

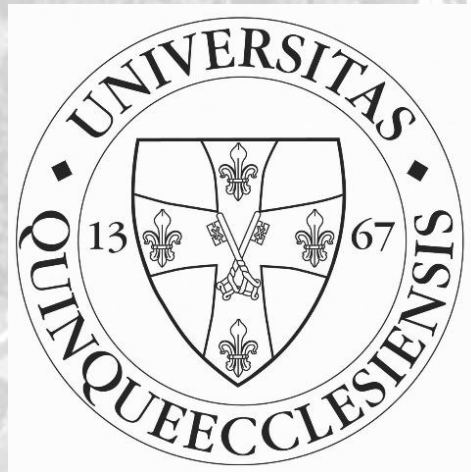
SAC meeting on Ångström FEL

Beam dynamics - EEX

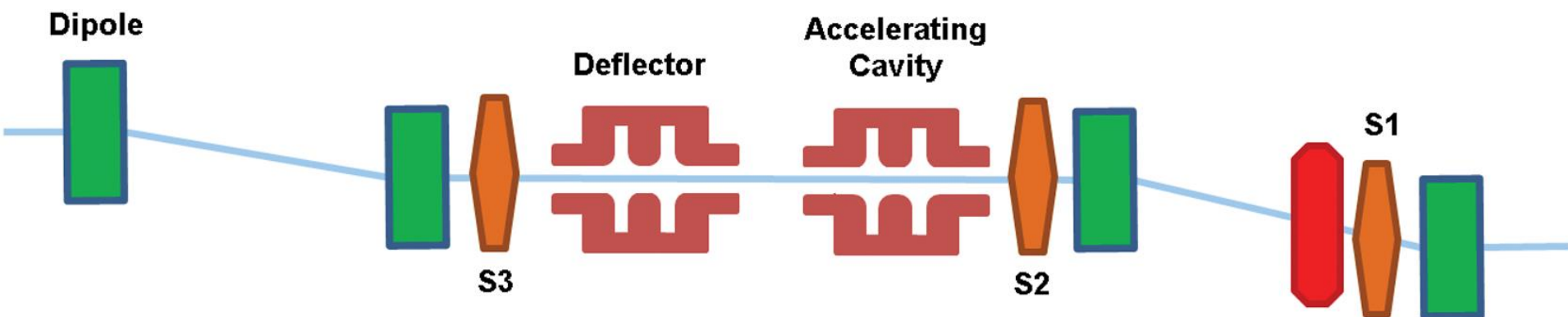
Zoltán Tibai¹ - Kévin Pepitone²

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²Uppsala University, Uppsala, Sweden



Emittance exchange



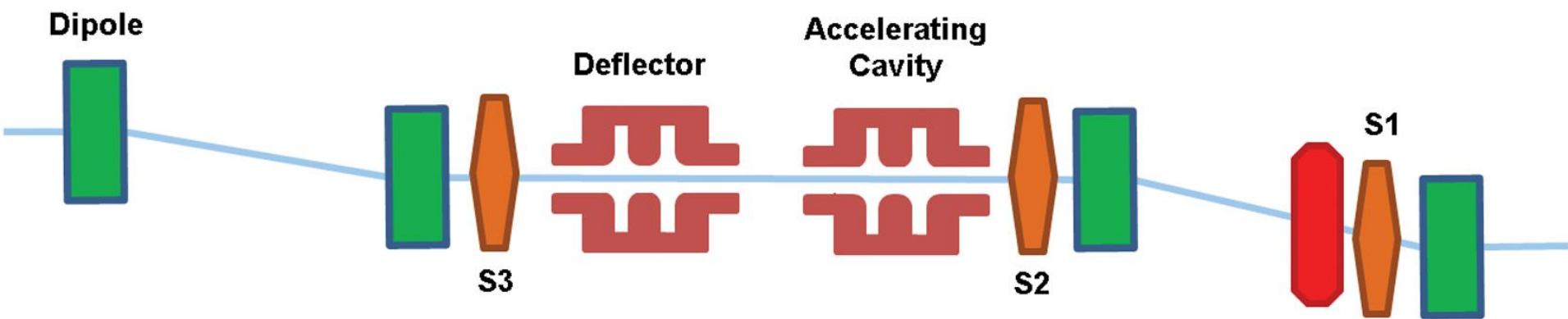
M. Cornacchia and P. Emma, *Phys. Rev. Spec. Acc. and Beams*, **5**, 084001 (2002)

P. Emma et al., *Phys. Rev. Spec. Acc. And Beams*, **9**, 100702 (2006)

E. A. Nanni and W. S. Graves, *Phys. Rev. Spec. Acc. and Beams*, **18, 084401 (2015)**

E. A. Nanni et. al., *Phys. Rev. Acc. And Beams*, **21**, 014401 (2018)

Emittance exchange



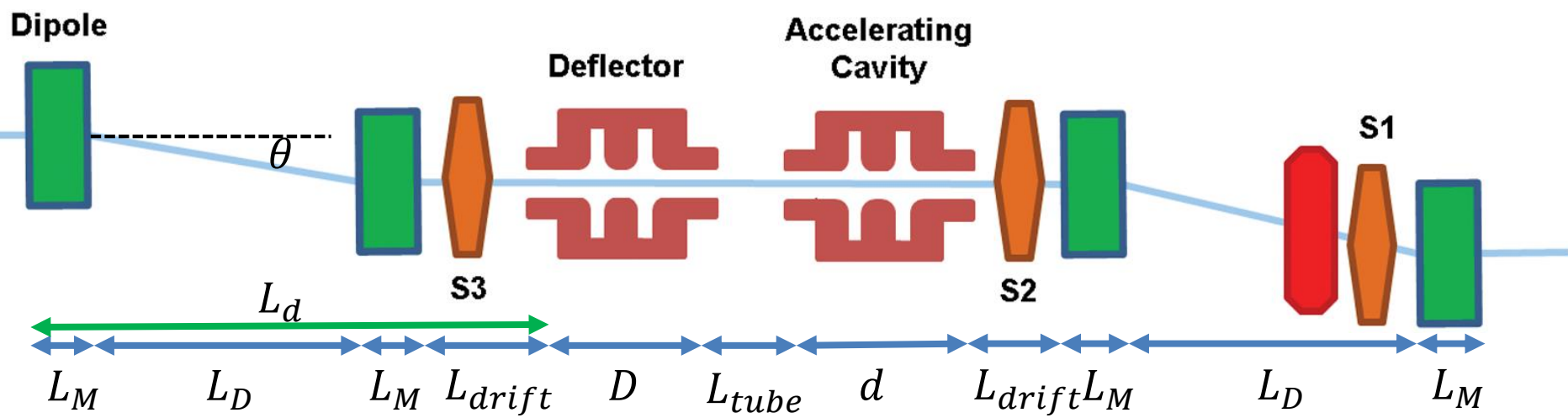
$$X = \begin{bmatrix} x \\ x' \\ y \\ y' \\ z \\ \frac{\Delta p}{p} \end{bmatrix}$$

