

The phenomenological cornucopia of $SU(3)$ exotica

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Exotics Charges Under SU(3)

Some scattered phenomenological examples of fields in various representations

- Scalar Fundamentals: Squarks (supersymmetry)-QCD pair production
- Fermion Octets: Gluinos (supersymmetry)-QCD pair production
- Scalar Octets: Monohar Wise(also weak doublets), Sgluons (R-symmetric supersymmetry) Gluon Fusion, QCD pair production
- Sextet Quarks: General Models -QCD pair production

There are many more possible interactions between hypothetical color-charged states and the Standard Model. Some with unusual and distinct collider signatures.

Therefore we attempt a catalogue of possible interactions so that the phenomenology can be systematically explored

Methodological Approach

- Use EFT interactions to catalogue all interactions between SM and Exotic Sector (Later build out with simplified models)

Contains SM fields

Contains Exotic fields

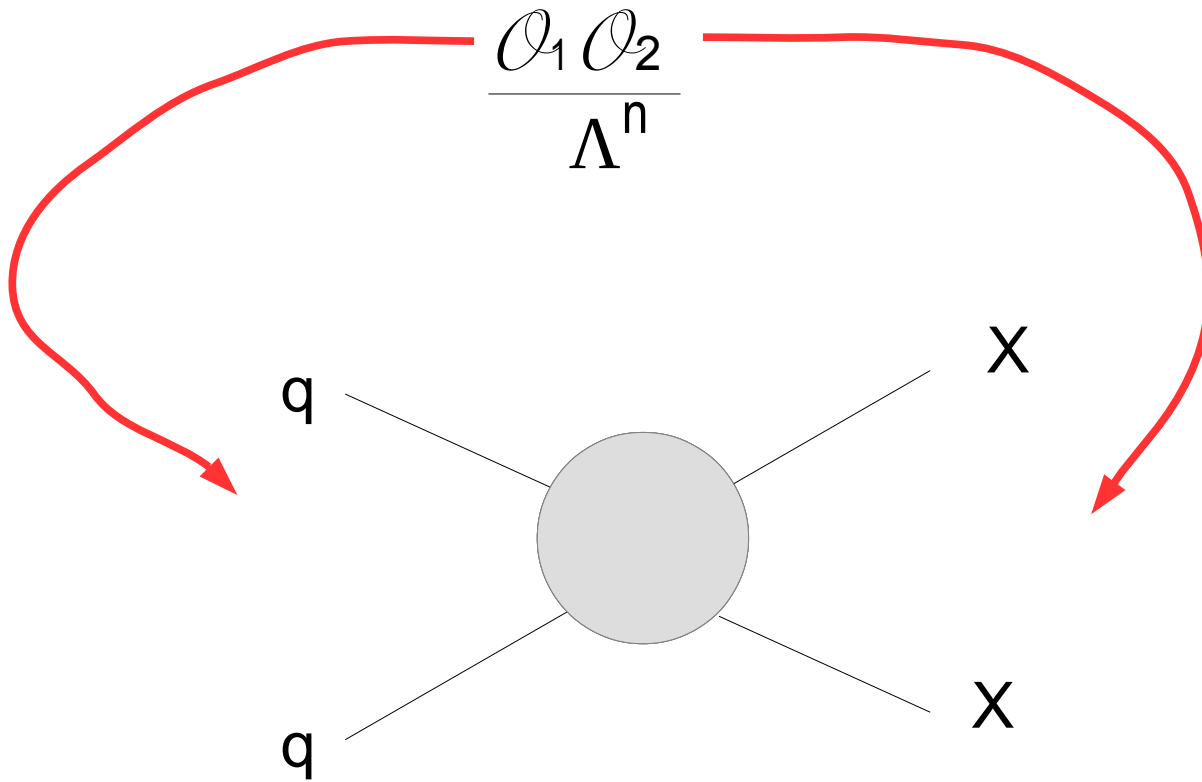
$$\frac{\mathcal{O}_1 \mathcal{O}_2}{\Lambda^n}$$

Effective Cut-Off

Appropriate power gives dim 4 operator

Interaction Term in Lagrangian

Example Diagram



Explore exotic states in various representations of SU(3)

For any Given type of exotic field attempt to find *all* effective operators up to a certain dimension in order to sample entire space of phenomenological signatures

First example exotic sextets (spin 0 or spin $\frac{1}{2}$) all effective operators up to dimension 6 (will include a few interesting examples of dimension 7 operators)

Build Operators using new Exotic Fields

ϕ : spin 0, sextet of SU(3), SU(2) singlet

ψ : spin $\frac{1}{2}$, sextet of SU(3), SU(2) singlet

Write down interaction which preserve all symmetries (Lorentz invariance, gauge symmetries of SM, CPT). For gauge invariance allow various possibilities for hypercharge.

Color Structure and the Construction of SU(3) singlets

To construct color invariants we use an iterative method exploiting the known tensor products of SU(3)

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}}_a \oplus \mathbf{6}_s,$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8},$$

$$\mathbf{6} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{10},$$

$$\mathbf{6} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \mathbf{15},$$

$$\mathbf{6} \otimes \mathbf{6} = \bar{\mathbf{6}}_s \oplus \mathbf{15}_a \oplus \mathbf{15}'_s,$$


$$\mathbf{6} \otimes \bar{\mathbf{6}} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27},$$

$$\mathbf{8} \otimes \mathbf{3} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15},$$

$$\mathbf{8} \otimes \bar{\mathbf{6}} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{24},$$

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1}_s \oplus \mathbf{8}_s \oplus \mathbf{8}_a \oplus \mathbf{10}_a \oplus \bar{\mathbf{10}}_a \oplus \mathbf{27}_s.$$

Interpreting Products

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$


Interpret these as fields in fundamental and antifundamental (quarks/antiquarks)

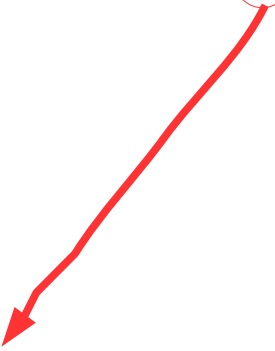
Interpreting Products

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

quarks/antiquarks



Singlet of SU(3): Higgs, photon, Z, W

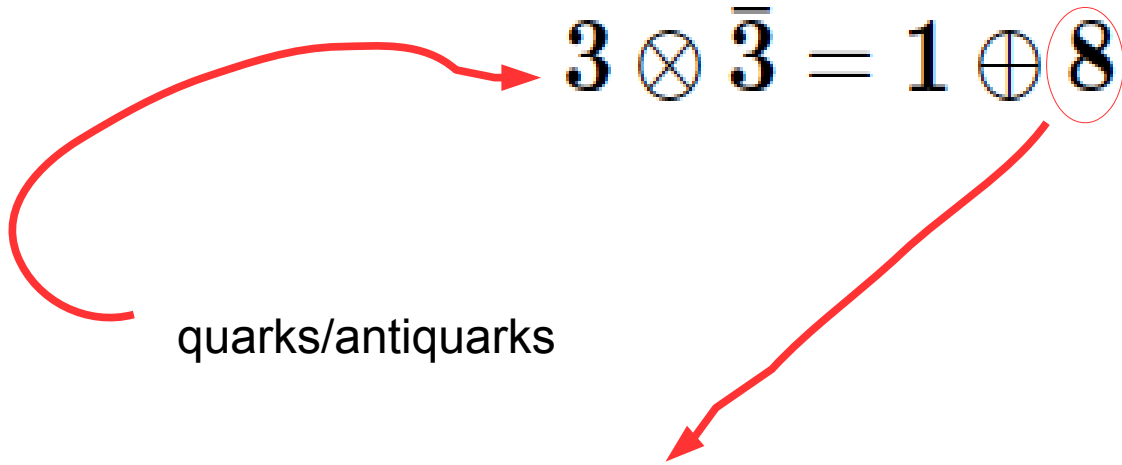


Interpreting Products

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

quarks/antiquarks

Octet of SU(3): gluon



Move all fields to left side of product equation to construct a singlet

$$3 \otimes \bar{3} \otimes 8$$

This is the invariant interaction term we can write in the Lagrangian

We can use this method to construct all color invariant terms in the Lagrangian that contain the new sextet fields

We use an interactive method to construct color invariants. First we find the operators with three color-charged fields

$$3 \otimes 3 = \bar{3}_a \oplus 6_s,$$

$$3 \otimes \bar{3} = 1 \oplus 8,$$

$$6 \otimes 3 = 8 \oplus 10,$$

$$6 \otimes \bar{3} = 3 \oplus 15,$$

$$6 \otimes 6 = \bar{6}_s \oplus 15_a \oplus 15'_s,$$

$$6 \otimes \bar{6} = 1 \oplus 8 \oplus 27,$$

$$8 \otimes 3 = 3 \oplus \bar{6} \oplus 15,$$

$$8 \otimes \bar{6} = 3 \oplus \bar{6} \oplus 15 \oplus 24,$$

$$8 \otimes 8 = 1_s \oplus 8_s \oplus 8_a \oplus 10_a \oplus \bar{10}_a \oplus 27_s$$

