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Gauged FN

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Understanding fermion masses in a gauged Froggatt-Nielsen model

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Introduction

Discovery of Higgs boson $\Rightarrow$ particle content of Standard Model complete

Still SM not complete description of nature:

Problems within SM

- no explanation of neutrino masses (no ν_R)
- many free parameters with huge hierarchies ($m_e/m_t \sim 10^{-6}$)
- fine-tuning of Higgs mass compared to Planck mass
- absence of CP-violation in strong interactions ($\theta_s < 10^{-6}$)
- Higgs too heavy for electroweak baryogenesis

Problems outside SM

- no dark matter in SM
- quantum theory of gravity

Need physics beyond the SM

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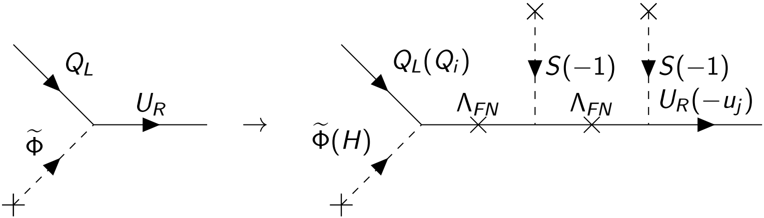
Froggatt-Nielsen model

New **flavon** symmetry (charge) to understand hierarchies of fermion masses and mixings

Yukawa interaction only an effective vertex



S scalar charged (-1) under new symmetry

vector-like Froggatt-Nielsen fermion: mass Λ_{FN}, same SM charges as corresponding SM fermions

each insertion gives a suppression $\dfrac{\langle S\rangle}{\Lambda_{FN}}$

flavon charge conservation $\Rightarrow$ Yukawa coupling

$$(Y^U)_{ij} \sim (g^U)_{ij} \left(\frac{\langle S\rangle}{\Lambda_{FN}}\right)^{|Q_i+u_j+H|}, \mathrm{(FN)\,assume}\,(g^U)_{ij} = \mathcal{O}(1)$$

Will argue two Higgs doublets needed when symmetry gauged

Yukawa Lagrangian in a general 2HDM:

$$
\begin{aligned}
-\mathcal{L}_Y = \quad & \overline{Q}_L \widetilde{\Phi}_1 Y_1^U U_R + \overline{Q}_L \Phi_1 Y_1^D D_R + \overline{L}_L \Phi_1 Y_1^L E_R \\
& + \overline{Q}_L \widetilde{\Phi}_2 Y_2^U U_R + \overline{Q}_L \Phi_2 Y_2^D D_R + \overline{L}_L \Phi_2 Y_2^L E_R + \mathrm{h.c.}
\end{aligned}
$$

Avoid tree-level Flavour Changing Neutral Currents (FCNCs) by $\mathbb{Z}_2$ symmetry:

left-handed doublets Q_L, L_L transforms as "+"

$\mathbb{Z}_2$-symmetry	Φ_1	Φ_2	U_R	D_R	E_R
Type-I (SM like)	$+$	$-$	$-$	$-$	$-$
Type-II (MSSM like)	$+$	$-$	$-$	$+$	$+$
Type-Y (flipped)	$+$	$-$	$-$	$+$	$-$
Type-X (lepton spec)	$+$	$-$	$-$	$-$	$+$

$\Rightarrow$ each fermion only couples to one Higgs field
$\Rightarrow$ no FCNCs at tree-level

left-handed fermion fields: $Q_L^i, (U_R^i)^c, (D_R^i)^c, L_L^i, (E_R^i)^c$

two Higgs fields: Φ_1, Φ_2

denote flavon charges by Q_i, u_i, d_i, L_i, e_i and $H_{1,2}$

Yukawa couplings at the electroweak scale:

$$(Y_a^U)_{ij}\overline{Q}_L^i\widetilde{\Phi}_a U_R^j \longrightarrow (g_a^U)_{ij}\left(\frac{\langle S\rangle}{\Lambda_{FN}}\right)^{|Q_i+u_j+H_a|}\overline{Q}_L^i\widetilde{\Phi}_a U_R^j$$

$$(Y_a^D)_{ij}\overline{Q}_L^i\Phi_a D_R^j \longrightarrow (g_a^D)_{ij}\left(\frac{\langle S\rangle}{\Lambda_{FN}}\right)^{|Q_i+d_j-H_a|}\overline{Q}_L^i\Phi_a D_R^j$$

$$(Y_a^L)_{ij}\overline{L}_L^i\Phi_a E_R^j \longrightarrow (g_a^L)_{ij}\left(\frac{\langle S\rangle}{\Lambda_{FN}}\right)^{|L_i+e_j-H_a|}\overline{L}_L^i\Phi_a E_R^j$$

