The Phase Jump Method for Efficiency Enhancement in Free-Electron Lasers

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Motivation

Efficiency as figure of merit for FELs

$$\text{Efficiency} = \frac{\text{Power of output radiation}}{\text{Power of injected electron beam}}$$

User demand: Future experiments at x-ray FELs

- **More photons** within a **shorter pulse** $\Rightarrow$ Higher power
- e.g. Single-shot coherent diffractive imaging of single biological molecules

Efficiency limit: Saturation of FEL process

$$\text{Efficiency at saturation} \approx \rho \sim 10^{-3}$$
$$1D \text{ saturation length} \approx \lambda_u/\rho$$

Efficiency enhancement: Overcome efficiency limit

- Principle: Sustain energy extraction process beyond initial saturation
- Methods: Undulator tapering, **phase jump method**

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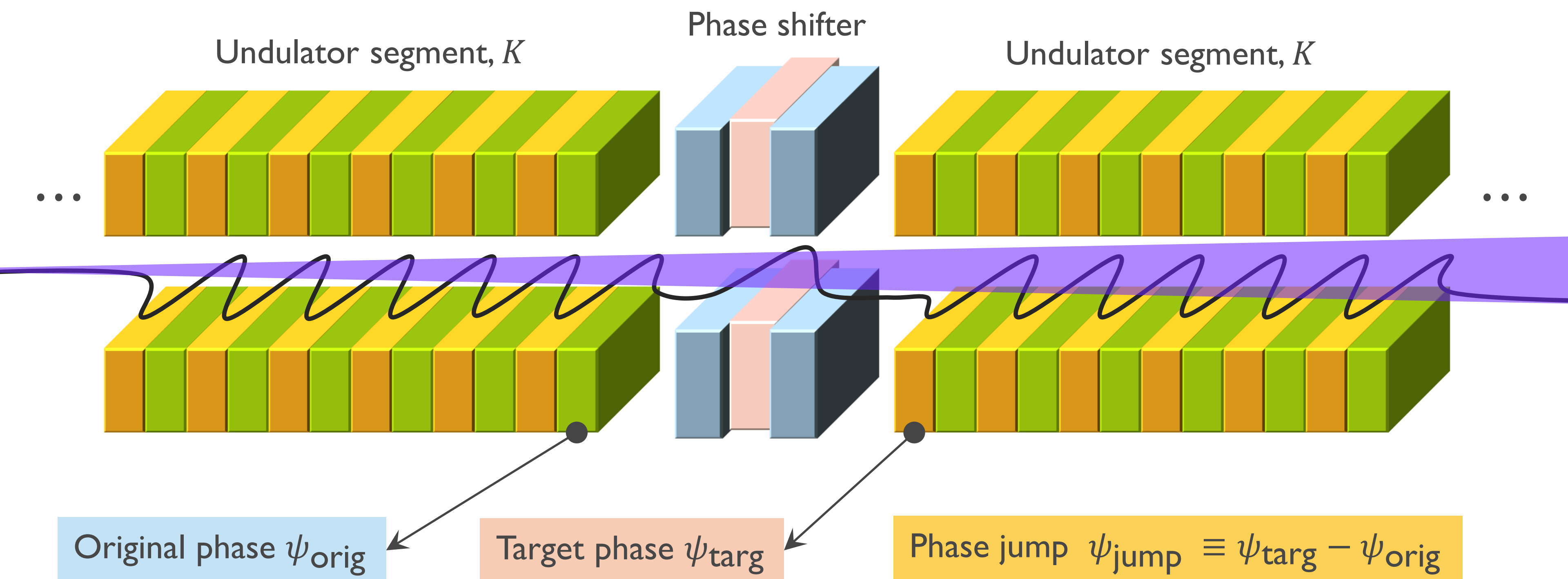
Phase Jump

Phase shifter = compact magnetic chicane or single-period undulator

Function: Adjust phase angle between electron beam and optical field



Undulator tapering: $K = K(z)$, $\psi_{jump} = 0$ (or a multiple of 2π)

