# BERRY-ESSEEN FOR SUMMANDS ZOLOTAREV–$\zeta$–CLOSE TO NORMAL

ABSTRACT FOR "ESSEEN 100 YEARS", SEPTEMBER 2018, UPPSALA

LUTZ MATTNER, UNIVERSITÄT TRIER

We prove for some absolute constant $c < \infty$ the bound

$$\left\|\widetilde{F^{*n}} - \Phi\right\|_\infty \;\leq\; \frac{c}{\sqrt{n}}\,\zeta_1(\widetilde{P} - \mathrm{N}) \vee \zeta_3(\widetilde{P} - \mathrm{N}) \qquad \text{for } n \geq 2$$

where $P$ with distribution function $F$ is any nondegenerate law on $\mathbb{R}$ with a finite third moment. Here $F^{*n}$ denotes the distribution function of the $n$th convolution power of $P$, a tilde $\widetilde{\ }$ indicates standardization, $\Phi$ is the distribution function of the standard normal law N, and $\zeta_r$ with $r \in \mathbb{N}$ denotes Zolotarev's $\zeta$-norm on the signed measures on $\mathbb{R}$ with vanishing moments of orders $0, \ldots, r-1$ and finite moments of order $r$. So for $r \in \{1, 2, 3\}$, we have

$$\begin{aligned}
\zeta_r(\widetilde{P} - \mathrm{N}) &:= \sup\left\{\int f \,\mathrm{d}\big(\widetilde{P} - \mathrm{N}\big) \;:\; f^{(r-1)} \text{ Lipschitz with constant } 1\right\}\\
&= \int \left|\int_{-\infty}^{x} \frac{(x-y)^{r-1}}{(r-1)!}\,\mathrm{d}\big(\widetilde{P} - \mathrm{N}\big)(y)\right| \mathrm{d}x\\
&\leq \tfrac{1}{r!}\int r|x|^{r-1}\left|\widetilde{F}(x) - \Phi(x)\right| \mathrm{d}x \;=:\; \tfrac{1}{r!}\varkappa_r(\widetilde{P} - \mathrm{N})\\
&\leq \tfrac{1}{r!}\int |x|^r \,\mathrm{d}\left|\widetilde{P} - \mathrm{N}\right|(x) \;=:\; \tfrac{1}{r!}\nu_r(\widetilde{P} - \mathrm{N}) \;\leq\; 1.8\,\nu_3(\widetilde{P}),\\
\zeta_1(\widetilde{P} - \mathrm{N}) \vee \zeta_3(\widetilde{P} - \mathrm{N}) \vee 1 &\leq \nu_3(\widetilde{P}),\\
\zeta_1(\widetilde{P} - \mathrm{N}) &= \varkappa_1(\widetilde{P} - \mathrm{N}) \;=\; \left\|\widetilde{F} - \Phi\right\|_1 \;\not\asymp\; \nu_1(\widetilde{P} - \mathrm{N}),\\
\zeta_1(\widetilde{P} - \mathrm{N}) \vee \zeta_3(\widetilde{P} - \mathrm{N}) &\not\asymp \varkappa_1(\widetilde{P} - \mathrm{N}) \vee \varkappa_3(\widetilde{P} - \mathrm{N}).
\end{aligned}$$

The bound presented hence improves, with its dependence on $\widetilde{P} - \mathrm{N}$ being linear through the rather weak norm $\zeta_1 \vee \zeta_3$, the three mutually incomparable earlier results of Sazonov (1972)–Zolotarev (1973) who have instead $\nu_1 \vee \nu_3$, Ulyanov (1976) who has $(\varkappa_1 \vee \varkappa_3) \vee (\varkappa_1 \vee \varkappa_3)^{1-2^{-n}}$, and Senatov (1998) who has an additional term $\frac{1}{n^\gamma}\left\|\widetilde{F} - \Phi\right\|_\infty$. As these three precursors and a few others beginning with a result of Paulauskas (1969), it is stronger up to a constant factor than the classical Berry (1941)–Esseen (1942) bound, which has $\nu_3(\widetilde{P})$ in place of $\zeta_1(\widetilde{P} - \mathrm{N}) \vee \zeta_3(\widetilde{P} - \mathrm{N})$.

Universität Trier, Fachbereich IV – Mathematik, 54286 Trier, Germany



[Version: 2018-09-_03_Lutz_Mattner_Abstract_for_Esseen_100_years.texSeptember 3, 2018 at 9:16]