$$X = \begin{bmatrix} y \\ y' \\ z \\ \frac{\Delta p}{p} \end{bmatrix}$$

$$X_{Astra,GPT} = \begin{bmatrix} y_n \\ y'_n \\ z_n \\ \frac{\Delta p_n}{p_n} \end{bmatrix}_{4 \times n} \quad n \approx 43000$$



Dogleg



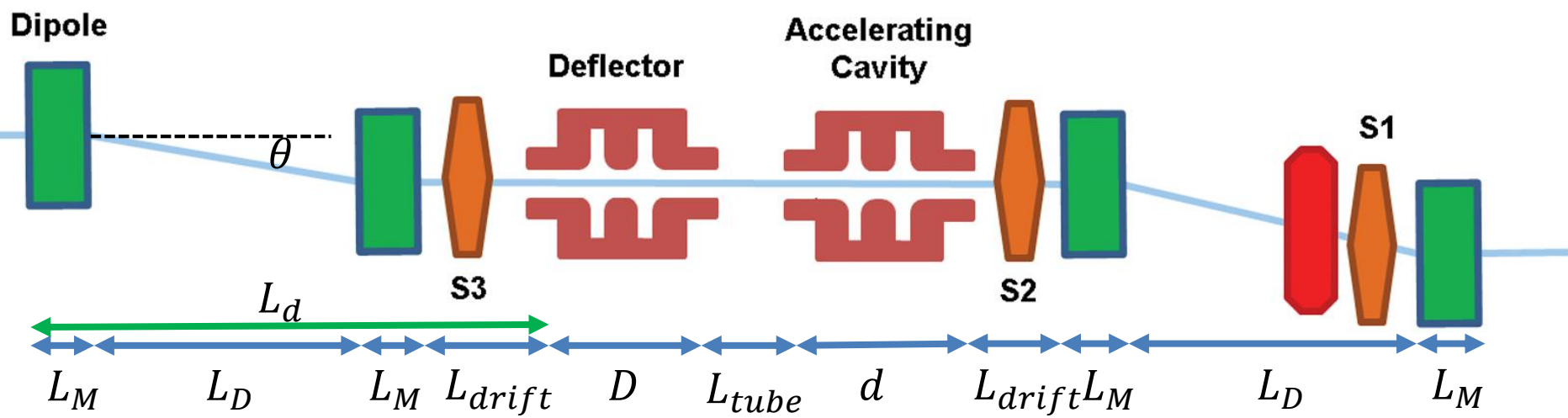
$$R_d = \begin{bmatrix} 0 & L_d & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \xi \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L_d = \frac{L_D}{\cos(\theta)^3} + \frac{2L_M}{\cos(\theta)} + L_{drift}$$

$$\eta = \frac{L_D \sin(\theta)}{\cos(\theta)^2} + \frac{2L_M}{\sin(\theta)} \left(\frac{1}{\cos(\theta)} - 1 \right)$$

$$\xi = \frac{L_D \sin(\theta)^2}{\cos(\theta)^3} + 2L_M \left(\frac{1}{\cos(\theta)} - \frac{\theta}{\sin(\theta)} \right) + \frac{L_d}{\gamma^2 \beta^2}$$

Deflector



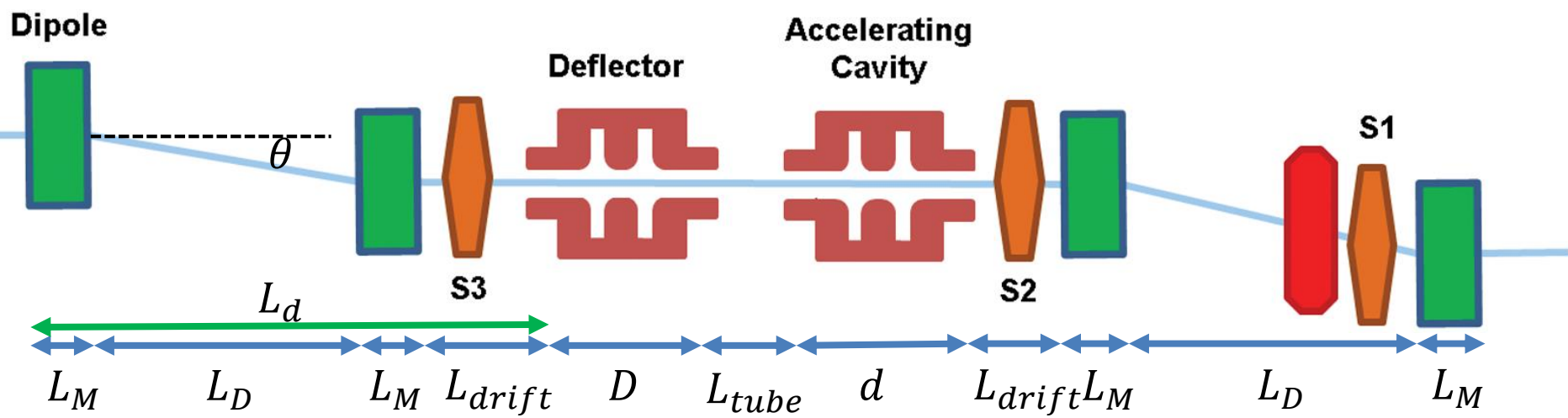
$$R_k = \begin{bmatrix} 1 & D & kD/2 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & kD/2 & \frac{1 + 2n^2}{12n^2} k^2 D & 1 \end{bmatrix}$$

$$k = \frac{\omega_{rf} V_{\perp}}{cE},$$

- V_{\perp} integrated deflection voltage
- ω_{rf} rf frequency
- E beam energy
- n cell number



