

Minimal anomaly free $U(1)$ extensions: constraints, prospects & outlooks

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Particle Theory Seminar @ Uppsala University

Plan of my talk

- What is gauge anomaly?
- Why anomaly free theory?
- Minimal $U(1)$ extensions
- Anomaly cancellation
- Green-Schwarz mechanism
- Prospects at HL-LHC & ILC
- Outlooks: collider & dark matter
- Summary and conclusions

Trivia: gauge anomaly

- Symmetry of Lagrangian \rightarrow conserved current \rightarrow conserved classically but not at the quantum level, the symmetry is anomalous
- Massless (QED) Dirac Lagrangian has chiral symmetry; the axial vector current is conserved

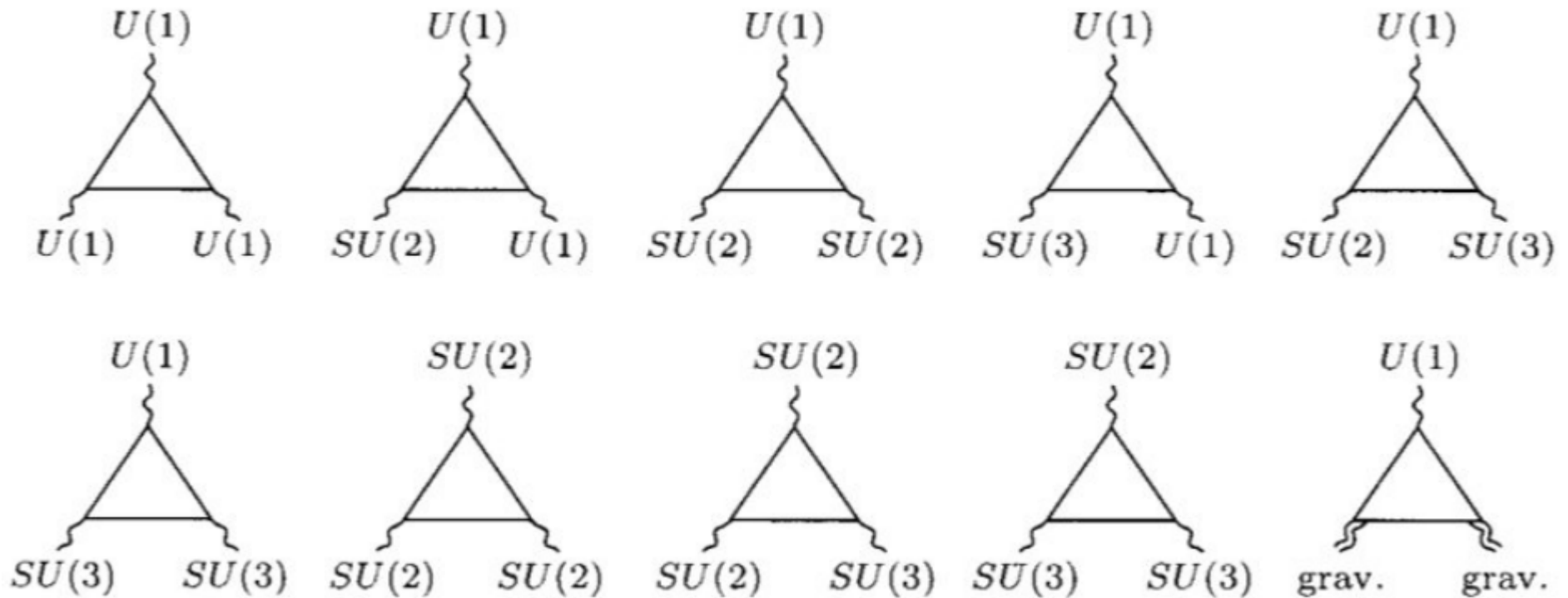
$$\text{Classically: } j^{\mu 5} = \bar{\psi} \gamma^{\mu} \gamma^5 \psi; \quad \partial_{\mu} j^{\mu 5} = 0$$

$$\text{Quantum level: } \partial_{\mu} j^{\mu 5} = -\frac{e^2}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}$$

- Violations of Ward identities or equivalently breaking of gauge symmetry
- Gauge symmetry is required to make the theory unitary and Lorentz invariant
- Gauge anomalies occur only in even dimensions. Triangle diagrams in 4D with 3 gauge bosons attached ($n = 1 + D/2$)
- Triangle is enough. This is correct to all orders in perturbation theory

Adler & Bardeen, Phys. Rev. 182, 1517 (1969)

Gauge anomaly



P.C. from Peskin and Schroeder

SM is anomaly free

U(1) extensions of the SM

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_z$$



$$SU(3)_c \times SU(2)_L \times U(1)_Y$$



$$SU(3)_c \times U(1)_{EM}$$

$$SU(3)_c \times SU(2)_L \times \underbrace{U(1)_1 \times U(1)_2}_{\text{Red bracket}}$$



$$SU(3)_c \times SU(2)_L \times U(1)_Y$$



$$SU(3)_c \times U(1)_{EM}$$

- Redefining gauge fields and rescaling gauge couplings, possible to make

$$U(1)_1 \times U(1)_2 \equiv U(1)_Y \times U(1)_z$$

- Gauge theory consists of several U(1)'s --> possibility of kinetic mixing
- Kinetic mixing can be rotated away at a given scale

U(1) extensions: mass generation

- A complex scalar gets TeV scale VEV

$$\langle \Phi \rangle \rightarrow M_{Z'} \sim \text{TeV}$$

- EWSB: Higgs mechanism

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_z$$



$$SU(3)_c \times SU(2)_L \times U(1)_Y$$



$$SU(3)_c \times U(1)_{EM}$$

- Proca Lagrangian: mass term of gauge bosons spoils Gauge invariance

- Stückelberg mechanism $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(mA_\mu + \partial_\mu\sigma)^2$

Invariant under: $A_\mu \rightarrow A_\mu + \partial_\mu\epsilon$ $\sigma \rightarrow \sigma - m\epsilon$

Axion

- Works for Abelian group but does not work for non-Abelian groups
- Introduced to address strong CP problem. U(1) Peccei-Quinn symmetry

$$\frac{g_s^2}{64\pi^2} \left(\Theta - \frac{\sigma}{f} \right) \epsilon^{\mu\nu\alpha\beta} G_{\mu\nu}^a G_{\alpha\beta}^a$$

Minimal models: assumptions

- Additional $U(1)$ group which is broken by the VEV of a complex scalar
- SM fermions are the only fermions that are charged under SM gauge group
- 3-generations of RH neutrinos which are SM singlets but charged under $U(1)$
- Neutrinos obtain masses via a Type-I seesaw scenario
- Gauge charges are generation independent and EWSB occurs as in the SM

Anomaly free conditions

$[\text{SU}(2)_L]^2 [\text{U}(1)_z]$	\longrightarrow	$\text{Tr}[\{T^i, T^j\} z] = 0$
$[\text{SU}(3)_c]^2 [\text{U}(1)_z]$	\longrightarrow	$\text{Tr}[\{T^a, T^b\} z] = 0$
$[\text{U}(1)_Y]^2 [\text{U}(1)_z]$	\longrightarrow	$\text{Tr}[Y^2 z] = 0$
$[\text{U}(1)_Y][\text{U}(1)_z]^2$	\longrightarrow	$\text{Tr}[Y z^2] = 0$
$[\text{U}(1)_z]^3$	\longrightarrow	$\text{Tr}[z^3] = 0$
Gauge-gravity anomaly	\longrightarrow	$\text{Tr}[z] = 0$

- 6 parameters and 4 independent equations \rightarrow 2 free charges
- Most general solution to the anomaly cancellation conditions is

$$Q_f = aY_f + b(B_f - L_f)$$

Gauge charges

$$z_l = -3z_q; \quad z_d = 2z_q - z_u; \quad z_e = -2z_q - z_u;$$

$$\frac{1}{3} \sum_{k=1}^n z_k = -4z_q + z_u; \quad \left(\sum_{k=1}^n z_k \right)^3 = 9 \sum_{k=1}^n z_k^3$$

For both Majorana and Dirac mass for neutrinos $z_k = -4z_q + z_u = \pm 1/2$


	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_z$
q_L	3	2	1/3	z_q
u_R	3	1	4/3	z_u
d_R	3	1	-2/3	$2z_q - z_u$
l_L	1	2	-1	$-3z_q$
e_R	1	1	-2	$-2z_q - z_u$
ν_R	1	1	0	z_k
H	1	2	+1	$-z_q + z_u$
φ	1	1	0	1

