Child-Langmuir law for photoinjectors

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Original Child-Langmuir law: history

- 1D diode problem,
- Infinite transverse sizes,

$$J_{1D} = \frac{4}{9}\varepsilon_0 \sqrt{\frac{2\,q}{m}}\,\frac{V^{3/2}}{D^2}$$

(1911)

- 2D case for large emitting area,

$$\frac{J_{2D}}{J_{1D}} = 1 + \frac{D}{4R}, \qquad \frac{D}{R} \leq 1$$

(2001)

- Empirical for photoinjectors

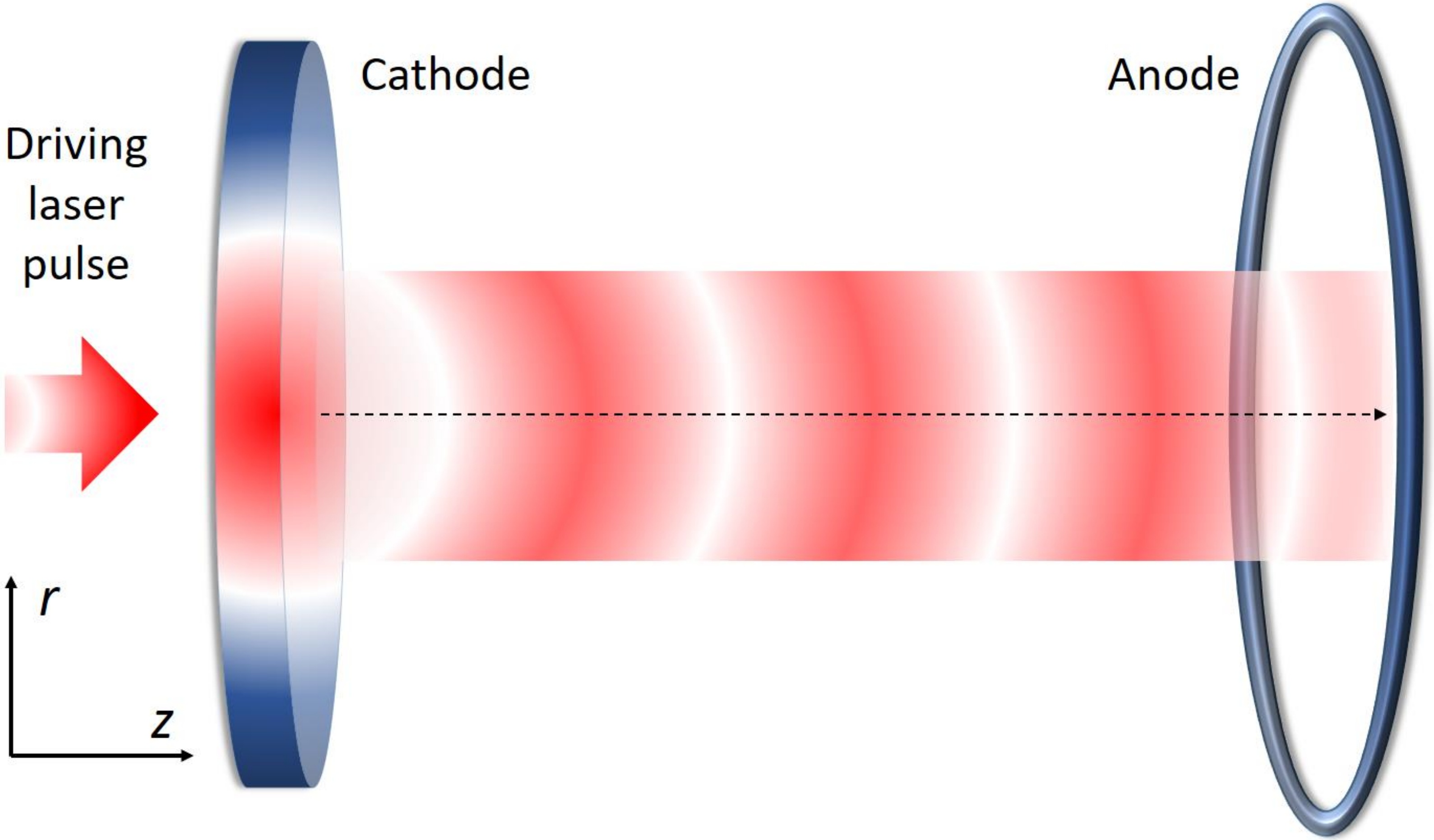
$$J_{ph} = \frac{4}{9}\varepsilon_0 \sqrt{\frac{2\,q}{m\,R}}\,E_{acc}^{3/2}$$