We use an interactive method to construct color invariants. First we find the operators with three or fewer color-charged fields

$$3 \otimes 3 = \bar{3}_a \oplus \textcircled{6}_s,$$

$$3 \otimes \bar{3} = 1 \oplus 8,$$

$$\textcircled{6} \otimes 3 = 8 \oplus 10,$$

$$\textcircled{6} \otimes \bar{3} = 3 \oplus 15,$$

$$\textcircled{6} \otimes \textcircled{6} = \textcircled{\bar{6}}_s \oplus 15_a \oplus 15'_s,$$

$$\textcircled{6} \otimes \textcircled{\bar{6}} = 1 \oplus 8 \oplus 27,$$

$$8 \otimes 3 = 3 \oplus \textcircled{\bar{6}} \oplus 15,$$

$$8 \otimes \textcircled{\bar{6}} = 3 \oplus \textcircled{\bar{6}} \oplus 15 \oplus 24,$$

$$8 \otimes 8 = 1_s \oplus 8_s \oplus 8_a \oplus 10_a \oplus \bar{10}_a \oplus 27_s$$

Identify 6 with sextets

To find all products with the exotic fields

From 7 products we find 5 distinct invariants

We construct singlets using the proper choice of Clebsch-Gordon coefficients

$$\mathbf{6} \otimes \bar{\mathbf{6}}$$

Two field invariant

$i \leftrightarrow j$ symmetric K_s^{ij} ← $\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}},$

J^{sia} ← $\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8},$

$t \leftrightarrow u$ symmetric S^{stu} ← $\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6},$

Generators of $\mathbf{6}$ $[t_{\mathbf{6}}^a]_s^t$ ← $\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}.$

Three field invariant

We now construct interaction in the Lagrangian

$$6 \otimes \bar{6}$$

Two field invariant

→

$$\begin{aligned} & \textcircled{3} \otimes \textcircled{3} \otimes \bar{6}, \\ & \textcircled{3} \otimes 6 \otimes \textcircled{8}, \\ & 6 \otimes 6 \otimes 6, \\ & 6 \otimes \bar{6} \otimes \textcircled{8}. \end{aligned}$$

Three field invariant

Interpret 3 as quark and 8 as gluon, we make now complete the Lorentz Structure of the operators. We have already found some interesting new operators

All operators with this color structure

$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8}$	Singlet (Lorentz + \mathcal{G}_{SM})			L	Y
	Generic	Specific	Coupling		
<i>Scalar</i> Φ_s	$(q\ell)\Phi G$	$J^{s ia} \Phi_s (\bar{q}_{Ri}^c \sigma^{\mu\nu} \ell_{RX}) G_{\mu\nu a}$	$\frac{1}{\Lambda_\Phi^2} \lambda_I^X$	-1	$\{\frac{1}{3}, \frac{4}{3}\}$
	$(\bar{\ell}q)\Phi G$	$J^{s ia} \Phi_s (\bar{L}_{LX} H \sigma^{\mu\nu} q_{Ri}) G_{\mu\nu a}$	$\frac{1}{\Lambda_\Phi^3} \lambda_I^X$	1	$\{-\frac{5}{3}, -\frac{2}{3}\}$
<i>Dirac</i> Ψ_s	$(q\Psi)G$	$J^{s ia} (\bar{q}_{Ri}^c \sigma^{\mu\nu} \Psi_s) G_{\mu\nu a}$	$\frac{1}{\Lambda_\Psi} \kappa_I$	0	$\{-\frac{2}{3}, \frac{1}{3}\}$
		$J^{s ia} (\bar{q}_{Ri}^c \Psi_s) B^{\mu\nu} G_{\mu\nu a}$	$\frac{1}{\Lambda_\Psi^3} \kappa_I$		
	$(q\Psi) H ^2 G$	$J^{s ia} (\bar{q}_{Ri}^c \sigma^{\mu\nu} \Psi_s) H ^2 G_{\mu\nu a}$			

$$3 \otimes 6 \otimes 8$$

We interpret this as an interaction between a quark, gluon and sextet the lowest dimension operators is

$$\frac{1}{\Lambda_\Psi} \kappa_I J^{s ia} (\bar{q}_{R I i}^c \sigma^{\mu\nu} \Psi_s) G_{\mu\nu a}$$

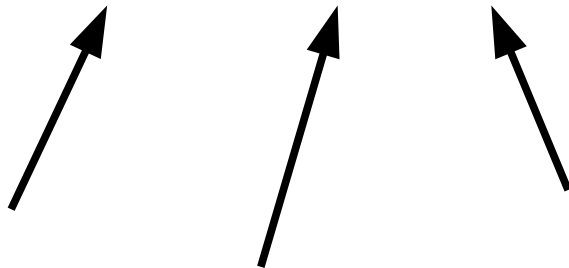
$$\left\{ -\frac{2}{3}, \frac{1}{3} \right\}$$

Possible hypercharges of sextet

quark

fermion sextet


gluon



$$3 \otimes 6 \otimes 8$$

We interpret this as an interaction between a quark, gluon and sextet the lowest dimension operators is

Lorentz indices contracted

$$\frac{1}{\Lambda_\Psi} \kappa_I J^{s ia} (\bar{q}_{R I i}^c \sigma^{\mu\nu} \Psi_s) G_{\mu\nu a} \quad \left\{ -\frac{2}{3}, \frac{1}{3} \right\}$$


Possible hypercharges of sextet

$$3 \otimes 6 \otimes 8$$

We interpret this as an interaction between a quark, gluon and sextet the lowest dimension operators is

$$\frac{1}{\Lambda_\Psi} \kappa_I J^{s ia} (\bar{q}_{Ri}^c \sigma^{\mu\nu} \Psi_s) G_{\mu\nu a}$$

Lorentz indices contracted

$\{-\frac{2}{3}, \frac{1}{3}\}$

Possible hypercharges of sextet

Color indices contracted

$$3 \otimes 6 \otimes 8$$

We interpret this as an interaction between a quark, gluon and sextet the lowest dimension operators is

Lorentz indices contracted

$$\frac{1}{\Lambda_\Psi} \kappa_I J^{s ia} (\bar{q}_{Ri}^c \sigma^{\mu\nu} \Psi_s) G_{\mu\nu a}$$

$$\left\{ -\frac{2}{3}, \frac{1}{3} \right\}$$

Possible hypercharges of sextet

Color indices contracted

Dimension 5 operator

$$3 \otimes 6 \otimes 8$$

We interpret this as an interaction between a quark, gluon and sextet the lowest dimension operators is

Lorentz indices contracted

$$\frac{1}{\Lambda_\Psi} \kappa_I \psi^{s ia} (\bar{q}_{Ri}^c \sigma^{\mu\nu} \Psi_s) G_{\mu\nu a}$$

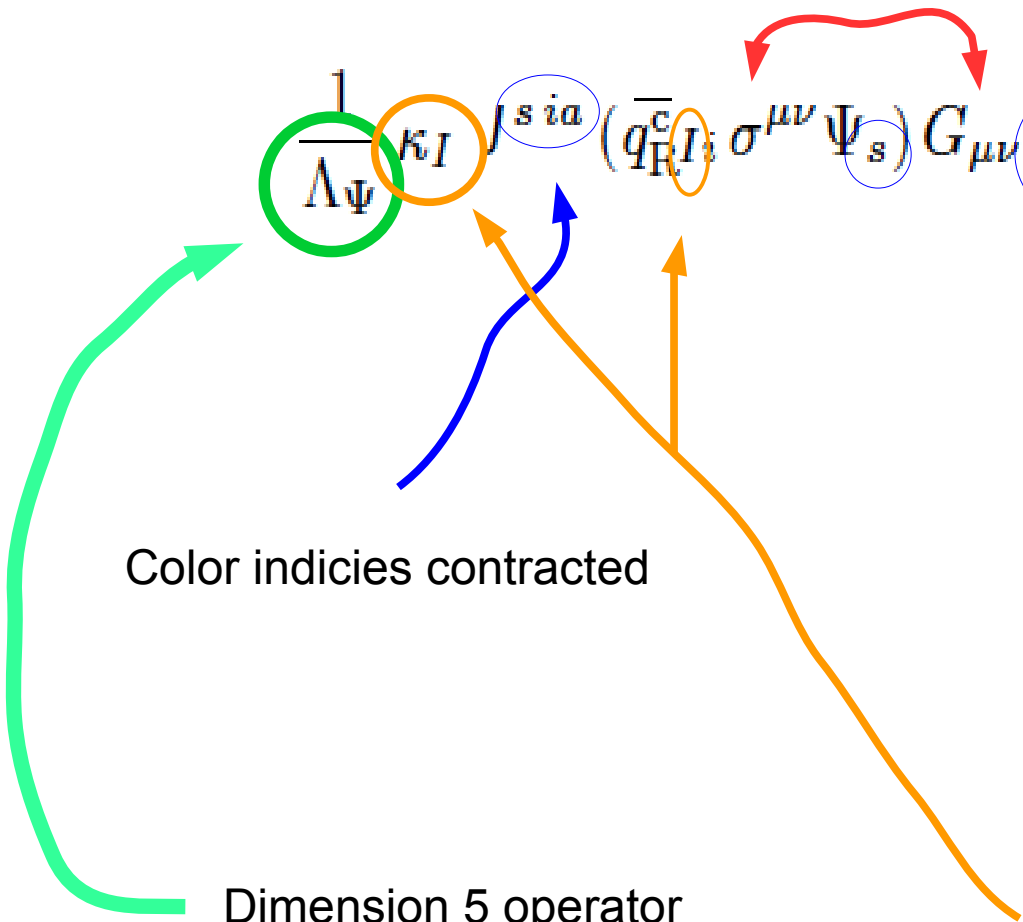
$$\left\{ -\frac{2}{3}, \frac{1}{3} \right\}$$