$(g_a^F)_{ij} \sim \mathcal{O}(1)$, with $F = U, D, L$ and $a = 1, 2$.

moduli gives number of insertions (S or S^*)

introduce $\epsilon = \frac{\langle S \rangle}{\Lambda_{FN}} \Rightarrow$ Yukawa matrices become

$$Y^U_{ij} = g^U_{ij}\epsilon^{|Q_i+u_j+H_a|} \quad Y^D_{ij} = g^D_{ij}\epsilon^{|Q_i+d_j-H_b|} \tag{1}$$

$a, b \in \{1, 2\}$ depending on type.

Bi-unitary transformations gives diagonal mass-matrices, $D^U, \ D^D$

$$\begin{aligned} Y^U &= (V^U_L)^\dagger D^U V^U_R \\ Y^D &= (V^D_L)^\dagger D^D V^D_R \end{aligned}$$

Froggatt-Nielsen procedure requires ordering of exponents

$$\begin{aligned} |Q_i + u_j + H_a| &\geq |Q_{i+1} + u_j + H_a|\,, \, |Q_i + u_{j+1} + H_a|\,, \\ |Q_i + d_j - H_b| &\geq |Q_{i+1} + d_j - H_b|\,, \, |Q_i + d_{j+1} - H_b|\,. \end{aligned}$$

then

$$\begin{aligned} (V^U_L)_{ij} &\sim \epsilon^{|Q_i-Q_j|}, & (V^U_R)_{ij} &\sim \epsilon^{|u_i-u_j|} \\ (V^D_L)_{ij} &\sim \epsilon^{|Q_i-Q_j|}, & (V^D_R)_{ij} &\sim \epsilon^{|d_i-d_j|} \end{aligned}$$

diagonal elements of mass matrices same as diagonal entries of Y

$$(D^U)_{ii} \sim \epsilon^{|Q_i+u_i+H_a|} \quad (D^D)_{ii} \sim \epsilon^{|Q_i+d_i-H_b|}$$

Finally CKM-matrix given by

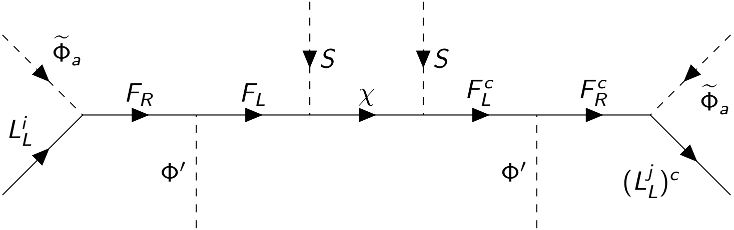
$$(V_{CKM})_{ij} = (V^U_L)_{ik}(V^{D\,\dagger}_L)_{kj} \sim \epsilon^{|Q_i-Q_j|}$$

Neutrino masses

Weinberg operator

$$-\mathcal{L}_{\nu}^{(5)} = \frac{1}{2}\frac{(\kappa_{ab})_{ij}}{\Lambda_{FN}}\left(\widetilde{\Phi}_a^{\dagger}\overline{L_L^{c}}^{j}\right)\left(\widetilde{\Phi}_b^{\dagger}L_L^{i}\right) + \mathrm{H.c.}$$

generated from FN-mechanism, χ Majorana FN-fermion, mass Λ_{FN}



$\mathbb{Z}_2$ symmetry $\Rightarrow a = b$, turns out only $a = 1$ works

$$(\kappa_{aa})_{ij} \longrightarrow (\kappa_{11}^{\nu})_{ij}\left(\frac{\langle S\rangle}{\Lambda_{FN}}\right)^{|L_i+H_1|+|L_j+H_1|}$$

both moduli have to be integers and $(\kappa_{11}^{\nu})_{ij} \sim \mathcal{O}(1)$ in FN-spirit.

