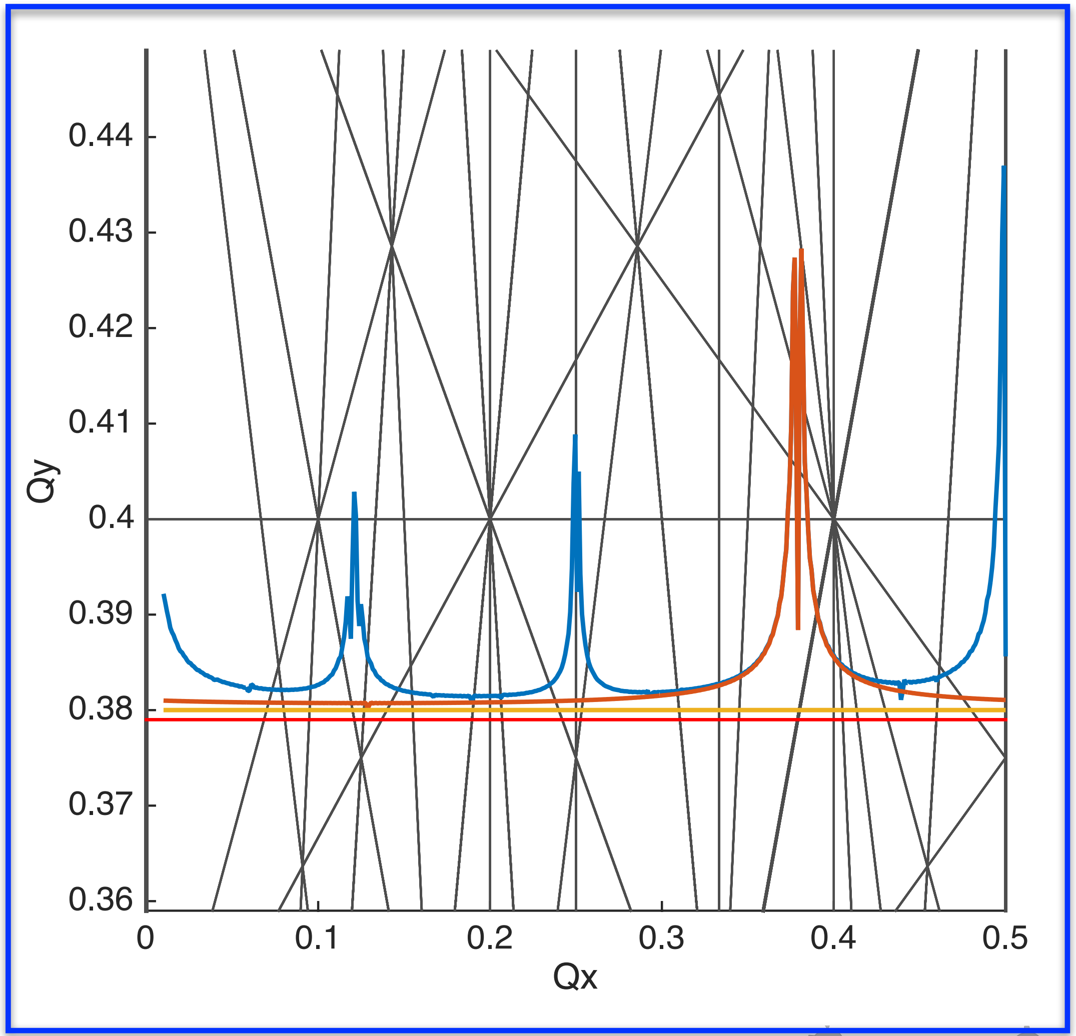


Resonances

Plot smear on top of tune diagram