k-parametrization

A model-independent approach

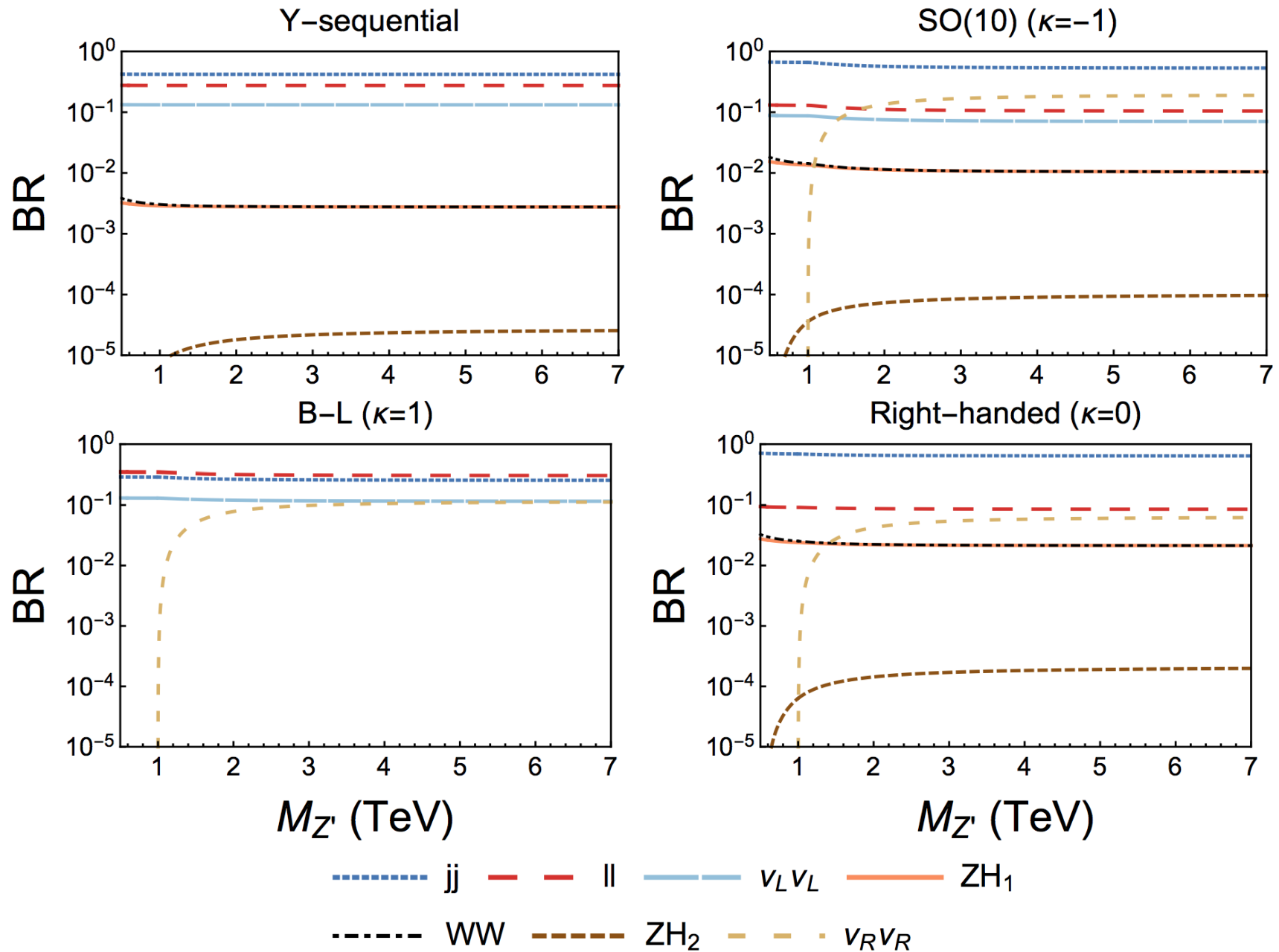
Model	$\kappa = z_q/z_u$
Gauged $B - L$	1
Y sequential	1/4
SO(10)-GUT	-1
Right-handed	0

Special case: no RHN is
Required to cancel anomalies

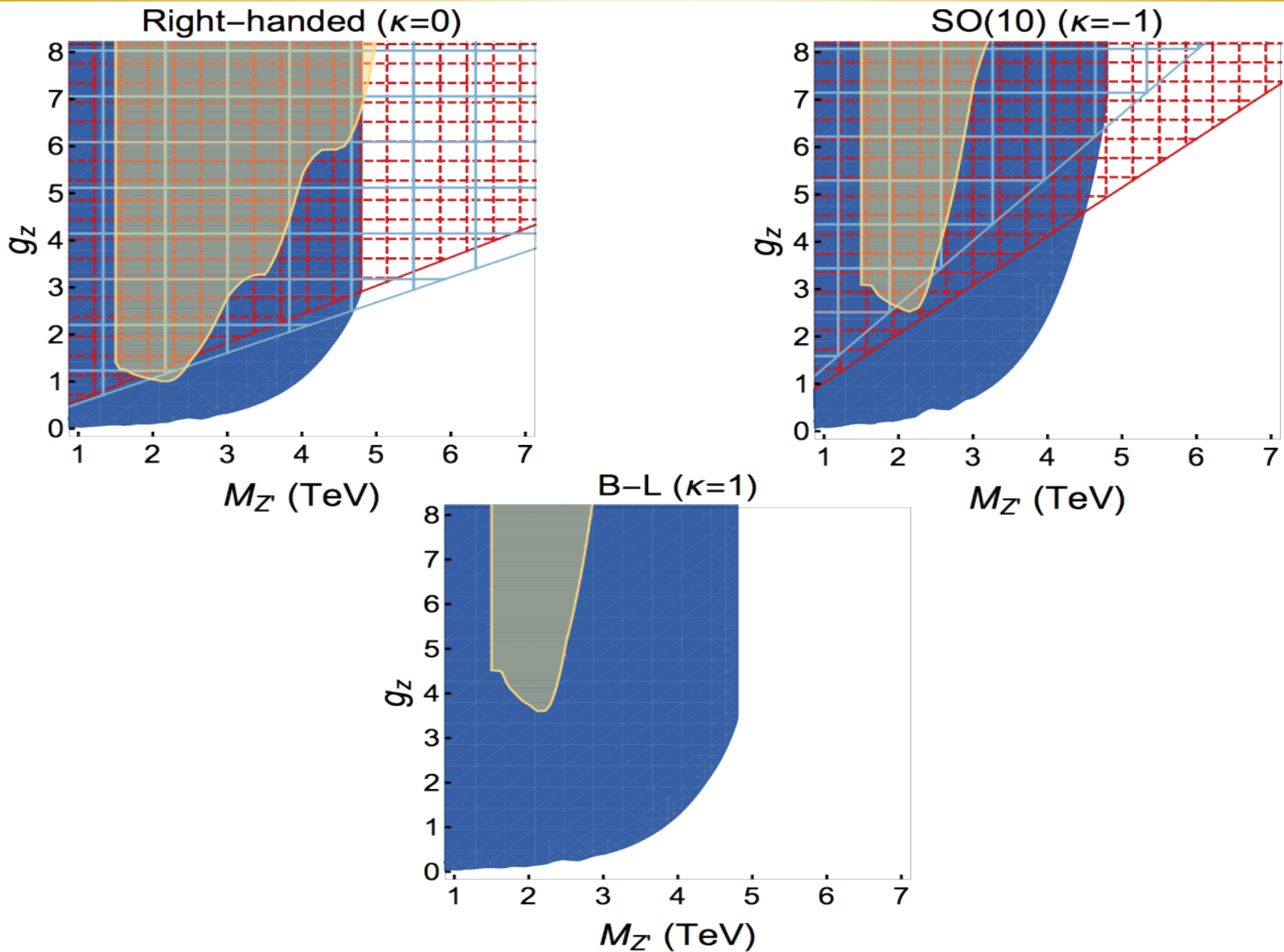


$$\sigma(M_{Z'}, g_z, \kappa) = g_z^2 \left\{ a_L^u(M_{Z'}) \left(\frac{\kappa}{1-4\kappa} \right)^2 + a_R^u(M_{Z'}) \left(\frac{1}{1-4\kappa} \right)^2 \right. \\ \left. + a_L^d(M_{Z'}) \left(\frac{\kappa}{1-4\kappa} \right)^2 + a_R^d(M_{Z'}) \left(\frac{2\kappa-1}{1-4\kappa} \right)^2 \right\}$$

Branching ratios

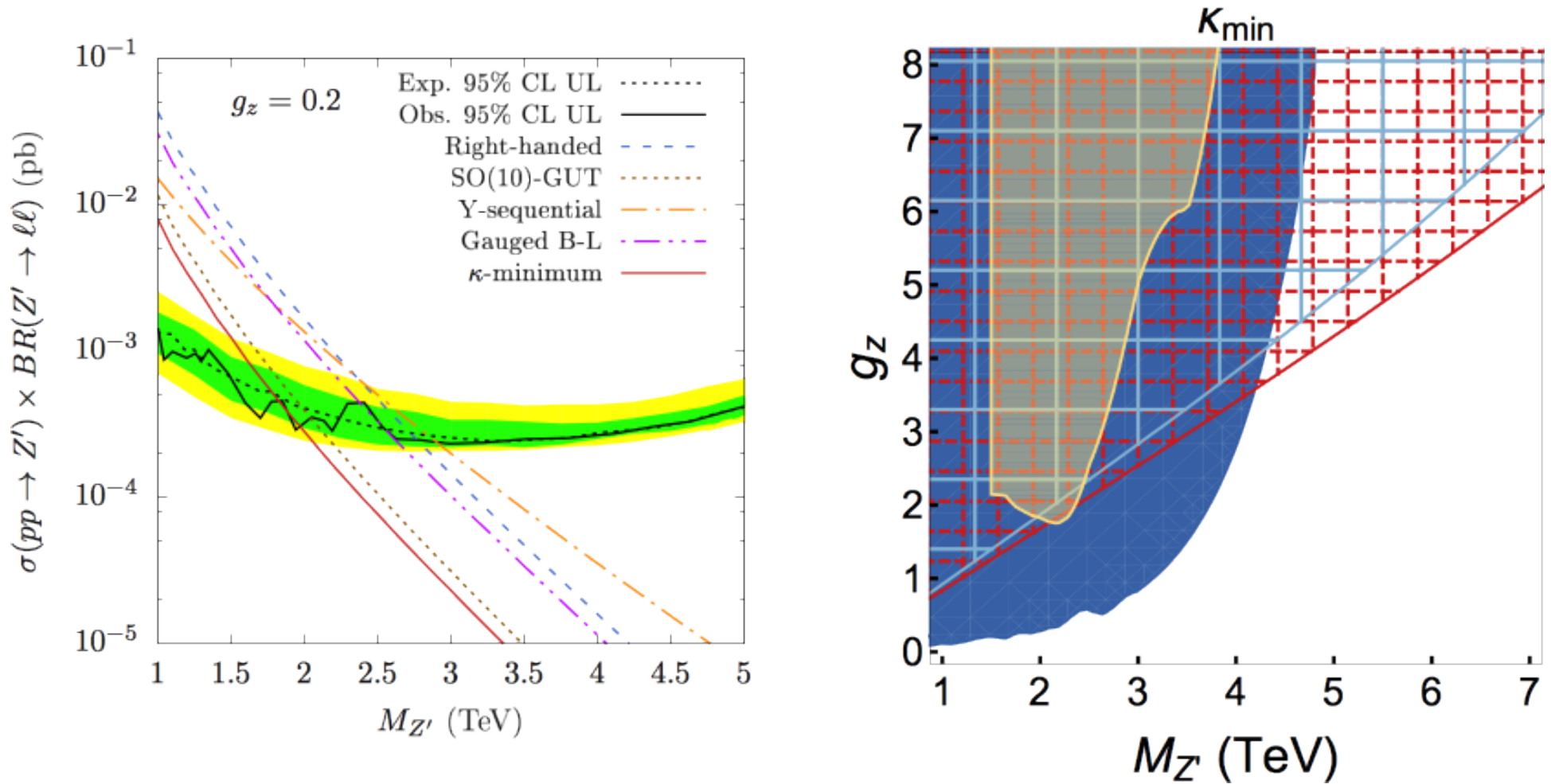


Exclusion limits



■ ATLAS dilepton □ Γ_Z □ T parameter ■ ATLAS dijet

Exclusion limits



κ_{min} is the value for which $\sigma \times BR$ is minimum

Green-Schwarz mechanism

- A mechanism to cancel anomaly by introducing gauge-variant operators in the effective action
- Heavy chiral fermions are integrated out and at low energy their remnants appear as effective operators