Phase jump method: K = constant, $\psi_{jump} \neq 0$ in general

Hamiltonian Mechanics

- Hamiltonian for FEL process

$$H(\psi,\eta) = ck_u\eta^2 + \frac{c\Omega^2}{2k_u}(1-\cos\psi)$$

- Hamilton equations

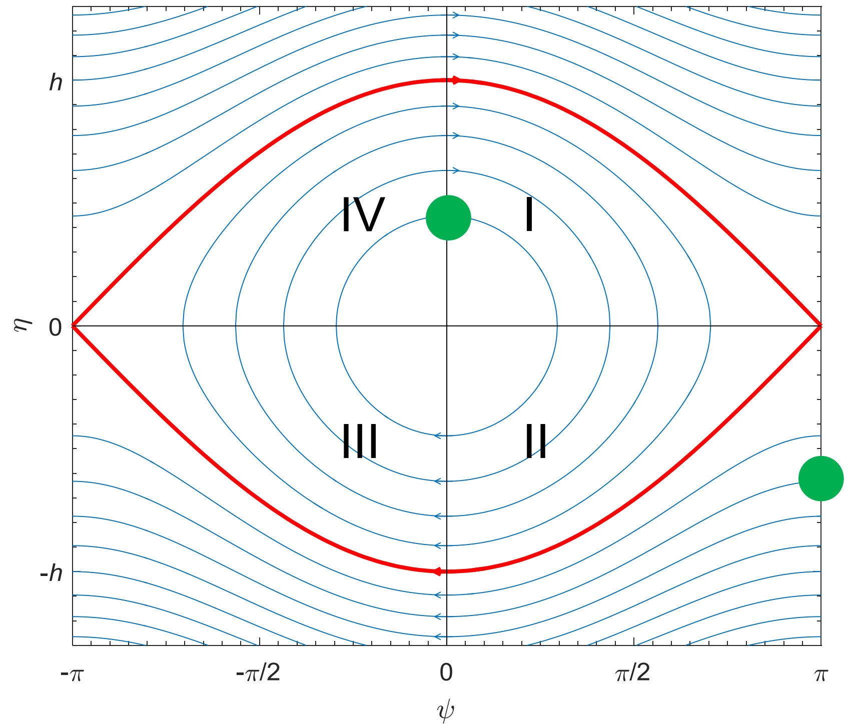
$$\frac{d\psi}{dt} = \frac{\partial H}{\partial \eta}$$
$$\frac{d\eta}{dt} = -\frac{\partial H}{\partial \psi}$$

- Equations of motion

$$\frac{d\psi}{dz} = 2k_u\eta$$
$$\frac{d\eta}{dz} = -\frac{\Omega^2}{2k_u}\sin\psi$$

- Pendulum equation

$$\frac{d^2\psi}{dz^2} + \Omega^2\sin\psi = 0$$



- Quadrants I & II: Particles decelerate

$$\psi > 0 \quad \Rightarrow \quad \frac{d\eta}{dz} < 0$$

- Quadrants III & IV: Particles accelerate

$$\psi < 0 \quad \Rightarrow \quad \frac{d\eta}{dz} > 0$$

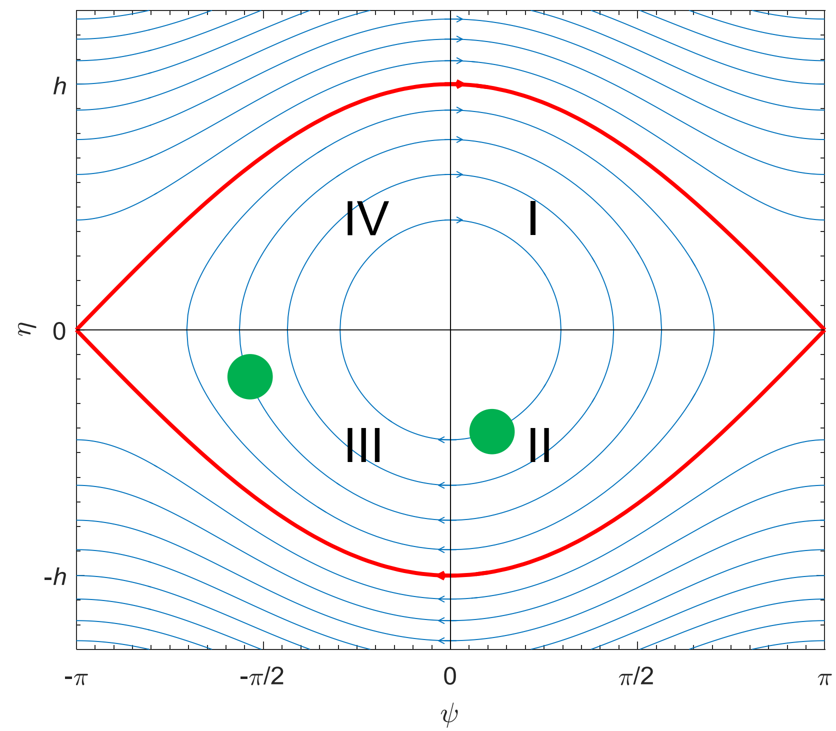
Average Particle

- **Microbunching** fully developed at initial saturation point
- Define **average particle** within the microbunch μ

$$\bar{\eta} \equiv \langle \eta \rangle_{\mu} = \frac{1}{N} \sum_{j \in \mu}^{N} \eta_j$$

$$\bar{\psi} \equiv \arg\langle e^{-i\psi} \rangle_{\mu} = -i \ln\left(\frac{1}{N} \sum_{j \in \mu}^{N} e^{-i\psi_j} \right)$$

- Keep average particle in deceleration quadrants (I & II)
- Keep average particle away from acceleration quadrants (III & IV)