Constraints from quark and lepton masses in type-II 2HDM for: $\tan\beta = 1$, Weinberg operator for neutrino masses, $\epsilon = 0.2 \simeq \sin\theta_C$, and $\Lambda_{FN} \sim 10^{14}$ GeV

$$
\begin{aligned}
n_t = Q_3 + u_3 + H_2 &= 0 \\
n_c = Q_2 + u_2 + H_2 &= 4 \\
n_u = Q_1 + u_1 + H_2 &= 7 \\
n_b = Q_3 + d_3 - H_1 &= 3 \\
n_s = Q_2 + d_2 - H_1 &= 5 \\
n_d = Q_1 + d_1 - H_1 &= 7 \\
n_\tau = L_3 + e_3 - H_1 &= 3 \\
n_\mu = L_2 + e_2 - H_1 &= 4 \\
n_e = L_1 + e_1 - H_1 &= 8 \\
Q_1 - Q_2 &= 1 \\
Q_2 - Q_3 &= 2 \\
L_2 - L_3 &= 0 \\
L_2 + H_1 &= 0
\end{aligned}
$$

note: definition of n depends on type

Yukawa matrices

$$Y_2^U \sim \begin{pmatrix} \epsilon^7 & \epsilon^5 & \epsilon^3 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^0 \end{pmatrix}, \quad Y_1^D \sim \begin{pmatrix} \epsilon^7 & \epsilon^6 & \epsilon^6 \\ \epsilon^6 & \epsilon^5 & \epsilon^5 \\ \epsilon^4 & \epsilon^3 & \epsilon^3 \end{pmatrix}, \quad Y_1^L \sim \begin{pmatrix} \epsilon^8 & * & * \\ * & \epsilon^4 & \epsilon^3 \\ * & \epsilon^4 & \epsilon^3 \end{pmatrix}$$

CKM matrix

$$V_{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

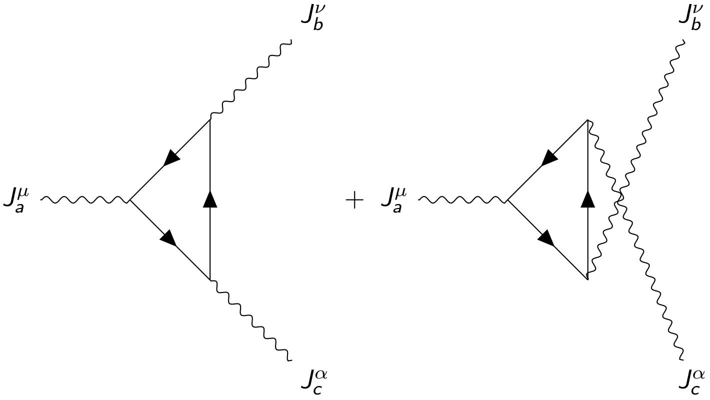
neutrino mass matrix

$$\kappa_{11} \sim \begin{pmatrix} * & * & * \\ * & \epsilon^0 & \epsilon^0 \\ * & \epsilon^0 & \epsilon^0 \end{pmatrix}$$

* denotes an element not determined a priori.

Anomalies

Breaks symmetries of classical Lagrangian at quantum level



Anomaly constraints $\mathcal{A}_{XYZ} = \frac{1}{2}\mathrm{tr}[T_X\{T_Y, T_Z\}]$