Possible hypercharges of sextet

Color indices contracted

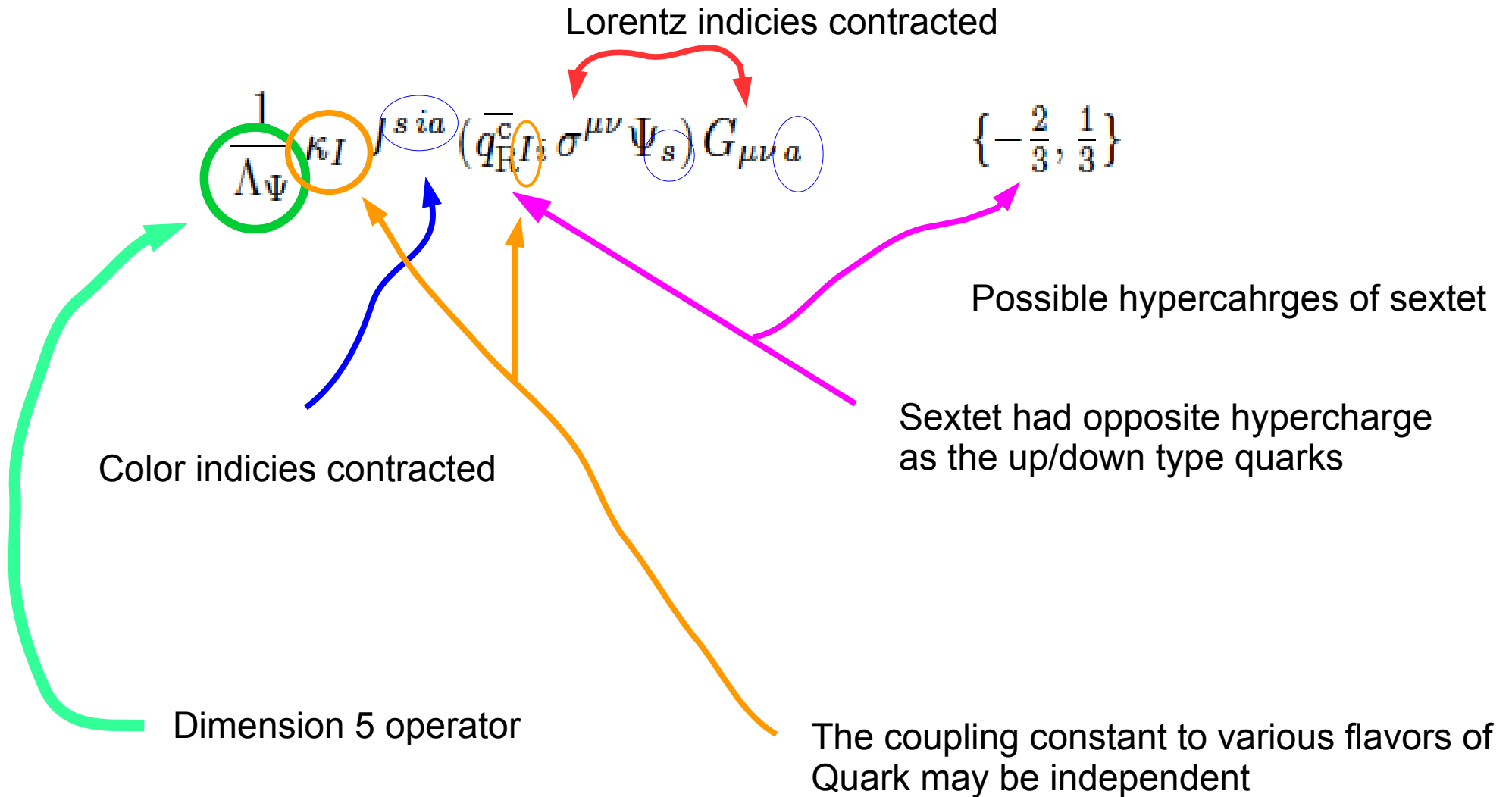
Dimension 5 operator

The coupling constant to various flavors of Quark may be independent

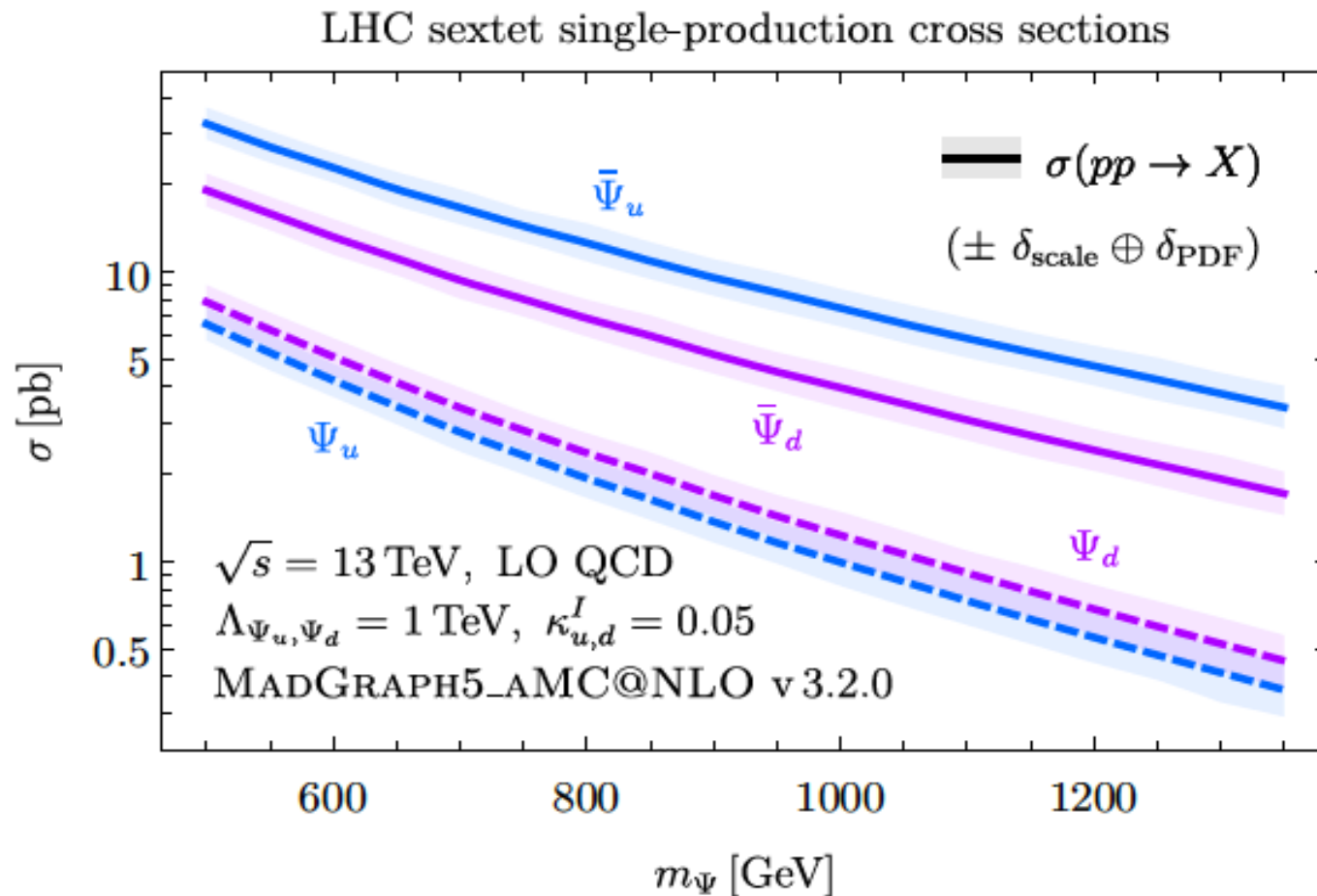


$$3 \otimes 6 \otimes 8$$

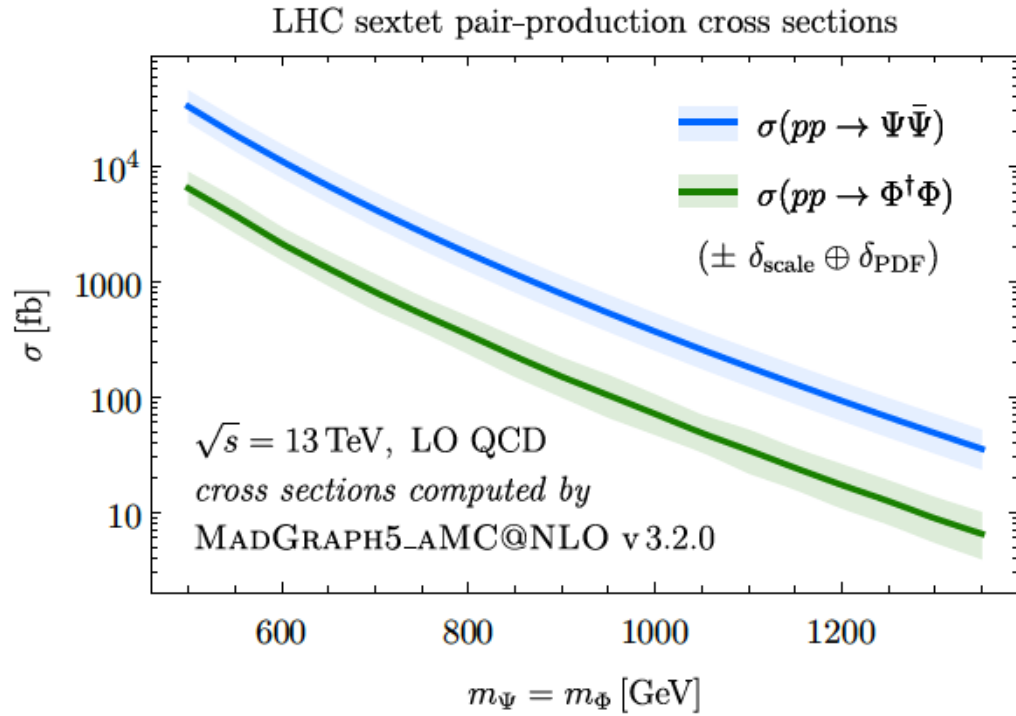
We interpret this as an interaction between a quark, gluon