(2014)

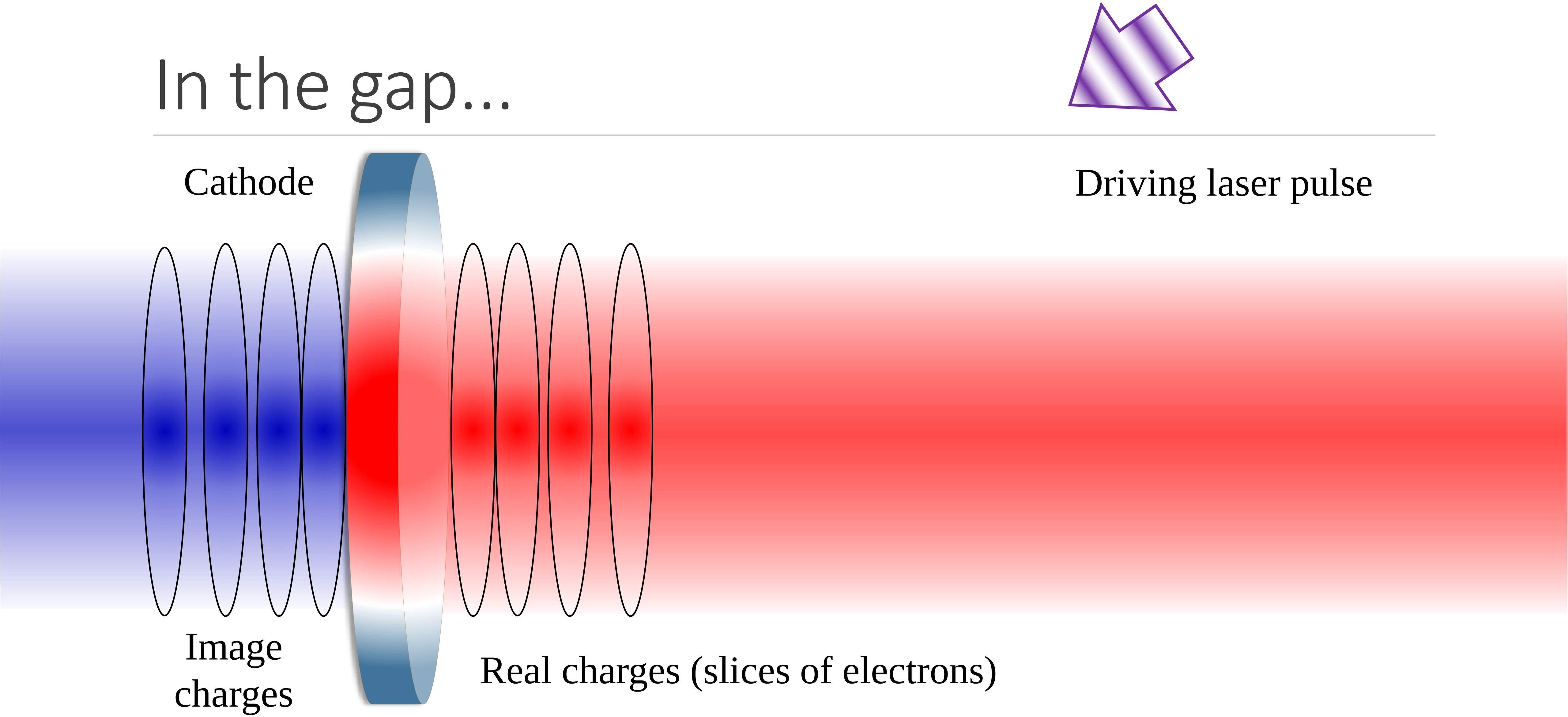
Photoinjector cathode

Principal differences from 1D:

- Longitudinal size of the extracted bunch is larger than the transverse size, $L >> R$
- Instead of a physical diode, an effective diode is to be considered