to identify resonances

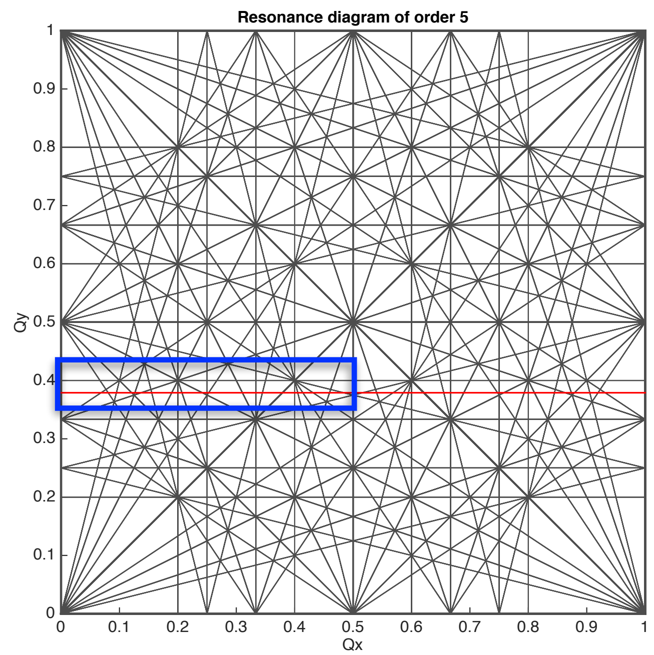


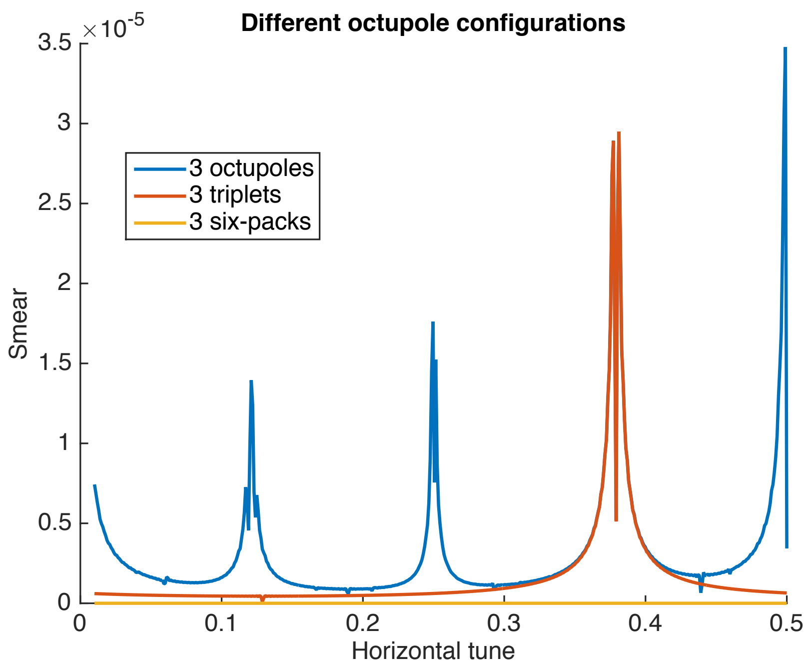