- Microbunched beam decelerates and radiates coherently
- Energy extraction continues beyond initial saturation point

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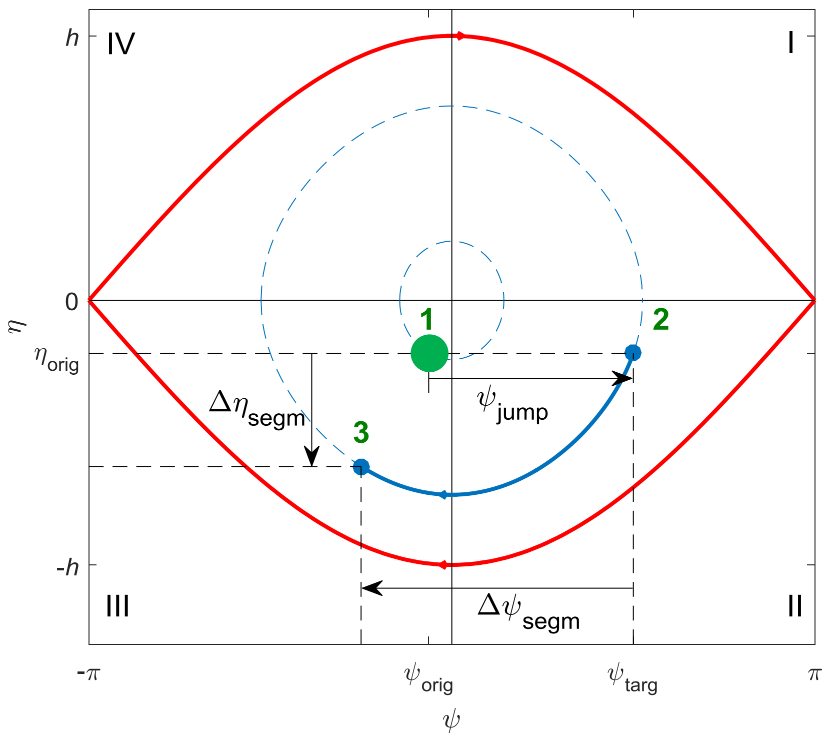
Microbunch Deceleration Cycle

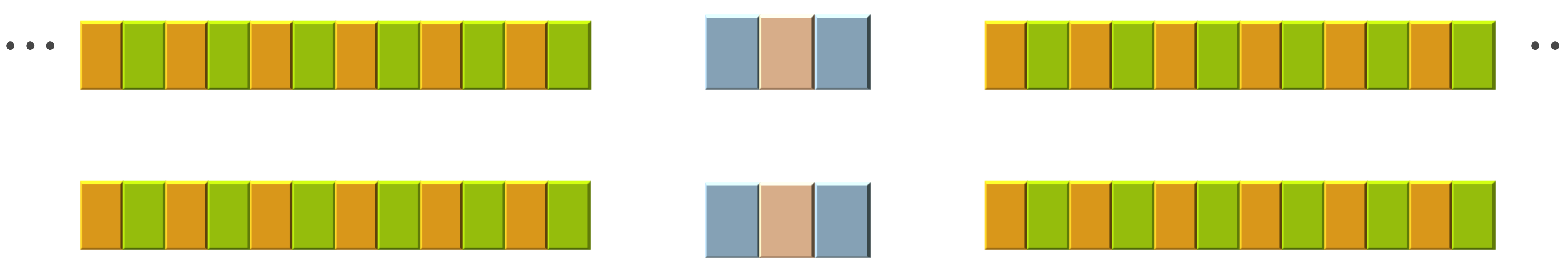


- Position **1**: Start point of drift section
  $(\bar{\psi}, \bar{\eta}) = (\psi_{orig}, \eta_{orig})$

- Position **2**: End point of drift section
  $(\bar{\psi}, \bar{\eta}) = (\psi_{targ}, \eta_{orig})$

- Positon **3**: End point of undulator segment
  $(\bar{\psi}, \bar{\eta}) = (\psi_{targ} + \Delta\psi_{segm}, \eta_{orig} + \Delta\eta_{segm})$

- Position **3** of the current cycle becomes Position **1** of the next cycle.



