

Significance of relativistic dynamics to the Dark Matter problem.

A. Deur

Uppsala, Sept. 24th 2020

A.D., C. Sargent, B. Terzić, ApJ 896 2, 94 (2020)

A.D., EPJC 79 10, 883 (2019)

A.D., EPJC 77 6, 412 (2017)

A.D., MNRAS, 438, 1535 (2014)

A.D., PLB, 676, 21 (2009)

Dark Matter

Dark Matter is ubiquitous in the universe.

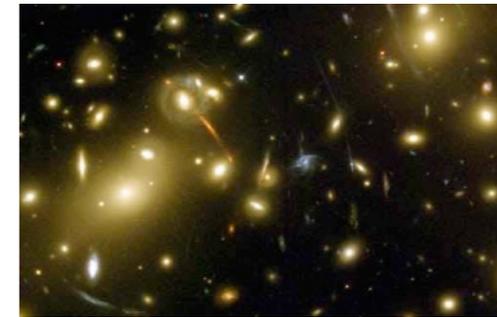
It is estimated to represent 85% of the matter in the universe and 27% of its mass.

What it is needed for:

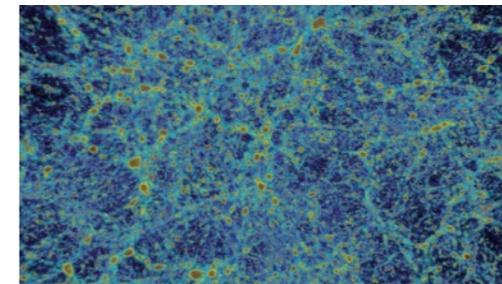
- Explains the fast rotation of disks galaxies.



- Keeps galaxies confined in clusters.

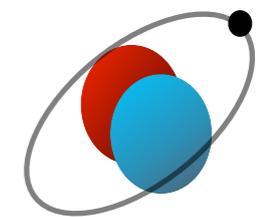


- Speed-up large structure formations.



- Needed for Cosmic Microwave Background anisotropy distributions

- Add necessary non-baryonic density explaining amount of deuterium created in primordial nucleosynthesis.

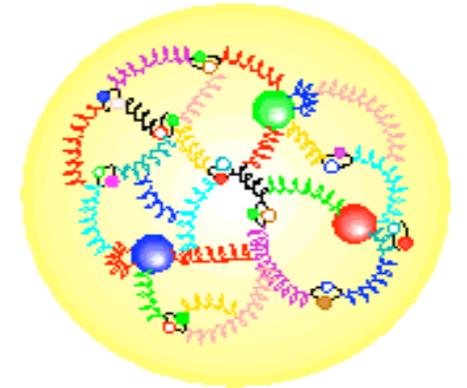


- Etc...

What it is remains a mystery.

Our Ariadne thread: Quantum Chromodynamics (QCD)

QCD: Theory of the strong force binding quarks into hadrons.



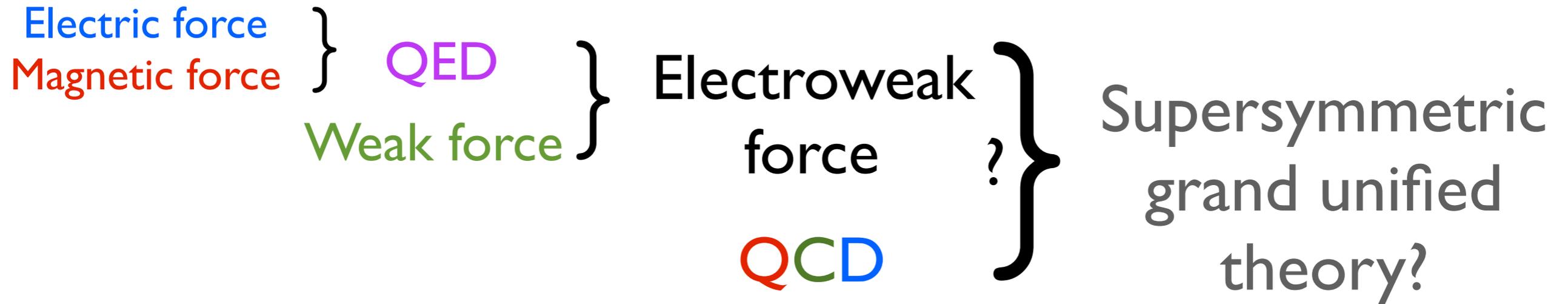
The vectors of the strong force (the gluons) carry color charges.

⇒ They interact with each other. Origin of quark and gluon confinement.

We will follow here our QCD/Ariadne thread.

⇒ Not an exhaustive survey of the possible solutions to the dark matter mystery.

What can QCD teach us about dark matter?



⇒ WIMPs

QCD is free to violate Charge-Parity (CP) symmetry. Yet, there is no evidence of violation. To force QCD to not be CP-violating, add new symmetry which is spontaneously broken.

⇒ Axions

What can QCD teach us about dark matter?

Electric force
Magnetic force



Weak force



force



Supersymmetric
grand unified

Most natural SUSY has not been found at LHC.

⇒ WIMPs

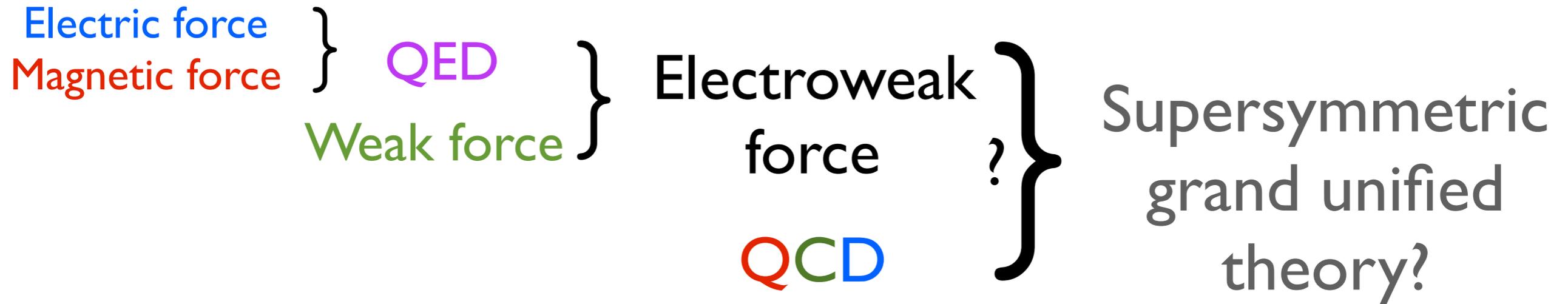
Direct searches (LUX, CDMS...) or production (LHC,...) are so far negative and most favored models are ruled out.

QCD is free to violate Charge-Parity (CP) symmetry. Yet, there is no evidence of violation. The WIMP miracle did not come to pass. new symmetry which is spontaneously broken.

⇒ Axions

Axions searches (ADMX, XENON) have also excluded favored phase space.

What can QCD teach us about dark matter?



⇒ WIMPs

QCD is free to violate Charge-Parity (CP) symmetry. Yet, there is no evidence of violation. To force QCD to not be CP-violating, add new symmetry which is spontaneously broken.

⇒ Axions

QCD and gravity have a similar underlying structure (similar field Lagrangians). Complex QCD effects (e.g. confinement) could not have been predicted.
⇒ Hard to guess the consequences of the similar Lagrangian of gravity.

⇒ Look for parallels between hadron phenomenology and dark matter observations.

Key facts of Strong Force

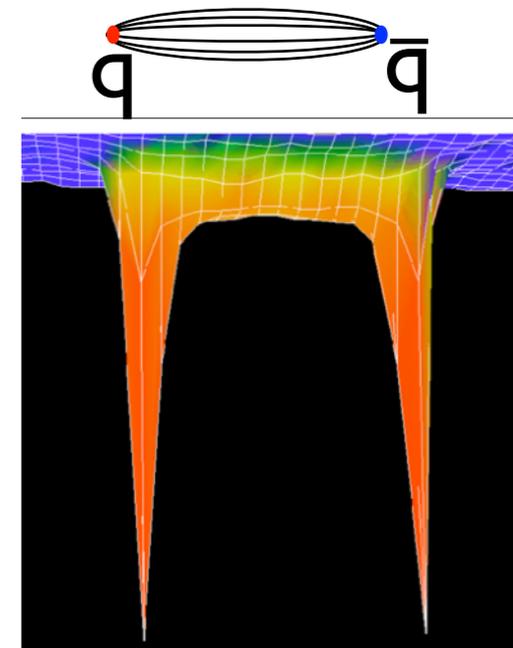
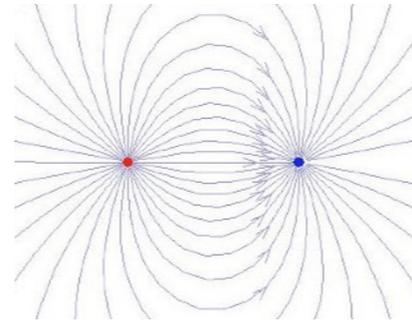
Facts

- Quarks and gluons are **confined** inside hadrons

Explanations

Strong force coupling is large & gluons are color-charged

⇒ field-lines collapse into **string-like flux-tubes**.



Evidences from lattice QCD:

Key facts of Strong Force

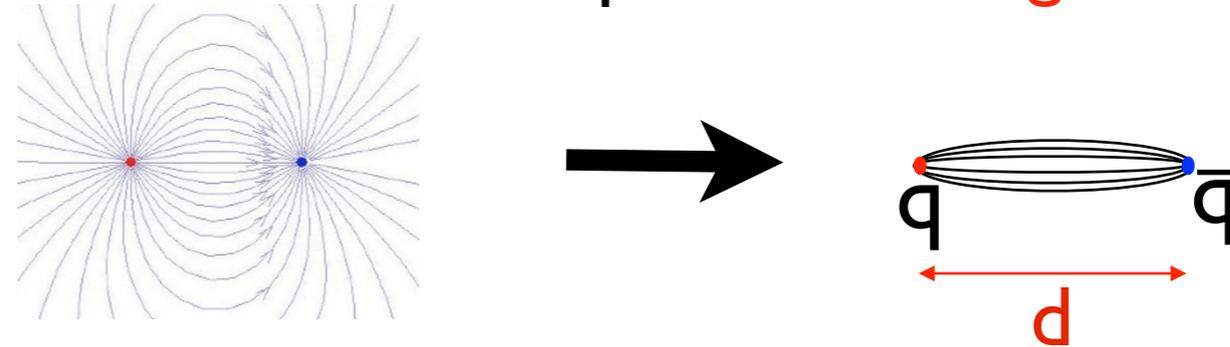
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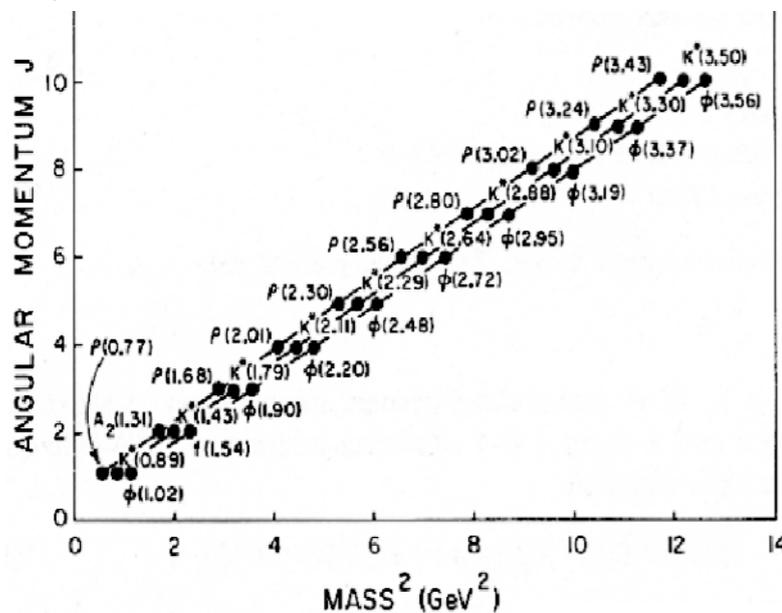
Quarks and gluons are **confined** inside hadrons

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Hadrons lie on **Regge trajectories**

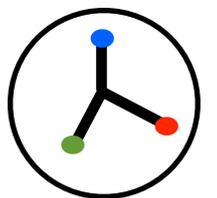
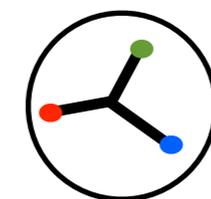


The more a hadron spins, the larger the binding energy (i.e. the mass) to compensate for the centrifugal force.