In the gap…



Cathode

Driving laser pulse

Image charges

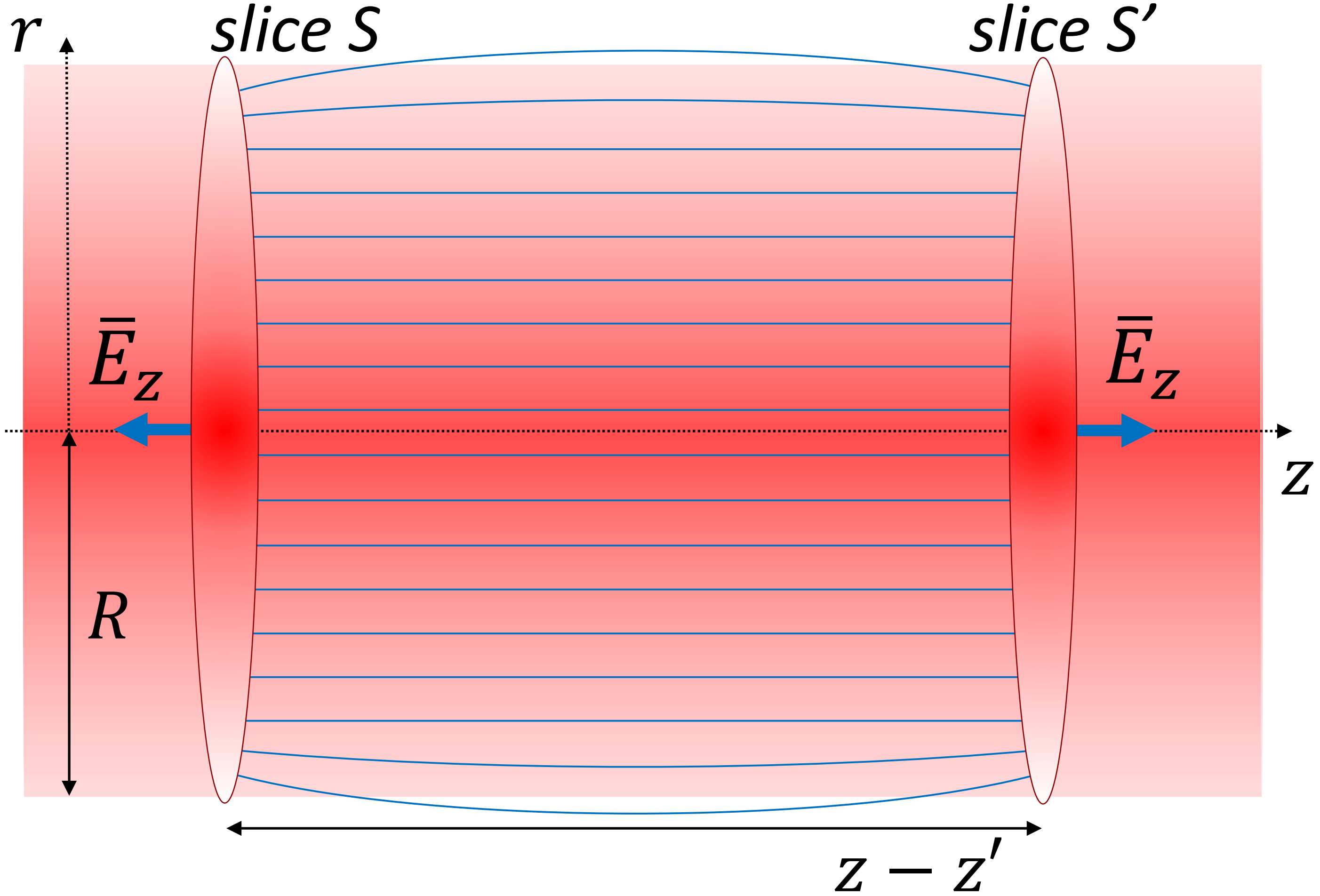
Real charges (slices of electrons)

Energy-wise interaction field

$$U = \frac{1}{4\pi\varepsilon_0}\int_S\int_{S'}\frac{\sigma(x,y)\,\sigma(x',y')}{|\vec{x}-\vec{x}'|}dS'dS$$

$$Q_{sl}\overline{E}_z = -\frac{\partial U}{(z-z')}$$

$$\overline{E}_z(z,z') = \frac{1}{4\pi\varepsilon_0 Q_{sl}}\int_S\int_{S'}\frac{\sigma(\vec{x})\,\sigma(\vec{x}')(z-z')}{|\vec{x}-\vec{x}'|^3}dS'dS$$