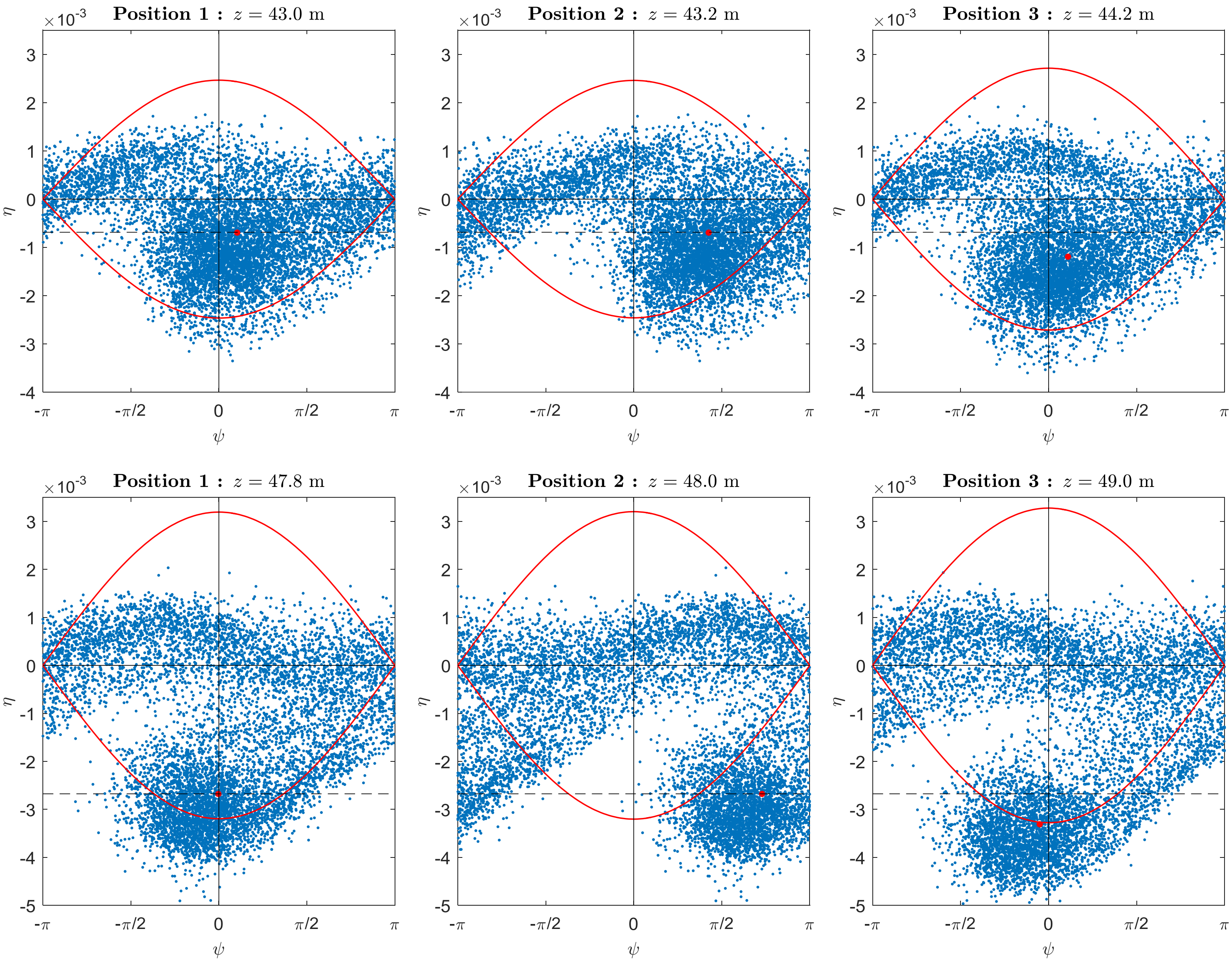
Undulator segment
K

Phase shifter
ψ_{jump}

Undulator segment
K

Microbunch Deceleration Cycle

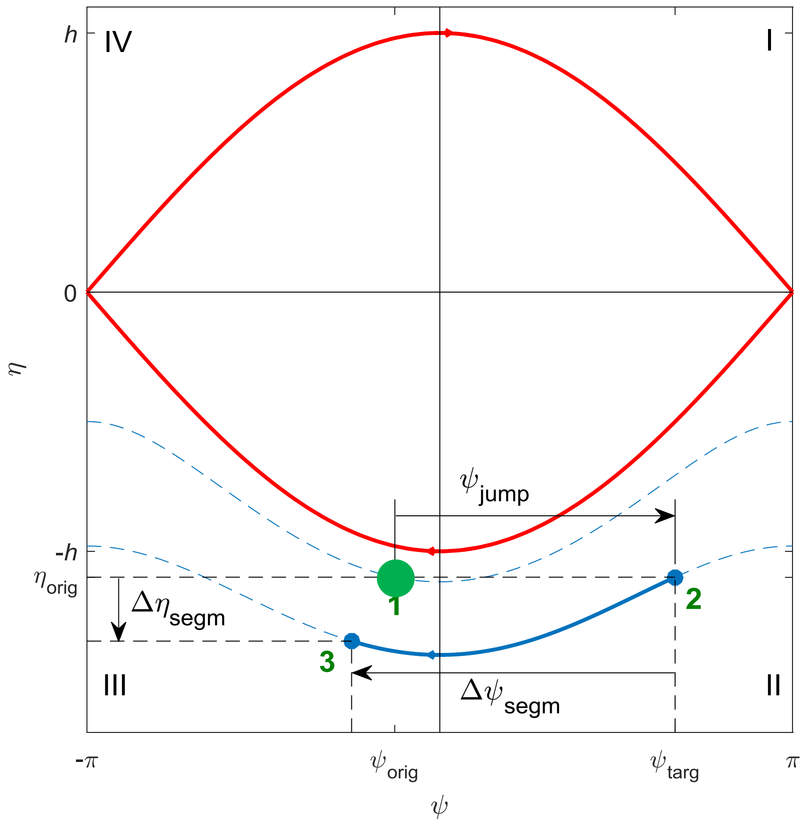
From numerical simulation



Bucket height increases with optical field amplitude

Average particle out of bucket, but not the end of the story!