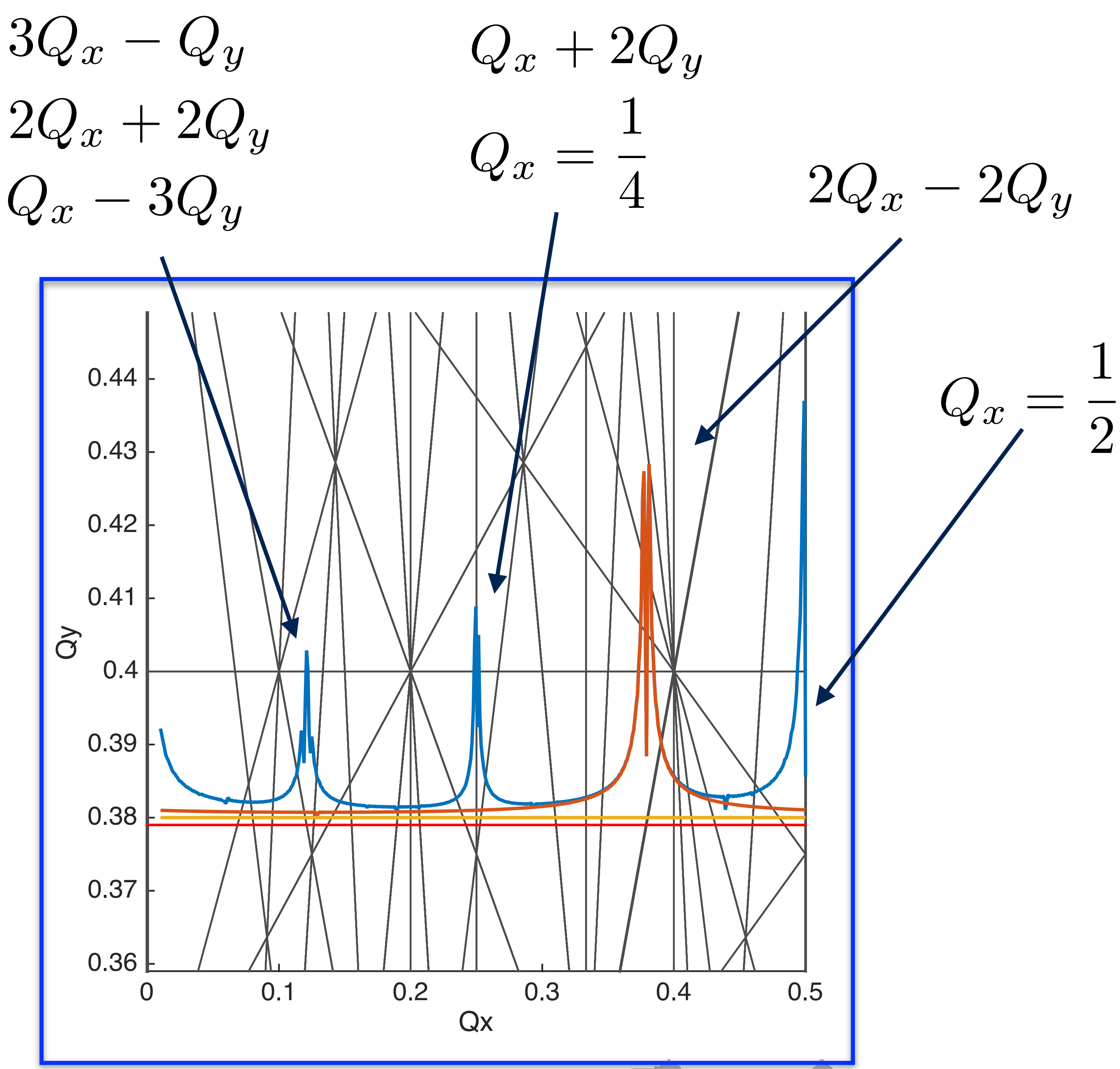


Resonances