and sextet the lowest dimension operators is



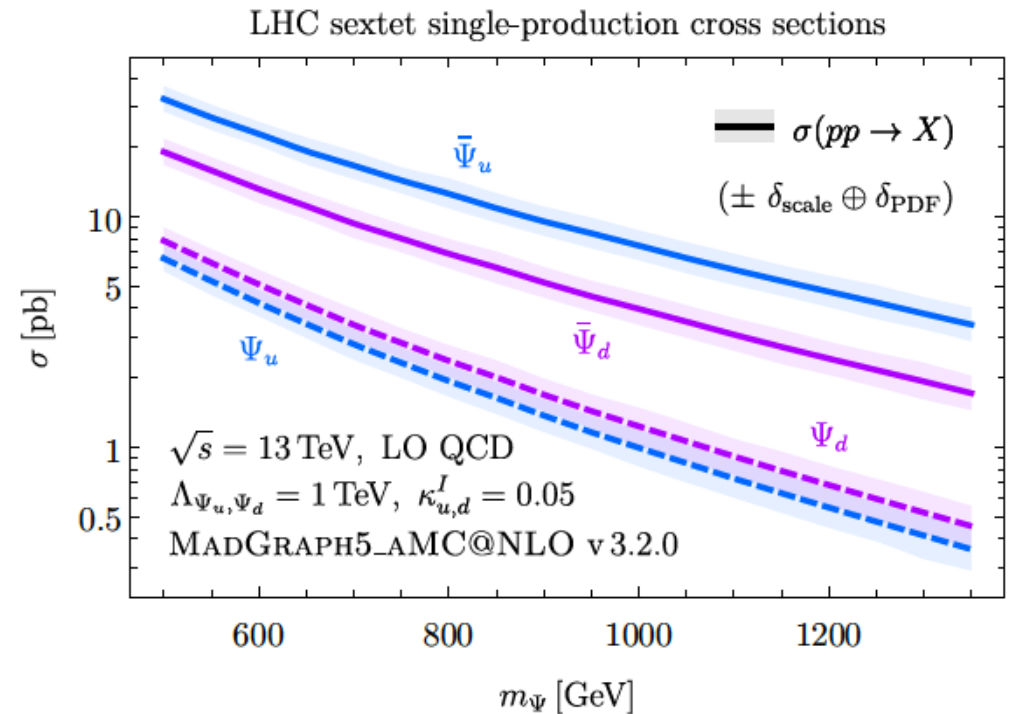
This operator leads to single sextet production through quark-gluon fusion



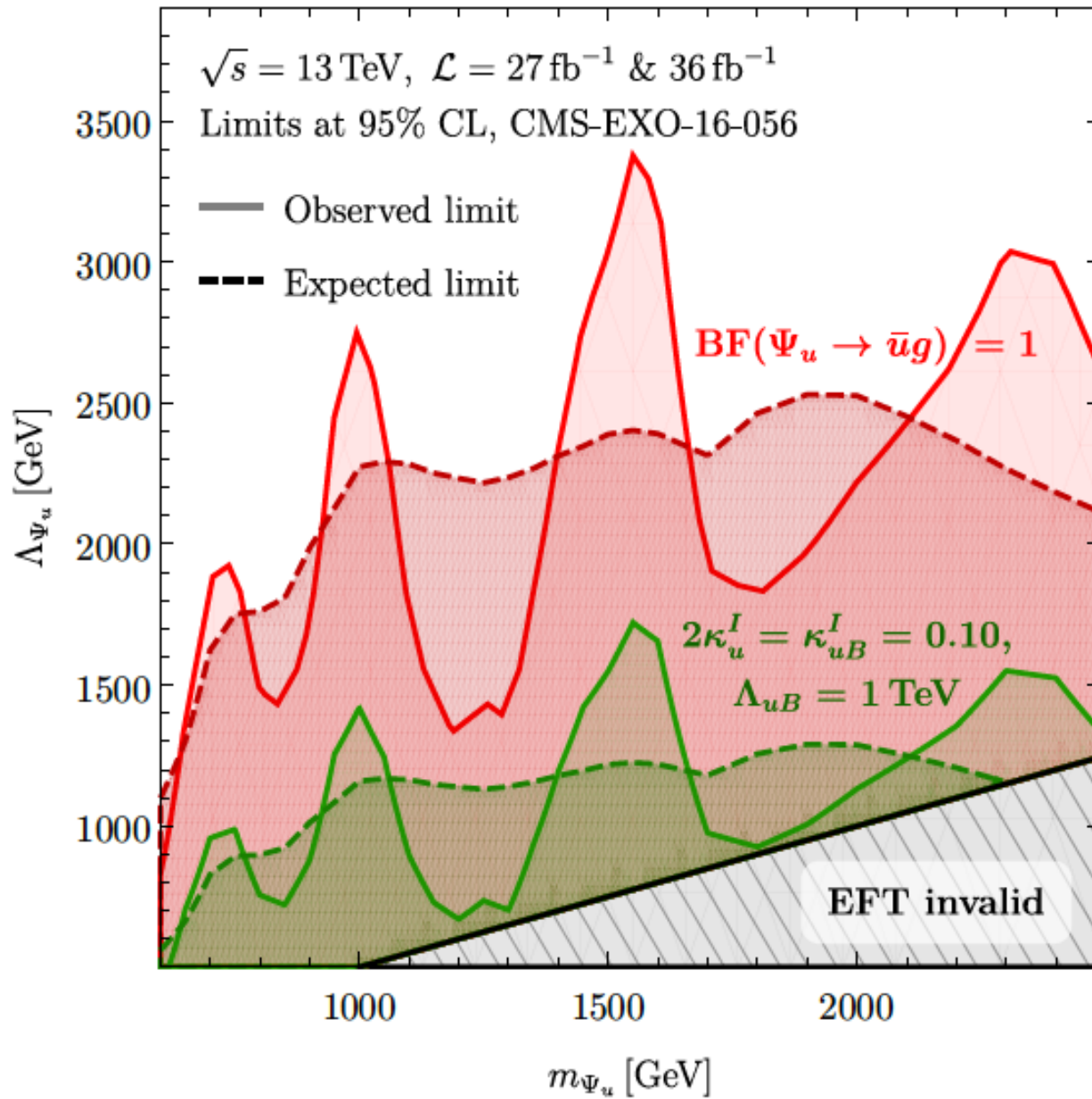
Differential quark-antiquarks PDF's ensure different production cross section for Up and down type sextets between sextets and anti-sextets