Out-of-bucket Regime



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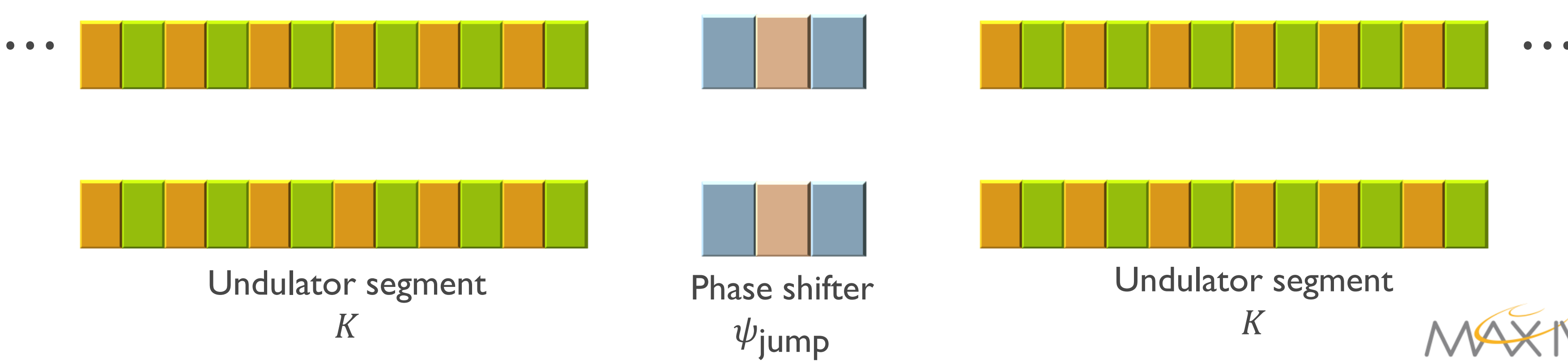
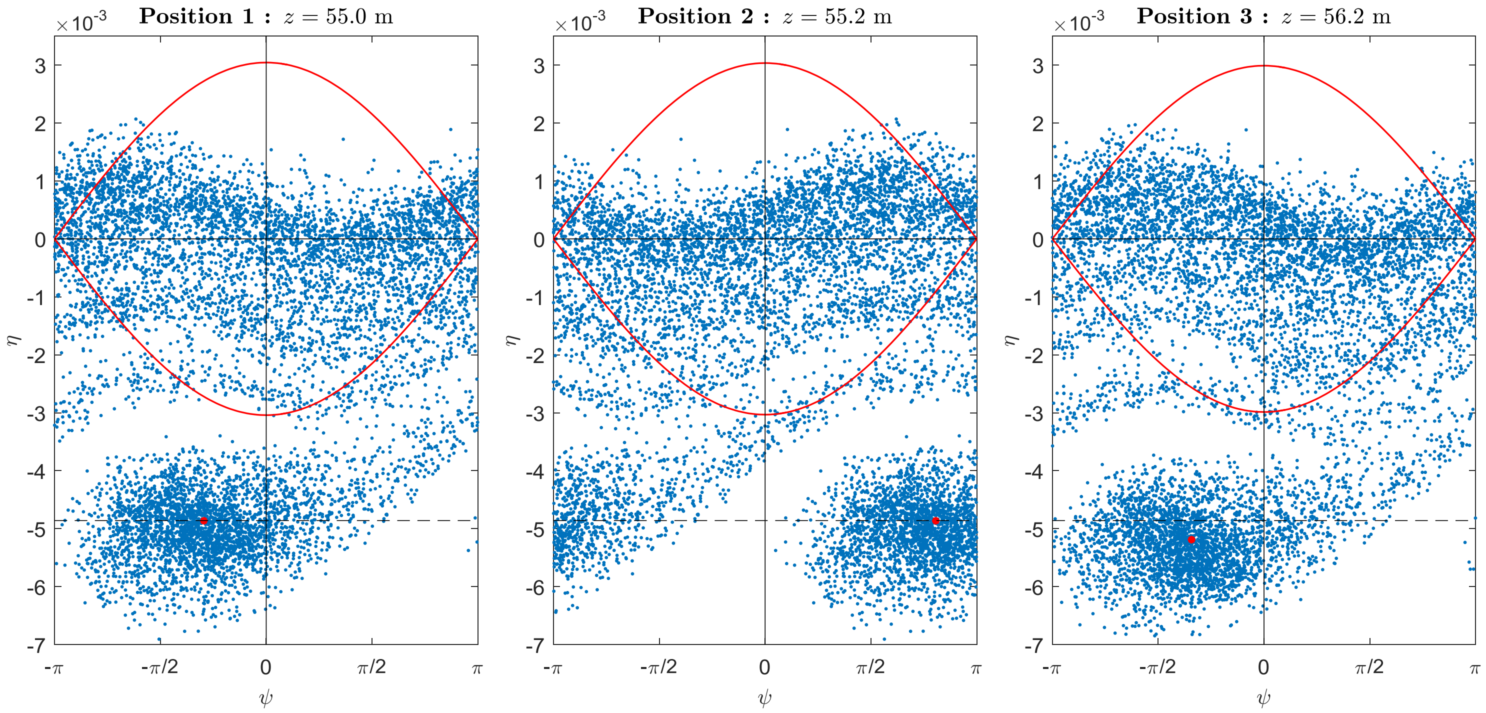