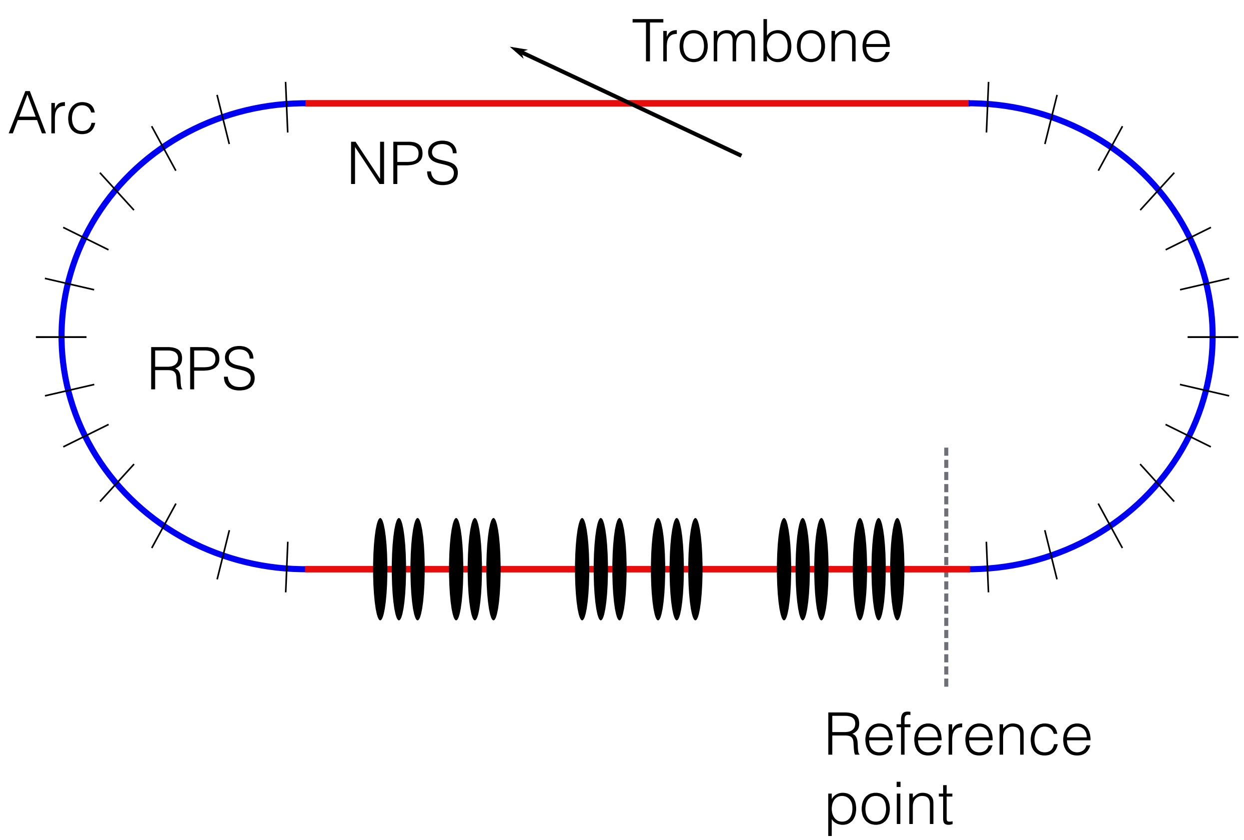
Plot smear on top of tune diagram to identify resonances