T_X generators of gauge group X in fundamental representation

hypercharge normalization $Y = 2(Q - T_3)$

Anomalies

$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ plus gravity

non-trivial anomaly constraints involving the $U(1)'$-charges

$$
\begin{aligned}
\mathcal{A}_{11'1'} &= 2\sum_{j=1}^{3}\left(Q_j^2 - 2u_j^2 + d_j^2 - L_j^2 + e_j^2\right) = 0 \\
\mathcal{A}_{111'} &= \frac{2}{3}\sum_{j=1}^{3}\left(Q_j + 8u_j + 2d_j + 3L_j + 6e_j\right) = 0 \\
\mathcal{A}_{331'} &= \frac{1}{2}\sum_{j=1}^{3}\left(2Q_j + u_j + d_j\right) = 0 \\
\mathcal{A}_{221'} &= \frac{1}{2}\sum_{j=1}^{3}\left(3Q_j + L_j\right) = 0 \\
\mathcal{A}_{1'1'1'} &= \sum_{j=1}^{3}\left(6Q_j^3 + 3u_j^3 + 3d_j^3 + 2L_j^3 + e_j^3 + \nu_j^3\right) = 0 \\
\mathcal{A}_{gg1'} &= \sum_{j=1}^{3}\left(6Q_j + 3u_j + 3d_j + 2L_j + e_j + \nu_j\right) = 0
\end{aligned}
$$

added three right handed SM singlets with flavon charges ν_j

Sum rules relate mass constraints and anomalies

type-II: $$n_d + n_s + n_b - n_e - n_\mu - n_\tau = \sum_{j=1}^{3}(Q_j + d_j - L_j - e_j) = \frac{8}{3}\mathcal{A}_{331'} - \frac{1}{4}\mathcal{A}_{111'} - \mathcal{A}_{221'} = 0$$

type-Y: $$n_u + n_c + n_t + n_e + n_\mu + n_\tau = \sum_{j=1}^{3}(Q_j + u_j + L_j + e_j) = -\frac{2}{3}\mathcal{A}_{331'} + \frac{1}{4}\mathcal{A}_{111'} + \mathcal{A}_{221'} = 0$$

type-X: $$n_u + n_c + n_t + n_d + n_s + n_b = \sum_{j=1}^{3}(2Q_j + u_j + d_j) = 2\mathcal{A}_{331'} = 0$$

type-I: (SM-like) symmetry, all three rules have to be satisfied (although only two independent)

Sum rules $\Rightarrow$ some n have to be negative for types I, X, Y:

suppression factor OK thanks to absolute value, but

leads to skewed Yukawa matrices which break ordering

$$\begin{aligned} |Q_i + u_j + H_a| &\geq |Q_{i+1} + u_j + H_a|\,,\ |Q_i + u_{j+1} + H_a|\,, \\ |Q_i + d_j - H_b| &\geq |Q_{i+1} + d_j - H_b|\,,\ |Q_i + d_{j+1} - H_b|\,. \end{aligned}$$

example $n_c = -4$

$$Y_a^U \sim \begin{pmatrix} \epsilon^7 & \epsilon^3 & \epsilon^3 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^5 & \epsilon^0 \end{pmatrix}$$

$\Rightarrow$ only type-II consistent

In addition: sum rules that do not have to be satisfied for a given $\mathbb{Z}_2$ symmetry specify the charges of the Higgs fields

For type-II model

$$n_u + n_c + n_t + n_d + n_s + n_b = 2\mathcal{A}_{331'} + 3(H_2 - H_1) = 3(H_2 - H_1) \ \in \mathbb{Z}$$

$$\Rightarrow H_2 - H_1 \in \mathbb{Z}/3$$

For definiteness remove mixing between $U(1)_Y$ and $U(1)'$ (in massless limit) by requiring

$$\sum_{j=1}^{3}(2Q_j - 4u_j + 2d_j - 2L_j + 2e_j) = 0$$

the trace of the hyper charge and flavon charge generators

In total this gives 10 mass, 4 mixing, and 6 anomaly constraints with 6*3+2 charges to solve for

Algebraic geometry and Mordell-Weil generators

- **Gröbner basis:** the most reduced set of equations with the same solutions. As many terms and variables as possible have been eliminated. (Like Gauss-Jordan elimination in linear algebra)
- **Mordell-Weil's theorem:** The set of rational points on an elliptic curve (i.e. smooth curve of genus 1) is finitely generated
- **Mordell-Weil generators:** The generators of the group of rational points