String Potential $\propto d \Rightarrow$ Ang. Mom. $\propto M^2 + \text{constant}$

Strong force not felt at large distances (but for residual effects: Yukawa potential, etc...)

Field lines (i.e. gluons) are inside the hadron.



Empirical parallels between cosmology and hadronic physics

Cosmology

Galaxies (or clusters of galaxies) have a larger mass than the sum of their known constituents.

Tully-Fisher relation:

$\log(M) = \gamma \log(v) + \varepsilon$ ($\gamma = 3.9 \pm 0.2$, $\varepsilon \sim 1.5$)
(M galaxy visible mass, v rotation speed)

Hadronic physics

2 quarks ~ 7 MeV, Pion mass 140 MeV
3 quarks ~ 9 MeV, Proton: 938 MeV

Regge trajectories:

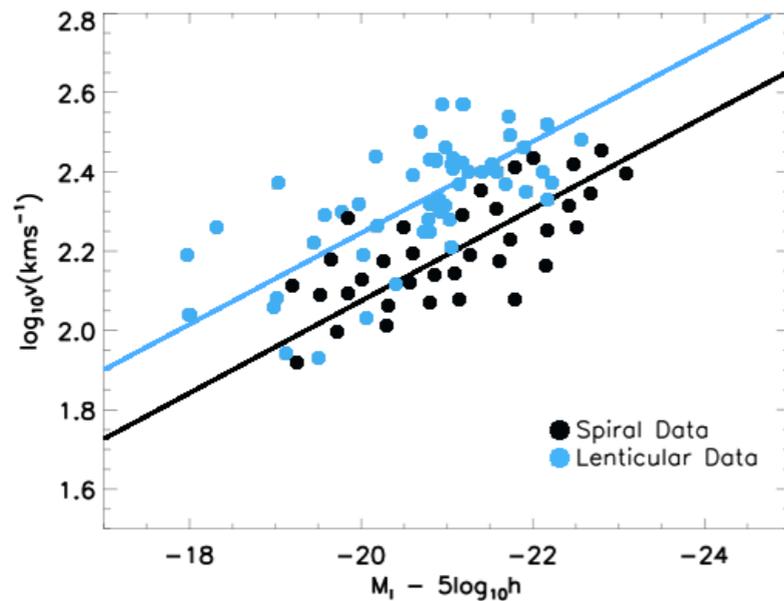
$\log(M) = c \log(J) + b$ ($c = 0.5$)
(M, hadron mass, J angular momentum)

Empirical parallels between cosmology and hadronic physics

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Tully-
log(
(M gal

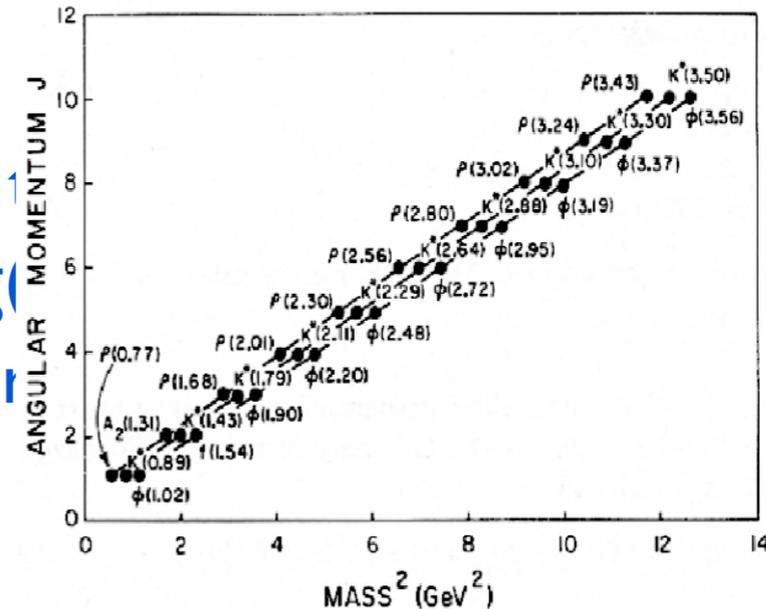


~ 1.5
speed)

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log(
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Negative pressure pervades the Universe and repels galaxies from each other.

Presently, gravity + dark energy ~ 0 at large distance (cancel in Universe's evolution). "Cosmic coincidence".

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Regge trajectories:

$\log(M) = c \log(J) + b$ ($c = 0.5$)
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Strong force ~ 0 outside hadrons (only residual Yukawa force).

Intriguing correspondence between key phenomena of hadronic physics and observations involving dark matter and dark energy.

It might be due to the similarities between gravity's underlying theory (General Relativity) and that of the strong force (QCD).

Theoretical parallels between gravity and the strong force

General Relativity

$$\mathcal{L}_{GR} = \frac{1}{16\pi G} \sqrt{-g} g^{\mu\nu} R_{\mu\nu}$$

(Annotations: $\sqrt{-g}$ is labeled "det $g_{\mu\nu}$ "; $g^{\mu\nu}$ is labeled "Metric tensor"; $R_{\mu\nu}$ is labeled "Curvature tensor")

Expand \mathcal{L}_{GR} in term of gravity field $\psi_{\mu\nu}$ by developing $g_{\mu\nu}$ around the flat metric:

$$g_{\mu\nu} \sim \eta_{\mu\nu} + G^{1/2} \psi_{\mu\nu} + \dots$$

$$\mathcal{L}_{GR} = [\partial\psi\partial\psi] + \sqrt{16\pi G} [\psi\partial\psi\partial\psi] + \frac{1}{16\pi G} [\psi^2\partial\psi\partial\psi] + \dots$$

(pure field Lagrangian)

Newton's gravity
(static case): $1/r$ potential

QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

(pure field Lagrangian)

Expand \mathcal{L}_{QCD} in term of gluon field ψ_{μ}^a :

$$\mathcal{L}_{QCD} = \frac{1}{4} [\partial\psi\partial\psi] + \sqrt{\pi\alpha_s} [\psi^2\partial\psi] - \pi\alpha_s [\psi^4]$$

(pure field Lagrangian)

Perturbative QCD
(static case): $1/r$ potential

Theoretical parallels between gravity and the strong force

General Relativity

$$\mathcal{L}_{GR} = \frac{1}{16\pi G} \sqrt{-g} g^{\mu\nu} R_{\mu\nu}$$

← Curvature tensor (pointing to $R_{\mu\nu}$)
 ← Metric tensor (pointing to $g^{\mu\nu}$)
 ← $\det g_{\mu\nu}$ (pointing to $\sqrt{-g}$)

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Field self-interaction terms

$$\partial\psi\partial\psi + G^{1/2} \psi\partial\psi\partial\psi + G\psi^2\partial\psi\partial\psi$$

Theoretical parallels between gravity and the strong force

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Differences between GR and QCD

- **G is very small** ($GM_p^2 = 5.9 \times 10^{-39}$)
- **Tensor field, spin: 2** (gravity always attracts \Rightarrow **gravity effects add up**)
- **α_s is large: ~ 0.1**
- **Gluon spin: 1** (QCD attracts or repulses, as for QED)

That $\sqrt{16\pi G}$ is small can be compensated by large ψ for massive enough systems (see QED and non-linear optics).

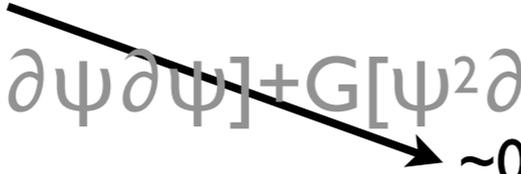
$\Rightarrow \sqrt{16\pi G} [\psi\partial\psi\partial\psi]$ can be significant.

\Rightarrow Open possibility for gravity self-interaction to generate similar effects to those seen in QCD

Near a proton $\sqrt{16\pi GM_p/r_p} = 1 \times 10^{-17}$ with M_p the proton mass and r_p its radius.

⇒ Self-interaction effects are negligible:

$$\mathcal{L}_{GR} = [\partial\psi\partial\psi] + \sqrt{G} [\psi\partial\psi\partial\psi] + G[\psi^2\partial\psi\partial\psi] + \dots$$

 ~ 0

For a typical galaxy: $\sqrt{16\pi GM/\text{size}_{\text{system}}} \sim 0.05$.

If GR's self-interaction is not negligible: causes an increase in gravitational force.

If galaxy dynamics analyzed using Newton's law, this increase would appear as a **missing mass** (Dark Matter).

Quantitative estimate of gravity's self-interaction

$$\mathcal{L}_{GR} = [\partial\psi\partial\psi] + \sum_{n=1} (16\pi G)^{n/2} [\psi^n \partial\psi\partial\psi] - \sum_{n=1} (16\pi G)^{n/2} \psi^{n-1} \psi_{\mu\nu} T^{\mu\nu}$$

Short hand for sum of possible Lorentz-invariant terms of form $[\partial\psi\partial\psi]$.

Explicitly given by the [Fierz-Pauli Lagrangian](#):

$$[\partial\psi\partial\psi] \equiv \frac{1}{2} \partial^\lambda \psi_{\mu\nu} \partial_\lambda \psi^{\mu\nu} - \frac{1}{2} \partial_\lambda \psi^\mu_\mu \partial^\lambda \psi^\nu_\nu - \partial^\lambda \psi_{\lambda\nu} \partial_\mu \psi^{\mu\nu} - \partial^\nu \psi^\lambda_\lambda \partial^\mu \psi_{\mu\nu}$$

Quantitative estimate of gravity's self-interaction

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For a stationary system, we approximate ψ as scalar (ψ_{00} component)

$$\mathcal{L}_{GR} = \partial\psi\partial\psi + \sqrt{16\pi G}\psi\partial\psi\partial\psi + 16\pi G\psi^2\partial\psi\partial\psi + \dots + \sqrt{16\pi G}\psi_{00}T^{00}$$

Use Feynman path-integral formalism on a lattice to obtain gravity's potential.