Accelerator cavity

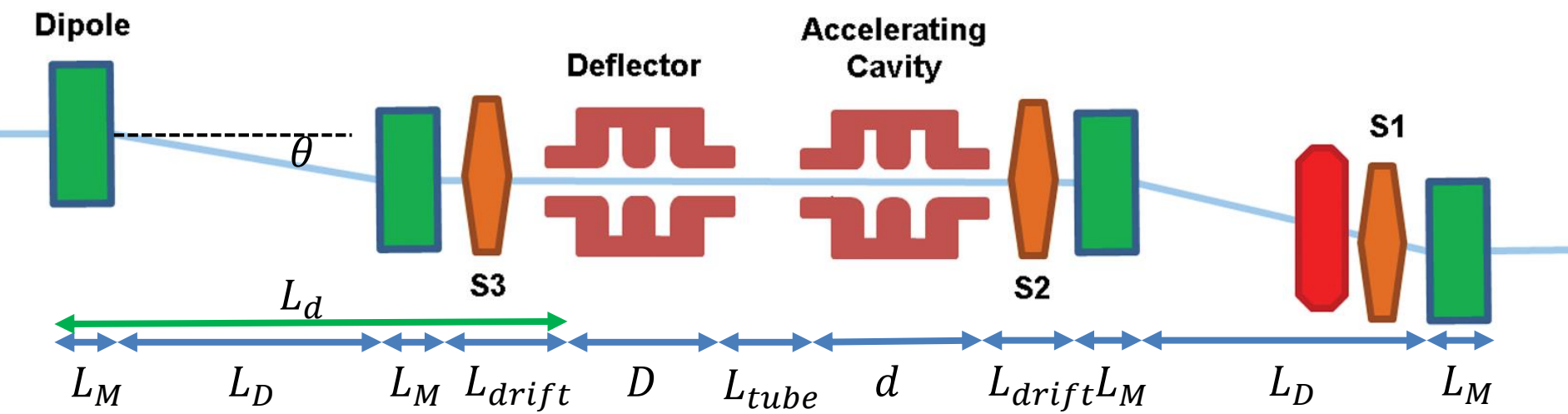


$$\chi = \frac{2\pi |e| V_{\parallel}}{\lambda p c},$$

- V_{\parallel} integrated acceleration voltage
- λ rf wavelength
- $p = \beta \gamma m_e c$

$$R_a = \begin{bmatrix} 1 & d & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\chi & 1 \end{bmatrix}$$

Emittance exchange

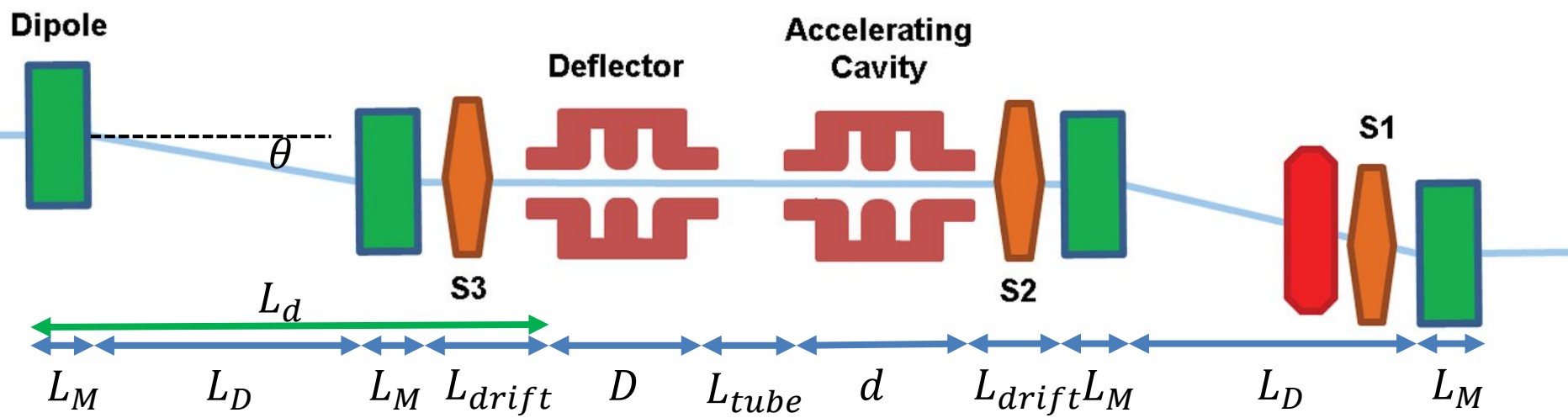


$$R_{EEX} = R_d R_a R_k R_d =$$

| | | | |
|--------------------|---|---|--|
| $\eta k \ominus 1$ | $2\eta k L + 2L + D + d + \eta k \ominus + \eta k d + \eta^2 \chi + \frac{1}{6} \eta^2 k^2 D$ | R_{13} | R_{14} |
| \ominus | $\eta k \oplus 1$ | R_{23} | R_{24} |
| R_{31} | R_{32} | $1 + \eta k + \ominus \chi + \frac{1}{6} \xi k^2 D$ | $2\xi k \eta + 2\xi + \ominus^2 k + \frac{1}{6} \xi^2 k^2 D$ |
| R_{41} | R_{42} | $\chi + \ominus k^2 D$ | $1 + \eta k + \ominus \chi + \frac{1}{6} \xi k^2 D$ |



Emittance exchange



If:

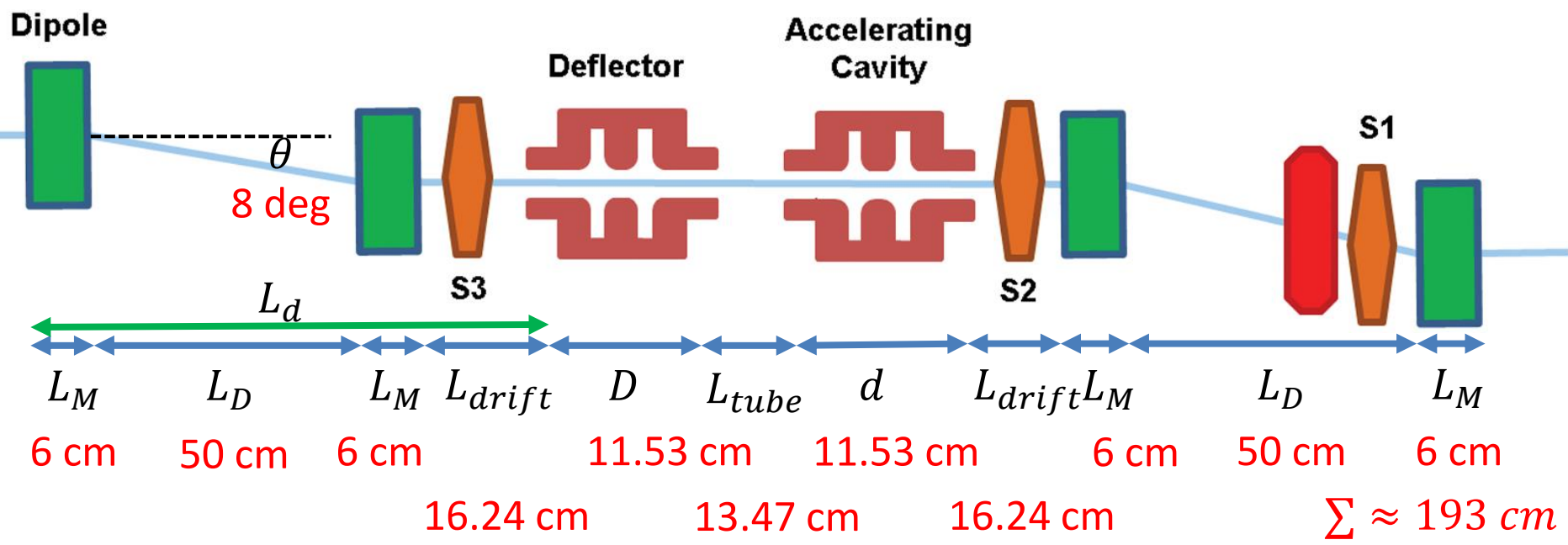
$$k = -\frac{1}{\eta}$$

$$\chi = -\frac{k^2 D}{4}$$

$$R_{EEX} = R_d R_a R_k R_d = \begin{bmatrix} 0 & 0 & -\frac{L+d}{\eta} & \eta - \frac{(L+d)\xi}{\eta} \\ 0 & 0 & -\frac{1}{\eta} & -\frac{\xi}{\eta} \\ -\frac{\xi}{\eta} & \eta - \frac{L\xi}{\eta} & 0 & 0 \\ \frac{1}{\eta} & -\frac{L}{\eta} & 0 & 0 \end{bmatrix}$$



Emittance exchange



$\omega_{rf} = 9.3 \text{ GHz}$

$d = 5.64 \text{ cm}$

$D = 10.8 \text{ cm}$

$\xi = 1.2 \text{ cm}$

$\eta = 7.9 \text{ cm}$

$k = -12.59 \text{ 1/m}$

$\chi = -4.57 \text{ 1/m}$

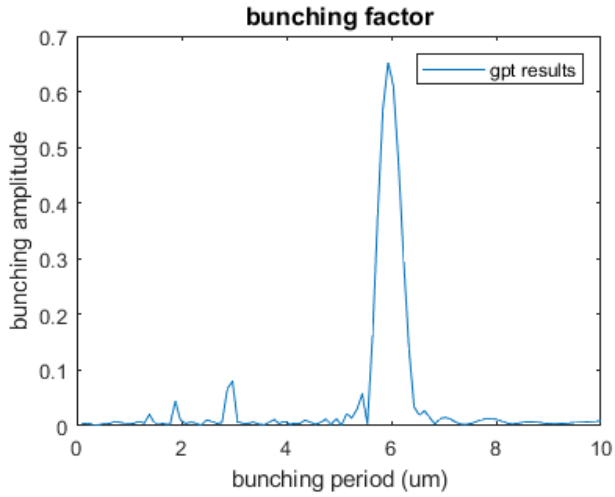
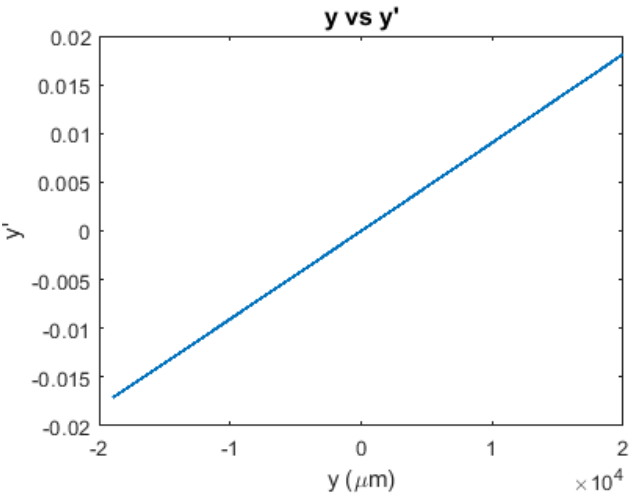
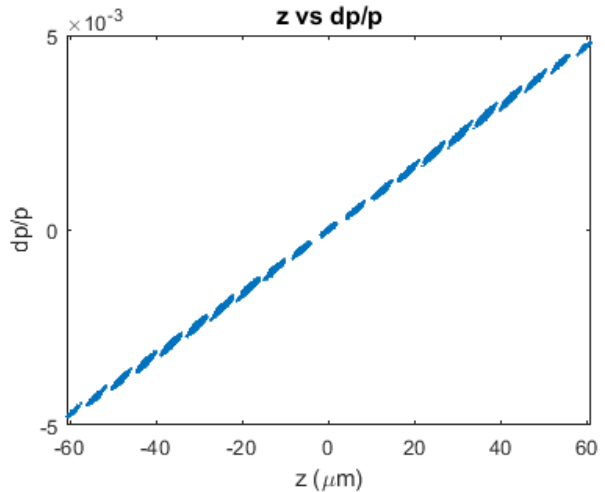
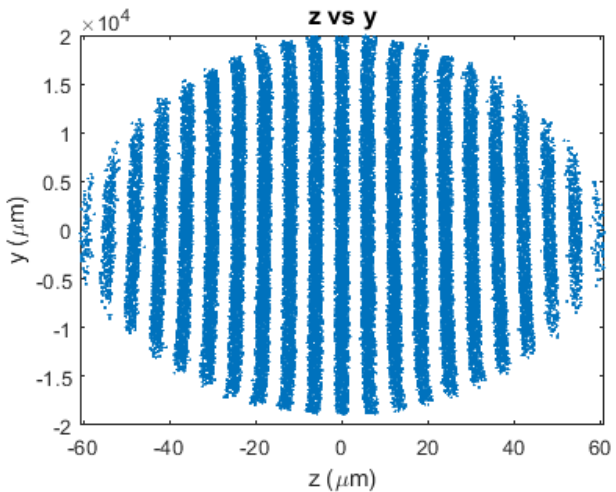
$L = 73.6 \text{ cm}$