$$\mathcal{L}_{GS} = \mathcal{L}_{kin} + \mathcal{L}_{PQ} + \mathcal{L}_{GCS}$$

$$\mathcal{L}_{kin} = -\frac{1}{4}F_z^{\mu\nu}F_{z,\mu\nu} + \frac{1}{2}(\partial^\mu a + M_I g_z B_z^\mu)(\partial_\mu a + M_I g_z B_{z,\mu})$$

$$\mathcal{L}_{PQ} = \frac{1}{96\pi^2 M_I} a \varepsilon_{\mu\nu\rho\sigma} (C_{zzz} g_z^3 F_z^{\mu\nu} F_z^{\rho\sigma} + C_{zzy} g_z^2 g' F_z^{\mu\nu} F_Y^{\rho\sigma} + C_{zyy} g_z g'^2 F_Y^{\mu\nu} F_Y^{\rho\sigma} + D_2 g_z g^2 \text{Tr}(F_W^{\mu\nu} F_W^{\rho\sigma}) + D_3 g_z g_S^2 \text{Tr}(F_S^{\mu\nu} F_S^{\rho\sigma})),$$

$$\mathcal{L}_{GCS} = \frac{1}{48\pi^2} \varepsilon_{\mu\nu\rho\sigma} (g'^2 g_z E_{zyy} B_Y^\mu B_z^\nu F_Y^{\rho\sigma} + g' g_z^2 E_{zzz} B_Y^\mu B_z^\nu F_z^{\rho\sigma} + g^2 g_z K_2 B_z^\mu \Omega_W^{\nu\rho\sigma} + g_S^2 g_z K_3 B_z^\mu \Omega_S^{\nu\rho\sigma}),$$

$$\Omega_{\nu\rho\sigma}^{S,W} = \frac{1}{3} \text{Tr} [A_\nu^{S,W} (F_{\rho\sigma}^{S,W} - [A_\rho^{S,W}, A_\sigma^{S,W}]) + (\text{cyclic perm.})]$$

Gauge charges in GS setup

- New U(1) is broken to the SM gauge group at some high scale through the Stuckelberg mechanism \rightarrow a massive Z'
- No right-handed neutrino and EWSB proceeds as usual like in the SM

Fields	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_z$
H	1	2	1	z_H
q_L	3	2	1/3	z_q
u_R	3	1	4/3	$z_u = z_q + z_H$
d_R	3	1	-2/3	$z_d = z_q - z_H$
ℓ_L	1	2	-1	z_ℓ
e_R	1	1	-2	$z_e = z_\ell - z_H$

- The general solution to the anomaly cancellation conditions is

$$Q_f^z = 3z_q B_f + z_\ell L_f + z_H \left\{ Y_f - (B - L)_f \right\}$$

Relations

Anomaly	Relation
$[\text{U}(1)_z]^3$	$C_{zzz} = -z_h^3 - 3z_h z_l^2 - z_l^3 + 3z_h^2(z_l + 6z_q)$
$[\text{U}(1)_z]^2 [\text{U}(1)_Y]$	$E_{zzy} = 4z_h(z_l + 3z_q)$ $C_{zzy} = 3E_{zzy}$
$[\text{U}(1)_z] [\text{U}(1)_Y]^2$	$E_{zyy} = 4(z_l + 3z_q)$ $C_{zyy} = \frac{3}{2}E_{zyy}$
$[\text{SU}(2)_L]^2 [\text{U}(1)_z]$	$K_2 = (6z_q + 2z_l)$ $D_2 = -\frac{3}{2}K_2$
$[\text{SU}(3)_C]^2 [\text{U}(1)_z]$	$K_3 = 0$ $D_3 = 0$

- One can ignore the gauge-gravity anomaly cancellation condition by assuming that gravity couples very weakly

$$[R]^2 [\text{U}(1)_z] \rightarrow \text{Tr} [z] \sim 3z_q + 2z_l - z_H$$

Various models

Non-chiral models ($z_H = 0$)

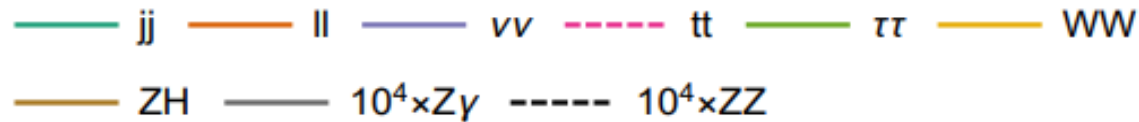
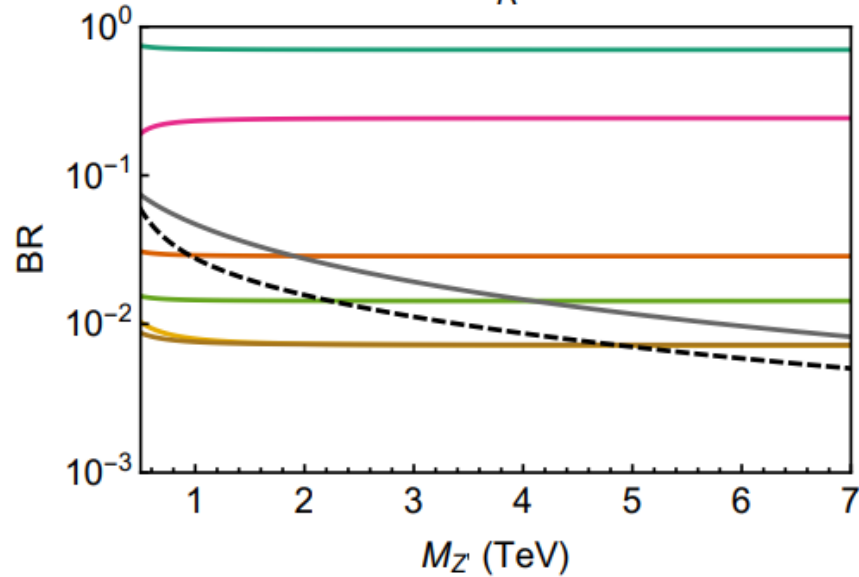
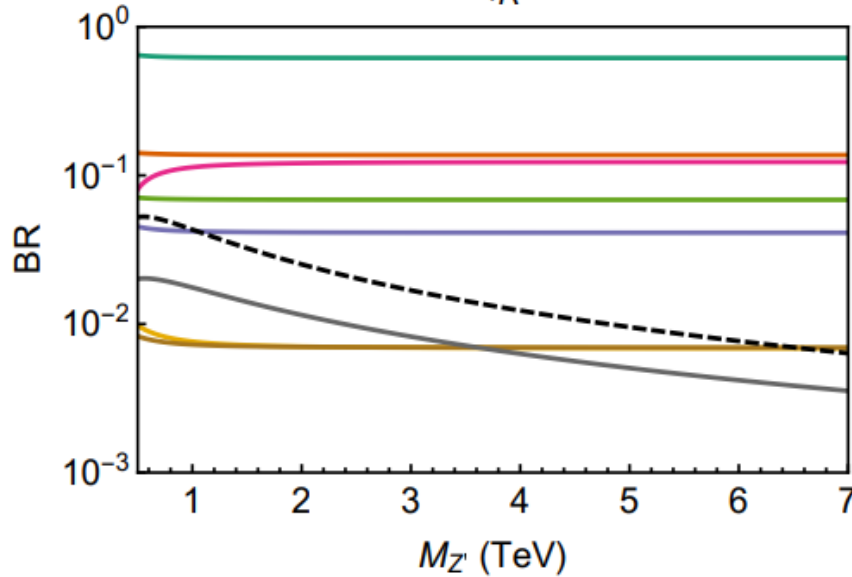
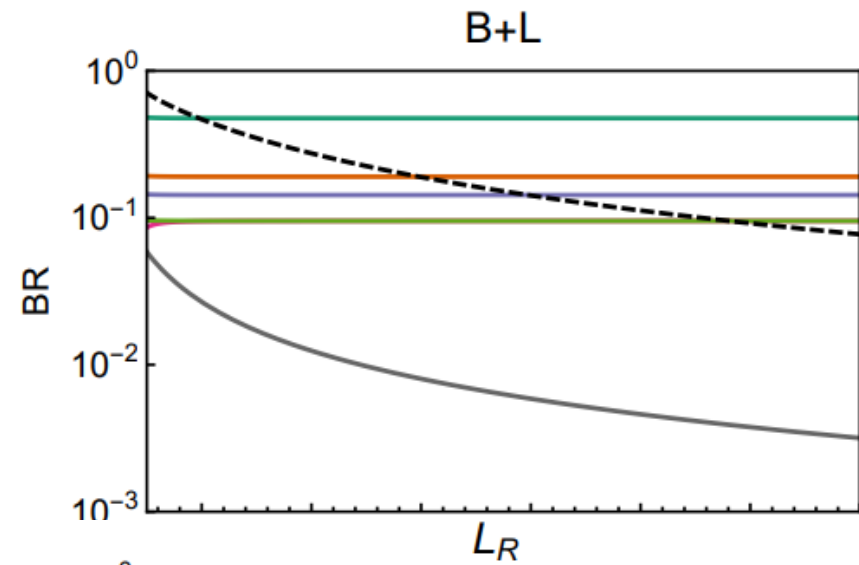
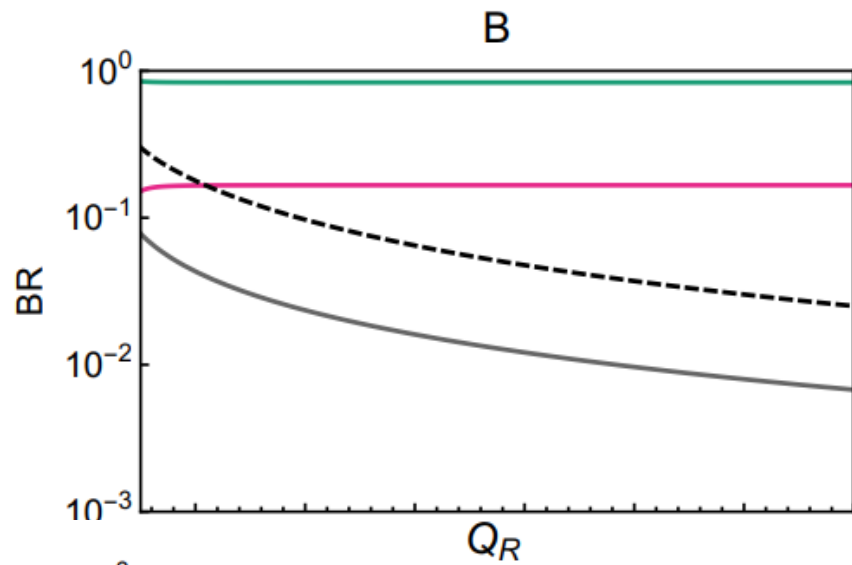
- **Leptophobic:**
 $Q_z^f = B_f; \quad z_q = 1/3, \quad z_\ell = 0, \quad z_H = 0$
- **Fermiophobic:**
 $Q_z^f = 0; \quad z_q = 0, \quad z_\ell = 0, \quad z_H = 0$
- **Gravity model:**
 $Q_z^f = z_\ell(L_f - 2B_f); \quad z_q = -(2/3)z_\ell; \quad z_H = 0$
- **Quarkphobic:**
 $Q_z^f = L_f; \quad z_q = 0, \quad z_\ell = 1, \quad z_H = 0$
- **B-L model:**
 $Q_z^f = B_f - L_f; \quad z_q = 1/3, \quad z_\ell = -1, \quad z_H = 0$
- **B+L model:**
 $Q_z^f = B_f + L_f; \quad z_q = 1/3, \quad z_\ell = 1, \quad z_H = 0$