Comparing LHC pair production from gluon fusion to single production from the new operator



Constraining $\bar{\Psi}_u$ as a dijet resonance



Now consider this same color structure but try to build out the operator with a spin 0 sextet

$$\frac{1}{\Lambda_{\Phi}^2} \lambda_I^X J^{s ia} \Phi_s (\bar{q}_{Ri}^c \sigma^{\mu\nu} \ell_{RX}) G_{\mu\nu a}$$

Now consider this same color structure but try to build out the operator with a spin 0 sextet

$$\frac{1}{\Lambda_{\Phi}^2} \lambda_I^X J^{s ia} \Phi_s (\bar{q}_{R I i}^c \sigma^{\mu\nu} \ell_{R X}) G_{\mu\nu a}$$

To contract Lorentz indices
We need to add a fermion

Now consider this same color structure but try to build out the operator with a spin 0 sextet

$$\frac{1}{\Lambda_{\Phi}^2} \lambda_I^X J^{s ia} \Phi_s (\bar{q}_{R I i}^c \sigma^{\mu\nu} \ell_{R X}) G_{\mu\nu a}$$

To contract Lorentz indices
We need to add a fermion

Now this is a dimension 6 operator

Now consider this same color structure but try to build out the operator with a spin 0 sextet

$$\frac{1}{\Lambda_\Phi^2} \lambda_I^X J^{s ia} \Phi_s (\bar{q}_{Ri}^c \sigma^{\mu\nu} \ell_{RX}) G_{\mu\nu a}$$

To contract Lorentz indices
 We need to add a fermion

To prevent proton decay
 The sextet has a lepton
 Number -1

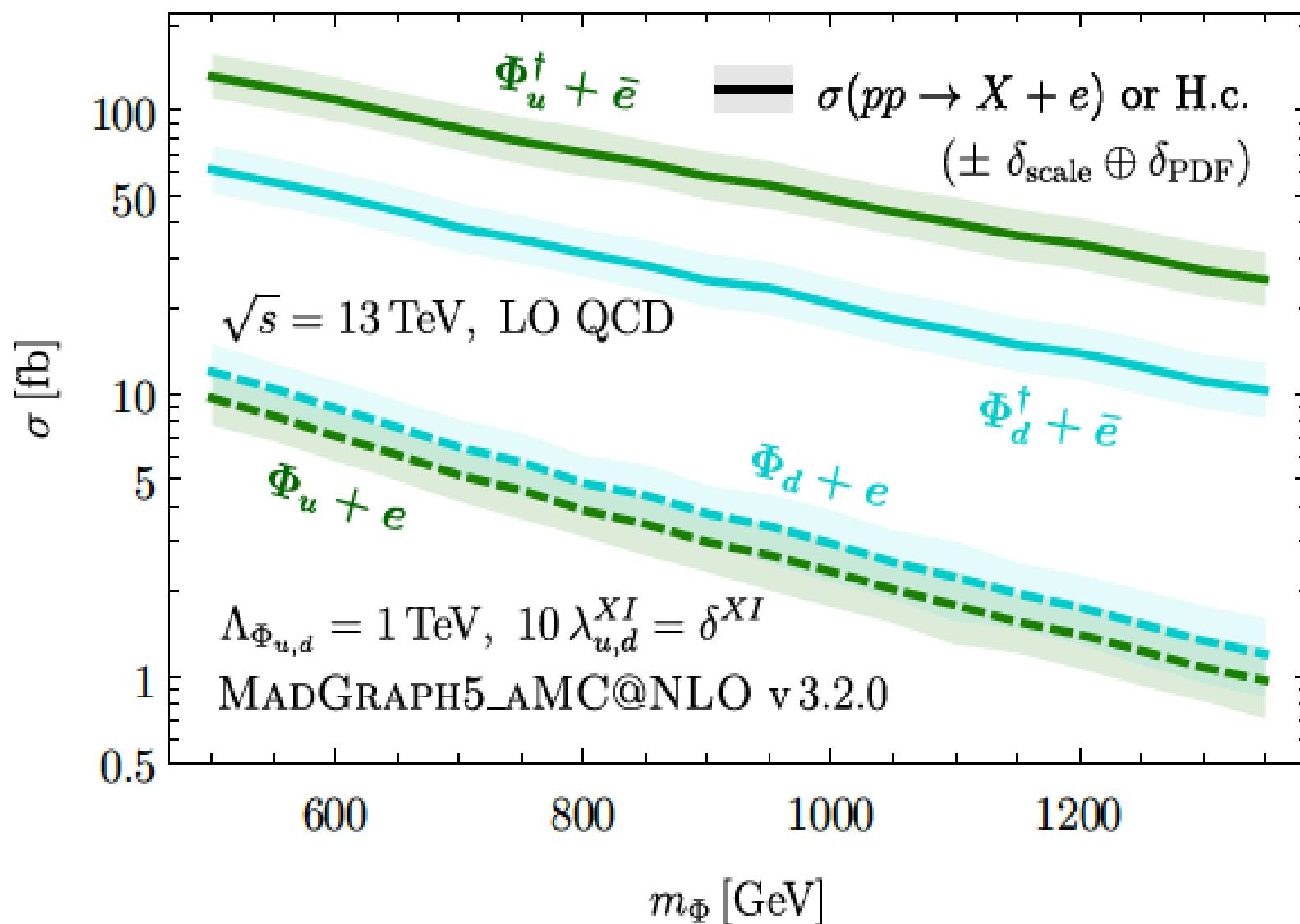
Now we the sextet must
 Have the opposite hypercharge
 As the sum of the lepton and quark
 charges

$\{\frac{1}{3}, \frac{4}{3}\}$

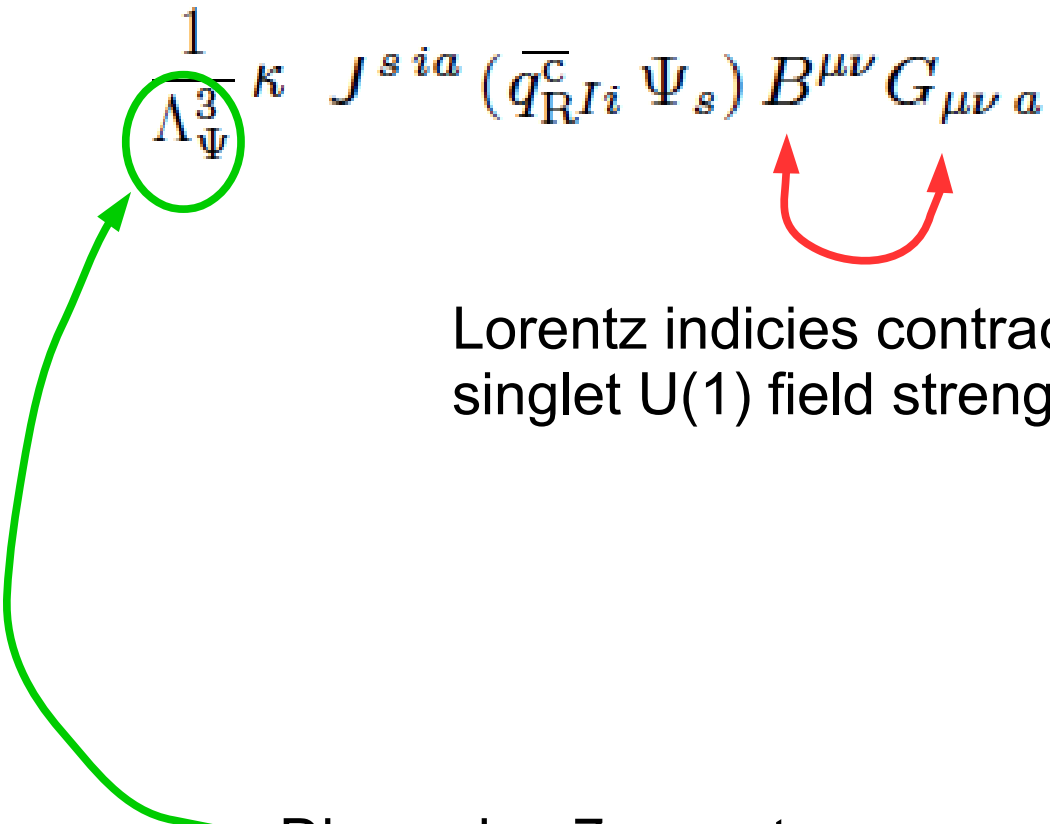
Now this is a dimension 6 operator

Now there is a process where a scalar sextet is produced in association with a lepton