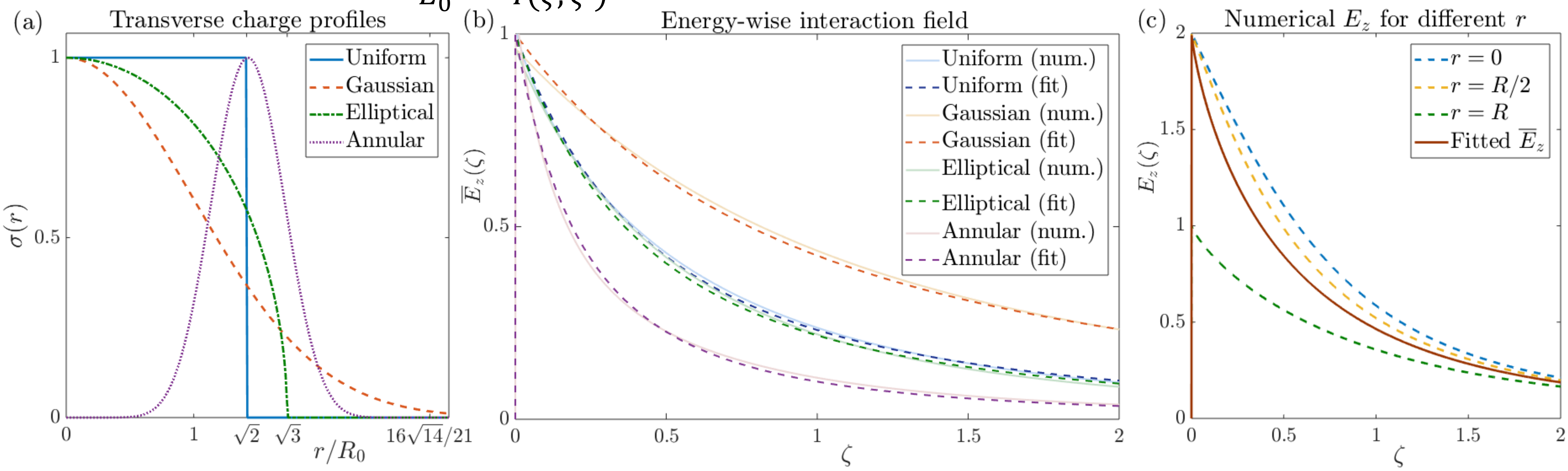


Energy-wise interaction field: fit

$$\overline{E}_z(\zeta,\zeta') = \underbrace{\frac{Q_{sl}\beta}{4\pi\varepsilon_0 R^2}}_{E_0}\underbrace{\frac{sgn(\zeta-\zeta')}{(1+\alpha|\zeta-\zeta'|)^2}}_{I(\zeta,\zeta')}$$

$$\zeta = \frac{z}{R}$$

$\alpha = 1.078,$
$\beta = 1.955$



Child-Langmuir law

$$E_{tot}^{SC}(\zeta) = E_0 \int_{-\infty}^{\infty} I(\zeta, \zeta') f(\zeta') d\zeta'$$

$$f(\zeta') = |\zeta'|^{-\frac{2}{3}}$$

Child-Langmuir law

$$E_{tot}^{SC}(\zeta) = E_0 \int_{-\infty}^{\infty} I(\zeta, \zeta')f(\zeta')d\zeta' \qquad f(\zeta') = |\zeta'|^{-\frac{2}{3}}$$

At the cathode

$$\underbrace{E_{acc}}_{\text{External field}} = G\,J(\Delta) \int_{-\infty}^{\infty} I(0, \zeta')f(\zeta')d\zeta' = G\,J(1) \int_{-1}^{1} I(0, \zeta')f(\zeta')d\zeta'$$

Child-Langmuir law

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<u>At the cathode</u>

$$\underbrace{E_{acc}}_{\text{External field}} = \underbrace{G\,J(\Delta)\int_{-\infty}^{\infty} I(0,\zeta')f(\zeta')d\zeta'}_{\text{SC field from an infinitely long bunch}} = G\,J(1)\int_{-1}^{1} I(0,\zeta')f(\zeta')d\zeta'$$

Child-Langmuir law

$$E_{tot}^{SC}(\zeta) = E_0 \int_{-\infty}^{\infty} I(\zeta,\zeta')f(\zeta')d\zeta'$$