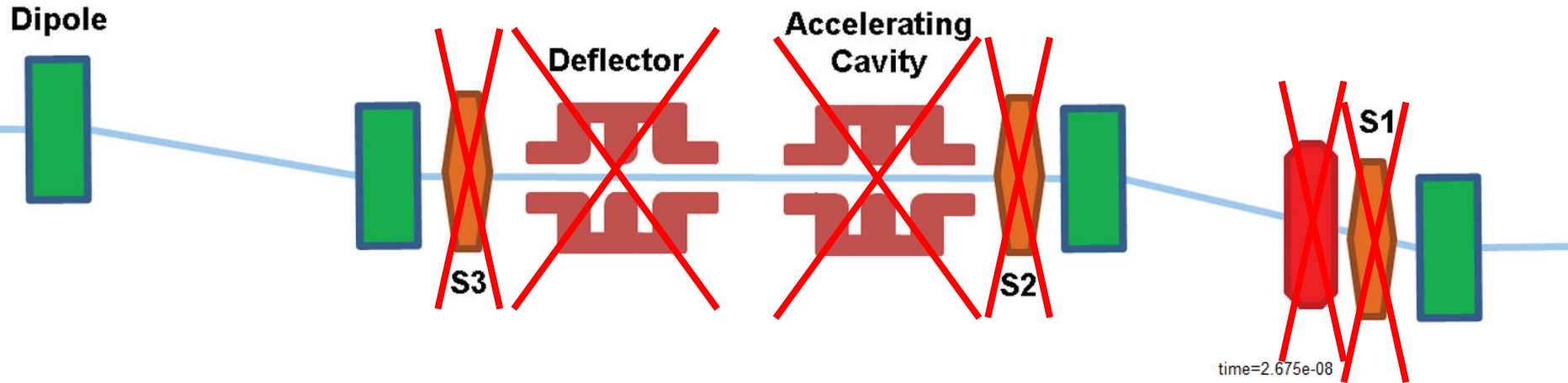
Analytical result of the setup

$$R_d R_a R_k R_d \begin{bmatrix} y_n \\ y'_n \\ z_n \\ \frac{\Delta p_n}{p_n} \end{bmatrix}_{4 \times n_{GPT}}$$