LHC sextet + e production cross sections



Higher Dimensional Operators

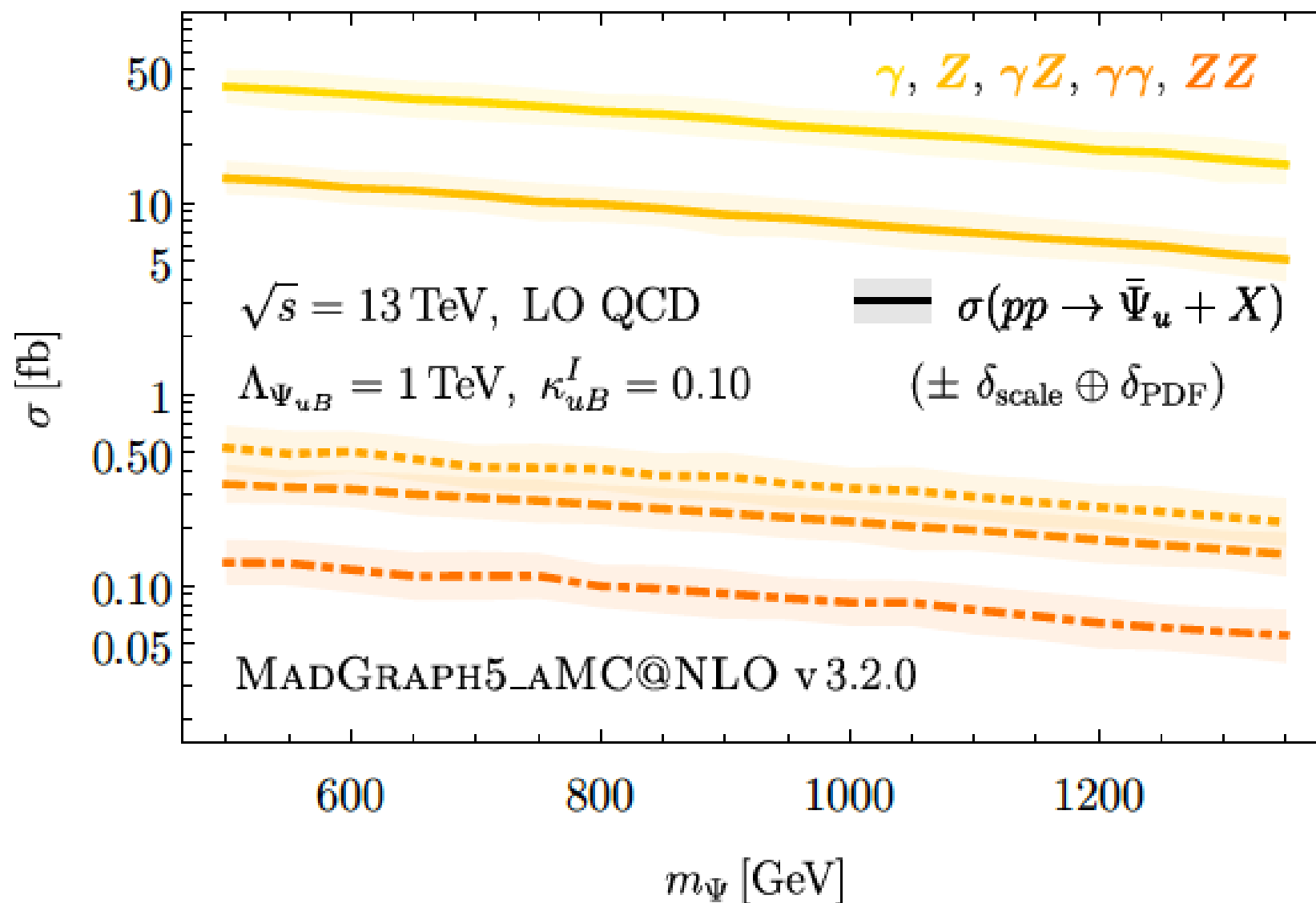
$$\frac{1}{\Lambda_\Psi^3} \kappa J^{s i a} (\bar{q}_{R i}^c \Psi_s) B^{\mu\nu} G_{\mu\nu a}$$


Lorentz indices contracted with color singlet U(1) field strength tensor

Dimension 7 operator

This operator allows the production of sextets in association with photon or Z

LHC $\bar{\Psi}_u + \gamma/Z$ production cross sections



$3 \otimes 3 \otimes \bar{6}$	Singlet (Lorentz + \mathcal{G}_{SM})			L	Y	
	Generic	Specific	Coupling			
Scalar Φ_s	$(qq')\Phi^\dagger$	$K_s^{ij} \Phi^{\dagger s} (\bar{q}_{Ri}^c q_{Rj})$	λ_{IJ}	0	$\{-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\}$	
		$K_s^{ij} \Phi^{\dagger s} (\bar{q}_{Ri}^c \sigma^{\mu\nu} q_{Rj}) B_{\mu\nu}$	$\frac{1}{\Lambda_\Phi^2} \lambda_{IJ}$			
		$K_s^{ij} \Phi^{\dagger s} (\bar{Q}_{Li}^c i\tau^2 Q_{Lj})$	λ_{IJ}		$\frac{1}{3}$	
		$K_s^{ij} \Phi^{\dagger s} (\bar{Q}_{Li}^c \sigma^{\mu\nu} i\tau^2 Q_{Lj}) B_{\mu\nu}$	$\frac{1}{\Lambda_\Phi^2} \lambda_{IJ}$			
		$K_s^{ij} \Phi^{\dagger s} (\bar{Q}_{Li}^c HH^\dagger Q_{Lj})$	$\frac{1}{\Lambda_\Phi^2} \lambda_{IJ}$			
		$K_s^{ij} \Phi^{\dagger s} (\bar{Q}_{Li}^c \sigma^{\mu\nu} t_2^A Q_{Lj}) W_{\mu\nu A}$				
	$(qq') H ^2\Phi^\dagger$	$K_s^{ij} \Phi^{\dagger s} (\bar{q}_{Ri}^c q_{Rj}) H ^2$		$\{-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\}$		
Dirac Ψ_s	$(qq')(\bar{\Psi}\ell)$	$K_s^{ij} (\bar{q}_{Ri}^c q_{Rj})(\bar{\Psi}^s \ell_{RX})$	$\frac{1}{\Lambda_\Psi^2} \kappa_{IJ}^X$	1	$\{-\frac{5}{3}, -\frac{2}{3}, \frac{1}{3}\}$	
		$K_s^{ij} (\bar{q}_{Ri}^c q_{Rj})(\bar{\Psi}^s H^\dagger L_{LX})$	$\frac{1}{\Lambda_\Psi^3} \kappa_{IJ}^X$			
		$K_s^{ij} (\bar{q}_{Ri}^c \sigma^{\mu\nu} q_{Rj})(\bar{\Psi}^s \sigma_{\mu\nu} H^\dagger L_{LX})$			$\frac{1}{\Lambda_\Psi^2} \kappa_{IJ}^X$	$\frac{4}{3}$
		$K_s^{ij} (\bar{Q}_{Li}^c i\tau^2 Q_{Lj})(\bar{\Psi}^s \ell_{RX})$	$-\frac{2}{3}$			
		$K_s^{ij} (\bar{Q}_{Li}^c i\tau^2 Q_{Lj})(\bar{\Psi}^s H^\dagger L_{LX})$	$\frac{1}{\Lambda_\Psi^3} \kappa_{IJ}^X$			$\{-\frac{2}{3}, \frac{1}{3}\}$
		$K_s^{ij} (\bar{Q}_{Li}^c H\gamma^\mu q_{Rj})(\bar{\Psi}^s \gamma_\mu \ell_{RX})$				
	$(\bar{\Psi}q)(q\ell)$	$K_s^{ij} (\bar{\Psi}^s \sigma^{\mu\nu} q_{Ri})(\bar{q}_{Rj}^c \sigma_{\mu\nu} \ell_{RX})$	$\frac{1}{\Lambda_\Psi^3} \kappa_{IJ}^X$	$\{-\frac{5}{3}, -\frac{2}{3}, \frac{1}{3}\}$		
		$K_s^{ij} (\bar{\Psi}^s \gamma^\mu q_{Ri})(\bar{Q}_{Lj}^c H\gamma_\mu \ell_{RX})$		$\{-\frac{2}{3}, \frac{1}{3}\}$		
	$(\bar{\Psi}q)(\bar{\ell}q)$	$K_s^{ij} (\bar{\Psi}^s \gamma^\mu q_{Ri})(\bar{L}_{LX} \gamma_\mu i\tau^2 Q_{Lj})$	$\frac{1}{\Lambda_\Psi^2} \kappa_{IJ}^X$	$\{\frac{1}{3}, \frac{4}{3}\}$		
		$K_s^{ij} (\bar{\Psi}^s \gamma^\mu q_{Ri})(\bar{\ell}_{RX} \gamma_\mu q_{Rj})$		$\{\frac{1}{3}, \frac{4}{3}, \frac{7}{3}\}$		