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<u>At the cathode</u>

$$\underbrace{E_{acc}}_{\text{External field}} = \underbrace{G\,J(\Delta)\int_{-\infty}^{\infty} I(0,\zeta')f(\zeta')d\zeta'}_{\text{SC field from an infinitely long bunch}} = \underbrace{G\,J(1)\int_{-1}^{1} I(0,\zeta')f(\zeta')d\zeta'}_{\text{SC field from a bunch with } \Delta = 1}$$

It allows us to use J_{2D} as a reference

Child-Langmuir law

$$E_{tot}^{SC}(\zeta) = E_0 \int_{-\infty}^{\infty} I(\zeta,\zeta')f(\zeta')d\zeta'$$

$$f(\zeta') = |\zeta'|^{-\frac{2}{3}}$$

$$I_0^{\infty} = \frac{8\,\pi}{3\sqrt{3}\alpha^{1/3}} \approx 5$$

<u>At the cathode</u>

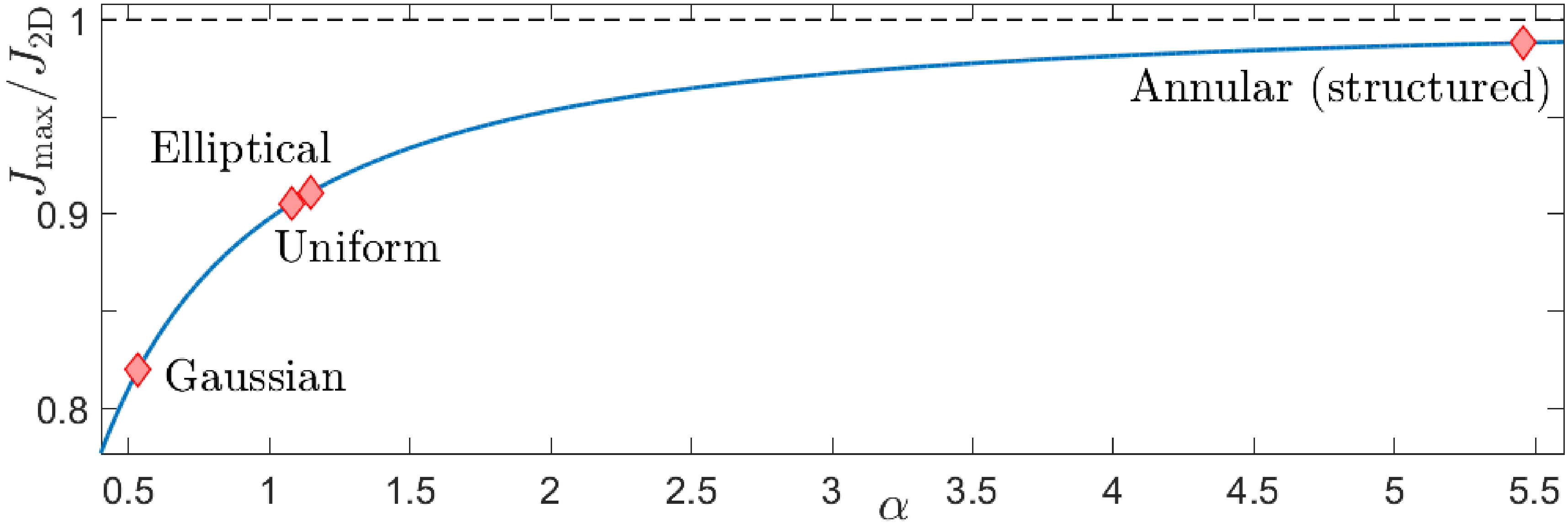
$$\Delta = \frac{L}{R}$$ - Aspect ratio of the extracted bunch

$$\underbrace{E_{acc}}_{\text{External field}} = \underbrace{G\,J(\Delta)\int_{-\infty}^{\infty} I(0,\zeta')f(\zeta')d\zeta'}_{\text{SC field from an infinitely long bunch}} = \underbrace{G\,J(1)\int_{-1}^{1} I(0,\zeta')f(\zeta')d\zeta'}_{\text{SC field from a bunch with } \Delta = 1}$$

It allows us to use J_{2D} as a reference

$$\boxed{\frac{J_{max}}{J(1)} = \frac{J(\infty)}{J_{2D}} = \frac{\int_{-1}^{1} I(0,\zeta')f(\zeta')d\zeta'}{I_0^{\infty}} = A(\alpha)}$$