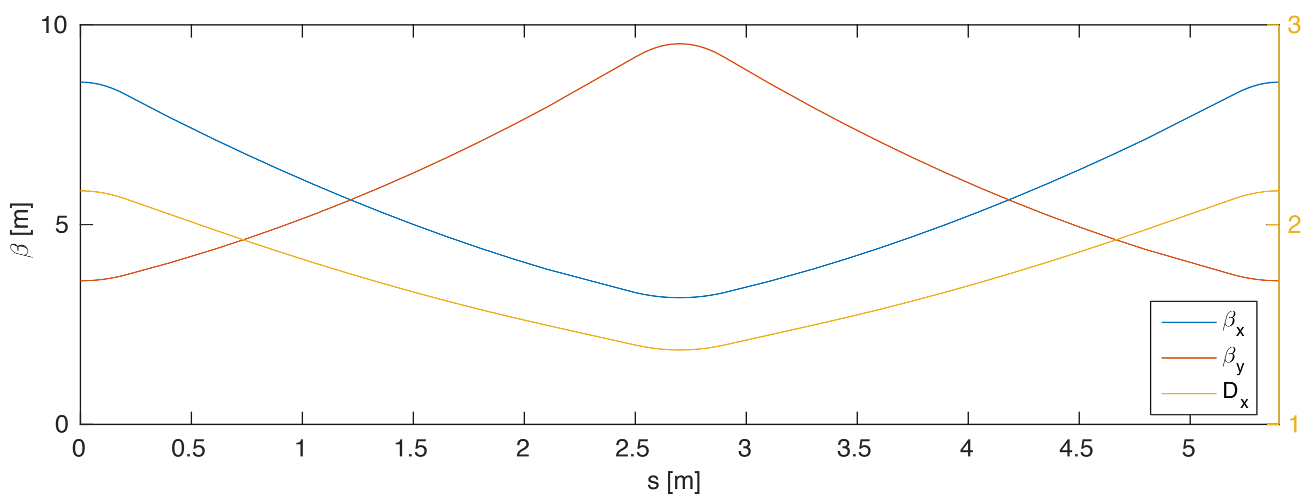


Simulation - extended model



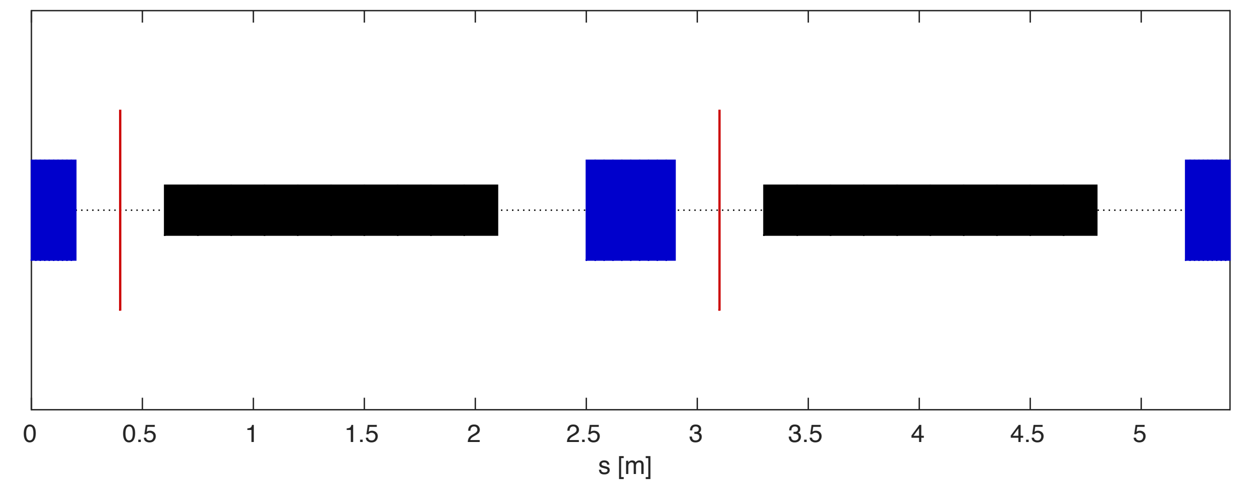
Two straight sections (NPS):

- a”trombone” for setting the overall tune
- a section containing the octupoles
- Simulate the three different octupole configurations

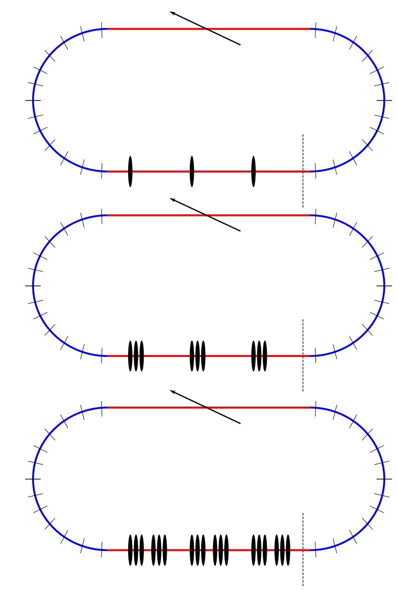


Each arc consists of 9 FODO cells.