$n_{GPT} = 43000$



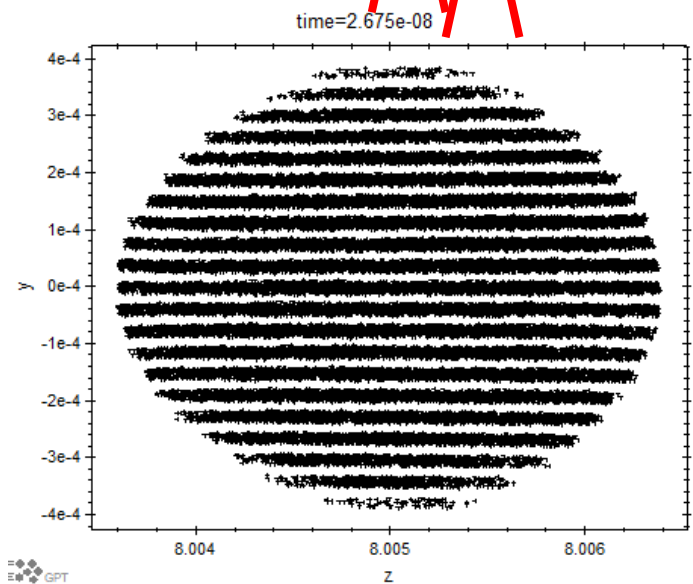
Analytical and numerical results



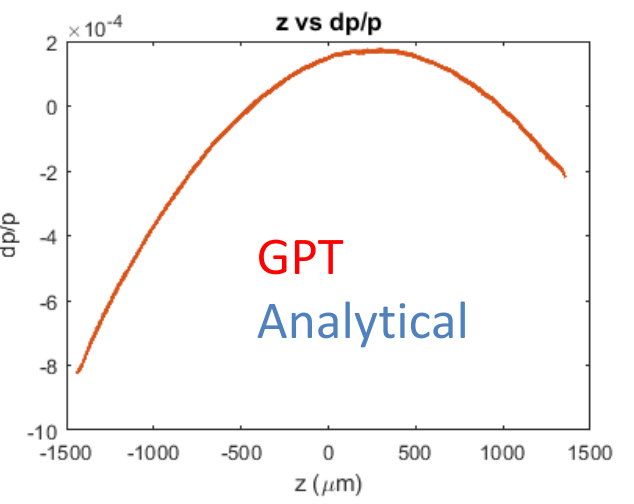
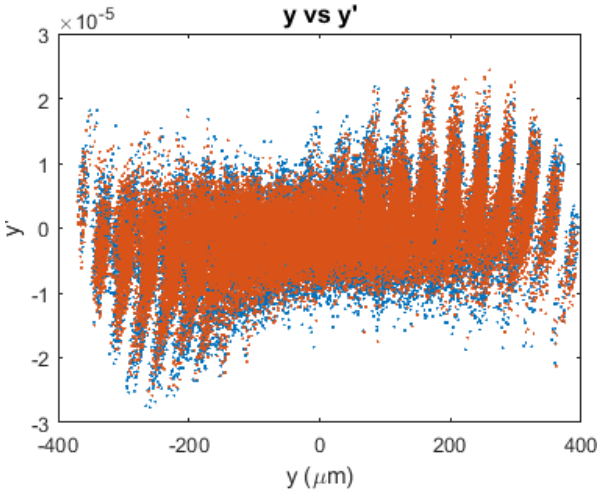
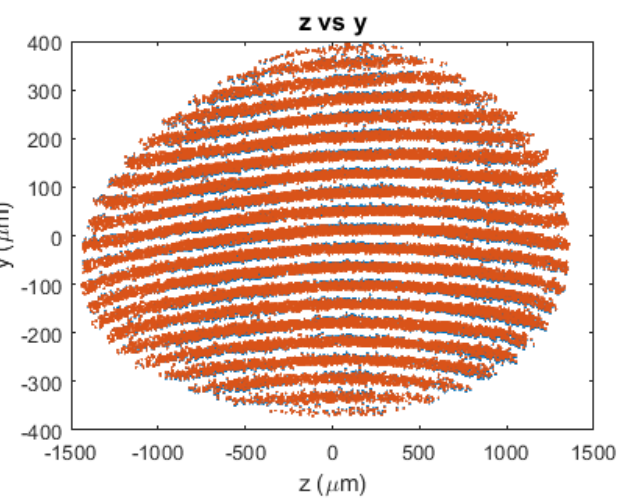
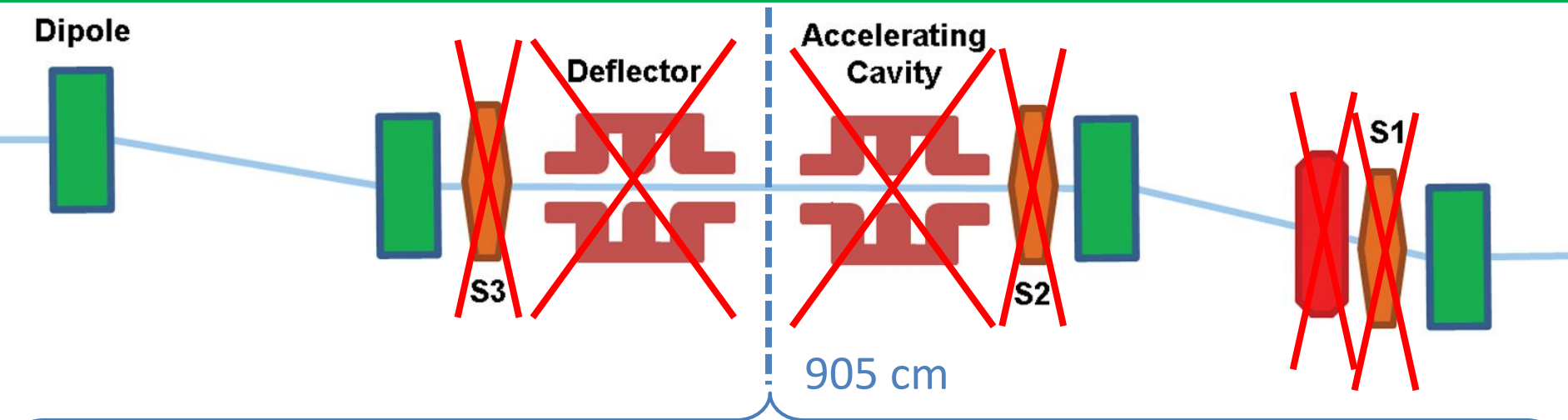
Analytical: $R_d R_{drift} R_d$

$$\begin{bmatrix} y_n \\ y'_n \\ z_n \\ \frac{\Delta p_n}{p_n} \end{bmatrix}_{4 \times 43000}$$