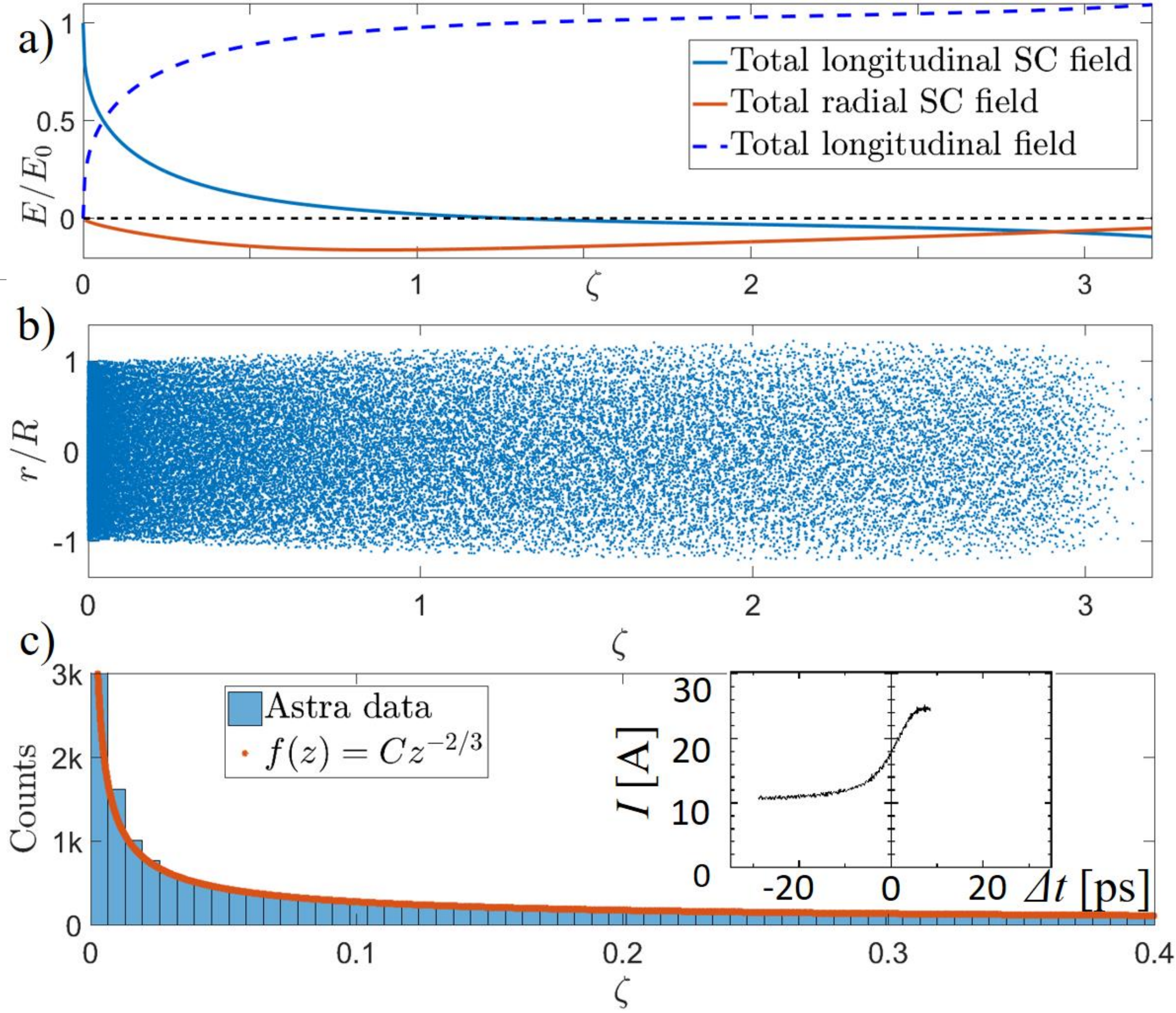
Child-Langmuir law



$$A(\alpha\Delta) = \frac{1}{I_0^{\infty}}\int_{-\Delta}^{\Delta} I(0,\zeta')f(\zeta')d\zeta' =$$

$$\frac{\sqrt{3}}{2\,\pi}\log\left[\frac{1+(\alpha\Delta)^{\frac{1}{3}}}{\sqrt{1-(\alpha\Delta)^{\frac{1}{3}}+(\alpha\Delta)^{\frac{2}{3}}}}\right]+\frac{2}{I_0^{\infty}(1+\alpha\Delta)}+\frac{3}{2\,\pi}\tan^{-1}\left[\frac{\sqrt{3}(\alpha\Delta)^{\frac{1}{3}}}{2-(\alpha\Delta)^{\frac{1}{3}}}\right]+\frac{3}{2}\Theta[(\alpha\Delta)^{\frac{1}{3}}-2]$$