Stationary system:

- 3D problem (ignore time dimension);
- Instantaneous potential given by 2-point Green's function;
- Results are classical: Path-integral weight: $e^{-iS/\hbar}$

$$S \stackrel{\text{def}}{=} \int \mathcal{L} d^4x = \tau \int \mathcal{L} d^3x = \tau S' \quad \text{with } \tau \rightarrow \infty.$$

$$e^{-iS'\tau/\hbar} \quad \text{with } \tau/\hbar \rightarrow \infty \equiv \hbar \rightarrow 0: \text{ classical case.}$$

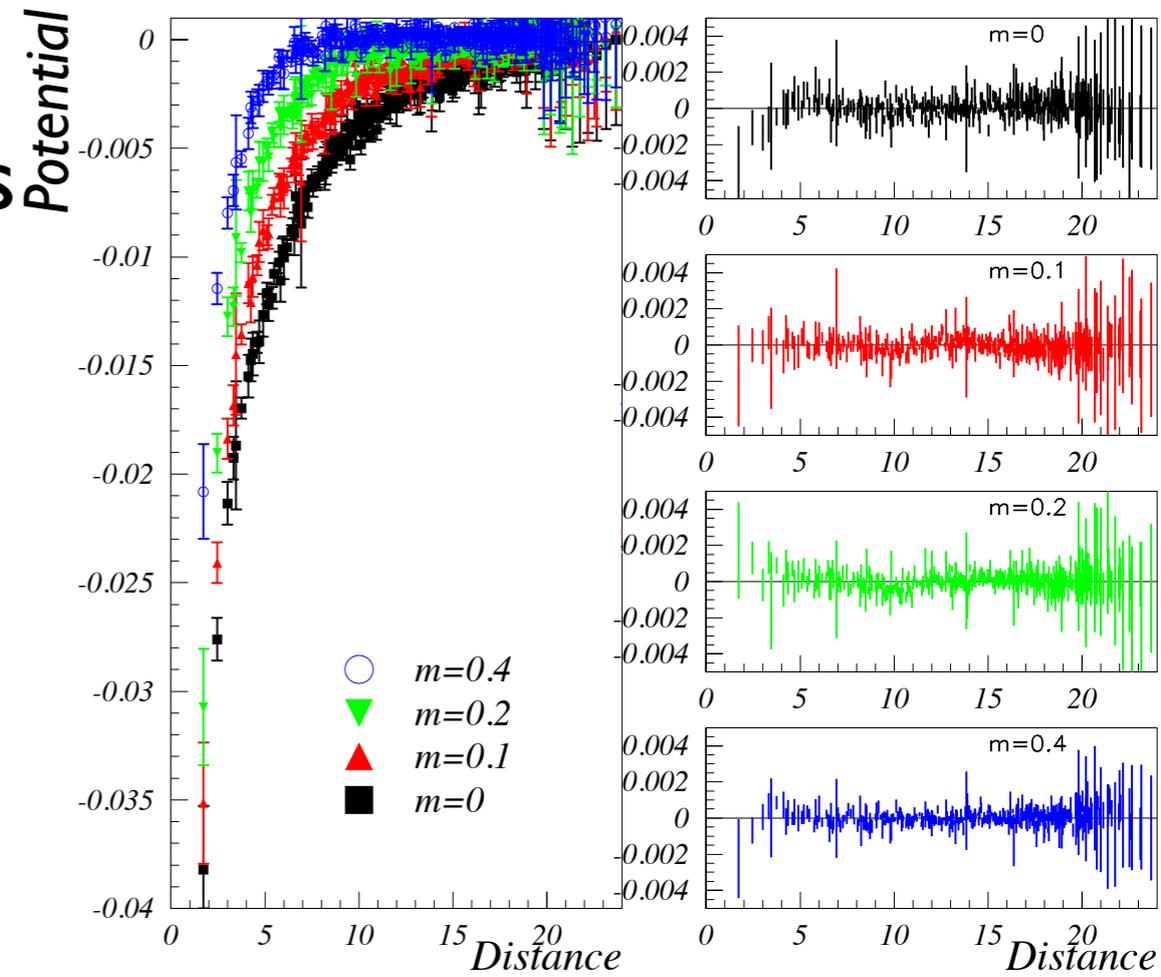
($\tau \rightarrow \infty$ equivalent to high temperature limit in condensed matter.

High temperature limit \equiv classical limit)

Verifications

Calculation:

Recover Newton and Yukawa potentials



Verifications

Calculation:

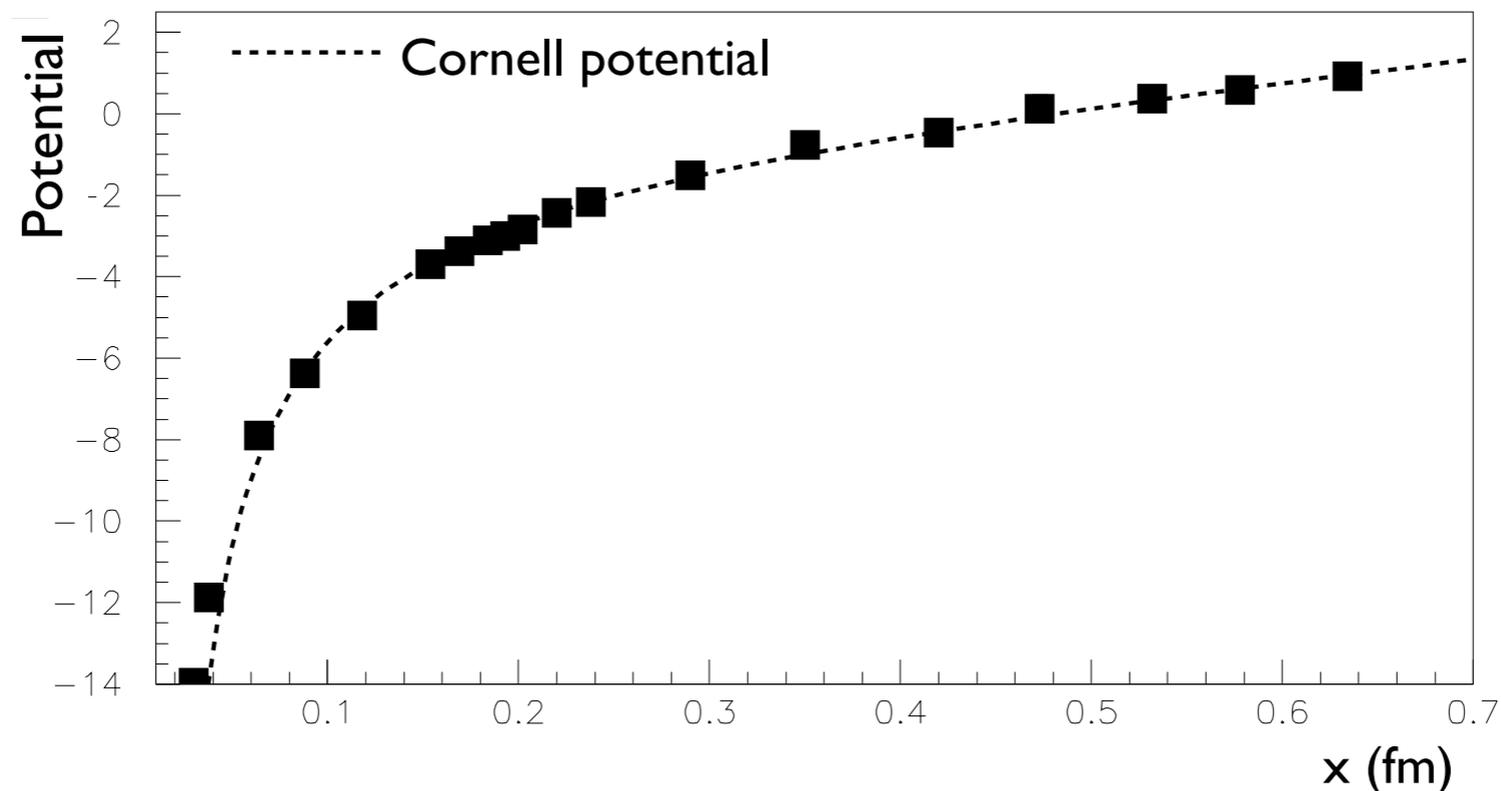
Recover Newton and Yukawa potentials (no self-coupling)

Apply the same approximation to \mathcal{L}_{QCD} (toy model):

Mass gap;

Running effective field mass similar to QCD's gluon effective mass from Schwinger-Dyson Equations.

Running coupling \Rightarrow Recover Cornell potential (potential for static quarks).



Verifications

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Recover Newton and Yukawa potentials (no self-coupling)

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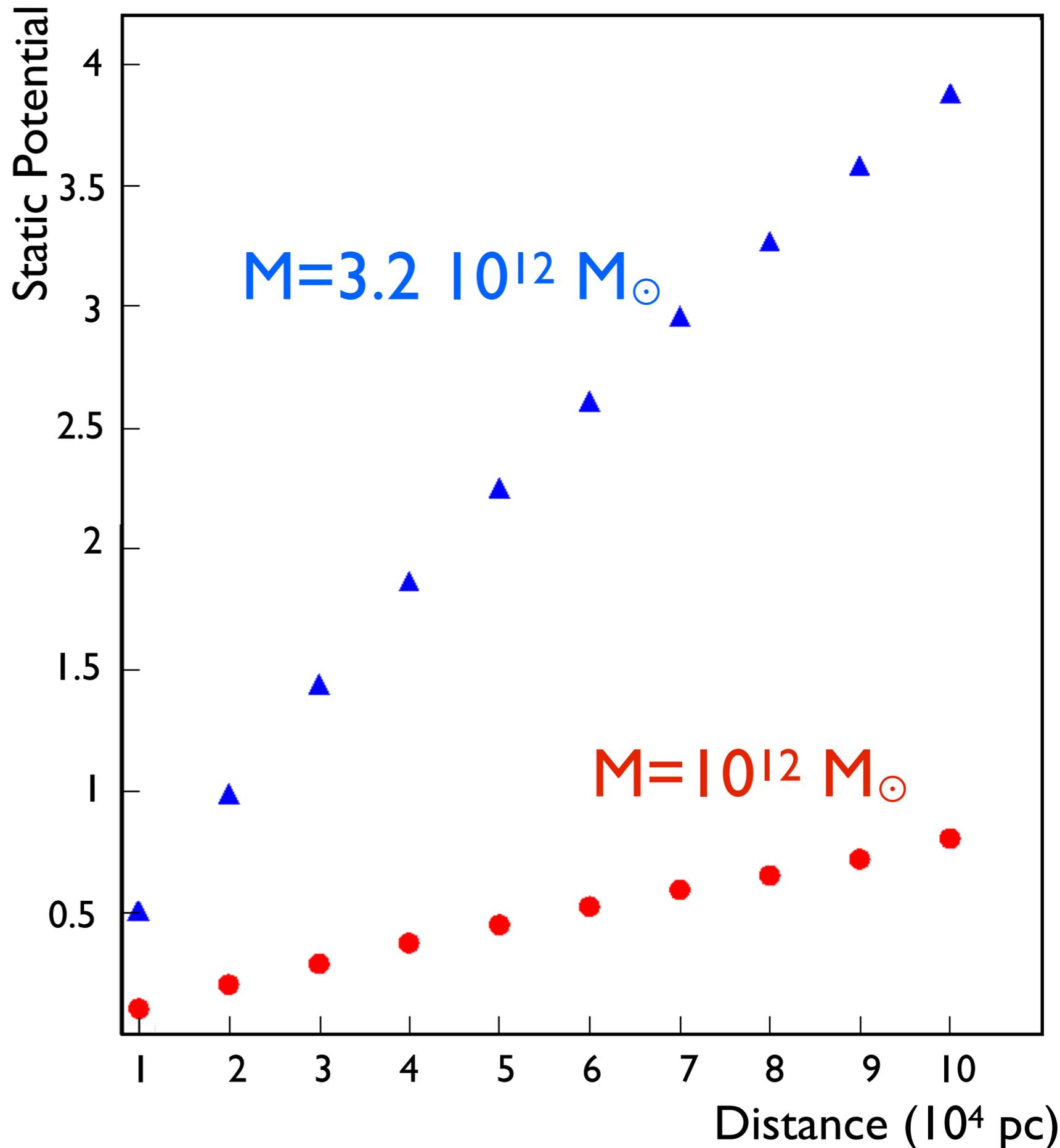
Method generates a model that recovers principal characteristics of static QCD.