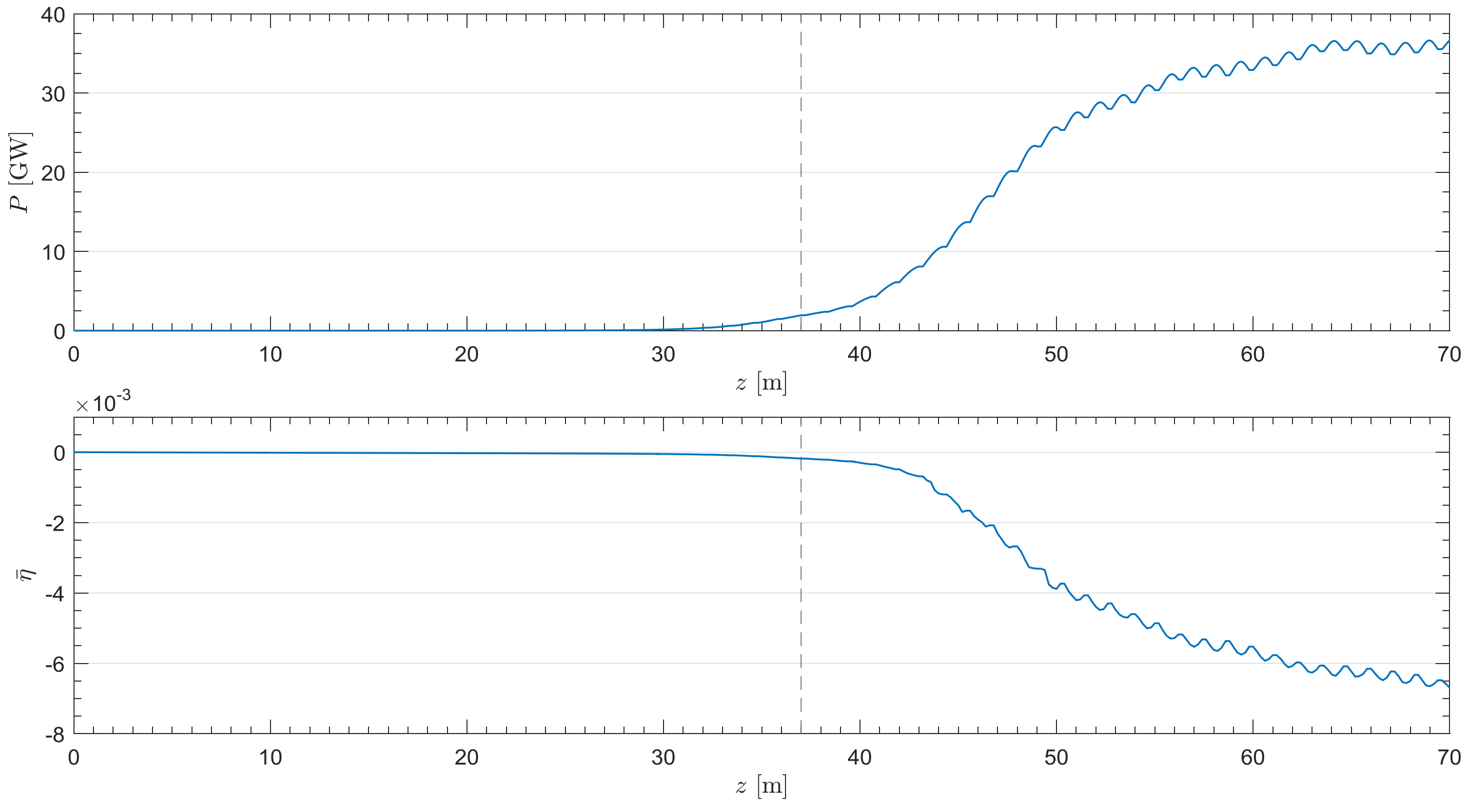
Out-of-bucket Regime

From numerical simulation



Radiation Power Evolution

From numerical simulation



- **Initial saturation:** $z = 37$ m , $P = 2$ GW.
- **Final saturation**: $z \approx 65$ m , $P = 36$ GW.