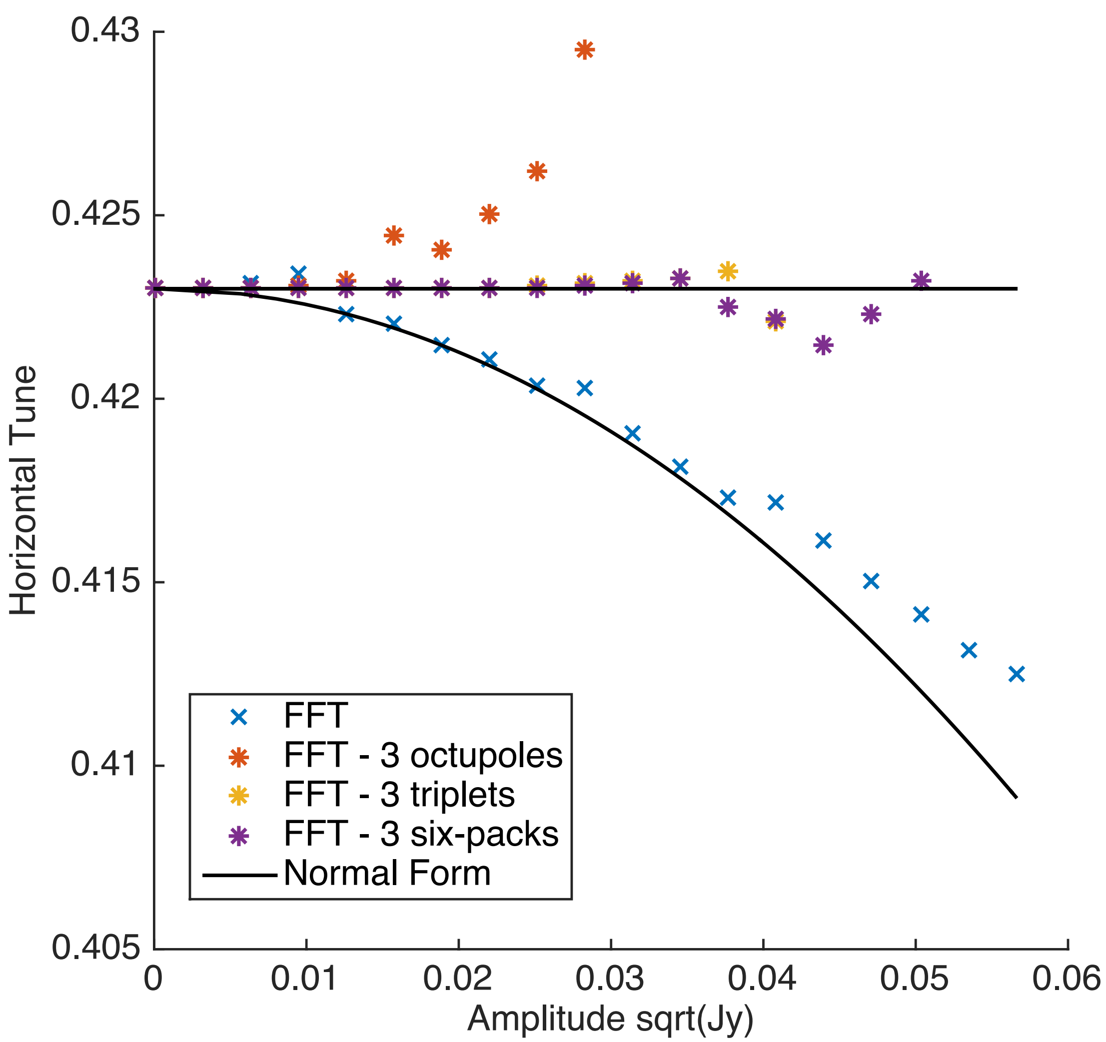
The FODO cells include:
- 2 dipole bends
- 2 sextupoles for chromaticity correction