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Gröbner basis:

$$
\begin{aligned}
Q_1 - 8/27 &= 0, & Q_2 + 19/27 &= 0, & Q_3 + 73/27 &= 0,\\
u_1 - 26/27 &= 0, & u_2 + 28/27 &= 0, & u_3 + 82/27 &= 0,\\
d_1 - 34/9 &= 0, & d_2 - 25/9 &= 0, & d_3 - 25/9 &= 0,\\
L_1 - 94/27 &= 0, & L_2 - 79/27 &= 0, & L_3 - 79/27 &= 0,\\
e_1 - 43/27 &= 0, & e_2 + 50/27 &= 0, & e_3 + 77/27 &= 0,\\
H_1 + 79/27 &= 0, & H_2 - 155/27 &= 0,
\end{aligned}
$$

and

$$
\begin{aligned}
&\nu_1 + \nu_2 + \nu_3 + 140/9 = 0,\\
&\nu_2^2 \cdot \nu_3 + 140/9 \cdot \nu_2^2 + \nu_2 \cdot \nu_3^2 + 280/9 \cdot \nu_2 \cdot \nu_3 + 19600/81 \cdot \nu_2 + \\
&+140/9 \cdot \nu_3^2 + 19600/81 \cdot \nu_3 + 95036/81 = 0.
\end{aligned}
$$

Elliptic curve $\rightarrow$ Weierstrass form

$$
E: \; y^2 + 2xy + \frac{95036}{81} y = x^3 + \frac{19519}{81} x^2 + \frac{12449716}{729} x
$$

this curve has rank one $\rightarrow$ the set of rational points is given by

$$E(\mathbb{Q}) = E(\mathbb{Q})_{\text{tors}} \oplus \mathbb{Z}P_1.$$

with

$$E(\mathbb{Q})_{\text{tors}} = \{(0 : -95036/81 : 1), (0 : 0 : 1), (0 : 1 : 0)\}$$
$$P_1 = (2041940/81 : 323674124/81 : 1)$$

mapping the point P_1 back to the original curve gives

$$(\nu_2, \nu_3) = \left(\frac{30795}{193}, -\frac{18344}{115}\right) \qquad \nu_1 = -\frac{140}{9} - \nu_2 - \nu_3 = -\frac{3116597}{199755}$$

Summarizing

Quarks

$$Y_2^U \sim \begin{pmatrix} \epsilon^7 & \epsilon^5 & \epsilon^3 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^0 \end{pmatrix} \qquad Y_1^D \sim \begin{pmatrix} \epsilon^7 & \epsilon^6 & \epsilon^6 \\ \epsilon^6 & \epsilon^5 & \epsilon^5 \\ \epsilon^4 & \epsilon^3 & \epsilon^3 \end{pmatrix}$$

$$V_{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

Leptons

$$Y_1^L \sim \begin{pmatrix} \epsilon^8 & 0 & 0 \\ 0 & \epsilon^4 & \epsilon^3 \\ 0 & \epsilon^4 & \epsilon^3 \end{pmatrix} \qquad \kappa_{11} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$U_{PMNS} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

Higgs

$$H_2 - H_1 = 26/3$$

$\mathbb{Z}_2$ symmetry follows from flavon symmetry

The same set up can be used to find solutions also for the case of type-I seesaw mechanism to generate neutrino masses

- have to introduce three $SU(2)_L$ singlet fields N_R^i, $i = 1, 2, 3$
- no longer possible to solve keeping $\mathcal{A}_{1'1'1'}$ and $\mathcal{A}_{gg1'}$
- assume solved by additional SM-neutral fermions

supersymmetric version

- complications from higgsinos contributing to anomalies
- sum rules modified $\Rightarrow$ type-II no longer consistent with anomalies
- have to invoke Green-Schwarz mechanism (string theoretic UV completion) to cancel some anomalies

$$\mathcal{A}'_{221'} = \mathcal{A}'_{331'} = \frac{3}{20}\mathcal{A}'_{111'}$$

Conclusions

- fermions masses and mixings can be understood in gauged Froggatt-Nielsen model
- flavon charges constrained by masses and mixings as well as anomalies
- Gröbner basis and Mordell-Weil generators powerful tools to find rational flavon charges
- sum rules that relate mass-constraints and anomalies $\Rightarrow$ type-II 2HDM favoured