Efficiency enhanced by 18 times

- Phase jumps not fully optimized for maximum final power.

Final Saturation

- Recall FEL equations of motion

$$\frac{d\psi}{dz} = 2k_u\eta$$
$$\frac{d\eta}{dz} = -\frac{\Omega^2}{2k_u}\sin\psi$$

- Rate of energy extraction determined by slope of trajectory

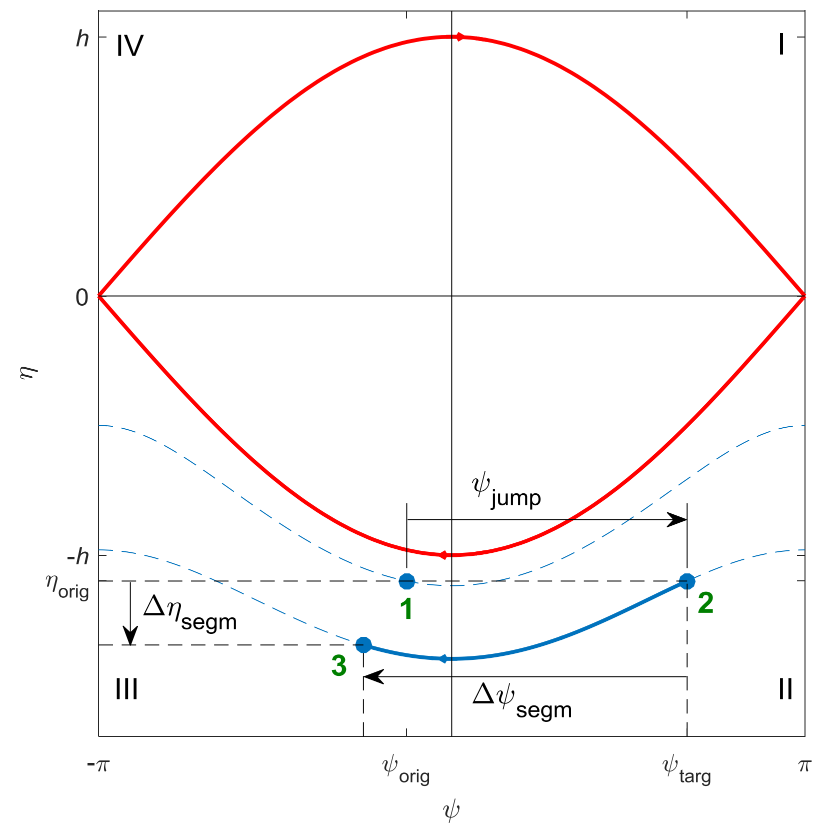
$$\left|\frac{d\eta}{d\psi}\right| = \frac{\Omega^2|\sin\psi|}{4{k_u}^2|\eta|} \propto \frac{1}{|\eta|}$$

$$\rightarrow 0 \text{ for large } |\eta|$$

- Impossibility to avoid Quadrant III

$$|\Delta\psi_{\text{segm}}| \approx 2k_u|\eta|L_{\text{segm}}$$

$$\text{Large } |\eta| \Rightarrow |\Delta\psi_{\text{segm}}| > \pi$$



- In the end, $\Delta\eta_{\text{segm}} < 0$ no longer achievable.