We can build out
The operators for
each tensor product

	<i>Scalar sextet Φ only</i>		<i>Dirac sextet Ψ only</i>		≥ 1 of each	
$SU(3)_c$ invariant	d_{\min}	Structure	d_{\min}	Structure	d_{\min}	Structure
$\mathbf{6} \otimes \bar{\mathbf{6}}$	4^\dagger	$\Phi^\dagger\Phi$	5^\dagger	$(\bar{\Psi}\Psi)$	4	$(\bar{\Psi}\ell)\Phi$
						$(\Psi\ell)\Phi^\dagger$
						$(\bar{\ell}\Psi)\Phi^\dagger$
$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}}$	4	$(qq')\Phi^\dagger$	6	$(qq')(\bar{\Psi}\ell)$		
	6	$(qq') H ^2\Phi^\dagger$		$(\bar{\Psi}q)(q\ell)$		
				$(\bar{\Psi}q)(\bar{\ell}q)$		
$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{8}$	6	$(q\ell)\Phi G$	5	$(q\Psi)G$		
		$(\bar{\ell}q)\Phi G$	7	$(q\Psi) H ^2G$		
$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6}$	5^\dagger	$\Phi\Phi\Phi$	6	$(\Psi\Psi)(\Psi\ell)$	5	$(\Psi\ell)\Phi\Phi$
				$(\Psi\Psi)(\bar{\ell}\Psi)$		$(\bar{\ell}\Psi)\Phi\Phi$
					6^\dagger	$(\Psi\Psi)\Phi$
$\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	6	$\Phi^\dagger\Phi GB$	5	$(\bar{\Psi}\Psi)G$	6	$(\bar{\Psi}\ell)\Phi G$
			7	$(\bar{\Psi}\Psi) H ^2G$		$(\Psi\ell)\Phi^\dagger G$
						$(\bar{\ell}\Psi)\Phi^\dagger G$

Iterating tensor products

Observation. If there exist invariant combinations of $n + 1$ and $m + 1$ fields transforming in the direct product representations $\mathbf{r}_1 \otimes \cdots \otimes \mathbf{r}_n \otimes \mathbf{p}$ and $\mathbf{q}_1 \otimes \cdots \otimes \mathbf{q}_m \otimes \mathbf{p}$ of $SU(3)$, then there exists an invariant combination of $n + m$ fields in the reducible representation $\mathbf{r}_1 \otimes \cdots \otimes \mathbf{r}_n \otimes \bar{\mathbf{q}}_1 \otimes \cdots \otimes \bar{\mathbf{q}}_m$.

Constructing 4 field invariants

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}}_a \oplus \mathbf{6}_s,$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8},$$

$$\mathbf{6} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{10},$$

$$\mathbf{6} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \mathbf{15},$$

$$\mathbf{6} \otimes \mathbf{6} = \bar{\mathbf{6}}_s \oplus \mathbf{15}_a \oplus \mathbf{15}'_s,$$

$$\mathbf{6} \otimes \bar{\mathbf{6}} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27},$$

$$\mathbf{8} \otimes \mathbf{3} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15},$$

$$\mathbf{8} \otimes \bar{\mathbf{6}} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{24},$$

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1}_s \oplus \mathbf{8}_s \oplus \mathbf{8}_a \oplus \mathbf{10}_a \oplus \bar{\mathbf{10}}_a \oplus \mathbf{27}_s$$

By iterating tensor products
We construct new invariant

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} = \mathbf{6} \otimes \mathbf{3}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{6}}$$

With coefficient

$$[t_3^a]_j^i \bar{J}_{sak}$$

Fundamentals contracted into 8

6-3-8 contraction

color-structure matters

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}}_a \oplus \mathbf{6}_s,$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8},$$

$$\mathbf{6} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{10},$$

$$\mathbf{6} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \mathbf{15},$$

$$\mathbf{6} \otimes \mathbf{6} = \bar{\mathbf{6}}_s \oplus \mathbf{15}_a \oplus \mathbf{15}'_s,$$

$$\mathbf{6} \otimes \bar{\mathbf{6}} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27},$$

$$\mathbf{8} \otimes \mathbf{3} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15},$$

$$\mathbf{8} \otimes \bar{\mathbf{6}} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{24},$$

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1}_s \oplus \mathbf{8}_s \oplus \mathbf{8}_a \oplus \mathbf{10}_a \oplus \bar{\mathbf{10}}_a \oplus \mathbf{27}_s$$

Iterating we get

$$\mathbf{6} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} = \bar{\mathbf{3}}_a$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{6}}$$

With coefficient

$$\bar{L}_{jkl} K_s^{li}$$

Fundamentals contracted into 3

3-3-6bar contraction

All four field color invariants

Invariant	Clebsch-Gordan coefficients						Notes
$3 \otimes 3 \otimes 6 \otimes 6$	$K_u^{ij} S^{ust}$						$\ni [\Pi_{3366}]^{ijst}$
$3 \otimes 3 \otimes \bar{6} \otimes 8$	$L^{ijk} \bar{J}_{ska}$	$K_s^{ik} [t_3^a]_k^j$	$K_r^{ij} [t_6^a]_s^r$	$Q^q{}^i{}_s V_q{}^{ja}$			$\ni [\Pi_{33\bar{6}8}]^{ij_s^a}$
$3 \otimes \bar{3} \otimes \bar{3} \otimes \bar{6}$	$\bar{L}_{jkl} K_s{}^{li}$			$[t_3^a]_j^i \bar{J}_{sak}$			$\ni [\Pi_{3\bar{3}\bar{3}\bar{6}}]^{i_jks}$
$3 \otimes \bar{3} \otimes 6 \otimes \bar{6}$	$\delta_j^i \delta_t^s$			$[t_3^a]_j^i [t_6^a]_t^s$			$\ni [\Pi_{3\bar{3}6\bar{6}}]^{i_j^s t}$
$3 \otimes 6 \otimes 6 \otimes \bar{6}$				$J^{sia} [t_6^a]_u^t$	$Q^q{}^i{}_u W_q{}^{st}$		$\ni [\Pi_{366\bar{6}}]^{i st}_u$
$3 \otimes 6 \otimes 8 \otimes 8$	$[t_3^a]_j^i J^{sjb}$		$J^{tia} [t_6^b]_t^s$	$J^{sic} \{f, d\}^{abc}$	$E_x{}^{is} G^{xab}$	$V_q{}^{ia} \chi^q{}^{sb}$	$\ni [\Pi_{3688}]^{i sab}$
$3 \otimes \bar{6} \otimes \bar{6} \otimes 8$	$K_s^{ij} \bar{J}_{sai}$			$J^{sia} \bar{S}_{stu}$			$\ni [\Pi_{3\bar{6}\bar{6}8}]^{i st}_a$
$6 \otimes 6 \otimes \bar{6} \otimes \bar{6}$	$\delta_u^s \delta_v^t$			$[t_6^a]_u^s [t_6^a]_v^t$			$\ni [\Pi_{66\bar{6}\bar{6}}]^{st}_{uv}$
$6 \otimes 6 \otimes 6 \otimes 8$				$S^{str} [t_6^a]_r^u$	$W_q{}^{st} \chi^q{}^{ua}$		$\ni [\Pi_{6668}]^{stu}_a$
$6 \otimes \bar{6} \otimes 8 \otimes 8$	$\delta_t^s \delta_b^a$			$[t_6^c]_t^s [t_8^c]_b^a$	$F^n{}_t H^{nab}$		$\ni [\Pi_{6\bar{6}88}]^{s t}_b^a$