\Rightarrow Confidence that method works for GR with self-coupling included

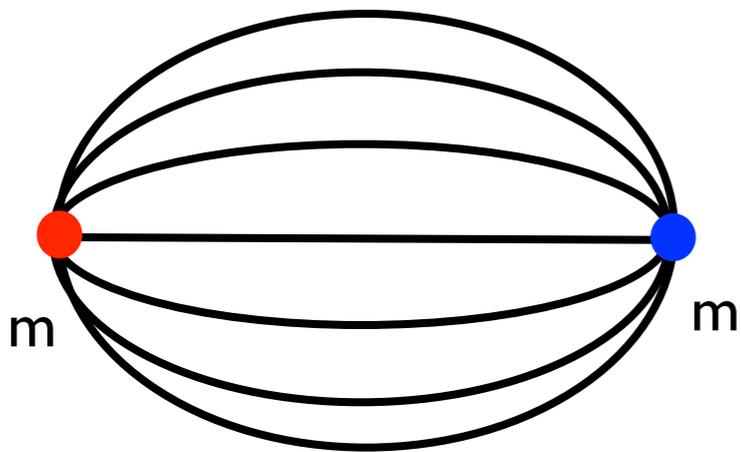
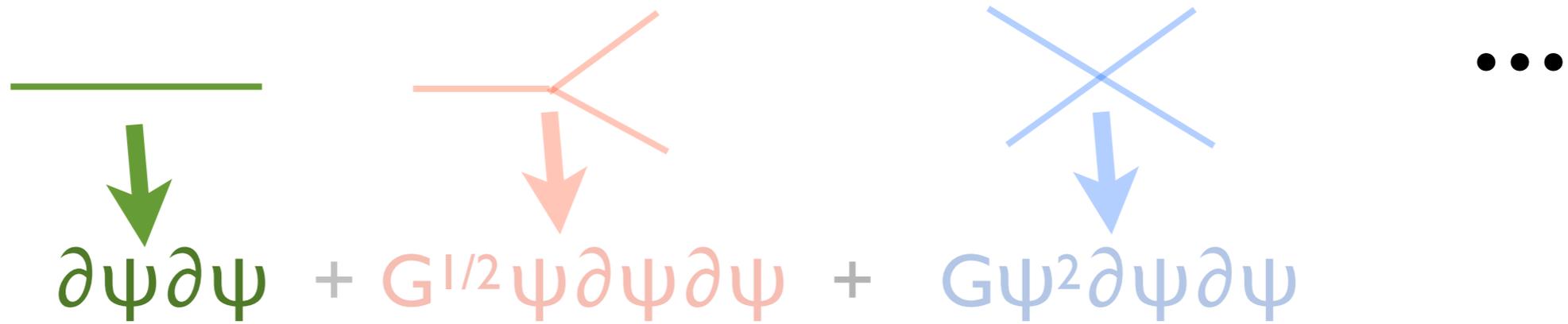
GR with self-coupling:

Full \mathcal{L}_{GR} for two point masses M :

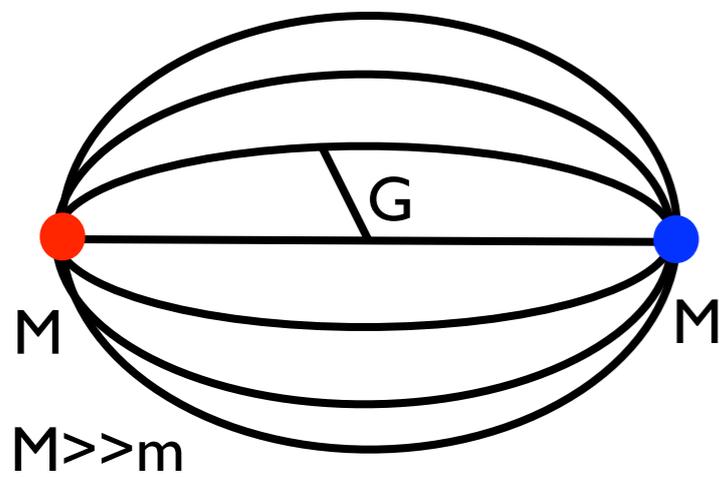
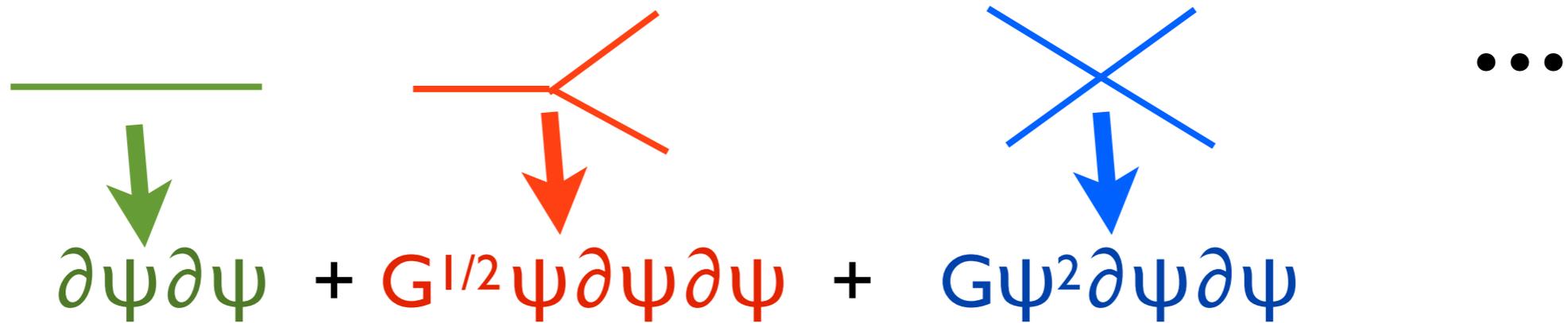
Linear potential, similar to QCD's Cornell potential



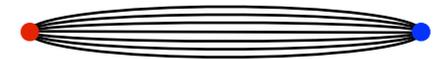
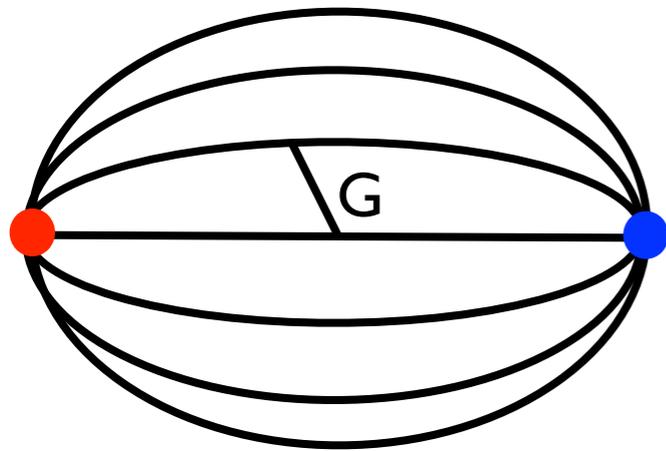
Interpretation



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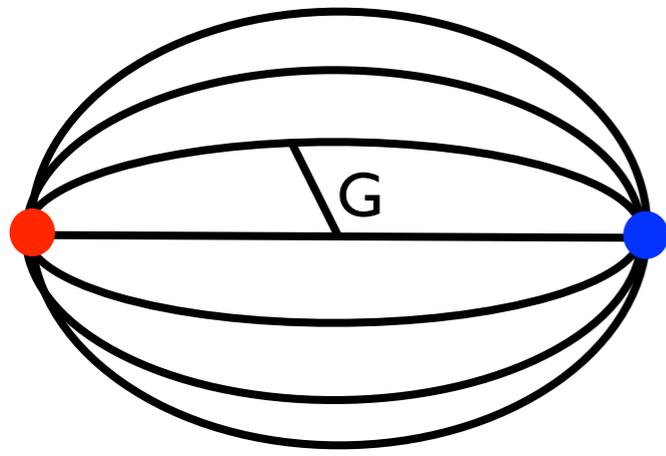
Interpretation



3D system becomes 1D
 \Rightarrow Force \sim constant

The system is more strongly bound than with $1/r^2$ force. Not accounting for this (i.e. using Newton's gravity) creates a **missing mass discrepancy** (i.e. a dark mass).

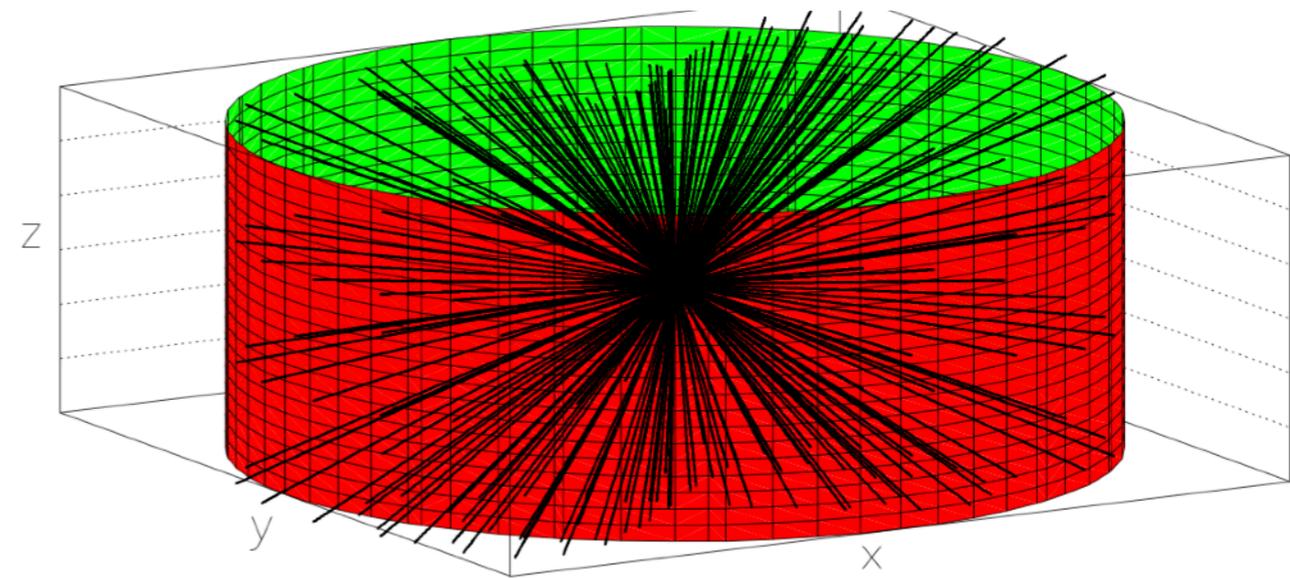
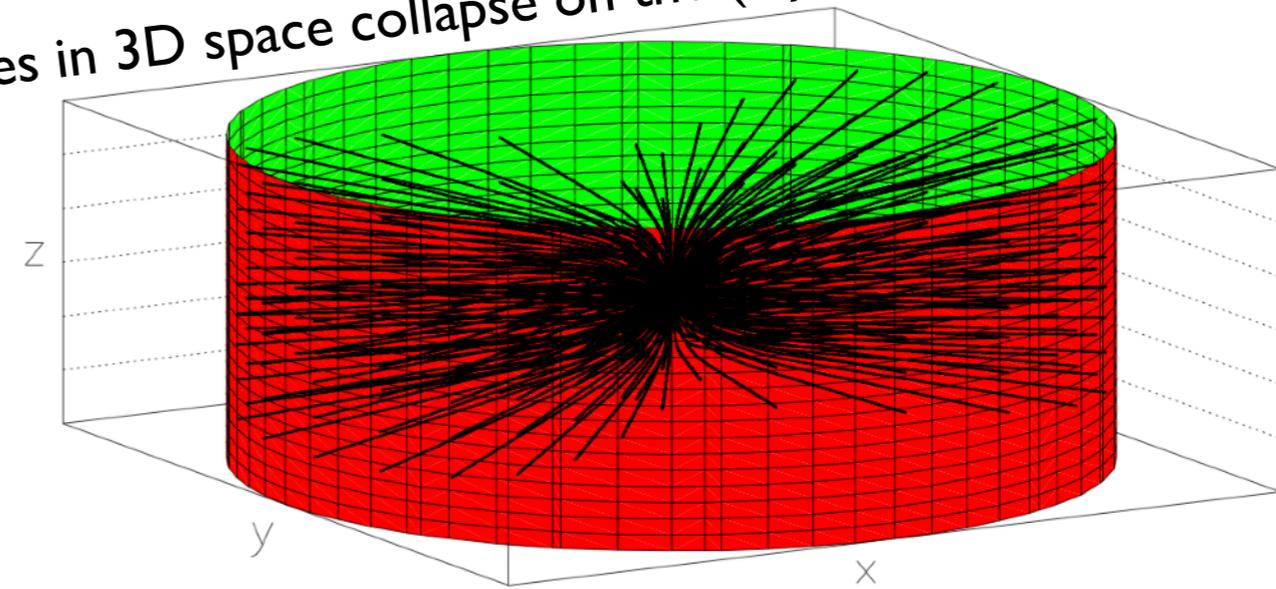
Consequences



3D system becomes 1D
 \Rightarrow Force \sim constant

• Likewise, for an homogeneous distribution of mass into a (x,y) plan, field lines are confined into the plan. 3D system becomes 2D \Rightarrow Force $\sim 1/r$