- No more net energy transfer from electrons to radiation

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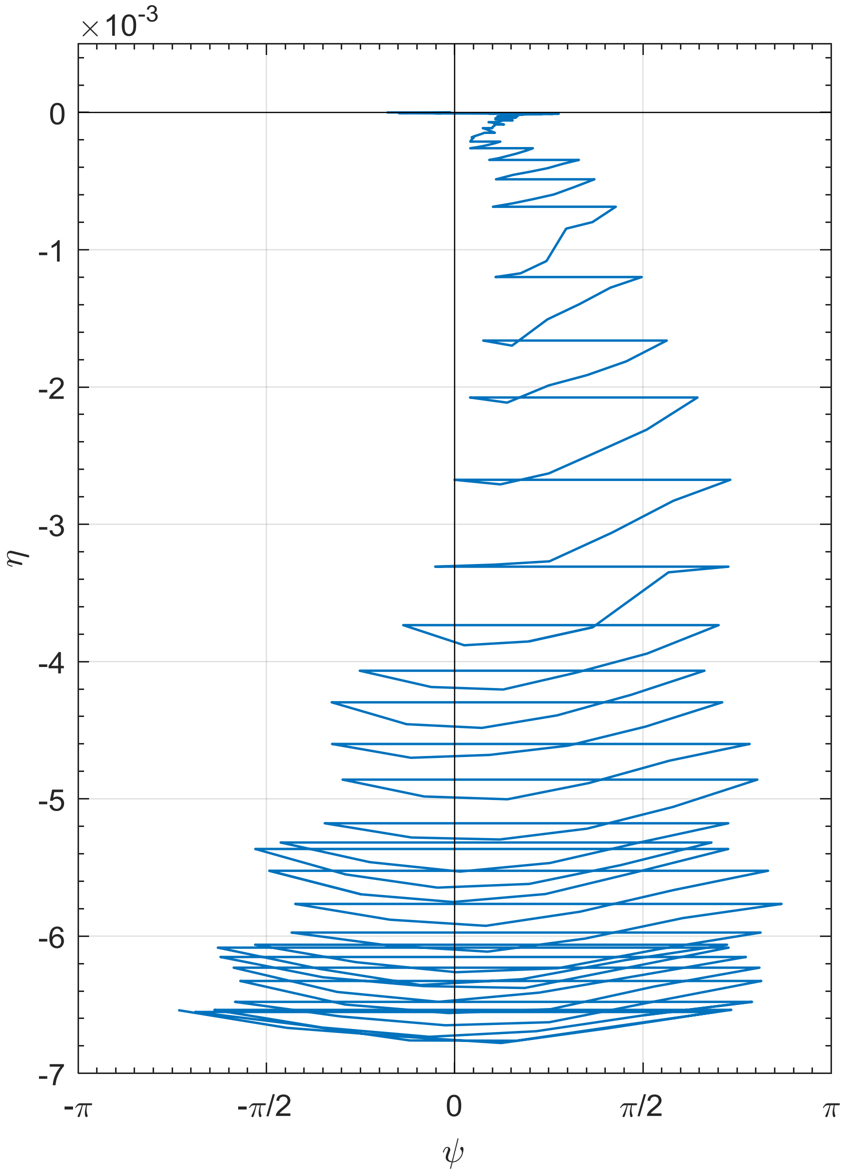
Trace of the Average Particle

From numerical simulation



- Horizontal line segment corresponds to transition from position **1** to position **2**.
- Vertical spacing between successive horizontal line segments is $|\Delta\eta_{\text{segm}}|$.
- $|\Delta\eta_{\text{segm}}|$ decreases with η.
- More negative η, more difficult to avoid the acceleration quadrants $(\psi < 0)$

$$\because \; |\Delta\psi_{\text{segm}}| \approx 2k_u|\eta|L_{\text{segm}}$$

- Final saturation at $\eta \approx -6 \times 10^{-3}$

Compare & Contrast

	Phase Jump Method	**Undulator Tapering**
Model	Alan's model – under development	Kroll-Morton-Rosenbluth (KMR) model
Particle to trace	Average particle within microbunch	Synchronous particle, analogous to that in RF acceleration
Knob to turn	ψ_{jump} of each phase shifter	K of each undulator segment
Bucket motion	Stationary	Moving towards lower energy
Deceleration mechanism	Microbunch deceleration cycle	Bucket deceleration
Energy extraction outside bucket	Possible (out-of-bucket regime)	Generally impossible
Rate of energy extraction	Determined by $d\eta/d\psi$	Determined by dK/dz
Main cause of final saturation	$d\eta/d\psi \rightarrow 0$ and $\Delta\psi_{\text{segm}} > \pi$	Weakening of refractive guiding and particle detrapping

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Summary & Outlook

- Developed a steady-state model of the phase jump method
- FEL efficiency enhanced by sustaining power growth beyond initial saturation
- Microbunch deceleration cycle of the average particle
- Energy extraction in the out-of-bucket regime
- Mechanism of final saturation
- Correlated the model with numerical simulations
- Compare and contrast: phase jump method vs undulator tapering
- Ongoing work: developing optimization schemes for the phase jumps
- Ongoing work: suppression of sidebands in FEL spectrum

Back-up Slide: Phase Space Coordinates

- Particle dynamics in longitudinal phase space described by **phase** and **energy**
- Particle energy = γmc^2
- FEL resonant condition

$$\lambda = \frac{\lambda_u}{2{\gamma_R}^2}\left(1+\frac{K^2}{2}\right) \quad \Leftrightarrow \quad \gamma_R = \sqrt{\frac{\lambda_u}{2\lambda}\left(1+\frac{K^2}{2}\right)}$$

- Undulator tapering: $K = K(z) \Rightarrow \gamma_R = \gamma_R(z)$
- Phase jump method: $K = \text{constant} \Rightarrow \gamma_R = \text{constant}$
- **Energy coordinate:** Relative energy deviation from resonant energy

$$\eta \equiv \frac{\gamma - \gamma_R}{\gamma_R}$$

- **Phase coordinate:** Phase with respect to ponderomotive potential

$$\psi \equiv (k + k_u)z - ckt + \phi \in [-\pi, \pi]$$