In the gap…

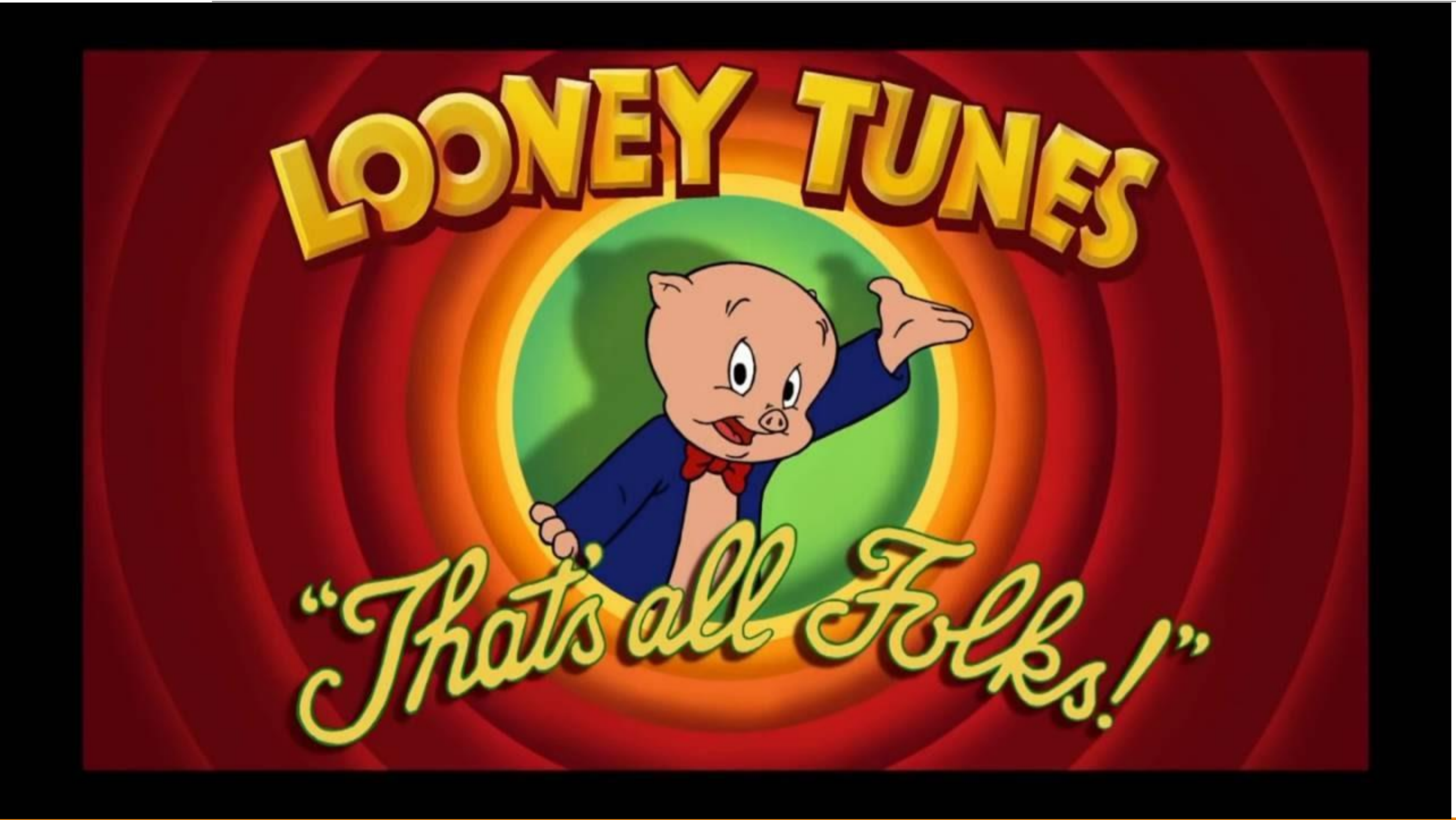

a)
1
0.5
0
E/E_0
Total longitudinal SC field
Total radial SC field
Total longitudinal field
0
1
ζ
2
3
b)
1
0
-1
r/R
0
1
2
3
ζ
c)
3k
2k
1k
0
Counts
Astra data
$f(z) = Cz^{-2/3}$
30
20
10
0
I [A]
-20
0
20
Δt [ps]
0
0.1
0.2
0.3
0.4
ζ