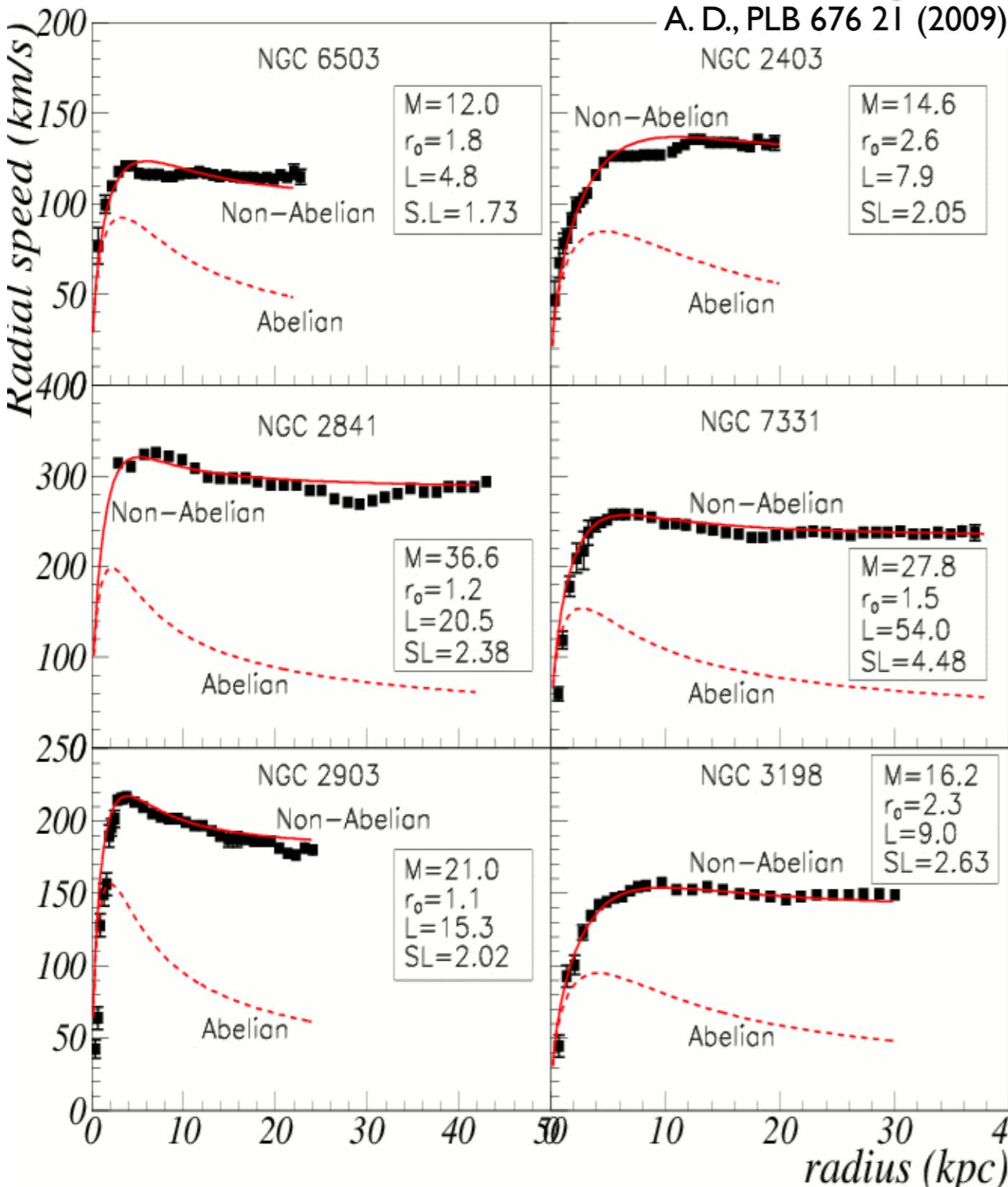
Field lines in 3D space collapse on the (x,y) plan.



• For an homogeneous spherical mass distribution, there is no effect (no preferred directions).
Force $\sim 1/r^2$

Consequences

*Very massive disk: $1/r$ force. For $\rho = \frac{M}{2\pi r_0^2} e^{-r/r_0}$ density profile \Rightarrow rotation speed of a disk becomes constant at large radius.



- Full calculation.
- - - No field self-interaction (Newton).
- data.

M: galaxy mass (10⁹ solar mass)
 r₀: Scale length. } Free parameters

L: galaxy luminosity (B-band)
 (10⁹ solar luminosity)

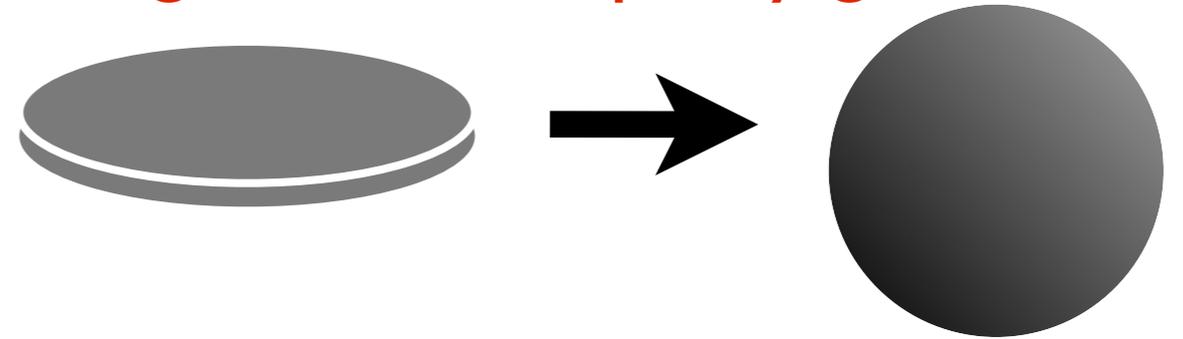
With only baryonic matter,
 expect $M \sim 2-3L$ and $r_0 = SL$

Consequences

* For a disk, $1/r$ force + $\rho = \frac{M}{2\pi r_0^2} e^{-r/r_0}$ density profile \Rightarrow rotation speed of a disk becomes constant at large radius. (prediction)

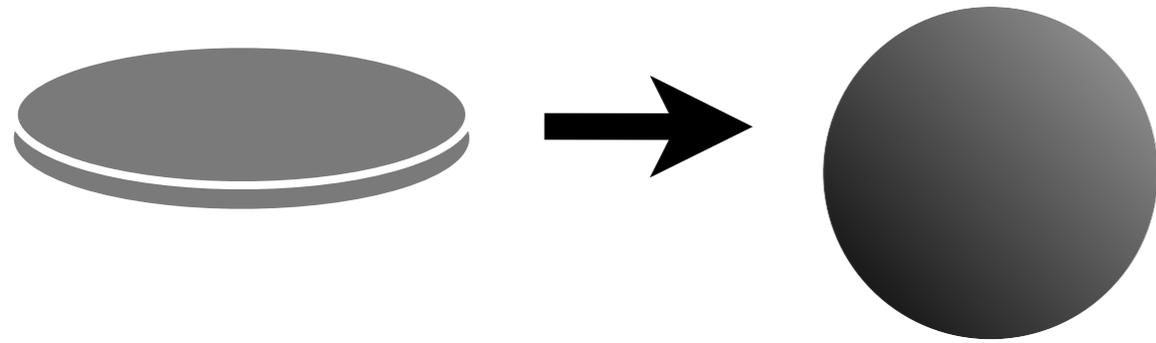


* Prediction (2009) for elliptical galaxies: missing mass discrepancy grows with ellipticity.



Dark Mass \propto ellipticity

Consequences



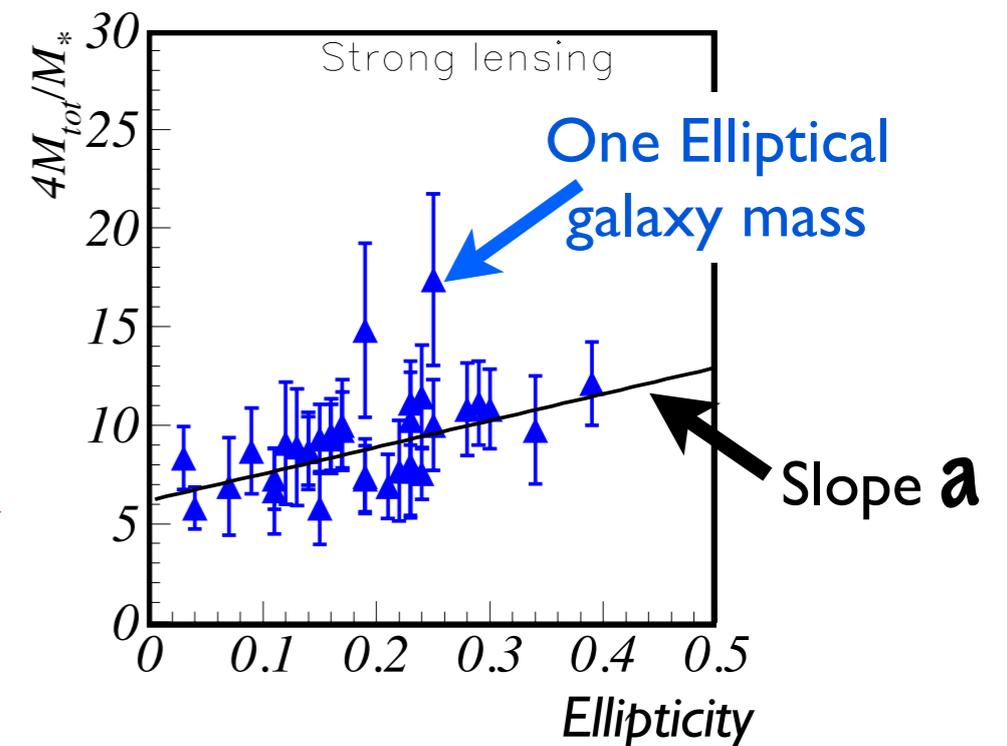
$$\text{Dark Mass} = \mathbf{a} \times \text{ellipticity} ?$$

Method:

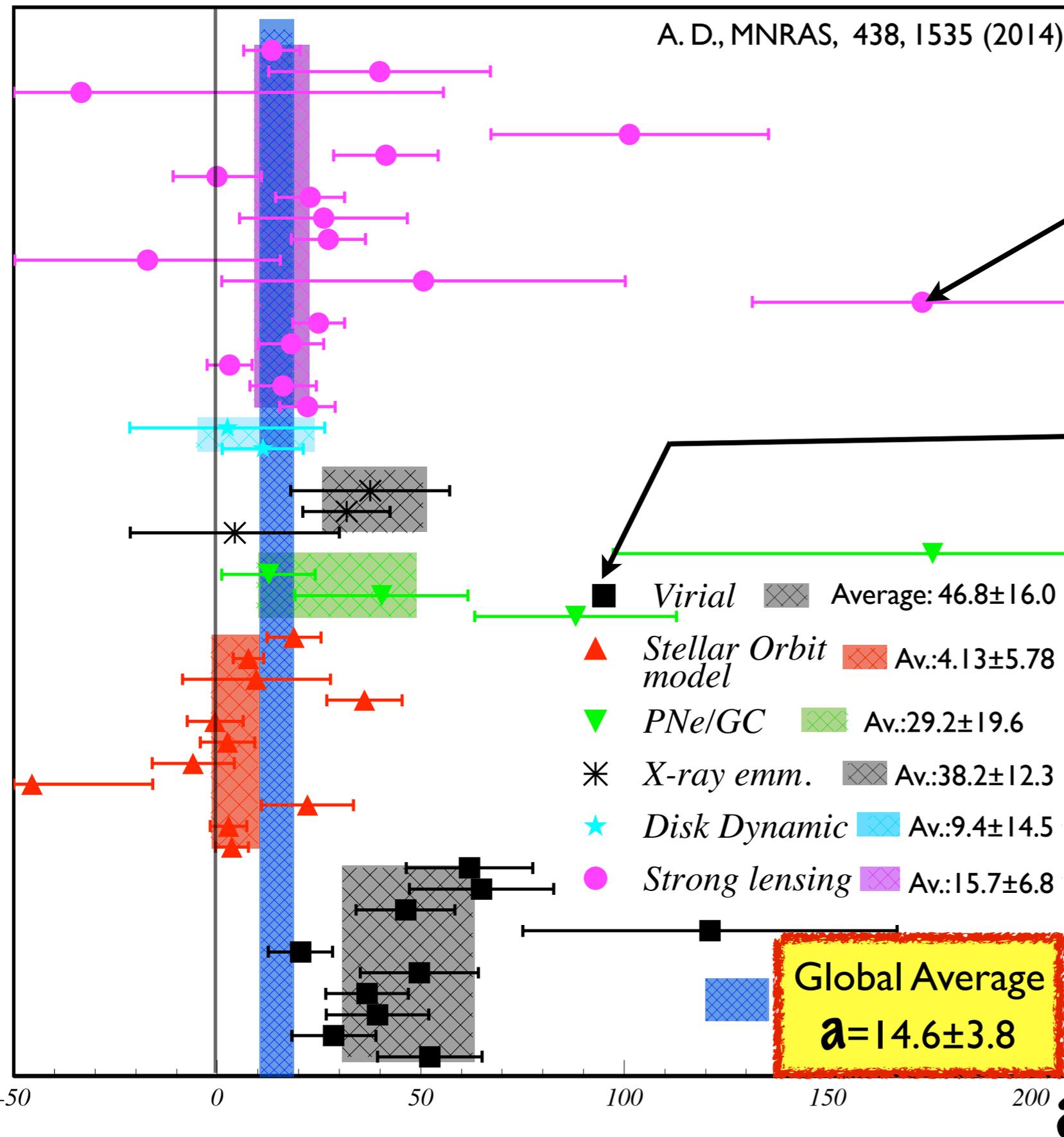
- Use 41 publications reporting dark masses for sets of elliptical galaxies (totaling 685 galaxies).
- Publications often use different methods to obtain dark mass.
- For each publication, extract \mathbf{a} .