Chiral models ($z_H \neq 0$)

- **Y-sequential:**
 $Q_f^z = z_H Y_f; \quad z_q = (1/3)z_H; \quad z_\ell = -z_H$
- **SO(10) GUT:**
 $Q_f^z = -(1/2)(B_f - L_f) + (1/5)Y_f; \quad z_q = -1/10; \quad z_\ell = 3/10; \quad z_H = 1/5$
- **Right-handed quark:**
 $Q_f^z = z_H(Y_f - B_f) + (z_\ell + z_H)L_f; \quad z_q = 0$
- **Right-handed lepton:**
 $Q_f^z = (3z_q - z_H)B_f + z_H(Y_f + L_f); \quad z_\ell = 0$
- **Gravity model:**
 $Q_f^z = -2z_\ell B_f + 3(z_q + z_\ell)L_f + 3(3z_q + 2z_\ell)Y_f$
 $z_H = 3z_q + 2z_\ell$
- **Right-handed:**
 $Q_f^z = z_H(Y_f - B_f + L_f); \quad z_q = 0; \quad z_\ell = 0$
- **Left-handed lepton:**
 $Q_f^z = 3z_q B_f + z_\ell(2Y_f + L_f); \quad z_H = z_\ell$
- **Axial lepton:**
 $Q_f^z = (3z_q + z_\ell/2)B_f + (z_\ell/2)(L_f - Y_f)$
 $z_H = -z_\ell/2$

Many possibilities, very rich phenomenology

Branching ratios



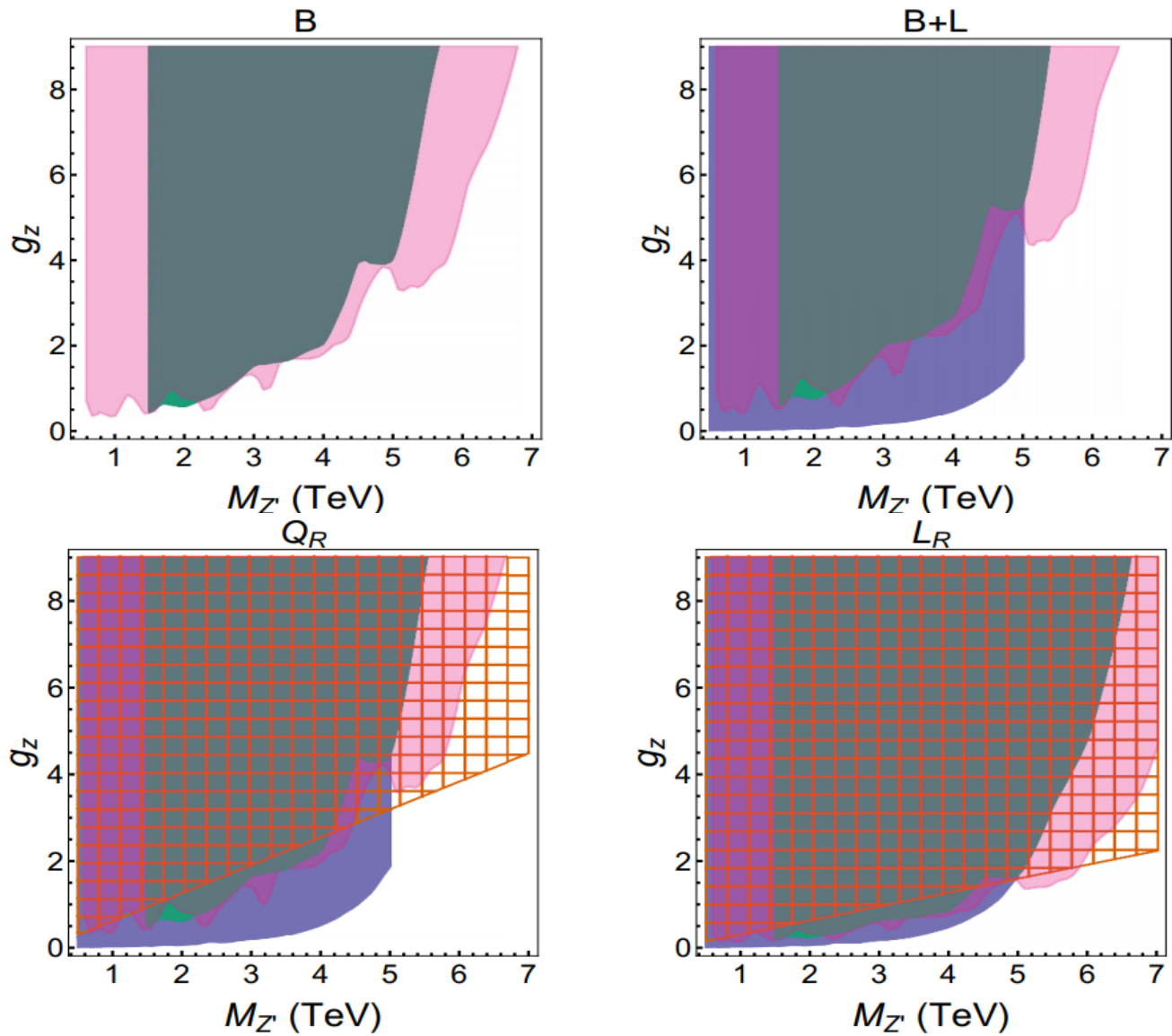
Unique signature

Axion couplings: $\mathcal{A}ZZ, \mathcal{A}Z'Z', \mathcal{A}\gamma\gamma, \mathcal{A}W^+W^-$

- Minimal setup axion is not physical, but simply a Goldstone boson
- Considering a more complicated Higgs sector, it is possible to furnish a physical axion and a Goldstone boson, through mixing with other scalar fields
- Unique signatures GS: $ZZ\gamma, ZZ'\gamma, Z'Z'\gamma$
- We are especially interested in $Z' \rightarrow ZZ$ and $Z' \rightarrow \gamma Z$ decays which, if observed, may give indications of the GS nature of the theory
- Production cross section can be written as

$$\sigma(M_{Z'}, g_z, z_q, z_H) = \frac{g_z^2}{4} \left[a^u(M_{Z'}) \left\{ z_q^2 + (z_q + z_H)^2 \right\} + a^d(M_{Z'}) \left\{ z_q^2 + (z_q - z_H)^2 \right\} \right]$$