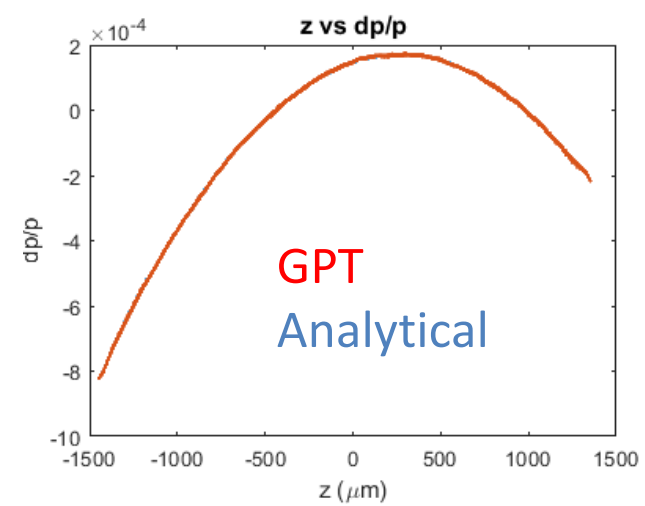
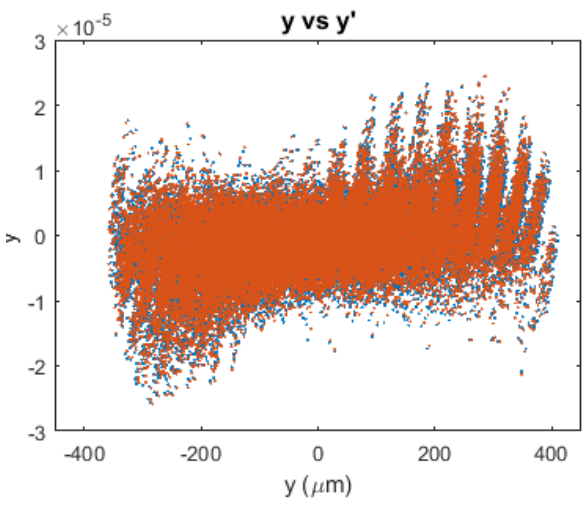
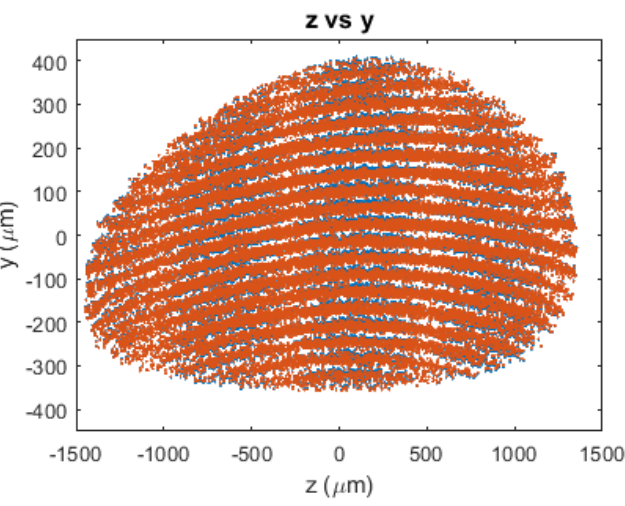
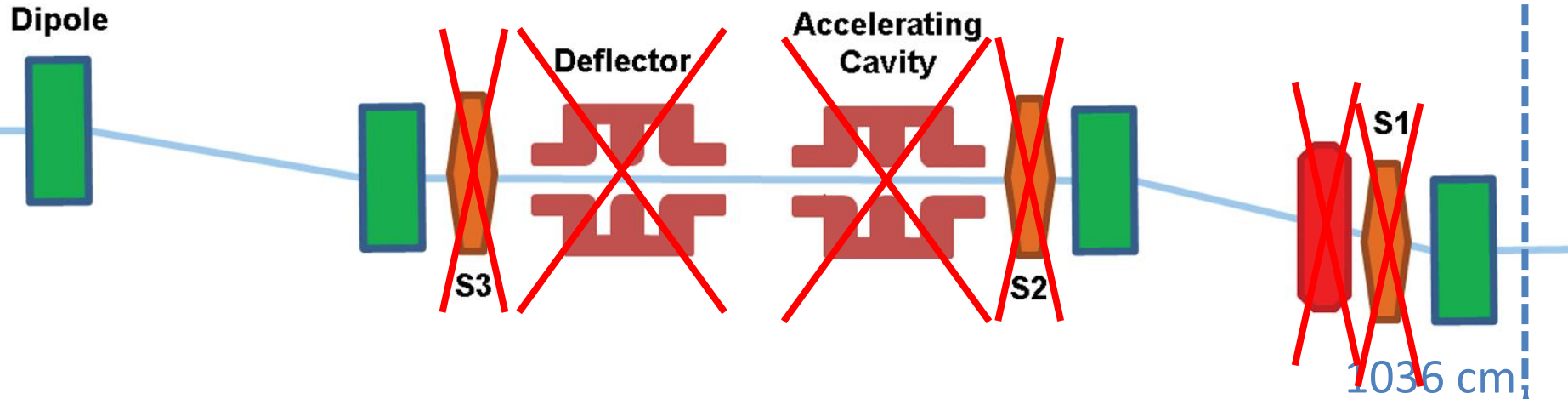
GPT (ASTRA):