One publication

- Average results of all publications.



Consequences



One symbol: one publication (many galaxies).

Different symbol type: different method of extracting the dark mass

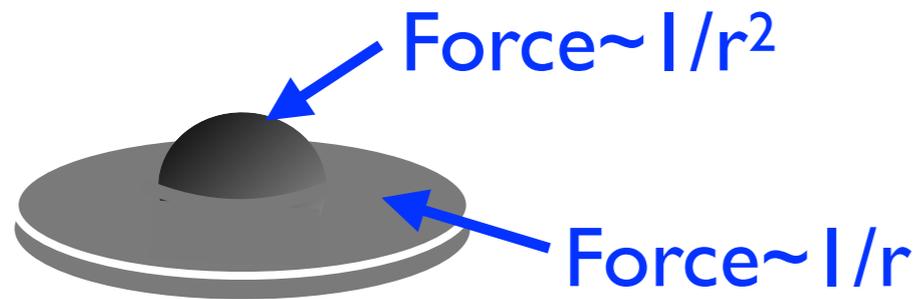
$a \neq 0 \Rightarrow \text{Dark Mass} \propto \text{ellipticity}$

Consequences

- * $1/r$ force + $\rho = \frac{M}{2\pi r_0^2} e^{-r/r_0}$ stellar density profile \Rightarrow rotation speed of a disk galaxy becomes constant at large galactic radius.
- * Elliptical galaxies: Dark Mass, varies with ellipticity.
- * Galactic missing mass discrepancy suppressed in (nearly spherical) galaxy bulges and grows with radius.

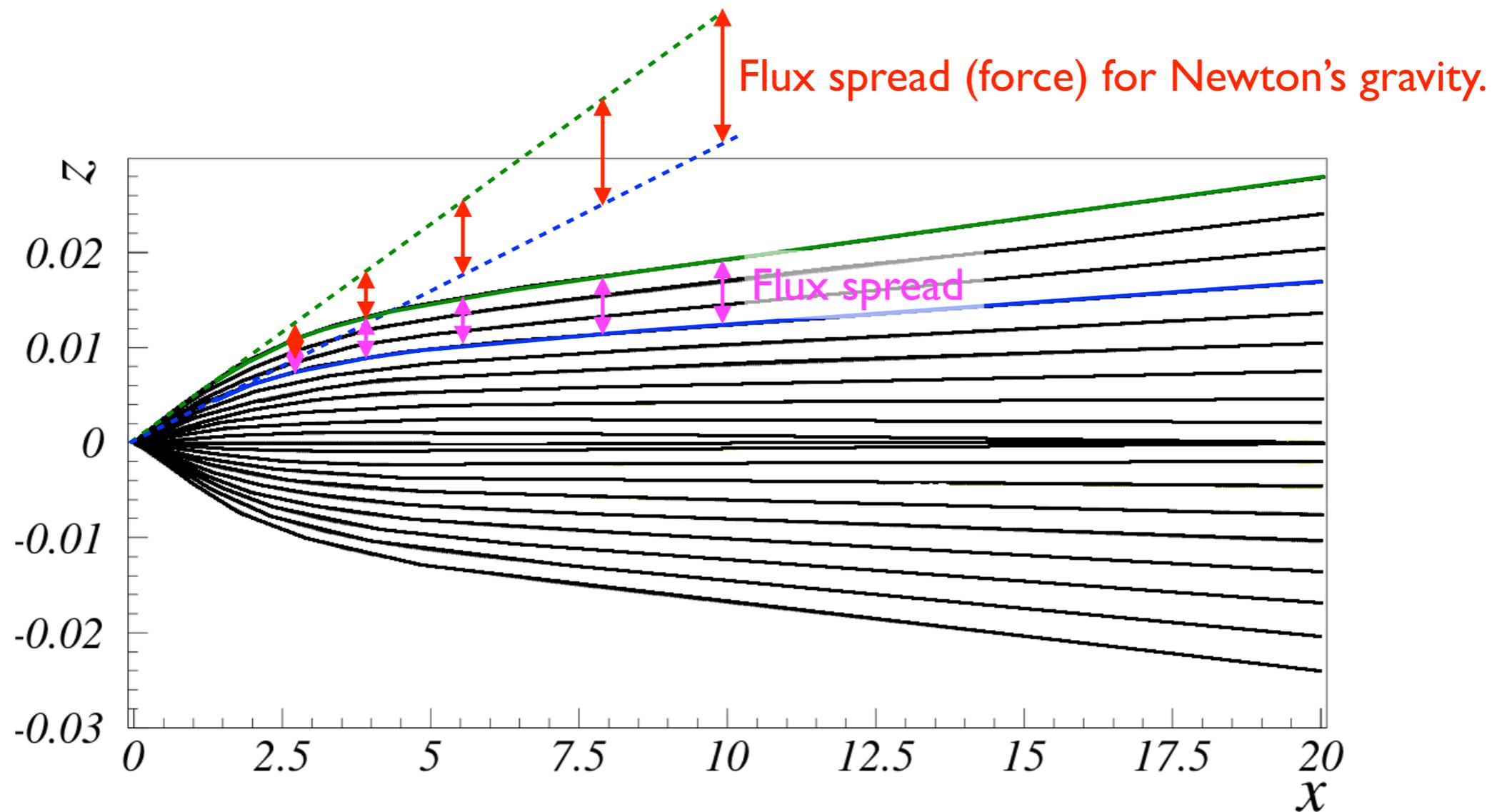
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Consequences

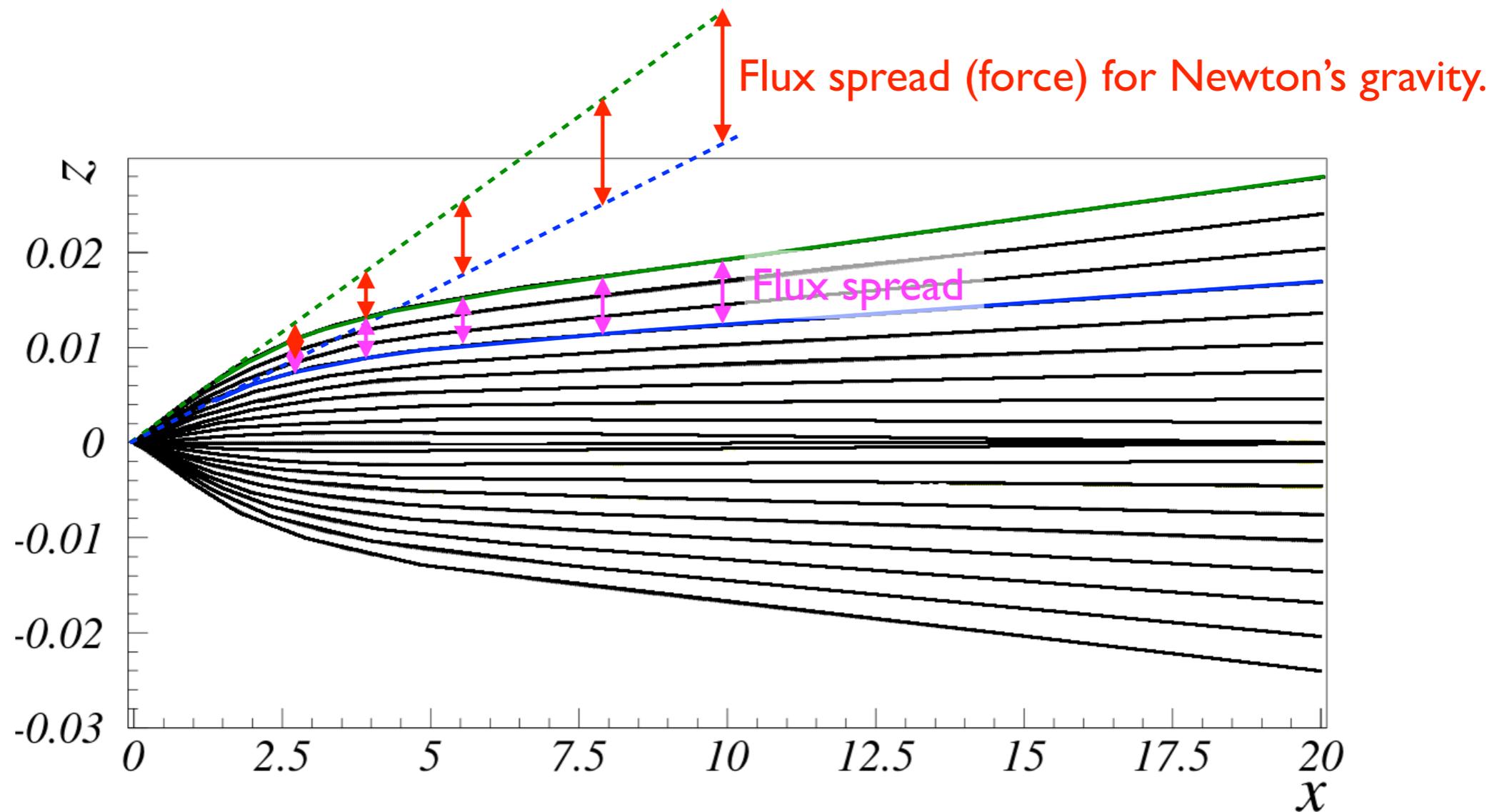
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} \propto missing mass discrepancy. Increases with distance.

Consequences

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Bending of field lines happens where the field is large (near the galactic center), but the effect remains small. Consequences start to appear at large distances due to lever-arm magnification.