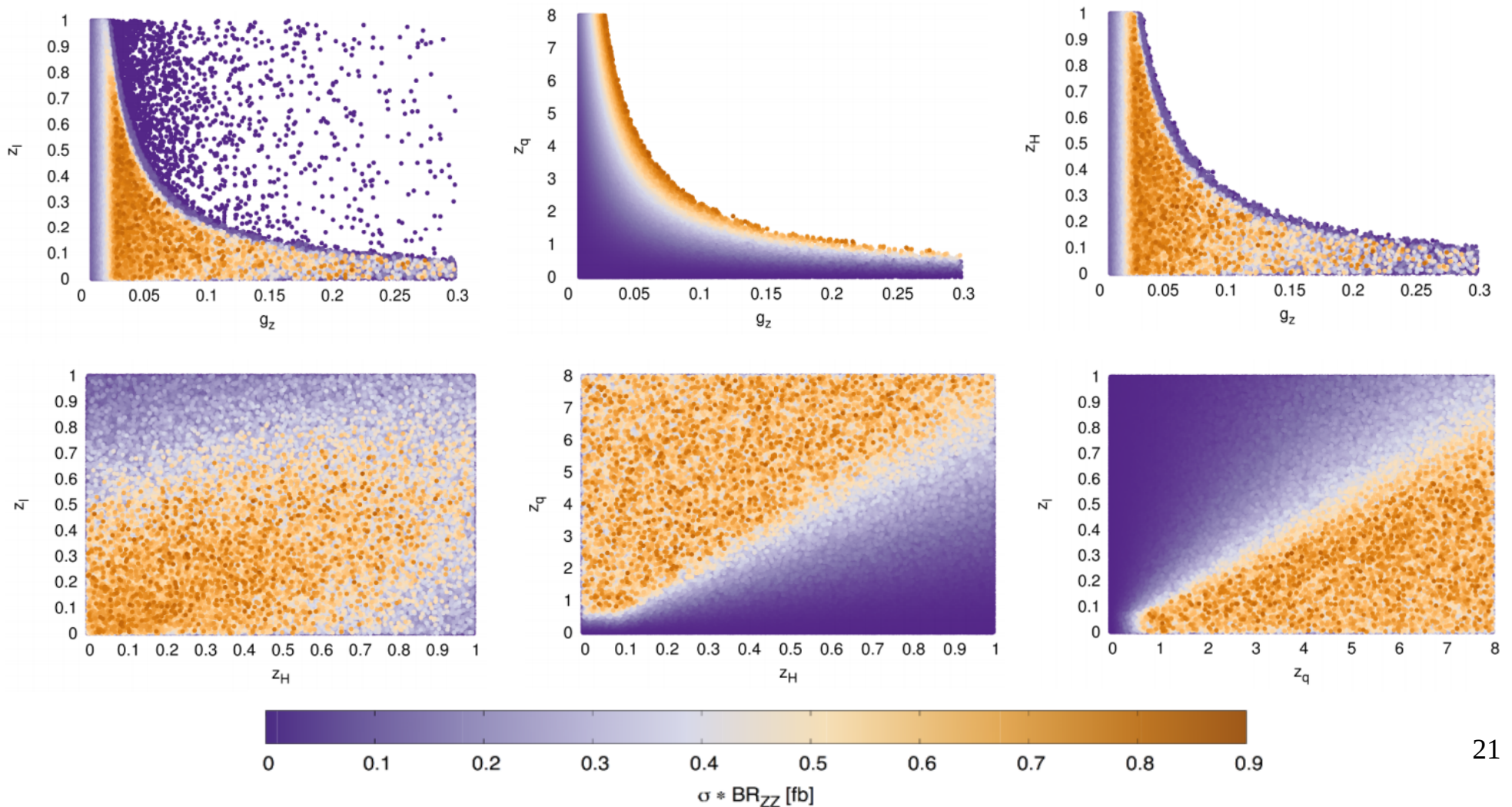
Exclusion limits: various models



■ ATLAS dilepton ■ ATLAS dijet ■ CMS dijet ■ T parameter

Prospect: $GS-Z'$ at HL-LHC

- Random Scan: $0.5 \text{ TeV} < M_{Z'} < 0.8 \text{ TeV}$, $0.01 < g_z < 0.3$, $0 < z_H, z_\ell < 1.0$, $0 < z_q < 8$,
- Constraints: ■ ATLAS dilepton ■ ATLAS dijet ■ CMS dijet □ T parameter



Prospect: $GS-Z'$ at HL-LHC

- Unique signatures: $Z' \rightarrow ZZ; Z' \rightarrow Z\gamma$
- Very small branching and therefore very challenging to observe
At 3000 fb^{-1} integrated luminosity
 $M_{Z'} \sim 0.5 \text{ TeV} \rightarrow \sigma \times \text{BR}(Z' \rightarrow ZZ) \sim 1.0 \text{ fb} \rightarrow N = 3000$
 $M_{Z'} \sim 0.5 \text{ TeV} \rightarrow \sigma \times \text{BR}(Z' \rightarrow Z\gamma) \sim 0.25 \text{ fb} \rightarrow N = 750$
- If there is $GS-Z'$, it is expected to show up in the dijet/dilepton channels before the discovery in the $ZZ/Z\gamma$ channels
- In 750 GeV context: Is an 'axizilla' possible for di-photon resonance?
- We have axion couplings $\mathcal{A}ZZ, \mathcal{A}Z'Z', \mathcal{A}\gamma\gamma, \mathcal{A}W^+W^-$
- Axionic dark matter, cosmological implications?

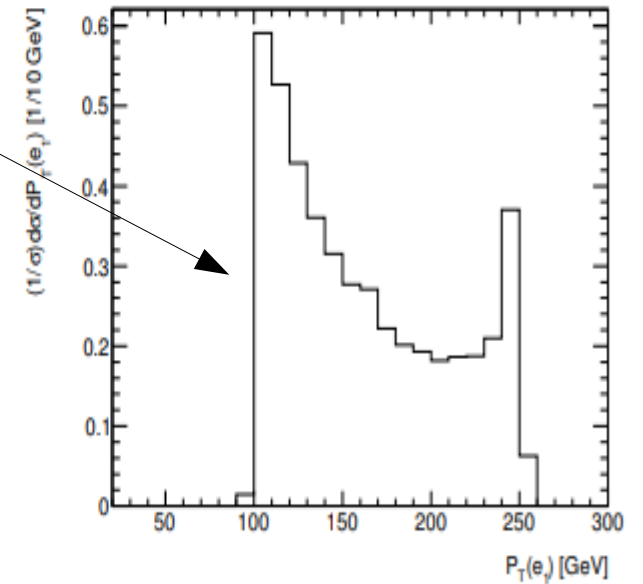
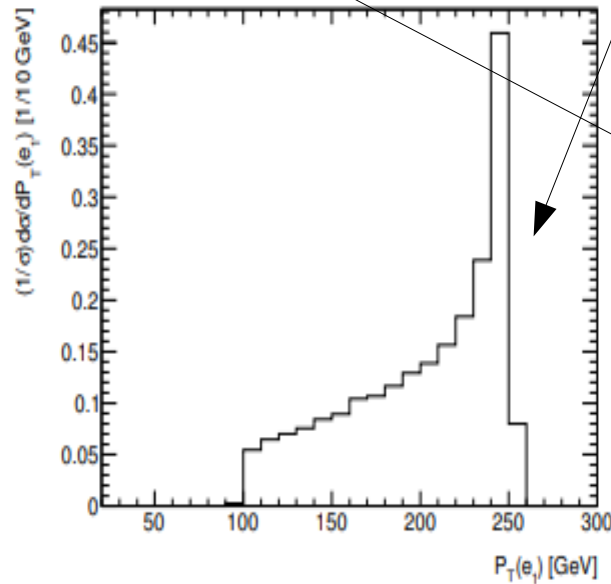
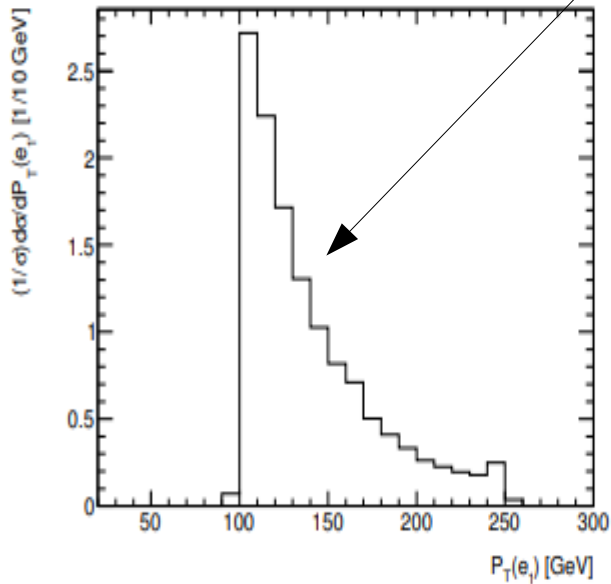
PLB 755 (2016) 190-195

Prospect: Z' at ILC

$$\sigma_{\text{tot}}(e^+e^- \rightarrow e^+e^-) = \sigma_{\text{SM}} + \kappa^2 \sigma_I(M_{Z'}) + \kappa^4 \sigma_{\text{BSM}}(M_{Z'})$$

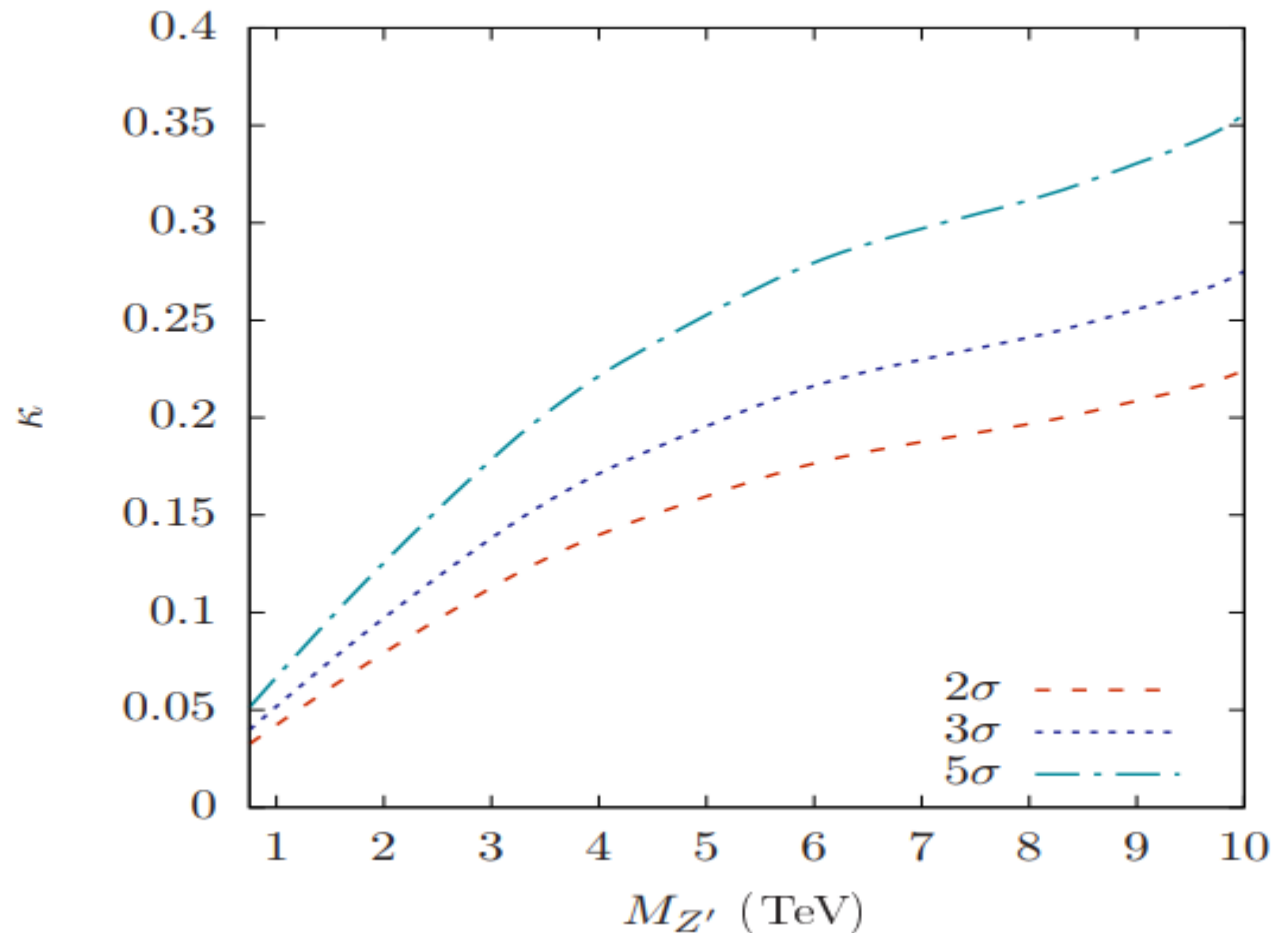
$M_{Z'} = 1 \text{ TeV}; \sqrt{s} = 0.5 \text{ TeV}$