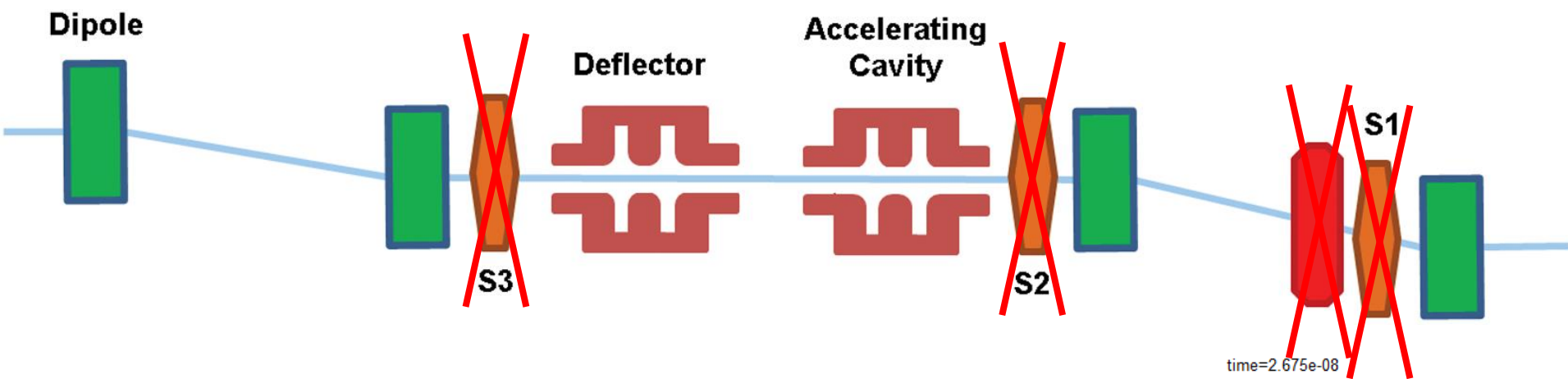
Analytical and numerical results



Analytical and numerical results



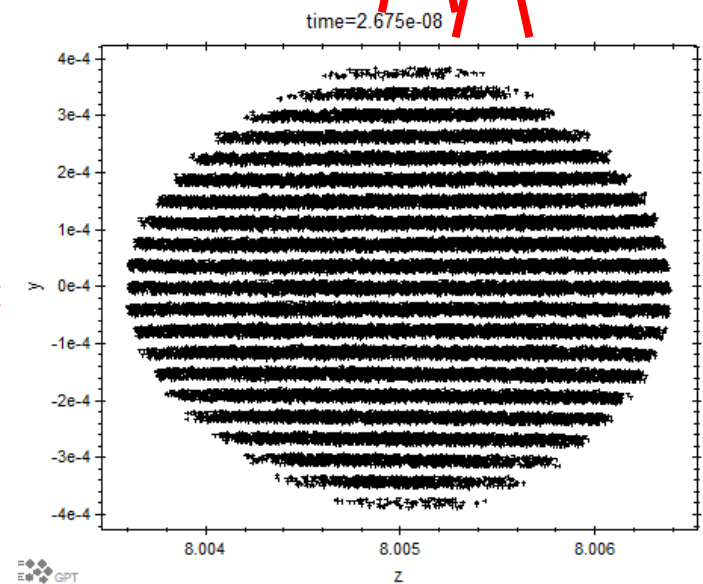
Analytical and numerical results



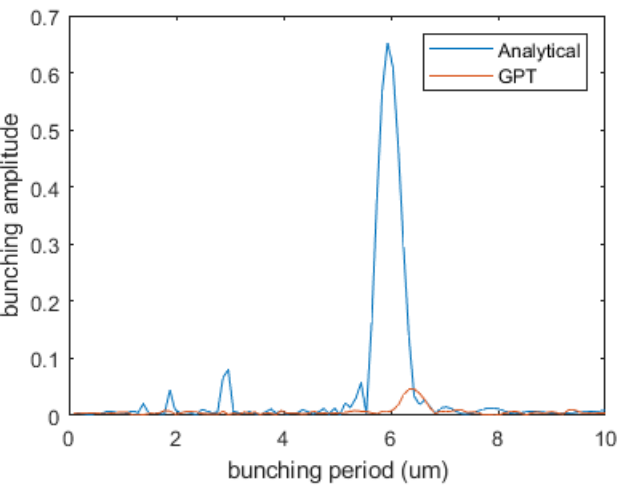
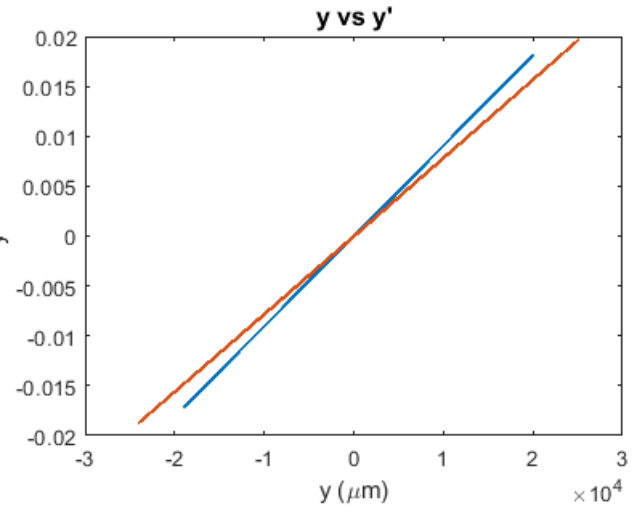
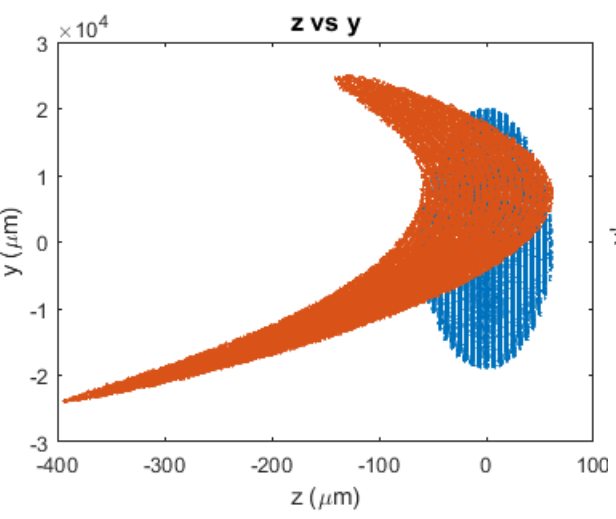
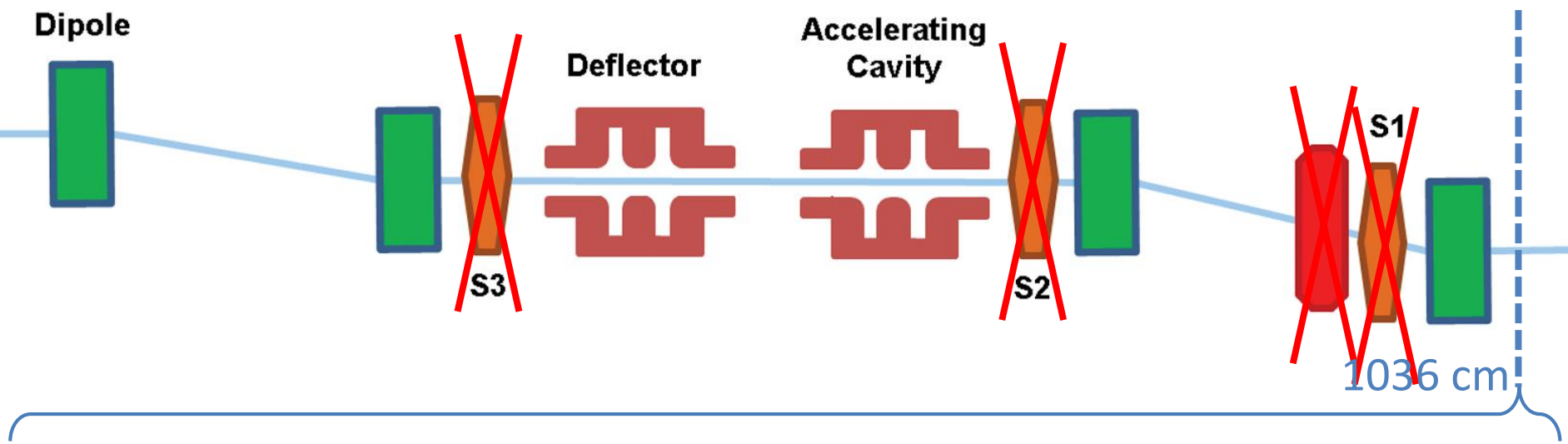
Analytical: $R_d R_a R_k R_d$

$$\begin{bmatrix} y_n \\ y'_n \\ z_n \\ \frac{\Delta p_n}{p_n} \end{bmatrix}_{4 \times 43000}$$

GPT (ASTRA):



Analytical and numerical results



Next steps

- Optimize sextupoles and octupole,
- Reoptimize the phase of the deflector and accelerator cavity,
- Make calculations with smaller grating period.



Thank you for your attention!

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