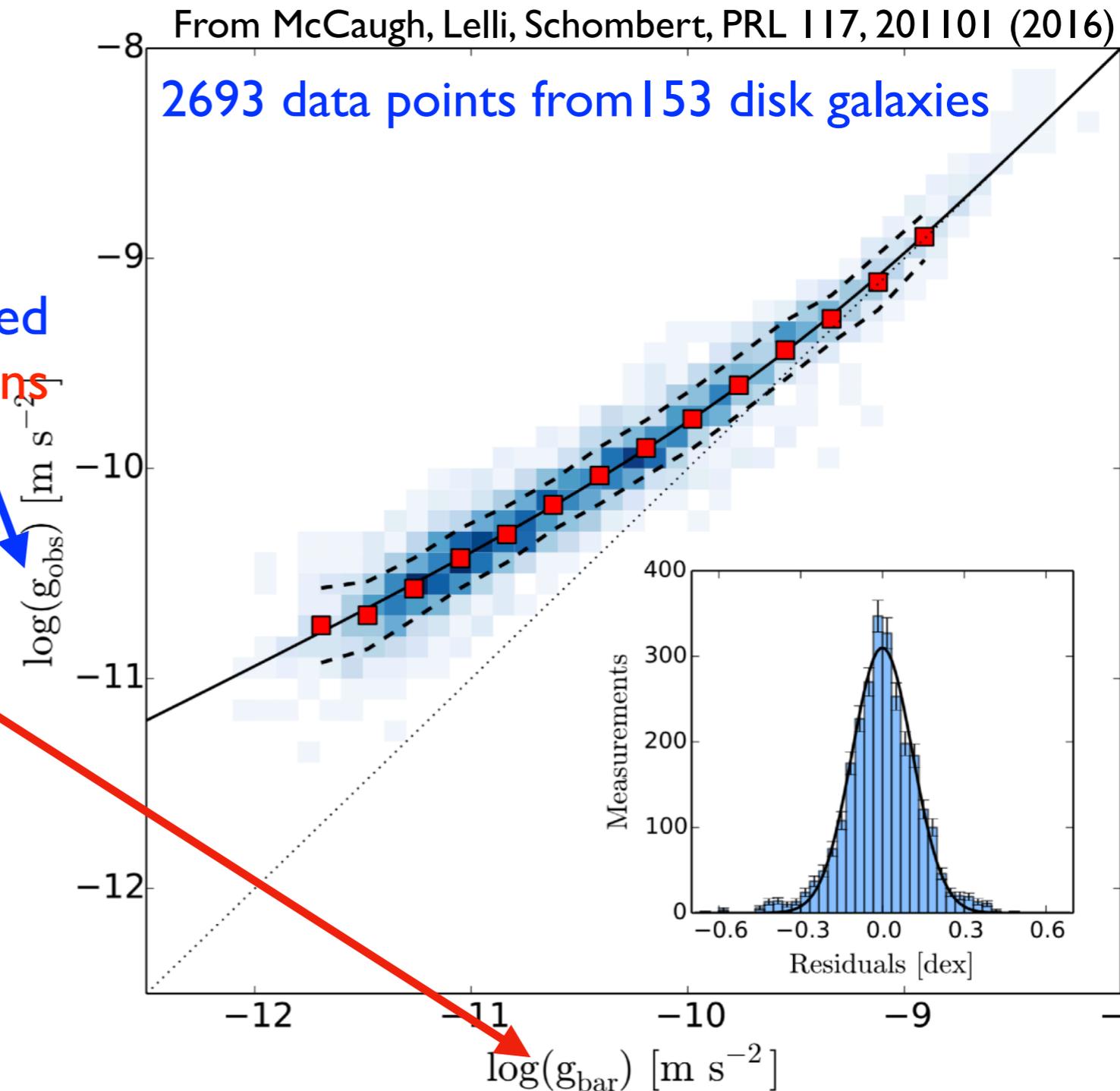
Consequences

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Empirical tight relation between **observed accelerations** and **computed accelerations from galactic baryonic mass**.

Surprising: total mass (mostly dark) should determine the dynamics.

Problem for Dark Matter models: no strict correlation between dark and baryonic distributions expected.



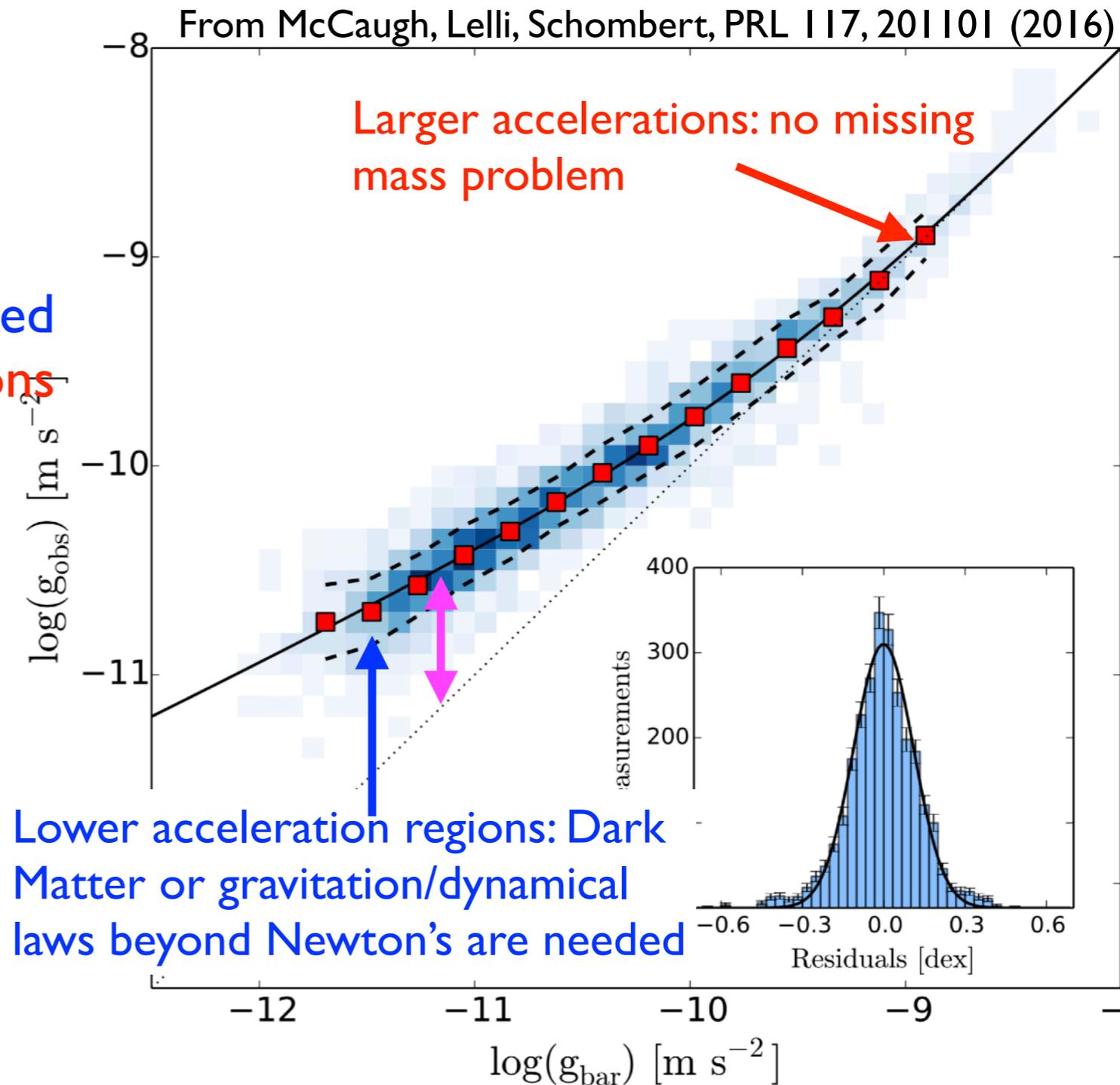
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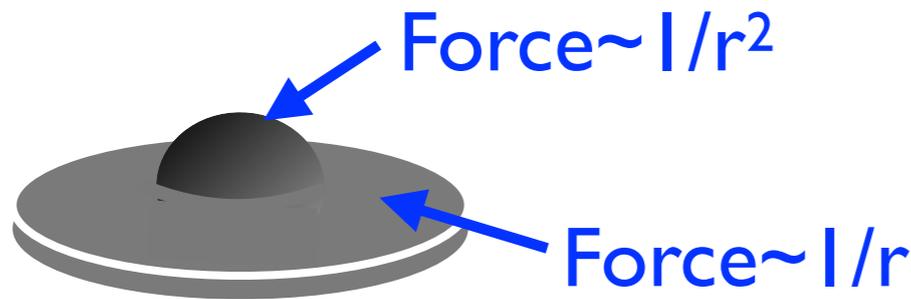
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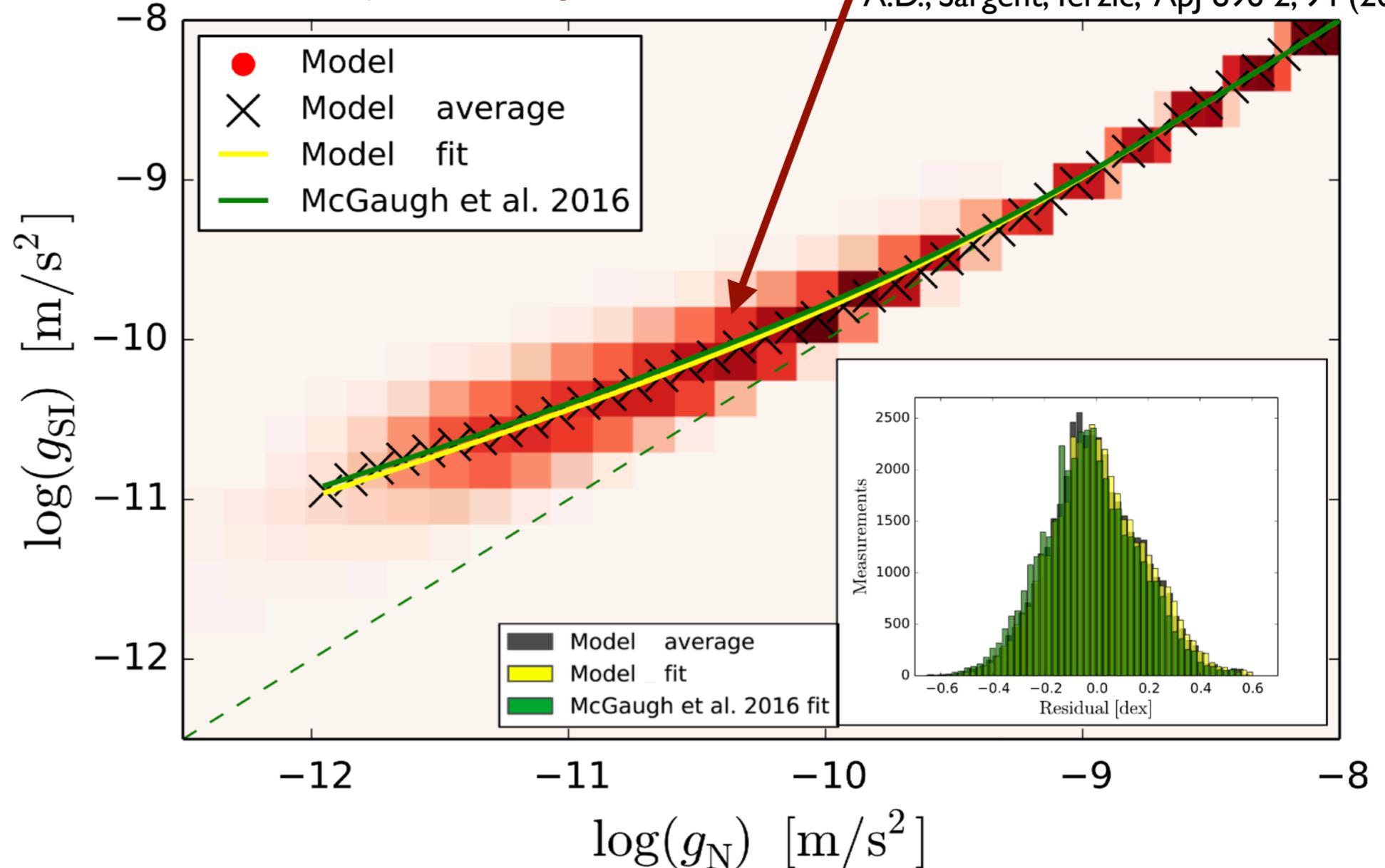
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Simple model with $1/r^2$ force in bulge and $1/r$ force in disk explains the data. Model spans galactic parameter space (many different disk galaxies).

No adjustable parameters!

A.D., Sargent, Terzić, ApJ 896 2, 94 (2020)



Consequences

- * $1/r$ force + $\rho = \frac{M}{2\pi r_0^2} e^{-r/r_0}$ stellar density profile \Rightarrow rotation speed of a disk galaxy becomes constant at large galactic radius.
- * Elliptical galaxies: Dark Mass, varies with ellipticity.
- * Galactic missing mass discrepancy suppressed in (nearly spherical) galaxy bulges and grows with radius.
- * Dark Energy.

Modification of Universe's evolution

General relativity+assuming **isotropic** and **homogeneous** universe:

$$\frac{2K}{a^2} + \frac{2\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} = 4\pi G(\rho - p)$$

with

K : space curvature sign,

a : scale factor,

ρ : density,

p : pressure.