Operators built from 4 field color-invariants

	Scalar sextet Φ only		Dirac sextet Ψ only		≥ 1 of each	
SU(3) _c invariant	d_{\min}	Structure	d_{\min}	Structure	d_{\min}	Structure
$3 \otimes 3 \otimes 6 \otimes 6$	5	$(qq')\Phi\Phi$	6	$(qq')(\Psi\Psi)$	7	$(qq')(\Psi\ell)\Phi$
	7	$(qq')\Phi H ^2\Phi$		$(q\Psi)(q'\Psi)$		$(q\ell)(q'\Psi)\Phi$
$3 \otimes 3 \otimes \bar{6} \otimes 8$	6	$(qq')\Phi^\dagger G$				
$3 \otimes \bar{3} \otimes \bar{3} \otimes \bar{6}$	7	$(\bar{q}q')(\bar{q}''\ell)\Phi$	6	$(\bar{q}q')(\bar{q}''\Psi)$		
		$(qq')^\dagger(q''\ell)\Phi$		$(qq')^\dagger(\bar{q}''\Psi)$		
$3 \otimes \bar{3} \otimes 6 \otimes \bar{6}$	5	$(\bar{q}q')\Phi^\dagger\Phi$	6	$(\bar{q}q')(\bar{\Psi}\Psi)$	7*	$(\bar{q}q')(\bar{\Psi}\ell)\Phi$
	7	$(\bar{q}q')\Phi^\dagger H ^2\Phi$		$(\bar{q}\Psi)(\bar{\Psi}q')$		$(\bar{q}\Psi)(q'\ell)\Phi^\dagger$
$3 \otimes 6 \otimes 6 \otimes \bar{6}$	6	$(q\ell) \Phi ^2\Phi$	6	$(q\Psi)(\bar{\Psi}\Psi)$	5	$(q\Psi)\Phi^\dagger\Phi$
		$(\bar{\ell}q) \Phi ^2\Phi$		$(\bar{\Psi}q)(\Psi\Psi)$		$(\bar{\Psi}q)\Phi\Phi$
					7*	$(q\Psi)(\Psi\ell)\Phi^\dagger$
						$(q\ell)(\bar{\Psi}\Psi)\Phi$
$3 \otimes 6 \otimes 8 \otimes 8$			7	$(q\Psi)GG$		
$3 \otimes \bar{6} \otimes \bar{6} \otimes 8$	7	$(q\ell)\Phi^\dagger\Phi^\dagger G$			6	$(\bar{\Psi}q)\Phi^\dagger G$
		$(\bar{\ell}q)\Phi^\dagger\Phi^\dagger G$				
$6 \otimes 6 \otimes \bar{6} \otimes \bar{6}$	6 [†]	$ \Phi ^4$			6	$(\bar{\Psi}\ell) \Phi ^2\Phi$
						$(\Psi\ell) \Phi ^2\Phi^\dagger$
						$(\bar{\ell}\Psi) \Phi ^2\Phi^\dagger$
	7				7	$(\bar{\Psi}\Psi) \Phi ^2 H ^2$
$6 \otimes 6 \otimes 6 \otimes 8$	7	$\Phi\Phi\Phi GB$			6	$(\Psi\Psi)\Phi G$
					7	$(\Psi\ell)\Phi\Phi G$
						$(\bar{\ell}\Psi)\Phi\Phi G$
$6 \otimes \bar{6} \otimes 8 \otimes 8$	6	$ \Phi ^2 GG$	7	$(\bar{\Psi}\Psi)GG$		

More interesting processes

	Scalar sextet Φ only		Dirac sextet Ψ only		≥ 1 of each	
SU(3) _c invariant	d_{\min}	Structure	d_{\min}	Structure	d_{\min}	Structure
$3 \otimes 3 \otimes 6 \otimes 6$	5	$(qq')\Phi\Phi$	6	$(qq')(\Psi\Psi)$	7	$(qq')(\Psi\ell)\Phi$
	7	$(qq')\Phi H ^2\Phi$		$(q\Psi)(q'\Psi)$		$(q\ell)(q'\Psi)\Phi$
$3 \otimes 3 \otimes \bar{6} \otimes 8$	6	$(qq')\Phi^\dagger G$				
$3 \otimes \bar{3} \otimes \bar{3} \otimes \bar{6}$	7	$(\bar{q}q')(\bar{q}''\ell)\Phi$	6	$(\bar{q}q')(\bar{q}''\Psi)$		
		$(qq')^\dagger(q''\ell)\Phi$		$(qq')^\dagger(\bar{q}''\Psi)$		
$3 \otimes \bar{3} \otimes 6 \otimes \bar{6}$	5	$(\bar{q}q')\Phi^\dagger\Phi$	6	$(\bar{q}q')(\bar{\Psi}\Psi)$	7*	$(\bar{q}q')(\bar{\Psi}\ell)\Phi$
	7	$(\bar{q}q')\Phi^\dagger H ^2\Phi$		$(\bar{q}\Psi)(\bar{\Psi}q')$		$(\bar{q}\Psi)(q'\ell)\Phi^\dagger$
$3 \otimes 6 \otimes 6 \otimes \bar{6}$	6	$(q\ell) \Phi ^2\Phi$	6	$(q\Psi)(\bar{\Psi}\Psi)$	5	$(q\Psi)\Phi^\dagger\Phi$
		$(\bar{\ell}q) \Phi ^2\Phi$		$(\bar{\Psi}q)(\Psi\Psi)$		$(\bar{\Psi}q)\Phi\Phi$
					7*	$(q\Psi)(\Psi\ell)\Phi^\dagger$ $(q\ell)(\bar{\Psi}\Psi)\Phi$
$3 \otimes 6 \otimes 8 \otimes 8$			7	$(q\Psi)GG$		
$3 \otimes \bar{6} \otimes \bar{6} \otimes 8$	7	$(q\ell)\Phi^\dagger\Phi^\dagger G$			6	$(\bar{\Psi}q)\Phi^\dagger G$
		$(\bar{\ell}q)\Phi^\dagger\Phi^\dagger G$				
$6 \otimes 6 \otimes \bar{6} \otimes \bar{6}$	6†	$ \Phi ^4$			6	$(\bar{\Psi}\ell) \Phi ^2\Phi$
						$(\Psi\ell) \Phi ^2\Phi^\dagger$
						$(\bar{\ell}\Psi) \Phi ^2\Phi^\dagger$
	7	$(\bar{\Psi}\Psi) \Phi ^2 H ^2$			7	
$6 \otimes 6 \otimes 6 \otimes 8$	7	$\Phi\Phi\Phi GB$			6	$(\Psi\Psi)\Phi G$
						$(\Psi\ell)\Phi\Phi G$
					7	$(\bar{\ell}\Psi)\Phi\Phi G$
$6 \otimes \bar{6} \otimes 8 \otimes 8$	6	$ \Phi ^2 GG$	7	$(\bar{\Psi}\Psi)GG$		

Associated production with quark
From quark-gluon fusion

Three body decay of sextet

Iterate again

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}}_a \oplus \mathbf{6}_s,$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8},$$

$$\mathbf{6} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{10},$$

$$\mathbf{6} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \mathbf{15},$$

$$\mathbf{6} \otimes \mathbf{6} = \bar{\mathbf{6}}_s \oplus \mathbf{15}_a \oplus \mathbf{15}'_s,$$

$$\mathbf{6} \otimes \bar{\mathbf{6}} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27},$$

$$\mathbf{8} \otimes \mathbf{3} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15},$$

$$\mathbf{8} \otimes \bar{\mathbf{6}} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{24},$$

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1}_s \oplus \mathbf{8}_s \oplus \mathbf{8}_a \oplus \mathbf{10}_a \oplus \bar{\mathbf{10}}_a \oplus \mathbf{27}_s$$

$$\mathbf{6} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} = \bar{\mathbf{3}}_a$$

$$\mathbf{6} \otimes \bar{\mathbf{3}} \otimes \mathbf{6} \otimes \bar{\mathbf{3}} = \bar{\mathbf{3}}_a$$

Five field invariant

	<i>Scalar sextet Φ only</i>		≥ 1 of each	
$SU(3)_c$ invariant	d_{\min}	Structure	d_{\min}	Structure
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{6}$	7	$(qq')(q'q''')\Phi$		
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{6}}$	7	$(qq')(\bar{q}'q''')\Phi^\dagger$		
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{6} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}}$	6	$(qq') \Phi ^2\Phi^\dagger$	7*	$(qq')(\bar{\Psi}\Psi)\Phi^\dagger$
				$(\bar{\Psi}q)(q'\Psi)\Phi^\dagger$
$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6}$	6	$(\bar{q}q')\Phi\Phi\Phi$	7	$(\bar{q}q')(\Psi\Psi)\Phi$
				$(\bar{q}\Psi)(q'\Psi)\Phi$
$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	7	$(\bar{q}q') \Phi ^2G$		
$\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6}$	7	$(q\ell)\Phi\Phi\Phi\Phi$		
		$(\bar{\ell}q)\Phi\Phi\Phi\Phi$		
$\mathbf{3} \otimes \mathbf{6} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}}$	7	$(q\ell) \Phi ^2\Phi^\dagger\Phi^\dagger$		
		$(\bar{\ell}q) \Phi ^2\Phi^\dagger\Phi^\dagger$		
$\mathbf{6} \otimes \mathbf{6} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$			7	$(\bar{\Psi}\Psi)\Phi^\dagger\Phi G$
$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{8} \otimes \mathbf{8}$	7	$\Phi\Phi\Phi GG$		
$\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}}$	7	$\Phi\Phi^\dagger\Phi^\dagger\Phi^\dagger\Phi^\dagger H ^2$		

Conclusion and Future Directions

Systematic exploration of color invariants leads to a complete catalogue of BSM operators with new and interesting collider phenomenology

Future Directions Include

- In depth phenomenological studies for sextet models including future colliders
- Building out UV completions
- Completing catalogue for other representations, including fields with nontrivial representations under $SU(3) \times SU(2)$