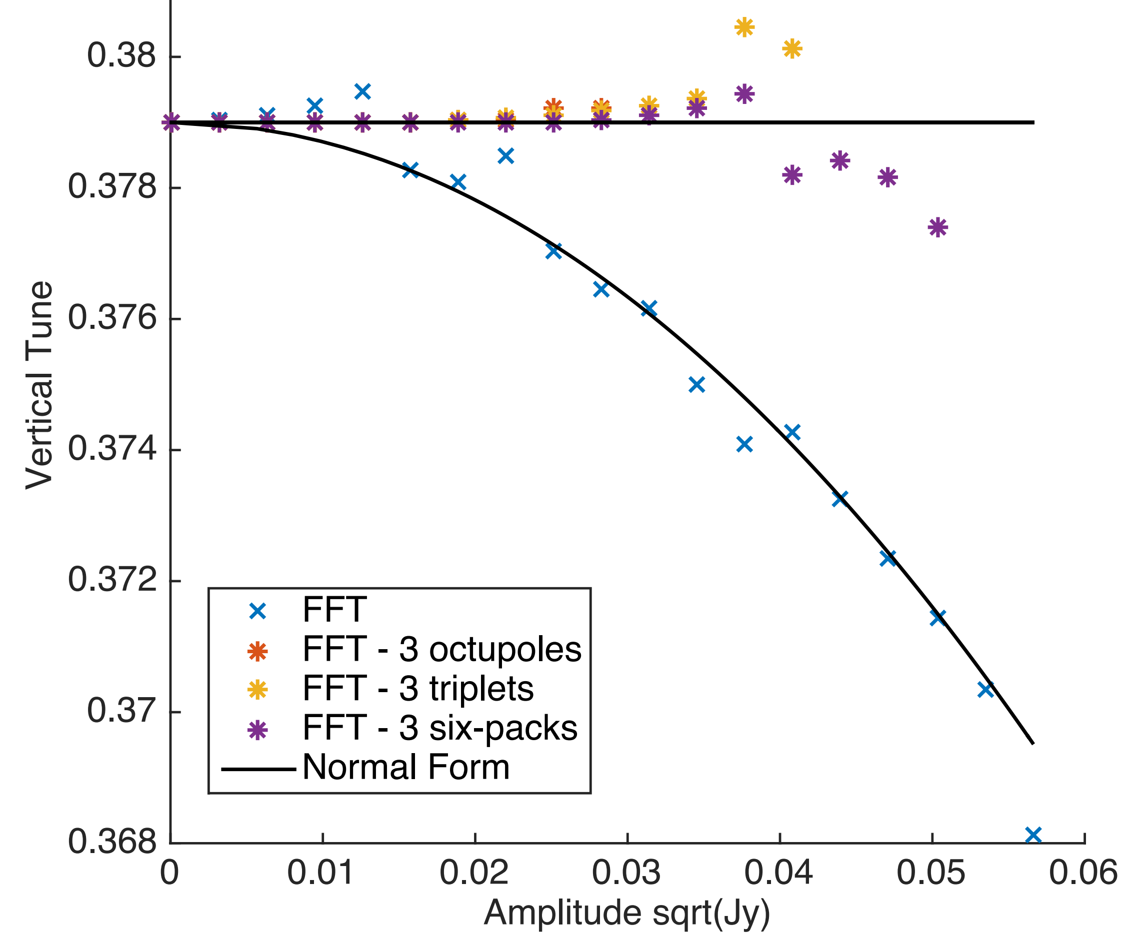


Simulation - results



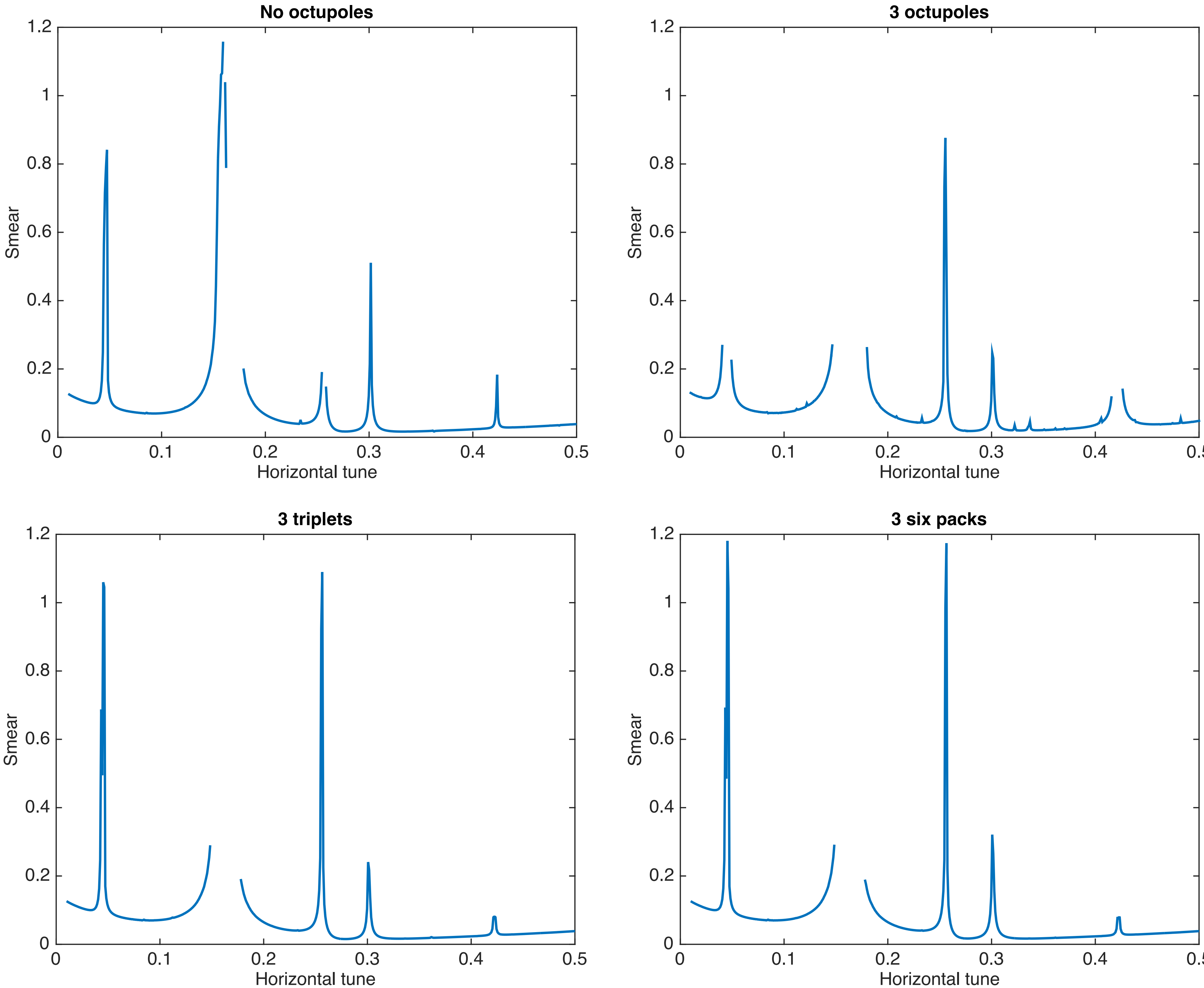
Tune-shift for the different octuple configurations:





Configuration with 3 octupoles reduces stability.
Using triplets are more stable and six packs even more so.

Simulation - smear plots



- Configuration with only three octupoles worsen stability and introduces some additional resonances.
- Triplets or six-packs do not add resonances.
- For this case resonances are dominated by the sextuplets.

Conclusions

- Powerful analytical method
- Code to treat Hamiltonians and normal forms
- Optimum placement of octupoles for tune-shift compensation

Future work

- Include resonant normal forms
- Apply method to other related issues
- Apply method to an actual machine
- …

Thank you for your attention!