Universe standard evolution equation

Lifting isotropy and homogeneity assumption, a factor appear in front of ρ .

Can be interpreted as field trapping effect.

⇒ Depletion factor $D(z)$ in front of ρ of matter.

(redshift $z \sim$ distance or Universe age⁻¹)

Construction of $D(z)$

Use known timeline of large structure formation and the known amounts of baryonic matter in these structures (galaxies, clusters, superclusters).

Construction of $D(z)$

Timeline:

Galaxies

- $z=15 \rightarrow z=0$: galaxies formation and evolution.
 - $z=10 \rightarrow z=2$: galaxies are highly asymmetric (e.g. filaments-like). **Field trapping**
 - $z=2 \rightarrow z=0$: galaxies evolve to more symmetric shapes; Elliptical/Spiral galaxy ratio increases. **Field release**

Groups, clusters and superclusters

- $z=10 \rightarrow z=2$: galaxies form groups and proto-clusters **Field trapping**
- $z=2 \rightarrow z=1.2$: proto-clusters evolve to clusters. **Field trapping**
- $z=1.2 \rightarrow z=0$: clusters arrange into more symmetric formation (sheets, filaments) **Field release**

Structure depletion functions:

$$D_g(z) \simeq \left[1 - \left(1 + e^{(z-z_{g0})/\tau_g} \right)^{-1} \right] + [A_g e^{-B_g z}]$$

Galaxies

$$D_c(z) \simeq \left[1 - \left(1 + e^{(z-z_{c0})/\tau_c} \right)^{-1} \right] + [A_c e^{-B_c z}]$$

Groups, clusters and superclusters

• z_0 is the redshift at the middle of the formation process of the specific structure type: $z_{g0} = 9 \pm 1$
 $z_{c0} = 5.6 \pm 1$

• τ is the typical duration for the specific formation process: $\tau_g = 3 \pm 0.5$
 $\tau_c = 2.2 \pm 0.5$

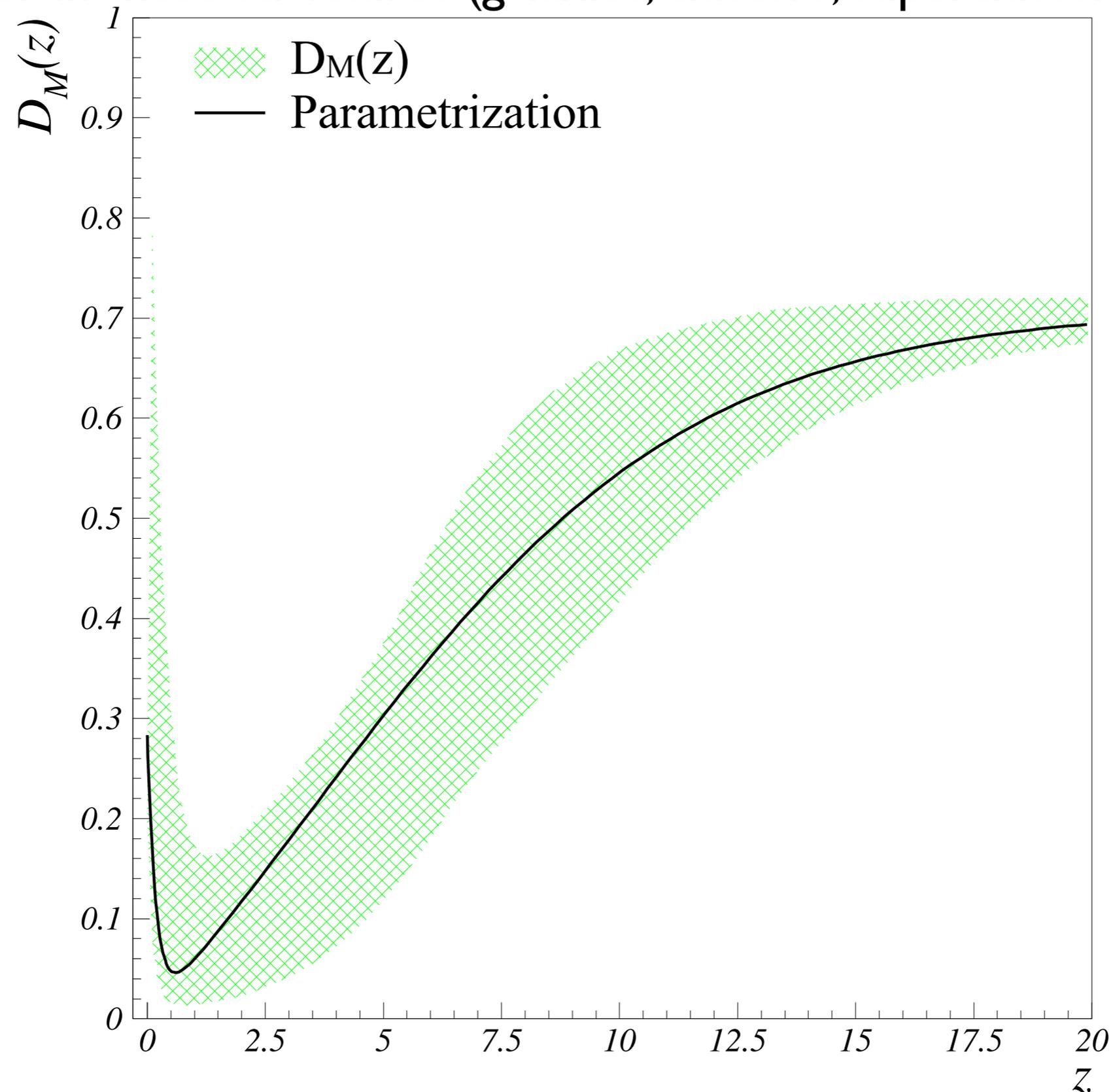
• A : matter in specific structure concerned by evolution to more symmetric shapes: $A_g = 10\% \pm 10\%$
 $A_c = 87\% \pm 87\%$

• B^{-1} : Typical duration of evolution to more symmetric shapes: $B_g^{-1} = 0.08 \pm 0.03$
 $B_c^{-1} = 0.21 \pm 0.10$

Chosen so $0 < D(0) < 1$

Construction of $D(z)$

Use known timeline of large structure formation and the known amounts of baryonic matter in these structures (galaxies, clusters, superclusters).



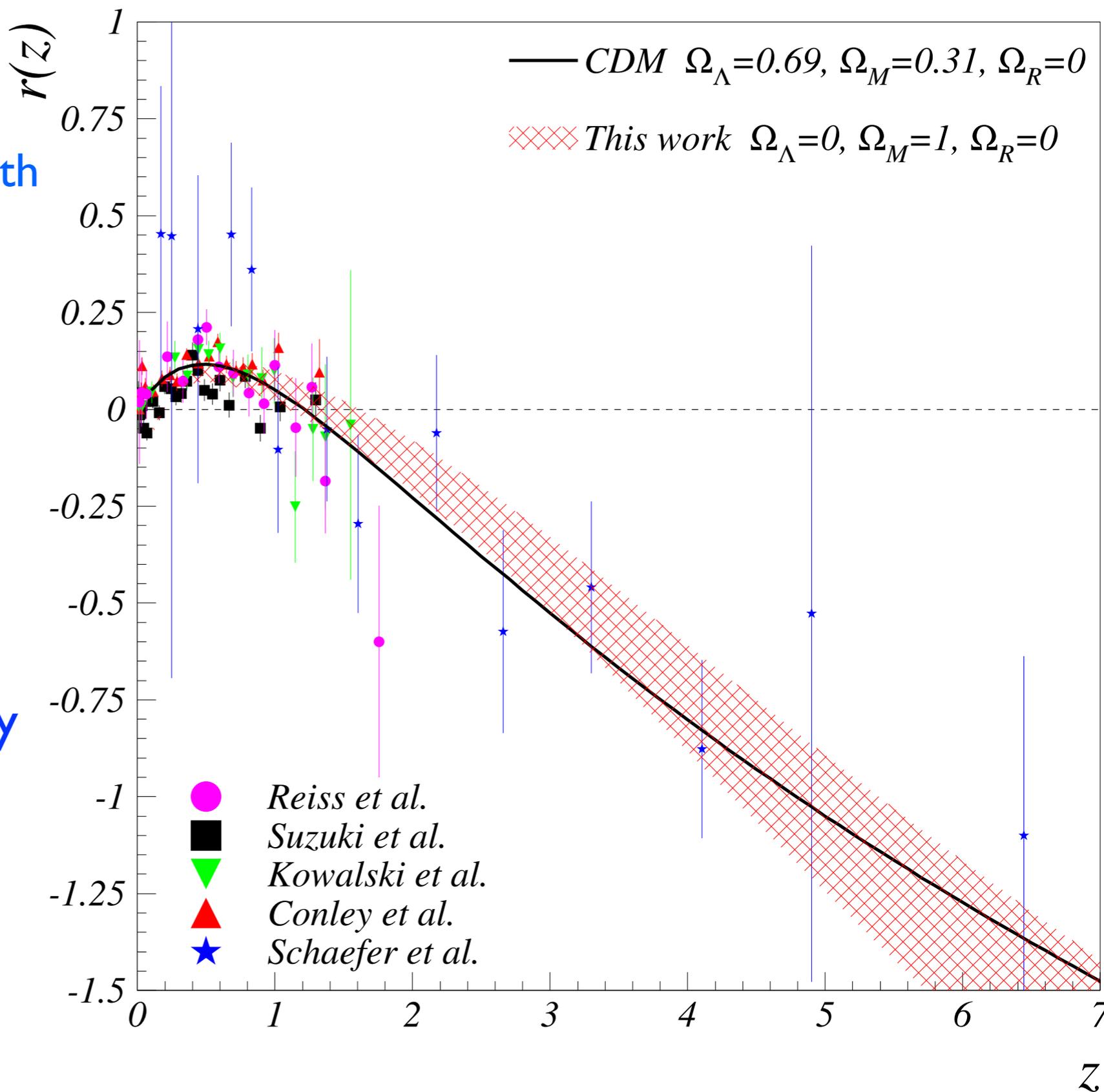
Comparison with supernova and γ -ray burst observations

Compute residual $r(z)$ between Luminosity distance computed with self-interaction and empty flat universe baseline.

\Rightarrow Good agreement with the observational evidence for dark energy.

Suggest that dark energy may be the consequence of the trapping of gravitational field lines in large structures.

Cosmological constant $\Lambda=0$



Relevance to dark energy phenomenology

- Supernova data
- Age of the Universe
13.5±0.3 Gyears.
- Agrees (by construction of $D(z)$) with large structure formation (galaxies, clusters, superclusters...)
- Main feature of cosmic microwave background and baryon acoustic peak observations (main feature explained. Detailed work not done)

Yields naturally the **Cosmic Coincidence** i.e. that presently, **dark energy repulsive effect** \approx **gravity's attraction**, while in the past, **attraction** \gg **repulsion** and in the future **repulsion** \gg **attraction** is expected.

Consequences

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- * Elliptical galaxies: Dark Mass, varies with ellipticity.
- * Galactic missing mass discrepancy suppressed in (nearly spherical) galaxy bulges and grows with radius.
- * Dark Energy.
- * Galaxy clusters contain $\sim 90\%$ of Dark Mass.
- * Straightforward explanation for the Bullet Cluster observation.

Summary

- Complex and important **non-perturbative** QCD effects have been observed. Not intuitive: would have been hard to predict, just looking at QCD's Lagrangian.
- Since **QCD and GR have a similar structure**, maybe similar effects also arise when large masses are involved (e.g. **galaxies**).
- Hadronic and galactic dynamics have **intriguing phenomenological parallels**.
- The **WIMP and axion search negative results**, the **trouble with SUSY**, make this natural explanation attractive.
- Numerical calculations show that field self-interactions can explain quantitatively several key phenomena of the Dark Universe:
 - * Flat rotation curves of disk galaxies.
 - * Correlation between the ellipticity of elliptical galaxies and their dark mass.
 - * Galactic missing mass correlation with baryonic mass.
 - * Dark Energy.
 - * Cluster dynamics.
 - * Bullet Cluster observation.

Thank you