$$\mathcal{L} \supset \kappa Z' e^+ e^-$$



$p_T(e_1), p_T(e_2) > 100 \text{ GeV}; |\eta(e_1)|, |\eta(e_2)| < 2.5; \Delta R(e_1, e_2) > 0.4$

Prospect: Z' at ILC



$$\sqrt{s} = 0.5 \text{ TeV}$$
$$\mathcal{L} = 100 \text{ fb}^{-1}$$

$$\sigma = \frac{\mathcal{N}_s}{\sqrt{\mathcal{N}_B}}$$

- Simple minded analysis with $p_T(e_1), p_T(e_2) > 200 \text{ GeV}$
- Can be improved by using multivariate analysis, optimal observable methods

Outlooks: RHN pair production & other...

	SU(3) _c	SU(2) _L	U(1) _Y	U(1) _X
q_L^i	3	2	1/6	$(1/6)x_H + (1/3)x_\Phi$
u_R^i	3	1	2/3	$(2/3)x_H + (1/3)x_\Phi$
d_R^i	3	1	-1/3	$-(1/3)x_H + (1/3)x_\Phi$
ℓ_L^i	1	2	-1/2	$(-1/2)x_H - x_\Phi$
e_R^i	1	1	-1	$-x_H - x_\Phi$
H	1	2	-1/2	$(-1/2)x_H$
N_R^j	1	1	0	$-x_\Phi$
Φ	1	1	0	$+2x_\Phi$

Table 1: Particle content of the minimal U(1)_X model, where $i, j = 1, 2, 3$ are indices. Without loss of generality, we fix $x_\Phi = 1$.

From PRD, 97, 115023 (2018)
and EPJC, 78, 696 (2018) by
A. Das, N. Okada and D. Raut

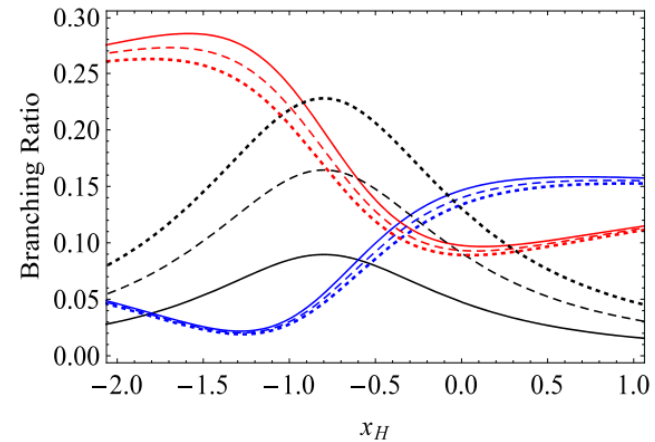


Figure 1: The branching ratios of Z' boson as a function of x_H with a fixed $m_{Z'} = 3$ TeV. The solid lines correspond to $m_{N^1} = m_{Z'}/4$ and $m_{N^{2,3}} > m_{Z'}/2$; the dashed (dotted) lines correspond to $m_{N^{1,2}} = m_{Z'}/4$ and $m_{N^3} > m_{Z'}/2$ ($m_{N^{1,2,3}} = m_{Z'}/4$). From top to bottom, the solid (red, black and blue) lines at $x_H = -1$ are the branching ratios to the first generations of jets (up and down quarks), RHNs, and charged leptons, respectively. The lines for the RHN final states correspond to the sum of the branching ratio to all possible RHNs.

- Three new particles:

$$Z', N_R, H_2$$

$$Z' \rightarrow N_R N_R; \quad Z' \rightarrow Z H_2; \quad \dots$$

- Displaced vertices, correlated collider searches are very important

Neutrino mass

Demanding U(1)_z invariance

- Majorana mass term: D = 5 Weinberg operator

$$\frac{1}{M} (\bar{\ell}_L^c H) (\ell_L H) \longrightarrow 4z_q = z_u$$

- In general: D > 5 effective operators $\tilde{\phi} = \{\phi, \phi^\dagger\}$

$$\frac{1}{M} \left(\frac{\tilde{\phi}}{M} \right)^n (\bar{\ell}_L^c H) (\ell_L H) \longrightarrow 8z_q - 2z_u = \pm n$$

- Dirac mass term: D = 4 renormalizable operator

$$\bar{\ell}_L \nu_R \tilde{H} \longrightarrow z_k = -4z_q + z_u$$

- In general: D > 5 effective operators

$$\left(\frac{\tilde{\phi}}{M} \right)^m \bar{\ell}_L \nu_R \tilde{H} \longrightarrow z_k + 4z_q - z_u = \pm m$$

- Majorana mass term: right-handed neutrino

$$M \left(\frac{\tilde{\phi}}{M} \right)^{n'} \bar{\nu}_R^c \nu_R \longrightarrow z_k = \pm \frac{n'}{2}$$

$$4z_q - z_u = \pm \frac{m'}{2}$$

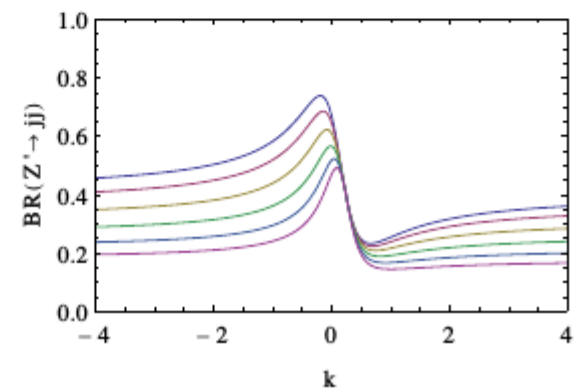
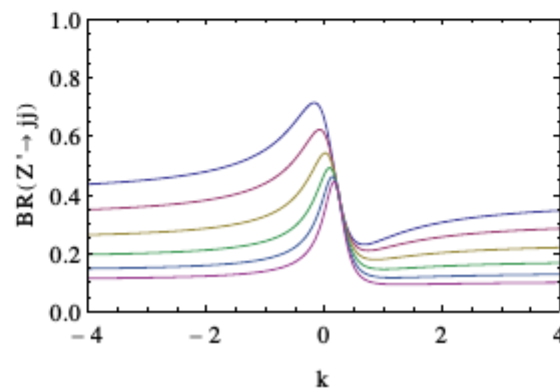
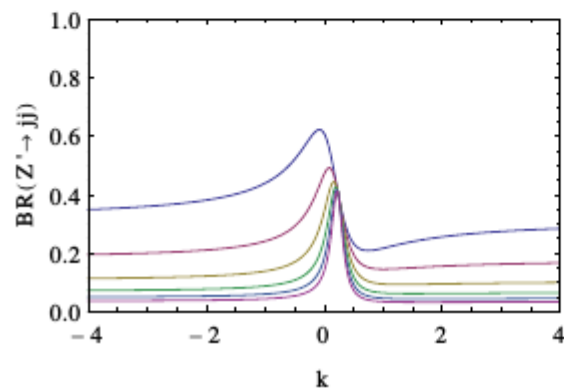
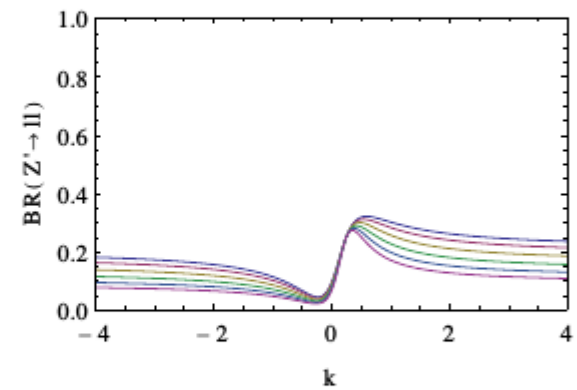
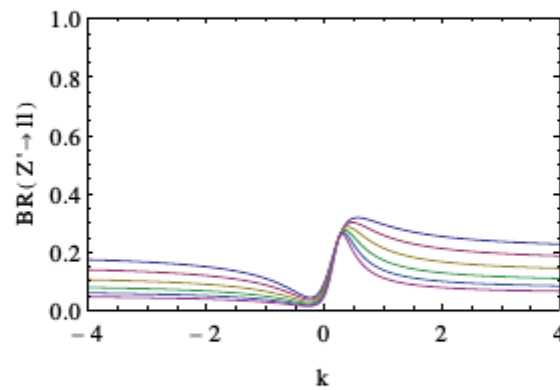
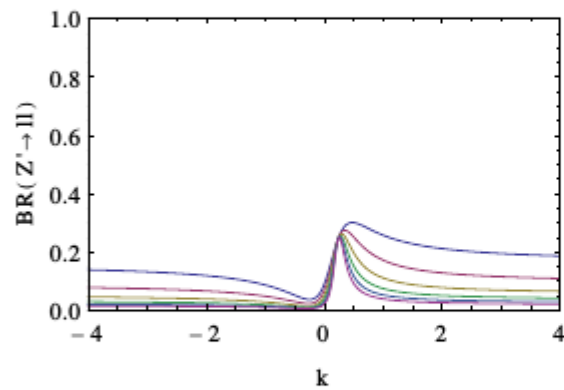
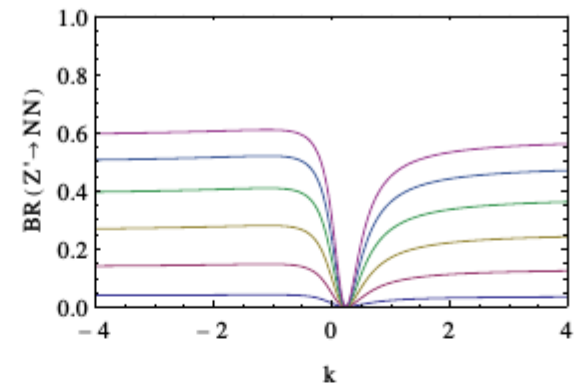
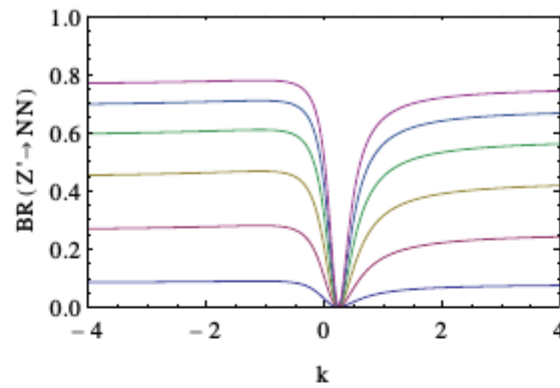
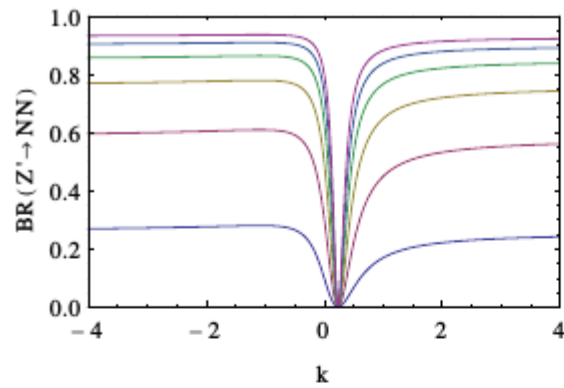
$$z_k = \pm \frac{n'}{2}$$