Summary

The model enabled us to:

- find the maximum current density for photocathodes,
- reveal its dependence on the transverse profile of the bunch,
- predict the max CD for surface-plasmon-enhanced photocathodes

Thank you for your attention!

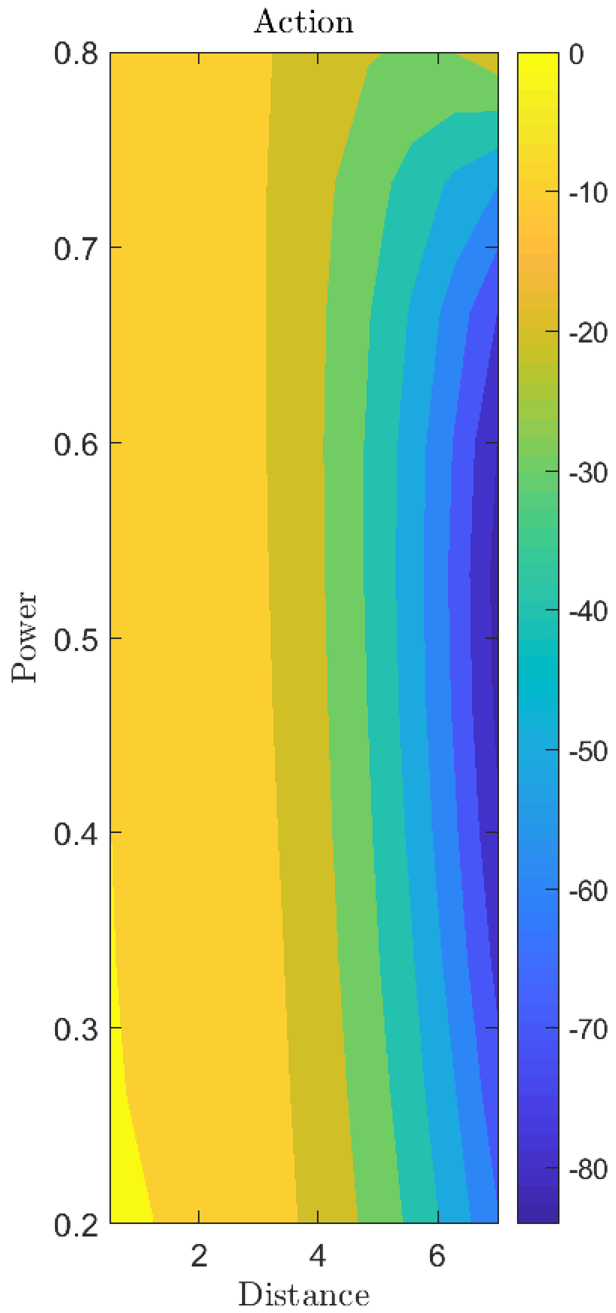


Together with

- Alan Mak,
- Kevin Pepitone and
- Vitaliy Goryashko

Backup: least action

- 1D density solution is somewhat valid in 3D case,
- Close to the cathode, the density power is ≈ 2/3,
- In the far zone, it becomes 1/2



Backup: radial motion

- Close to the cathode, radial fields are mostly compensated by the image charge
- Further away, at $\zeta \approx 1,$ the radial field reaches its maximum
- After propagation to $\zeta = 3,$ the bunch radius increases by roughly 30%

$$-2\phi\rho'' + (E_{acc} - E_z^{SC})\rho' = E_r^{SC}$$

$$\rho = \rho_0 + \frac{\rho_0}{12}\left\{\frac{\pi}{2} - \arctan\frac{1}{\sqrt{2\,\zeta}} + \sqrt{\frac{\zeta}{2}}\left[\ln(1+2\zeta) - 2\right]\right\}$$