Branching ratio: Z'

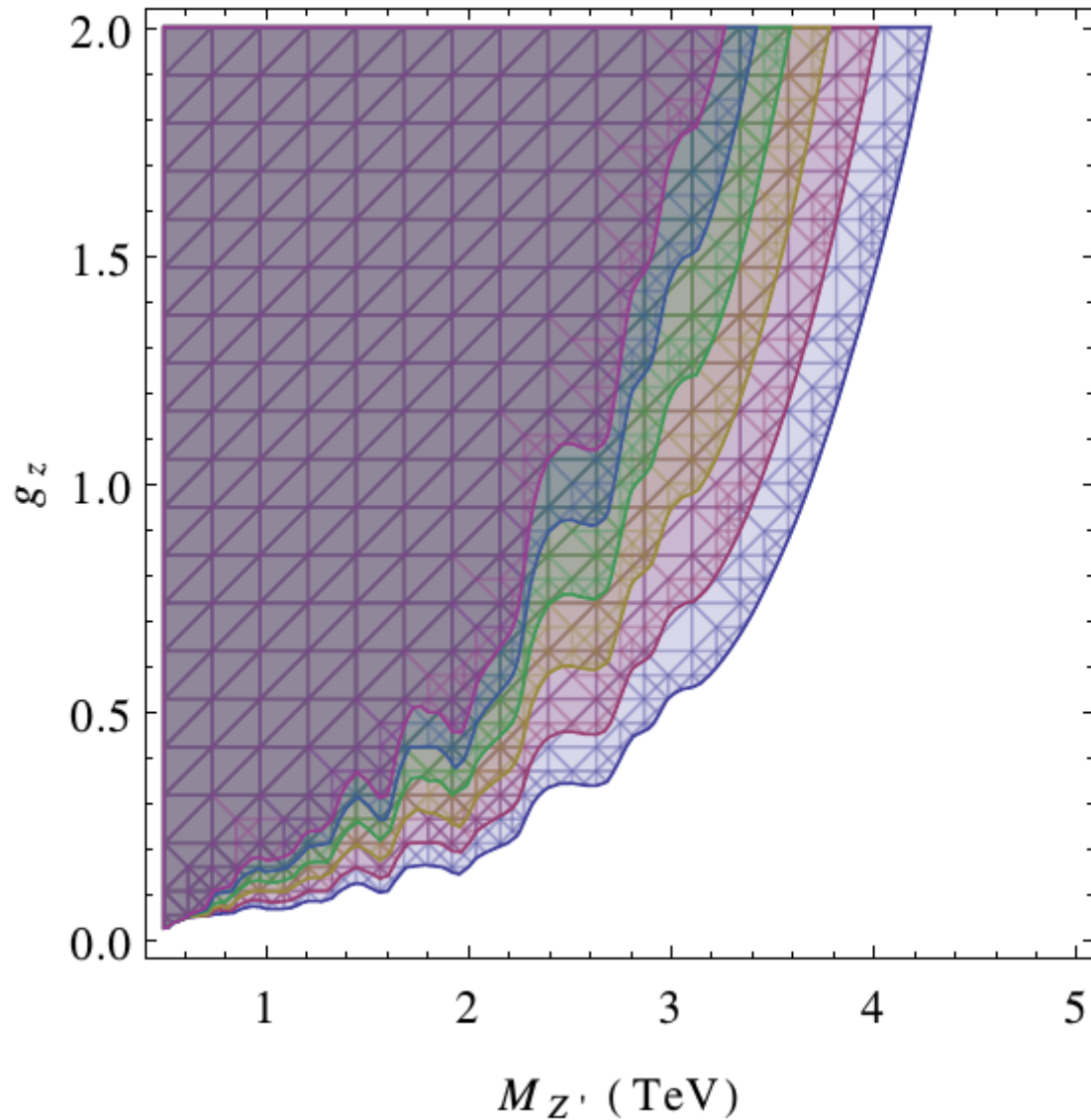
$m' = 1$

$m' = 2$

$m' = 3$

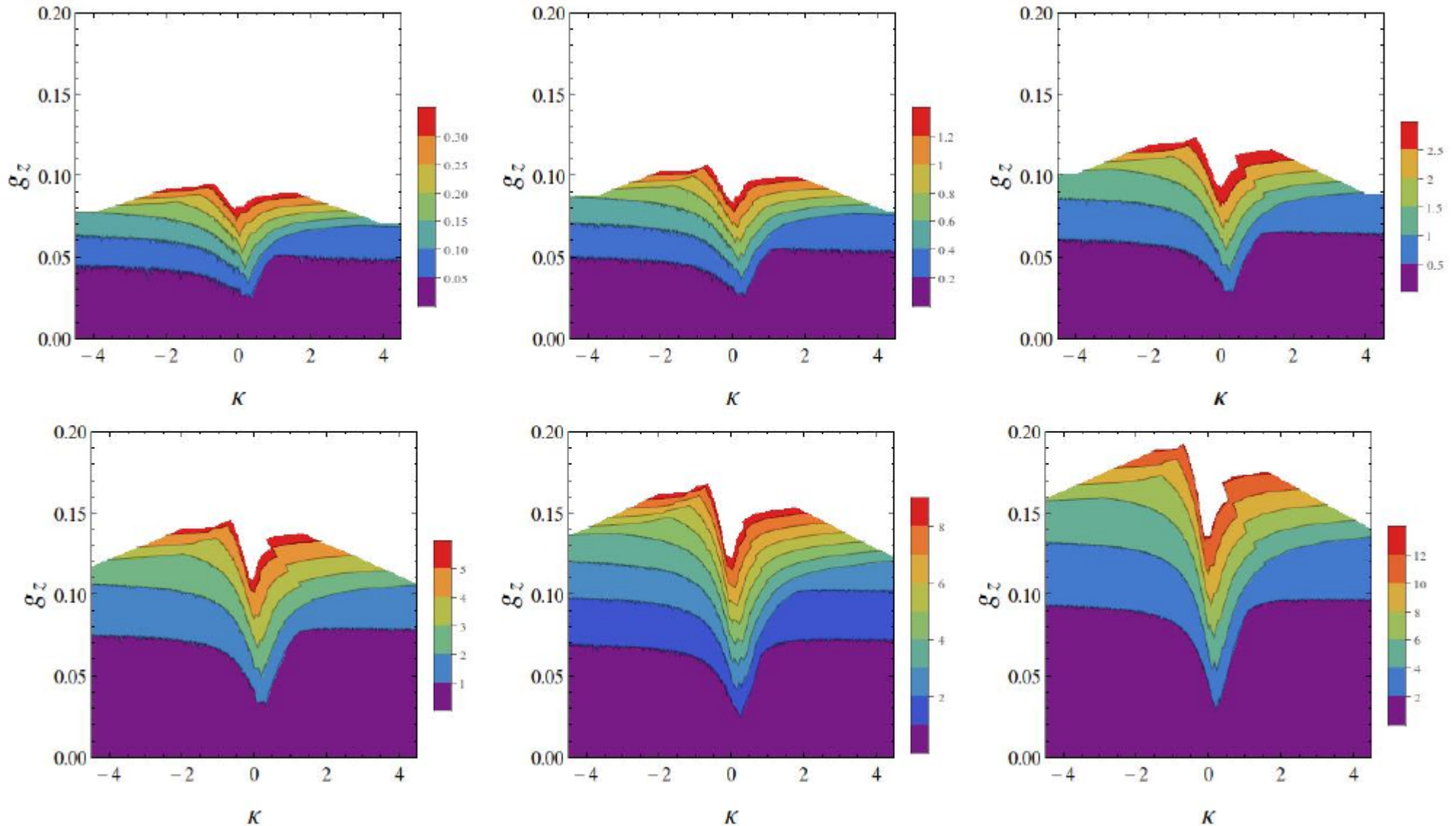


Exclusion from dilepton data



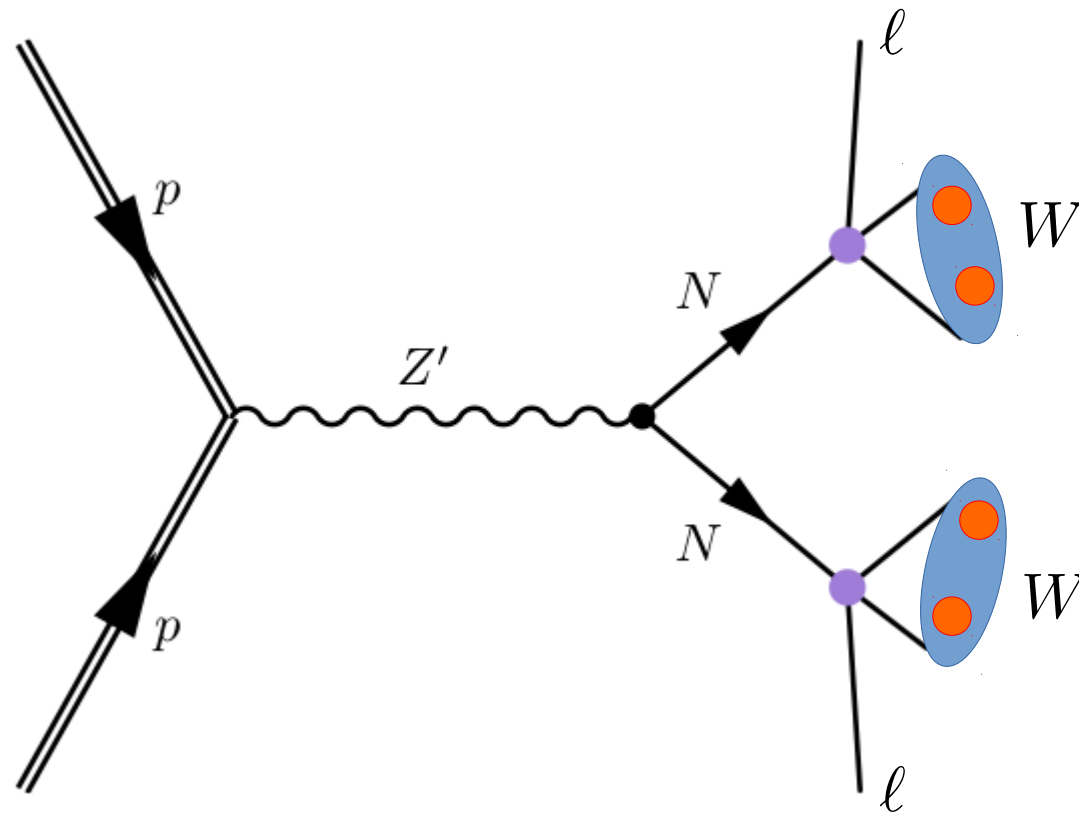
$$m' = 1$$

Allowed by the dilepton data



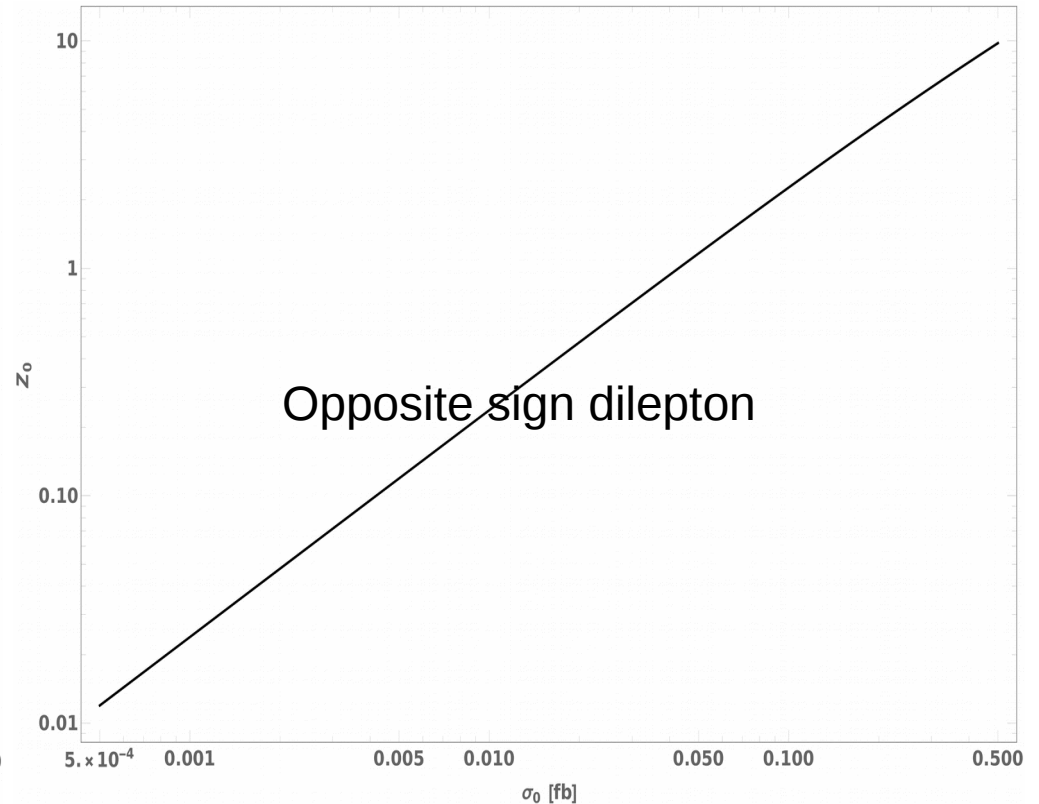
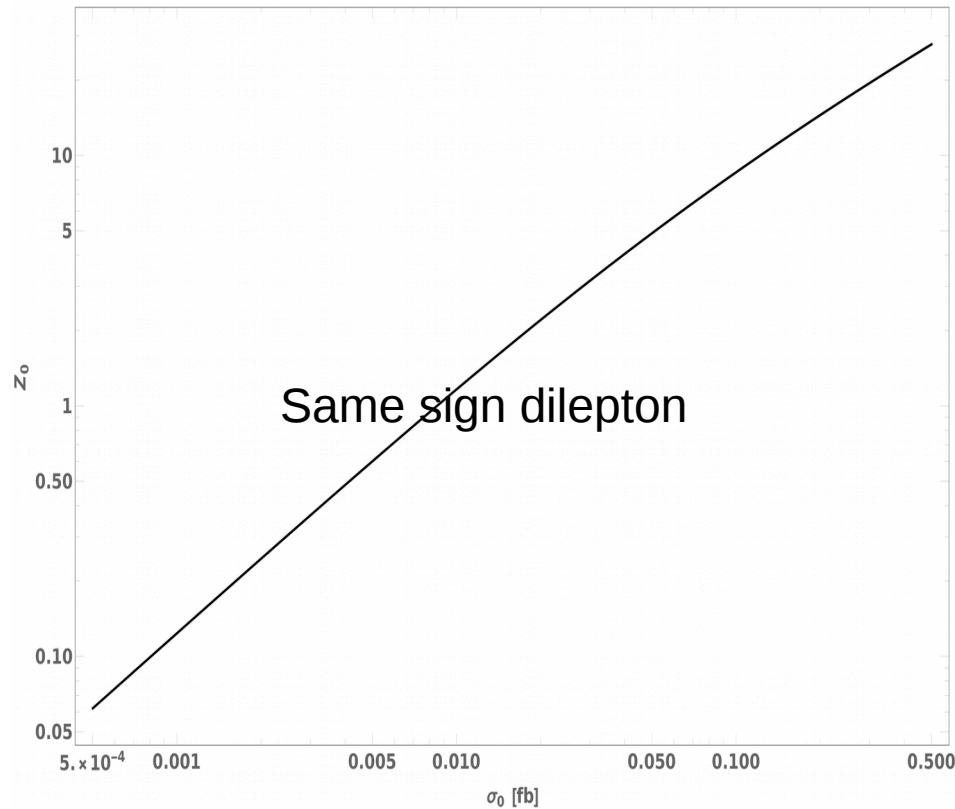
$$m' = 1; \quad M_{Z'} = 1.5 \text{ TeV}$$

$Z' \rightarrow NN$ channel



Same-sign OR opposite-sign dilepton +
two-boosted W -jet (two prong)

Sensitivity @ HL-LHC



- We have to translate cross sections in terms of model parameters
- Plan to investigate symmetric and asymmetric channels with other decay modes of $N \rightarrow Z\nu, H\nu$
- Multivariate analysis to improve the sensitivity especially lower mass region

Outlooks: dark matter models

- *Anomaly-Free Dark Matter Models are not so Simple*
by Ellis, Fairbairn, Tunney, JHEP, 08 053 (2017)
- *Phenomenological Constraints on Anomaly-Free Dark Matter Models*
by Ellis, Fairbairn, Tunney, arXiv:1807.02503
- They have considered fermionic dark matter particles to cancel gauge anomalies
- Can we think of a next-to-minimal anomaly free setup with DM and RHN to have seesaw mechanism?
- Mass generation with combined Higgs-Stückelberg mechanism can lead to massive axions. Can they be DM?

Summary and conclusions

- We consider minimal anomaly free $U(1)$ extensions of the SM
- Anomalies are cancelled by introducing chiral fermions or through the Green-Schwarz mechanism
- Using EWPT constraints, 13 TeV LHC data, , we constrain minimal anomaly free $U(1)$ extensions of the SM
- We introduce a model independent parametrization for these class of models
- **Project-1: (ongoing)** New signatures like RHN pair productions, correlated collider analysis with Z' , RHN and heavy scalar
- **Project-2:** Combined Higgs-Stuckelberg mechanism, axion (light or heavy) DM, in context of Green-Schwarz mechanism
- **Project-3:** Anomaly free $U(1)$ dark matter models with RHN to have seesaw mechanism